Inventory Management II

Lot Sizing

Lecture 5
ESD.260 Fall 2003

Caplice

Outline

- EOQ Model
- Sensitivity Analysis
- Discounting
  - All Units
  - Incremental
  - One Time Special

Assumptions: Basic EOQ Model

- Demand
  - **Constant** vs Variable
  - **Known** vs Random
  - **Continuous** vs Discrete
- Lead time
  - **Instantaneous**
  - Constant or Variable (deterministic/stochastic)
- Dependence of items
  - **Independent**
  - Correlated
  - Indentured
- Review Time
  - **Continuous** vs Periodic
- Number of Echelons
  - **One** vs Many
- Capacity / Resources
  - **Unlimited** vs Limited
- Discounts
  - **None**
  - All Units or Incremental
- Excess Demand
  - **None**
  - All orders are backordered
  - Lost orders
  - Substitution
- Perishability
  - **None**
  - Uniform with time
- Planning Horizon
  - Single Period
  - Finite Period
  - **Infinite**
- Number of Items
  - **One**
  - Many

Notation

TC = Total Cost (dollar/time)

D = Average Demand (units/time)

C_o = Ordering Cost (dollar/order)

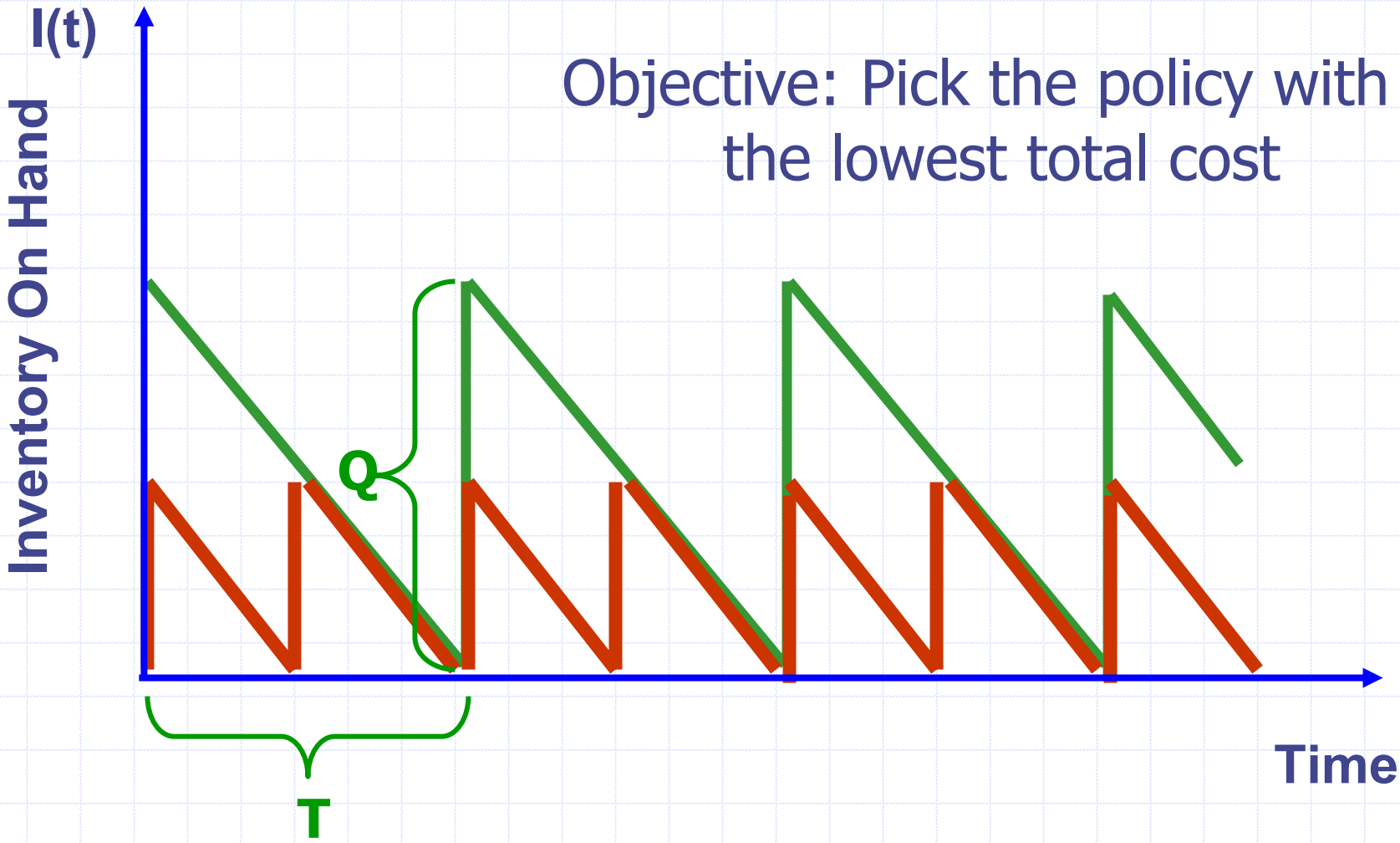
C_h = Holding Cost (dollars/dollars held/time)

Cp = Purchase Cost (dollars/unit)

Q = Order Quantity (units/order)

T = Order Cycle Time (time/order)

Lot Sizing: Many Potential Policies



Objective: Pick the policy with the lowest total cost

What is the total cost?

