Review

ESD.260 Fall 2003

Demand Forecasting

Accuracy and Bias Measures

1. Forecast Error: $e_t = D_t - F_t$
2. Mean Deviation: $$MD = \frac{\sum_{t=1}^{n} e_t}{n}$$
3. Mean Absolute Deviation $$MAD = \frac{\sum_{t=1}^{n} |e_t|}{n}$$
4. Mean Squared Error: $$MSE = \frac{\sum_{t=1}^{n} e_t^2}{n}$$
5. Root Mean Squared Error: $$RMSE = \sqrt{\frac{\sum_{t=1}^{n} e_t^2}{n}}$$
6. Mean Percent Error: $$MPE = \frac{\sum_{t=1}^{n} \frac{e_t}{D_t}}{n}$$
7. Mean Absolute Percent Error: $$MAPE = \frac{\sum_{t=1}^{n} \frac{|e_t|}{D_t}}{n}$$

MD – cancels out the over and under – good measure of bias not accuracy

MAD – fixes the cancelling out, but statistical properties are not suited to probability based dss

MSE – fixes cancelling out, equivalent to variance of forecast errors, HEAVILY USED statistically appropriate measure of forecast errors

RMSE – easier to interpret (proportionate in large data sets to MAD) MAD/RMSE = SQRT(2/pi) for e~N

Relative metrics are weighted by the actual demand

MPE – shows relative bias of forecasts

MAPE – shows relative accuracy

Optimal is when the MSE of forecasts -> Var(e) – thus the forecsts explain all but the noise.

What is good in practice (hard to say) MAPE 10% to 15% is excellent, MAPE 20%-30% is average CLASS?

The Cumulative Mean

Generating Process:

$$D_t = L + n_t$$

where: $n_t \sim iid(\mu = 0, \sigma^2 = V[n])$

Forecasting Model:

$$F_{t+1} = (D_1 + D_2 + D_3 + \ldots + D_t) / t$$

Stationary model – mean does not change – pattern is a constant

Not used in practice – is anything constant?

Thought though is to use as large a sample siDe as possible to

The Naïve Forecast

Generating Process:

$$D_t = D_{t-1} + n_t$$

where: $n_t \sim iid(\mu = 0, \sigma^2 = V[n])$

Forecasting Model:

$$F_{t+1} = D_t$$

The Moving Average

Generating Process:

$$D_t = L + n_t \quad ; \; t < t_s$$

$$D_t = L + S + n_t \quad ; \; t \geq t_s$$

where: $n_t \sim \text{iid}\,(\mu = 0 \,,\, \sigma^2 = V[n])$

Forecasting Model:

$$F_{t+1} = (D_t + D_{t-1} + \ldots + D_{t-M+1}) / M$$

where M is a parameter

Exponential Smoothing

$$F_{t+1} = \alpha D_t + (1-\alpha) F_t$$

Where: $0 < \alpha < 1$

An Equivalent Form:

$$F_{t+1} = F_t + \alpha e_t$$

Holt's Model for Trended Data

Forecasting Model:

$$F_{t+1} = L_{t+1} + T_{t+1}$$

Where: $L_{t+1} = \alpha D_t + (1-\alpha)(L_t + T_t)$

and: $T_{t+1} = \beta(L_{t+1} - L_t) + (1- \beta)T_t$

Winter's Model for Trended/Seasonal Data

$$F_{t+1} = (L_{t+1} + T_{t+1})\, S_{t+1-m}$$

$$L_{t+1} = \alpha(D_t/S_t) + (1-\alpha)(L_t + T_t)$$

$$T_{t+1} = \beta(L_{t+1} - L_t) + (1-\beta)T_t$$

$$S_{t+1} = \gamma(D_{t+1}/L_{t+1}) + (1-\gamma)\, S_{t+1-m}$$

Notes from Homework 1

- Problem 1
  - Did not used the model which yielded the lowest MSE
  - Remove outliers
- Problem 2
  - Setting initial values for level (L) and trend (T)
  - The more data you use, the more accurate are these initial values
  - Penalty for waiting too long
  - If initial values are off by a lot, the model will take a longer time to "adjust" itself
- Problem 3
  - Initializing seasonality indexes

Inventory Management

Bottomline

Inventory is not bad. Inventory is good.

Inventory is an important tool which, when correctly used, can reduce total cost and improve the level of service performance in a logistics system.

Fundamental Purpose of Inventory

To Reduce Total System Cost

- To buffer uncertainties in:
  - supply,
  - demand, and/or
  - transportation

  the firm carries **safety stocks**.

- To capture scale economies in:
  - purchasing,
  - production, and/or
  - transportation

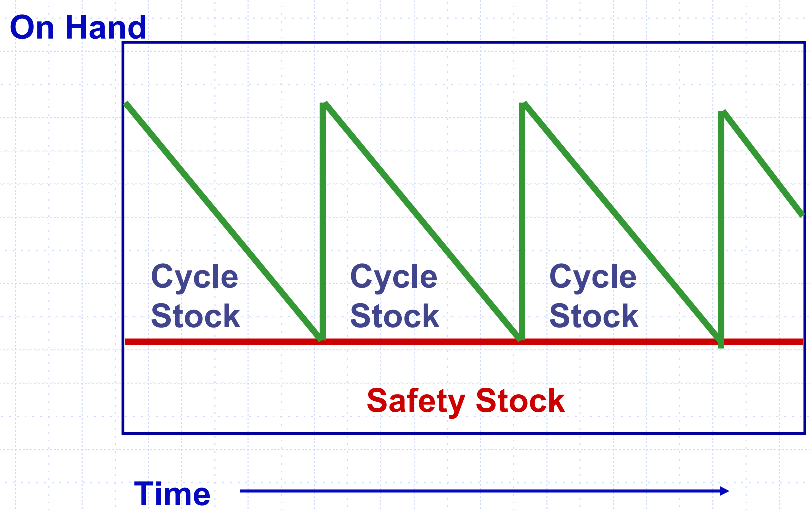
  the firm carries **cycle stocks**.

