Chapter 11 Process Capability and Process Performance Metrics

Introduction

- Traditionally, process capability/performance index studies are conducted to assess a process relative to specification criteria.
- The equations for process capability/ performance indices are quite simple but are very sensitive to the input value for standard deviation (σ). Unfortunately, there can be differences of opinion on how to determine standard deviation in a given situation.
- The equations presented in this chapter apply to normally distributed data.

11.1 S4/IEE Application Examples: Process Capability/Performance Metrics

- Satellite-level metric: The last three years' ROI for a company was reported monthly in a control chart. No special causes or trends were identified. Monthly ROIs were plotted on a normal probability plot, where a null hypothesis for normality was not rejected. The capability/performance of the system was reported on the probability plot as a best-estimate 80% interval, which described common-cause variability. Organizational goals were set to improve this metric. A strategic plan was created that was in alignment with the organizational goal to improve this metric, and 30,000-foot-level operational metrics were then chosen that would be the focus of improvement efforts. S^4/IEE projects were then chosen to improve these metrics.

11.1 S4/IEE Application Examples: Process Capability/Performance Metrics

- Transactional 30,000-foot-level metric: One random paid invoice was selected each day from last year's invoices where the number of days beyond the due date was measured and reported (i.e., days sales outstanding [DSO]). The DSO for each sample was reported in an XmR control chart, where no reason was identified for a couple of special-cause data points. These data were plotted on a normal probability plot, where a null hypothesis for normality was rejected. A lognormal plot fit the data well. An XmR chart of the lognormal data did not indicate any special-cause conditions. The lognormal probability plot was used to estimate the proportion of invoices beyond 30, 60, and 90 days. An S^4/IEE project was initiated to improve the DSO metric.

11.1 S4/IEE Application Examples: Process Capability/Performance Metrics

- Transactional 30,000-foot-level metric: The mean and standard deviation of all DSOs were tracked using two XmR charts with a weekly subgrouping, where the standard deviation values had a log transformation. No special causes were identified. The long-term capability/performance of the process was reported as percentage nonconformance beyond 30, 60, and/or 90 days, using variance of components techniques or a statistical program that reports this metric under their $\bar{x}$ and s process capability/ performance option.
- Manufacturing 30,000-foot-level metric (KPOV): One random sample of a manufactured part was selected each day over the last year. The diameter of the part was measured and plotted in a control chart. No special causes were identified. A null hypothesis for normality could not be rejected. The long-term process capability/ performance metric was reported as the estimated ppm rate beyond the specification limits.

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11.1 S4/IEE Application Examples: Process Capability/Performance Metrics

- Transactional and manufacturing 30,000-foot-level cycle-time metric (a lean metric): One transaction was randomly selected each day over the last year, where the time from order entry to fulfillment was measured. The differences between these times relative to their due date were reported in an XmR chart. No special causes were identified. A null hypothesis for normality could not be rejected. The long-term process capability/performance metric was reported as the estimated ppm rate beyond the due date for the transactions.
- Transactional and manufacturing 30,000-foot-level inventory metric or satellite-level TOC metric (a lean metric): Inventory was tracked monthly using a control chart. No special causes were identified. A null hypothesis for normality could not be rejected. The long-term process capability/performance nonconformance rate was reported as an equal to or less than frequency of occurrence level of 80% for month-to-month inventory levels and the associated monetary implications.

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11.1 S4/IEE Application Examples: Process Capability/Performance Metrics

- Manufacturing 30,000-foot-level quality metric: The number of printed circuit boards produced weekly for a high-volume manufacturing company is similar. The weekly failure rate of printed circuit boards is tracked on an XmR. No special causes were identified. The centerline ppm rate of the XmR chart was reported as the capability/performance of the process.
- Transactional 50-foot-level metric (KPIV): An S^4/IEE project to improve the 30,000-foot-level metrics for DSOs identified a KPIV to the process, the importance of timely calling customers to ensure that they received a company's invoice. A control chart tracked the time from invoicing to when the call was made, where one invoice was selected hourly. No special cause was identified. A hypothesis for normality could not be rejected. The long-term percentage of instances beyond the objective "specification," identified during the S^4/IEE project was the reported capability/performance of this input variable.

11.1 S4/IEE Application Examples: Process Capability/Performance Metrics

- Product DFSS: An S^4/IEE product DFSS project was to reduce the 30,000-foot-level MTBF (mean time between failures) of a product by its vintage. A control chart tracked the product MTBF by product vintage. The capability/ performance of the system was reported on the probability plot as a best-estimate 80% interval, which described common-cause variability and what might be expected for MTBF rates in the future unless something were done differently to change the design process. Categories of problems were tracked over the long haul in a Pareto chart to identify improvement opportunities for newly developed products.
- S^4/IEE infrastructure 30,000-foot-level metric: A steering committee uses a control chart to track the duration of projects. The process capability/performance metric of the system was reported as a best-estimate 80% interval on a probability plot.

11.2 Definitions

- Inherent Process Variation. The portion of process variation due to common causes only. This variation can be estimated from control charts by $\bar{R}/d_2$, among other things (e.g.,$\bar{s}/c_4$).
- Total Process Variation. Process variation due to both common and special causes. This variation may be estimated by s, the sample standard deviation, using all of the individual readings obtained from either a detailed control chart or a process study; that is,

$$\hat{\sigma} = s = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}}$$

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11.2 Definitions

- Process Capability. The 6σ range of a process's inherent variation; for statistically stable processes only, where σ is usually estimated by $\bar{R}/d_2$. ["Process capability is broadly defined as the ability of a process to satisfy customer expectations" Bothe (1997).]
- Process Performance. The 6σ range of a process's total variation, where σ is usually estimated by s, the sample standard deviation.

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11.2 Definitions

C_p The capability index, defined as the tolerance width divided by the process capability, irrespective of process centering.

C_{pk} The capability index, which accounts for process centering. It relates the scaled distance between the process mean and the closest specification limit to half the total process spread.

P_p The performance index, defined as the tolerance width divided by the process performance, irrespective of process centering. It should be used only to compare to C_p and C_{pk} and to measure and prioritize improvement over time.

P_{pk} The performance index, which accounts for process centering. It should be used only to compare to C_p and C_{pk} and to measure and prioritize improvement over time.

11.3 Misunderstandings

- Misapplication of the capability index and the performance index.
- Use randomly collected data as if they are sequential. (Computer software)

→ Reporting estimated process capability/performance nonconformance rates for both continuous and attribute data.

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11.4 Confusion: Short-term v.s. Long-term Variability

Opinion 1:

- Process capability describes the "capability". (the best could currently be expected to work) It considers short-term variability with an assumed adjustment of 1.5σ to compensate for drifts to get long-term variability.
- Std. deviation within subgroups estimates the short-term variability; the std. deviation of all the data combined estimates the long-term variability.
- Process capability indices C_p and C_{pk} assess the potential short-term variability by using a short-term std. deviation.
- Process performance indices P_p and P_{pk} assess the overall long-term variability by using a long-term std. deviation.

11.4 Confusion: Short-term v.s. Long-term Variability

Opinion 2:

- Process capability describes how well a process is executing relative to the needs of the customer. "Short-term" and "long-term" are not typically considered separately.
- Std. deviation estimates the overall variability of a process.
- Process capability indices C_p and C_{pk} address the needs of customers by using a total std. deviation.
- Process performance indices P_p and P_{pk} are not typically used.

11.5 Calculating Standard Deviation

- Desired result can be obtained only if data are collected in an appropriate fashion.
- 3 sources of continuous data:
  - Situation 1: An $\bar{x}$ and R control chart with subgroups of sample size of 5.
  - Situation 2: An X chart with individual measurements.
  - Situation 3: A random sample of measurements from population.
- No method is correct for obtaining an estimate of σ in all 3 situations.

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11.5 Calculating Standard Deviation

Calculating Std. Deviation

- Method 1: Long-term: $\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}}$
- Method 2: Short-term: $\hat{\sigma} = \frac{\bar{R}}{d_2}$
- Method 3: Short-term: $\hat{\sigma} = \frac{\bar{s}}{c_4}$
- Method 4: Short-term: $\hat{\sigma} = 1.047\ (Moving\ \tilde{R})$
- Method 5: Short-term: $\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{\overline{MR}}{1.128}$
- Method 6: Short-term: $\hat{\sigma} = \frac{s_p}{c_4(d)}$

$$s_p = \sqrt{\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} \left(x_{ij} - \bar{x}_i\right)^2}{\sum_{i=1}^{m} (n_i - 1)}}$$

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11.5 Calculating Standard Deviation

Method 1: Long-term Estimate of σ

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}}$$

- Situation 1: Valid only when a process is stable.
- Situation 2: This approach can give an estimate of process variability from the customer's point of view.
- Situation 3: This is the only method that makes sense.

