Chapter 9
Six Sigma Measurements

Introduction

- This chapter summarizes metrics that are often associated with Six Sigma. (not all applicable)
  - Better select metrics and calculation techniques
  - Good communication tools with other organizations, suppliers, and customers.

9.1 Converting Defect Rates (DPMO or ppm) to Sigma Quality Level Units

- Table S: Conversion Between ppm and Sigma
  - Centered distribution with double sided spec. limits
  - With and without 1.5 sigma shifting
  - Six sigma level (with 1.5σ shift) can be approximated by

$$Sigma\ quality\ level = 0.8406 + \sqrt{29.37 - 2.221 * \ln(ppm)}$$

- World-class organizations are those considered to be at 6σ performance in the short term or 4.5σ in the long term.
- Average companies are said to show 4σ performance.
- The difference means 1826 times fewer defects, and increasing profit by at least 10%.

9.2 Six Sigma Relationships: Nomenclature/Basic Relationships

- Number of operation steps $= m$
- Defects $= D$
- Unit $= U$
- Opportunities for a defect $= O$
- Yield $= Y$

Basic Relationships

- Total opportunities: $TOP = U \times O$
- Defects per unit: $DPU = \frac{D}{U}$
- Defects per opportunity: $DPO = \frac{DPU}{O} = \frac{D}{U \times O}$
- Defects per million opportunity: $DPMO = DPO \times 10^6$

9.2 Six Sigma Relationships: Yield Relationships

- Throughput yield: $Y_{TP} = e^{-DPU}$
- Defect per unit: $DPU = -\ln(Y_{TP})$
- Rolled Throughput yield: $Y_{RT} = \prod_{i=1}^{m} Y_{TPi}$
- Total defect per unit: $TDPU = -\ln(Y_{RT})$
- Normalized yield: $Y_{norm} = \sqrt[m]{Y_{RT}}$
- Defect per normalized unit: $DPU_{norm} = -\ln(Y_{norm})$

9.2 Six Sigma Relationships: Standardized Normal Dist. Relationships

- $Z_{equiv} \cong Z \sim N(0; 1)$
- Z long-term: $Z_{LT} = Z_{equiv}$
- Z short-term relationship to Z long-term with 1.5 standard deviation shift: $Z_{ST} = Z_{LT} + 1.5_{shift}$
- Z Benchmark: $Z_{benchmark} = Z_{Y_{norm}} + 1.5$

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9.3 Process Cycle Time

- Process Cycle Time: the time it takes for a product to go through an entire process.
- Real process time includes the waiting and storage time between and during operations. (inspection, shipping, testing, analysis, repair, waiting time, storage, operation delays, and setup times)
- Theoretical process time (TAKT time)= $\frac{Real\ daily\ op.time}{req.daily\ production}$
- Possible solutions:
  - Improved work methods
  - Changed production sequence
  - Transfer of inspection to production employees
  - Reduction in batch size

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9.3 Process Cycle Time

- Benefits of reducing real process cycle time:
  - Reducing the number of defective units
  - Improving process performance
  - Reducing inventory/production costs
  - Increasing internal/external customer satisfaction
  - Improving production yields
  - Reducing floor space requirements
- Quality assessments should be made before implementing process-cycle-time reduction changes.

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9.4 Yield

- Yield is the area under the probability density curve between tolerances.
- From the Poisson distribution, this equates to the probability with zero failures.

$$Y = P(x = 0) = \frac{e^{-\lambda}\lambda^{x}}{x!} = e^{-\lambda} = e^{-DPU}$$

- λ is the mean of Poisson distribution and x is the number of failure.

9.5 Example 9.1: Yield

- Five defects are observed in 467 units produced.
- $DPU = 5/467 = 0.01071$

$$Y = P(x = 0) = e^{-DPU} = e^{-0.1071} = 0.98935$$

9.6 Z Variable Equivalent

- The Poisson distribution can be used to estimate the Z variable.
- Z_{equiv}=abs(NORM.S.INV(DPU))
- Z long-term: $Z_{LT} = Z_{equiv}$
- Z short-term: $Z_{ST} = Z_{LT} + 1.5_{shift}$

9.7 Example 9.2: Z Variable Equivalent

- For the previous example, $DPU = 5/467 = 0.01071$
- Z_{equiv}=abs(NORM.S.INV(0.1071))=2.30
- Z long-term: $Z_{LT} = Z_{equiv} = 2.30$
- Z short-term: $Z_{ST} = Z_{LT} + 1.5_{shift} = 2.30 + 1.5 = 3.8$
- Ppm rate = 10,724 (Table S)

9.8 Defects per Million Opportunities (DPMO)

- A defect-per-unit calculation can give additional insight into a process by including the number of opportunities for failure. (DPO or DPMO)
- Consider a process where defects were classified by characteristic type and the number of opportunities for failure (OP) were noted for each characteristic type.
- The number of defects (D) and units (U) are then monitored for the process over some period of time.