Dimensions of Inventory Modeling

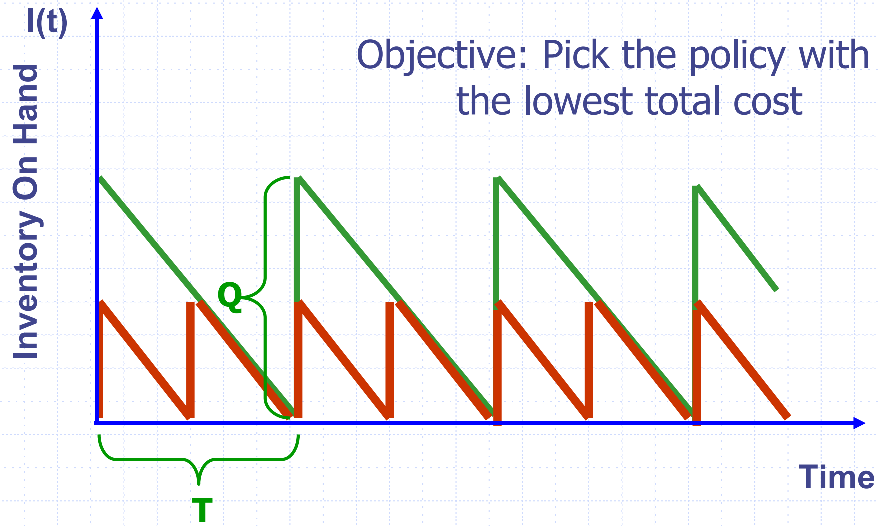
- Demand
  - Constant vs Variable
  - Known vs Random
  - Continuous vs Discrete
- Lead time
  - Instantaneous
  - Constant or Variable (deterministic/stochastic)
- Dependence of items
  - Independent
  - Correlated
  - Indentured
- Review Time
  - Continuous
  - Periodic
- Discounts
  - None
  - All Units or Incremental
- Excess Demand
  - None
  - All orders are backordered
  - Lost orders
  - Substitution
- Perishability
  - None
  - Uniform with time
- Planning Horizon
  - Single Period
  - Finite Period
  - Infinite

Lot sizing

Cycle Stock & Safety Stock



Lot Sizing: Many Potential Policies



Relevant Costs

- What makes a cost relevant?
- Components
  - Purchase Cost
  - Ordering Cost
  - Holding Cost
  - Shortage Cost

Notation

TC = Total Cost (dollar/time)

D = Average Demand (units/time)

C_o = Ordering Cost (dollar/order)

C_h = Holding Cost (dollars/dollars held/time)

Cp = Purchase Cost (dollars/unit)

Q = Order Quantity (units/order)

T = Order Cycle Time (time/order)

Economic Order Quantity (EOQ)

$$TC(Q) = C_o\left(\frac{D}{Q}\right) + C_h C_p\left(\frac{Q}{2}\right)$$

$$TC[Q] = \frac{C_o D}{Q} + \frac{C_h C_p Q}{2}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h C_p}}$$

$$TC^* = \sqrt{2DC_o C_h C_p}$$

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From TC [Q] to Q*

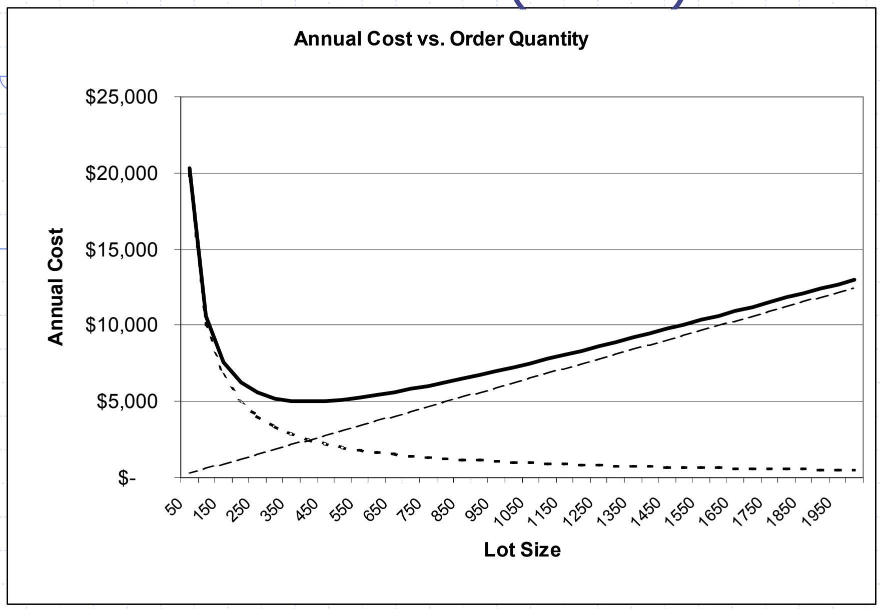
Take the derivate and set it to 0

The Effect of Non-Optimal Q

Q	DC_o/Q	$C_hC_pQ/2$	TC
2000	$500	$12,500	$13,000
500	$2,000	$3,125	$5,125
400	**$2,500**	**$2,500**	**$5,000**
200	$5,000	$1,250	$6,250
20	$50,000	$125	$50,125

So, how sensitive is TC to Q?