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11.5 Calculating Standard Deviation

Method 2: Short-term Estimate of σ

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

- Situation 1: This estimator alleviates the problem of σ being inflated by special causes. Rational subgroup should be used (consecutively produced units). Inefficient with large subgroup.
- Situation 2: This calculation is not directly possible.
- Situation 3: This calculation is not possible because the sequence of production is not known.

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11.5 Calculating Standard Deviation

Method 3: Short-term Estimate of σ

$$\hat{\sigma} = \frac{\bar{s}}{c_4}$$

- Situation 1: This estimator alleviates the problem of σ being inflated by special causes. Rational subgroup should be used (consecutively produced units). More efficient than Method 2.
- Situation 2: This calculation is not directly possible.
- Situation 3: This calculation is not possible because the sequence of production is not known.

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11.5 Calculating Standard Deviation

Method 4: Short-term Estimate of σ

$$\hat{\sigma} = 1.047(Moving\ \tilde{R})$$

- Situation 1: This approach is not applicable.
- Situation 2: Valid alternative. Research has indicated that this approach gives good results for a wide variety of out-of-control patterns.
- Situation 3: This calculation is not possible because the sequence of production is not known.

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11.5 Calculating Standard Deviation

Method 5: Short-term Estimate of σ

$$\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{\overline{MR}}{1.128}$$

- Situation 1: This approach is not applicable.
- Situation 2: Valid alternative.
- Situation 3: This calculation is not possible because the sequence of production is not known.

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11.5 Calculating Standard Deviation

Method 6: Short-term Estimate of σ

$$\hat{\sigma} = \frac{s_p}{c_4(d)}$$

Where $s_p = \sqrt{\frac{\sum_{i=1}^{m} \sum_{j=1}^{n} \left(x_{ij} - \bar{x}_i\right)^2}{\sum_{i=1}^{m} (n_i - 1)}}$, $\mathrm{d} = \left(\sum_{i=1}^{m} n_i\right) - m + 1$

- Situation 1: If all groups are weighted the same regardless of the number of observations, the $\bar{s}$ (or $\bar{R}$) approach is preferred. If variation is weighted according to subgroup size, the pooled approach is appropriate.
- Situation 2: This calculation is not directly possible.
- Situation 3: This calculation is not possible.

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11.5 Calculating Standard Deviation

	Method 1 LT	Method 2 ST	Method 3 ST	Method 4 ST	Method 5 ST	Method 6 ST
	s	$\bar{R}/d_2$	$\bar{s}/c_4$	$1.047(\widetilde{MR})$	$\overline{MR}/1.128$	$s_p/(c_4(d))$
$\bar{x}$ and R	Valid if process stable	Valid	Valid	N.A.	N.A.	Valid
XmR	Valid (from customer)	Not possible	Not possible	Valid	Valid	Not possible
Random	Valid	Not possible	Not possible	Not possible	Not possible	Not possible

11.5 Calculating Standard Deviation

Other Methods

- Long-term or short-term estimate of σ. Variance components analysis using total variability from all considered components.
- Short-term or long-term estimate of σ. Single factor analysis of variance (ANOVA)
- Short-term or long-term estimate of σ. Two factor analysis of variance (ANOVA)

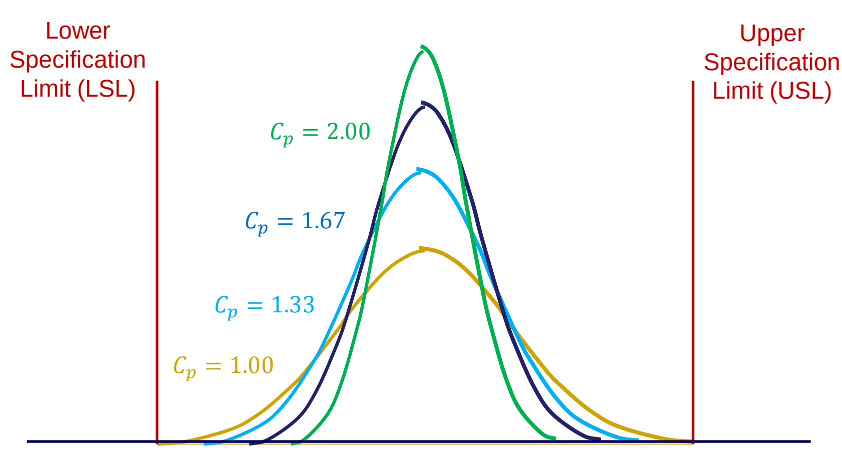
11.6 Process Capability Indices: C_p and C_{pk}

- C_p represents the allowable tolerance interval spread in relation to the process capability (6σ under Normal dist.)

$$C_p = \frac{USL - LSL}{6\sigma}$$

- Process centering is not taken into account.

11.6 Process Capability Indices: C_p and C_{pk}



11.6 Process Capability Indices: C_p and C_{pk}

- C_{pk} is used concurrently to consider the spread and mean shift of the process.

$$C_{pk} = min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right]$$

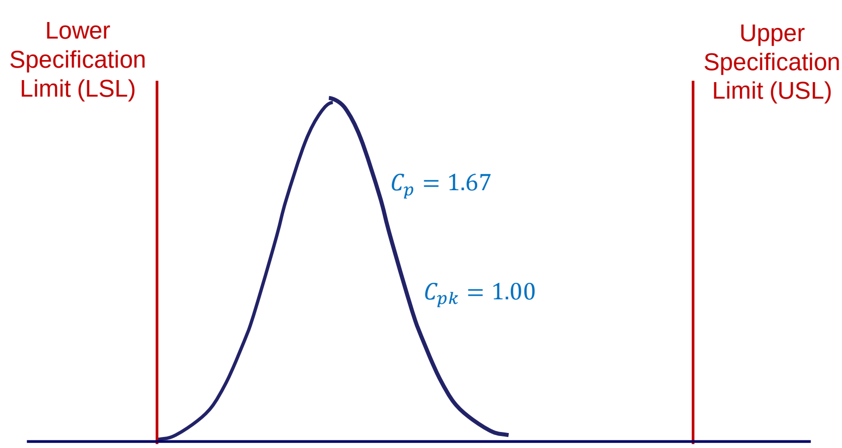
- The relationship of C_{pk} to C_p is

$$C_{pk} = C_p(1-k)$$

- The k factor quantifies the amount by which the process is off center.

$$k = \frac{|m-\mu|}{(USL-LSL)/2} \quad where\ \ m = \left[\frac{USL+LSL}{2}\right], 0 \le k \le 1$$

11.6 Process Capability Indices: C_p and C_{pk}



11.6 Process Capability Indices: C_p and C_{pk}

Strategies for estimating σ:

- Short-term view of σ. From an $\bar{x}$ and R chart,

$$\hat{\sigma} = s = \frac{\bar{R}}{d_2}, \hat{\mu} = \bar{\bar{x}}$$

- Long-term view of σ. Total standard deviation from a variance components analysis.
- A minimum acceptable process capability index often recommended is 1.33 (or 4σ level)
- Motorola Six Sigma program proposes striving to obtain a minimum individual process step C_p value of 2.0 and a C_{pk} value of 1.5.

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11.7 Process Performance Indices: P_p and P_{pk}

- Sometimes referred to as long-term process capability/ performance indices.
- Relationship between P_p and P_{pk} is similar to that between C_p and C_{pk}
- P_p represents the allowable tolerance interval spread in relation to the process performance (6σ under Normal dist.)

$$P_p = \frac{USL - LSL}{6\sigma}$$

- Process centering is not taken into account.

