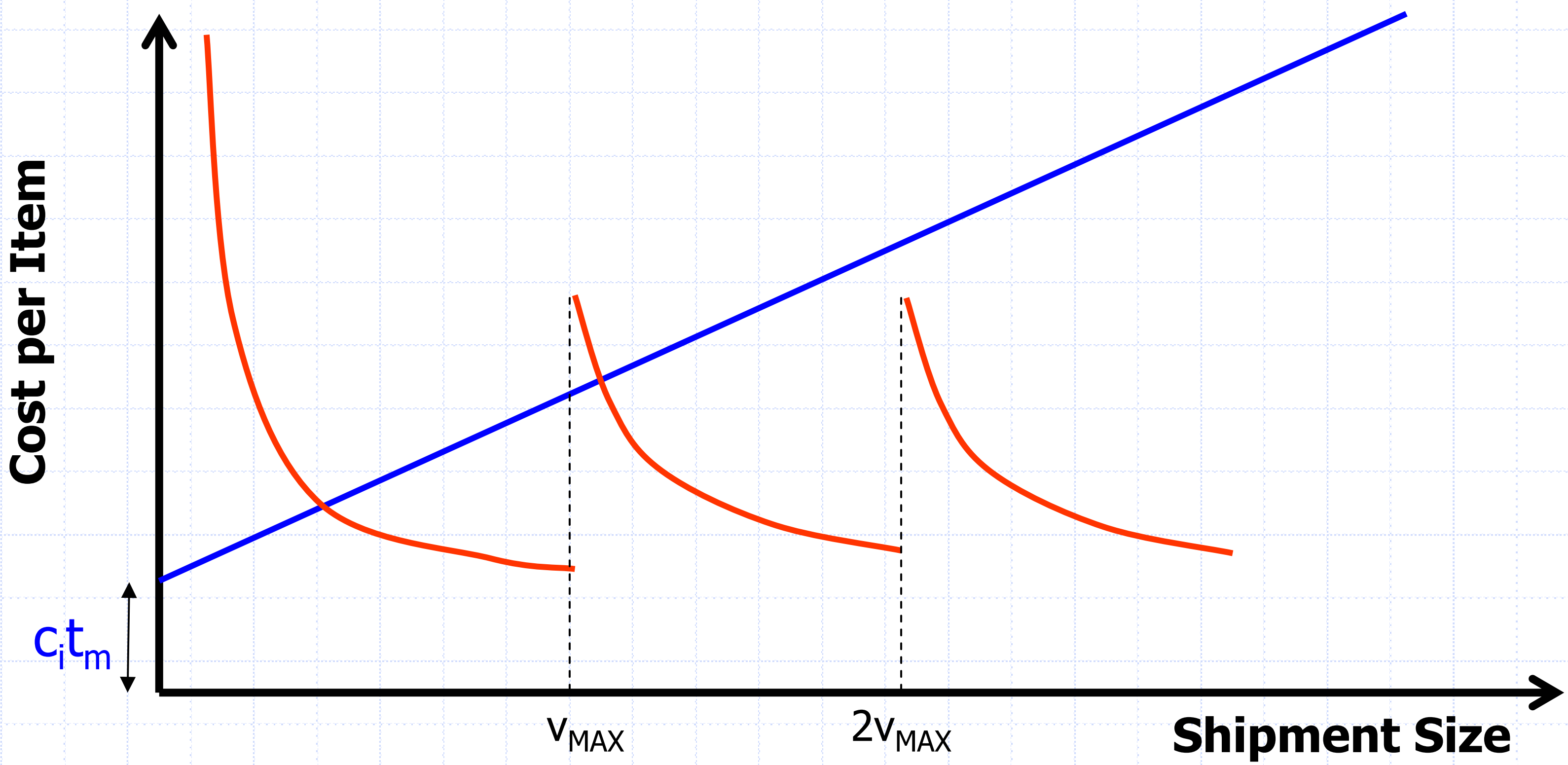
# Transportation II

Lecture 16
ESD.260 Fall 2003

Caplice

# One to One System

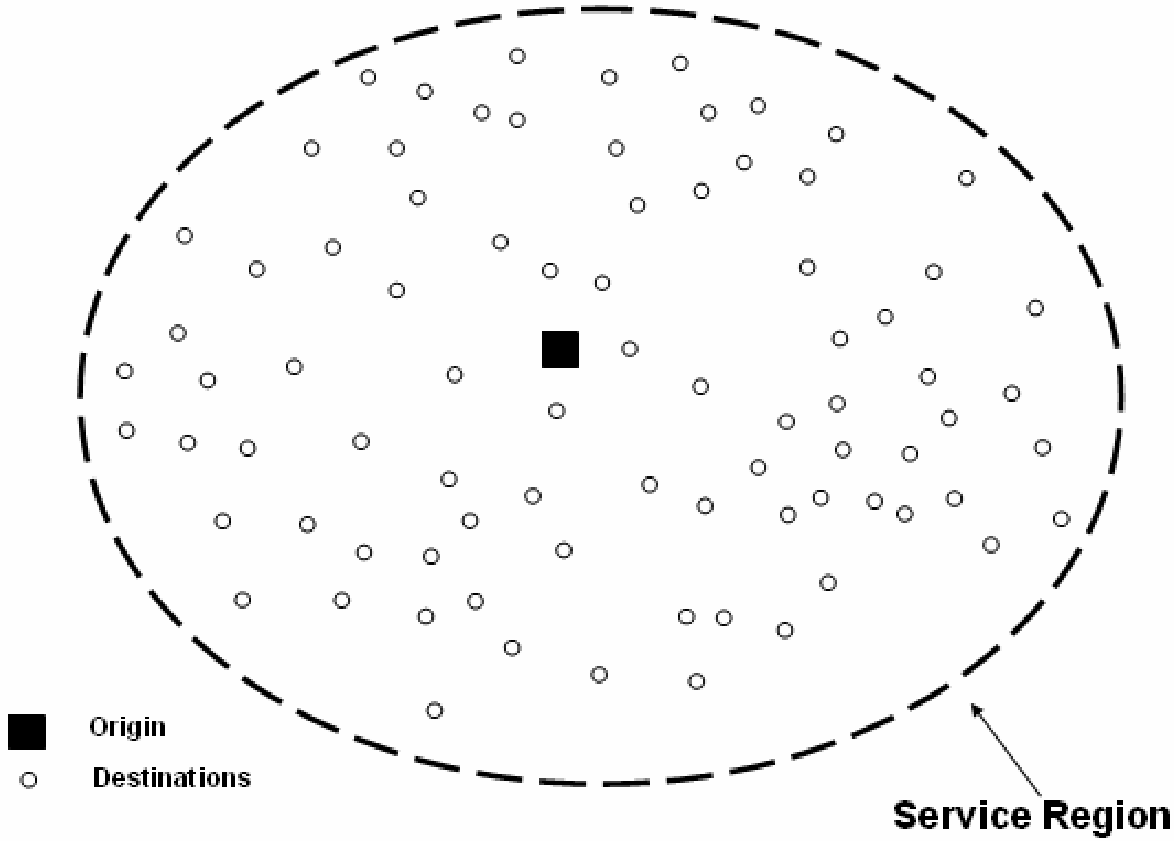
$$LC(\$/item) = c_r H_{MAX} + c_i H_{MAX} + c_i t_m + c_s\left(\frac{1+n_s}{\bar{v}}\right) + c_d\left(\frac{d}{\bar{v}}\right) + c_{vs}$$



# One to Many System

**Single Distribution Center:**
- Products originate from one origin
- Products are demanded at many destinations
- All destinations are within a specified Service Region

