



Inventory Management V

Finite Planning Horizon

Lecture 9
ESD.260 Fall 2003

Caplice



Assumptions: Basic FPH Model

- ◆ Demand
 - **Constant** vs Variable
 - **Known** vs Random
 - Continuous vs **Discrete**
- ◆ Lead time
 - **Instantaneous**
 - Constant or Variable (deterministic/stochastic)
- ◆ Dependence of items
 - **Independent**
 - Correlated
 - Indentured
- ◆ Review Time
 - Continuous vs Periodic
- ◆ Number of Echelons
 - **One** vs Many
- ◆ Capacity / Resources
 - **Unlimited** vs Limited
- ◆ Discounts
 - **None**
 - All Units or Incremental
- ◆ Excess Demand
 - **None**
 - All orders are backordered
 - Lost orders
 - Substitution
- ◆ Perishability
 - **None**
 - Uniform with time
- ◆ Planning Horizon
 - Single Period
 - **Finite Period**
 - Infinite
- ◆ Number of Items
 - **One**
 - Many

Example

When should I order and for how much?

Costs

$D = 2000$ items per year

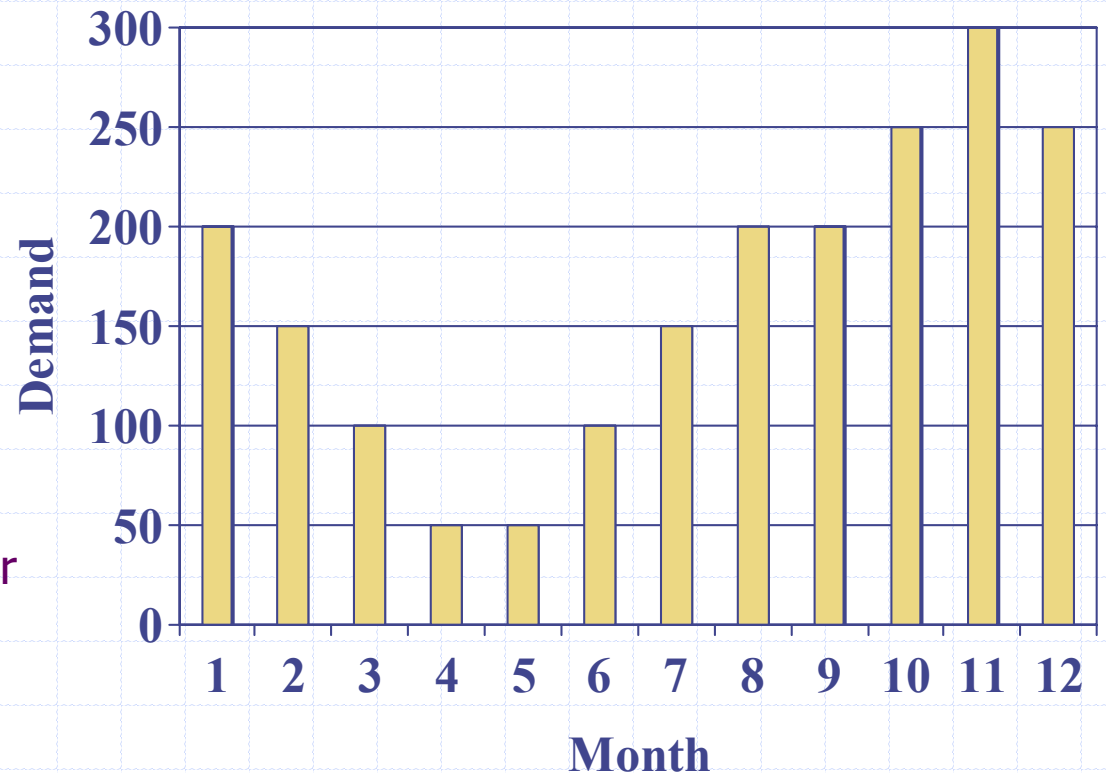
$C_o = \$500.00$ per order

$C_p = \$50.00$ per item

$Ch = 24\%$ per item per year

$Ch_p = (Ch C_p) / N$
= \$1 per month per item

$N =$ number of periods per year



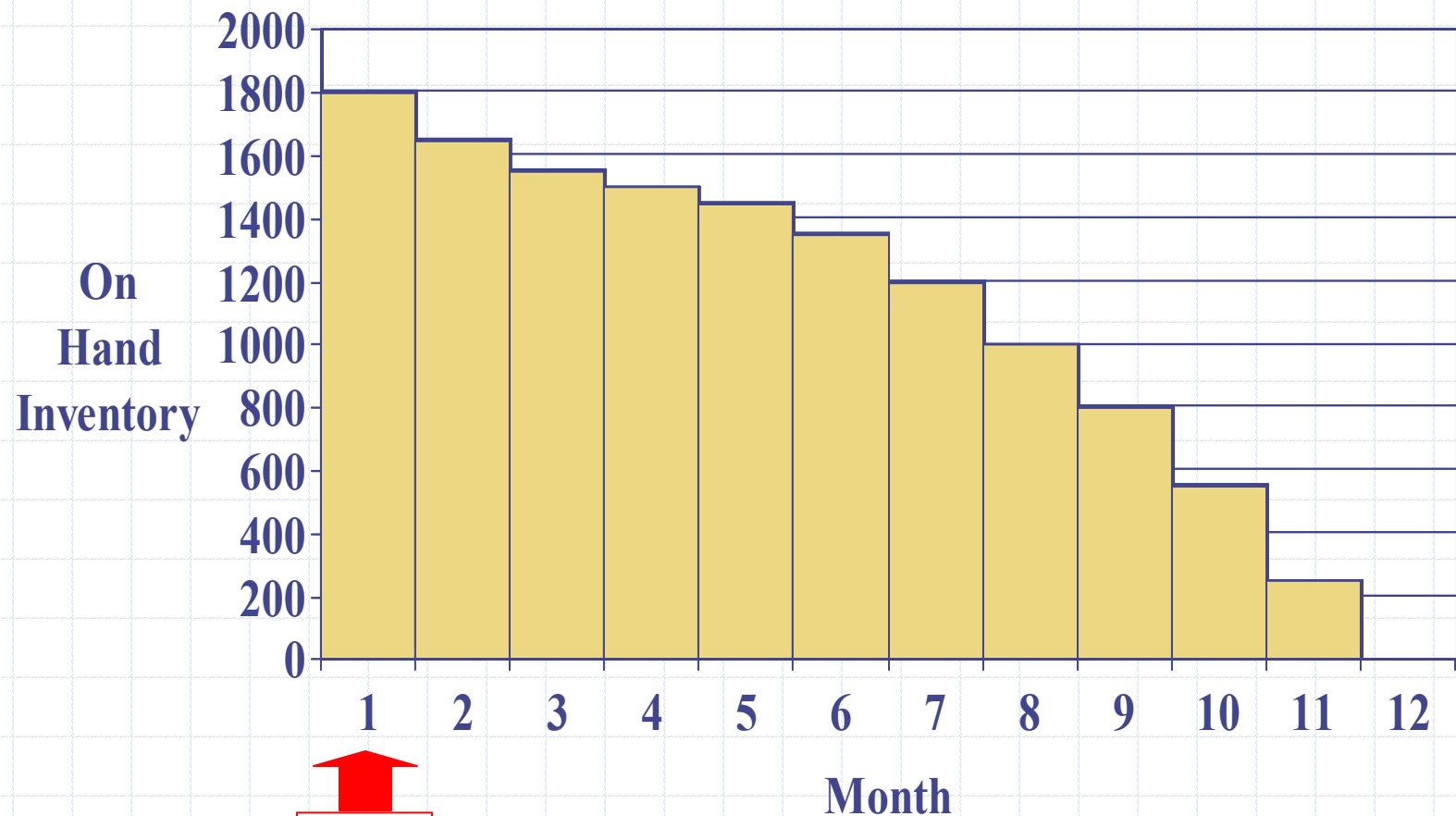
More Assumptions

- Demand is required and consumed on first day of the period
- Holding costs are not charged on items used in that period
- Holding costs are charged for inventory ordered in advance of need