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11.7 Process Performance Indices: P_p and P_{pk}

- P_{pk} is used concurrently to consider the spread and mean shift of the process.

$$P_{pk} = min\left[\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right]$$

- The relationship of P_{pk} to P_p is

$$P_{pk} = P_p(1-k)$$

- The k factor quantifies the amount by which the process is off center.

$$k = \frac{|m-\mu|}{(USL-LSL)/2} \quad where\ \ m = \left[\frac{USL+LSL}{2}\right], 0 \le k \le 1$$

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11.7 Process Performance Indices: P_p and P_{pk}

Strategies for estimating σ:

- From an $\bar{x}$ and R chart,

$$\hat{\sigma} = s = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}}, \hat{\mu} = \bar{\bar{x}}$$

- P_p and P_{pk} indices are no longer used in the Motorola University training program.

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11.8 Process Capability and The Z Distribution

- Capability can be represented using the distance of the process average from specification limits in standard units.
- Unilateral Tolerance:

$$Z = \frac{USL - \mu}{\sigma} \text{ or } Z = \frac{\mu - LSL}{\sigma}$$

- Bilateral Tolerance:

$$Z_{USL} = \frac{USL - \mu}{\sigma}, Z_{LSL} = \frac{\mu - LSL}{\sigma}, Z_{min} = \min[Z_{USL}, Z_{LSL}]$$

- An expression for C_{pk} in terms of Z is $C_{pk} = Z_{min}/3$
- The difference between a minimum C_p value of 2.0 and a C_{pk} value of 1.5 represents the 1.5σ mean shift.

11.9 Capability Ratios

- Capability ratios are used to describe processes.
- Capability ratio

$$CR = \frac{1}{C_p}$$

- Performance ratio

$$PR = \frac{1}{P_p}$$

11.10 C_{pm} Index

- C_p and C_{pk} do not adequately address the issue of process centering, Taguchi advocated an alternative metric, C_{pm}

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\mu - T)^2 + \sigma^2}} \quad where\ T \text{ is the target}$$

- The equation for C_{pm} is based on the reduction of variation from the target value as the guiding principle to quality improvement. It is consistent with the philosophy of loss function (monetary loss to the customer and society in general when products do not meet the target exactly.
- More importance is given to target (T), less to specification.
- Variation from target comes from the process and centering.

11.11 Example 11.1: Process Capability/Performance Indices

S4/IEE Application Examples

- Transactional: Five sequentially paid invoices were randomly selected each hour. The number of days past the invoice due date was tracked using a $\bar{x}$ and R chart.
- Cycle time (manufacturing and transactional): Each hour 5 sequential transactions were randomly selected. Cycle time for completing the transactions was tracked using a $\bar{x}$ and R chart.

11.11 Example 11.1: Process Capability/Performance Indices

From Example 10.2

- A grinding machine is to produce treads for a hydraulic system of an aircraft to a diameter of 0.4037±0.0013".
- Measurements were taken every hour on 5 parts using a visual comparator with accuracy of .0001".
- The data, sample means, sample ranges, sample standard deviations, and sum of squares within the sample are recorded in Table 11.1. (All data are expressed in units of 0.0001" in excess of 0.4000".) → Specification limits will be considered 24 to 50.

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11.11 Example 11.1: Process Capability/Performance Indices

Sample #						Mean	Range	s	SS
1	36	35	34	33	32	34.00	4	1.5811	10.0
2	31	31	34	32	30	31.60	4	1.5166	9.2
3	30	30	32	30	32	30.80	2	1.0954	4.8
4	32	33	33	32	35	33.00	3	1.2247	6.0
5	32	34	37	37	35	35.00	5	2.1213	18.0
6	32	32	31	33	33	32.20	2	0.8367	2.8
7	33	33	36	32	31	33.00	5	1.8708	14.0
8	23	33	36	35	36	32.60	13	5.5045	121.2
9	43	36	35	24	31	33.80	19	6.9785	194.8
10	36	35	36	41	41	37.80	6	2.9496	34.8
11	34	38	35	34	38	35.80	4	2.0494	16.8
12	36	38	39	39	40	38.40	4	1.5166	9.2
13	36	40	35	26	33	34.00	14	5.1478	106.0
14	36	35	37	34	33	35.00	4	1.5811	10.0
15	30	37	33	34	35	33.80	7	2.5884	26.8
16	28	31	33	33	33	31.60	5	2.1909	19.2
17	33	30	34	33	35	33.00	5	1.8708	14.0
18	27	28	29	27	30	28.20	3	1.3038	6.8
19	35	36	29	27	32	31.80	9	3.8341	58.8
20	33	35	35	39	36	35.60	6	2.1909	19.2
Total						671.00	124.00	49.9532	702.40
Avg.						63.90	6.20	2.50	

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11.11 Example 11.1: Process Capability/Performance Indices

Method 1: Long-term Estimate of σ: Using Individual Data

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)}} = \sqrt{\sum_{i=1}^{100} \frac{(x_i - 33.55)^2}{(100-1)}} = 3.52874$$

$$\hat{\sigma}_{adjusted} = \frac{\hat{\sigma}}{c_4} = \frac{3.52874}{.9975} = 3.53776$$

$$P_p = \frac{USL - LSL}{6\sigma} = \frac{50 - 24}{6(3.53776)} = 1.22$$

$$P_{pk} = min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] = \min[1.55, 0.90] = 0.90$$

11.11 Example 11.1: Process Capability/Performance Indices

Method 1:

$$Z_{USL} = \frac{USL - \hat{\mu}}{\hat{\sigma}} = \frac{50.00 - 33.55}{3.53776} = 4.65$$

$$Z_{LSL} = \frac{\hat{\mu} - LSL}{\hat{\sigma}} = \frac{33.55 - 24.00}{3.53776} = 2.70$$

$$ppm_{USL} = \Phi(Z_{USL}) \times 10^6 = \Phi(4.65) \times 10^6 = 1.7$$

$$ppm_{LSL} = \Phi(Z_{LSL}) \times 10^6 = \Phi(2.70) \times 10^6 = 3472.7$$

$$ppm_{total} = ppm_{USL} + ppm_{LSL} = 1.7 + 3472.7 = 3474.4$$

11.11 Example 11.1: Process Capability/Performance Indices

Method 2: Short-term Estimate of σ: Using $\bar{R}$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{6.2}{2.326} = 2.66552$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{50 - 24}{6(2.66552)} = 1.63$$

$$C_{pk} = min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] = \min[2.06, 1.19] = 1.19$$

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11.11 Example 11.1: Process Capability/Performance Indices

Method 2:

$$Z_{USL} = \frac{USL - \hat{\mu}}{\hat{\sigma}} = \frac{50.00 - 33.55}{2.66552} = 6.17$$

$$Z_{LSL} = \frac{\hat{\mu} - LSL}{\hat{\sigma}} = \frac{33.55 - 24.00}{2.66552} = 3.58$$

$$ppm_{USL} = \Phi(Z_{USL}) \times 10^6 = \Phi(6.17) \times 10^6 = 0$$

$$ppm_{LSL} = \Phi(Z_{LSL}) \times 10^6 = \Phi(3.58) \times 10^6 = 170$$

$$ppm_{total} = ppm_{USL} + ppm_{LSL} = 0 + 170 = 170$$

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11.11 Example 11.1: Process Capability/Performance Indices

Method 3: Short-term Estimate of σ: Using $\bar{s}$

$$\hat{\sigma} = \frac{\bar{s}}{c_4} = \frac{2.4977}{0.9400} = 2.6571$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{50 - 24}{6(2.6571)} = 1.63$$

$$C_{pk} = min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] = \min[2.06, 1.20] = 1.20$$

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11.11 Example 11.1: Process Capability/Performance Indices

Method 3:

$$Z_{USL} = \frac{USL - \hat{\mu}}{\hat{\sigma}} = \frac{50.00 - 33.55}{2.6571} = 6.19$$

$$Z_{LSL} = \frac{\hat{\mu} - LSL}{\hat{\sigma}} = \frac{33.55 - 24.00}{2.6571} = 3.59$$

$$ppm_{USL} = \Phi(Z_{USL}) \times 10^6 = \Phi(6.19) \times 10^6 = 0$$

$$ppm_{LSL} = \Phi(Z_{LSL}) \times 10^6 = \Phi(3.59) \times 10^6 = 170$$

$$ppm_{total} = ppm_{USL} + ppm_{LSL} = 0 + 170 = 170$$

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11.11 Example 11.1: Process Capability/Performance Indices

Method 4: Short-term Estimate of σ: from median of moving ranges

$$\hat{\sigma} = 1.047(Moving\ \tilde{R})$$

Not Applicable.

