
Single Period Inventory Models



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Outline

- ☐ Single period inventory decisions
 - ☐ Calculating the optimal order size
 - Numerically
 - ☐ Using spreadsheet
 - ☐ Using simulation
 - Analytically
 - ☐ The profit function
 - For specific distributions
 - ☐ Level of Service
 - ☐ Extensions:
 - Fixed costs
 - Risks
 - Initial inventory
 - Elastic demand
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Single Period Ordering

- ☐ Seasonal items
 - ☐ Perishable goods
 - ☐ News print
 - ☐ Fashion items
 - ☐ Some high tech products
 - ☐ Risky investments
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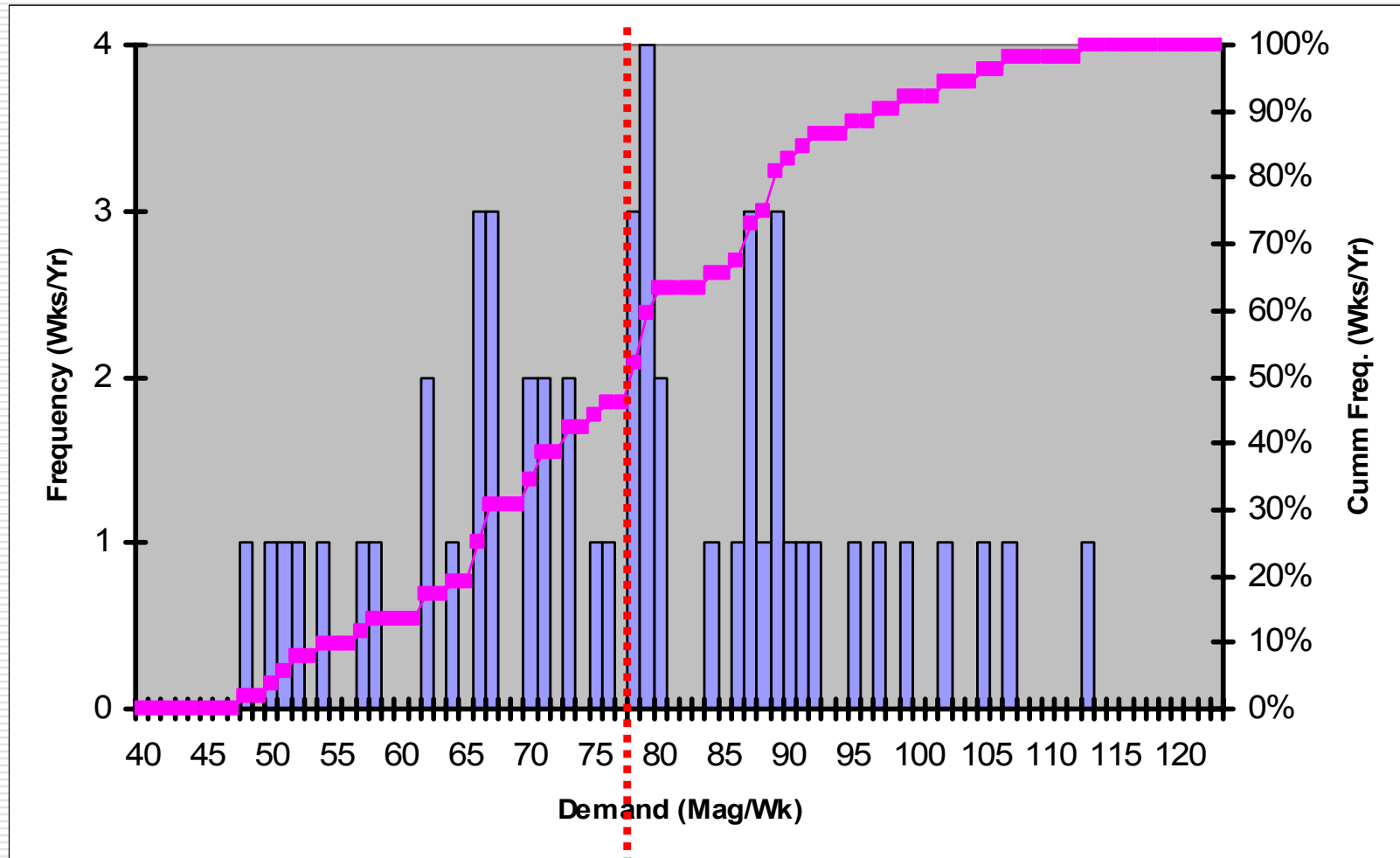
Selling Magazines

□ Weekly demand:

