Inventory Management IV

Safety Stock

Lecture 7
ESD.260 Fall 2003

Caplice

Assumptions: Basic EOQ Model

- Demand
  - Constant vs **Variable**
  - Known vs **Random**
  - **Continuous** vs Discrete
- Lead time
  - Instantaneous
  - **Constant** or Variable (deterministic/stochastic)
- Dependence of items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous** vs Periodic
- Number of Echelons
  - **One** vs Many
- Capacity / Resources
  - **Unlimited** vs Limited
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand
  - None
  - **All orders are backordered**
  - **All orders are lost**
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - Finite Period
  - **Infinite**
- Number of Items
  - **One**
  - Many

Fundamental Purpose of Inventory

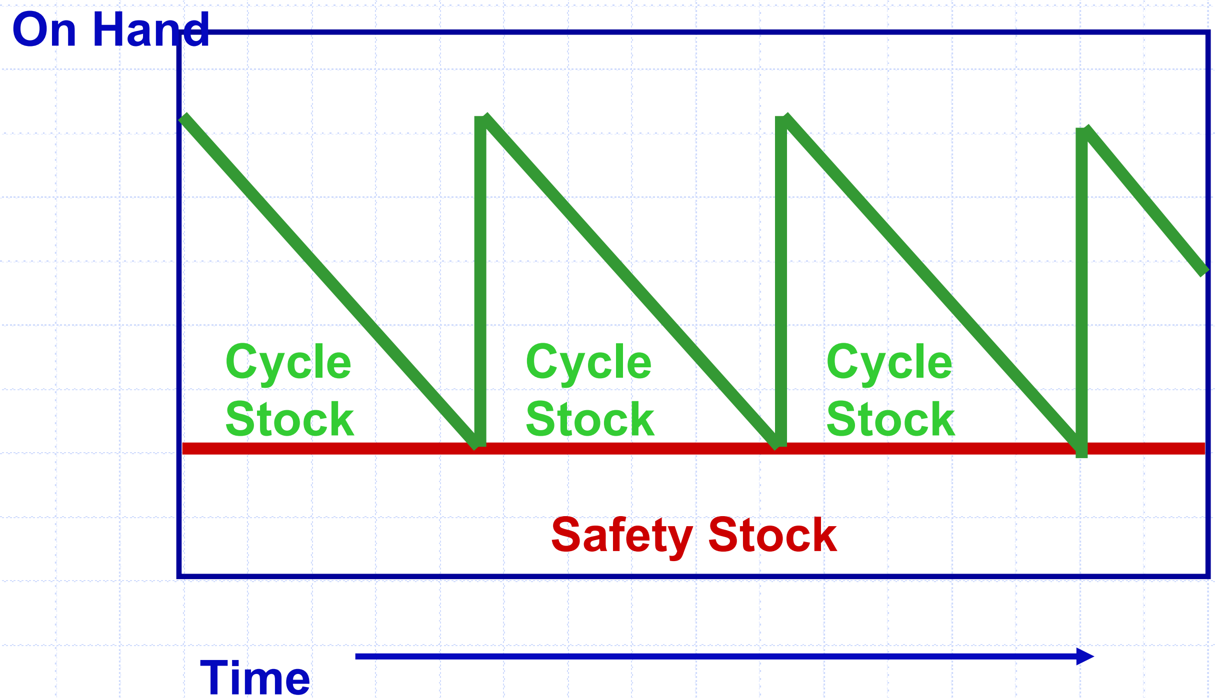
To buffer uncertainties in:
- **supply,**
- **demand, and/or**
- **transportation**

the firm carries safety stocks.

To capture scale economies in:
- **purchasing,**
- **production, and/or**
- **transportation**