Method 5: Short-term Estimate of σ: from $\overline{MR}$

$$\hat{\sigma} = \frac{\overline{MR}}{d_2} = \frac{\overline{MR}}{1.128}$$

Not Applicable.

11.11 Example 11.1: Process Capability/Performance Indices

Method 6: Short-term Estimate of σ: Pooled Standard Deviation

$$s_p = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\left(x_{ij} - \overline{x_i}\right)^2}{\sum_{i=1}^{m}(n_i - 1)}} = \sqrt{\frac{702.4}{20(4)}} = 2.963106$$

$$\mathrm{d} = \left(\sum_{i=1}^{m} n_i\right) - m + 1 = (20 \times 5) - 20 + 1 = 81$$

$$\hat{\sigma} = \frac{s_p}{c_4(d)} = \frac{2.963106}{0.9969} = 2.97238$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{50 - 24}{6(2.97238)} = 1.46$$

$$C_{pk} = min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] = \min[1.84, 1.07] = 1.07$$

11.11 Example 11.1: Process Capability/Performance Indices

Method 6:

$$Z_{USL} = \frac{USL - \hat{\mu}}{\hat{\sigma}} = \frac{50.00 - 33.55}{2.97238} = 5.53$$

$$Z_{LSL} = \frac{\hat{\mu} - LSL}{\hat{\sigma}} = \frac{33.55 - 24.00}{2.97238} = 3.21$$

$$ppm_{USL} = \Phi(Z_{USL}) \times 10^6 = \Phi(5.53) \times 10^6 = 0$$

$$ppm_{LSL} = \Phi(Z_{LSL}) \times 10^6 = \Phi(3.21) \times 10^6 = 657$$

$$ppm_{total} = ppm_{USL} + ppm_{LSL} = 0 + 657 = 657$$

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11.11 Example 11.1: Process Capability/Performance Indices

	Method 1 LT		Method 2 ST	Method 3 ST	Method 4 ST	Method 5 ST	Method 6 ST
	s		$\bar{R}/d_2$	$\bar{s}/c_4$	$1.047\widetilde{MR}$	$\overline{MR}/1.128$	$s_p/(c_4(d))$
$\hat{\sigma}$	3.5378	$\hat{\sigma}$	2.6655	2.6571	NA	NA	2.9724
P_p	1.22	C_p	1.63	1.63			1.46
P_{pk}	0.90	C_{pk}	1.19	1.20			1.07
Z_{USL}	4.65	Z_{USL}	6.17	6.19			5.53
Z_{LSL}	2.70	Z_{LSL}	3.58	3.59			3.21
ppm	3474	ppm	170	170			657

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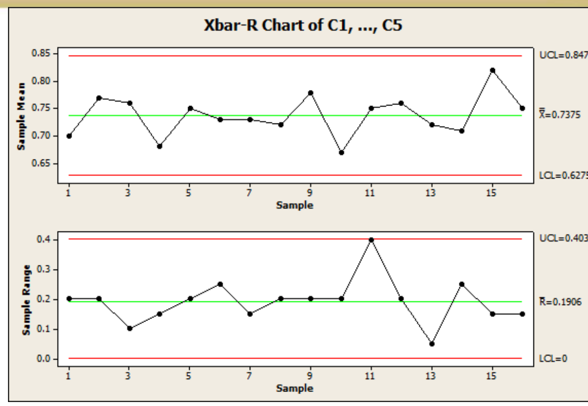
11.12 Example 11.2: Process Capability/Performance Indices Study

#						$\bar{x}$	R
1	0.65	0.70	0.65	0.65	0.85	0.70	0.20
2	0.75	0.85	0.75	0.85	0.65	0.77	0.20
3	0.75	0.80	0.80	0.70	0.75	0.76	0.10
4	0.60	0.70	0.70	0.75	0.65	0.68	0.15
5	0.70	0.75	0.65	0.85	0.80	0.75	0.20
6	0.60	0.75	0.75	0.85	0.70	0.73	0.25
7	0.75	0.80	0.65	0.75	0.70	0.73	0.15
8	0.60	0.70	0.80	0.75	0.75	0.72	0.20
9	0.65	0.80	0.85	0.85	0.75	0.78	0.20
10	0.60	0.70	0.60	0.80	0.65	0.67	0.20
11	0.80	0.75	0.90	0.50	0.80	0.75	0.40
12	0.85	0.75	0.85	0.65	0.70	0.76	0.20
13	0.70	0.70	0.75	0.75	0.70	0.72	0.05
14	0.65	0.70	0.85	0.75	0.60	0.71	0.25
15	0.90	0.80	0.80	0.75	0.85	0.82	0.15
16	0.75	0.80	0.75	0.80	0.65	0.75	0.15



LSL=0.500
USL=0.900

$\bar{\bar{x}} = 0.7375$
$\bar{R} = 0.1906$

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11.12 Example 11.2: Process Capability/Performance Indices Study

Method 2: Short-term Estimate of σ: Using $\bar{R}$

$$\hat{\sigma} = \frac{\bar{R}}{d_2} = \frac{0.1906}{2.326} = 0.0819$$

$$Z_{USL} = \frac{USL - \hat{\mu}}{\hat{\sigma}} = \frac{0.900 - 0.7375}{0.0819} = 1.9831$$

$$Z_{LSL} = \frac{\hat{\mu} - LSL}{\hat{\sigma}} = \frac{0.7375 - 0.500}{0.0819} = 2.8983$$

$$C_{pk} = \frac{Z_{min}}{3} = \frac{1.9831}{3} = 0.6610$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{0.900 - 0.500}{6(0.0819)} = 0.8136$$

$$ppm_{USL} = \Phi(Z_{USL}) \times 10^6 = \Phi(1.98) \times 10^6 = 23{,}679$$

$$ppm_{LSL} = \Phi(Z_{LSL}) \times 10^6 = \Phi(2.90) \times 10^6 = 1{,}876$$

$$ppm_{total} = ppm_{USL} + ppm_{LSL} = 23{,}679 + 1{,}876 = 25{,}555$$

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11.12 Example 11.2: Process Capability/Performance Indices Study

Method 1: Long-term Estimate of σ: Using Individual Data

$$\hat{\sigma} = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{\bar{x}})^2}{(n-1)}} = \sqrt{\sum_{i=1}^{80} \frac{(x_i - 0.7375)^2}{(80-1)}} = 0.0817$$

$$P_p = \frac{USL - LSL}{6\sigma} = \frac{0.900 - 0.500}{6(0.0817)} = 0.8159$$

$$P_{pk} = min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] = \min[0.6629, 0.9688] = 0.6629$$

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11.12 Example 11.2: Process Capability/Performance Indices Study

- Process capability and process performance metrics are noted to be almost identical.
- Calculation for ST variability were slightly larger which is not reasonable. Using $\bar{s}$, the $\hat{\sigma} = 0.0811$.

	Method 1 LT		Method 2 ST
	s		$\bar{R}/d_2$
$\hat{\sigma}$	0.0817	$\hat{\sigma}$	0.0819
P_p	0.8159	C_p	0.8136
P_{pk}	0.6629	C_{pk}	0.6610
Z_{USL}		Z_{USL}	1.98
Z_{LSL}		Z_{LSL}	2.90
ppm		ppm	25555

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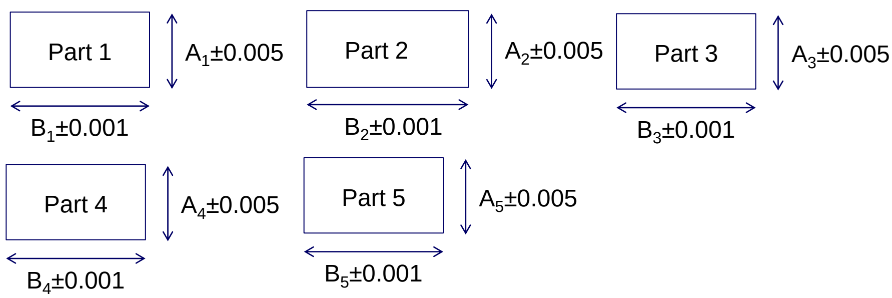
11.13 Example 11.3: Process Capability/Performance Index Needs

- Management has given the mandate that process steps are to have process indices of $C_p \geq 2.0$ and $C_{pk} \geq 1.5$.
- Consider the five parts are to be manufactured and then assembled.



