MUSIC-Lite: Efficient MUSIC Using Approximate Computing: An OFDM Radar Case Study

Rajat Bhattacharjya, Arnab Sarkar, Biswadip Maity, and Nikil Dutt, *Life Fellow, IEEE*

***Abstract*—Multiple signal classification (MUSIC) is a widely used direction of arrival (DoA)/angle of arrival (AoA) estimation algorithm applied to various application domains, such as autonomous driving, medical imaging, and astronomy. However, MUSIC is computationally expensive and challenging to implement in low-power hardware, requiring exploration of tradeoffs between accuracy, cost, and power. We present MUSIC-lite, which exploits approximate computing to generate a design space exploring accuracy-area-power tradeoffs. This is specifically applied to the computationally intensive singular value decomposition (SVD) component of the MUSIC algorithm in an orthogonal frequency-division multiplexing (OFDM) radar use case. MUSIC-lite incorporates approximate adders into the iterative CORDIC algorithm that is used for hardware implementation of MUSIC, generating interesting accuracy-area-power tradeoffs. Our experiments demonstrate MUSIC-lite's ability to save an average of 17.25% on-chip area and 19.4% power with a minimal 0.14% error for efficient MUSIC implementations.**

***Index Terms*—Approximate computing, CORDIC, multiple signal classification (MUSIC), orthogonal frequency-division multiplexing (OFDM) radar, singular value decomposition (SVD).**

I. INTRODUCTION

MULTIPLE signal classification (MUSIC) is a widely used signal processing technique applied to direction of arrival (DoA) or angle of arrival (AoA) estimation problems [1]. MUSIC finds multiple applications in diverse fields, such as localization in autonomous vehicles [2], signal analysis in biomedical sensors [3], seismic monitoring systems [4], and joint radar-communication systems [5].

MUSIC relies on matrix decomposition techniques like eigenvalue decomposition (EVD) and singular value decomposition (SVD) during the signal processing stage. Traditionally, these matrix decomposition techniques are computationally expensive and power-hungry [6], [7] thereby negatively impacting the deployment of MUSIC on resource-constrained embedded platforms. Therefore, we need solutions that can implement MUSIC efficiently on low-power hardware. Existing works focus on making MUSIC implementation efficient by reducing the number of snapshots [8], or using a less-precise covariance matrix [9]. However, these approaches suffer from high-error rates.

In this letter, we present MUSIC-lite that exploits approximate computing [10] to reduce the computational complexity of MUSIC, while generating power-efficient solutions with minimal loss of accuracy. We apply MUSIC-lite to an orthogonal frequency-division multiplexing (OFDM) radar pipeline case study that deploys approximate arithmetic circuits (adders) to generate a design space trading off power, accuracy and cost (area). The OFDM radar pipeline case study highlights several benefits: 1) the versatility of the OFDM waveform—used in both communication and radar systems—allows for a unified and comprehensive analysis using MUSIC-lite to develop efficient joint radar-communication systems across multiple application domains, including automotive, healthcare, and military applications and 2) OFDM systems exhibit high-spectral efficiency, adaptability to varying bandwidth requirements, and robustness to Doppler shifts, making them invaluable across diverse applications and challenging environments.

We reduce the complexity of MUSIC by leveraging hardware-efficient approximations in SVD. Due to the inherent error-resilience of SVD [11], introducing approximations can substantially reduce on-chip power consumption and area. To achieve this, we utilize the CORDIC algorithm (CA) [12] for the hardware implementation of SVD, as it primarily relies on iterative shifts and additions. By integrating approximate adders into CA, we further enhance the hardware efficiency of the SVD process.

Our experimental results show that MUSIC-lite saves 17.25% on-chip area and 19.4% power with an average minimal error of 0.14% in the positive SNR region. To the best of our knowledge, MUSIC-lite is the first to explore the use of approximate arithmetic circuits in MUSIC for area and power efficiency.

II. BACKGROUND

We begin with a brief background on 1) MUSIC; 2) SVD implementation using the Golub–Kahan algorithm; and 3) hardware realization of the Golub–Kahan algorithm using CORDIC.