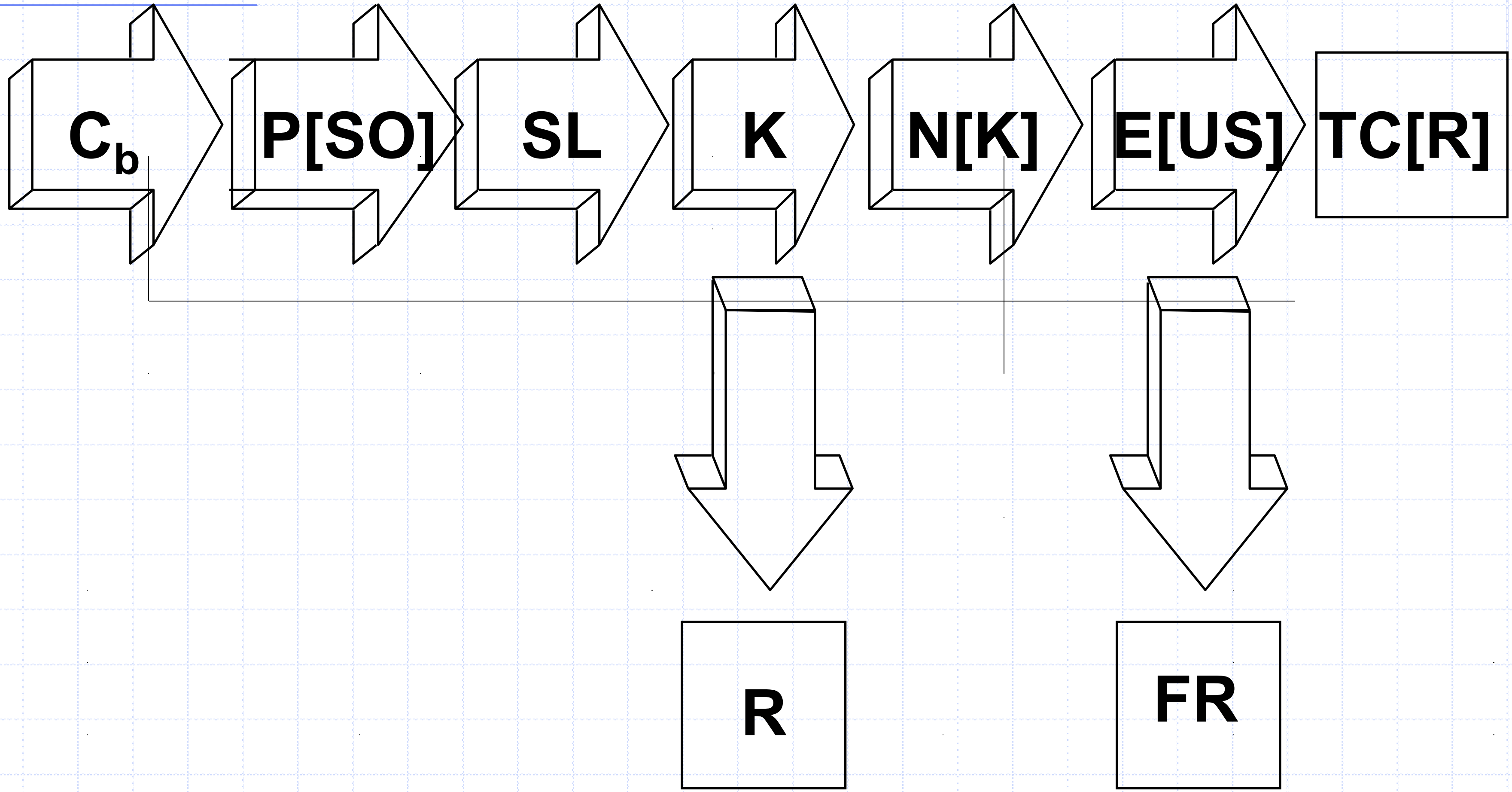
the firm carries cycle stocks.

Cycle Stock and Safety Stock



What should my inventory policy be?
(how much to order when)
What should my safety stock be?
What are my relevant costs?

Preview: Safety Stock Logic



Determining the Reorder Point

$$R = d' + k\sigma$$

Reorder Point

Estimated demand over the lead time (F_t)