90	48	87	78	58	71	102	87	66	79	97	75	89
57	86	95	67	89	70	113	52	84	62	91	71	66
99	73	92	66	67	89	87	64	70	54	67	88	62
79	79	105	76	73	78	50	107	80	78	51	79	80

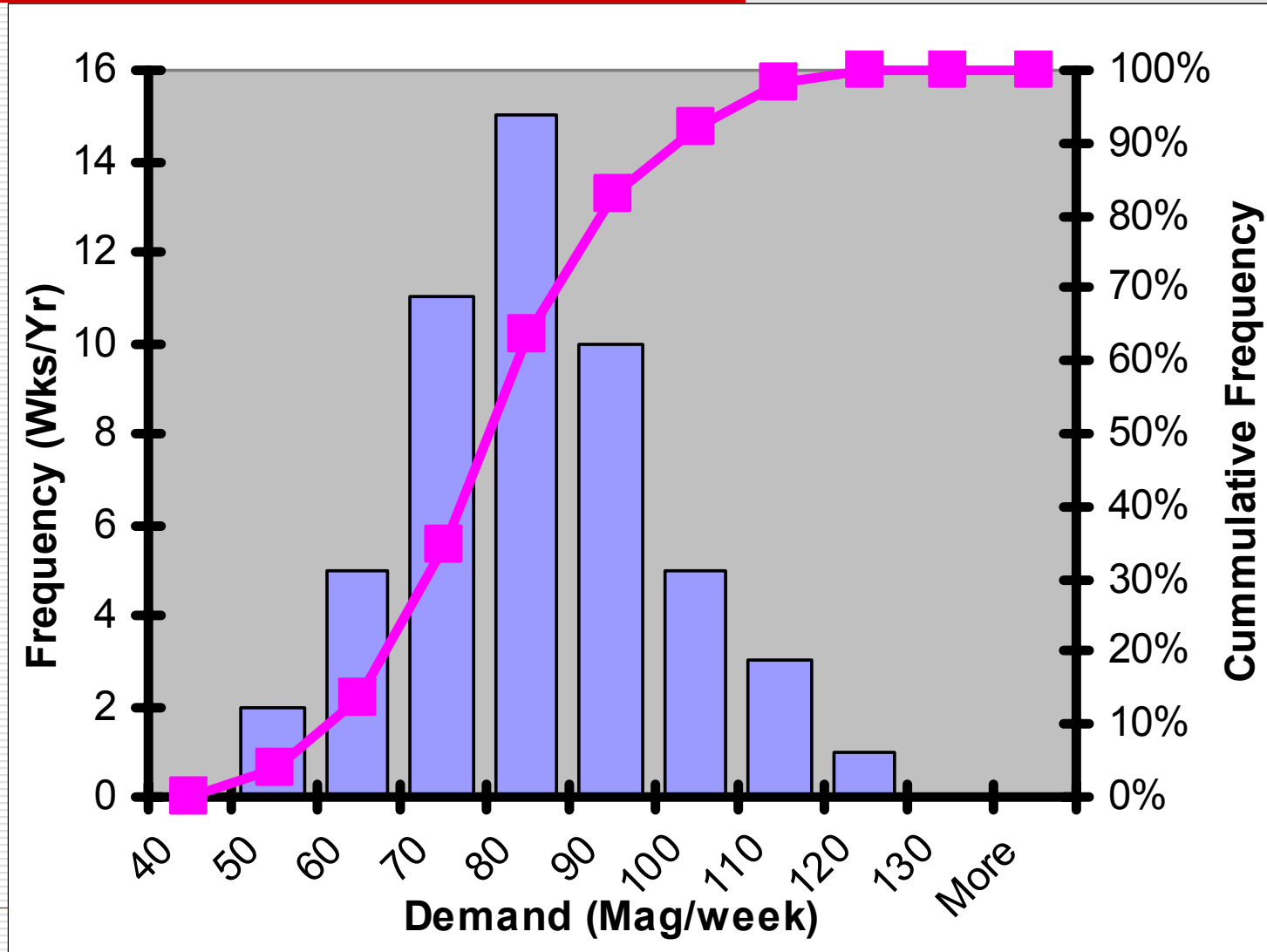
- Total: 4023 magazines
 - Average: 77.4 Mag/week
 - Min: 51; max: 113 Mag/week
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Detailed Histogram



Average=77.4 Mag/wk

Histogram



The Ordering Decision (Spreadsheet)