Five Basic Approaches

1. The One-Time Buy
2. Lot For Lot
3. Simple EOQ
4. The Silver Meal Algorithm
5. Optimal Procedures
 - ◆ Wagner-Whitin (Dynamic Programming)
 - ◆ Mixed Integer Programming

Approach: One-Time Buy

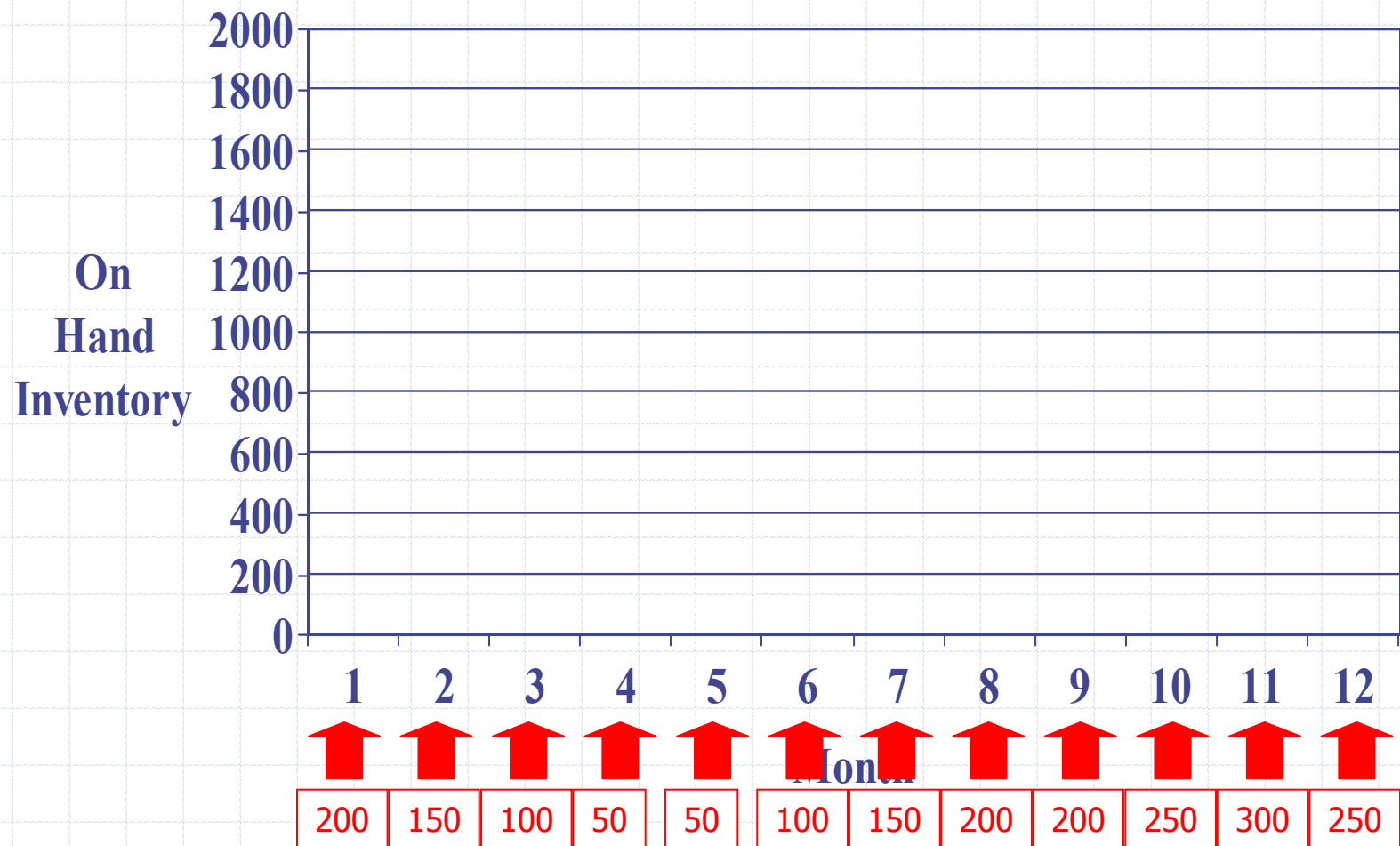


2000

Approach: One-Time Buy

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	2000	\$1800	\$500	\$2300
2	150	0	\$1650	\$0	\$1650
3	100	0	\$1550	\$0	\$1550
4	50	0	\$1500	\$0	\$1500
5	50	0	\$1450	\$0	\$1450
6	100	0	\$1300	\$0	\$1300
7	150	0	\$1200	\$0	\$1200
8	200	0	\$1000	\$0	\$1000
9	200	0	\$800	\$0	\$800
10	250	0	\$550	\$0	\$550
11	300	0	\$250	\$0	\$250
12	250	0	\$0	\$0	\$0
Totals:	2000	2000	\$13100	\$500	\$13600

