Chapter 28 Understanding the Creation of Full and Fractional Factorial 2^k DOEs

Introduction

- This chapter provides a conceptual explanation of two-level factorial experiments.
- It uses a non-manufacturing example to illustrate the application of this techniques.

28.1 S^4/IEE Application Examples: DOE

- Transactional 30,000-foot-level metric: An S^4/IEE project was to reduce DSO for an invoice. Wisdom of the organization and passive analysis led the creation of a DOE experiment that considered factors: size of order (large versus small), calling back within a week after mailing invoice (yes versus no), prompt paying customer (yes versus no), origination department (from passive analysis: least DSO versus highest DSO average), stamping "past due" on envelope (yes versus no).

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28.1 S^4/IEE Application Examples: DOE

- Transactional and manufacturing 30,000-foot-level metric: An S^4/IEE project was to improve customer satisfaction for a product or service. Wisdom of the organization and passive analysis led to the creation of a DOE experiment that considered factors: type of service purchased (A versus B), size of order (large versus small), department that sold service (from passive analysis: best versus worst), experience of person selling/delivering service (experienced versus new).

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28.1 S^4/IEE Application Examples: DOE

- Manufacturing 30,000-foot-level metric: An S^4/IEE project was to improve the process capability/performance metrics for the diameter of a plastic part from an injection-molding machine. Wisdom of the organization and passive analysis led to the creation of a DOE experiment that considered factors: temperature (high versus low), pressure (high versus low), hold time (long versus short), raw material (high side of tolerance versus low side of tolerance), machine (from passive analysis: best performing versus worst performing), and operator (from passive analysis: best versus worst).

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28.1 S^4/IEE Application Examples: DOE

- Manufacturing 30,000-foot-level metric: An S^4/IEE project was to improve the process capability/performance metrics for the daily defect rate of a printed circuit board assembly diameter of a plastic part from an injection-molding machine. Wisdom of the organization and passive analysis led to the creation of a DOE experiment that considered factors: board complexity (complicated versus less complicated), manufacturing line (A versus B), processing temperature (high versus low), solder type (new versus current), and operator (A versus B).

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28.1 S^4/IEE Application Examples: DOE

- Product DFSS: An S^4/IEE project was to improve the process capability/performance metrics for the number of notebook computer design problems uncovered during the product's life. A DOE test procedure assessing product temperature was added to the test process, where factors and their levels would be various features of the product. Each trial computer configuration would experience, while operational, an increase in temperature until failure occurs. The temperature at failure would be the response for the DOE (as measured with temperature sensing devices that are placed on various components within computer). Wisdom of the organization and passive analysis led to the creation of a DOE experiment that considered factors: hard drive size (large versus small), speed of processor (fast versus slow), design (new versus old), test case (high stress on machine processing versus low stress on machine processing), modem (high speed versus low speed).

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28.1 S^4/IEE Application Examples: DOE

- Product DFSS: An S^4/IEE project was to improve the process capability/performance metrics for the number of daily problem phone calls received within a call center. Passive analysis indicated that product setup was the major source of calls for existing products! services. A DOE test procedure assessing product setup time was added to the test process for new products. Wisdom of the organization and passive analysis led to the creation of a DOE experiment that considered factors: features of products or services, where factors and their levels would be various features of the product/service, along with various operator experience, include as a factor special setup instruction sheet in box (sheet included versus no sheet included).

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28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

- This section discusses 2-level full factorial experimental designs. The next section illustrates why fractional factorial design matrices work.
- When executing a full factorial experiment, a response is achieved for all combinations of factor levels. For analyzing 3 factors, 8 trials ($2^3 = 8$) to address all combinations.
- The plus/minus notation illustrates the high/low level of the factors.
- In this experiment design, each factor is executed at its high and low level an equal number of times.

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28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

2-Level Full Factorial Experiment Design

Trial No.	Factor Designation			Experiment Response
	A	B	C	
1	+	+	+	x_1
2	+	+	-	x_2
3	+	-	+	x_3
4	+	-	-	x_4
5	-	+	+	x_5
6	-	+	-	x_6
7	-	-	+	x_7
8	-	-	-	x_8

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28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

- The best estimate factor effects can be assessed by noting the difference in the average outputs of the trials. The calculation of the factor A effect is

$$[(\bar{x}_{[A^+]}) - (\bar{x}_{[A^-]})] = \frac{x_1 + x_2 + x_3 + x_4}{4} - \frac{x_5 + x_6 + x_7 + x_8}{4}$$

The result is an estimate of the average response change from the high to low level of A.