11.13 Example 11.3: Process Capability/Performance Index Needs

- The tolerances of the dimensions on the parts were achievable using conventional manufacturing practices.
- Also consider C_p and C_{pk} are expressions of long-term capability (equating to P_p and P_{pk})
- Consider the five parts are to be manufactured and then assembled.

11.13 Example 11.3: Process Capability/Performance Index Needs

Q: For the purposes of achieving final measurements that meet customer needs, should each dimension be given equivalent monitoring effort relative to the process indices?

A: It may not be necessary.

- Assume that the 0.005 tolerance (A dimension) is easy to achieve consistently with the current process, only periodic measurements may be needed.
- The tolerance of B dimension may require a special operation. Assume that the B dimensions are in control/ predictable and the processes had a mean equal to the nominal specification ($C_p = C_{pk}$), but C_p values for these dimensions ranged from 1.0 to 1.33 for the 5 parts.

11.13 Example 11.3: Process Capability/Performance Index Needs

Q: Whether a new process should be developed in order to reduce the B-dimensional variability?

A: It may not be necessary.

- Assume that the important dimension from a customer's perspective is the overall dimension of B.

Part 1	Part 2	Part 3	Part 4	Part 5
B_1±0.001	B_2±0.001	B_3±0.001	B_4±0.001	B_5±0.001

B±0.005

- Assume that B dimensions are distributed normally, the mean value of each part equaled the nominal spec., and a 3σ limit equaled the tolerance of 0.001 ($C_p = C_{pk} = 1.0$)

11.13 Example 11.3: Process Capability/Performance Index Needs

- An overall tolerance (3σ limit) can be determined as:

$$B\ tolerance = \sqrt{(.001)^2 + (.001)^2 + (.001)^2 + (.001)^2 + (.001)^2}$$
$$= \sqrt{.000005} = .00224$$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{0.010}{2(.00224)} = 2.23$$

- Because $C_p \geq 2.0$, the process index target is met on the overall dimension.
- Care must be taken not to spend resources unwisely by striving for tightening the tolerances that may not benefit the customer.

11.14 Process Capability Confidence Interval

- The confidence interval for C_{pk} and P_{pk} is difficult to calculate directly.
- The 100(1−α)% confidence interval for C_p is

$$\hat{C}_p\sqrt{\frac{\chi_{1-\frac{\alpha}{2};n-1}^{2}}{n-1}} \leq C_p \leq \hat{C}_p\sqrt{\frac{\chi_{\frac{\alpha}{2};n-1}^{2}}{n-1}}$$

11.15 Example 11.4: Confidence Interval for Process Capability

- An organization wants to create a table that can be used to give a quick 95% confidence interval for a population C_p given the sample size and calculated C_p.
- Consider C_p value of 1.0 with a sample size (n) of 10,

$$\hat{C}_p\sqrt{\frac{\chi_{1-\frac{\alpha}{2};n-1}{}^2}{n-1}} \le C_p \le \hat{C}_p\sqrt{\frac{\chi_{\frac{\alpha}{2};n-1}{}^2}{n-1}}$$

$$\sqrt{\frac{\chi_{.975;9}{}^2}{10-1}} \le C_p \le \sqrt{\frac{\chi_{.025;9}{}^2}{10-1}}$$

$$0.55 \le C_p \le 1.45$$

11.15 Example 11.4: Confidence Interval for Process Capability

n	C_p Lower Confidence Multiple	C_p Upper Confidence Multiple
10	0.5478	1.4538
20	0.6847	1.3149
30	0.7439	1.2556
40	0.7788	1.2208
50	0.8025	1.1971
60	0.8199	1.1798
70	0.8334	1.1663
80	0.8443	1.1555
90	0.8532	1.1465
100	0.8608	1.1389

11.16 Process Capability/Performance for Attribute Data

S4/IEE Application Examples

- Transactional and manufacturing 30,000-foot-level metric: A company had a large number of transactions completed daily, where the number of daily transactions was similar. The number of defective recorded transactions were measured and reported. An XmR chart of the defective had no special causes. The capability/performance of the process was reported as the centerline of the chart.
- Transactional and manufacturing 30,000-foot-level metric: The number of daily transactions is approximately the same, but not exact. The number of defects in filling out invoices is unsatisfactory. An XmR chart of the defect rate had no special causes. The capability/performance of the process was reported as the centerline of the chart.

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11.16 Process Capability/Performance for Attribute Data

- For attribute charts, capability is defined as the average proportion or rate of nonconforming product. (AIAG, 1995)
- If desired, capability can be expressed as the proportion conforming to specification (i.e., $1 - \bar{p}$).
- For a preliminary estimate, use historical data, but exclude data points associated with special causes.
- For a formal process capability study, new data should be run, preferably for 25 or more periods, with the points all reflecting statistical control. The $\bar{p}$ for these consecutive in-control periods is a better estimate of the process's current capability.

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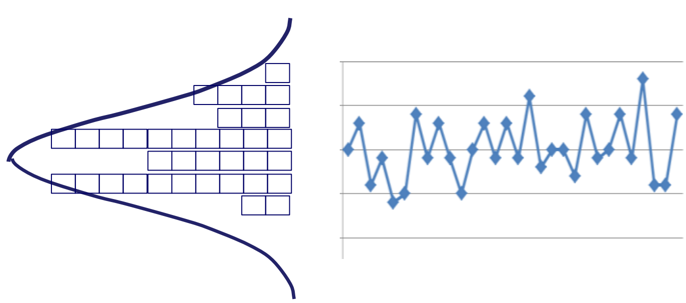
11.17 Describing a Predictable Process Output when No Specification Exists

- Specification requirements are needed to determine C_p, C_{pk}, P_p, and P_{pk}. Sometimes, the output of a process does not have a specification.
- Alternative: To describe the overall response as expected percentage of occurrences.
- The approach can be applied to non-normal distributions, such as Weibull for cycle time.

11.17 Describing a Predictable Process Output when No Specification Exists

Step 1

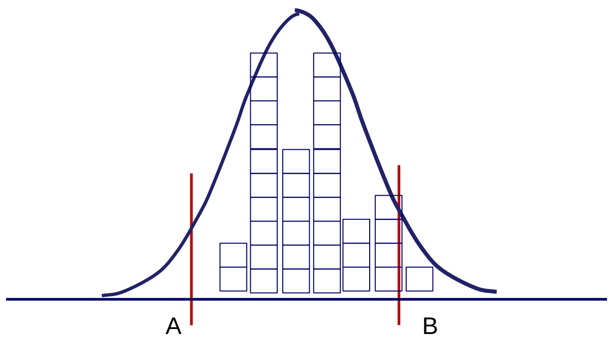
- Convert time-sequenced data (such as X chart) to a distribution of non-sequenced data.



11.17 Describing a Predictable Process Output when No Specification Exists

Step 2

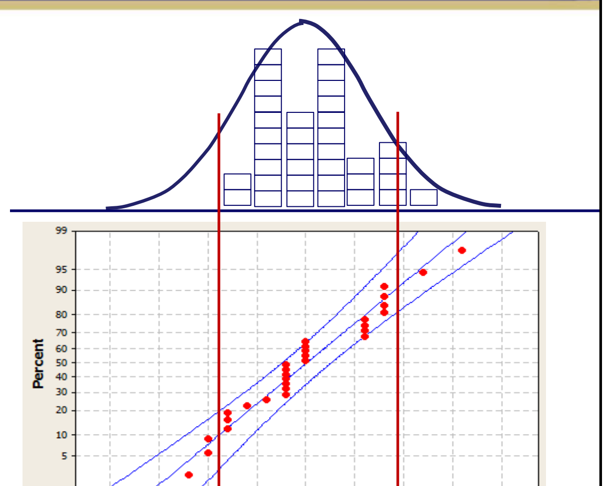
- Decide what percentage of occurrence are expected between the response levels.



11.17 Describing a Predictable Process Output when No Specification Exists

Step 3

- Determine the response levels using a probability plot.



11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

- Specification requirements are needed to determine C_p, C_{pk}, P_p, and P_{pk}. Sometimes, the output of a process does not have a specification.
- Alternative: To describe the overall response as expected percentage of occurrences.
- The approach can be applied to non-normal distributions, such as Weibull for cycle time.