TC = Purchase + Order + Holding + Shortage Cost

Which costs are relevant?

- Purchase Costs
- Ordering Costs
- Holding Costs
- Shortage Costs

$$TC(Q) = C_o\left(\frac{D}{Q}\right) + C_h C_p\left(\frac{Q}{2}\right)$$

Economic Order Quantity (EOQ)

$$TC[Q] = \frac{C_o D}{Q} + \frac{C_h C_p Q}{2}$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h C_p}}$$

$$TC^* = \sqrt{2DC_o C_h C_p}$$

Simple EOQ Example

Annual demand of widgets is 2,000. The cost of ordering is $500. Widgets are procured for $50 each and are sold for $95 each. Holding cost is estimated to be 25%.

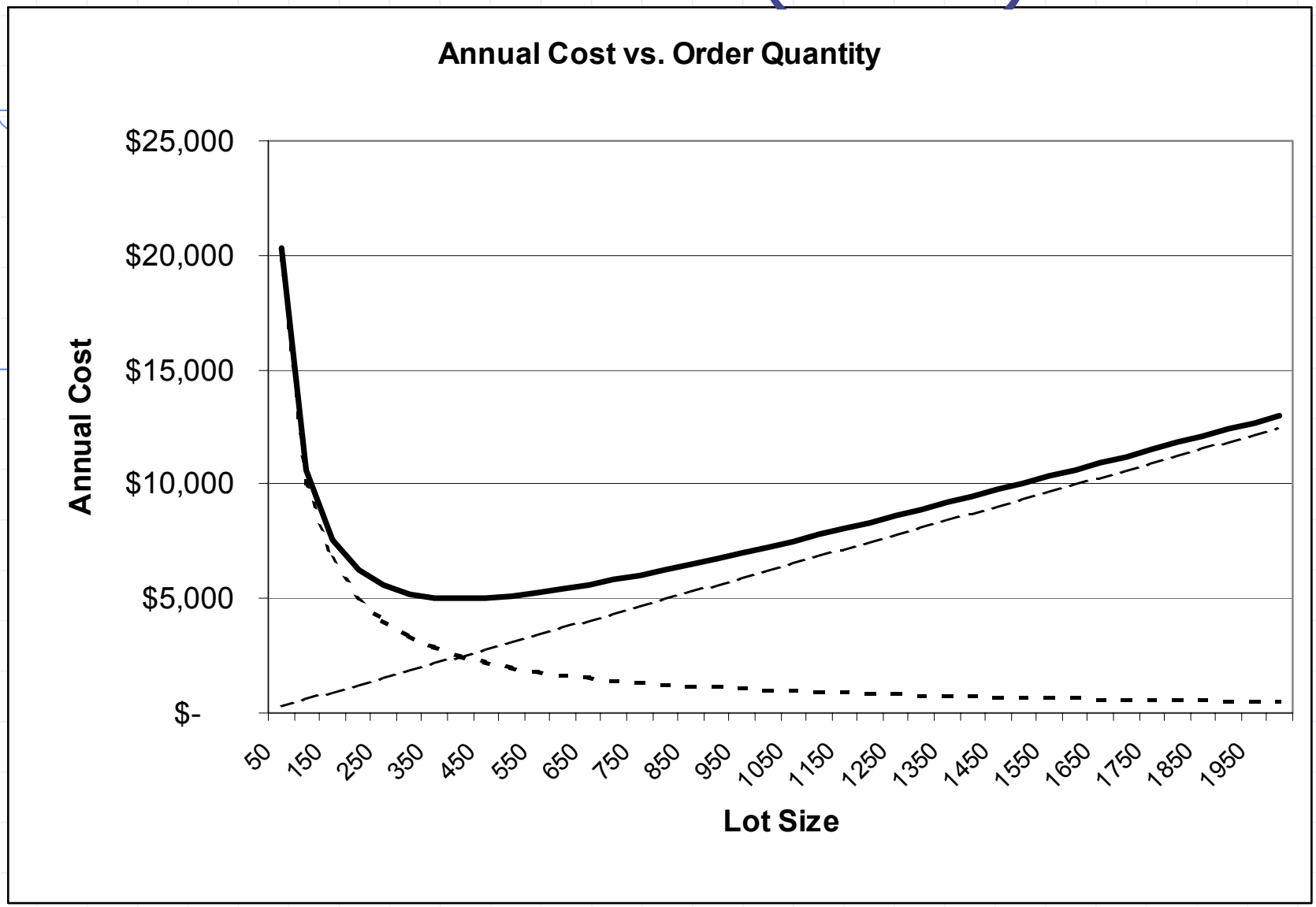
What is the EOQ, Total Cost, and Cycle Time?

