Impact of Sea-Level Rise on Sea Water Intrusion in Coastal Aquifers

by Adrian D. Werner¹ and Craig T. Simmons²

Abstract

Despite its purported importance, previous studies of the influence of sea-level rise on coastal aquifers have focused on specific sites, and a generalized systematic analysis of the general case of the sea water intrusion response to sea-level rise has not been reported. In this study, a simple conceptual framework is used to provide a first-order assessment of sea water intrusion changes in coastal unconfined aquifers in response to sea-level rise. Two conceptual models are tested: (1) flux-controlled systems, in which ground water discharge to the sea is persistent despite changes in sea level, and (2) head-controlled systems, whereby ground water abstractions or surface features maintain the head condition in the aquifer despite sea-level changes. The conceptualization assumes steady-state conditions, a sharp interface sea water-fresh water transition zone, homogeneous and isotropic aquifer properties, and constant recharge. In the case of constant flux conditions, the upper limit for sea water intrusion due to sea-level rise (up to 1.5 m is tested) is no greater than 50 m for typical values of recharge, hydraulic conductivity, and aquifer depth. This is in striking contrast to the constant head cases, in which the magnitude of salt water toe migration is on the order of hundreds of meters to several kilometers for the same sea-level rise. This study has highlighted the importance of inland boundary conditions on the sea-level rise impact. It identifies combinations of hydrogeologic parameters that control whether large or small salt water toe migration will occur for any given change in a hydrogeologic variable.

Introduction

Sea water intrusion (or salt water intrusion) is the encroachment of saline water into fresh ground water regions in coastal aquifer settings. It has been studied extensively for well over a century (Werner and Gallagher 2006; Voss and Souza 1987; Huyakorn et al. 1987; Pinder and Cooper 1970; Herzberg 1901; Ghyben 1888). An exhaustive review on this topic is provided by Bear et al. (1999), and the reader is referred to that text for further information on this topic.

It is well known that sea water intrusion is affected by both natural and anthropogenic processes. In particular, sea-level rise associated with climate change (by way of changes to atmospheric pressure, expansion of oceans and seas as they warm, and melting of ice sheets and glaciers) is one potentially significant process that is expected to play a role in sea water intrusion. The Intergovernmental Panel on Climate Change (IPCC 2001) predicts that by 2100, global warming will lead to a sea-level rise of between 110 and 880 mm, and it is generally understood that sea-level rise is expected to result in the inland migration of the mixing zone between fresh and saline water (FAO 1997). This is because the rise in sea water levels leads to increased saline water heads at the ocean boundary, and enhanced sea water intrusion is the logical consequence. Despite the perhaps somewhat obvious qualitative claim that sea water intrusion will be exacerbated as global mean sea level rises, previous accounts rarely provide quantitative guidance on this matter, that is, how far inland sea water migration may be expected to
occur as sea level rises and what hydrogeologic parameters control that migration. Furthermore, despite its purported potential importance, surprisingly few quantitative studies have investigated the impact of sea-level rise on sea water intrusion at all (Meisler et al. 1984; Oude Essink 1999; Sherif and Singh 1999; Bobba 2002). This limited number of studies report only on site-specific observations and site-specific numerical modeling studies, and it is very difficult to draw general conclusions about the nature of the impact. For example, using numerical simulations, Sherif and Singh (1999) noted that a 500-mm rise in the Mediterranean Sea level will cause additional intrusion of 9.0 km in the Nile Delta Aquifer in Egypt but that this same rise in sea water level in the Bay of Bengal will only cause an additional increase in sea water intrusion of 0.4 km—a vastly different result. These differences are not explained in physical or more generalized terms because a systematic comparison between these sites is not made. Bobba (2002) conducted numerical simulations of salt water intrusion into the Godavari delta, India. In that study, simulations were used to demonstrate an apparent risk of mixing salt water with fresh water due to sea-level rise, but it is very difficult to delineate the effects of vertical infiltration of sea water into the aquifer vs. sea water intrusion as we define it here (the lateral landward migration of the sea water-fresh water interface). The complexity of the numerical model did not permit an easy analysis of the latter effect, and the study does not formulate any quantitative conclusions about the extent of sea water intrusion induced by sea-level rise. It is readily apparent that while earlier studies consistently point to the likely deleterious impacts of sea-level rise on sea water intrusion in either a quantitative or a semiquantitative manner, their highly site-specific nature (and hence inherent complexity with respect to boundary conditions, hydrogeology, climate, and ground water pumping extraction patterns) means that it is virtually impossible to draw generalized conclusions about the nature of the impact of sea-level rise on sea water intrusion in quantitative terms. More problematic still is that while many of these studies use complex quantitative methods to study these processes, most do not draw quantitative conclusions about the impact of sea-level rise—quite possibly the result of an inherent inability to unravel the various processes contributing to both ground water hydraulic head and salinity observations.