- Assume: each magazine sells for: \$15
- Cost of each magazine: \$8

	Order:	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160
d/wk	Prob.															
40	0.00	\$140	\$210	\$280	\$200	\$120	\$40	-\$40	-\$120	-\$200	-\$280	-\$360	-\$440	-\$520	-\$600	-\$680
50	0.04	\$140	\$210	\$280	\$350	\$270	\$190	\$110	\$30	-\$50	-\$130	-\$210	-\$290	-\$370	-\$450	-\$530
60	0.10	\$140	\$210	\$280	\$350	\$420	\$340	\$260	\$180	\$100	\$20	-\$60	-\$140	-\$220	-\$300	-\$380
70	0.21	\$140	\$210	\$280	\$350	\$420	\$490	\$410	\$330	\$250	\$170	\$90	\$10	-\$70	-\$150	-\$230
80	0.29	\$140	\$210	\$280	\$350	\$420	\$490	\$560	\$480	\$400	\$320	\$240	\$160	\$80	\$0	-\$80
90	0.19	\$140	\$210	\$280	\$350	\$420	\$490	\$560	\$630	\$550	\$470	\$390	\$310	\$230	\$150	\$70
100	0.10	\$140	\$210	\$280	\$350	\$420	\$490	\$560	\$630	\$700	\$620	\$540	\$460	\$380	\$300	\$220
110	0.06	\$140	\$210	\$280	\$350	\$420	\$490	\$560	\$630	\$700	\$770	\$690	\$610	\$530	\$450	\$370
120	0.02	\$140	\$210	\$280	\$350	\$420	\$490	\$560	\$630	\$700	\$770	\$840	\$760	\$680	\$600	\$520
130	0.00	\$140	\$210	\$280	\$350	\$420	\$490	\$560	\$630	\$700	\$770	\$840	\$910	\$830	\$750	\$670
Exp. Profit:		\$140	\$210	\$280	\$350	\$414	\$464	\$482	\$457	\$403	\$334	\$257	\$177	\$97	\$17	-\$63

Expected Profits



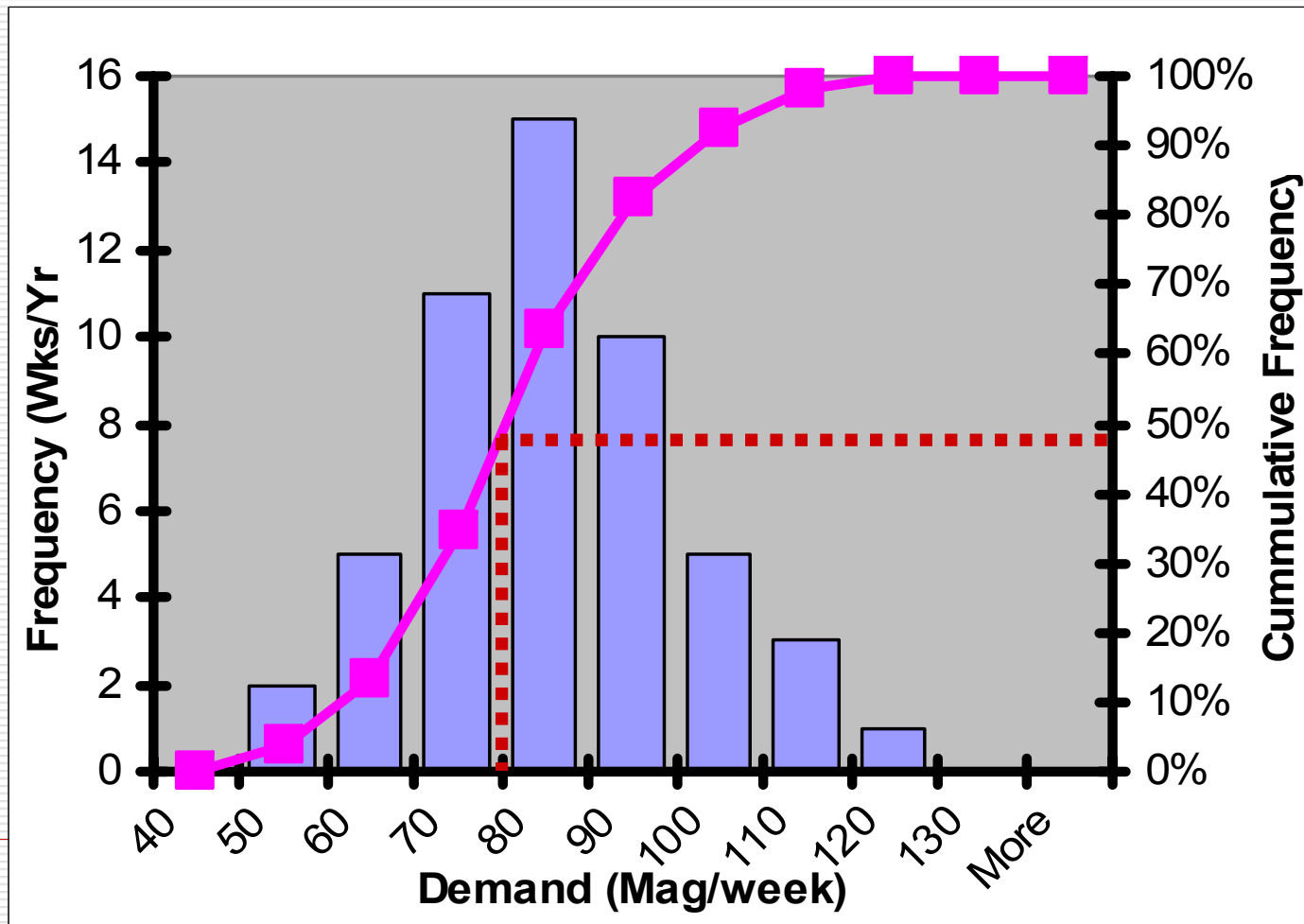
Optimal Order (Analytical)