Safety Stock

k = SS factor

σ = RMSE

$$k = \frac{(R - d')}{\sigma}$$

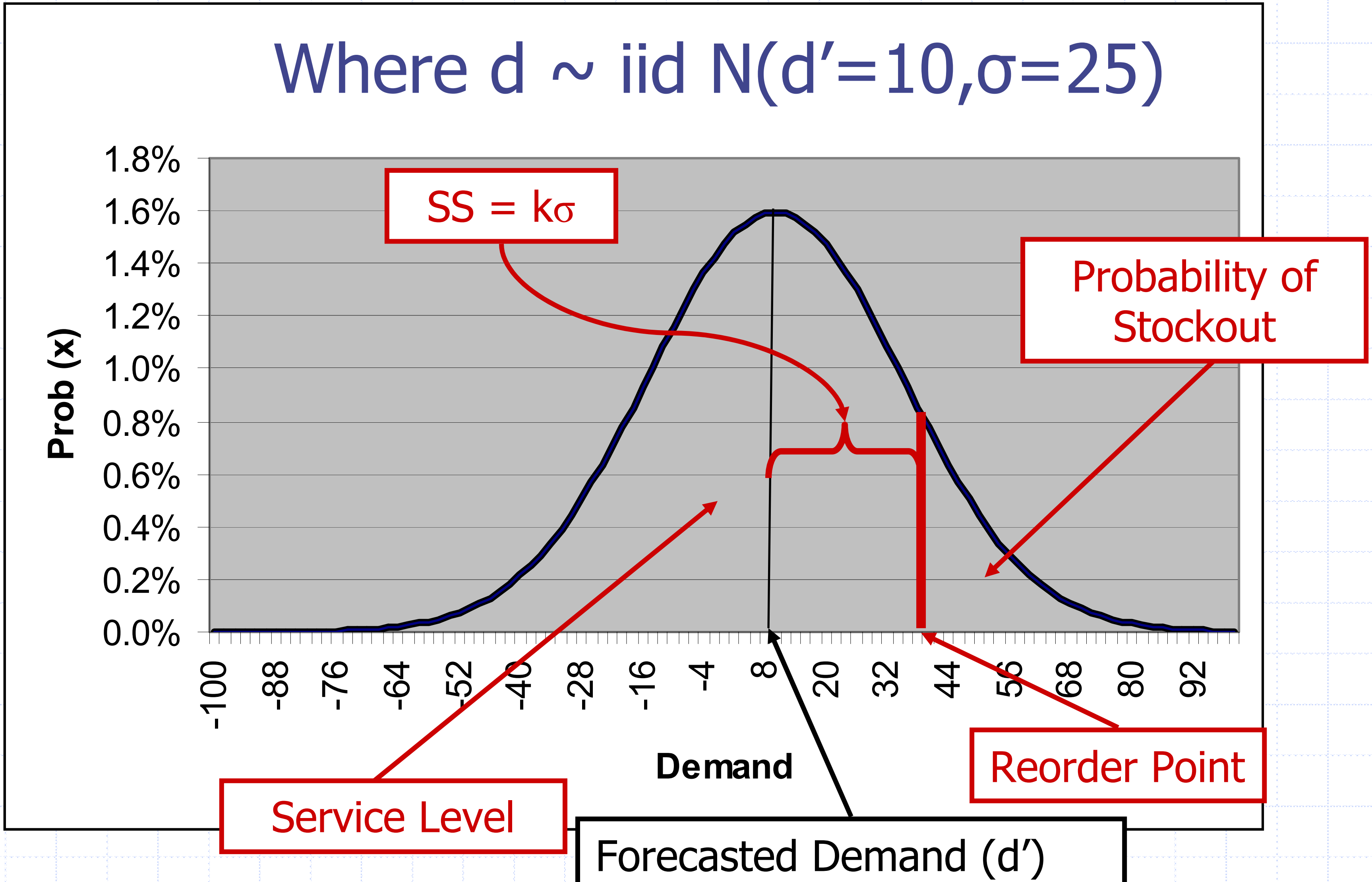
Note

1. We usually pick k for desired stock out probability
2. Safety Stock = R – d’

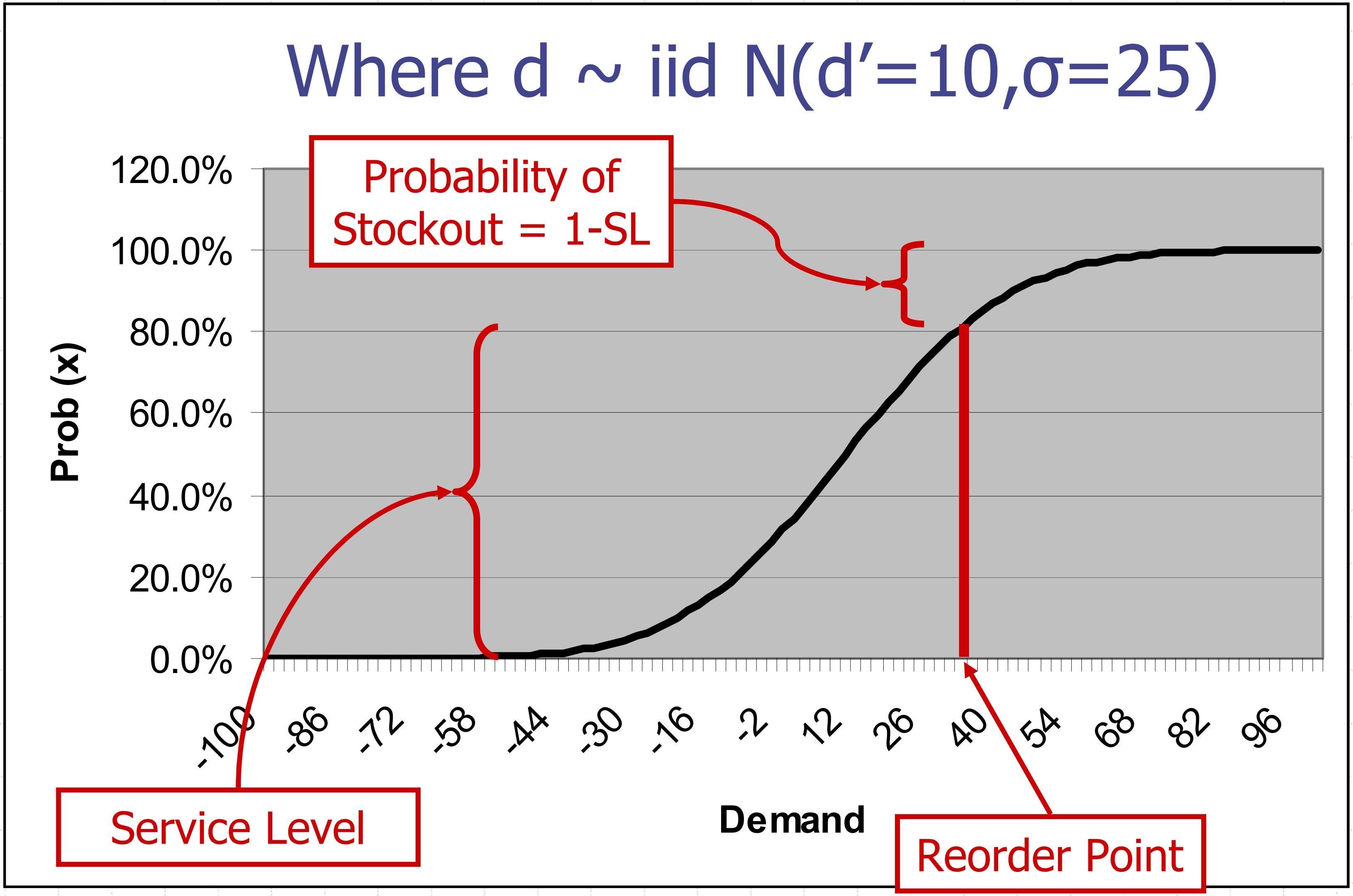
Define Some Terms

- Safety Stock Factor (k)
  - Amount of inventory required in terms of standard deviations worth of forecast error
- Stockout Probability = P[d > R]
  - The probability of running out of inventory during lead time
- Service Level = P[d ≤ R] = 1- P[SO]
  - The probability of NOT running out of inventory during lead time

Service Level and Stockout Probability



Cumulative Normal Distribution



Finding SL from a Given K

If I select a K = 0.42

Using a Table of Cumulative Normal Probabilities . . .