Q* = 400 units

TC* = $5,000 / year

T* = 0.20 years or 73 days

Total Cost versus Lot (Order) Size



The Effect of Non-Optimal Q

Q	DC_o/Q	$C_hC_pQ/2$	TC
2000	$500	$12,500	$13,000
500	$2,000	$3,125	$5,125
400	**$2,500**	**$2,500**	**$5,000**
200	$5,000	$1,250	$6,250
20	$50,000	$125	$50,125

So, how sensitive is TC to Q?

Sensitivity of EOQ

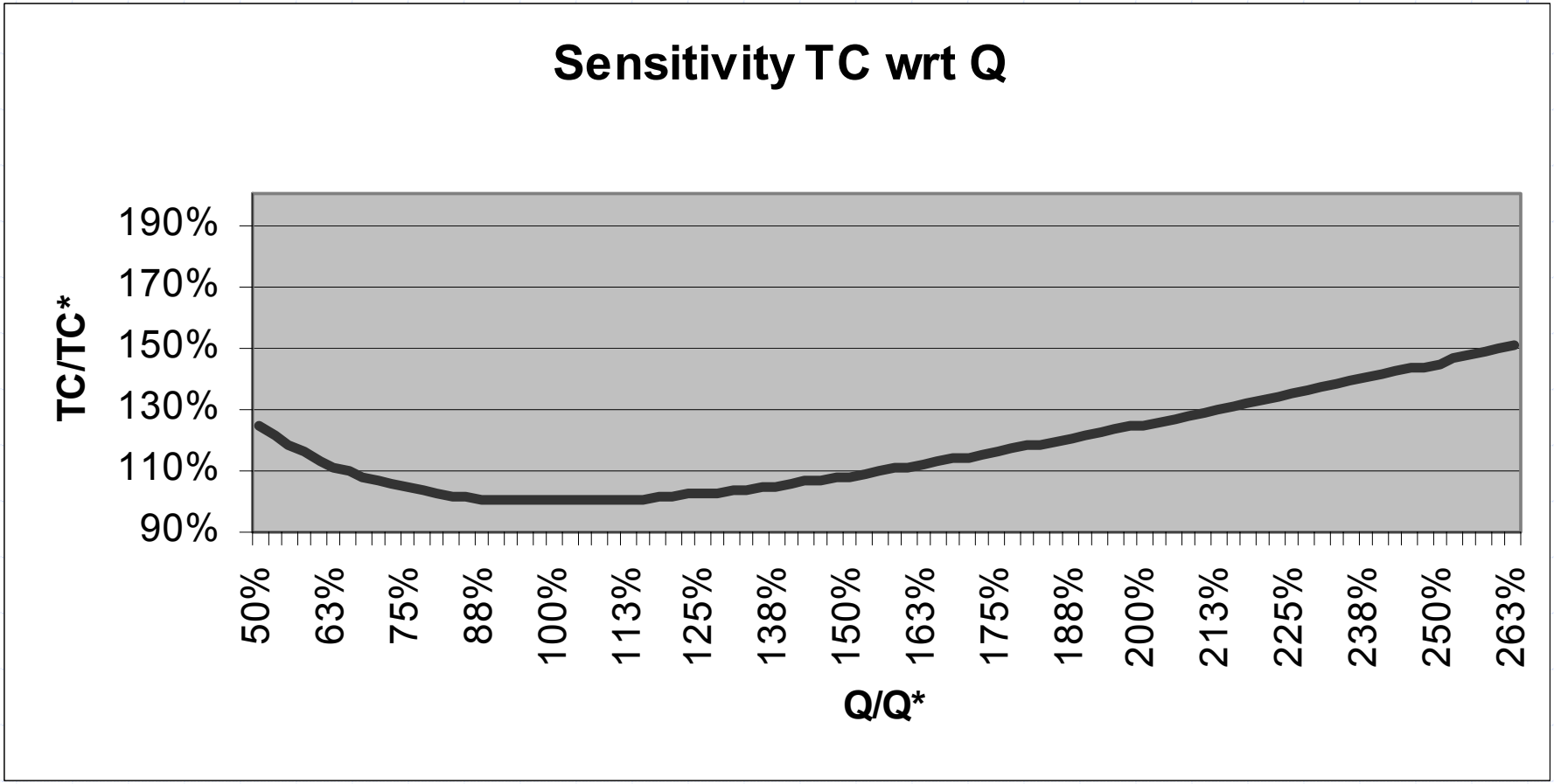
- How does TC change with Q?

$$TC(Q) = C_o\left(\frac{D}{Q}\right) + C_p C_h\left(\frac{Q}{2}\right)$$

$$TC(Q^*) = \sqrt{2C_o C_p C_h D}$$

$$\frac{TC(Q)}{TC(Q^*)} = \frac{1}{2}\left[\frac{Q^*}{Q} + \frac{Q}{Q^*}\right]$$