- The optimal order is Q^*
- At Q^* the probability of selling one more magazine is the probability that demand is greater than Q^*
- The expected profit from ordering the (Q^*+1) st magazine is:
 - If demand is high and we sell it:
 - $(\text{REV}-\text{COST}) \times \text{Pr}(\text{ Demand is higher than } Q^*)$
 - If demand is low and we are stuck:
 - $(-\text{COST}) \times \text{Pr}(\text{ Demand is lower or equal to } Q^*)$
- The optimum is where the total expected profit from ordering one more magazine is zero:
 - $(\text{REV}-\text{COST}) \times \text{Pr}(\text{ Demand } > Q^*) - \text{COST} \times \text{Pr}(\text{ Demand } \leq Q^*) = 0$

$$\text{Pr}(\text{ Demand } \leq Q^*) = \frac{\text{REV}-\text{COST}}{\text{REV}}$$

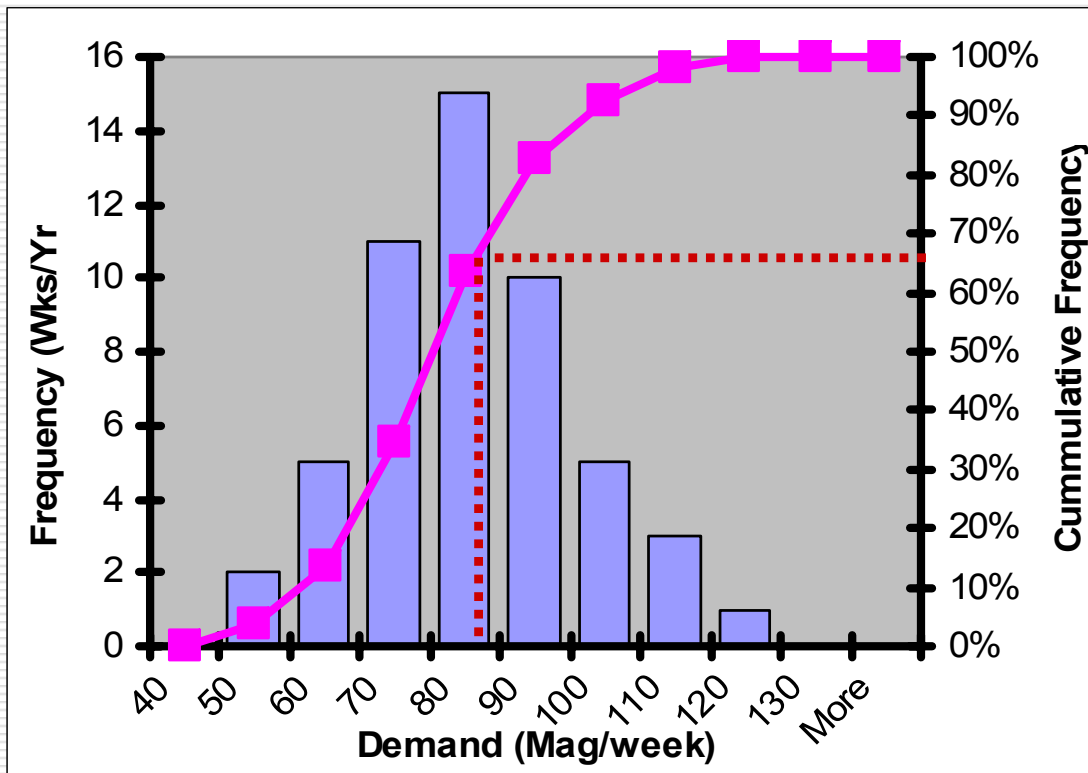
Optimal Order

The “critical ratio”: $\Pr(\text{Demand} \leq Q^*) = \frac{\text{REV}-\text{COST}}{\text{REV}} = \frac{15-8}{15} = 0.47$



