# Fast approach for unsteady flow routing in complex river networks based on performance graphs

Arturo S. Leon<sup>1</sup>, Elizabeth A. Kanashiro<sup>2</sup> and Juan A. González-Castro<sup>3</sup>

#### ABSTRACT

We present a new model for unsteady flow routing through dendritic and looped river networks based on performance graphs. The model builds upon the application of Hydraulic Performance Graph (HPG) to unsteady flow routing introduced by González-Castro (2000) and adopts the Volume Performance Graph (VPG) introduced by Hoy and Schmidt (2006). The HPG of a channel reach graphically summarizes the dynamic relation between the flow through and the stages at the ends of the reach under gradually varied flow (GVF) conditions, while the VPG summarizes the corresponding storage. Both, the HPG and VPG are unique to a channel reach with a given geometry and roughness, and can be computed decoupled from unsteady boundary conditions by solving the GVF equation for all feasible conditions in the reach. Hence, in the proposed approach, the performance graphs can be used for different boundary conditions without the need to recompute them. Previous models based on the performance graph concept were formulated for routing through single channels or channels in series. The new approach expands on the use of HPG/VPGs and adds the use of rating performance graphs

<sup>&</sup>lt;sup>1</sup>Assistant Professor, School of Civil and Construction Engineering, Oregon State University, 220 Owen Hall, Corvallis, OR 97331-3212, USA. E-mail: arturo.leon@oregonstate.edu (Corresponding author)

<sup>&</sup>lt;sup>2</sup>Hydraulic Engineer, Ausenco-Vector, Calle Esquilache 371, San Isidro, Lima - Perú. E-mail: Akemi.Kanashiro@ausenco.com

<sup>&</sup>lt;sup>3</sup>Principal Engineer, Hydro-Data Management, South Florida Water Management District, 3301 Gun club Rd., West Palm Beach, FL 33406, USA. E-mail: jgonzal@sfwmd.gov

for unsteady flow routing in dentritic and looped networks. We exemplify the applicability of the proposed model to subcritical unsteady flow routing through a looped network and contrast its simulation results with those from the well-known unsteady HEC-RAS model. Our results show that the present extension of application of the HPG/VPGs appears to inherit the robustness of the HPG routing approach in González-Castro (2000). The unsteady flow model based on performance graphs presented here can be extended to include supercritical flows.

**Keywords:** Dendritic network, Flooding; Flow routing, Hydraulic routing; Looped network; Modeling, River hydraulics; Simulation, Unsteady flow

#### INTRODUCTION

Most free surface flows (also called open-channel flows) are unsteady and nonuniform. Hence, in many applications the spatial and temporal variation of water stages and flow discharges need to be determined. Unsteady flows in river systems are typically simulated using one-dimensional models although two and three-dimensional models are now being used more frequently. In a one-dimensional framework, unsteady flows in rivers are typically simulated by the Saint-Venant equations, the pair of partial differential equations representing conservation of mass and momentum for a control volume are:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

$$\frac{1}{g}\frac{\partial V}{\partial t} + \frac{\partial}{\partial x}\left(\frac{V^2}{2g}\right) + \cos\theta\frac{\partial y}{\partial x} + S_f - S_o = 0 \tag{2}$$

where x = distance along the channel in the longitudinal direction; t = time; Q = discharge; A = cross-sectional area; y = flow depth normal to x;  $\theta =$  angle between the longitudinal bed slope and a horizontal plane; g = acceleration of gravity;  $S_o =$  bed slope and  $S_f =$  friction slope. In Eq. (2) [momentum equation], the first, second,

third, fourth and fifth terms represent the local acceleration, convective acceleration, pressure gradient, friction and gravity terms, respectively.

The Saint-Venant equations are typically solved for appropriate initial and boundary conditions to simulate the spatial and temporal variation of water stages and flow discharges resulting from flood routing. At present, no analytical solution for the Saint-Venant equations is known, except for special conditions (e.g., dam break flow over a dry bed in a frictionless and horizontal channel). Hence, solutions of general openchannel flow conditions such as those found in practical applications are sought numerically. Solutions to the full dynamic, one dimensional Saint-Venant Equations and their quasi-steady, noninertia (or diffusion), and kinematic wave approximations (details on these approximations can be found for example in Yen 1986) have been sought based on several numerical schemes and methods (e.g., Abbot and Basco 1989). As emphasized in González-Castro (2000) and González-Castro and Yen (2012), despite the wide array of methods available for the solution of the Saint-Venant equations, the lack of robustness and accuracy issues still pose a problem.

Another important issue with unsteady flow models is computational burden, especially when an unsteady model is used for optimization problems such as real-time operation of regulated river systems (e.g., Leon et al. 2012). In this case hundreds or even thousands of runs need to be performed for each operational decision ( $\sim$  30 minutes), which would require numerically efficient models for unsteady flow routing or a large number of computer processors (clusters). Even if the simulations are run on computer clusters, there is no guarantee that hydraulic routing models will work for all range of conditions (e.g., low stage flows up to flows in the floodplains). In the authors' experience, under some simulation conditions most of the existing routing models fail to converge to a solution. In particular, the widely known unsteady HEC-RAS model (Hydrologic Engineering Center 2010b), which has been found to converge for a range

of conditions, fails to converge under some conditions. When HEC-RAS fails to converge, it proceeds with the simulation based on assumed pre-specified conditions (e.g., critical flow), which may yield questionable results.