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28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

2-Level Full Factorial Experiment Design with Interaction

Trial No.	Factor Designation							Experiment Response
	A	*B*	*C*	*AB*	*BC*	*AC*	*ABC*	
1	+	+	+	+	+	+	+	x_1
2	+	+	-	+	-	-	-	x_2
3	+	-	+	-	-	+	-	x_3
4	+	-	-	-	+	-	+	x_4
5	-	+	+	-	+	-	-	x_5
6	-	+	-	-	-	+	+	x_6
7	-	-	+	+	-	-	+	x_7
8	-	-	-	+	+	+	-	x_8

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28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

- Interaction effects are a measurement of factor levels working together to affect a response.
- All interaction effects can be assessed given these 8 trials with 3 factors.
- 2-factor interaction effects are noted similarly. For AB interaction, the best estimate of the effect is

$$[(\bar{x}_{[AB^+]}) - (\bar{x}_{[AB^-]})] = \frac{x_1 + x_2 + x_7 + x_8}{4} - \frac{x_3 + x_4 + x_5 + x_6}{4}$$

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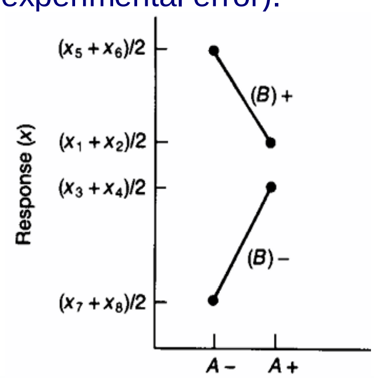
28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

- A question of concern in a factorial experiment is whether the calculated effects are large enough to be considered statistically significant (compared to experimental error).
- If a 2-factor interaction is found statistically significant, more information about the interaction can be obtained from an interaction plot. It is noted that there are 4 combinations of the levels of the *AB* factors (++, +-, -+, --).
- The 4 average of the combination are plotted on the interaction plot.



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28.2 Conceptual Explanation:
2-Level Full Factorial Exp. and 2-Factor Interactions

- If there is no interaction between factors, the lines on an interaction plot will be parallel. The overall effect initially determined for the interaction $\left[\left(\bar{x}_{[AB^{+}]}\right)-\left(\bar{x}_{[AB^{-}]}\right)\right]$ is a measure of the lack of parallelism of the lines.

28.3 Conceptual Explanation:
Saturated 2-Level DOE

- When many factors are considered, full factorials can yield a very large test sample size, whereas a saturated fractional factorial experiment can require a much reduced sample size.
- The basic saturated fractional factorial experiment design is illustrated in the following table (Table 28.3).
- In this table, the calculated interaction columns give the levels of 4 additional factors, making the total number of 2-level factor considerations 7 in 8 trials.

28.3 Conceptual Explanation: Saturated 2-Level DOE

Saturated Fractional Factorial Experiment: 8 Trails, 7 Factors

Trial No.	Factor Designation							Experiment Response
	A	*B*	*C*	*D* ~~*AB*~~	*E* ~~*BC*~~	*F* ~~*AC*~~	*G* ~~*ABC*~~	
1	+	+	+	+	+	+	+	x_1
2	+	+	-	+	-	-	-	x_2
3	+	-	+	-	-	+	-	x_3
4	+	-	-	-	+	-	+	x_4
5	-	+	+	-	+	-	-	x_5
6	-	+	-	-	-	+	+	x_6
7	-	-	+	+	-	-	+	x_7
8	-	-	-	+	+	+	-	x_8

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28.3 Conceptual Explanation: Saturated 2-Level DOE

- The disadvantage of the saturated fractional factorial experiment is the confounding of two-factor interaction effects and main effects. [There is confounding of the *AB* interaction and the main effect *D*; there is also confounding of factor *D* and other two-factor interactions because of the introduction of the additional factors *D*, *E*, *F*, and *G*.]
- Because each column can now have more than one meaning, it is assigned a *contract column number* (Tables M1 to M5, and N1 to N3)

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- Consider that a high school administration wants to reduce absenteeism of students.
- Factors can affect school absenteeism:
  - Student: age, sex, ethnic background, etc.
  - School: location, teacher, class, etc.
  - Time: day of week, class period, etc.
- After an initial regression assessment, consider using DOE approach with the following factors (Observations/Improve):
  - A: Day of the week; B: Call-back when absent
  - C: School E: Mentor if missed a lot
  - D: Class period F: Contract if missed a lot
  - G: Sex of student

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- Consider now how to address the levels for each factor. [We should always ask whether the additional level is helpful for addressing the problem at hand.]
- Consider the number of trials if the following levels were assigned to each factor:
  - *A*: Day of the week: Monday vs. Friday
  - *B*: Call-back when absent: Yes vs. No
  - *C*: School: Location 1, 2, 3, 4
  - *D*: Class period: 1, 2, 3
  - *E*: Mentor if missed a lot: Yes vs. No
  - *F*: Contract if missed a lot: Yes vs. No
  - *G*: Sex of student: Male vs. Female

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- The total number of combinations for a full factorial is 384.
- To reduce the number of trials, consider altering the number of levels to 2 (schools with the best and worst attendance record).
- A 2-level assessment would reduce the number of trials to 128 for a full factorial design.
- This could be further reduced to 64, 32, 16, or 8 trials using a fractional factorial structure.
- Various fractional factorial experiments give varying resolution of factor information. Resolution is related to the management of 2-factor interactions.

