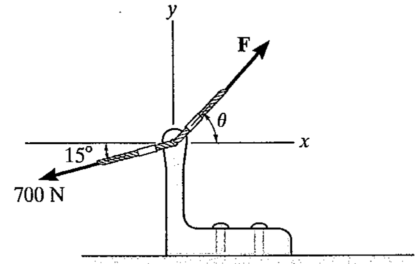
2–1.

If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

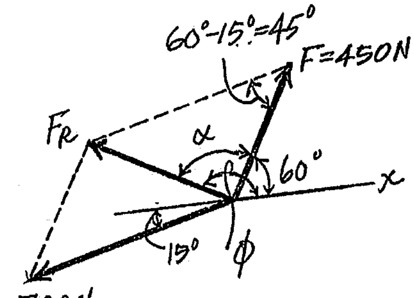
$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

This yields

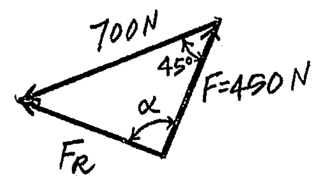
$$\frac{\sin\alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$

Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60° = 95.19° + 60° = 155° \qquad \textbf{Ans.}$$



(a)

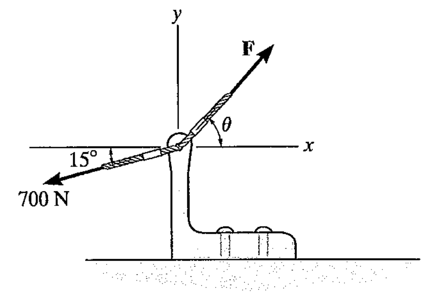


(b)

Ans:
$F_R = 497$ N
$\phi = 155°$

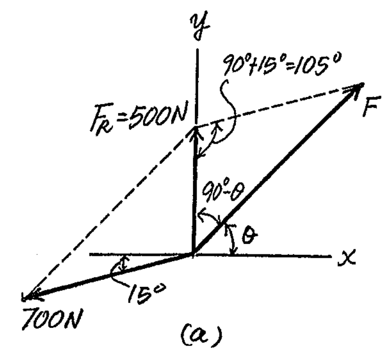
2–2.

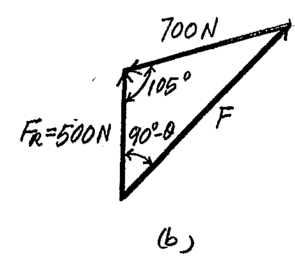
If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.





Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

$$= 959.78 \text{ N} = 960 \text{ N} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b*, and using this result, yields

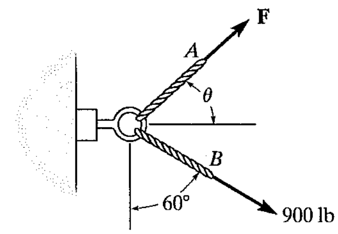
$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$\theta = 45.2^\circ \qquad \textbf{Ans.}$$

Ans:
$F = 960$ N
$\theta = 45.2^\circ$

2–9.

If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope A and the corresponding angle θ.



SOLUTION

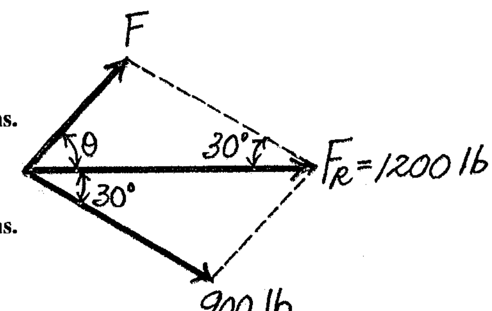
Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,

Trigonometry. Applying the law of cosines by referring to Fig. *b*,

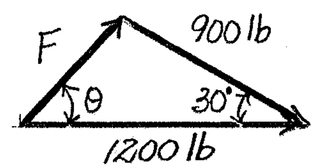
$$F = \sqrt{900^2 + 1200^2 - 2(900)(1200)\cos 30^\circ} = 615.94 \text{ lb} = 616 \text{ lb}$$ **Ans.**

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin\theta}{900} = \frac{\sin 30^\circ}{615.94}; \qquad \theta = 46.94^\circ = 46.9^\circ$$ **Ans.**



(a)

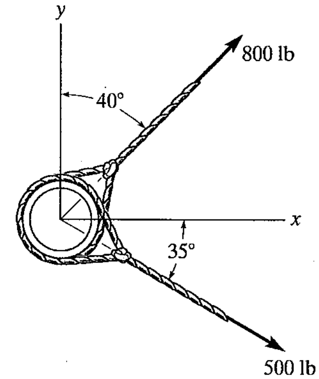


(b)

Ans:
$F = 616$ lb
$\theta = 46.9^\circ$

2–10.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. a,
Trigonometry. Applying the law of cosines by referring to Fig. b,

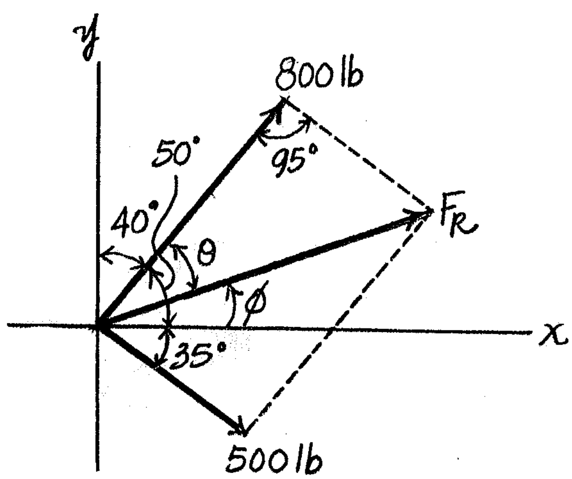
$$F_R = \sqrt{800^2 + 500^2 - 2(800)(500)\cos 95^\circ} = 979.66 \text{ lb} = 980 \text{ lb} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. b,