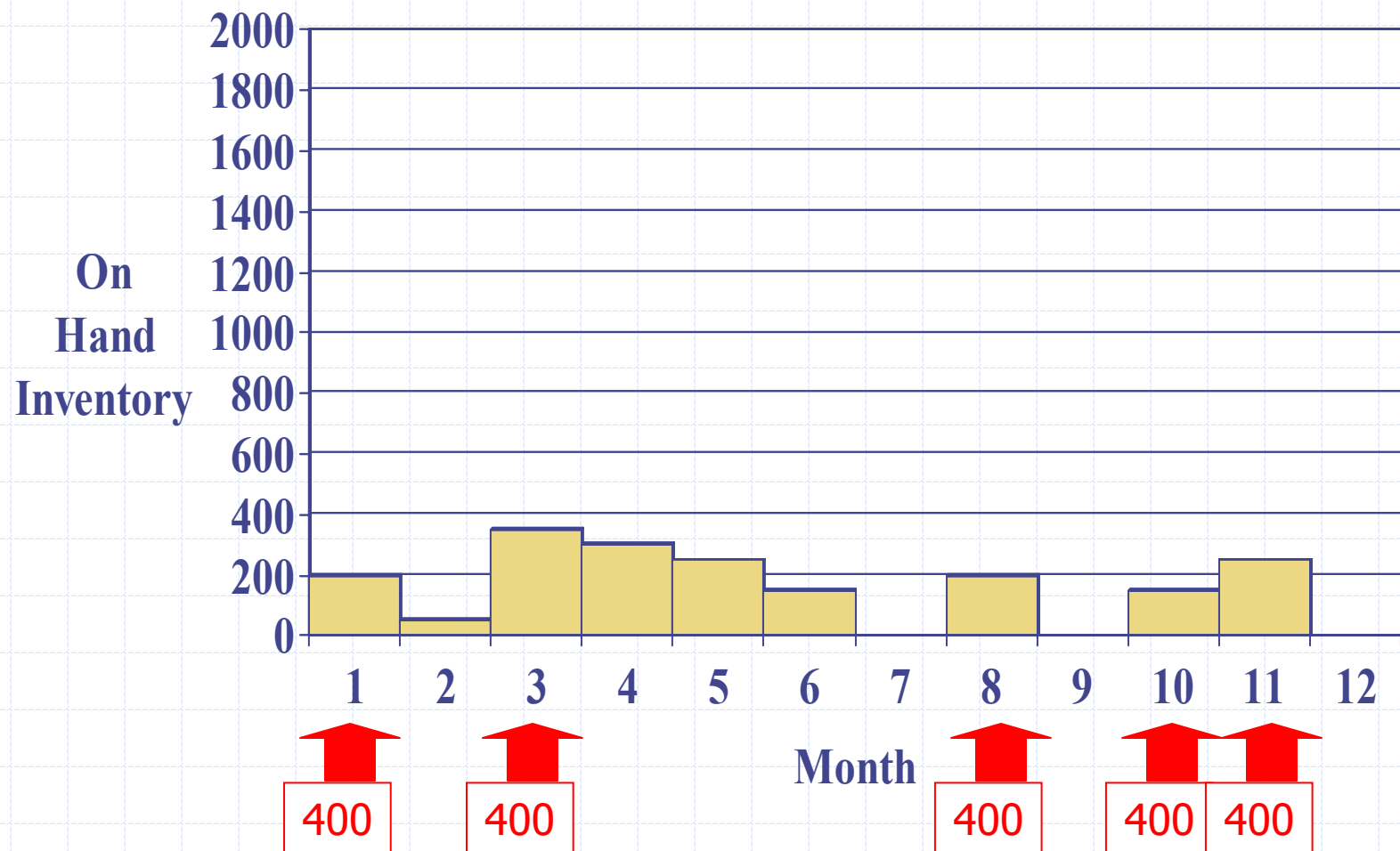
Approach: Lot for Lot



Approach: Lot for Lot

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	200	\$0	\$500	\$500
2	150	150	\$0	\$500	\$500
3	100	100	\$0	\$500	\$500
4	50	50	\$0	\$500	\$500
5	50	50	\$0	\$500	\$500
6	100	100	\$0	\$500	\$500
7	150	150	\$0	\$500	\$500
8	200	200	\$0	\$500	\$500
9	200	200	\$0	\$500	\$500
10	250	250	\$0	\$500	\$500
11	300	300	\$0	\$500	\$500
12	250	250	\$0	\$500	\$500
Totals:	2000	2000	\$0	\$6000	\$6000