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- When reducing the number of levels to 2, quantitative factors (such as pressure) would be modeled as a linear relationship. For qualitative factors (such as suppliers, operators, or machines), consider choosing the sources that represent the extremes.
- For purpose of illustration, consider only 3 of the 2-level factors:
  - Day of week: Monday vs. Friday
  - Call-back when absent: Yes vs. No
  - School: 1 vs. 2
- 8 trials can assess all possible combinations of the 3 factors.

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- For purpose of illustration, consider only 3 of the 2-level factors:
  - Day of week: Monday vs. Friday
  - Call-back when absent: Yes vs. No
  - School: 1 vs. 2
- 8 trials can assess all possible combinations of the 3 factors.

Factor	Level	
	–	+
A: Day of week	Friday	Monday
B: Call-back when absent	Yes	No
C: School	1	2

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- One experiment design might be to select 800 students randomly from two schools.
- Students would then be randomly placed into one of the eight trial categories, 100 students in each trial.
- The total number of days absent from each category would be the response for the analysis.
- This approach offers some advantages over the regression analysis. New factors are assessed that could improve the process. Effects from happenstances are lessened.
- The eight-trial combinations are as follows,

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

Trial No.	Factor Designation			Response
	A	*B*	*C*	
1	+	+	+	x_1
2	+	+	-	x_2
3	+	-	+	x_3
4	+	-	-	x_4
5	-	+	+	x_5
6	-	+	-	x_6
7	-	-	+	x_7
8	-	-	-	x_8

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- Consider trial 2, the response would be the total absenteeism of 100 students on Monday with no call-back for school 1.
- Also, there are equal number of trials at each level for each factor, that is 4 trials had *C* at "+" and 4 trials had *A* at "-".
- The effect of a factor would be the average at the "+" level minus the average at the "-" level.
- Consider that this experiment yielded the following results:

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

Trial No.	Factor Designation			Response
	A	*B*	*C*	
1	+	+	+	198
2	+	+	-	203
3	+	-	+	169
4	+	-	-	172
5	-	+	+	183
6	-	+	-	181
7	-	-	+	94
8	-	-	-	99

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- The estimated main effects are:

Factor	Effect
A: Day of week	+46.25
B: Call-back when absent	+57.75
C: School	-2.75

- By observation, the magnitude of the school effect seems small.
- The sign of the other two factors indicates which level of the factor is best. In this case, lower number are best; hence, Friday and call-back are best.
- However, this model does not address the interaction.

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- As mentioned earlier, a 2-factor interaction causes a lack of parallelism between the two lines in a two factor interaction plot.
- When sample data are plotted, the lines typically will not be exactly parallel. The question is whether the degree of out-of-parallelism is large enough to be considered as result of true interaction opposed to chance. This issue is addressed through the calculation of an interaction effect.
- An interaction contrast column is created by multiplying the level designations of all main effect contrast columns:

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

Trial No.	Factor Designation							Response
	A	*B*	*C*	*AB*	*BC*	*AC*	*ABC*	
1	+	+	+	+	+	+	+	x_1
2	+	+	-	+	-	-	-	x_2
3	+	-	+	-	-	+	-	x_3
4	+	-	-	-	+	-	+	x_4
5	-	+	+	-	+	-	-	x_5
6	-	+	-	-	-	+	+	x_6
7	-	-	+	+	-	-	+	x_7
8	-	-	-	+	+	+	-	x_8

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- Again, there are 4 "+" and 4 "-" in each contrast column. The magnitude of the effect from an interaction contrast column relative to other contrast column effects can be used to assess the likelihood of an interaction.
- Hence, not all 2-factor interaction plots are needed, only those 2-factor interactions with sufficiently large effects.
- Entering the responses in the format with interactions:

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

Trial No.	Factor Designation							Response
	A	*B*	*C*	*AB*	*BC*	*AC*	*ABC*	
1	+	+	+	+	+	+	+	198
2	+	+	-	+	-	-	-	203
3	+	-	+	-	-	+	-	169
4	+	-	-	-	+	-	+	172
5	-	+	+	-	+	-	-	183
6	-	+	-	-	-	+	+	181
7	-	-	+	+	-	-	+	94
8	-	-	-	+	+	+	-	99

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- The interaction effects are determined in a similar fashion to main effects. :

Factor	Effect
A: Day of week	+46.25
B: Call-back when absent	+57.75
C: School	-2.75
AB: Day*call-back	27.75
BC: Call-back*school	1.25
AC: Day*school	-1.25
ABC: Day*call-back*school	-2.25

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- This summary indicates that A, B, and AB effects are large relative to the other effects.
- A 2-factor interaction plot shows which factor levels are most beneficial.