In the last three decades, Ben Chie Yen's research group at the University of Illinois at Urbana-Champaign extended the concept of delivery curves introduced by Bakhmeteff (1932). Yen's group proposed a general approach to summarize the dynamic relation between the water surface elevations (stages) or depths at the ends of a channel reach (e.g., rivers or canals) for different constant discharges under gradually varied flow (GVF) conditions (e.g., Yen and González-Castro 2000; Schmidt 2002). This approach was called the Hydraulic Performance Graph (HPG). The HPG is a set of curves of constant discharge known as hydraulic performance curves (HPCs). Each HPC defines the locus of the upstream and downstream water depths in a channel reach for a given constant flow discharge. An example of an HPG for a mild-sloped channel is depicted in Fig. 1. The HPG shown in Fig. 1 has few HPCs; in actual applications, the number of HPCs must be set based on a precision goal. This must be decided based on a convergence analysis, which consists of successive refinement of the resolution of the set of HPCs to a resolution such that the solution for the conditions of interest (e.g., stage and flow hydrographs at a given station) becomes nearly independent of the number of HPCs used. The procedure to determine the optimal resolution of HPCs to ensure that a prescribed accuracy is afforded, is outside the scope of this paper.

HPGs can be used to summarize gradually varied subcritical and supercritical flows. However, they have been mostly applied to summarize subcritical GVF in channel reaches with steep, mild, adverse and horizontal slopes (see methodology in Yen and González-Castro 2000; and Schmidt 2002). The construction of the HPG for each reach may involve hundreds of GVF simulations, each simulation corresponding to one discrete point on the HPG. When using a one-dimensional model for constructing the performance graphs, any GVF model can be used. In the present application, the steady HEC-RAS model was used to generate the HPG's/VPG's.

HPGs have been applied to solve problems in open-channel flows including the (a) evaluation of hydraulic performance of floodplain channels under pre- and postbreached levee conditions (González-Castro and Yen 1996), (b) assessment of the carrying capacity of channel systems in series (Yen and González-Castro 2000), and (c) theoretical development of discharge ratings based on the hydrodynamics of unsteady and nonuniform flows (Schmidt 2002). González-Castro (2000) assessed the applicability of HPG's for unsteady flow routing in single prismatic channels and channel systems in series with successful results. The unsteady approach of González-Castro (2000) assumes that the flow is steady at the different time steps of the simulation. More recently, Hoy and Schmidt (2006) relied on the Volume Performance Graph (VPG) instead of a finite-difference scheme like the four-point implicit finite difference scheme used by González-Castro (2000) to satisfy the reach-wise mass conservation during routing. The VPG approach is equivalent to enforcing Eq. (1) [conservation of mass] in a reach (see details in Hoy and Schmidt 2006). An example of a VPG for a mild-sloped channel is depicted in Fig. 2. The HPG and VPG are unique to a channel reach with a given geometry and roughness, and can be computed decoupled from unsteady boundary conditions by solving the GVF equation for all feasible conditions in the reach. They are essentially a fingerprint of all gradually varied flow conditions in a channel reach. Consequently, HPG/VPGs need to be revised only when geomorphic changes modify the geometry or roughness characteristics of the channel (Yen and González-Castro 2000). A significant advantage of the HPG approach with respect to other routing models is that the results are little sensitive to space and time discretization (González-Castro 2000). Even though the performance graph approach is robust, previous models based on the HPG/VPG engine were formulated only for

a single channel or channels in series that are not suitable for routing in dentritic and looped networks.

The present work extends the application of the performance graph approach for unsteady flow routing in river networks. In a similar fashion to the unsteady approach of González-Castro (2000), the model introduced here, to which we refer to as OSU Unsteady Routing (OSU UR) model, assumes that the flow is timewise steady at the different time steps of the routing. The application presented in this paper is limited to subcritical flows; however it can be extended to supercritical flows. Besides relying on the HPG/VPG concept, the OSU Unsteady Routing model also makes use of what we refer to as Rating Performance Graph (RPG). RPGs graphically summarize the dynamic relation between the flow through and the stages upstream and downstream of an in-line structure. An example of a RPG is depicted in Fig. 3. RPG's are conceptually similar to look-up tables such as those utilized to characterize the dynamics of hydraulic structures in the FEQ model of Franz and Melching (1997b). However, RPG's are described with an adaptive spacing so as to capture changes smoothly, which leads to better interpolation estimates. Further details on RPGs are presented in the next section. The paper is organized as follows: (1) formulation of the OSU Unsteady Routing model; (2) the OSU Unsteady Routing model and the unsteady HEC-RAS model are applied to inflow hydrograph routing for a test case in the Applications Guide of the HEC-RAS model (Hydrologic Engineering Center 2010a); and (3) the results of these models are compared and discussed.

#### FORMULATION OF THE OSU UNSTEADY ROUTING MODEL

As mentioned earlier, the OSU Unsteady Routing model is built upon the performance graphs approach. HPG's and VPG's are obtained for each channel reach for as many flows and downstream boundary conditions as necessary to cover the region of possible pairs of upstream and downstream stages in the reach. The maximum water stages are given by the elevations of the channel banks, floodplain levees, or other topographic features. The HPG/VPGs of a system can also include the inundation of urban areas (see Fig. 4).