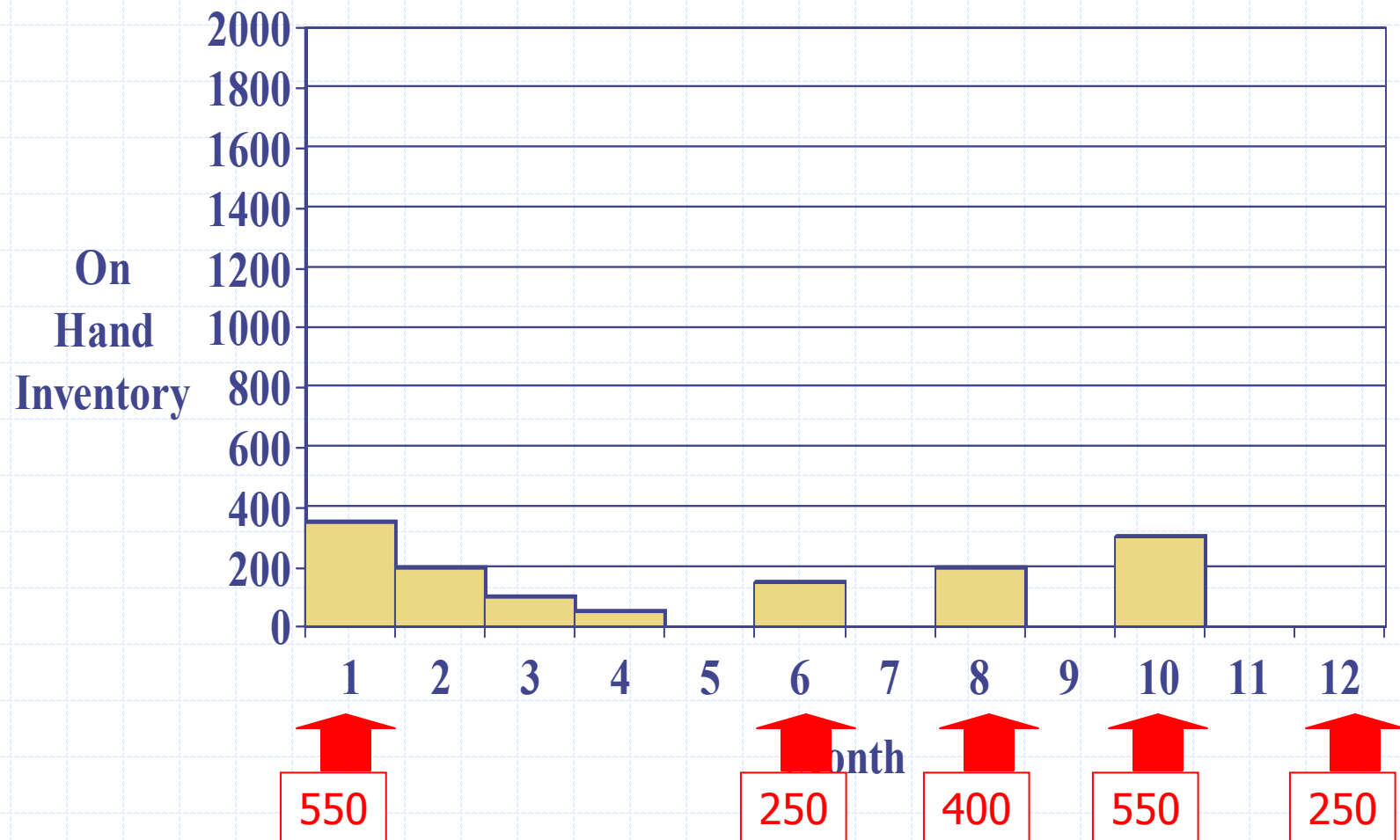
Approach: EOQ



Approach: EOQ

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	400	\$200	\$500	\$700
2	150	0	\$50	\$0	\$50
3	100	400	\$350	\$500	\$850
4	50	0	\$300	\$0	\$300
5	50	0	\$250	\$0	\$250
6	100	0	\$150	\$0	\$150
7	150	0	\$0	\$0	\$0
8	200	400	\$200	\$500	\$700
9	200	0	\$0	\$0	\$0
10	250	400	\$150	\$500	\$650
11	300	400	\$250	\$500	\$750
12	250	0	\$0	\$0	\$0
Totals:	2000	2000	\$1900	\$2500	\$4400

Approach: Silver-Meal Algorithm



Approach: Silver-Meal Algorithm

Mon	Dmd	Lot Qty	Order Cost	Holding Cost	Lot Cost	Mean Cost
1st	Buy:					
1	200	200	\$500	\$0	\$500	\$500
2	150	350	\$500	\$150	\$650	\$325
3	100	450	\$500	\$150+\$200	\$850	\$283
4	50	500	\$500	\$150+\$200+\$150	\$1000	\$250
5	50	550	\$500	\$150+\$200+\$150+\$200	\$1200	\$240
6	100	650	\$500	\$150+\$200+\$150+\$200+\$500	\$1700	\$283
2nd	Buy:					
6	100	100	\$500	\$0	\$500	\$500
7	150	250	\$500	\$150	\$650	\$325
8	200	450	\$500	\$150+\$400	\$1050	\$350