Characteristic type	Defects	Units	Opportunities
Description	D	U	OP

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9.8 Defects per Million Opportunities (DPMO)

Characteristic type	Defects	Units	Opportunities
Description	D	U	OP

Total Opportunities	Defects per Unit	Defects per Opportunity	Defects per Million Opportunities
$TOP = U \times OP$	$DPU = D/U$	$DPO = D/TOP$	$DPMO = DPO \times 10^6$

- Summations could be determined for the defects (D) and total opportunities (TOP) columns.
- The overall DPO and DPMO could then be calculated from these summations.

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9.8 DPMO Application in Electronic Industry

- Soldering of components onto printed circuit board.
- The total number of opportunities for failure could be the number of components times the number of solder joints.
- With a DPMO metric, a uniform measurement applies to the process, not just to the product.

9.9 Example 9.3: DPMO

Charc.	D	U	OP	TOP	DPU	DPO	DPMO
A	21	327	92	30084	0.0642	0.000698	698
B	10	350	85	29750	0.0286	0.000336	336
C	8	37	43	1591	0.2162	0.005028	5028
D	68	743	50	37150	0.0915	0.001830	1830
E	74	80	60	4800	0.9250	0.015417	15417
F	20	928	28	25984	0.0216	0.000770	770
	201			**129359**		**0.001554**	**1554**

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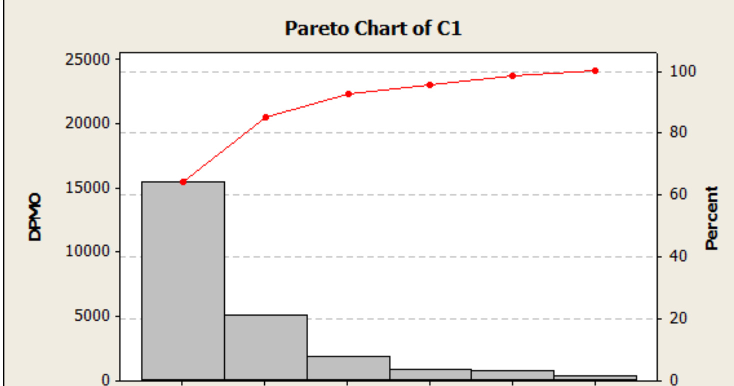
9.9 Example 9.3: DPMO



C1	E	C	D	F	A	B
DPMO	15417	5028	1830	770	698	336
Percent	64.0	20.9	7.6	3.2	2.9	1.4
Cum %	64.0	84.9	92.5	95.7	98.6	100.0

9.10 Rolled Throughput Yield

- Reworks within an operation have no value and comprise what is termed "hidden factory".
- Rolled throughput yield (RTY) measurements can give visibility to process steps that have high defect rates and/or rework needs.
- To determine the rolled throughput yield (Y_{RT}):
  - Calculate the throughput yield ($Y_{TP} = e^{-DPU}$) for each process steps.
  - Multiply these Y_{TP} of all steps for a process ($Y_{RT} = \prod Y_{TP}$).
  - A cumulative rolled throughput yield (Y_{RT}) up to a step is the product of the Y_{TP} of all previous steps.

9.11 Example 9.4: Rolled Throughput Yield

- A process has 10 operation steps, $Y_1 \sim Y_{10}$

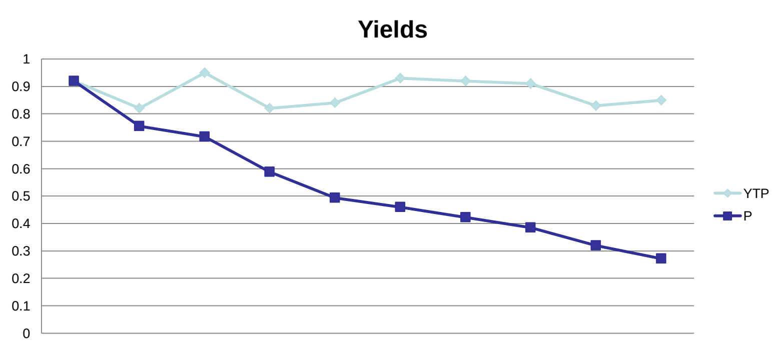
$Y_1 = 0.92$ → $Y_2 = 0.82$ → $Y_3 = 0.95$ → $Y_4 = 0.82$ → $Y_5 = 0.84$ →