**Assumptions:**
- Vehicles are homogenous
- Same capacity, $v_{MAX}$
- Fleet size is constant



# LCF for One to Many System

- Cost Function
  - Storage (Rent) Holding Costs
  - Inventory Holding Costs
  - Transportation Costs
  - Handling Costs
- Key Cost Drivers?
  -
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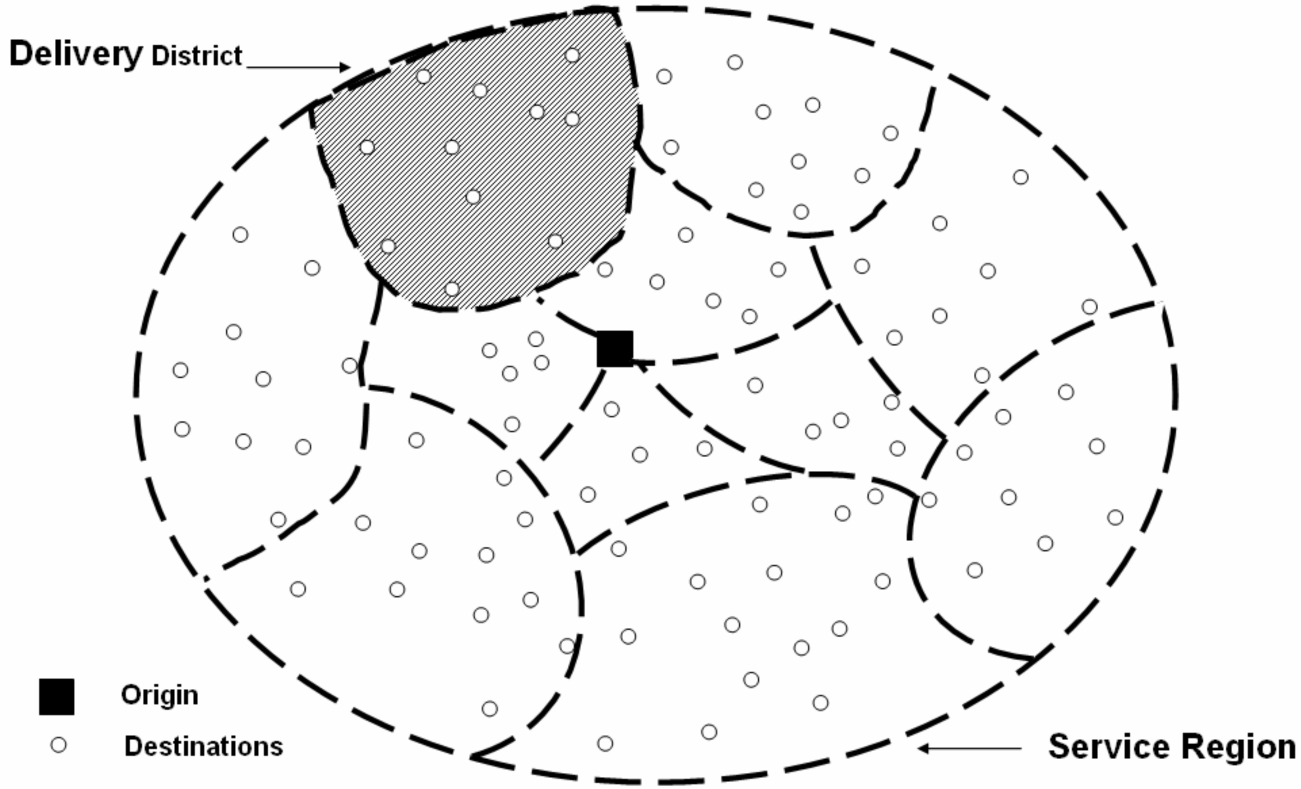
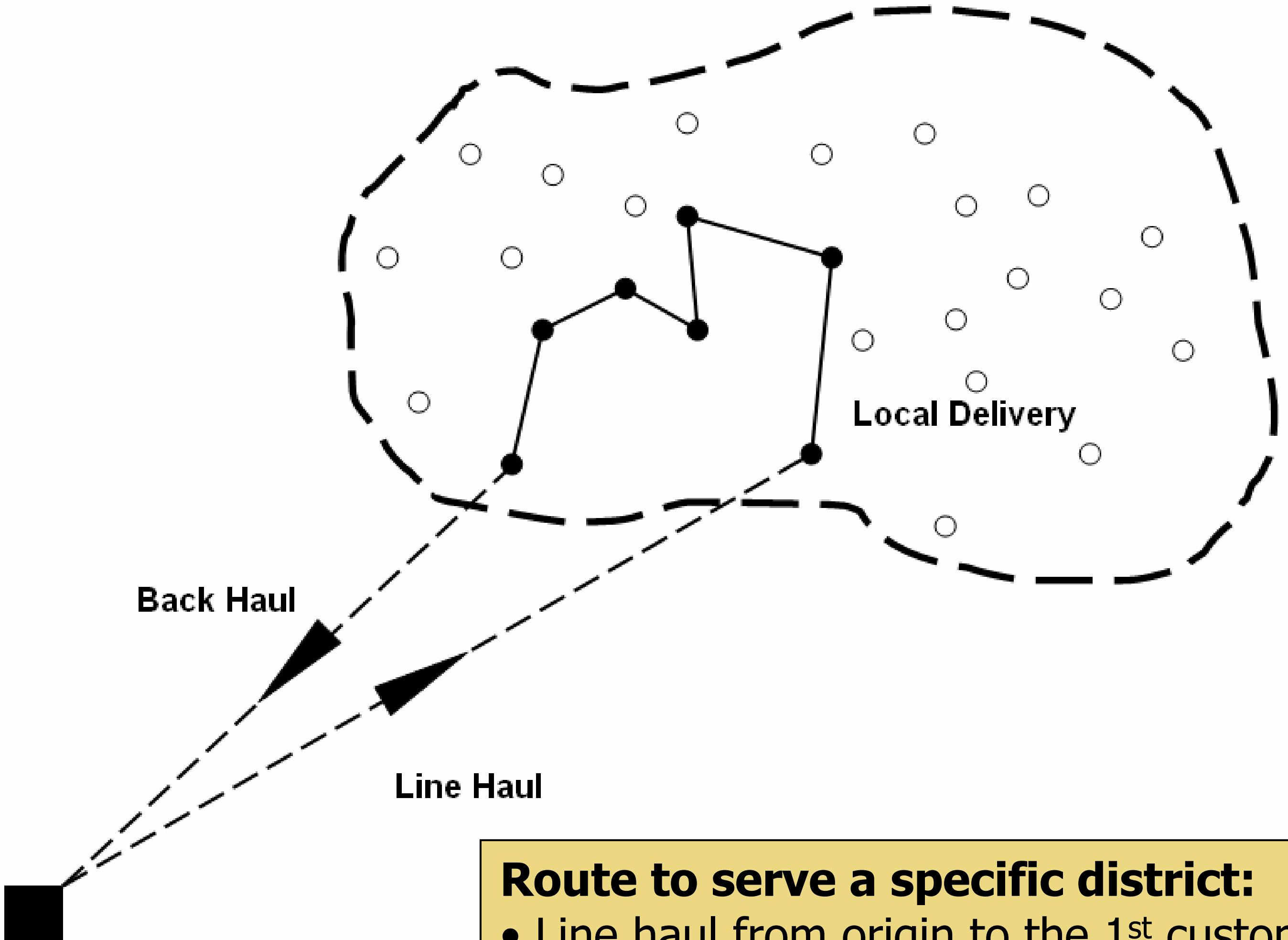
# One to Many System