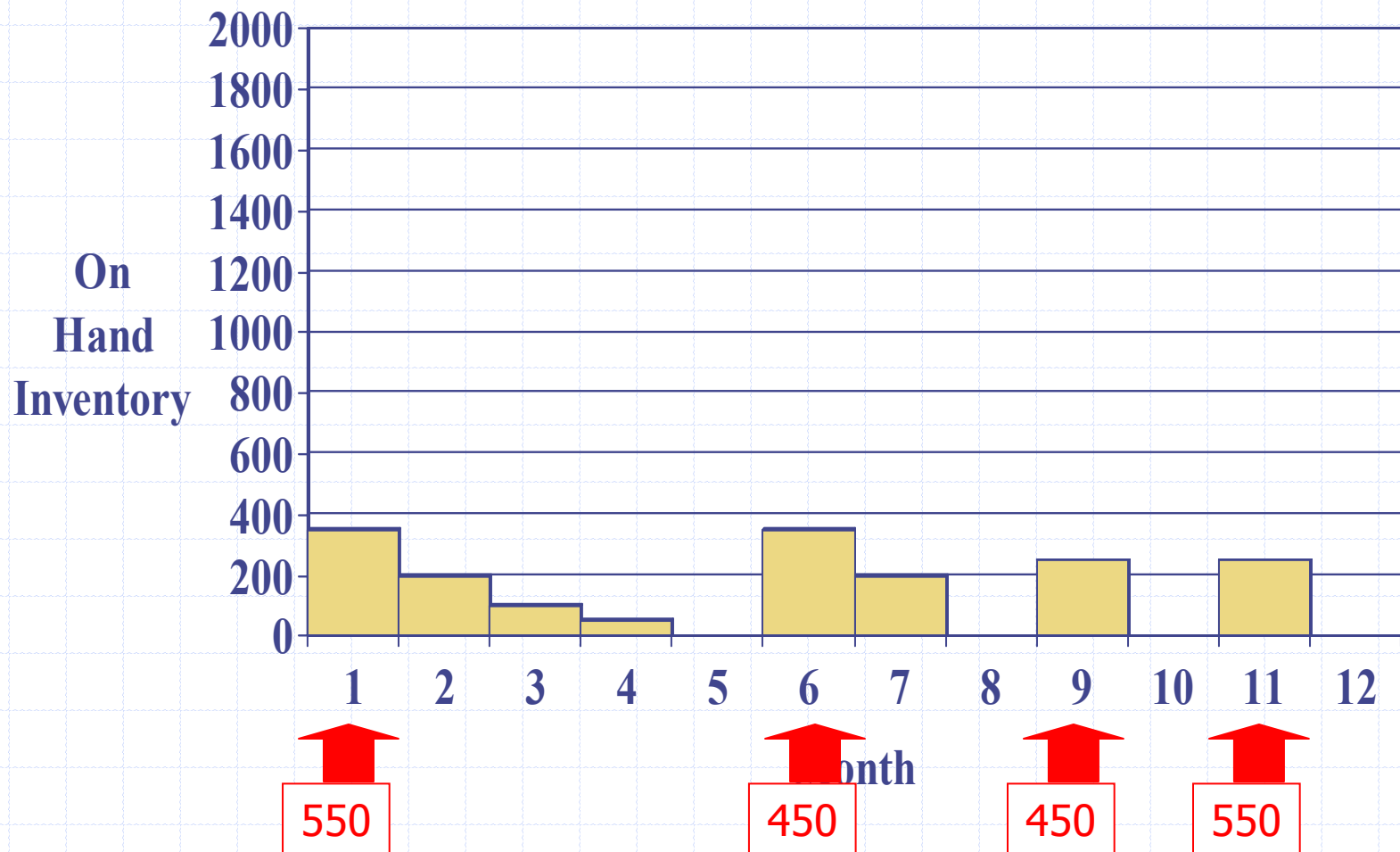
Approach: Silver-Meal Algorithm

Mon	Dmd	Lot Qty	Order Cost	Holding Cost	Lot Cost	Mean Cost
3rd	Buy:					
8	200	200	\$500	\$0	\$500	\$500
9	200	400	\$500	\$200	\$700	\$350
10	250	650	\$500	\$200+\$500	\$1200	\$400
4th	Buy:					
10	250	250	\$500	\$0	\$500	\$500
11	300	550	\$500	\$300	\$800	\$400
12	250	800	\$500	\$300+\$500	\$1300	\$433
5th	Buy:					
12	250	250	\$500	\$0	\$500	\$500

Approach: Silver-Meal Algorithm

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	550	\$350	\$500	\$850
2	150	0	\$200	\$0	\$200
3	100	0	\$100	\$0	\$100
4	50	0	\$50	\$0	\$50
5	50	0	\$0	\$0	\$0
6	100	250	\$150	\$500	\$650
7	150	0	\$0	\$0	\$0
8	200	400	\$200	\$500	\$700
9	200	0	\$0	\$0	\$0
10	250	550	\$300	\$500	\$800
11	300	0	\$0	\$0	\$0
12	250	250	\$0	\$500	\$500
Totals:	2000	2000	\$1350	\$2500	\$3850