$Y_6 = 0.93$ → $Y_7 = 0.92$ → $Y_8 = 0.91$ → $Y_9 = 0.83$ → $Y_{10} = 0.85$

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10
Y_{TP}	0.92	0.82	0.95	0.82	0.84	0.93	0.92	0.91	0.83	0.85
Π	0.92	0.754	0.717	0.588	0.494	0.459	0.422	0.384	0.319	0.271

9.11 Example 9.4: Rolled Throughput Yield

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10
Y_{TP}	0.92	0.82	0.95	0.82	0.84	0.93	0.92	0.91	0.83	0.85
Π	0.92	0.754	0.717	0.588	0.494	0.459	0.422	0.384	0.319	0.271



9.13 Yield Calculation

- A process had D defects and U units within a period of time for operation step (m).

Operation	Defects	Units	DPU	Operation Throughput Yield
Step Number	D	U	$DPU = D/U$	$Y_{TPi} = e^{-DPU}$
$\sum$	Sum of defects	Sum of units	Sum of DPUs	$Y_{RT} = \prod_{i=1}^{m} Y_{TPi}$
				$TDPU = -\ln(Y_{RT})$

9.14 Example 9.6: Yield Calculation

Op.	Defects	Units	DPU	Y_{TP}
1	5	523	0.00956	0.99049
2	75	851	0.08813	0.91564
3	18	334	0.05389	0.94753
4	72	1202	0.05990	0.94186
5	6	252	0.02381	0.97647
6	28	243	0.11523	0.89116
7	82	943	0.08696	0.91672
8	70	894	0.07830	0.92469
9	35	234	0.14957	0.86108
10	88	1200	0.07333	0.92929
RTY				0.47774
TDPU				0.73868

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9.15 Example 9.7: Normal Transformation (Z Value)

	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Y8	Y9	Y10
Y_{TP}	0.92	0.82	0.95	0.82	0.84	0.93	0.92	0.91	0.83	0.85
Z	1.405	0.915	1.645	0.915	0.994	1.476	1.405	1.341	0.954	1.036

9.16 Normalized Yield and Z Value for Benchmarking

- Typically, yields for each of the m steps differ.
- Rolled throughput yield (Y_{RT}) gives an overall yield for the process.
- A normalized yield (Y_{norm}) for the process is

$$Y_{norm} = \sqrt[m]{Y_{RT}}$$

- The defects per normalized unit (DPU_{norm}) is

$$DPU_{norm} = -\ln(Y_{norm})$$

- $Z_{equiv} \cong Z \sim N(0;1)$ Z=abs(norm.s.inv(DPU_{norm}))
- Z long-term: $Z_{LT} = Z_{equiv}$
- Z short-term : $Z_{ST} = Z_{LT} + 1.5_{shift}$
- Z Benchmark: $Z_{benchmark} = Z_{Y_{norm}} + 1.5$

9.17 Example 9.8: Normalized Yield and Z Value for Benchmarking

- Previous example had a RTY of .47774 ($Y_{RT} = .47774$).
- A normalized yield (Y_{norm}) for the process is

$$Y_{norm} = \sqrt[10]{0.47774} = 0.92879$$

- The defects per normalized unit (DPU_{norm}) is

$$DPU_{norm} = -\ln(Y_{norm}) = -\ln(0.92879) = 0.07387$$

- Z=abs(norm.s.inv(DPU_{norm}))=1.45
- $Z_{equiv} \cong Z = 1.45$
- Z long-term: $Z_{LT} = Z_{equiv} = 1.45$
- Z short-term : $Z_{ST} = Z_{LT} + 1.5_{shift} = 2.95$
- Z Benchmark: $Z_{benchmark} = Z_{Y_{norm}} + 1.5 = 2.95$

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9.18 Six Sigma Assumptions

- The characterization of process parameters by a normal distribution.
- A frequent drift/shift of the process mean by 1.5σ.
- The process mean and standard deviation are known.
- Defects are randomly distributed throughout units.
- Part/process steps are independent.

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