**Finding the estimated total distance:**
- Divide the Service Region into Delivery Districts
- Estimate the distance required to service each district



# One to Many System




**Route to serve a specific district:**

- Line haul from origin to the 1st customer in the district
- Local delivery from 1st to last customer in the district
- Back haul (empty) from the last customer to the origin

# An Aside: Routing & Scheduling

- Problem:
  - How do I route vehicle(s) from origin(s) to destination(s) at a minimum cost?
  - A HUGE literature and area of research
- One type of classification (by methodology)

1. One origin, one destination, multiple paths
   Shortest Path Problem
2. Single path to reach all the destinations
   Minimum Spanning Tree
3. Many origins, many destinations, constrained supply
   Transportation Method (LP)
4. One origin, many destinations, sequential stops -
   Traveling Salesman Problem
   Vehicle Routing Problem
   - Stops may require delivery & pick up
   - Vehicles have different capacity (capacitated)
   - Stops have time windows
   - Driving rules restricting length of tour, time, number of stops

# One to Many System

- Find the estimated distance for each tour, $d_{TOUR}$
  - Capacitated Vehicle Routing Problem
  - Cluster-first, Route-second Heuristic

$$d_{TOUR} \approx 2d_{LineHaul} + d_{Local}$$

$d_{LineHaul}$ = Distance from origin to center of gravity of delivery district

$d_{Local}$ = Local delivery between c customers in district (TSP)

# One to Many System

- What can we say about the expected TSP distance to cover n stops in district of area A?
  - Hard bound and some network specific estimates:

$$E\left[d_{TSP}\right] \leq 1.15\sqrt{nA}$$

$$E\left[d_{TSP}\right] \approx k\sqrt{nA}$$

For n>25 over Euclidean space, k=.7124

For straight line (Manhattan Metric), k=.7650

Density, δ, number of stops per area

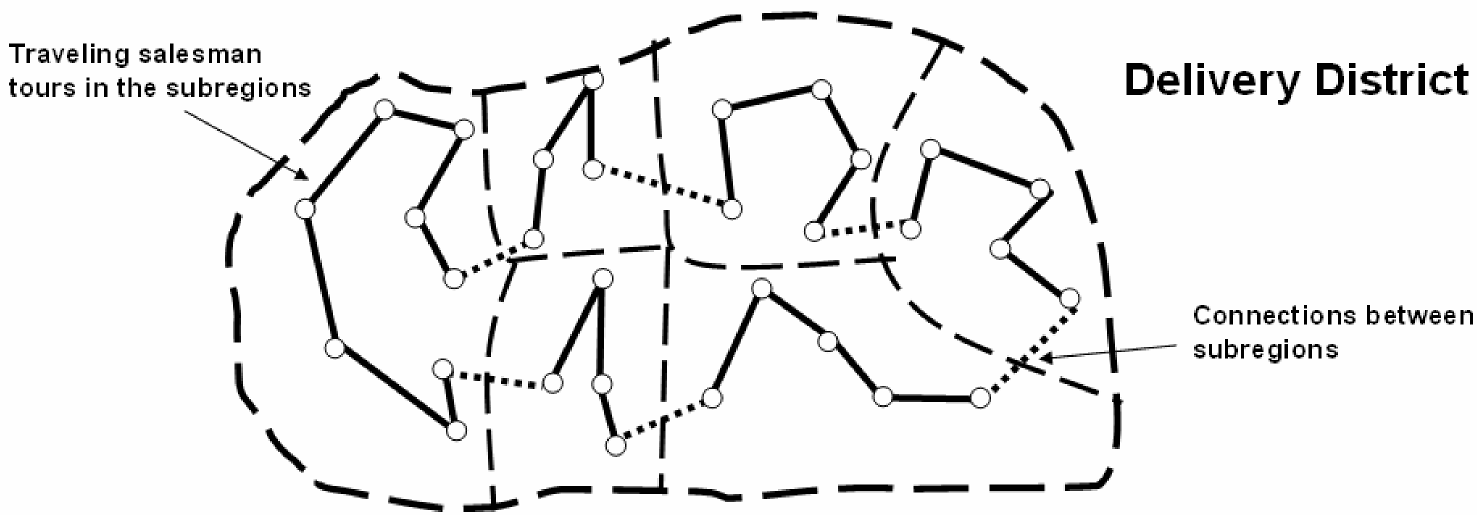
Average distance per stop, $d_{stop}$

$$\delta = n / A$$

$$d_{stop} = \frac{d_{TSP}}{n} = k \cdot \frac{\sqrt{nA}}{n} = \frac{k}{\sqrt{\delta}}$$