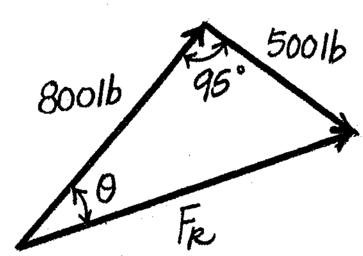
$$\frac{\sin\theta}{500} = \frac{\sin 95^\circ}{979.66}; \qquad \theta = 30.56^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\phi = 50^\circ - 30.56^\circ = 19.44^\circ = 19.4^\circ \qquad \textbf{Ans.}$$



(a)

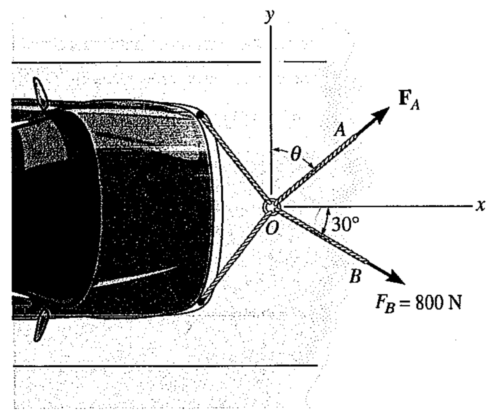


(b)

Ans:
$F_R = 980$ lb
$\phi = 19.4^\circ$

2–26.

Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



SOLUTION

$\xrightarrow{+} F_{R_x} = \Sigma F_x;$ $\quad F_{R_x} = F_A \sin\theta + 800\cos 30° = 1250$

$+\uparrow F_{R_y} = \Sigma F_y;$ $\quad F_{R_y} = F_A \cos\theta - 800\sin 30° = 0$

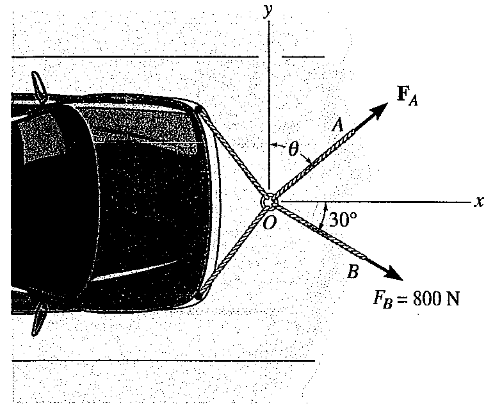
$\theta = 54.3°$ **Ans.**

$F_A = 686$ N **Ans.**

Ans:
$\theta = 54.3°$
$F_A = 686$ N

2–27.

Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.



SOLUTION

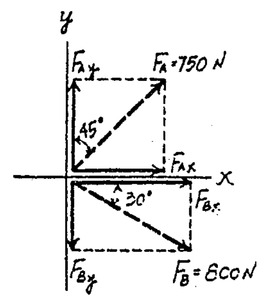
Scalar Notation: Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 750 \sin 45° + 800 \cos 30°$$

$$= 1223.15 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 750 \cos 45° - 800 \sin 30°$$

$$= 130.33 \text{ N} \uparrow$$



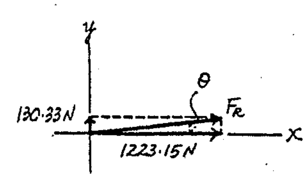
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN}$$ **Ans.**

The directional angle θ measured counterclockwise from positive x axis is

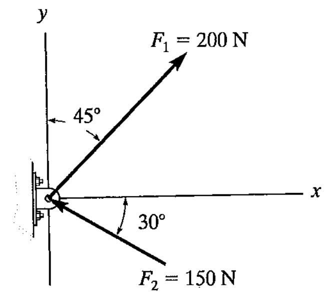
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08°$$ **Ans.**



Ans:
$F_R = 1.23$ kN
$\theta = 6.08°$

***2–32.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. *a*,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 \sin 45° - 150 \cos 30° = 11.518 \text{ N} \rightarrow$$

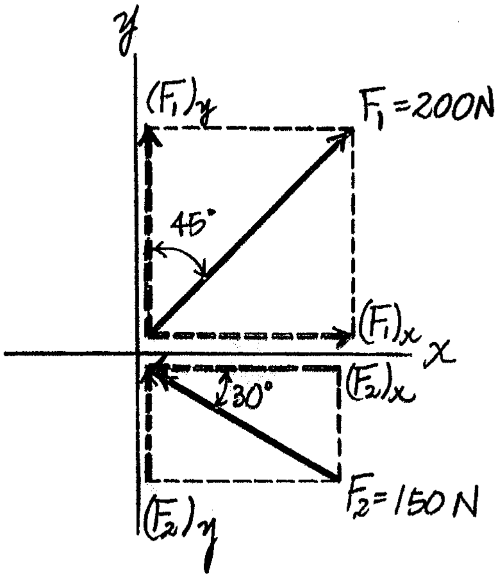
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 200 \cos 45° + 150 \sin 30° = 216.42 \text{ N} \uparrow$$

Referring to Fig. *b*, the magnitude of the resultant force F_R is

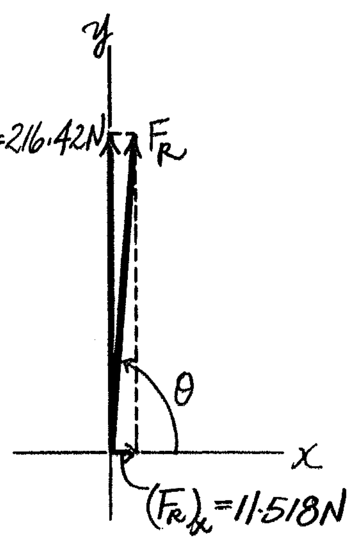
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{11.518^2 + 216.42^2} = 216.73 \text{ N} = 217 \text{ N} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{216.42}{11.518}\right) = 86.95° = 87.0° \qquad \textbf{Ans.}$$



(a)

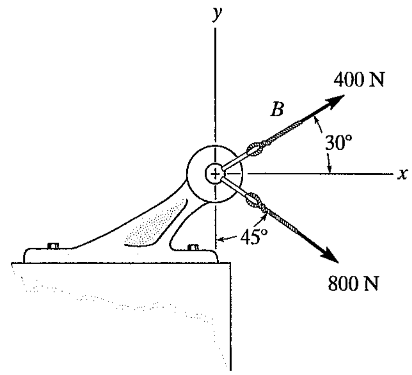


(b)