1) *MUSIC*—for a uniform linear antenna array—acquires signals, constructs the covariance matrix, performs the SVD to extract the signal and noise subspace, and then finally performs a peak search by finding minima in the noise subspace. Fig. 2 outlines the MUSIC flow (using CORDIC for SVD). Mathematically, MUSIC spectra is represented by

$$P_{MU}(\theta) = \frac{1}{a^*(\theta)E_N E_N^H a(\theta)} \tag{1}$$

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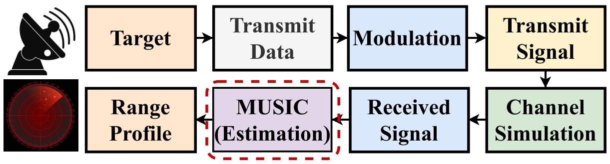

Fig. 1. OFDM radar processing pipeline. We approximate the computationally intensive MUSIC block highlighted in red.

where $P_{MU}(\theta)$ is the MUSIC spectra, E_N is the matrix whose columns are the eigenvectors of the noise subspace, and $a(\theta)$ are the elements of the array steering matrix. Equation (1) generates large peaks observed at the direction/AoA (θ), as shown in the MUSIC Spectrum of Fig. 2.

2) *SVD* is the most computationally intensive part of MUSIC for calculating the singular values as stated in Section I. Hence, we direct our focus on SVD. A fast and numerically stable method to calculate SVD is the Golub–Kahan algorithm [13]. The Golub–Kahan algorithm begins by transforming the given matrix A into a bidiagonal form. It then diagonalizes this bidiagonal matrix. The resulting diagonal elements are the singular values of A. The transformation of the matrix to its bidiagonal and diagonal form is achieved using Givens rotations.

3) *The CA* efficiently realizes the Golub–Kahan algorithm in hardware by using shifts and additions for the Givens rotations. CA iteratively shifts and updates the x and y coordinates to achieve the required rotation as shown below

$$x_{i+1} = x_i - d_i \cdot y_i \cdot 2^{-i}$$

$$y_{i+1} = y_i + d_i \cdot x_i \cdot 2^{-i}$$

$$z_{i+1} = z_i - d_i \cdot \arctan\left(2^{-i}\right).$$

   a) x_{i+1}: The x-coordinate after the ith iteration.
   b) y_{i+1}: The y-coordinate after the ith iteration.
   c) z_{i+1}: The angle accumulator after the ith iteration.
   d) d_i: Direction factor.

Since CA involves iterative additions for convergence, approximating the addition operations enables the exploration of power-efficient design points that can also meet the accuracy requirements of the SVD algorithm.

III. MUSIC-LITE METHODOLOGY

We present our MUSIC-lite methodology (Fig. 2) for the OFDM radar case study (Fig. 1), with the goal of demonstrating the utility of MUSIC-lite in supporting the design space exploration of alternative MUSIC implementation across different accuracy-power-area design points for end-to-end analysis of the OFDM radar pipeline. Table I shows the various parameters associated with the OFDM radar pipeline in our study.

The MUSIC-lite methodology (Fig. 2) can be broken down into three major steps: 1) functional validation; 2) hardware implementation; and 3) design space exploration.