Total Cost versus Lot (Order) Size



Minimum point is relatively flat : there is a range / small changes in parameters may change the optimal Q

Insights from EOQ

- There is a direct trade off between lot size and total inventory
- Total cost is relatively insensitive to changes
  - Very robust with respect to changes in:
    - Q – rounding of order quantities
    - D – errors in forecasting
    - C_h, C_o, C_p – errors in cost parameters
- Thus, EOQ is widely used despite its highly restrictive assumptions

Introduce Discounts to Lot Sizing

- Types of discounts
  - All units discount
  - Incremental discount
  - One time only discount

- How will different discounting strategies impact your lot sizing decision?

All Units Discount

Unit Price [C_{pi}]	Price Break Quantity [PBQ_I]
$50.00	0
$45.00	500
$40.00	1000

All Units Discount

Need to introduce purchase cost into TC function

$$TC[Q, C_{pi}] = DC_{pi} + \frac{C_o D}{Q} + \frac{C_h C_{pi} Q}{2}$$

All Units Discount: Method

Same Example:
D=2000 Units/yr
C_h=.25
C_o=$500

C_{pi} Price Breaks:
$50 for 0 to <500 units
$45 for 500 to <1000 units
$40 for 1000+ units

1	C_{pi}	$40.00	$45.00	$50.00
2	PBQ	1000	500	0
3	EOQ[C_{pi}]	447	421	400
4	Q_{pi}	1000	500	400
5	DC_{pi}	$80,000	$90,000	$100,000
6	C_oD/Q_{pi}	$1,000	$2,000	$2,500
7	$C_hC_{pi}Q_{pi}/2$	$5,000	$2,812	$2,500
8	TC[Q_{pi}]	$86,000	$94,812	$105,000

Method :

Start with lowest price ($40)

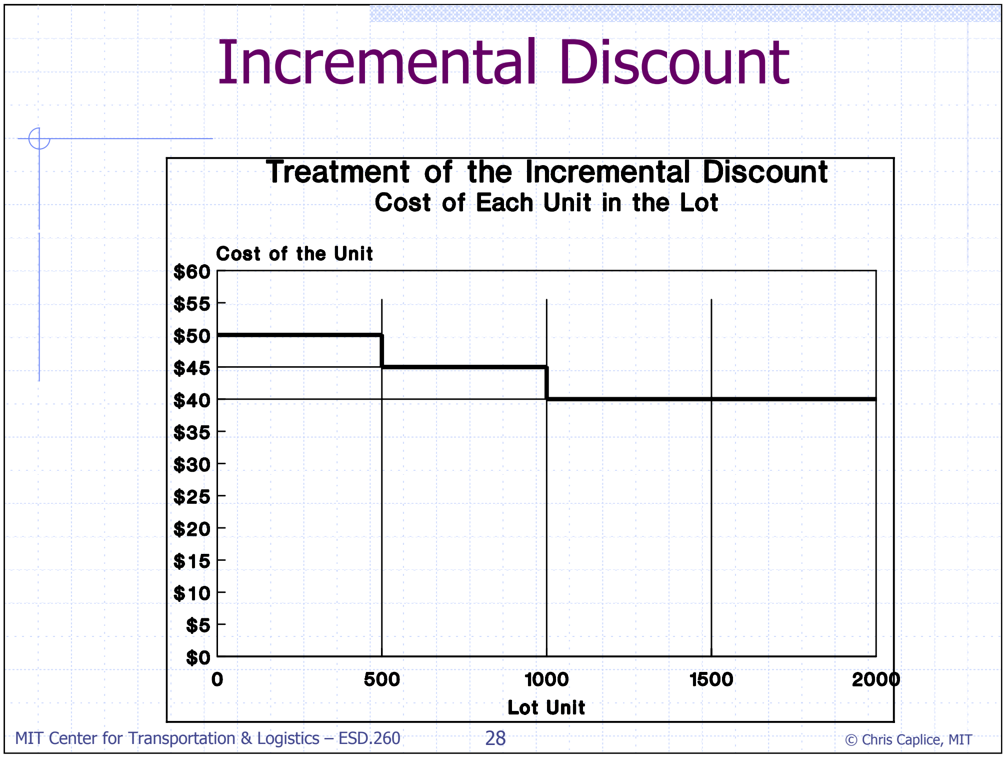
Find EOQ at that price point and price break quantity (EOQ cpi + PBQ)

Find Qpi = max [PBQ, EOQcpi]

Find total cost using new price point (TCqpi)

Go to next price point

If the EOQ was 1,200 – the optimal quantity fall between the range, I can't do better. So we can stop the calculations



Insight :

As oppose to the previous where there is a range
The cost I have to incur to be able to get to the next price level is like a fixed cost

Incremental Discount

	Index	i=3	i=2	i=1
1	C_{pi}	$40.00	$45.00	$50.00
2	PBQ_i	1000	500	0
3	F_i	$7500	$2500	$0
4	$EOQ[C_{pi}]$	1789	1033	400
5	Q_{pi}	1789		400
6	C_{pe}	$44.19		$50.00
7	DC_{pe}	$88,384.57		$100,000
8	C_oD/Q_{pi}	$558.97		$2,500
9	$(C_h C_{pe} Q_{pi})/2$	$9,882.50		$2,500
10	$TC[Q_{pi}]$	$98,826.04		$105,000