We contend that there is a clear and urgent need to explore inward sea water-fresh water interface migration in response to changing global mean sea level in unconfined coastal aquifers. In this article, we explore this in an intentionally simple and somewhat generalized manner in order to provide insights into the nature of the magnitude of the interface migration process and its hydrogeologic controls. We use simple conceptual models and idealized steady-state analytical solutions. While our analysis is intentionally simple, we make the observation that the important elements of a realistic hydrogeologic conceptual model are included but without site-specific details, assumptions, or simplifications. This allows us to elucidate the major hydrogeologic controls on inward interface migration within a systematic “cause and effect” sensitivity analysis framework. Two conceptual models are tested and may be considered to provide realistic end-members on the likely range of behavior that may be expected in field settings: (1) flux-controlled systems, in which ground water discharge to the sea is persistent despite changes in sea level (i.e., ground water–level rise is commensurate with sea-level rise and hydraulic gradients are maintained), and (2) head-controlled systems, whereby hydrogeologic controls (e.g., surface water bodies or borefields) maintain the inland head in the aquifer despite sea-level changes. We quantify sea water-fresh water interface migration due to sea-level rise as a function of key hydrogeologic variables including aquifer thickness, recharge rate, hydraulic conductivity, and the rate of ground water discharge to the sea.

Theory

The theoretical analysis presented in this section is a reprise of treatments presented previously by Bruggeman and Custodio (1987), Custodio (1987), and Falkland (1991), and the reader is referred to these references for an exhaustive analysis. The equations that follow are based upon well-understood and commonly accepted analytical expressions for sea water intrusion problems on both continents and elongated islands. For completeness in our treatment and approach, the appropriate equations in our formulation are presented concisely here before we describe their unique application to the particular case of sea-level rise phenomena. We intentionally adopt an approach that is analytical in nature as a way of providing the simplest starting approach and because the mathematical formulations allow us to more clearly identify the primary controlling factors in the problem. Furthermore, by using a sharp interface approximation provided by the analytical theory, we avoid the complicating effects of introducing dispersion, which is often difficult to quantify, into this preliminary analysis. Dispersion would be required for explicit solute transport simulation in more complex variable density ground water flow and solute transport numerical models.

The simplest conceptual models of sea water intrusion adopt the Ghyben-Herzberg approximation of hydrostatic equilibrium between two immiscible fluids of different density (the one-dimensional [1D] case is considered here), given as (Herzberg 1901; Ghyben 1888):

$$z = \frac{\rho_f}{\rho_s - \rho_f} h = zh \tag{1}$$

where $z = z(x)$ is the depth of the salt water-fresh water interface below mean sea level (L), $\rho_f$ is the fresh water density [ML$^{-3}$], $\rho_s$ is the salt water density [ML$^{-3}$], $h = h(x)$ is the water table elevation above mean sea level [L], and $z$ is the density ratio $\rho_f/(\rho_s - \rho_f)$. $z$ is commonly assumed to be 40 but varies between 33 and 50 for typical densities of fresh ground water and sea water. The
Ghyben-Herzberg approximation is commonly applied in combination with the Dupuit approximation, and the outflow face along the coast is neglected, resulting in an underestimation of the interface depth (van der Veer 1977), although it is known to be a useful first approximation for situations where the transition zone is thin relative to the aquifer thickness.

In this study, both the Ghyben-Herzberg and the Dupuit-Forchheimer approximations are adopted to examine the situation of a homogeneous, isotropic confined coastal aquifer subject to constant aquifer recharge under steady-state conditions. The boundary value problem being solved comprises a specified head at the coast, no flow at some distance inland, uniform recharge, and interface flow, with an a priori unknown interface position. Figure 1 illustrates the basic physical setting and defines the parameters of the theoretical framework. The equation for ground water discharge to the coast in the presence of a salt water-fresh water interface ($0 < x < x_T$; where $x_T$ is the position of the toe of the salt water wedge) is (Custodio 1987; Falkland 1991):

$$q(x) = q_0 - Wx = K(h + z)\frac{dh}{dx} \quad (2)$$

where $q_0$ is uniform discharge to the sea per unit length of coastline (and is the consequence of the net recharge occurring between the coastline and the inland no-flow boundary) [L^2T^{-1}], $W$ is uniform net recharge [LT^{-1}], $x$ is taken from the submarine aquifer outcrop [L], and $K$ is hydraulic conductivity [LT^{-1}]. Note that Equation 2 is an unconventional form of Darcy’s law, whereby $q$ and $x$ are positive in opposite directions (Figure 1); the choice is made for convenience as it allows for talking about a positive discharge toward the coast as well as a toe position at a positive $x$ position. Changes in the extent of sea water intrusion are commonly defined according to movements in the position of the toe of the salt water wedge $x_T$ [L].