Overall, the limitations of the OSU UR model are the same as those of unsteady flow routing models based on the Saint-Venant equations. In addition, the present version of the OSU UR model does not have the capability to model dry-bed flows. Furthermore, the performance graphs approach applies to flow routing when the contribution of the local acceleration to the momentum balance is negligible. This condition is met by flows in many natural and man-made systems. As noted by Henderson (1966), even for a steep river with a "very fast-rising flood the local acceleration term is small compared to the gravity and friction terms. Actually, the relative contribution of the local acceleration with respect to the pressure gradient is in the order of the Froude number squared (Henderson 1966). Typically, the maximum Froude number of mild-slope unregulated river systems and regulated river systems is much smaller than one. A flow chart of the OSU Unsteady Routing model is presented in Fig. 5, which comprises six main modules. A brief description of these modules is presented next.

#### **Module I: Definition of river network**

In this module, data of nodes and river reaches that define the river system network are read and stored for later use. In the OSU Unsteady Routing model, a river system is represented as a network where all components of the river system are defined by river reaches and nodes. A river reach is defined by its upstream and downstream nodes and should have similar geometric properties along the reach (e.g., prismatic channel) as to ensure GVF conditions. Whether a channel reach is changing gradually enough so that flow through it can be treated as flow in a prismatic channel must be assessed based on a more general form of the ordinary differential equation (ODE) for GVF conditions. González-Castro (2000) discusses this issue based on the following more general ODE for GVF in non prismatic channels:

$$\frac{dy}{dx} = \frac{S_o - S_f + \frac{Fr^2}{\cos^2\theta} \frac{D}{T} \frac{dT}{dx}}{\cos\theta - Fr^2}$$
(3)

The third term in the numerator of Eq. (3) accounts for changes in nonprismatic channels. From this equation, it is clear that for a canal to behave as prismatic

$$S_o - S_f \gg \frac{Fr^2}{\cos^2\theta} \frac{D}{T} \frac{dT}{dx}$$
(4)

where T = free surface width, D = hydraulic depth (= A/T) and Fr = Froude number. The criterion in Eq. (4) is met by subcritical flows in canals with mild bed slopes for which  $Fr^2/\cos^2\theta = \mathcal{O}(0.1)$  and  $D/T = \mathcal{O}(0.1)$ , even when  $dT/dx = \mathcal{O}(So - Sf)$ .

The flow direction in a river reach is assumed to be from its upstream node to its downstream node as shown in Fig. 6. In Fig. 6, the subscript j and superscript n represent the river reach index and the discrete-time index, respectively, y and Qwith the subscripts u and d denote the water depths and discharges at the upstream and downstream ends of the river reach, respectively. A negative flow discharge in a river reach indicates that reverse flow occurs in that river reach. A node, may have inflowing and/or outflowing river reaches. A river reach is denoted as inflowing (or outflowing) when it conveys to (or from) the node. A node in the proposed model refers to point nodes that have no storage. A node is used at the location of hydraulic structures, connection of reaches and boundary conditions. It is worth mentioning that in the OSU Routing model, a reservoir is not represented by a node but by one or a series of reaches. The treatment of boundary conditions is presented in the Module IV section.

#### **Module II: Computation of the Performances Graphs**

In this module, HPGs and VPGs for all the reaches of the river system and RPG's for the hydraulic structures are computed and stored for later use. In the present application, we constructed the system's performance graphs (PGs) using the HEC-RAS steady module. The subcritical flow mode of the steady HEC-RAS model allows multiple steady subcritical flow simulations (e.g., using multiple discharges) with a fixed downstream water depth as initial condition. This allows a rapid construction of the performance graphs.

Once the PGs are constructed, they are plotted to ensure that they are free of numerically induced errors. Numerical errors may result in the superposition of PCs or in PCs that display oscillatory patterns (see Fig. 7). For the discrete points of a HPC that present apparent problems, the simulations must be repeated with more stringent criteria. These requires decreasing  $\Delta x$  (interpolation between cross-sections) and adjusting convergence parameters for the GVF simulations. This initial screening of the PGs results in the elimination of the aforementioned oscillatory patterns and superposition of HPCs. For a detailed description on the construction of HPGs the reader is referred to Yen and González-Castro (2000) and Schmidt (2002).

#### **Module III: Initial conditions**

The initial conditions in the OSU Unsteady Routing model for each river reach in the steady case can be specified by two of the variables describing the reach's HPG, i.e., the water stages at the reach's end, or one of the stages and the flow discharge in the reach. In the case that the simulation starts from non steady conditions, the initial conditions must be defined as the combination of the stages at the reach's end and the flow at one of its ends, or the flows at the reach's ends and the stage at one of its ends.

#### **Module IV: Boundary conditions**

The following types of boundaries are supported by the OSU Unsteady Routing model:

- External Boundary Condition (EBC), which is defined at the most upstream and downstream ends of the river system. An EBC can have either an inflowing or outflowing river reach connected to the node. An external boundary includes an inflow hydrograph, a stage hydrograph or a rating curve.
- Internal Boundary Condition (IBC), which is defined at internal nodes whenever two or more reaches meet. Three types of IBCs are supported in the OSU Unsteady Routing model. These are:
  - A fixed in-line structure BC (e.g., weirs or dams with fixed position of gates, see Fig. 8a). A single RPG is built for this BC. The water depth immediately upstream of the in-line structure is computed using the RPG built for this structure. For building the RPG of an in-line structure (fixed and mobile), the most upstream and downstream cross sections of the simulation must be kept close to the in-line structure to avoid large errors of conservation of mass. The criteria for the selection of cross sections for the construction of look-up tables can be found in Franz and Melching (1997a). This criteria also applies to the construction of RPGs.