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Cpe (equivalent price)

Quantity	Cpe
0 <= Q <= 500	$50
500 <= Q <= 1000	[$50*(500) + $45*(Q-500)] / Q
1000 < Q	[$50*500 + $45*(Q-500) +$40*(Q-1000)] / Q

Method

Start with i=1

Find fixed cost

F1= 0

Fi= Fi-1 + (Cpi-1 – Cpi) * PBQi

EOQ at Cpi

If EOQ cpi is within range, then Qpi

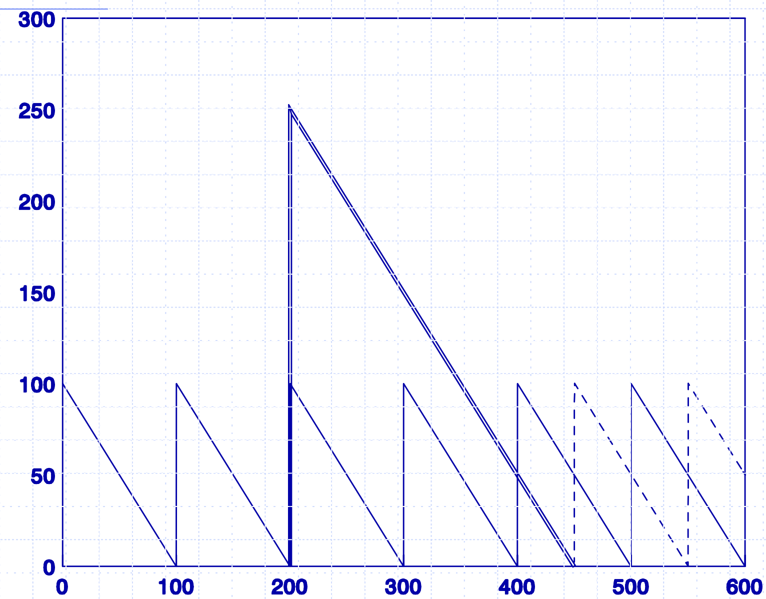
Otherwise, stop – go to the next I

Find Cpe = [Cpi * Qpi * Fi] / Qpi

Find TC

Next I

One Time Discount



Similar to a price increase where we order more right before the price increase

One Time Discount

- Let,
  - C_{pg} = One time deal purchase price ($/unit)
  - Q_g = One time special order quantity (units)
  - TC_{sp}=TC over time covered by special purchase ($)
- Then,

$$TC_{sp}[Q_g/D] = C_{pg}Q_g + C_h C_{pg}\frac{Q_g}{2}\frac{Q_g}{D} + C_o$$

One Time Discount

$$TCsp[Q_g/D] = C_{pg}Q_g + C_h C_{pg}\frac{Q_g}{2}\frac{Q_g}{D} + C_o$$

$$TC_{nsp}[Q_g/D] = C_{pg}Q_w + C_p(Q_g - Q_w) + C_h C_{pg}\frac{Q_w}{2}\frac{Q_w}{D} + C_h C_p\frac{Q_w}{2}\frac{(Q_g - Q_w)}{D} + C_o\frac{Q_g}{Q_w}$$

One Time Discount

$$Q_g^* = \frac{(C_p - C_{pg})D}{C_{pg} C_h} + \frac{C_p Q_w}{C_{pg}}$$

$$SAVINGS[Q_g^*] = \frac{C_o C_{pg}}{C_p} \left[\frac{Q_g^*}{Q_w} - 1 \right]^2$$

Notes from Homework 2

- Problem 1
  - Explore impact of reducing the ordering cost on the total system operating costs.
- Problem 2
  - Explored mechanics of prices discounts on lot sizing
  - Critical Cpi – how low the price need to be
  - Critical PBQi – how low quantity need to be
- Problem 3
  - All units discount and "added a minimum dollar value"

Safety Stock

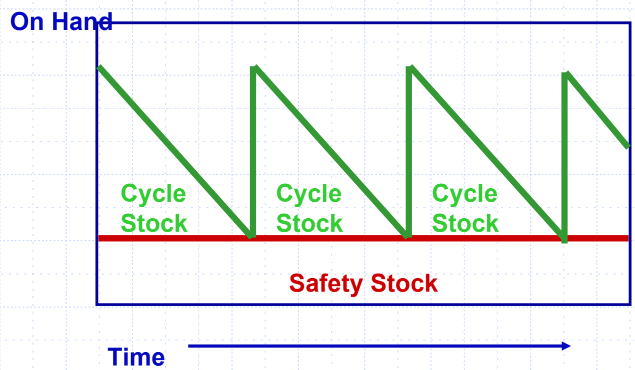
Assumptions: Basic EOQ Model

- Demand
  - Constant vs **Variable**
  - Known vs **Random**
  - **Continuous** vs Discrete
- Lead time
  - Instantaneous
  - **Constant** or Variable (deterministic/stochastic)
- Dependence of items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous** vs Periodic
- Number of Echelons
  - **One** vs Many
- Capacity / Resources
  - **Unlimited** vs Limited
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand
  - None
  - **All orders are backordered**
  - **All orders are lost**
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - Finite Period
  - **Infinite**
- Number of Items
  - **One**
  - Many