A. Functional Validation

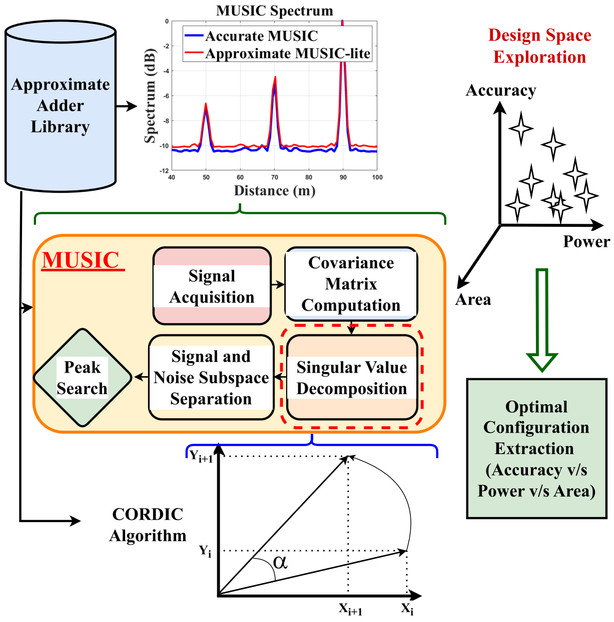

Fig. 2. Approximation using MUSIC-lite.

TABLE I
OFDM SYSTEM PARAMETERS FOR FIG. 1

System Parameter/Block	Value/Property
Carrier frequency	30 GHz
Number of subcarriers	32
Number of symbols	16
Subcarrier spacing	960 kHz
Elementary OFDM symbol duration	1.04 μs
Cyclic prefix duration	0.26 μs
Total symbol duration	1.3 μs
Modulation	4-QAM
Target position	50 m
Target velocity	20 m/s
Channel	Additive White Gaussian Noise Channel

First, at the software level, we introduce behavioral models of approximate adders in the radar pipeline. We use MATLAB to perform our experiments. We use the EvoApprox Library [14] to source the approximate adders. We select the approximate adders based on their individual error metrics (e.g., error percentage (EP), worst-case error (WCE), mean absolute error (MAE) [15]). We run the entire pipeline 100 times for an SNR range of −5 to 15 dB with a step size of 5. This exploration enables us to understand the effect of approximations at different noise levels and thereby provides a distinct boundary between conditions supporting approximations and vice-versa.

The accuracy metric is the estimated target's range in meters when deploying various approximate adders for MUSIC in the OFDM radar pipeline. If the approximated output level (target's range) falls within a user-defined quality window, we then proceed with hardware implementation.

B. Hardware Implementation

We evaluate the improvements in on-chip area and power efficiency by implementing a CORDIC core in hardware using Verilog HDL. Once we have the adders that fall within the application's acceptable quality window from the previous step, we incorporate them into the CORDIC core which is responsible for performing the SVD in MUSIC. Finally, we

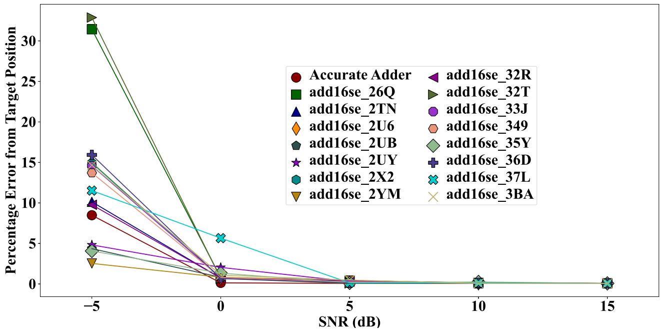

Fig. 3. Accuracy statistics using various adders (accurate and approximate). Range averaged for 100 runs per SNR for each adder.

synthesize the approximated CORDIC cores using Synopsys Design Compiler with a 45-nm technology node.

Next, the design space of the OFDM radar pipeline is explored by varying the approximate adders.

C. *Design Space Exploration*

Now that we have both accuracy and hardware statistics from Sections III-A and III-B, we can identify optimal design points and the tradeoffs associated with them. To determine the optimality of a design point, we consider user-defined quality constraints for both accuracy and hardware statistics (area, power). Thus, based on a predefined accuracy threshold and area and power budget, we make selections of approximate designs satisfying all three requirements.

IV. EVALUATION

We now present the efficacy of MUSIC-lite in the end-to-end OFDM radar case study. We approximate the addition operations inside the SVD (CA) of MUSIC and then obtain results for the target's range across an end-to-end pipeline as shown in Fig. 1. The target is situated at a distance of 50 m. We evaluate 15 16-bit sign-extended approximate adders from the EvoApprox Library [14] with diverse accuracy metrics [15].