Under simple conditions, good rating equations are probably easier to manage than both RPG's and look-up tables. However, for the operation of multi-type hydraulic structures under complex flow conditions (e.g., downstream flow conditions ranging from low to high water stages, and/or viceversa), Relying on RPGs (look-up tables) may expedite convergence during the simulation of unsteady flow. Both RPGs and look-up tables can be constructed based on measurements of flow discharge and water stages and/or results from computational fluid dynamics simulations. As pointed out by Franz and Melching (1997a; 1997b), the use of look-up tables (or RPGs) can be used to model the dynamics of a variety of hydraulic structures such weirs, culverts, spillways, bridges and canal expansions and contractions. The analyst only needs an adequate description of the relation between the flow through the structure, and the water-surface elevation downstream and upstream of the structure. For an application of RPGs to the opening and closing of gates the reader is referred to Leon et al. (2012).

• A mobile in-line structure BC (e.g., spillways and culverts with control gates, and pumping stations with pumps of different capacities or operated at variable speeds). Dams with a combination of tainter and roller gates are often found in canal networks and dams. In some cases, the roller gates at dams can be raised (underflow gate) or lowered (overflow gate). These gates are often operated using pre-defined operating rules that specify the number of gates to open and percentage of openings while addressing issues such as scour, safety, outdraft, etc. Complex arrays and operations can be handled by using a group of RPGs (e.g., one for each gate opening) for each gate or pre-specified combinations of gates depending on how they will be operated. The RPGs will need to encompass the entire range of gate openings and all possible ranges of downstream and upstream water levels. Naturally, a given flow discharge can be passed through the dam with more than one combination of operational settings. If there is any pre-specified order for the op-

eration of gates, this should be linked to the order of use of the RPGs. For the application of RPGs to mobile in-line structures, the reader is referred to Leon et al. (2012).

• Another IBC is a node without a hydraulic structure which is used to connect two or more river reaches with different roughness or bed slopes, or where an abrupt bed drop or canal expansion occurs. This node is denoted as a junction BC and its schematic is depicted in Fig. 8b.

#### **Module V: Evaluation of time step** ( $\Delta t$ )

This module estimates the time step for advancing the solution to the next time level. In the PG approach, the reach-wise averaged flow discharge  $[1/2 * (Q_u + Q_d)]$ and  $y_d$  are used to determine  $y_u$ . Hence, the time step must be long enough to ensure that disturbances generated at the ends of the reaches have enough time to arrive to the opposite end. This can be expressed as

$$\Delta t > \operatorname{Max}\left\{\frac{\Delta x_j}{\|\overline{u}_j^n\| + \overline{c}_j^n}\right\} \forall j$$
(5)

where j is a reach index, and  $\overline{u}^n$  and  $\overline{c}^n$  are the average reach velocity and gravity wave celerity at time level n, respectively. The average reach-wise gravity wave celerity and velocity are defined as  $\overline{c} = (c_u + c_d)/2$  and  $\overline{u} = (u_u + u_d)/2$ , respectively. A  $\Delta t$  smaller to that of Eq. (5) would mean that disturbances in some of the reaches don't have enough time to travel from one end of the reach to the other, which would violate one of the HPG/VPG assumptions (HPG/VPG use reach-wise averaged flow variables). The time step presented in Eq. (5) can be expressed as

$$\Delta t = k \operatorname{Max} \left\{ \frac{\Delta x_j}{\|\overline{u}_j^n\| + \overline{c}_j^n} \right\} \forall j$$
(6)

where k must be set larger than 1. Numerical tests we performed to evaluate the sensitivity of k for both simulations in section "Application to a looped river network" showed that the results are nearly insensitive to k. The simulation results presented in this paper were generated using k = 3.

#### Module VI: River system hydraulic routing

This module assembles and solves a non-linear system of equations to perform the hydraulic routing of the river system. These equations are assembled based on information summarized in the reaches HPGs and VPGs, RPGs at nodes with hydraulic structures, continuity and compatibility of water stages at junctions, and the system's initial and boundary conditions. The compatibility of water stages at a junction is a simplification of the energy equation ignoring losses and assuming that the differences in velocity heads  $[u^2/(2g)]$  upstream and downstream of the junction are negligible.

In general for a river network consisting of N reaches, there is a total of 3N unknowns at each time level, namely the flow discharge at the upstream and downstream end of each reach and the water depth at the downstream end of each reach, hence 3N equations are required. The water depth at the upstream end  $(y_u)$  of each reach is estimated from the reach HPG using the water depth at its downstream end  $(y_d)$  and the spatially averaged flow discharge  $[Q = (Q_u + Q_d)/2]$ . This can be represented as

$$y_u^n = \operatorname{HPG}[y_d^n, \frac{1}{2}(Q_u^n + Q_d^n)], \forall j$$
(7)

The application of Eq. (7) requires an interpolation process for determining  $y_u$ . Two cases of interpolation are possible. These are depicted in Figs. 9 and 10. As illustrated in these figures, the locus of the upstream and downstream water depth for a given constant flow discharge is denoted as hydraulic performance curve (HPC).

The first case of interpolation is used whenever  $y_{d_x}$  is located between two criti-

cal downstream water depths  $(y_{d_{c1}} \text{ and } y_{d_{c2}})$  as shown in Fig. 9. In this interpolation case (Fig. 9), the discrete points  $(y_{d_a}, y_u(Q_1, y_{d_a})), (y_{d_b}, y_u(Q_1, y_{d_b})), (y_{d_{c_1}}, y_{u_{c_1}})$  and  $(y_{d_{c_2}}, y_{u_{c_2}})$  are known and  $y_{u_x}$  for a given  $y_{d_x}$  and  $Q_x$  is sought.