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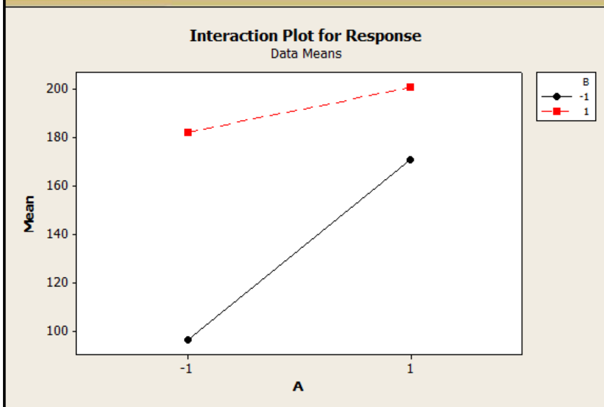
28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process



- The plot indicates the call-back program (*B*) helps more on Friday (-A) than on Monday (+A).

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- The number of trials can increase dramatically if we follow a similar procedure for an increased number of factors.
- Consider seven 2-level factors, the number of trials is 128.
  - *A*: Day of the week: Monday vs. Friday
  - *B*: Call-back when absent: Yes vs. No
  - *C*: School: Location 1 vs. 2
  - *D*: Class period: 1 vs. 2
  - *E*: Mentor if missed a lot: Yes vs. No
  - *F*: Contract if missed a lot: Yes vs. No
  - *G*: Sex of student: Male vs. Female

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- A 128-trial 7-factor experiment contains very high factor interactions, 3-factor, 4-factor, 5-factor, 6-factor, and 7-factor.
- Typically, we assume that any interactions above two are small. Hence, we don't need this many trials.
- A fractional DOE is an alternative to a full factorial DOE.
- Consider again the 8-trial, 2-level full factorial design with all interactions. An assignment of the 4 additional factors to the interaction contrast columns yields the following:

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

Trial No.	Factor Designation							Output
	A	*B*	*C*	*D*	*E*	*F*	*G*	
1	+	+	+	+	+	+	+	x_1
2	+	+	-	+	-	-	-	x_2
3	+	-	+	-	-	+	-	x_3
4	+	-	-	-	+	-	+	x_4
5	-	+	+	-	+	-	-	x_5
6	-	+	-	-	-	+	+	x_6
7	-	-	+	+	-	-	+	x_7
8	-	-	-	+	+	+	-	x_8

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- This 7-factor, 8-trial, 2-level saturated design minimizes sample size, assesses main factors, and confounds 2-factor interactions with main effects.
- From the analysis of main effects, the expectation is that class period 2 has more absenteeism than class period 1.

Factor	Effect	Significance
A: Day of week (1 vs. 2)	+46.25	Friday is best
B: Call-back when absent (Yes vs. No)	+57.75	Call-back is best
C: School (1 vs. 2)	-2.75	Not significant
D: Class period (1 vs. 2)	27.75	Significant
E: Mentor (Yes vs. No)	1.25	Not significant
F: Contract (Yes vs. No)	-1.25	Not significant
G: Sex (Male vs. Female)	-2.25	Not significant

28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- The example illustrates a DOE for non-manufacturing app.
- For 8 trials, the extremes discussed are:
  - Three 2-level factors (full factorial)
  - Seven 2-level factors (saturated factorial)
- The confounding of 2-factor interaction:
  - Full factorial: all interactions are determined
  - Saturated factorial: 2-factor interactions are confounded with main effects
- The resolution designation:
  - Three 2-level factors (full factorial): V+
  - Seven 2-level factors (saturated factorial): III

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28.4 Example 28.1: Applying DOE to a Non-Manufacturing Process

- There are other choices for the number of factors in eight trials instead of three or seven factors.
- There could be 4, 5, or 6 two-level factors examined in 8 trials, and can lead to other resolution levels.
- Resolution IV designs confound 2-factor interactions with each other, but not with main effects.
- Resolution V designs do not confound 2-factor interactions with other 2-factor interactions or with main effects.
- Table M shows various resolution alternatives, number of factors for 8, 16, 32, and 64 trials, and design matrices.