Salvage Value

Salvage value = \$4/Mag. Critical Ratio = $\frac{REV - COST}{REV - SLV} = \frac{15 - 8}{15 - 4} = 0.64$



The Profit Function

- ❑ Revenue from sold items
 - ❑ Revenue or costs associated with unsold items. These may include revenue from salvage or cost associated with disposal.
 - ❑ Costs associated with not meeting customers' demand. The lost sales cost can include lost of good will and actual penalties for low service.
 - ❑ The cost of buying the merchandise in the first place.
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The Profit Function

$$E[\text{Sales}] = Q \cdot \int_{x=Q}^{\infty} f(x)dx + \int_{x=0}^Q x \cdot f(x)dx$$

$$E[\text{Unsold}] = \int_{x=0}^Q (Q - x) \cdot f(x)dx = Q - E[\text{Sales}]$$

$$E[\text{Lost Sales}] = \int_{x=Q}^{\infty} (x - Q) \cdot f(x)dx = \mu - E[\text{Sales}]$$

$$E[\text{Profit}] = R \cdot E[\text{Sales}] + S \cdot E[\text{Unsold}] - L \cdot E[\text{Lost Sales}] - C \cdot Q$$

The Profit Function – Simple Case

$$E[Profit] = R \cdot E[Sales] - C \cdot Q$$

Optimal Order:

$$\frac{d}{dQ} E[Profit] = (1 - F(Q)) \cdot R - C = 0$$

$$\frac{d}{dQ} E[Sales] = 1 - F(Q)$$

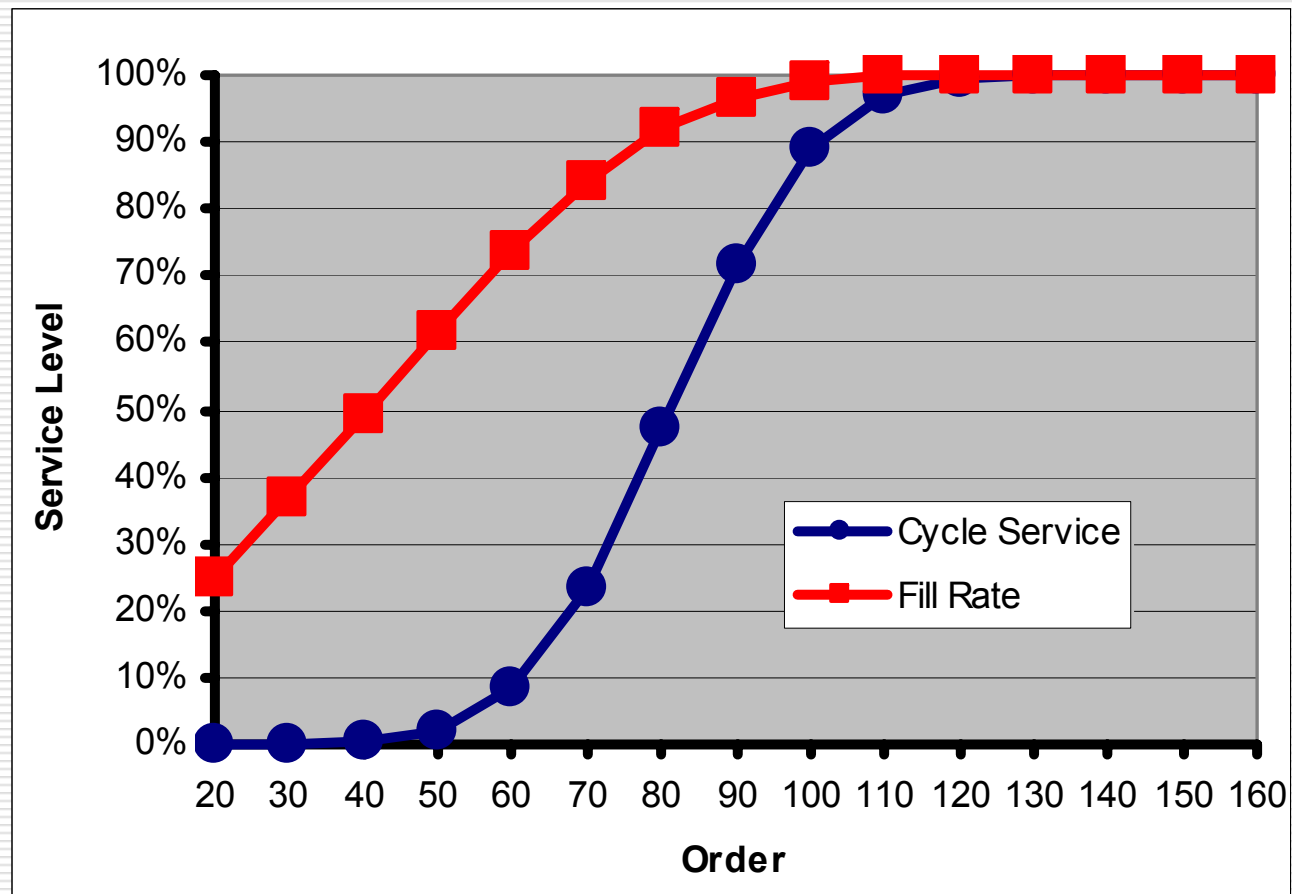
$$F[Q^*] = \frac{R - C}{R} \quad \text{and:} \quad Q^* = F^{-1} \left[\frac{R - C}{R} \right]$$