Sensitivity of EOQ



Sensitivity of EOQ

- What if our estimate of demand was off?
  - E = Actual Demand / Estimated Demand
  - Q′ = Estimated EOQ
  - Q* = Actual EOQ, given actual demand that occurred

$$E = \frac{D}{D'} = \frac{Actual}{Estimate}$$

$$Q' = \sqrt{\frac{2D'C_o}{C_h C_p}}$$

$$\frac{Q^*}{Q'} = \sqrt{E}$$

$$\frac{TC'}{TC^*} = \frac{1+E}{2\sqrt{E}}$$

Sensitivity of EOQ to Parameter Error

So, for D’ = 2000 units/year, if the actual demand was …

D	E	Q*/Q'	TC'/TC*
200	0.10	0.32	1.74
1,000	0.50	0.71	1.06
1,500	0.75	0.87	1.01
1,800	0.90	0.95	1.00
2,000	1.00	1.00	1.00
3,000	1.50	1.22	1.02
4,000	2.00	1.41	1.06
20,000	10.00	3.16	1.74

Insights from EOQ

- There is a direct trade off between lot size and total inventory
- Total cost is relatively insensitive to changes
  - Very robust with respect to changes in:
    - Q – rounding of order quantities
    - D – errors in forecasting
    - C_h, C_o, C_p – errors in cost parameters
- Thus, EOQ is widely used despite its highly restrictive assumptions

Introduce Discounts to Lot Sizing

- Types of discounts
  - All units discount
  - Incremental discount
  - One time only discount
- How will different discounting strategies impact your lot sizing decision?

All Units Discount

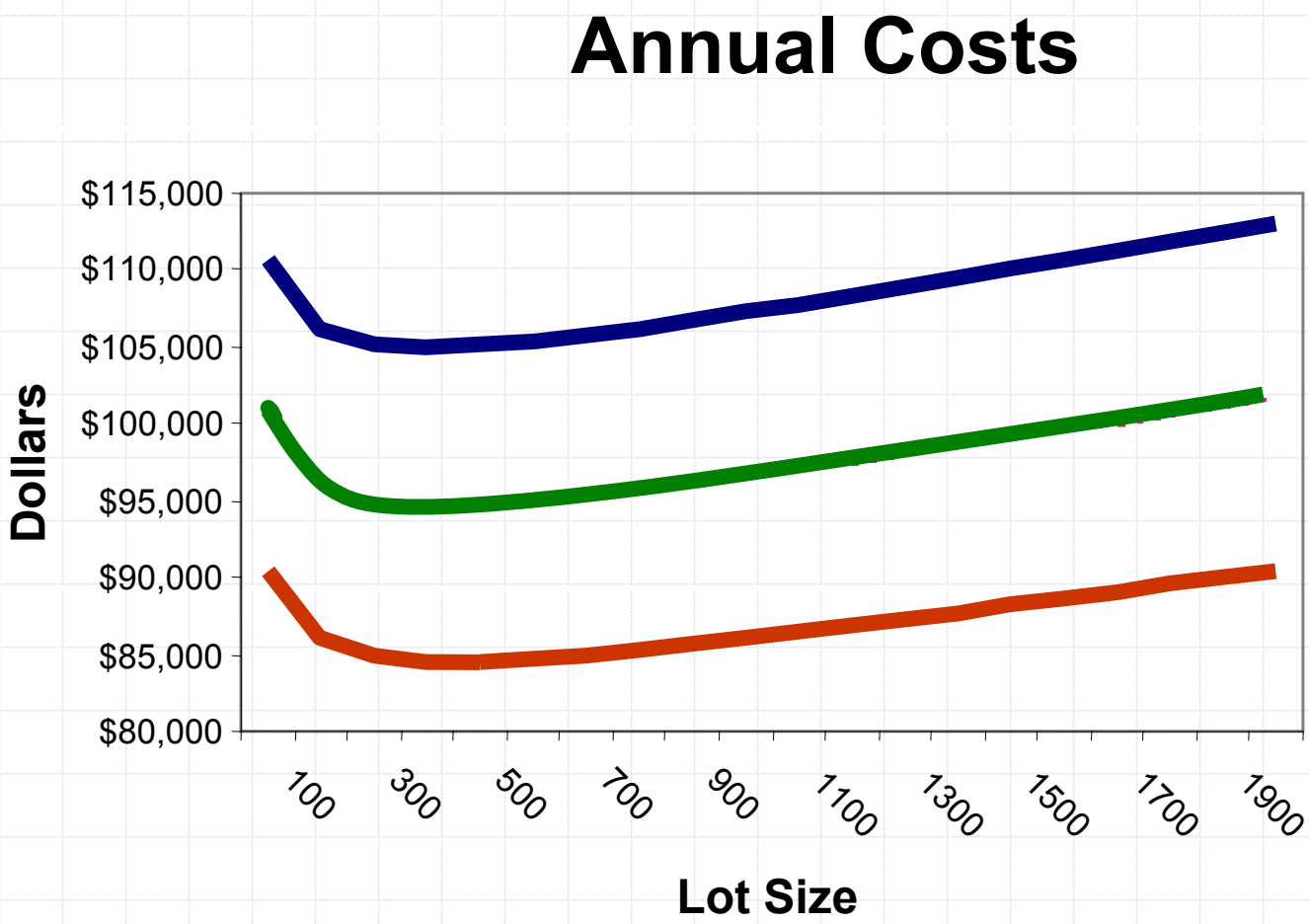
Unit Price $[C_{pi}]$	Price Break Quantity $[PBQ_I]$
$50.00	0
$45.00	500
$40.00	1000