The second interpolation case is used for all conditions other than the first case (Fig. 10). In the second case interpolation case (Fig. 10), the discrete points  $(y_{da}, y_u(Q_1, y_{da}))$ ,  $(y_{da}, y_u(Q_2, y_{da}))$ ,  $(y_{db}, y_u(Q_1, y_{db}))$  and  $(y_{db}, y_u(Q_2, y_{db}))$  are known and  $y_{ux}$  for a given  $y_{dx}$  and  $Q_x$  is sought.

The interpolation procedure to determine the upstream water depth  $y_{u_x}$  for a given  $Q_x$  and  $y_{d_x}$  for the first case (Fig. 9) can be summarized as follows:

1. Determine coefficient  $c_1$  as:

$$c_1 = \frac{Q_x - Q_1}{Q_2 - Q_1}$$

2. Determine slope  $s_1$  of HPC for  $Q_1$  as:

$$s_1 = \frac{y_u(Q_1, y_{d_a}) - y_{u_{c_1}}}{y_{d_a} - y_{d_{c_1}}}$$

3. Determine slope  $s_2$  of HPC for  $Q_2$  as:

$$s_2 = \frac{y_u(Q_2, y_{d_a}) - y_{u_{c_2}}}{y_{d_a} - y_{d_{c_2}}}$$

4. Determine slope  $s_x$  of HPC for  $Q_x$  as:

$$s_x = s_1(1 - c_1) + s_2c_1$$

5. Determine  $y_u(Q_x, y_{da})$  for  $Q_x$  and  $y_{da}$  as:

$$y_u(Q_x, y_{d_a}) = [1 - c_1][y_u(Q_1, y_{d_a})] + c_1[y_u(Q_2, y_{d_a})]$$

6. Determine  $y_{u_x}$  for  $Q_x$  and  $y_{d_x}$  as:

$$y_{u_x} = y_u(Q_x, y_{d_a}) - (y_{d_a} - y_{d_x})s_x$$

The interpolation procedure to determine the upstream water depth  $y_{u_x}$  for a given  $Q_x$  and  $y_{d_x}$  for the second case (Fig. 10) can be summarized as follows:

1. Determine coefficient  $c_1$  as:

$$c_1 = \frac{Q_x - Q_1}{Q_2 - Q_1}$$

2. Determine coefficient  $c_2$  as:

$$c_2 = \frac{y_{dx} - y_{da}}{y_{db} - y_{da}}$$

3. Determine  $y_u(Q_x, y_{da})$  for  $Q_x$  and  $y_{da}$  as:

$$y_u(Q_x, y_{d_a}) = y_u(Q_1, y_{d_a}) + c_1[y_u(Q_2, y_{d_a}) - y_u(Q_1, y_{d_a})]$$

4. Determine  $y_u(Q_x, y_{db})$  for  $Q_x$  and  $y_{db}$  as:

$$y_u(Q_x, y_{db}) = y_u(Q_1, y_{db}) + c_1[y_u(Q_2, y_{db}) - y_u(Q_1, y_{db})]$$

5. Determine  $y_{u_x}$  for  $Q_x$  and  $y_{d_x}$  as:

$$y_{u_x} = y_u(Q_x, y_{d_a}) + c_2[y_u(Q_x, y_{d_b}) - y_u(Q_x, y_{d_a})]$$

As mentioned earlier, 3N equations are required for a river system of N reaches. For illustration purposes without loosing generality, these equations are formulated for the simple network system depicted in Fig. 11. The network system presented in Fig. 11 has eight reaches and therefore has twenty four unknowns (3x8).

The reach-wise conservation of mass provides one equation for each reach. This equation is typically discretized as follows:

$$\frac{I^n + I^{n+1}}{2} - \frac{O^n + O^{n+1}}{2} = \frac{S^{n+1} - S^n}{\Delta t}, \forall j$$
(8)

where I = inflow, O = outflow, S = storage, n = value at the current time level, t and n + 1 = value at the next time level,  $t + \Delta t$ . It is worth mentioning that the storage S is not considered an unknown because it can be related to  $y_d$  and Q through the VPG (VPG relates S,  $y_d$  and Q ( $Q_u + Q_d$ )/2).

For our simple network (Fig. 11), the application of Eq. (8) provides a total of eight equations. For an inflow hydrograph (external boundary),  $(I^n + I^{n+1})/2$  is the average flow discharge computed from the hydrograph (integrated volume divided by  $\Delta t$ ). The water storage (volume of water) at any time in a river reach is determined from the reach VPG using its downstream water depth and the spatially averaged flow discharge ( $Q_u + Q_d$ )/2 as input values, as

$$S^{n} = \mathbf{VPG} \left[ y_{d}^{n}, \frac{1}{2} (Q_{u}^{n} + Q_{d}^{n}) \right], \forall j$$

$$\tag{9}$$

For more details on the VPG approach, the reader is referred to Hoy and Schmidt

(2006). The application of Eq. (9) requires an interpolation process similar to that of the HPG presented earlier (see Eq. 7). Also, for the network in Fig. 11, five continuity equations are available (nodes B, C, D, E and G). For instance at node C the continuity equation is given by

$$Q_{d2} = Q_{u3} + Q_{u6} \tag{10}$$

It can be also noticed that the system under study has three external boundary conditions (A, F, H). These external boundary conditions (EBCs) could be for instance inflow hydrographs [Q(t)], stage hydrographs [y(t)], overflow structures with hydraulic controls for which there is a flow-stage relation, or any other boundary pre-specified by the user. For each of these EBCs, a flow variable (e.g., Q(t), y(t)) or an equation is available.