# One to Many System

- Length of local tours
  - Number of customer stops, c, times $d_{stop}$ over entire region
  - Exploits property of TSP being sub-divided –
    - TSP of disjoint sub-regions ≥ TSP over entire region



# One to Many System

- Finding the total distance traveled on all, l, tours:

$$E\left[d_{TOUR}\right] = 2d_{LineHaul} + \frac{ck}{\sqrt{\delta}}$$

$$E\left[d_{AllTours}\right] = lE\left[d_{TOUR}\right] = 2ld_{LineHaul} + \frac{nk}{\sqrt{\delta}}$$

- The more tours I have, the shorter the line haul distance
- Minimize number of tours by maximizing vehicle capacity

$$l = \left[\frac{D}{v_{MAX}}\right]^{+}$$

$$E\left[d_{AllTours}\right] = 2\left[\frac{Q}{v_{MAX}}\right]^{+} d_{LineHaul} + \frac{nk}{\sqrt{\delta}}$$

$[x]^+$ is lowest integer value greater than x – a step function

Estimate this with continuous function:

$E([x]^+) \sim E(x) + ½$

# One to Many System

- So that expected distance is:

$$E\left[d_{AllTours}\right] = 2\left[\frac{E\left[D\right]}{v_{MAX}} + \frac{1}{2}\right] d_{LineHaul} + \frac{E\left[n\right]k}{\sqrt{\delta}}$$

- Note that if each delivery district has a different density, then:

$$E\left[d_{AllTours}\right] = 2\sum_i \left[\frac{E\left[D_i\right]}{v_{MAX}} + \frac{1}{2}\right] d_{LineHaul_i} + k\sum_i \frac{E\left[n_i\right]}{\sqrt{\delta_i}}$$

- For identical districts, the transportation cost becomes:

$$TransportCost = c_s\left[E\left[n\right] + \frac{E\left[D\right]}{v_{MAX}} + \frac{1}{2}\right] + c_d\left(2\left[\frac{E\left[D\right]}{v_{MAX}} + \frac{1}{2}\right] d_{LineHaul} + \frac{E\left[n\right]k}{\sqrt{\delta}}\right) + c_{vs}E\left[D\right]$$

# One to Many System

- Fleet Size
  - Find minimum number of vehicles required, M
  - Base on, W, amount of required work time
    - $t_w$ = available worktime for each vehicle per period
    - s = average vehicle speed
    - l = number of shipments per period
    - $t_l$ =loading time per shipment
    - $t_s$ = unloading time per stop

$$Mt_w \geq W = \frac{d_{AllTours}}{s} + lt_l + nt_s$$

$$W = \left(\frac{2d_{LineHaul}}{s} + t_l\right)\left[\frac{E[D]}{v_{MAX}} + \frac{1}{2}\right] + E[n]\left(\frac{k}{\sqrt{\delta}} + t_s\right)$$

# One to Many System

- Note that W is a linear combination of two random variables, n and D

$$E[aX+bY]=aE[X]+bE[Y]$$

$$Var[aX+bY]=a^2Var[X]+b^2Var[Y]+2abCov[X,Y]$$

- Substituting in, we can find E[W] and Var[W]

$$a=\left(\frac{2d_{LineHaul}}{s}+t_l\right)\left[\frac{1}{v_{MAX}}\right]$$

$$b=\left(\frac{k}{\sqrt{\delta}}+t_s\right)$$

Given a service level, SL

$P[W<Mt_w]=SL$   Thus,

$M= (E[W] + k(SL)\ StDev[W])/t_w$