Ans:
$F_R = 217$ N
$\theta = 87.0°$

2–33.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 400 \cos 30° + 800 \sin 45° = 912.10 \text{ N} \rightarrow$$

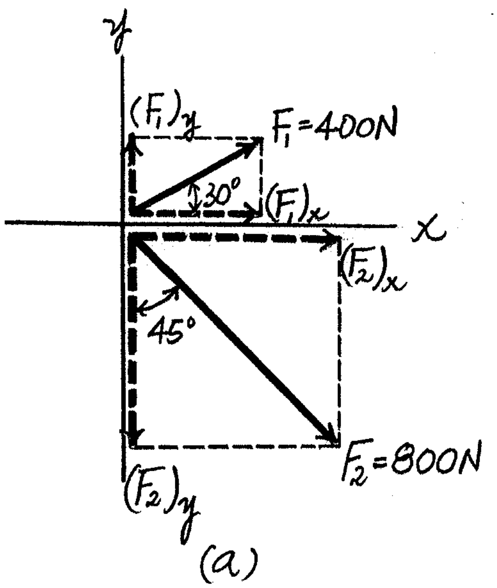
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 400 \sin 30° - 800 \cos 45° = -365.69 \text{ N} = 365.69 \text{ N}\downarrow$$

Referring to Fig. b, the magnitude of the resultant force is

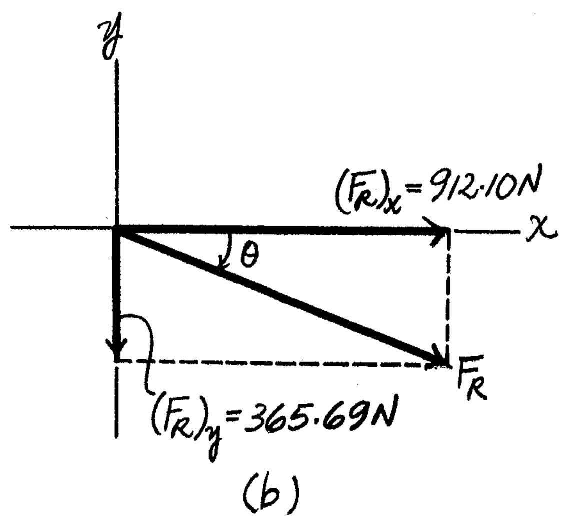
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{912.10^2 + 365.69^2} = 982.67 \text{ N} = 983 \text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{365.69}{912.10}\right) = 21.84° = 21.8° \qquad \textbf{Ans.}$$



(a)

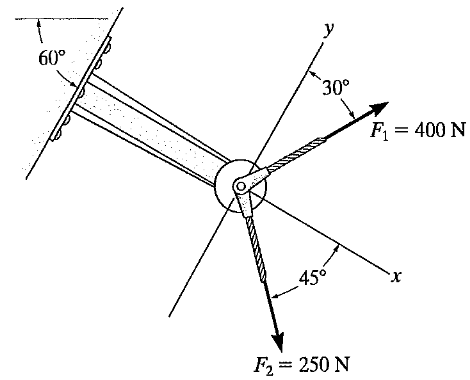


(b)

Ans:
$F_R = 983$ N
$\theta = 21.8°$

2–34.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



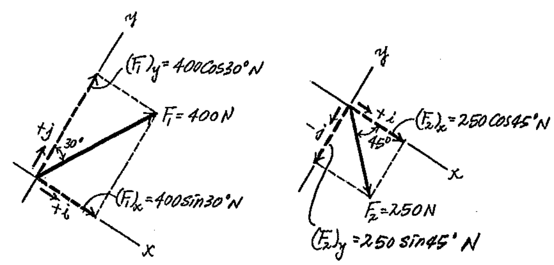
SOLUTION

$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

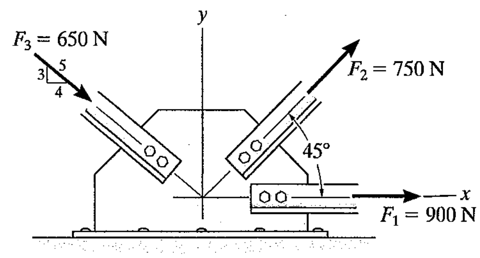
$$= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N}$$ **Ans.**



Ans:
$\mathbf{F}_1 = \{200\mathbf{i} + 346\mathbf{j}\}$ N
$\mathbf{F}_2 = \{177\mathbf{i} - 177\mathbf{j}\}$ N

***2–36.**

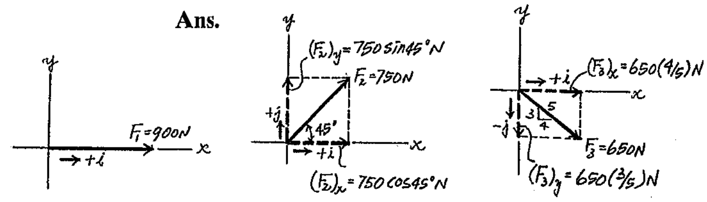
Resolve each force acting on the *gusset plate* into its x and y components, and express each force as a Cartesian vector.



$$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = \{750 \cos 45°(+\mathbf{i}) + 750 \sin 45°(+\mathbf{j})\}\text{ N}$$
$$= \{530\mathbf{i} + 530\mathbf{j}\}\text{ N}$$ **Ans.**

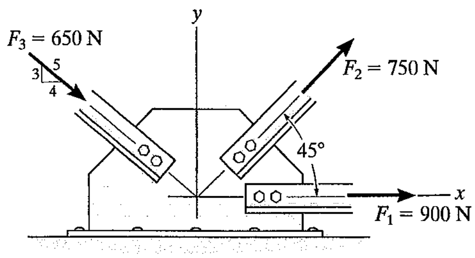
$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\}\text{ N}$$
$$= \{520\,\mathbf{i} - 390\mathbf{j})\}\text{ N}$$ **Ans.**



Ans:
$\mathbf{F}_1 = \{900\mathbf{i}\}$ N
$\mathbf{F}_2 = \{530\mathbf{i} + 530\mathbf{j}\}$ N
$\mathbf{F}_3 = \{520\mathbf{i} - 390\mathbf{j}\}$ N

2–37.