All Units Discount

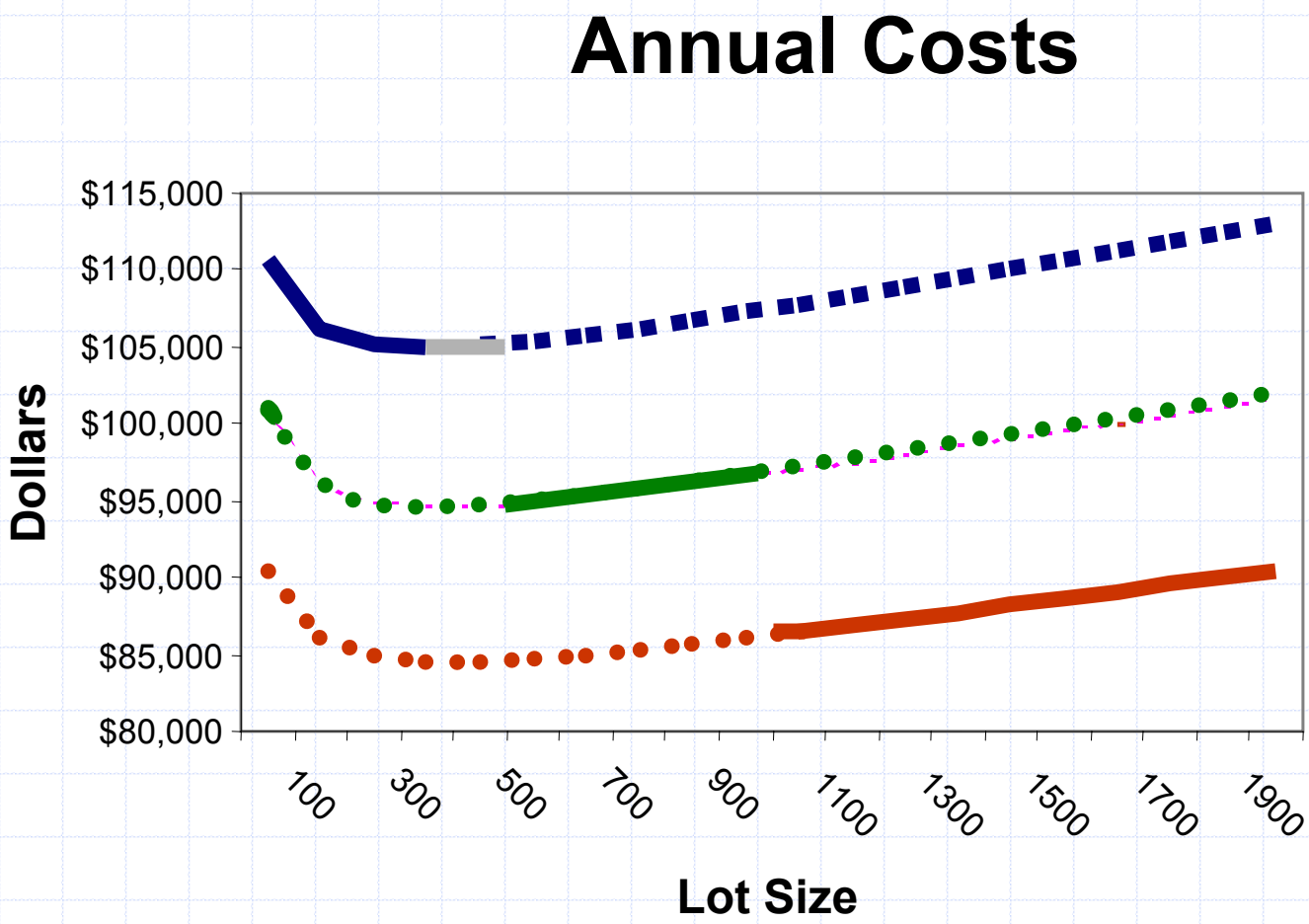
Need to introduce purchase cost into TC function