Approach: Optimization (MILP)



Approach: Optimization (MILP)

Decision Variables:

Q_i = Quantity purchased in period i
 Z_i = Buy variable = 1 if $Q_i > 0$, = 0 o.w.
 B_i = Beginning inventory for period i
 E_i = Ending inventory for period i

Data:

D_i = Demand per period, $i = 1, \dots, n$
 C_o = Ordering Cost
 C_{hp} = Cost to Hold, \$/unit/period
 M = a very large number....

MILP Model

Objective Function:

- Minimize total relevant costs

Subject To:

- Beginning inventory for period 1 = 0
- Beginning and ending inventories must match
- Conservation of inventory within each period
- Nonnegativity for Q , B , E
- Binary for Z

Approach: Optimization (MILP)

$$\text{Min } TC = \sum_{i=1}^n C_O Z_i + \sum_{i=1}^n C_{HP} E_i$$

Objective Function

s.t.

$$B_1 = 0$$

$$B_i - E_{i-1} = 0 \quad \forall i = 2, 3, \dots, n$$

$$E_i - B_i - Q_i = D_i \quad \forall i = 1, 2, \dots, n$$

$$M Z_i - Q_i \geq 0 \quad \forall i = 1, 2, \dots, n$$

$$B_i \geq 0 \quad \forall i = 1, 2, \dots, n$$

$$E_i \geq 0 \quad \forall i = 1, 2, \dots, n$$

$$Q_i \geq 0 \quad \forall i = 1, 2, \dots, n$$

$$Z_i = \{0, 1\} \quad \forall i = 1, 2, \dots, n$$

Beginning & Ending
Inventory Constraints

Conservation of
Inventory Constraints

Ensures buys occur
only if $Q > 0$

Non-Negativity &
Binary Constraints

Approach: Optimization (MILP)

Microsoft Excel - fhip.xls

File Edit View Insert Format Tools Data Window WBI Help

Arial 10 B I U \$ % , +.00 -.00

A1 =

	A	B	C	D	E	F	G	H	I
1									
2	FPHIP								
3		Chp=	\$ 1.00	Total Cost=	\$ 13,600.00				
4		Co=	\$ 500.00				"Big M"=	100000	
5									
6		Period	Demand	"Zi"	Lot Size	Hold Cost	Order Cost	"MZi"	
7		1	200	1	2000	\$ 1,800.00	\$ 500.00	100000	
8		2	150	0	0	\$ 1,650.00	\$ -	0	
9		3	100	0	0	\$ 1,550.00	\$ -	0	
10		4	50	0	0	\$ 1,500.00	\$ -	0	
11		5	50	0	0	\$ 1,450.00	\$ -	0	
12		6	100	0	0	\$ 1,350.00	\$ -	0	
13		7	150	0	0	\$ 1,200.00	\$ -	0	
14		8	200	0	0	\$ 1,000.00	\$ -	0	
15		9	200	0	0	\$ 800.00	\$ -	0	
16		10	250	0	0	\$ 550.00	\$ -	0	
17		11	300	0	0	\$ 250.00	\$ -	0	
18		12	250	0	0	\$ -	\$ -	0	
19			2000		2000	\$ 13,100.00	\$ 500.0		
20									
21									

Sheet1 Sheet2 Sheet3

Ready

Comparison of Approaches

Month	Demand	OTB	L4L	EOQ	S/M	OPT
1	200	2000	200	400	550	550
2	150		150			
3	100		100	400		
4	50		50			
5	50		50			
6	100		100		250	450
7	150		150			
8	200		200	400	400	
9	200		200			450
10	250		250	400	550	
11	300		300	400		550
12	250		250		250	
Total Cost		\$13,600	\$6,000	\$4,400	\$3,850	\$3,750