Fundamental Purpose of Inventory

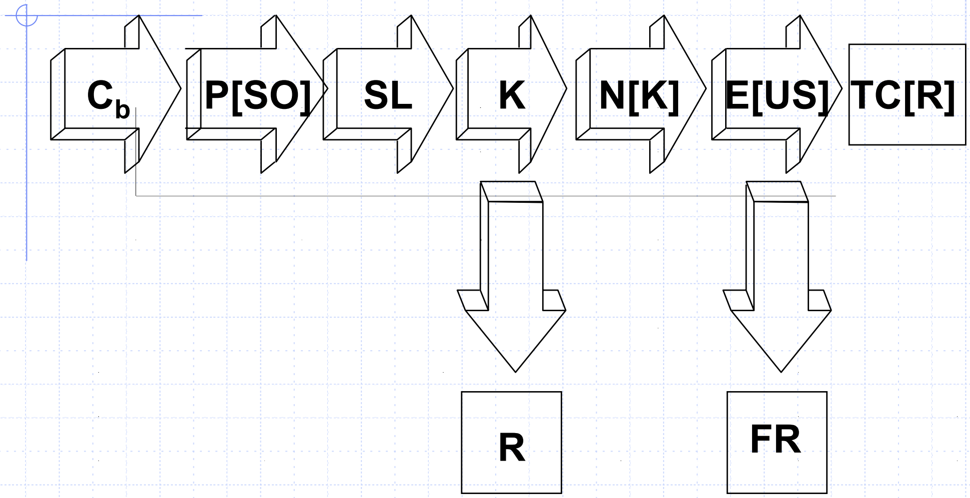
Firm carries safety stock to buffer uncertainties in:
- supply,
- demand, and/or
- transportation

Cycle Stock and Safety Stock



What should my inventory policy be?
(how much to order when)
What should my safety stock be?
What are my relevant costs?

Preview: Safety Stock Logic



Determining the Reorder Point

$$R = d' + k\sigma$$

Reorder Point

Estimated demand over the lead time (F_t)

Safety Stock

k = SS factor

σ = RMSE

$$k = \frac{(R - d')}{\sigma}$$

Note

1. We usually pick k for desired stock out probability
2. Safety Stock = R – d'

Define Some Terms

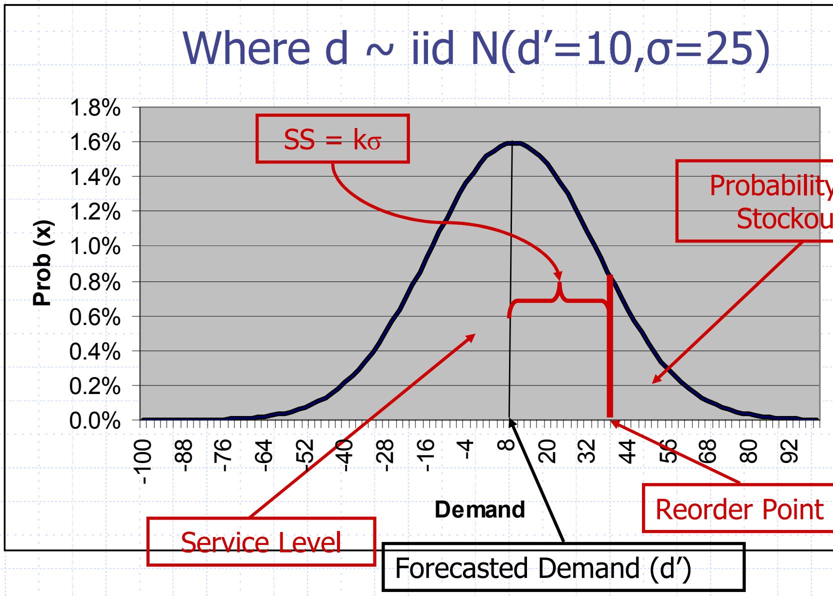
- Safety Stock Factor (k)

  Amount of inventory required in terms of standard deviations worth of forecast error
- Stockout Probability = P[d > R]

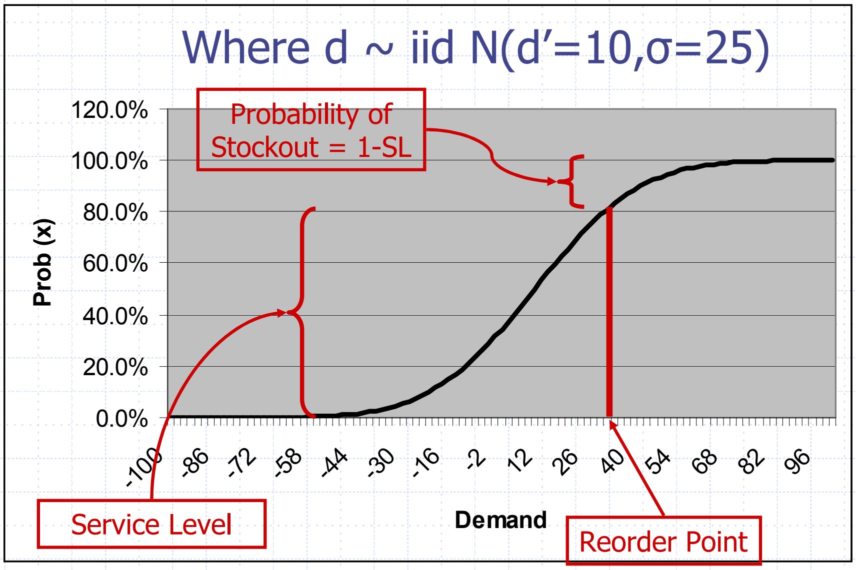
  The probability of running out of inventory during lead time
- Service Level = P[d ≤ R] = 1- P[SO]

  The probability of NOT running out of inventory during lead time

Service Level and Stockout Probability



Cumulative Normal Distribution



Finding SL from a Given K

Using a Table of Cumulative Normal Probabilities . . .