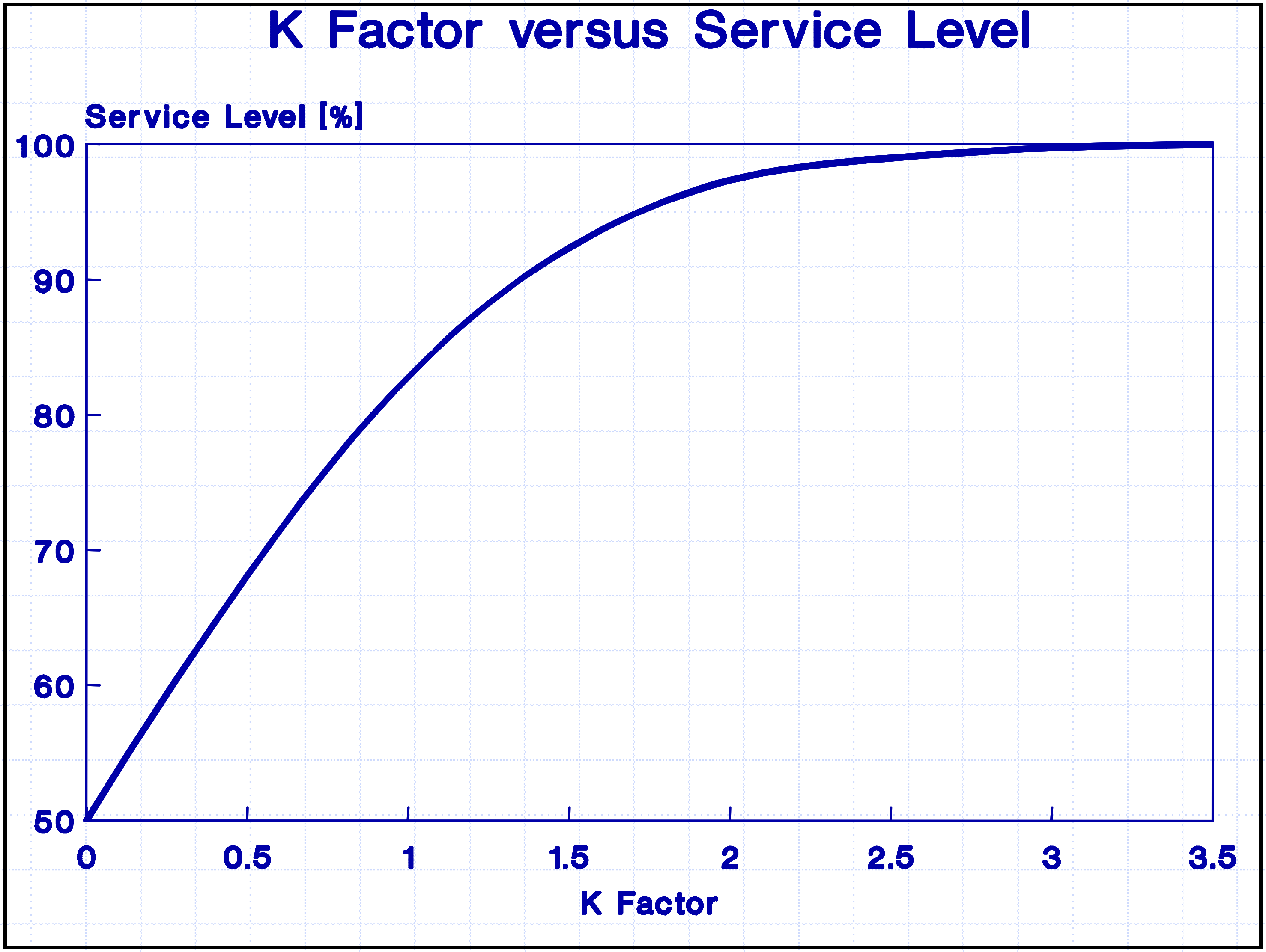
K	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
0.5	0.6915	0.6950	0.6985	0.7019	0.7054
0.6	0.7257	0.7291	0.7324	0.7357	0.7389
0.7	0.7580	0.7611	0.7642	0.7673	0.7704
0.8	0.7881	0.7910	0.7939	0.7967	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	238	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	485	0.8508

. . . then my Service Level is this value.

Or, in Excel, use the function:

SL=**NORMDIST**(kσ+d', d', σ, TRUE)

K Factor versus Service Level



Safety Stock and Service Level

Example:

if d ~ iid Normal (d'=100,σ=10)

What should my SS & R be?