11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

S^4/IEE Application Examples

- Satellite-level metric: The last three years' ROI for a company was reported monthly in a control chart. No special causes or trends were identified. Monthly ROIs were plotted on a normal probability plot, where a null hypothesis for normality was not rejected. The capability/performance of the system was reported on the probability plot as a best-estimate 80% frequency of occurrence interval, which described common-cause variability. Goals were set to improve this metric. A strategic plan was created that was in alignment with the goal to improve this metric. The 30,000-foot-level operational metrics were then chosen that would be the focus of improvement efforts. S^4/IEE projects were then chosen to improve these metrics.

11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

- Transactional 30,000-foot-level metric: One random paid invoice was selected each day from last year's invoices where the number of days beyond the due date was measured and reported (i.e., days sales outstanding [DSO]). The DSO for each sample was reported in an XmR control chart, where no reason was identified for a couple of special-cause data points. These data were plotted on a normal probability plot, where a null hypothesis for normality was rejected. A lognormal plot fit the data well. An XmR chart of the lognormal data did not indicate any special-cause conditions. A lognormal probability plot was used to describe the capability/ performance of the process as a best-estimate 80% frequency of occurrence interval, which described common-cause variability.

11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

- Transactional and manufacturing 30,000-foot-level cycle-time metric (a lean metric): One transaction was randomly selected each day over the last year, where the time from order entry to fulfillment was measured. The differences between these times relative to their due date were reported in an XmR chart. No special causes were identified. A null hypothesis for normality could not be rejected. The long-term process capability/performance metric was reported as the best-estimate 80% interval, which described the common-cause variability of the system.
- Transactional and manufacturing 30,000-foot-level inventory metric or satellite-level TOC metric (a lean metric): Inventory was tracked monthly using a control chart. No special causes were identified. A null hypothesis for normality could not be rejected. The long-term process capability/performance nonconformance rate was reported as the best-estimate 80% frequency of occurrence interval for month-to-month inventory levels, along with associated monetary implications.

11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

- Transactional 50-foot-level metric (KPIV): An S^4/IEE project to improve the 30,000-foot-level metrics for DSOs identified a KPIV to the process, the importance of timely calling customers to ensure that they received a company's invoice. A control chart tracked the time from invoicing to when the call was made, where one invoice was selected hourly. No special cause was identified. A hypothesis for normality could not be rejected. The long-term process capability/performance metric was reported as the best-estimate 80% interval, which described common-cause variability.

11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

- Product DFSS: An S^4/IEE product DFSS project was to reduce the 30,000-foot-level MTBF (mean time between failures) of a product by its vintage. A control chart tracked the product MTBF by product vintage. The capability/ performance of the system was reported on the probability plot as a best-estimate 80% interval, which described common-cause variability and what might be expected for MTBF rates in the future unless something were done differently to change the design process. Categories of problems were tracked over the long haul in a Pareto chart to identify improvement opportunities for newly developed products.
- S^4/IEE infrastructure 30,000-foot-level metric: A steering committee uses a control chart to track the duration of projects. The capability/ performance of the system was reported on the probability plot as a best-estimate 80% interval.

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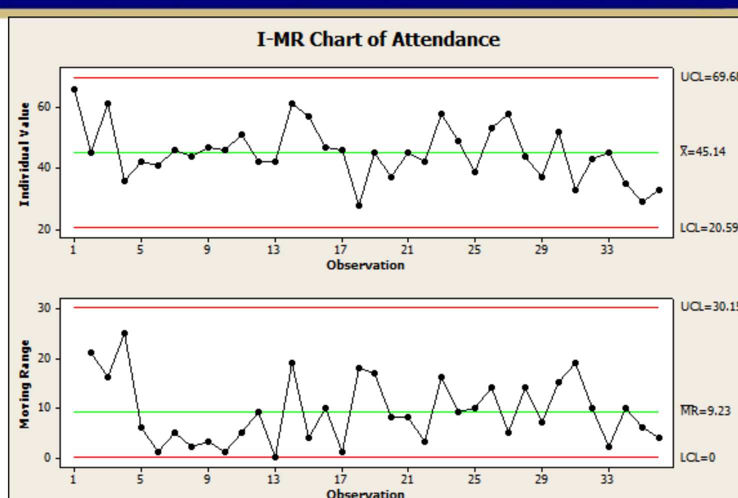
11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

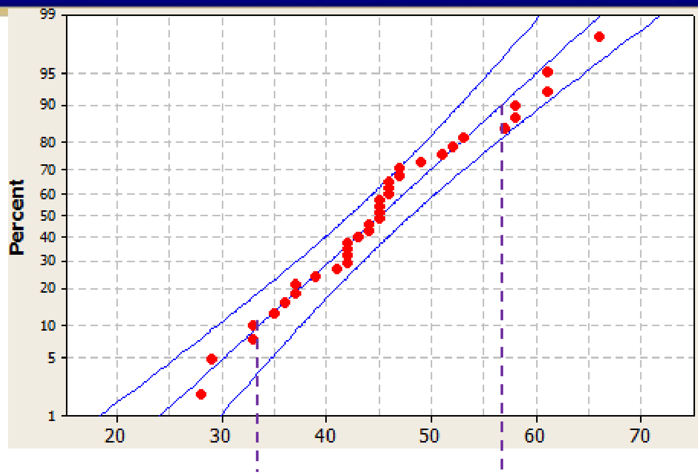
- Attendance at monthly professional society meetings was recorded for the past 3 years.
- An XmR chart indicates that the process is stable.
- A process capability/performance 80% frequency of occurrence metric could be used to describe the expected response variability from the process.
- An estimation that 80% of the time attendance would be between 34 and 57.

11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists

66	46	45	44
45	51	37	37
61	42	45	52
36	42	42	33
42	61	58	43
41	57	49	45
46	47	39	35
44	46	53	29
47	28	58	33



11.18 Example 11.5: Describing a Predictable Process Output when No Specification Exists



11.19 Process Capability/Performance Metrics from XmR Chart of Subgroup Means and Standard Deviation

S^4/EEE Application Examples

- Transactional 30,000-foot-level metric: The mean and standard deviation of all DSOs were tracked using two XmR charts with a weekly subgrouping, where the standard deviation values had a log transformation. No special causes were identified. The long-term capability/ performance of the process was reported as percentage nonconformance beyond 30, 60, and/or 90 days, using variance of components techniques or a statistical program that reports this metric under their $\bar{x}$ and s process capability option.

11.19 Process Capability/Performance Metrics from XmR Chart of Subgroup Means and Standard Deviation

- For infrequent subgrouping/sampling, use XmR control charts to track mean and standard deviation.
- Report an estimated nonconformance percentage or ppm rate that addresses the underlying distribution shape for the population, rather than reporting C_p, C_{pk}, P_p, and P_{pk}.
- Use variance components techniques or computer program that combines within and between variability from the original data set when determining an overall long-term ppm.

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11.20 Process Capability/Performance Metrics for Nonnormal Distribution

S4/IEE Application Examples

- Transactional 30,000-foot-level metric: One random paid invoice was selected each day from last year's invoices where the number of days beyond the due date was measured and reported (i.e., days sales outstanding [DSO]). The DSO for each sample was reported in an XmR control chart, where no reason was identified for a couple special-cause data points. These data were plotted on a normal probability plot, where a null hypothesis for normality was rejected. A lognormal plot fit the data well. An XmR chart of the lognormal data did not indicate any special-cause conditions. The lognormal probability plot was used to estimate the proportion of invoices beyond 30, 60, and 90 days. An S4/IEE project was initiated to improve the DSO metric.

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11.20 Process Capability/Performance Metrics for Nonnormal Distribution

- Transactional 30,000-foot-level metric: The mean and standard deviation of all DSOs were tracked using two XmR charts with a weekly subgrouping, where the standard deviation values had a log transformation. No special causes were identified. The long-term capability/performance of the process was reported as percentage nonconformance beyond 30, 60, and/or 90 days, using variance of components techniques or a statistical program that reports this metric under their $\bar{x}$ and s process capability option.

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11.20 Process Capability/Performance Metrics for Nonnormal Distribution

- The capability/performance measures described above are for normal distribution.
- Nonnormality is common for measurements such as flatness, roundness, and particle contamination.
- One approach to address nonnormal data is to normalize the data using a transformation.
- A transformation can be appropriated if the nonnormal distribution is known.
- There could be problems if it has to be estimated from preliminary data.