If I select a K = 0.42

K	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
0.5	0.6915	0.6950	0.6985	0.7019	0.7054
0.6	0.7257	0.7291	0.7324	0.7357	0.7389
0.7	0.7580	0.7611	0.7642	0.7673	0.7704
0.8	0.7881	0.7910	0.7939	0.7967	
				0.8238	
				0.8485	0.8508

. . . then my Service Level is this value.

Or, in Excel, use the function:
SL=**NORMDIST**(ko+d', d', σ, TRUE)

Safety Stock and Service Level

Example:

if d ~ iid Normal (d'=100,σ=10)

What should my SS & R be?

P[SO]	SL	k	Safety Stock	Reorder Point
.50	.50	0	0	100
.10	.90	1.28	13	113
.05	.95	1.65	17	117
.01	.99	2.33	23	123

So, how do I find Item Fill Rate?

- Fill Rate
  - Fraction of demand met with on-hand inventory
  - Based on each replenishment cycle

$$FillRate = \frac{OrderQuantity - E[UnitsShort]}{OrderQuantity}$$

- But, how do I find Expected Units Short?
  - More difficult
  - Need to calculate a partial expectation:

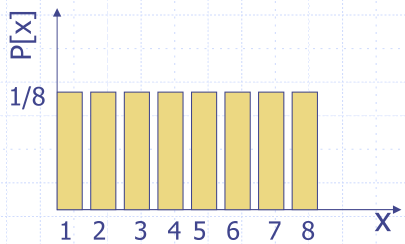
Expected Units Short

Consider both continuous and discrete cases

Looking for expected units short per replenishment cycle.

$$E[US]=\sum_{x=R}^{\infty}(x-R)\,p[x]$$

$$E[US]=\int_{R}^{\infty}(x_o-R)\,f_x(x_o)\,dx_o$$



What is E[US] if R=5?

For normal distribution we have a nice result:

E[US] = σN[k]

Where N[k] = Normal Unit Loss Function

Found in tables or formula

The N[k] Table

A Table of Unit Normal Loss Integrals

K	.00	.01	.02	.03	.04
0.0	.3989	.3940	.3890	.3841	.3793
0.1	.3509	.3464	.3418	.3373	.3328
0.2	.3069	.3027	.2986	.2944	.2904
0.3	.2668	.2630	.2592	.2555	.2518
0.4	.2304	.2270	.2236	.2203	.2169
0.5	.1978	.1947	.1917	.1887	.1857
0.6	.1687	.1659	.1633	.1606	.1580
0.7	.1429	.1405	.1381	.1358	.1334
0.8	.1202	.1181	.1160	.1140	.1120
0.9	.1004	.09860	.09680	.09503	.09328
1.0	.08332	.08174	.08019	.07866	.07716

Item Fill Rate

$$FillRate = \frac{OrderQuantity - E[UnitsShort]}{OrderQuantity} = FR$$

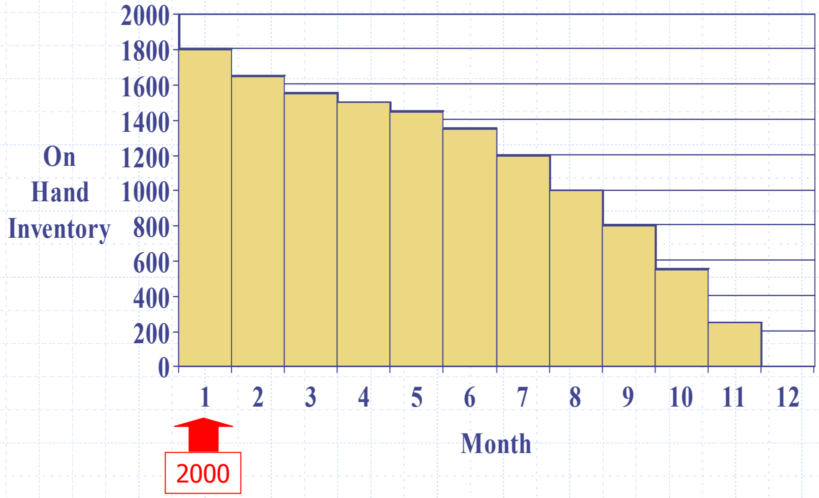
$$FR = 1 - \frac{E[US]}{Q} = 1 - \frac{\sigma N[k]}{Q}$$

$$N[k] = \frac{(1 - FR)Q}{\sigma}$$

So, now we can look for the k that achieves our desired fill rate.