For this example, so far sixteen equations are available and eight more equations are needed. Six of the remaining eight equations can be obtained by enforcing water stage compatibility conditions at nodes that connect two or more river reaches and that don't have any hydraulic structure associated to the node. In this case, if a node is connected to k river reaches, k-1 water stage compatibility conditions are available for the junction node. These conditions enforce the same elevation for the water stages immediately upstream and downstream of the node (Fig. 8b). For instance at node C, two water stage compatibility conditions are available as

$$z_{d2} + y_{d2} = z_{u3} + y_{u3}$$
$$z_{d2} + y_{d2} = z_{u6} + y_{u6}$$
(11)

In Eq. (11),  $z_d$  and  $z_u$  are the reach bottom elevations immediately downstream and upstream of a junction node, respectively. In the case of an abrupt change in channel geometry or abrupt bed drop at the junction node, the energy equation instead of the water stage compatibility condition should be used. If a hydraulic structure is associated to the node, the equation is obtained from the RPG of the hydraulic structure. In our example, the last two equations are obtained from RPG's at in-line-structures (nodes B and E in Fig. 11). The treatment of fixed and mobile in-line structures in the OSU Unsteady Routing model are the same. The only difference is that a single RPG is used for a fixed in-line structure while as a group of RPG's (depending on discrete gate position's) are required for a mobile in-line structure. For instance at node B, the water stage upstream of the structure is obtained from the RPG of the structure as follows (see Fig. 11)

$$y_{d_1} = \operatorname{RPG}[y_{u_2}, \frac{1}{2}(Q_{d_1} + Q_{u_2})]$$
(12)

The application of Eq. (12) requires an interpolation process similar to that of the HPG for a river reach presented earlier (Eq. 7). For solving the resulting non-linear system of equations, the OSU UR model uses the Open Source C/C++ MINPACK code. MINPACK solves systems of nonlinear equations, or carries out the least squares minimization of the residual of a set of linear or nonlinear equations. For more details on this library the reader is referred to Moré et al. (1984)

#### **APPLICATION TO A LOOPED RIVER NETWORK**

For illustrating the use of the OSU Unsteady Routing model, this model has been applied to a looped river system adapted from an example in the Applications Guide of the HEC-RAS model (Hydrologic Engineering Center 2010a). The plan view of this system is depicted in Fig. 12 and the geometric characteristics of the twenty six reaches that compose the system are presented in Table 1. To assess the OSU Unsteady Routing model two test cases were simulated. For each case, the results of the OSU Unsteady Routing model were compared with the results from the unsteady HEC-RAS model version 4.0. For this comparison, an inflow hydrograph and a rating curve boundary conditions were specified at the most upstream (node 1) and most downstream (node 26) ends of the system, respectively (Figs. 13 and 14, respectively). The inflow hydrograph (Fig. 13) for the first case represents a slow flood-wave, whereas the one for the second test case represents a fast flood-wave. All performance graphs used in the applications (HPGs and VPGs) were generated using the steady HEC-RAS model.

To determine the initial conditions in the two subcritical flow test cases, a steadyflow discharge of 1.395 m<sup>3</sup>/s with a water depth of 1.65 m at the downstream end of reach 26 (see Fig. 12) was used. These values of water depth and flow discharge were used for determining the water depths and flow discharges at the ends of every reach. The water depth and flow discharge at the ends of each reach are the necessary initial conditions.

#### Case 1: slow flood-wave

The simulated flow and stage hydrographs at two locations obtained with the OSU Unsteady Routing model are compared with the results obtained with HEC-RAS in Figs. 15 and 16, respectively. As can be observed in these figures, the difference between the OSU Unsteady Routing model results and the HEC-RAS results are rather small.

To evaluate the discrepancies in flow discharge, water stage and volume between the OSU Unsteady Routing and the HEC-RAS model results, the following relative differences were defined

$$E_Q(\%) = 100 \left( \frac{Q_{\text{OSU Unsteady}} - Q_{\text{HEC-RAS}}}{Q_{\text{Inflow Hidromax}} - Q_{\text{Inflow Hidromin}}} \right)$$
$$E_{WS}(\%) = 100 \left( \frac{WS_{\text{OSU Unsteady}} - WS_{\text{HEC-RAS}}}{WS_{\text{HEC-RAS}max} - WS_{\text{HEC-RAS}min}} \right)$$
$$E_V(\%) = 100 \left( \frac{V_{\text{OSU Unsteady}} - V_{\text{HEC-RAS}}}{V_{\text{HEC-RAS}}} \right)$$
(13)

where  $E_Q$ ,  $E_{WS}$  and  $E_V$  are the relative differences in flow discharge, water stage and cumulative outflow volume, respectively. Note in Eq. (13) that the discrepancies in flow discharge and water stage are defined as the difference of results between the two models normalized by the range of flow discharges or water stages.