From Equation 2, $h$ ($0 < x < x_T$) can be determined in areas of the aquifer that contain the salt wedge through direct integration and assuming a fixed head at the coast, as (Custodio 1987):

$$h = \sqrt{\frac{2q_0x - Wx^2}{K(1 + x)}} \quad (3)$$

The toe of the sea water wedge $x_T$ [L] is situated at the intersection of the salt water-fresh water interface and the horizontal aquifer basement and is given as (Custodio 1987):

$$x_T = q_0 \frac{W}{W^2} - \sqrt{\frac{q_0^2}{W^2} - \frac{K(1 + x)z_0}{Wz^2}} \quad (4)$$

where $z_0$ is the depth below mean sea level of the aquifer bottom [L]. A characteristic of the approach adopted here is that sea-level rise is assumed to influence $z_0$ in Equation 4 (i.e., $z_0$ is increased by an amount equal to the sea-level rise), and we neglect changes in the position of the coastline (which is taken to be the fixed origin of our coordinate system).

In the conceptual model presented here, a water table mound occurs and a maximum water table height $h_m$ occurs at a distance $x_m = q_0/W$, giving $q(x_m) = 0$ (i.e., a no-flow condition). From Equation 4, it is apparent that $x_m$ is always greater than or equal to $x_T$ (i.e., the no-flow boundary is at or inland of the sea water wedge toe) because $q_0$ and $W$ are both positive in this analysis. We consider the problem domain $0 < x < x_m$ because the situation of $x_T = x_m$ (i.e., the interface toe encounters the inland no-flow boundary) will result in the toe of the wedge penetrating the entire aquifer beyond $x_m$ (if the aquifer exists here) under the current conceptualization. The situation $x_T = x_m$ arises where the head at the wedge toe ($h_T$), derived from Equation 1 as $h_T = z_0/z$, is equal to $h_m$. The value of $q_0$ that results in $h_m = z_0/z$ is therefore the minimum coastal discharge $q_{min}$ producing a salt water-fresh water interface that remains inland of the no-flow boundary (a condition referred to as “interface limit” in the following) and is given from a combination of Equation 3 and $x_m = q_{min}/W$ as:

$$q_{min} = \frac{WK(1 + x)z_0^2}{x^2} \quad (5)$$

Sea water intrusion predictions in this study typically involved the determination of $x_T$, given a known $h$ at some distance $x$. In order to resolve this, an additional equation is necessary to describe the water table height inland of $x_T$ to account for the base of the aquifer, determined through integration of a simple steady-state mass balance and Darcy’s law as:

$$h = \sqrt{\frac{2}{K} \left( \frac{x-x_T}{2} \left( q_0 - W \left( x + x_T \right) \right) + \left( h_T + z_0 \right)^2 \right)} - z_0 \quad x \geq x_T \quad (6)$$

where $h_T$ is the water table height at the toe of sea water wedge. Equations 3, 4, and 6 were solved iteratively for most of the analyses undertaken in this study (depending on the variable of interest), although it is acknowledged

Figure 1. Salt water-fresh water interface and conceptual model for analysis.
that the method of modified Girinskii potentials devised by Strack (1976) allows for a noniterative solution.

Alternative formulations of the sea water-fresh water interface include the solution of Glover (1959); however, this does not account for recharge that is required in our analysis. Furthermore, Custodio (1987) compares these methods and indicates that Equation 2 is sufficient for calculations of the salt water wedge penetration (i.e., \( x_T \)). van der Veer (1977) formulated a two-dimensional solution for the salt water-fresh water interface and demonstrated that only very small differences exist between the two-dimensional solution and the 1D solution (Equation 2), except immediately near the coast. Therefore, the 1D solution is considered appropriate in this analysis.