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45° = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45° = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

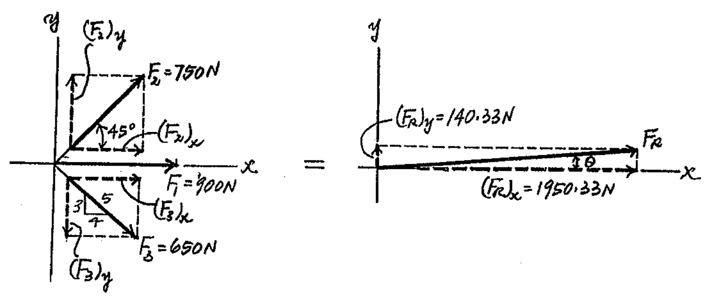
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$

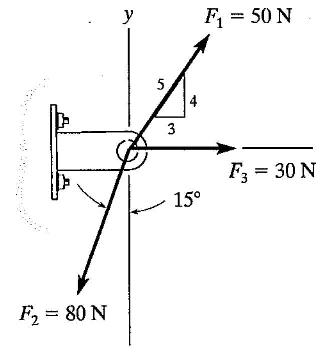


(a)

Ans:
$F_R = 1.96$ kN
$\theta = 4.12°$

2–38.

Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



SOLUTION

Cartesian Notation. Referring to Fig. *a*,

$$\mathbf{F}_1 = (F_1)_x\,\mathbf{i} + (F_1)_y\,\mathbf{j} = 50\left(\frac{3}{5}\right)\mathbf{i} + 50\left(\frac{4}{5}\right)\mathbf{j} = \{30\,\mathbf{i} + 40\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -(F_2)_x\,\mathbf{i} - (F_2)_y\,\mathbf{j} = -80\sin 15°\,\mathbf{i} - 80\cos 15°\,\mathbf{j}$$

$$= \{-20.71\,\mathbf{i} - 77.27\,\mathbf{j}\}\text{ N}$$

$$= \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$F_3 = (F_3)_x\mathbf{i} = \{30\,\mathbf{i}\} \qquad \textbf{Ans.}$$

Thus, the resultant force is

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (30\mathbf{i} + 40\,\mathbf{j}) + (-20.71\mathbf{i} - 77.27\mathbf{j}) + 30\mathbf{i}$$

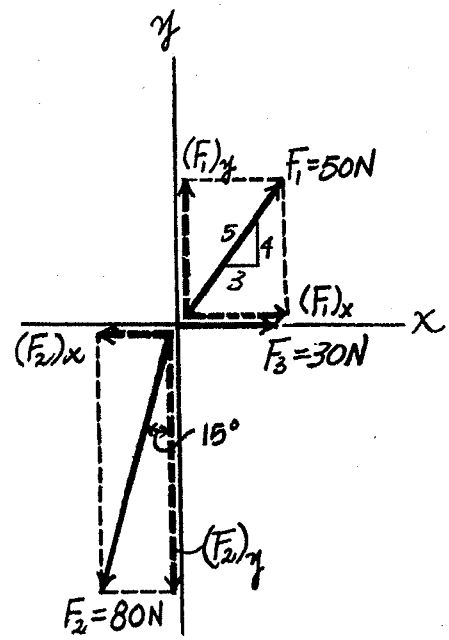
$$= \{39.29\,\mathbf{i} - 37.27\,\mathbf{j}\}\text{ N}$$

Referring to Fig. *b*, the magnitude of $\mathbf{F}_R$ is

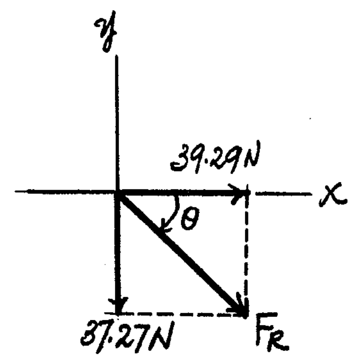
$$F_R = \sqrt{39.29^2 + 37.27^2} = 54.16\text{ N} = 54.2\text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left(\frac{37.27}{39.29}\right) = 43.49° - 43.5° \qquad \textbf{Ans.}$$



(a)

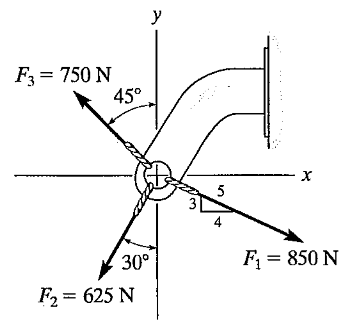


(b)

Ans:
$\mathbf{F}_1 = \{30\mathbf{i} + 40\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}$ N
$F_3 = \{30\,\mathbf{i}\}$
$F_R = 54.2$ N
$\theta = 43.5°$

2–42.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = \frac{4}{5}(850)\,\mathbf{i} - \frac{3}{5}(850)\,\mathbf{j}$$

$$= \{680\,\mathbf{i} - 510\,\mathbf{j}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = -625\sin 30^\circ\,\mathbf{i} - 625\cos 30^\circ\,\mathbf{j}$$

$$= \{-312\,\mathbf{i} - 541\,\mathbf{j}\}\text{ N}$$ **Ans.**

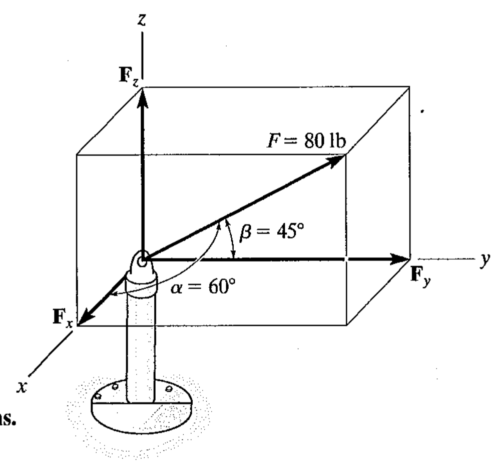
$$\mathbf{F}_3 = -750\sin 45^\circ\,\mathbf{i} + 750\cos 45^\circ\,\mathbf{j}$$

$$= \{-530\,\mathbf{i} + 530\,\mathbf{j}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{680\mathbf{i} - 510\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-312\mathbf{i} - 541\mathbf{j}\}$ N
$\mathbf{F}_3 = \{-530\mathbf{i} + 530\mathbf{j}\}$ N

***2–60.**

The force **F** has a magnitude of 80 lb and acts within the octant shown. Determine the magnitudes of the x, y, z components of **F**.