Finite Horizon Planning

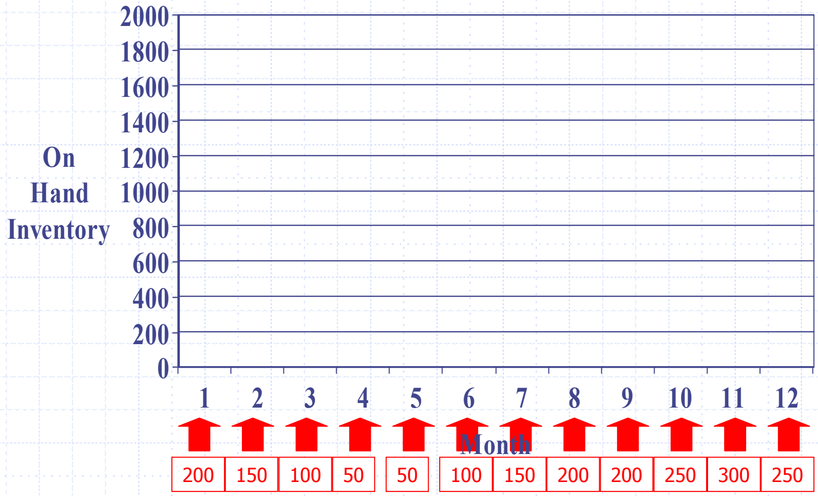
Approach: One-Time Buy



Approach: One-Time Buy

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	2000	$1800	$500	$2300
2	150	0	$1650	$0	$1650
3	100	0	$1550	$0	$1550
4	50	0	$1500	$0	$1500
5	50	0	$1450	$0	$1450
6	100	0	$1300	$0	$1300
7	150	0	$1200	$0	$1200
8	200	0	$1000	$0	$1000
9	200	0	$800	$0	$800
10	250	0	$550	$0	$550
11	300	0	$250	$0	$250
12	250	0	$0	$0	$0
Totals:	2000	2000	$13100	$500	$13600

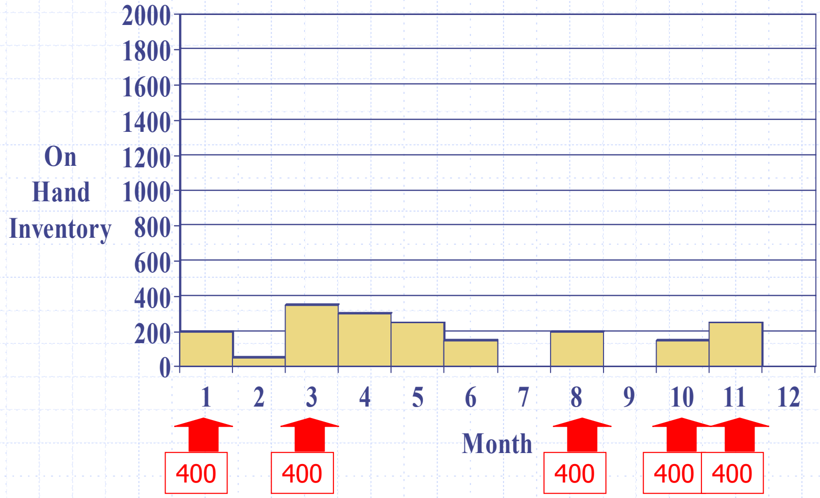
Approach: Lot for Lot



Approach: Lot for Lot

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	200	$0	$500	$500
2	150	150	$0	$500	$500
3	100	100	$0	$500	$500
4	50	50	$0	$500	$500
5	50	50	$0	$500	$500
6	100	100	$0	$500	$500
7	150	150	$0	$500	$500
8	200	200	$0	$500	$500
9	200	200	$0	$500	$500
10	250	250	$0	$500	$500
11	300	300	$0	$500	$500
12	250	250	$0	$500	$500
Totals:	2000	2000	$0	$6000	$6000

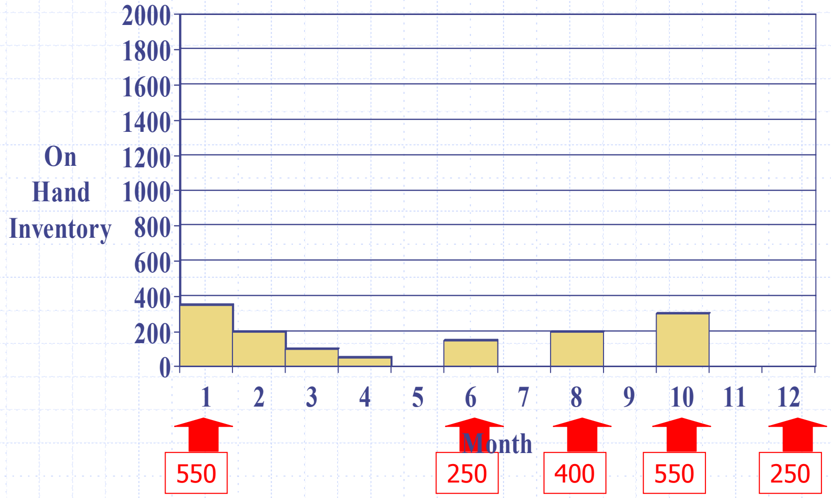
Approach: EOQ



Approach: EOQ

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	400	$200	$500	$700
2	150	0	$50	$0	$50
3	100	400	$350	$500	$850
4	50	0	$300	$0	$300
5	50	0	$250	$0	$250
6	100	0	$150	$0	$150
7	150	0	$0	$0	$0
8	200	400	$200	$500	$700
9	200	0	$0	$0	$0
10	250	400	$150	$500	$650
11	300	400	$250	$500	$750
12	250	0	$0	$0	$0
Totals:	2000	2000	$1900	$2500	$4400