The relative differences defined by Eq. (13) are shown in Figs. 17, 18 and 19 for the flow discharge, water stage and cumulative outflow volume (V) at the downstream end of reaches 9 and 18, respectively. The relative differences in flow discharge ranged from -0.6 to 0.6 %, in water stage varied from -0.25 to 0.30 % and in cumulative outflow volumes varied from -1.0 to 1.5 %. While the discrepancies between the results of HEC-RAS and the OSU routing model are negligible, the maximum discrepancies in discharge and stage occur near the time of the peak discharge (see Figures 17, 18). The local acceleration term may be responsible for the discrepancies, as the local acceleration term is more important in a sudden rising and falling of the flow (i.e., near peak flow). The unsteady HEC-RAS model accounts for the local acceleration term, while as the OSU routing model neglects this term.

Note that the relative differences in flow discharge and cumulative outflow volume for reaches 9 and 18 shown in Figures 17 and 19 nearly complement each other. Complementary results are to be expected when the model of reference (HEC-RAS in this case) is exact and when the two branches of the loop are identical (same number of reaches, all with identical geometric and roughness). In this application example the two branches of the loop are not identical. The relative differences in flow discharge and cumulative outflow volume for the case of slow flood-wave (Figures 17 and 19) were near complimentary likely because the model of reference (HEC-RAS) was highly accurate in this case.

#### **Case 2: fast flood-wave**

The flow and stage hydrographs at two locations simulated with the OSU Unsteady Routing and the HEC-RAS models are shown in Figs. 20 and 21, respectively. Simulation results obtained with both models appear to be similar. However the differences are noticeably more significant for this case than those for slow flood-wave conditions of case 1. Figs. 22, 23 and 24, show the differences in flow discharge, water stage and cumulative outflow volume between OSU Unsteady Routing and HEC-RAS (according with Eq. 13) at the downstream end of reaches 9 and 18. As shown in Figs. 22, 23 and 24, the relative difference in flow discharge ranged from -1.5 to 0.7 %, in water stage from -0.7 to 0.5 %, and in cumulative outflow volume from -1.0 to 5.5 %.

Fig. 25 shows the plot of the flow discharge versus water stage (i.e., rating curve) at the downstream end of reach 18 for the fast flood-wave case (case 2). This figure shows a typical looped rating curve having greater flows at lower stages in the rising limb and smaller flows at higher stages in the receding limb. The rating curve for the slow flood-wave case is similar to that of the fast flood-wave case; however, it is not shown due to space limitations.

To assess the robustness of the OSU Unsteady Routing model with respect to time discretization, we carried out the simulations with three different time resolutions (t = 10 s, 30 s and 60 s). Fig. 26 shows the results of numerical accuracy of OSU Unsteady Routing model due to time discretization for the upstream end of reach 14 and downstream end of reach 23. This figure appears to show that time discretization does not affect significantly the results. With regard to space and water depth discretization ( $\Delta x$ 

and  $\Delta y$ , respectively), the maximum value of  $\Delta x$  or  $\Delta y$  used to obtain a good accuracy is system dependent and should be obtained for each system by iteration. For instance, for  $\Delta x$ , two values of  $\Delta x$  can be used to check if the simulated results of discharge and water stage are similar for both discretizations. The process can be repeated to find the largest  $\Delta x$  that produces the desired accuracy and in turn the minimum CPU time. The reader is referred to González-Castro (2000) for an in-depth discussion on spatial discretization.

The results for the Central Processing Unit (CPU) times obtained using a Dell Precision T3500 2.67GHz, 1.00 GB of RAM for the OSU Unsteady Routing and the HEC-RAS models are presented in Table 2. The CPU time in Table 2 included the time of pre-processing and computational engine but not that of post-processing. The pre-processing in the OSU Routing model involves the reading of performance graphs (from .dat files into matrices) while as in HEC-RAS, it involves pre-processing typically demands more time and it depends on the user-specified outputs. As can be observed in Table 2, the results obtained with the OSU Unsteady Routing model is about 700% faster than that of the HEC-RAS model. In the OSU Unsteady Routing model, for the slow flood-wave, the time step ( $\Delta t$ ) ranged from 8.16 to 12.62 s while as for the fast flood-wave, the time step ranged from 8.31 to 12.62 s. For both flood-waves, the HEC-RAS model was simulated using a time step of 10 seconds.

#### CONCLUSIONS

In this paper we present a computationally efficient model for unsteady flow routing through river networks with dendritic, looped or a combination of dendritic and looped topologies. The application of the OSU UR model focused on routing of subcritical flows; however, the model can be extended to include reaches susceptible to transition from subcritical to supercritical flow (and vice versa) during routing. The model builds upon the application of Hydraulic Performance Graph (HPG) to unsteady flow routing introduced by González-Castro (2000) and adopts the Volume Performance Graph (VPG) introduced by Hoy and Schmidt (2006). Moreover, in the OSU UR model we extend the concept of performance graphs to ratings and introduce the Rating Performance Graphs (RPGs) which graphically summarize the dynamic relation between the flow through and the stages upstream and downstream of in-line structures. The OSU Unsteady Routing model solves a system of nonlinear equations assembled based on information summarized in the systems' HPG's, VPG's and RPG's, continuity in junctions, water stage compatibility at junctions of reaches, and the system's initial and boundary conditions. We exemplify the applicability of the OSU Unsteady Routing model to a looped network and contrast its simulation results with those from the wellknown unsteady HEC-RAS model. The key findings are as follows:

- Results show that agreement between OSU Unsteady Routing and HEC-RAS models is very good for slow and fast flood-wave conditions, with better agreement for slow flood-wave conditions.
- 2. The use of HPGs, VPGs and RPGs for unsteady flow routing in a river system as proposed herein is a relatively robust and numerically efficient approach because the momentum and storage for all river reaches are computed prior to the system routing based on the momentum and mass conservation principles of GVF and most of the computations for the system routing only involves interpolation steps to satisfy the prescribed BC's. It is worth mentioning that the OSU UR model provided an accurate solution for the looped system right the first time, while the unsteady HEC-RAS model required few adjustments to get the model to run properly. In addition, when using the performance graphs approach, instabilities or other problems due to discretization and numerical inaccuracies are removed as the system's HPGs, VPGs and RPGs are being

constructed.