SOLUTION

$1 = \cos^2 60° + \cos^2 45° + \cos^2 \gamma$

Solving for the positive root, $\gamma = 60°$

$F_x = 80 \cos 60° = 40.0$ lb **Ans.**

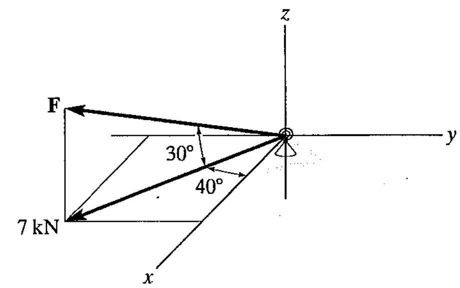
$F_y = 80 \cos 45° = 56.6$ lb **Ans.**

$F_z = 80 \cos 60° = 40.0$ lb **Ans.**

Ans:
$F_x = 40.0$ lb
$F_y = 56.6$ lb
$F_z = 40.0$ lb

2–62.

Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the x–y plane is 7 kN.



SOLUTION

Coordinate Direction Angles. The unit vector of **F** is

$$\mathbf{u}_F = \cos 30° \cos 40°\mathbf{i} - \cos 30° \sin 40°\mathbf{j} + \sin 30° \,\mathbf{k}$$

$$= \{0.6634\mathbf{i} - 0.5567\mathbf{j} + 0.5\,\mathbf{k}\}$$

Thus,

$\cos\alpha = 0.6634;\quad \alpha = 48.44° = 48.4°$ **Ans.**

$\cos\beta = -0.5567;\quad \beta = 123.83° = 124°$ **Ans.**

$\cos\gamma = 0.5;\quad \gamma = 60°$ **Ans.**

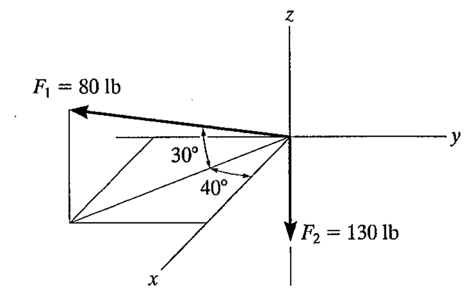
The magnitude of **F** can be determined from

$F\cos 30° = 7;\quad F = 8.083\text{ kN} = 8.08\text{ kN}$ **Ans.**

Ans:
$\alpha = 48.4°$
$\beta = 124°$
$\gamma = 60°$
$F = 8.08\text{ kN}$

2–63.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

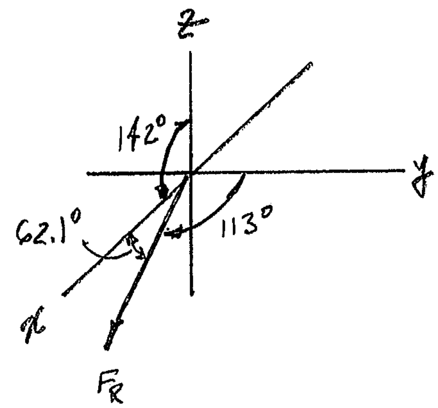
$\mathbf{F}_1 = \{80 \cos 30° \cos 40°\mathbf{i} - 80 \cos 30° \sin 40°\mathbf{j} + 80 \sin 30°\mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb



$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114 \text{ lb}$ **Ans.**

$\alpha = \cos^{-1}\left(\dfrac{53.1}{113.6}\right) = 62.1°$ **Ans.**

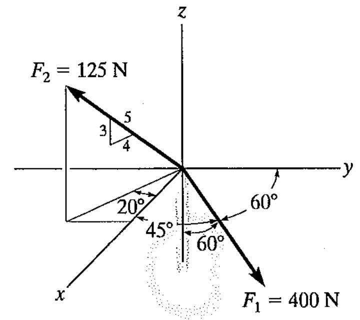
$\beta = \cos^{-1}\left(\dfrac{-44.5}{113.6}\right) = 113°$ **Ans.**

$\gamma = \cos^{-1}\left(\dfrac{-90.0}{113.6}\right) = 142°$ **Ans.**

Ans:
$F_R = 114$ lb
$\alpha = 62.1°$
$\beta = 113°$
$\gamma = 142°$

2–69.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

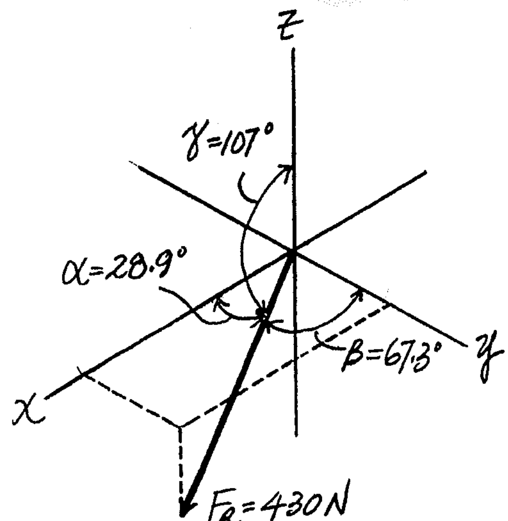
Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$\mathbf{F}_1 = 400\,(\cos 45°\mathbf{i} + \cos 60°\mathbf{j} - \cos 60°\mathbf{k}) = \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\}$ N

$\mathbf{F}_2 = 125\left[\frac{4}{5}(\cos 20°)\mathbf{i} - \frac{4}{5}(\sin 20°)\mathbf{j} + \frac{3}{5}\mathbf{k}\right] = \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} + \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$
$$= \{376.81\mathbf{i} + 165.80\mathbf{j} - 125.00\mathbf{k}\}\text{ N}$$



The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{376.81^2 + 165.80^2 + (-125.00)^2}$$
$$= 430.23\text{ N} = 430\text{ N} \quad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{376.81}{430.23}; \quad \alpha = 28.86° = 28.9° \quad \textbf{Ans.}$$