Level of Service

- ❑ Cycle Service – The probability that there will be a stock-out during a cycle
 - ❑ Cycle Service = $F(Q)$
 - ❑ Fill Rate - The probability that a specific customer will encounter a stock-out
 - ❑ $E[\text{Fill Rate}] = \frac{E[\text{Sales}]}{E[\text{Demand}]}$
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Level of Service

REV=\$15
COST=\$7



Normal Distribution of Demand

$$X \sim N(\mu, \sigma)$$

$$E[\text{sales}] = Q - \sigma \cdot (z \cdot \Phi(z) + \phi(z))$$

$$z = \frac{Q - \mu}{\sigma}$$

$$E[\text{Profit}] = (R - C) \cdot Q - R \cdot \sigma \cdot [z \cdot \Phi(z) + \phi(z)]$$

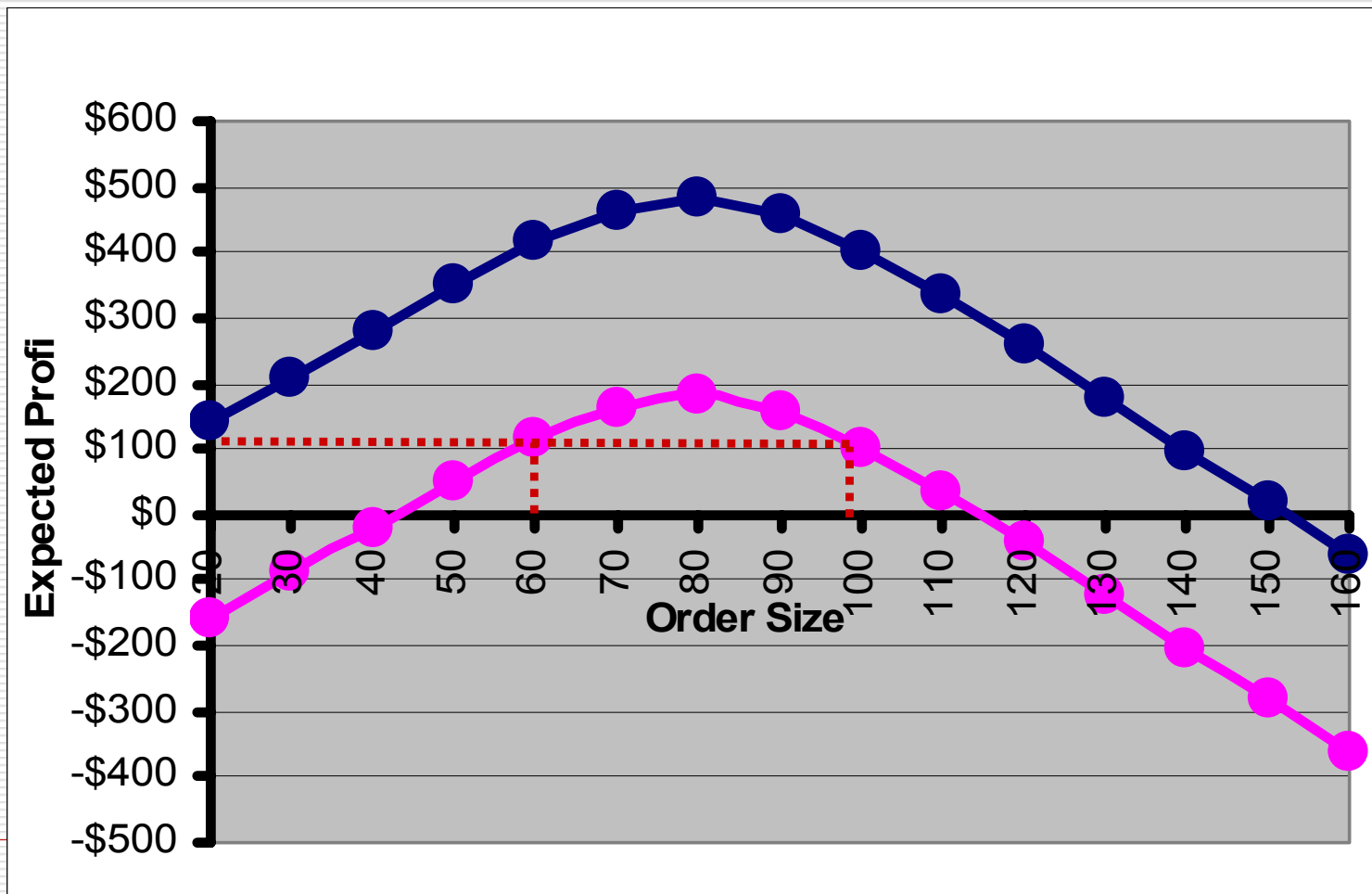


$$\begin{aligned} Q^* &= \text{NORMINV}\left(\frac{R - C}{R}\right) = \\ &= \text{NORMINV}\left(\frac{15 - 8}{15}\right) = 76 \text{ Mags} \end{aligned}$$

Incorporating Fixed Costs

REV=\$15
COST=\$7

With fixed costs of \$300/order:



Risk of Loss

REV=\$15
COST=\$7



$300/(15-7)=37.5$. Below that there is no possibility to make money even if we sell ALL the order.