P[SO]	SL	k	Safety Stock	Reorder Point
.50	.50	0	0	100
.10	.90	1.28	13	113
.05	.95	1.65	17	117
.01	.99	2.33	23	123

Service Level

- What is exactly is Service Level?

Service Level	**IS**	**IS NOT**
Probability that a stockout does not occur during lead time		
The fraction of demand satisfied from stock		
Probability that stock is available		
Fraction of order cycles during which no SO's occur		
Probability that all demand will be satisfied from on hand stock w/in any given replenishment cycle		
Fraction of time that stock is available in the system		
Fill rate for or availability of an item		

So, how do I find Item Fill Rate?

- Fill Rate
  - Fraction of demand met with on-hand inventory
  - Based on each replenishment cycle

$$FillRate = \frac{OrderQuantity - E[UnitsShort]}{OrderQuantity}$$

- But, how do I find Expected Units Short?
  - More difficult
  - Need to calculate a partial expectation:

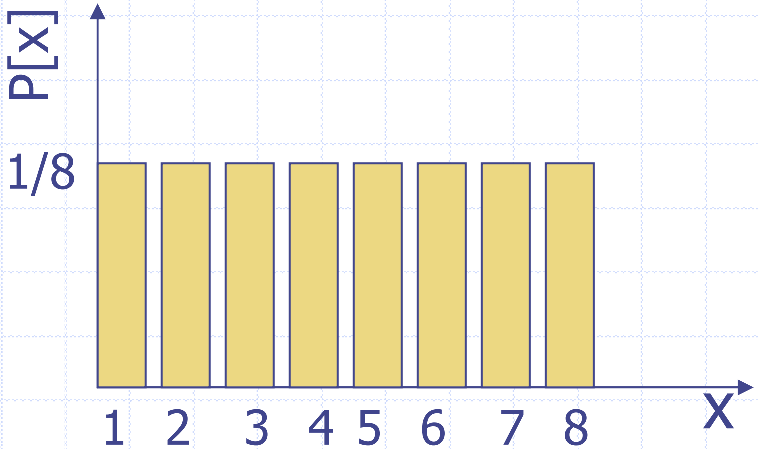
Expected Units Short

Consider both continuous and discrete cases

Looking for expected units short per replenishment cycle.

$$E[US] = \sum_{x=R}^{\infty} (x - R)\, p[x]$$

$$E[US] = \int_{R}^{\infty} (x_o - R)\, f_x(x_o)\, dx_o$$



What is E[US] if R=5?

For normal distribution we have a nice result:

E[US] = σN[k]

Where N[k] = Normal Unit Loss Function

Found in tables or formula

The N[k] Table

A Table of Unit Normal Loss Integrals

K	.00	.01	.02	.03	.04
0.0	.3989	.3940	.3890	.3841	.3793
0.1	.3509	.3464	.3418	.3373	.3328
0.2	.3069	.3027	.2986	.2944	.2904
0.3	.2668	.2630	.2592	.2555	.2518
0.4	.2304	.2270	.2236	.2203	.2169
0.5	.1978	.1947	.1917	.1887	.1857
0.6	.1687	.1659	.1633	.1606	.1580
0.7	.1429	.1405	.1381	.1358	.1334
0.8	.1202	.1181	.1160	.1140	.1120
0.9	.1004	.09860	.09680	.09503	.09328
1.0	.08332	.08174	.08019	.07866	.07716

Item Fill Rate

$$FillRate = \frac{OrderQuantity - E[UnitsShort]}{OrderQuantity} = FR$$

$$FR = 1 - \frac{E[US]}{Q} = 1 - \frac{\sigma N[k]}{Q}$$

$$N[k] = \frac{(1 - FR)Q}{\sigma}$$

So, now we can look for the k that achieves our desired fill rate.

Item Fill Rate Example

Find fill rate for R=250, 300, 350, and 500 where, Q = 500 units and demand ~ N(d'=200, σ=100)

R	k	SL	P[SO]	N[k]	N[k]s	FR
250	0.5	.69	.31	.1978	19.78	.9604
300	1.0	.84	.16	.0833	8.33	.9833
350	1.5	.93	.07	.0293	2.93	.9941
500	3.0	.999	.001	.0003	.03	.9999

Note:

- Fill rate is usually much higher than service level
- Fill rate depends on both R and Q
- Q determines the number of exposures for an item