Work-in-Progress: Worst-Case Execution-Time Measurement Techniques for Nonlinear Model Predictive Controllers

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***Abstract*—Real-time safety-critical systems using nonlinear model predictive control (NMPC) require guaranteed worst-case execution time (WCET) bounds. Measuring the WCET of NMPC is challenging. We compare three model-informed ways of generating WCET measurement tests. The first two involve uniform partitioning of the state space and simulation; the third uses the concept of complexity certification for active-set solvers. We validate our approach on two benchmarks: an inverted cart pendulum and motion planning of a bicycle. Our results demonstrate a trade-off between the number of tests and the degree of WCET underestimation.**

***Index Terms*—Nonlinear MPC, worst-case execution time, complexity certification**

I. Introduction

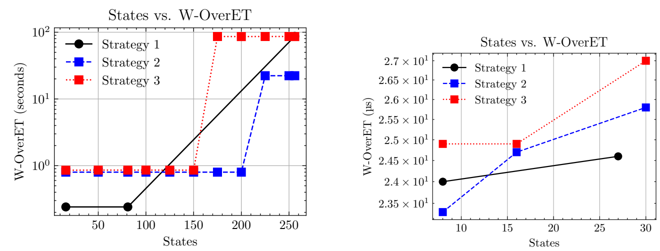
WCET measurement begins with obtaining a representative test set. Common approaches include *exhaustive exploration* of partitioned state space and *fixed-step simulation* from an initial state. The former becomes computationally expensive with more state variables, while the latter can miss corner cases, leading to WCET underestimation. We study a new strategy using a complexity certification algorithm within a parametric optimization framework. Our contributions include (1) approximating the NMPC via a parametric linear program, (2) a test-generation strategy using complexity certification (for the linearized MPC) to identify critical execution paths, and (3) empirical validation on benchmarks.

II. Complexity Certification

Arnström's recent work, [1], on implicit linear MPC introduces a complexity certification algorithm for generating test sets for OCPs, formulated as *parametric constrained optimization problem* of the form:

$$\min_x \quad \frac{1}{2}x^\intercal Hx + (f^\intercal + \theta^\intercal f_\theta^\intercal)x \quad \text{s.t.} \quad Ax \leq b + W\theta \tag{1}$$

Here, H, A, W are all matrices, f and f_θ are vectors, x represents the optimization variables, and θ represents parameters that define specific instances of the problem. The algorithm takes the description of such a parametric problem and outputs a list of parameter values that are guaranteed to include the "hardest" instance for a specific solver. The main challenge in applying this technique to NMPC is translating NMPC problems to a problem like in Equation (1). This translation often involves approximations in the objective function or the system dynamics. In short, we roughly follow [2] with some small approximations, to move towards the form in (1). Essentially, this means we first *discretize* the problem, then *translate* the objective function (using an approximate Hessian), and finally *linearize* the constraints on the dynamics of the system.



(a) Cart pendulum (b) Motion planning

Fig. 1: WCET metrics plotted against number of states

III. Preliminary Results

Figure 1 shows the WCET metrics for two benchmarks. Strategy-1 with finite state space partition misses critical scenarios outside the chosen partitions, leading to underestimation as seen by the initial point(s) on the black line. Strategy 2 with fixed-step simulation potentially misses corner cases outside the chosen simulation horizon, indicated by an initially low WCET value followed by a spike on the blue line. This strategy too may result in underestimation of WCET. On the contrary, strategy 3 (red line) using complexity certification successfully (a) identifies critical cases confirmed by high WCET metrics, and (b) reaches critical cases sooner than the other two strategies, minimizing the risk of underestimation. All strategies result in test sets with 100% code coverage, so code coverage is not a good indicator of a test set being good for WCET measurement of NMPC. Our preliminary recommendation is to use strategy 3 as much as possible and to complement it with strategy 2 (and with some states from strategy 1) for a well-rounded test set.

References

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