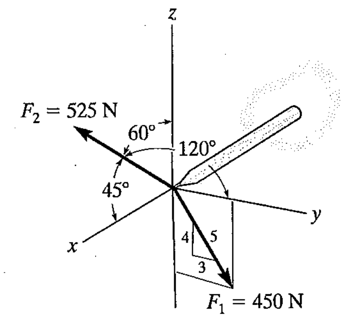
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{165.80}{430.23}; \quad \beta = 67.33° = 67.3° \quad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-125.00}{430.23}; \quad \gamma = 106.89° = 107° \quad \textbf{Ans.}$$

Ans:
$F_R = 430$ N
$\alpha = 28.9°$
$\beta = 67.3°$
$\gamma = 107°$

2–70.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

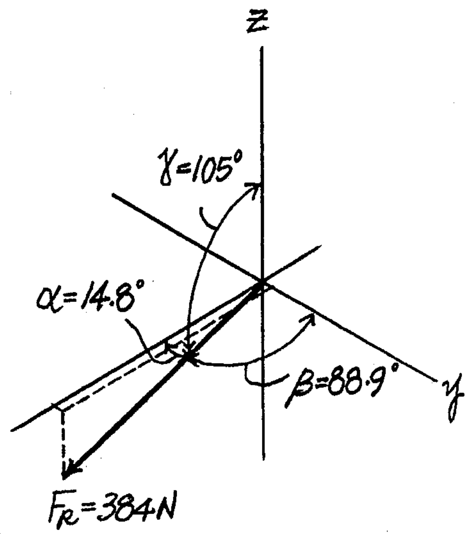
Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 450\left(\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = \{270\mathbf{j} - 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 525\,(\cos 45°\mathbf{i} + \cos 120°\mathbf{j} + \cos 60°\mathbf{k}) = \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{270\mathbf{j} - 360\mathbf{k}\} + \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}$$
$$= \{371.23\mathbf{i} + 7.50\mathbf{j} - 97.5\mathbf{k}\}\text{ N}$$



The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{371.23^2 + 7.50^2 + (-97.5)^2}$$
$$= 383.89\text{ N} = 384\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{371.23}{383.89}; \qquad \alpha = 14.76° = 14.8° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{7.50}{383.89}; \qquad \beta = 88.88° = 88.9° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-97.5}{383.89}; \qquad \gamma = 104.71° = 105° \qquad \textbf{Ans.}$$

Ans:

$F_R = 384$ N

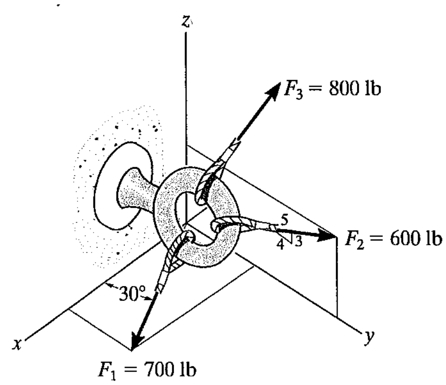
$\cos\alpha = \dfrac{371.23}{383.89}$; $\alpha = 14.8°$

$\cos\beta = \dfrac{7.50}{383.89}$; $\beta = 88.9°$

$\cos\gamma = \dfrac{-97.5}{383.89}$; $\gamma = 105°$

2–81.

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 60°$ and $\gamma_3 = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



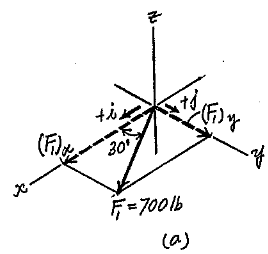
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

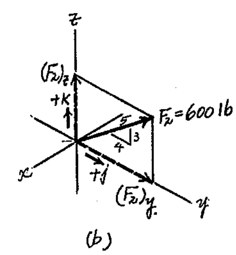
$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 60°\mathbf{j} + 800\cos 45°\mathbf{k} = [-400\mathbf{i} + 400\mathbf{j} + 565.69\mathbf{k}]\text{ lb}$$



Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (-400\mathbf{i} + 400\mathbf{j} + 565.69\mathbf{k})\\ &= [206.22\mathbf{i} + 1230\mathbf{j} + 925.69\mathbf{k}]\text{ lb}\end{aligned}$$



The magnitude of $\mathbf{F}_R$ is

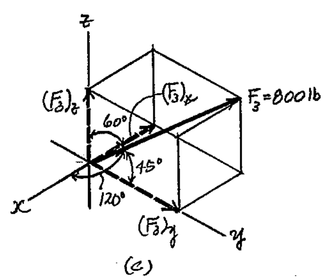
$$\begin{aligned}F_R &= \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}\\ &= \sqrt{(206.22)^2 + (1230)^2 + (925.69)^2} = 1553.16\text{ lb} = 1.55\text{ kip} \qquad \textbf{Ans.}\end{aligned}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{206.22}{1553.16}\right) = 82.4° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{1230}{1553.16}\right) = 37.6° \qquad \textbf{Ans.}$$

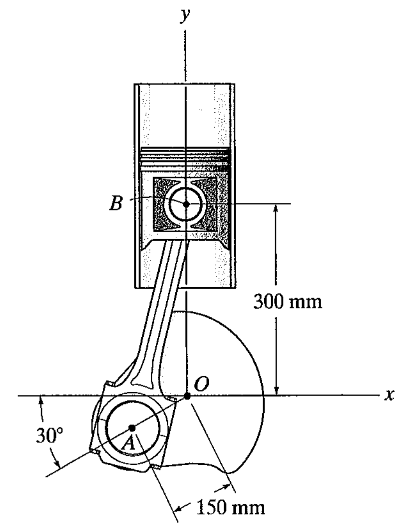
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{925.69}{1553.16}\right) = 53.4° \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.55$ kip
$\alpha = 82.4°$
$\beta = 37.6°$
$\gamma = 53.4°$

2–86.

Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

Position Vector. The coordinates of points A and B are $A(-150 \cos 30°, -150 \sin 30°)$ mm and $B(0, 300)$ mm respectively. Then

$$\mathbf{r}_{AB} = [0 - (-150 \cos 30°)]\mathbf{i} + [300 - (-150 \sin 30°)]\mathbf{j}$$

$$= \{129.90\mathbf{i} + 375\mathbf{j}\} \text{ mm}$$

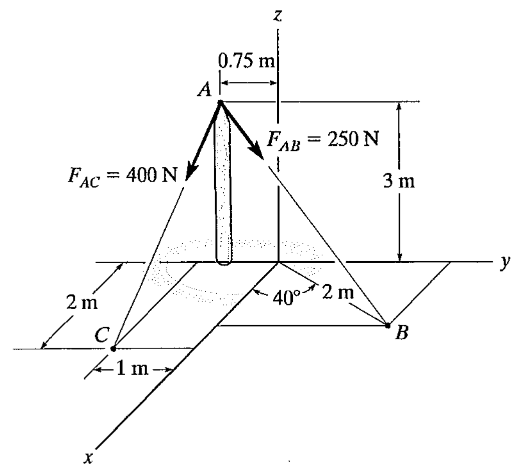
Thus, the magnitude of $\mathbf{r}_{AB}$ is

$$r_{AB} = \sqrt{129.90^2 + 375^2} = 396.86 \text{ mm} = 397 \text{ mm}$$ **Ans.**

Ans:
$r_{AB} = 397$ mm

***2–92.**

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Unit Vectors. The coordinates for points A, B and C are $(0, -0.75, 3)$ m, $B(2\cos 40°, 2\sin 40°, 0)$ m and $C(2, -1, 0)$ m respectively.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(2\cos 40° - 0)\mathbf{i} + [2\sin 40° - (-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2\cos 40° - 0)^2 + [2\sin 40° - (-0.75)]^2 + (0-3)^2}}$$

$$= 0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2-0)\mathbf{i} + [-1-(-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2-0)^2 + [-1-(-0.75)]^2 + (0-3)^2}}$$

$$= 0.5534\mathbf{i} - 0.0692\mathbf{j} - 0.8301\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 250\,(0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k})$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\}\text{ N}$$

$$= \{97.3\mathbf{i} + 129\mathbf{j} - 191\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 400\,(0.5534\mathbf{i} - 0.06917\mathbf{j} - 0.8301\mathbf{k})$$

$$= \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}\text{ N}$$

$$= \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}\text{ N}$$ **Ans.**

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} + \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}$$

$$= \{318.67\mathbf{i} + 101.63\mathbf{j} - 522.58\,\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{318.67^2 + 101.63^2 + (-522.58)^2}$$

$$= 620.46\text{ N} = 620\text{ N}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{318.67}{620.46}; \qquad \alpha = 59.10° = 59.1°$$ **Ans.**

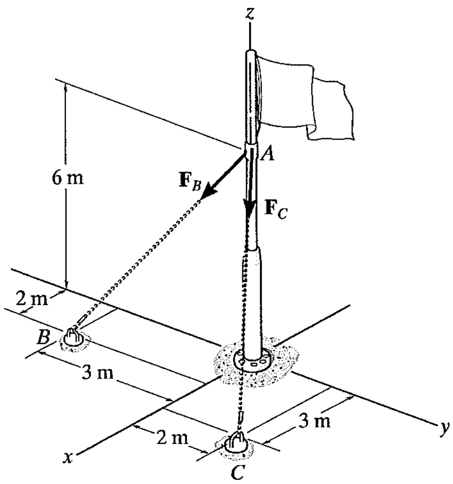
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{101.63}{620.46}; \qquad \beta = 80.57° = 80.6°$$ **Ans.**

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-522.58}{620.46}; \qquad \gamma = 147.38° = 147°$$ **Ans.**

Ans:
$\mathbf{F}_{AB} = \{97.3\mathbf{i} - 129\mathbf{j} - 191\mathbf{k}\}$ N
$\mathbf{F}_{AC} = \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}$ N
$F_R = 620$ N
$\cos\alpha = 59.1°$
$\cos\beta = 80.6°$
$\cos\gamma = 147°$

2–94.

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

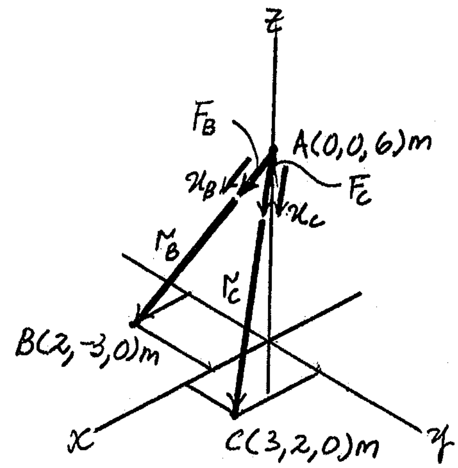
$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$



(a)

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0° \qquad \textbf{Ans.}$$

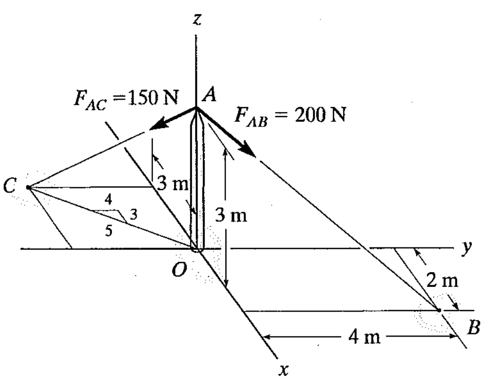
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157° \qquad \textbf{Ans.}$$

Ans:
$F_R = 1.17$ kN
$\alpha = 68.0°$
$\beta = 96.8°$
$\gamma = 157°$

***2–100.**

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



SOLUTION

Unit Vector. The coordinates for points A, B and C are $A(0, 0, 3)$ m, $B(2, 4, 0)$ m and $C(-3, -4, 0)$ m respectively

$$\mathbf{r}_{AB} = (2-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = \{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 4^2 + (-3)^2}} = \frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3-0)\mathbf{i} + (-4-0)\mathbf{j} + (0-3)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (-3)^2}} = -\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\mathbf{u}_{AB} = 200\left(\frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}\right)$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\mathbf{u}_{AC} = 150\left(-\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}\right)$$

$$= \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}\text{ N}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} + \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}$$

$$= \{-2.896\mathbf{i} + 45.66\mathbf{j} - 188.59\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(-2.896)^2 + 45.66^2 + (-188.59)^2}$$

$$= 194.06\text{ N} = 194\text{ N} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{-2.896}{194.06}; \quad \alpha = 90.86^\circ = 90.9^\circ \qquad \textbf{Ans.}$$