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11.20 Process Capability/Performance Metrics for Nonnormal Distribution

- A general approach for transforming data is a Box-Cox transformation (Box, 1977), where values (Y) are transformed to the power of λ (i.e., Y^{λ}).
  - $\lambda = -2$ Y transformed = $1/Y^2$
  - $\lambda = -0.5$ Y transformed = $1/\sqrt{Y}$
  - $\lambda = 0$ Y transformed = $\ln(Y)$
  - $\lambda = 0.5$ Y transformed = $\sqrt{Y}$
  - $\lambda = 2$ Y transformed = Y^2
- Maximum likelihood value for λ is when the residual sum of squares from fitted model is minimized.

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11.20 Process Capability/Performance Metrics for Nonnormal Distribution

- An alternative approach when data can be represented by a probability plot is to use the .135 and 99.865 percentile from the plot to describe the spread of the data.
- This method is applicable when all individual measurements are combined from an in-control/predictable process to determine a long-term capability/performance.
- The capability/performance indices from this procedure are termed “equivalent indices” because they use the equivalent percentile points from the normal distribution.

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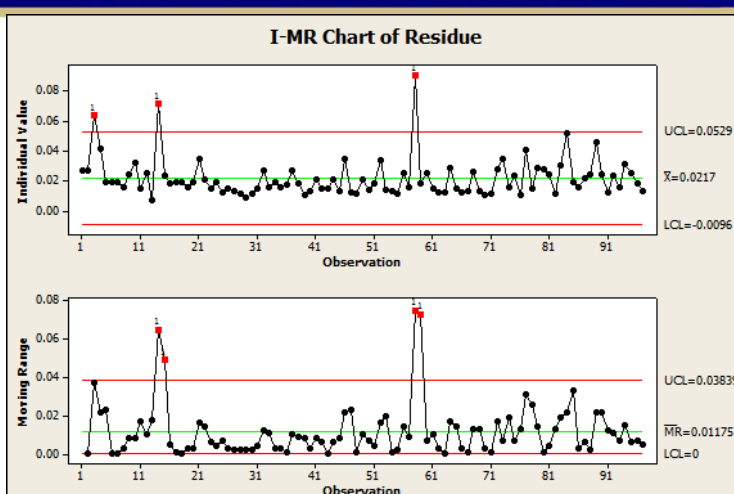
11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

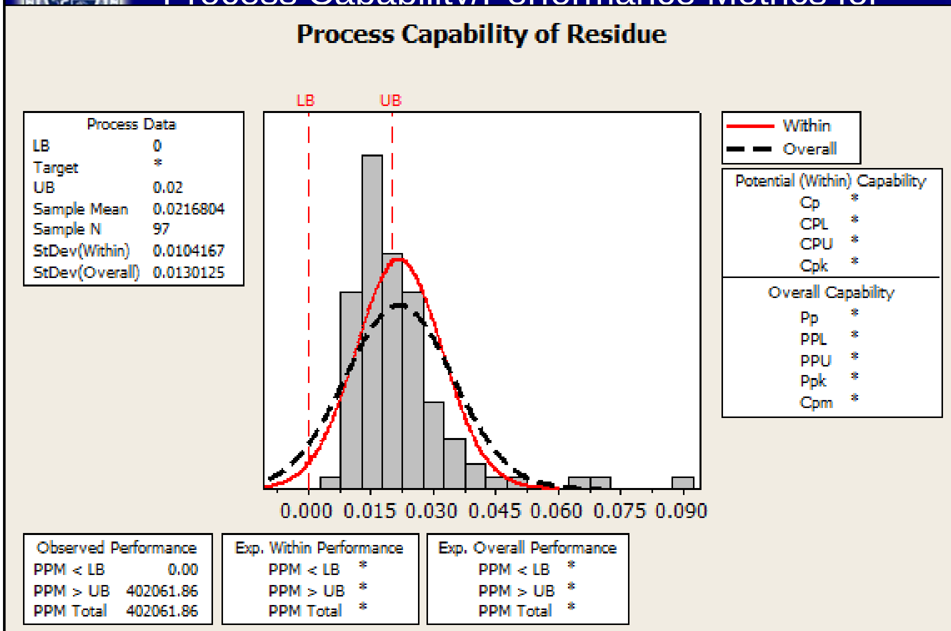
- A chemical process has residue with a specification upper limit of 0.02. There is no lower limit; but a natural bound 0.

11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

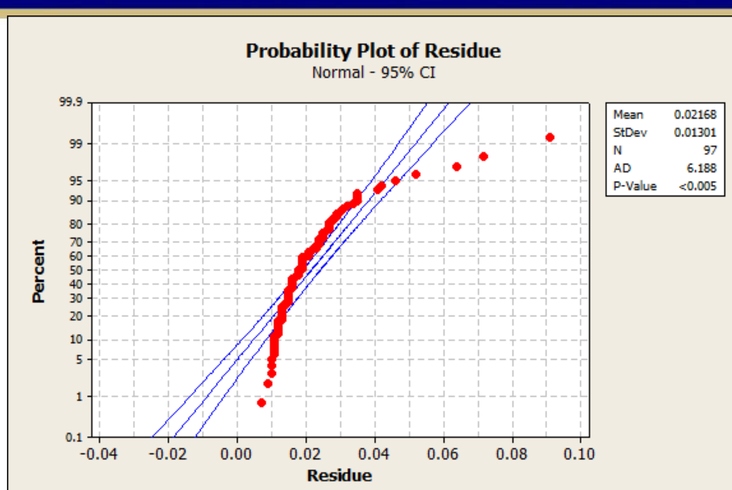
- XmR chart w/o transformation
- No reason for the special-cause points.



11.21 Example 11.6: Process Capability/Performance Metrics for



11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation



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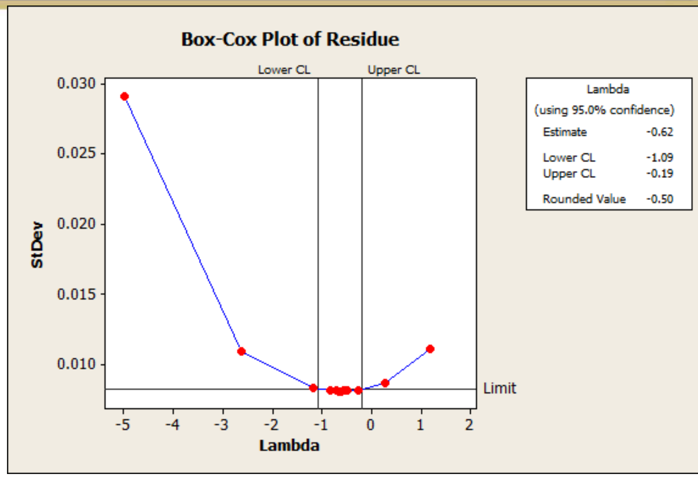
11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

- Box-Cox plot.
- The best estimate for λ is -0.62 (book -0.674)



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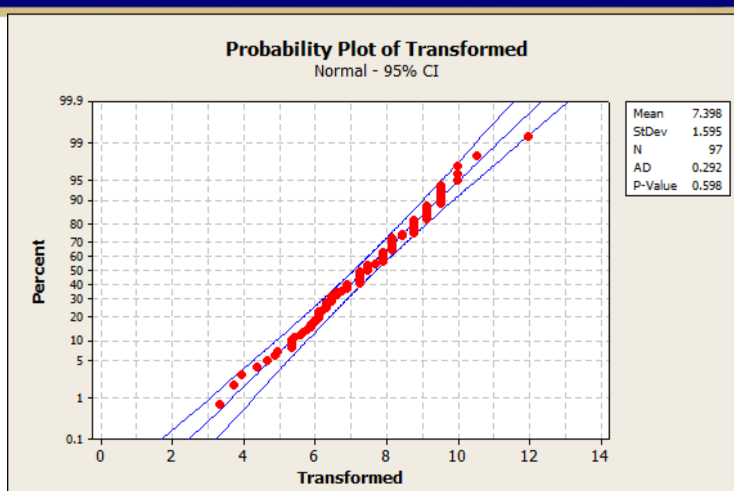
11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

