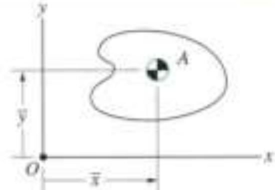
Properties of Areas and Lines

Areas



The coordinates of the centroid of the area A are

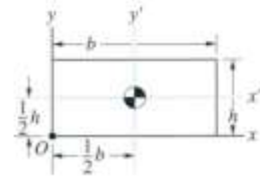
$$\bar{x} = \frac{\int_A x\,dA}{\int_A dA}, \qquad \bar{y} = \frac{\int_A y\,dA}{\int_A dA}.$$

The moment of inertia about the x axis I_x, the moment of inertia about the y axis I_y, and the product of inertia I_{xy} are

$$I_x = \int_A y^2\,dA, \qquad I_y = \int_A x^2\,dA, \qquad I_{xy} = \int_A xy\,dA.$$

The polar moment of inertia about O is

$$J_O = \int_A r^2\,dA = \int_A (x^2 + y^2)\,dA = I_x + I_y.$$

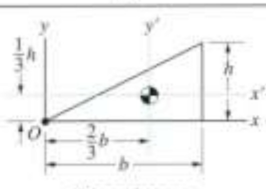


Rectangular area

Area $= bh$

$I_x = \frac{1}{3}bh^3$, $\qquad I_y = \frac{1}{3}hb^3$, $\qquad I_{xy} = \frac{1}{4}b^2h^2$

$I_{x'} = \frac{1}{12}bh^3$, $\qquad I_{y'} = \frac{1}{12}hb^3$, $\qquad I_{x'y'} = 0$

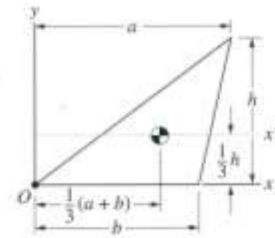


Triangular area

Area $= \frac{1}{2}bh$

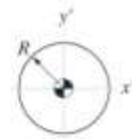
$I_x = \frac{1}{12}bh^3$, $\qquad I_y = \frac{1}{4}hb^3$, $\qquad I_{xy} = \frac{1}{8}b^2h^2$

$I_{x'} = \frac{1}{36}bh^3$, $\qquad I_{y'} = \frac{1}{36}hb^3$, $\qquad I_{x'y'} = \frac{1}{72}b^2h^2$



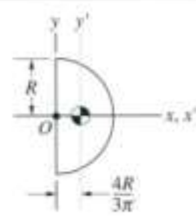
Triangular area

Area $= \frac{1}{2}bh$ $\qquad I_x = \frac{1}{12}bh^3$, $\qquad I_{x'} = \frac{1}{36}bh^3$



Circular area

Area $= \pi R^2$ $\qquad I_{x'} = I_{y'} = \frac{1}{4}\pi R^4$, $\qquad I_{x'y'} = 0$



Semicircular area

Area $= \frac{1}{2}\pi R^2$ $\qquad I_x = I_y = \frac{1}{8}\pi R^4$, $\qquad I_{xy} = 0$

$I_{x'} = \frac{1}{8}\pi R^4$, $\qquad I_{y'} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)R^4$, $\qquad I_{x'y'} = 0$