$$TC[Q, C_{pi}] = DC_{pi} + \frac{C_o D}{Q} + \frac{C_h C_{pi} Q}{2}$$

All Units Discount



All Units Discount



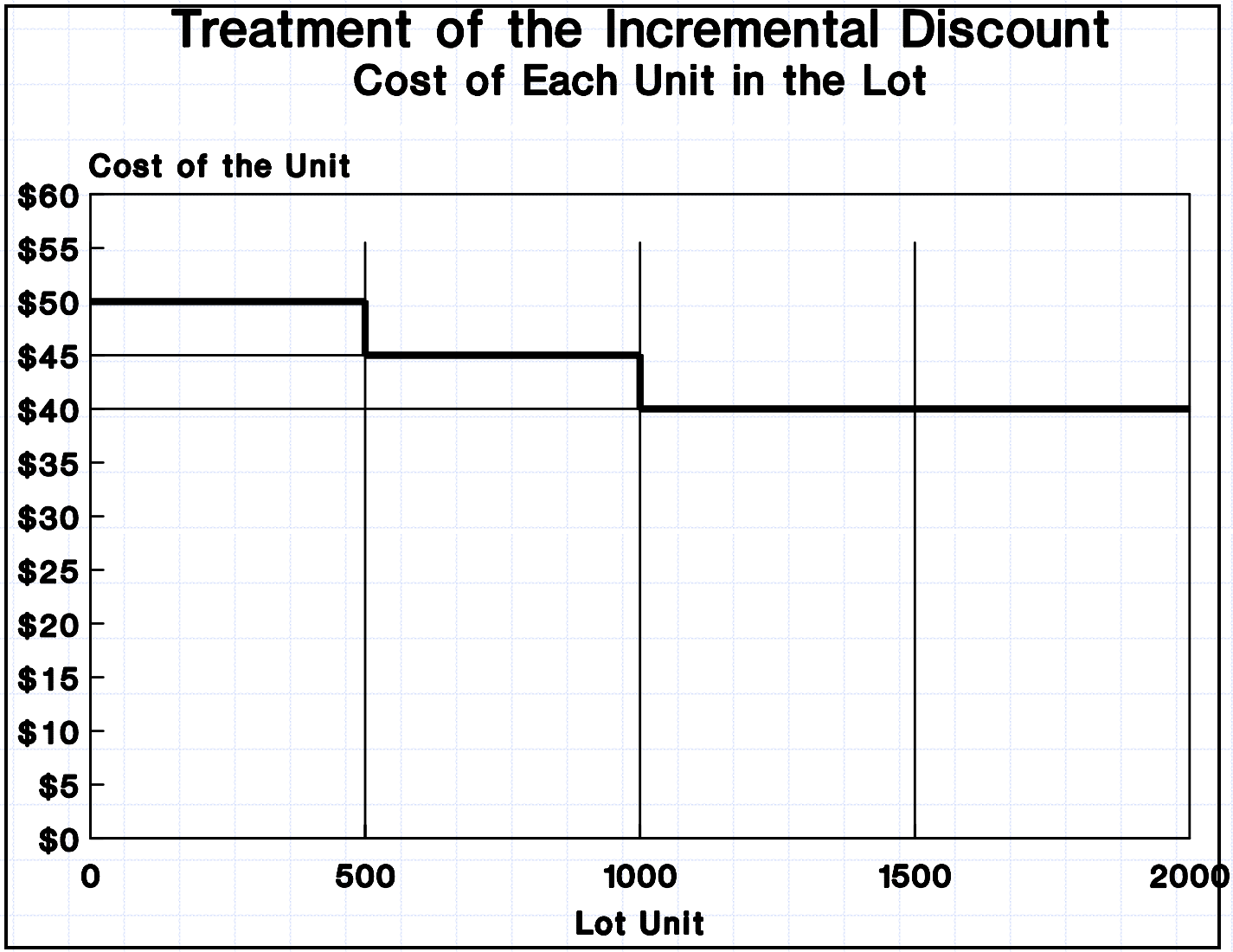
All Units Discount: Method

- Same Example:
  - D=2000 Units/yr
  - C_h=.25
  - C_o=$500

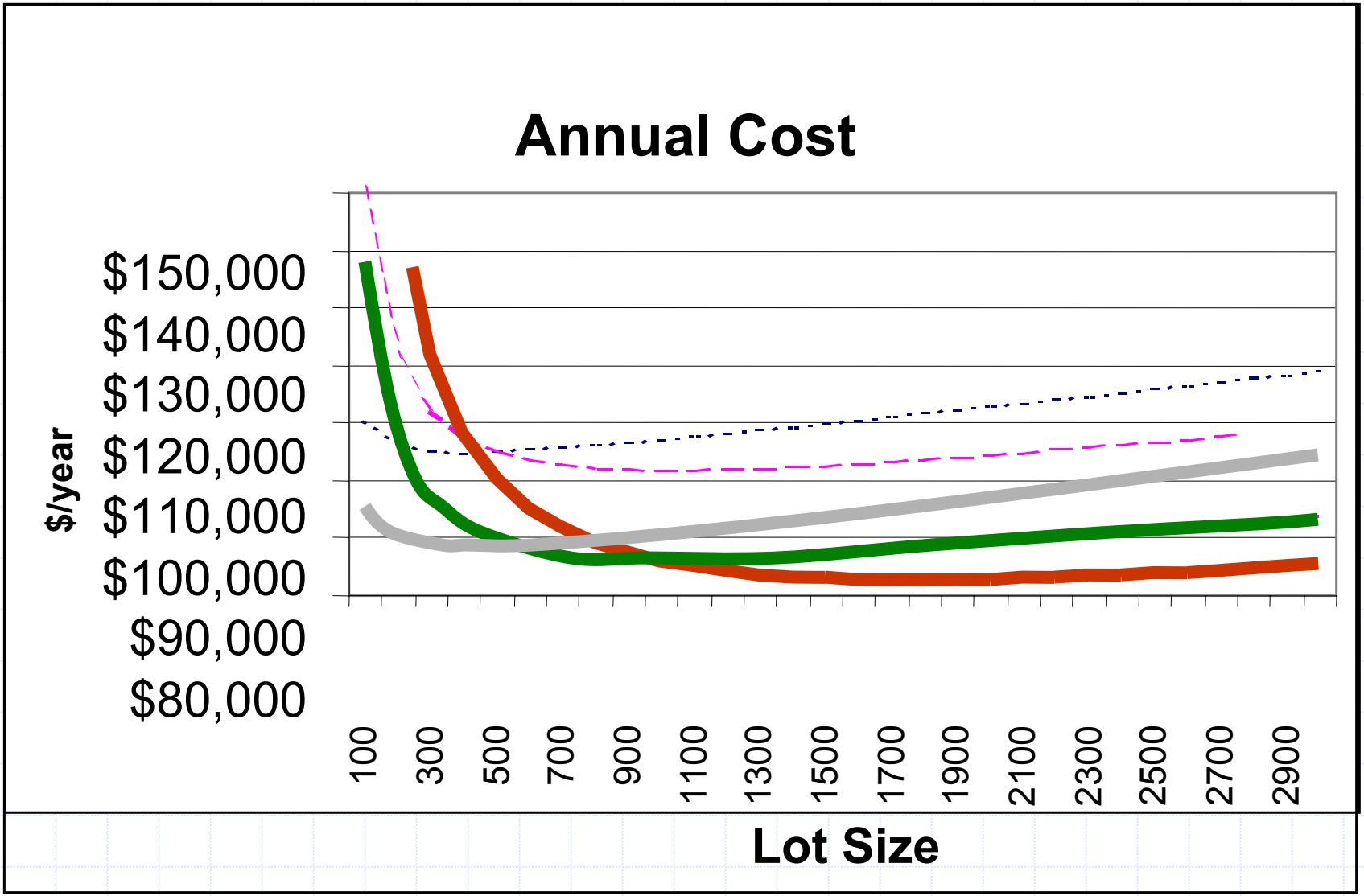
C_{pi} Price Breaks:
- $50 for 0 to <500 units
- $45 for 500 to <1000 units
- $40 for 1000+ units

1	C_{pi}	$40.00	$45.00	$50.00
2	PBQ	1000	500	0
3	EOQ[C_{pi}]	447	421	400
4	Q_{pi}	1000	500	400
5	DC_{pi}	$80,000	$90,000	$100,000
6	C_oD/Q_{pi}	$1,000	$2,000	$2,500
7	$C_hC_{pi}Q_{pi}/2$	$5,000	$2,812	$2,500
8	TC[Q_{pi}]	$86,000	$94,812	$105,000