A. *Accuracy Analysis*

Fig. 3 shows that our approximations have negligible impact on application accuracy. The figure shows the percentage error for range estimation (y-axis) while using both accurate and approximate adders across an SNR range of -5 to $+15$ dB (x-axis). The range values obtained per SNR are averaged across 100 runs per adder.

We make four major observations.

1) We see that for negative SNR, all adders (including the accurate adder) perform poorly with high error. However, even in this case we see that three approximate adders perform better than the accurate adder, with the best-approximate adder (`add16se_2YM`) showing only a 2.534% error from the target's actual position.
2) We observe that at SNR $=$ 0 dB, the accurate adder performs best with a deviation of 0.13% from the target's range, and among all approximate adders, `add16se_32T` performs best with a percentage error of 0.66%.
3) As we move toward higher-positive SNR, all adders more or less perform the same (including the accurate adder) in estimating target's range, with the highest-percentage error being 0.421% among all adders.
4) For higher-positive SNR (5 to 15 dB), among all approximate adders, `add16se_33J` performs the best with an average EP of 0.088%, although it has an inherent error probability of 0.25.

These observations lead to the conclusion that since there are many dynamic interactions happening over an end-to-end system, such a study is critical for deploying approximate adders in evaluating end-to-end application accuracy. Overall, Fig. 4 shows that approximate adders have negligible impact on application accuracy, obtaining similar statistics as with using accurate adders, with an EP of 0.14% (from the target's position) for practical radar scenarios that have positive SNR.

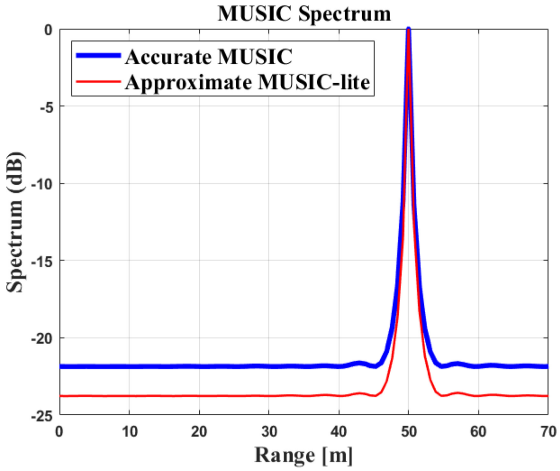

Fig. 4. Target's range profile at SNR = 10 dB with accurate adder; and approximate adders `add16se_2U6` and `add16se_2TN`.

B. *Hardware Evaluation*

Our hardware evaluation uses Verilog HDL to describe a CORDIC core with approximate addition operations. We synthesize the core using Synopsys Design Compiler with the NanGate 45-nm Open Cell Library to obtain area and power statistics. The core runs at 500-MHz clock frequency. The carry-lookahead adder (CLA) is selected as the baseline accurate adder to match the high-frequency requirement of 500 MHz.

Fig. 5 shows the area and power statistics obtained for the CORDIC core using both accurate and approximate adders. We make 2 major observations: 1) all approximate adders perform better than the baseline (CLA) adder in consideration and 2) the approximate adder `add16se_3BA` provides the best area and power savings, saving 21.74% area and 25% power compared to CLA. On average, we see that all the approximate adders save 17.25% on-chip area and 19.4% power when compared to CLA.

C. *Design Space Exploration*

To study the relationship between accuracy and hardware statistics, MUSIC-lite constructs a design space and explores optimal design points, i.e., adders with diverse accuracy and hardware metrics applied to this scenario so as to satisfy user-defined quality constraints. Fig. 6 shows the design space of relationships between accuracy, area and power when different adders are applied to our end-to-end OFDM radar scenario.