- Normal probability plot of the transformed residue data, Box-Cox λ is -0.62



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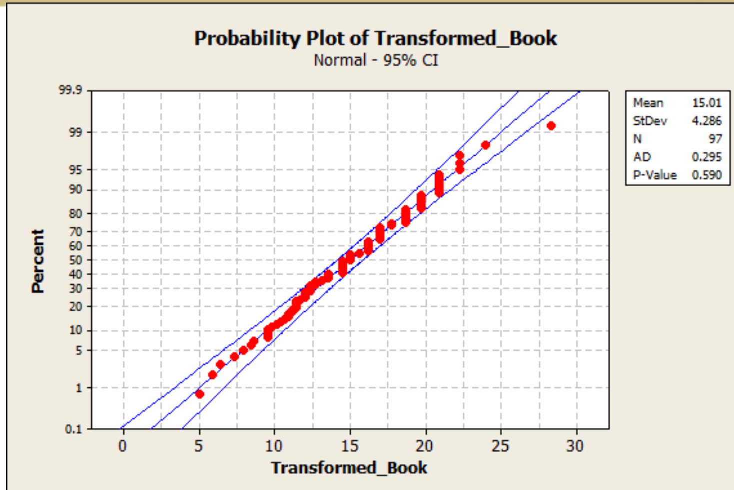
11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

- Normal probability plot of the transformed residue data, Box-Cox λ is -0.674



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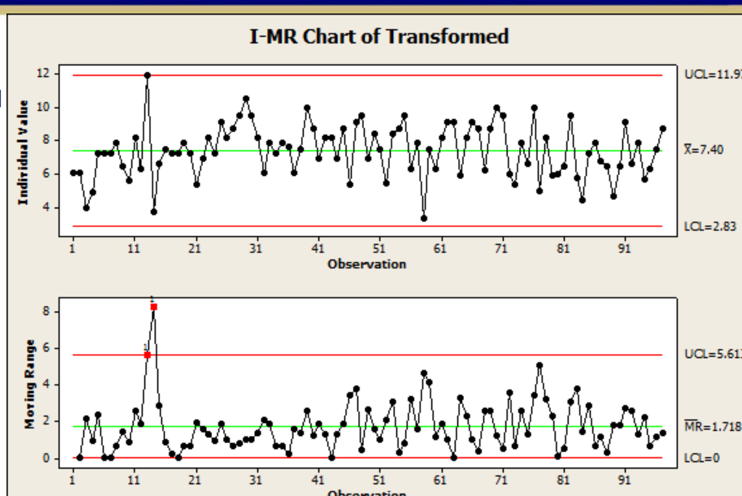
11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

- XmR charts of the transformed residue data, Box-Cox λ is -0.62



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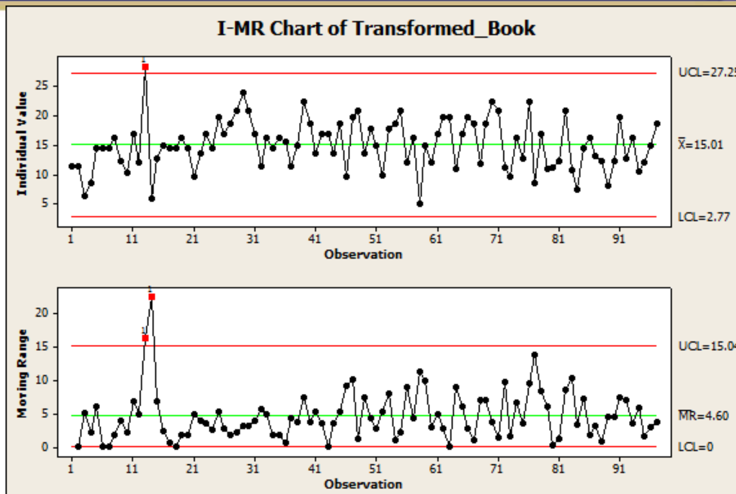
11.21 Example 11.6: Process Capability/Performance Metrics for Nonnormal Distribution: Box-Cox Transformation

- XmR charts of the transformed residue data, Box-Cox λ is -0.674



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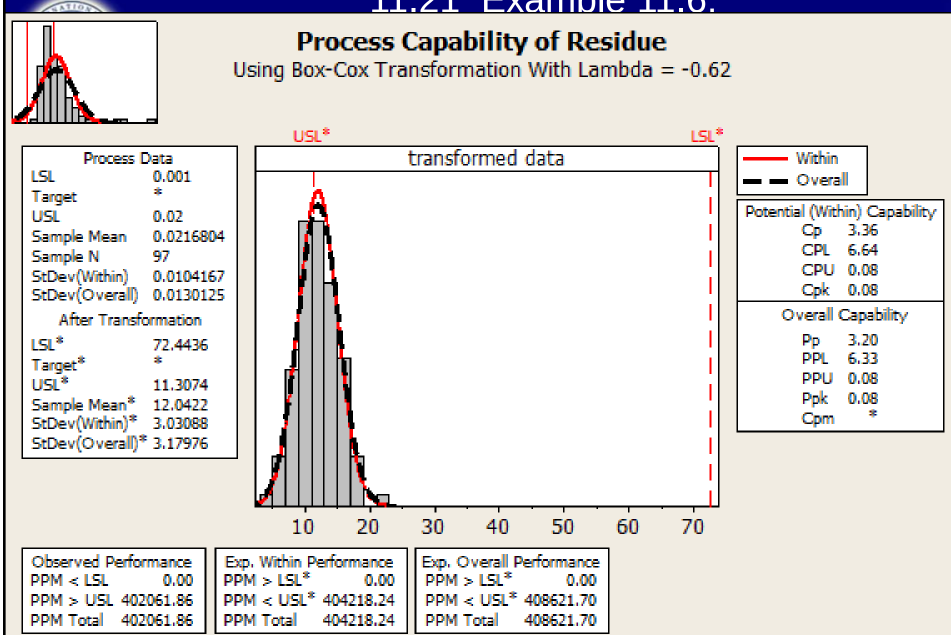
11.21 Example 11.6:



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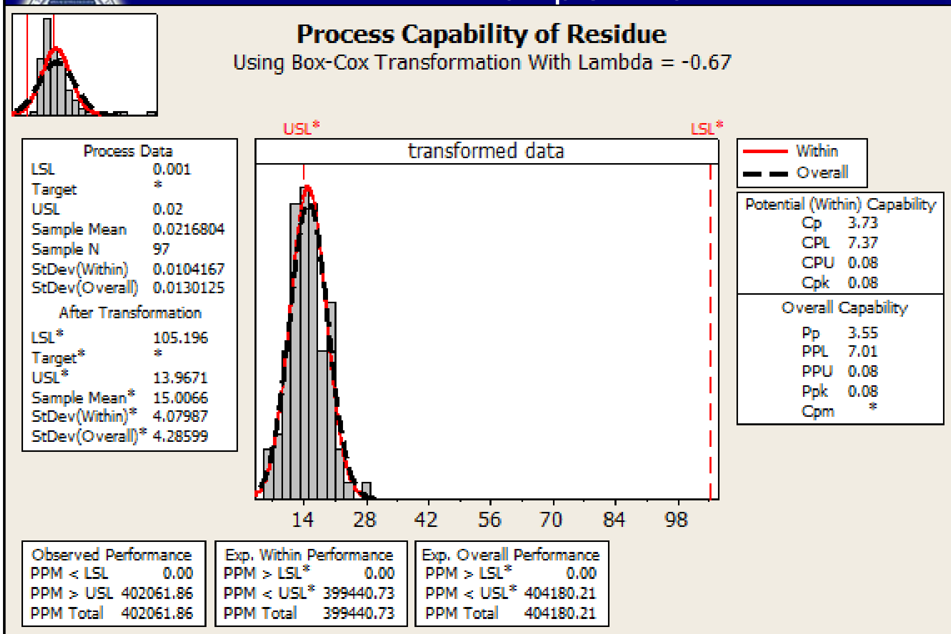
11.21 Example 11.6:



11.22 Implementation Comments

- "The key to effective use of any process measure continues to be the level of understanding of what the measure truly represents."
- Graphical analysis should be used in conjunction with process measures.
- All capability/performance assessments should be confined to a single process characteristics.

11.6 Process Capability Indices: C_p and C_{pk}