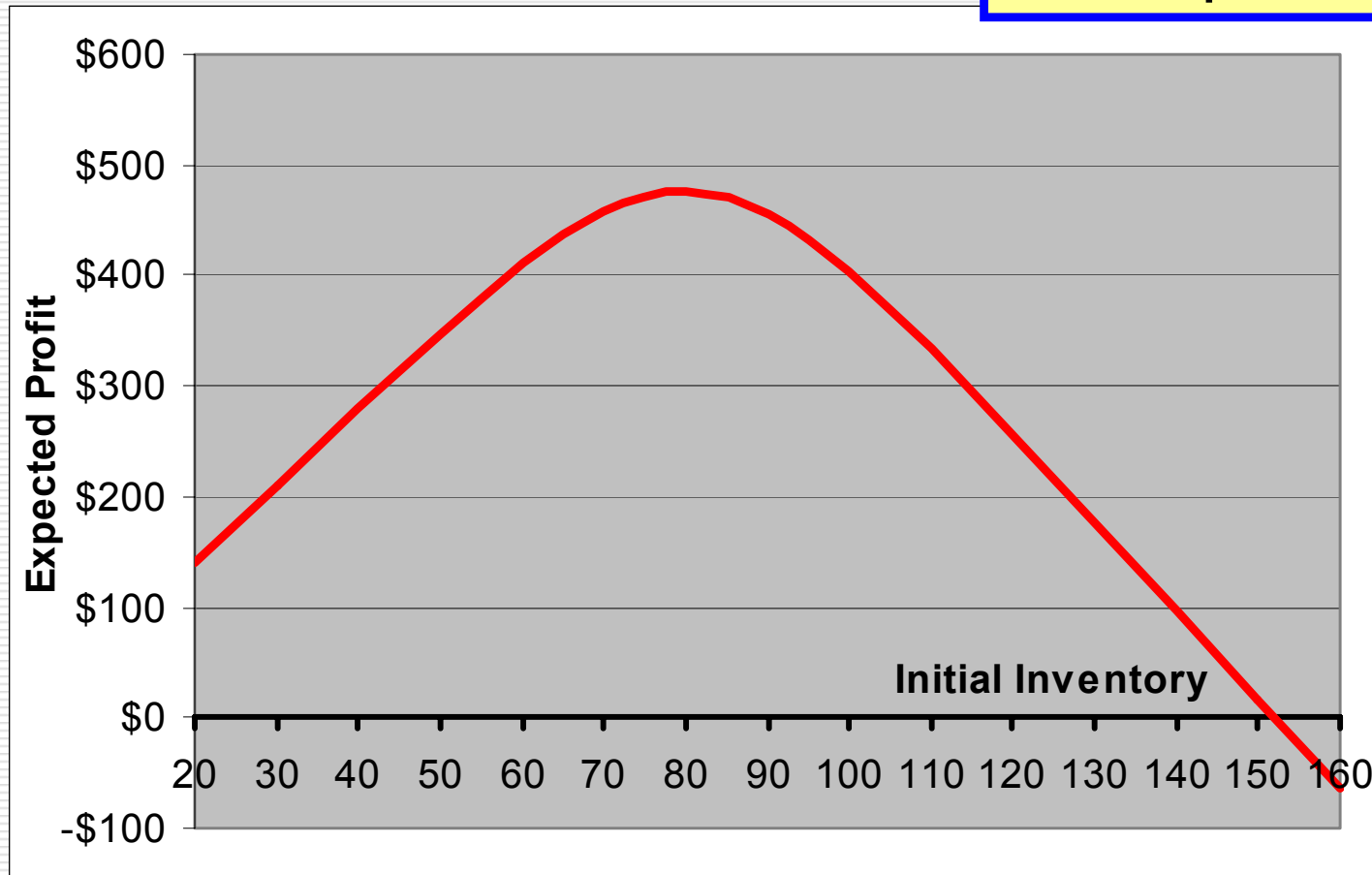
Ordering with Initial Inventory

- Given initial Inventory: Q_0 , how to order?
 1. Calculate Q^* as before
 2. If $Q_0 < Q^*$, order ($Q^* < Q_0$)
 3. If $Q_0 \geq Q^*$, order 0
- With fixed costs, order only if the expected profits from ordering are more than the ordering costs
 1. Set Q_{cr} as the smallest Q such that $E[\text{Profits with } Q_{cr}] > E[\text{Profits with } Q^*] - F$
 2. If $Q_0 < Q_{cr}$, order ($Q^* < Q_0$)
 3. If $Q_0 \geq Q_{cr}$, order 0

Ordering with Fixed Costs and Initial Inventory

REV=\$15
COST=\$7

Example: $F = \$150$