Incremental Discount



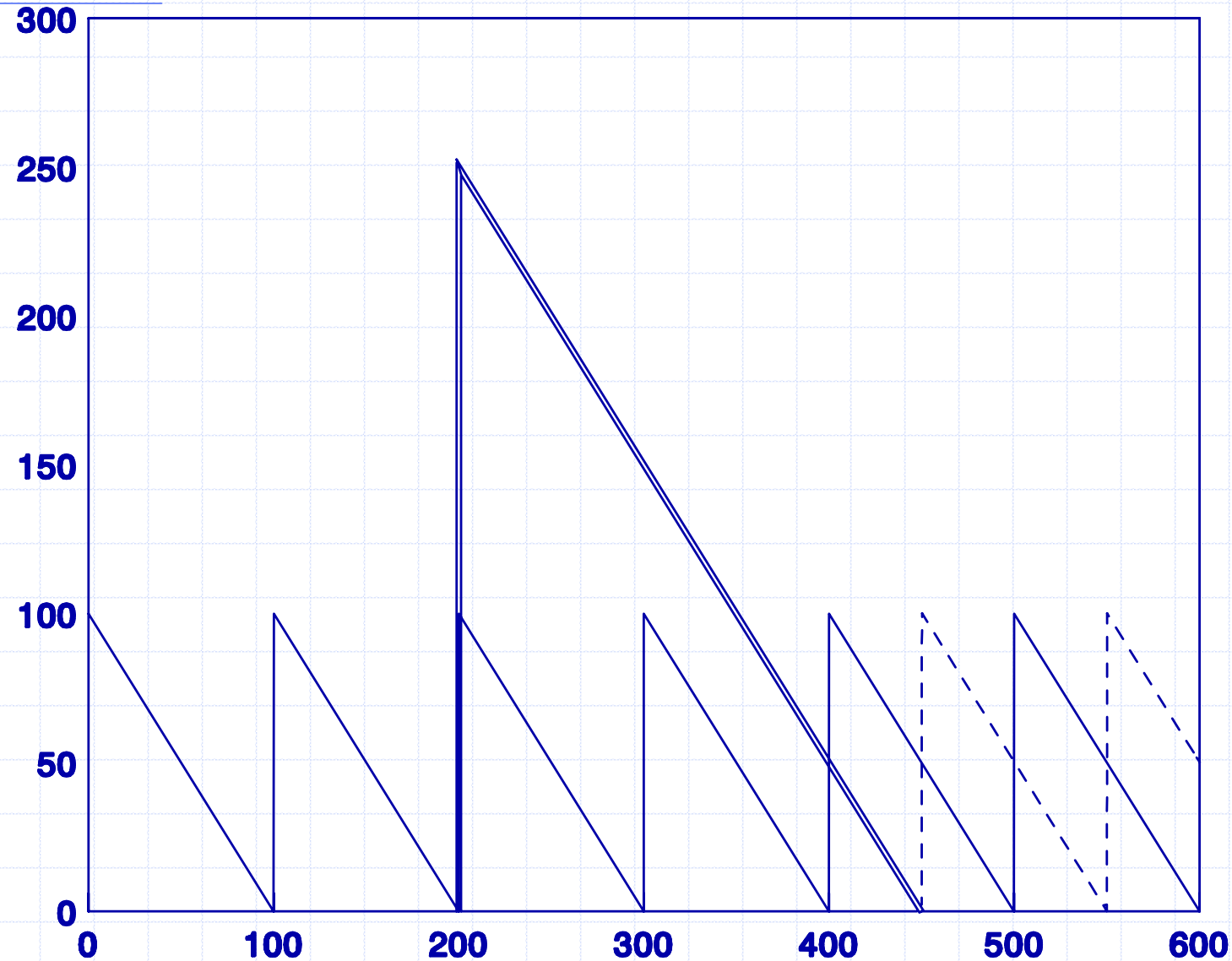
Incremental Discount



Incremental Discount

	Index	i=3	i=2	i=1
1	C_{pi}	$40.00	$45.00	$50.00
2	PBQ_i	1000	500	0
3	F_i	$7500	$2500	$0
4	$EOQ[C_{pi}]$	1789	1033	400
5	Q_{pi}	1789		400
6	C_{pe}	$44.19		$50.00
7	DC_{pe}	$88,384.57		$100,000
8	C_oD/Q_{pi}	$558.97		$2,500
9	$(C_h C_{pe} Q_{pi})/2$	$9,882.50		$2,500
10	$TC[Q_{pi}]$	$98,826.04		$105,000

One Time Discount



One Time Discount

- Let,
  - C_{pg} = One time deal purchase price ($/unit)
  - Q_g = One time special order quantity (units)
  - TC_{sp}=TC over time covered by special purchase ($)
- Then,

$$TC_{sp}[Q_g/D] = C_{pg}Q_g + C_hC_{pg}\frac{Q_g}{2}\frac{Q_g}{D} + C_o$$

One Time Discount

$$TCsp[Q_g/D] = C_{pg}Q_g + C_h C_{pg}\frac{Q_g}{2}\frac{Q_g}{D} + C_o$$

$$TC_{nsp}[Q_g/D] = C_{pg}Q_w + C_p(Q_g - Q_w) + C_h C_{pg}\frac{Q_w}{2}\frac{Q_w}{D} + C_h C_p \frac{Q_w}{2}\frac{(Q_g - Q_w)}{D} + C_o\frac{Q_g}{Q_w}$$

One Time Discount

$$Q_g^* = \frac{(C_p - C_{pg})D}{C_{pg} C_h} + \frac{C_p Q_w}{C_{pg}}$$

$$SAVINGS[Q_g^*] = \frac{C_o C_{pg}}{C_p} \left\lceil \frac{Q_g^*}{Q_w} - 1 \right\rceil^2$$

One Time Discount

A quick numerical example...

D = 2000 units / year
C_o = \$500.00 / order
C_h = 25%
C_p = \$50.00 / unit
Q^* = 400 units
TC^* = \$5000.00 / year

C_{pg} = \$45.00 / unit
Q^*_g = 1333
Savings* = \$2448.25