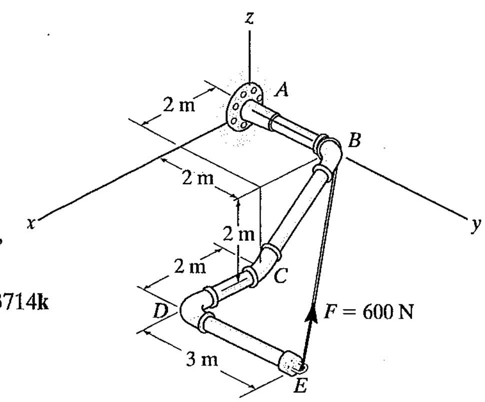
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{45.66}{194.06}; \quad \beta = 76.39^\circ = 76.4^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-188.59}{194.06}; \quad \gamma = 166.36^\circ = 166^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 194$ N
$\cos\alpha = 90.9^\circ$
$\cos\beta = 76.4^\circ$
$\cos\gamma = 166^\circ$

2–113.

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector **F** is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600\big(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}\big) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}]\text{ N}$$

Vector Dot Product: The magnitude of the component of **F** parallel to segment DE of the pipe assembly is

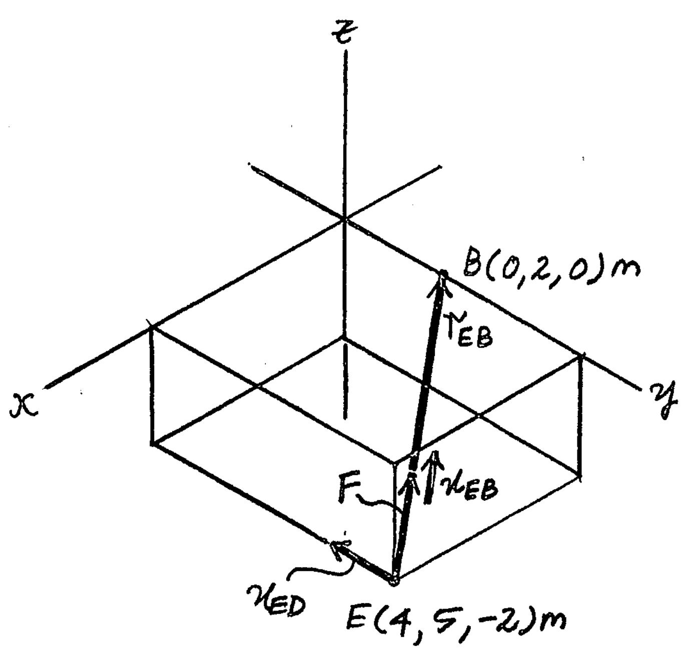
$$(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} = \big(-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}\big)\cdot\big(-\mathbf{j}\big)$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334\text{ N}$$ **Ans.**

The component of **F** perpendicular to segment DE of the pipe assembly is

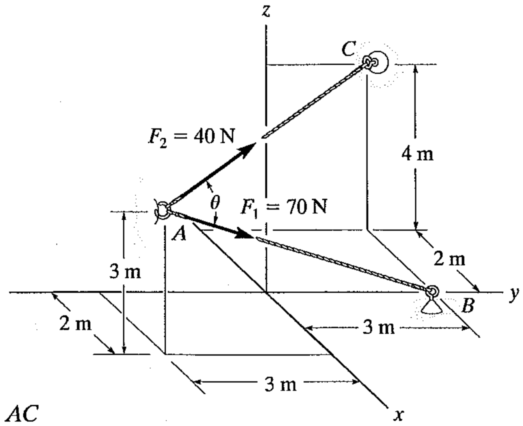
$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498\text{ N}$$ **Ans.**



Ans:
$(F_{ED})_{||} = 334$ N
$(F_{ED})_{\perp} = 498$ N

2–114.

Determine the angle θ between the two cables.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB}\cdot\mathbf{u}_{AC} = \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right)\cdot\left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= \left(-\frac{2}{7}\right)\left(-\frac{4}{\sqrt{53}}\right) + \frac{6}{7}\left(\frac{6}{\sqrt{53}}\right) + \left(-\frac{3}{7}\right)\left(\frac{1}{\sqrt{53}}\right)$$

$$= \frac{41}{7\sqrt{53}}$$

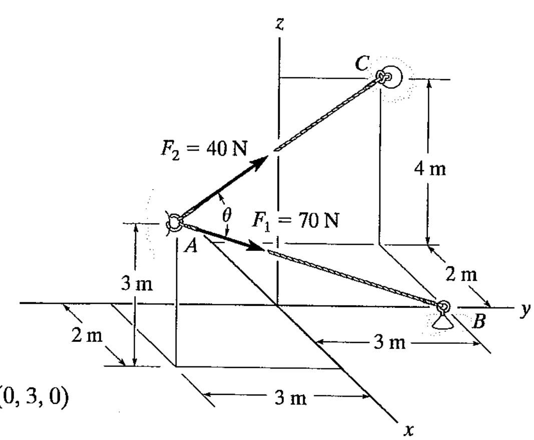
Then

$$\theta = \cos^{-1}(\mathbf{u}_{AB}\cdot\mathbf{u}_{AC}) = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) = 36.43° = 36.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 36.4°$

2–115.

Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

Force Vector, For $\mathbf{F}_1$,

$$\mathbf{F}_1 = F_1\,\mathbf{u}_{AB} = 70\left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) = \{-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}\}\text{ N}$$

Projected Component of $\mathbf{F}_1$. Along AC, it is

$$(F_1)_{AC} = \mathbf{F}_1 \cdot \mathbf{u}_{AC} = (-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= (-20)\left(-\frac{4}{\sqrt{53}}\right) + 60\left(\frac{6}{\sqrt{53}}\right) + (-30)\left(\frac{1}{\sqrt{53}}\right)$$

$$= 56.32\text{ N} = 56.3\text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{AC}$.

Ans:
$(F_1)_{AC} = 56.3$ N