- If initial inventory is LE 46, order up to 80
- If initial inventory is GE 47, order nothing

Elastic Demand

□ $\mu = D(P); \sigma = f(\mu)$

□ Procedure:

1. Set P

2. Calculate μ

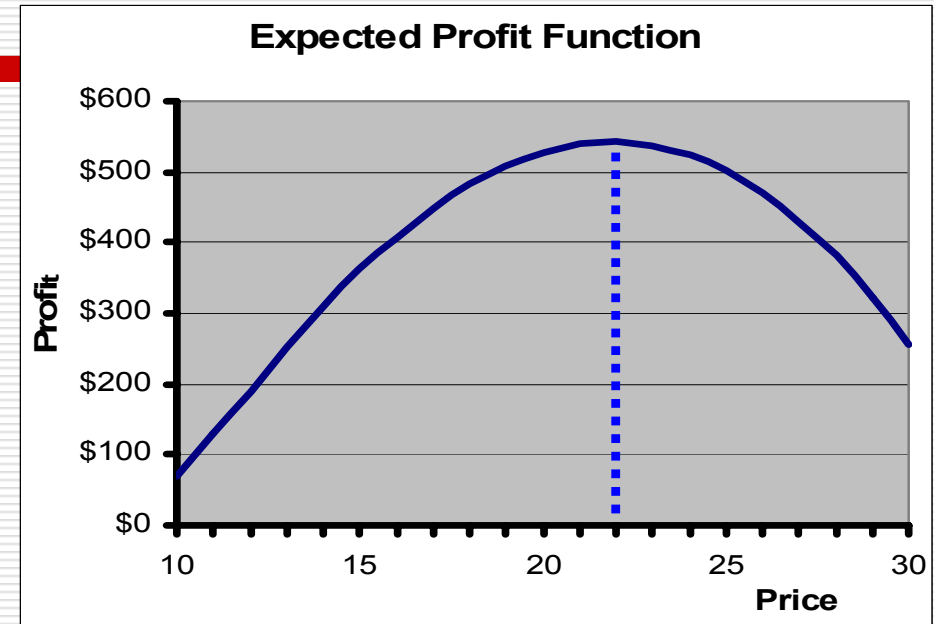
3. Calculate σ

4. $Q^* = F^{-1}\left(\frac{P-C}{P}\right)$

5. Calculate optimal expected profits as a function of P .

Rev = \$15
Cost = \$8
 $\mu(p) = 165 - 5 \cdot p$
 $\sigma = \mu/2$

$P^* = \$22$
 $Q^* = 65 \text{ Mag}$
 $\mu(p) = 56 \text{ Mag}$
 $\sigma = 28$
Exp. Profit = \$543



Any Questions?

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