Approach: Silver-Meal Algorithm



Approach: Silver-Meal Algorithm

Mon	Dmd	Lot Qty	Order Cost	Holding Cost	Lot Cost	Mean Cost
1st	Buy:					
1	200	200	$500	$0	$500	$500
2	150	350	$500	$150	$650	$325
3	100	450	$500	$150+$200	$850	$283
4	50	500	$500	$150+$200+$150	$1000	$250
5	50	550	$500	$150+$200+$150+$200	$1200	$240
6	100	650	$500	$150+$200+$150+$200+$500	$1700	$283
2nd	Buy:					
6	100	100	$500	$0	$500	$500
7	150	250	$500	$150	$650	$325
8	200	450	$500	$150+$400	$1050	$350

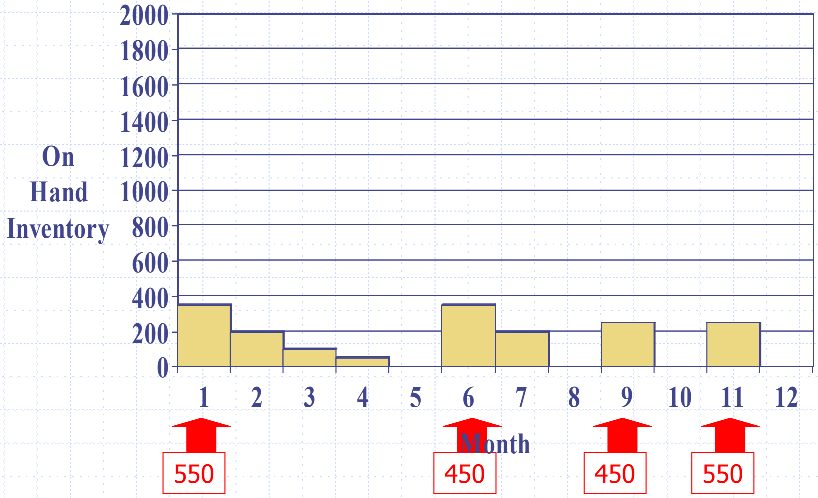
Approach: Silver-Meal Algorithm

Mon	Dmd	Lot Qty	Order Cost	Holding Cost	Lot Cost	Mean Cost
3rd	Buy:					
8	200	200	$500	$0	$500	$500
9	200	400	$500	$200	$700	$350
10	250	650	$500	$200+$500	$1200	$400
4th	Buy:					
10	250	250	$500	$0	$500	$500
11	300	550	$500	$300	$800	$400
12	250	800	$500	$300+$500	$1300	$433
5th	Buy:					
12	250	250	$500	$0	$500	$500

Approach: Silver-Meal Algorithm

Month	Demand	Order Quantity	Holding Cost	Ordering Cost	Period Costs
1	200	550	$350	$500	$850
2	150	0	$200	$0	$200
3	100	0	$100	$0	$100
4	50	0	$50	$0	$50
5	50	0	$0	$0	$0
6	100	250	$150	$500	$650
7	150	0	$0	$0	$0
8	200	400	$200	$500	$700
9	200	0	$0	$0	$0
10	250	550	$300	$500	$800
11	300	0	$0	$0	$0
12	250	250	$0	$500	$500
Totals:	2000	2000	$1350	$2500	$3850

Approach: Optimization (MILP)



Approach: Optimization (MILP)

Decision Variables:

- Qi = Quantity purchased in period i
- Zi = Buy variable = 1 if Q_i>0, =0 o.w.
- Bi = Beginning inventory for period I
- Ei = Ending inventory for period I

Data:

- D_i = Demand per period, i = 1,,n
- C_o = Ordering Cost
- C_{hp} = Cost to Hold, $/unit/period
- M = a very large number….

MILP Model

Objective Function:

- Minimize total relevant costs

Subject To:

- Beginning inventory for period 1 = 0
- Beginning and ending inventories must match
- Conservation of inventory within each period
- Nonnegativity for Q, B, E
- Binary for Z

Approach: Optimization (MILP)

$$Min\ TC = \sum_{i=1}^{n} C_O Z_i + \sum_{i=1}^{n} C_{HP} E_i$$

$s.t.$

$$B_1 = 0$$
$$B_i - E_{i-1} = 0 \qquad \forall i = 2,3,\ldots n$$
$$E_i - B_i - Q_i = D_i \qquad \forall i = 1,2,\ldots n$$
$$MZ_i - Q_i \ge 0 \qquad \forall i = 1,2,\ldots n$$
$$B_i \ge 0 \qquad \forall i = 1,2,\ldots n$$
$$E_i \ge 0 \qquad \forall i = 1,2,\ldots n$$
$$Q_i \ge 0 \qquad \forall i = 1,2,\ldots n$$
$$Z_i = \{0,1\} \qquad \forall i = 1,2,\ldots n$$

- Objective Function
- Beginning & Ending Inventory Constraints
- Conservation of Inventory Constraints
- Ensures buys occur only if Q>0
- Non-Negativity & Binary Constraints

Comparison of Approaches

Month	Demand	OTB	L4L	EOQ	S/M	OPT
1	200	2000	200	400	550	550
2	150		150			
3	100		100	400		
4	50		50			
5	50		50			
6	100		100		250	450
7	150		150			
8	200		200	400	400	
9	200		200			450
10	250		250	400	550	
11	300		300	400		550
12	250		250		250	
Total Cost		$13,600	$6,000	$4,400	$3,850	$3,750

Notes from Homework 3

- Problem 1
  - Critique an item being ordered
  - Did not know the backorder cost (5 or 10)
- Problem 2
  - Split between back order and lost sales
- Problem 3
  - Silver-Meal vs. MILP
- Problem 4
  - MRP
- Problem 5
  - Padded lead time