Application of Theory to Sea-Level Rise Problem

A parsimonious approach to the analysis of sea-level rise is adopted, whereby interface movements caused by sea-level rise are taken as the change between the different steady-state conditions (i.e., pre- and post-sea-level rise). This represents potentially the worst-case scenario and avoids the more onerous calculations of interface dynamics and the timing of sea water intrusion advancement. We consider the endmember where the intrusion is assumed to occur laterally as a result of the head boundary condition change. Given a lack of agreement on sea-level rise projections for the 21st century (Ball et al. 2001), the upper value for the rise in sea level of 880 mm, as given by the IPCC (2001) for the year 2100, is used as a base case for this assessment, although we test the sensitivity of the problem to sea-level rise up to 1500 mm.

The aforementioned theoretical framework is systematically applied to the problem of sea-level rise using typical parameter ranges. We illustrate how various factors (\( W \), \( q_0 \), \( z_0 \), and \( K \)) influence movement in the steady-state location of the sea water wedge toe (i.e., \( x_T \)) due to changes in sea level. Numerous sets of \( \Delta x_T \) predictions were produced, whereby a single parameter was incrementally varied. In order to constrain the range of predictions, the parametric study adopted a base case set of parameters typical of a shallow, alluvial aquifer in a setting of moderate-to-high rainfall (Table 1). Sensitivity analyses of the magnitude of sea-level change and the inland boundary condition were also undertaken.

In exploring the impact of sea-level rise on sea water intrusion, two different inland boundary conditions have been adopted: flux controlled and head controlled. In case 1 (flux controlled), the ground water discharge to the sea is constant despite sea-level rise. Some comment on this case is warranted from a physical point of view to justify this condition. In some settings, there may be a capacity for the water table to rise at the same rate as the sea-level change to maintain the conditions of constant discharge to the sea and, as will be shown in this study, minimize intrusion associated with sea-level rise. However, in other coastal aquifer settings, surface controls such as drains, wetlands, streams/rivers and ground water evapotranspiration, and unmanaged ground water abstraction may inhibit water table rise that would otherwise be expected to compensate for the sea-level rise effect. We call this latter scenario case 2 (head controlled), and this is associated with the situation whereby the ground water level at some distance from the coast is constant despite rising sea level. In our case, a distance of 2 km was adopted that is sufficiently far inland to be a reasonable point at which to define the boundary condition.

Results

Case 1: Flux-Controlled Systems

Flux-controlled systems include those in which ground water discharge to the sea (\( q_0 \)) is maintained to control the position of the sea water-fresh water interface despite a rise in sea level. The problem domain used in this study is bounded at the inland extent by a specified flux (i.e., \( q_i \)) boundary condition, given as \( q_i = q_0 - WL \), where \( L \) is the horizontal distance from the point of coastal discharge to the inland boundary. In order to maintain \( q_i \) and \( q_0 \) despite sea-level change, the head at \( x = L \) (i.e., \( h_l \)) must rise.

We first examine \( \Delta x_T \) as a function of sea-level rise. The results from applying Equations 3, 4, and 6 to the Table 1 parameters, except with variations to \( W \) and sea-level rise, are shown in Figure 2. \( \Delta x_T \) vs. sea-level rise predictions for alternative \( z_0 \) and \( K \) values are illustrated in Figure 3.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
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<tr>
<td>Base Case Parameters (derived from Werner and Gallagher 2006)</td>
</tr>
<tr>
<td>( W ) (mm/year)</td>
</tr>
<tr>
<td>80</td>
</tr>
</tbody>
</table>

Figure 2. Case 1 (flux-controlled systems): \( \Delta x_T \) vs. sea-level rise, with variation in \( W \).
The rise in $h_i$ (at $x = 2$ km) required to maintain $q_0$, despite sea-level rise, is 0.096 m for a 100-mm sea-level rise and 1.44 m for a 1500-mm sea-level rise (using the Table 1 parameters). The rise in $h_i$ associated with sea-level rise is not dependent on $W$ or $K$ but is slightly larger for a 50-m-deep aquifer (i.e., 0.099 and 1.48 m for sea-level rises of 100 and 1500 mm, respectively).

A systematic analysis of the sensitivity of $\Delta x_T$ to the parameters of Table 1 was undertaken assuming a sea-level rise of 880 mm and $a = 40$. The analysis parameter sets are similar to those of Table 2, except interface limits and parameter ranges for parameter sets 1 to 4 can be defined in terms of $h_i$ rather than $q_0$. Parameter sets 5, 6, and 7 adopted an inland head of 2 m rather than the specified flux of case 1. Results for illustrative examples from this analysis are given in Figure 7.