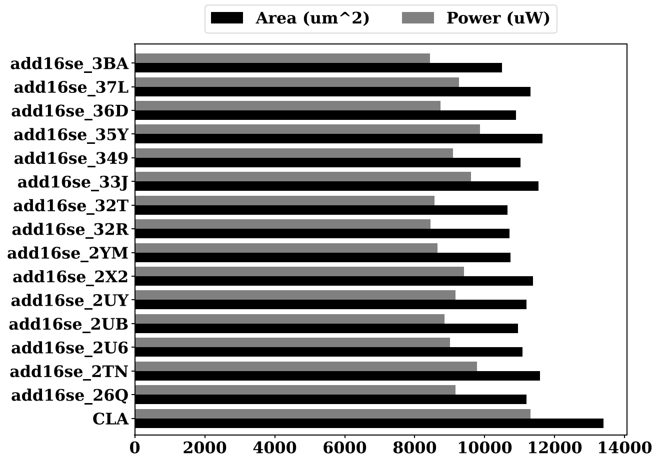

Fig. 5. Area and power statistics for CORDIC core using both accurate and approximate adders.

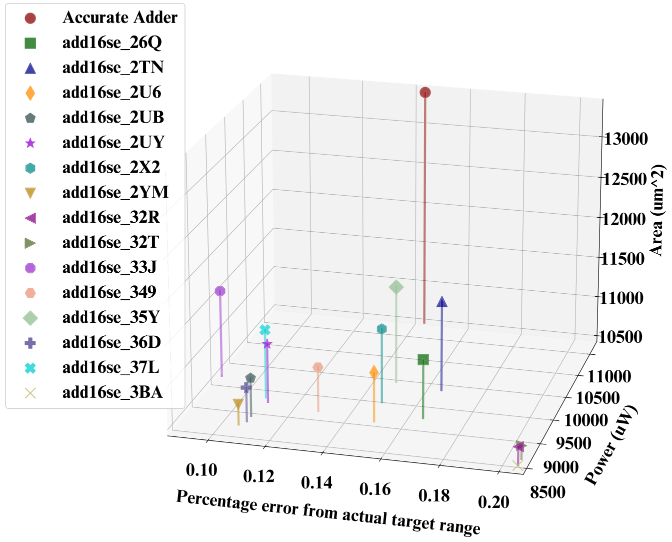

Fig. 6. Design space exploration. Relationship between accuracy, area, and power while using accurate and approximate adders in the presented scenario.

The accuracy metric is the average percentage error from the target's actual distance (50 m) in the positive SNR range (5 to 15 dB). The area and power metrics are the same as reported in Fig. 5. The design space in Fig. 6 can be used to evaluate different OFDM radar use cases. On one hand, if the user sets a quality constraint where maximum percentage error is 0.1% in the positive SNR region, then from the DSE we see that adder `add16se_33J` fulfills the criteria with a percentage error of 0.088%. However, we observe a tradeoff in area and power needs, since the area and power savings are 13.9% and 14.6%, respectively, which are low compared to other adders. On the other hand, if the user sets a multiobjective optimization of a percentage error below 0.12%, and an area and power savings above 18% and 21%, respectively, then we have the adders `add16se_2UB`, `add16se_2YM`, `add16se_36D` fulfilling those criteria.

Therefore, based on accuracy requirements, coupled with an area and power budget, many observations can be made and MUSIC-lite enables exploration of such optimal design points.

V. CONCLUSION AND FUTURE WORK

We presented MUSIC-lite, an approach exploiting approximations in MUSIC to enable design space exploration of alternative accuracy-area-power tradeoffs for end-to-end OFDM radar pipelines. The core contribution of MUSIC-lite is the use of approximate adders for the computationally intensive SVD block (CORDIC core) in MUSIC for an end-to-end OFDM radar pipeline. Our case study for an end-to-end OFDM radar pipeline demonstrated that MUSIC-lite can uncover an interesting design space that system designers can use to explore tradeoffs and design alternatives. For instance, our experiments show that MUSIC-lite helps save 17.25% on-chip area and 19.4% power with a minimal percentage error of 0.14% on average in positive SNR. Our ongoing and future work considers integrating other algorithms, such as [16] and [17] with MUSIC-lite across an end-to-end pipeline, to evaluate the combined effect on accuracy and hardware efficiency.

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