### Discussion

It is useful to consider the consistency of our approach with those of previous published studies. Using representative parameters for the Nile Delta case analyzed by Sherif and Singh (1999) ($L = 150$ km, $z_0 = 400$ m, $K = 100$ m/d, $h_i = 14$ m [above sea level], $W = 0$ mm/year) and in our method proposed here, we obtained reasonably consistent estimates of sea water intrusion in response to sea-level rise compared to their study (i.e., our analysis and that of Sherif and Singh [1999]) both...

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### Table 2

Parameter Sets Used in the Sensitivity Analysis of Case 1

<table>
<thead>
<tr>
<th>Parameter Set</th>
<th>$z_0$ (m)</th>
<th>$W$ (mm/year)</th>
<th>$K$ (m/d)</th>
<th>$q_0$ (m$^2$/d)</th>
<th>Conditions Producing Interface Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>40</td>
<td>10</td>
<td>0.16–0.87</td>
<td>$q_{min} = 0.159$ m$^2$/d</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>80</td>
<td>10</td>
<td>0.23–0.98</td>
<td>$q_{min} = 0.225$ m$^2$/d</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>120</td>
<td>10</td>
<td>0.28–1.08</td>
<td>$q_{min} = 0.275$ m$^2$/d</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>80</td>
<td>10</td>
<td>0.39–1.38</td>
<td>$q_{min} = 0.375$ m$^2$/d</td>
</tr>
<tr>
<td>5</td>
<td>4–65</td>
<td>80</td>
<td>10</td>
<td>0.50</td>
<td>$z_0^1 = 66.7$ m</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>80</td>
<td>0.01–46</td>
<td>0.50</td>
<td>$K^1 = 46.7$ m/d</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>0–374</td>
<td>10</td>
<td>0.50</td>
<td>$W^1 = 374$ mm/year</td>
</tr>
</tbody>
</table>

$^1$Maximum allowable values.
indicate toe migration on the order of 5 km inland for a 500-mm sea-level rise). This simple comparison with the study of Sherif and Singh (1999) indicates that the results presented here are not inconsistent with a more complex numerical model. However, while this demonstrates the applicability of our approach and is suggestive of its appropriateness in sea-level rise analyses, there are limited sea-level rise case studies that provide for an opportunity for detailed quantitative comparisons. We are careful not to suggest here that our approach is validated based upon this single comparison.

The results of this study have implications for the management of coastal aquifers subject to sea-level rise. The resulting sea water intrusion may be somewhat mitigated if water resource managers are able to allow ground water levels to rise commensurate with sea-level rise. Coastal aquifer management philosophies that adopt “trigger-level” approaches (e.g., Werner and Gallagher 2006; DHEC 2003) may need to account for this effect.

Further work is required to assess the effects of spatial (geologic heterogeneity) and temporal heterogeneity on the sea-level rise intrusion problem. Assessments of the landward migration of the coastal boundary, the transient evolution of the migrating salt water-fresh water interface relative to changing seaward boundary conditions, and

Figure 4. Illustrative examples (a to d) of parameter sensitivity analyses for case 1.
the inclusion of dispersion and hence wide transition zones remain other important areas for future analyses. Also, evapotranspiration is expected to be a significant process in shallow water table aquifers that are subject to sea-level rise. It is likely that the behavior of systems in which evapotranspiration is significant would lie between that of the two endmember cases (head controlled and flux controlled) presented here. More complex variable density ground water flow and solute transport numerical modeling analyses, informed by site-specific case studies, are required to analyze more complicated situations.

Conclusions
The physical controls and quantitative bounds on expected sea water intrusion behavior under sea-level rise have not been explicitly articulated in previous studies but are implied by the physics and mathematical representation of the sharp interface sea water intrusion problem. It is readily apparent from the results presented here that the conceptual approach employed (constant head or constant flux) has a major impact on the results. Indeed, our analysis highlights that flux-controlled systems are associated with minimum sea water intrusion as a result of sea-level rise and that head-controlled systems are associated with maximum sea water intrusion as a result of sea-level rise.

In the case of constant flux conditions, the upper limit for sea water intrusion due to sea-level rise (up to 1500 mm) is no greater than 50 m for typical values of recharge, hydraulic conductivity, and aquifer depth. This is in striking contrast to the constant head cases, in which the magnitude of salt water toe migration is on the order of hundreds of meters to in excess of a kilometer for the same sea-level rise. Our sensitivity analysis work identifies conditions under which major changes in the salt water toe are incurred for very small changes in key hydrogeologic variables. In physical terms, these conditions are invoked as the steady-state toe position approaches the location of the ground water table mound. Thus, the quantification of the position of the toe relative to any recharge-dependent water table mounding should be considered in sea water intrusion assessments that consider the effect of sea-level rise.

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References