- 3. The application examples presented here suggest that, overall, the proposed model is computationally more efficient than and affords a numerical accuracy comparable to the unsteady HEC-RAS model for unsteady flow routing through river networks. It is clear that the CPU time for pre-computing the PGs can be computationally demanding but this is done only once. The advantage of the OSU UR model may be significant when it is used for optimization problems such as real-time operation of regulated river systems. In this case hundreds or even thousands of runs would be needed for each operational decision that may be as short as 30 minutes.
- 4. Further assessment of the proposed approach includes evaluating the sensitivity of results to the algorithms for interpolation from the HPGs, VPGs and RPGs, and to discretization over space prescribed for the routing.
- 5. The authors are considering assessing the effect of BCs on the accuracy and robustness of the OSU UR model as future work.

#### ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support of NSF Idaho EPSCoR Program and the National Science Foundation under award number EPS-0814387. The first author also would like to thank the financial support of the School of Civil and Construction Engineering at Oregon State University. We gratefully acknowledge the collegiality of the anonymous reviewers; their constructive and insightful comments certainly helped us improve the manuscript.

#### REFERENCES

Abbot, M. B. and Basco, D. R. (1989). Computational Fluid Dynamics. An Introduction for Engineers. Longman Scientific and Technical, New York.

Bakhmeteff, B. A. (1932). Hydraulics of Open Channels. McGraw-Hill, New York.

- Franz, D. D. and Melching, C. S. (1997a). "Approximating the hydraulic properties of open channels and control structures, during unsteady flow using the Full Equations Utility (FEQUTL) program." *Water-Resources Investigations Report No. 97-4037*, U.S. Geological Survey.
- Franz, D. D. and Melching, C. S. (1997b). "Full Equations (FEQ) model for the solution of the full, dynamic equations of motion for one-dimensional unsteady flow in open channels and through control structures." *Water-Resources Investigations Report No. 96-4240*, U.S. Geological Survey.
- González-Castro, J. A. (2000). "Applicability of the hydraulic performance graph for unsteady flow routing." Ph.D. thesis, Univ. of Illinois at Urbana-Champaign, Illinois.
- González-Castro, J. A. and Yen, B. C. (1996). "Hydraulic performance of open channel breaching." ASCE, 334–339.
- González-Castro, J. A. and Yen, B. C. (2012). "Open-channel unsteady flow routing by hydraulic performance graph." Submitted for possible publication in the J. Hydraul. Eng.
- Henderson, F. M. (1966). Open Channel Flow. Macmillan Publishing Co., Inc., New York.
- Hoy, M. A. and Schmidt, A. R. (2006). "Unsteady flow routing in sewers using hydraulic and volumetric performance graphs." Proc. World Environmental and Water Resources Congress, Nebraska, 1–10.
- Hydrologic Engineering Center (2010a). *HEC-RAS, River Analysis System. Applications Guide. Version 4.1.* U.S. Army Corps of Engineers, Davis, California.
- Hydrologic Engineering Center (2010b). *HEC-RAS, River Analysis System, User's Manual. Version 4.1.* U.S. Army Corps of Engineers, Davis, California.
- Leon, A. S., Kanashiro, E. A., Valverde, R., and V., S. (2012). "A dynamic framework for intelligent control of river flooding - a case study." *Journal of Water Resources*

Planning and Management, Accepted.

- Moré, J. J., Sorensen, D. C., Hillstrom, K. E., and Garbow, B. S. (1984). *The MINPACK Project*. Sources and Development of Mathematical Software, W. J. Cowell, ed., Prentice-Hall, 88-111.
- Schmidt, A. R. (2002). "Analysis of Stage-Discharge Relations for Open-Channel Flows and Their Associated Uncertainties." Ph.D. thesis, Univ. of Illinois at Urbana-Champaign, Urbana, IL.
- Yen, B. C. (1986). *Hydraulics of sewers*. Advances in Hydroscience 14, 1-115, Academic Press, London.
- Yen, B. C. and González-Castro, J. A. (2000). "Open-channel capacity determination using hydraulic performance graph." J. Hydraul. Eng., 126(2), 112–122.

#### NOTATION

The following symbols are used in this paper:

EBC = external boundary condition;

 $\overline{c}$  = average gravity wave celerity;

HPG = hydraulic performance graph;

$$I = inflow;$$

$$\forall i = \text{for all } i;$$

N = set of nodes;

NB = set of downstream boundary nodes;

NS = set of source nodes;

$$O = \text{outflow};$$

 $Q_{d_i}$  = flow discharge at downstream end of reach j;

 $Q_{u_i}$  = flow discharge at upstream end of reach j;

$$S = \text{storage};$$

 $\overline{u}$  = average reach velocity;

 $\Delta x$  = length of river reach;

 $y_{d_j}$  = water depth at downstream end of reach j;

 $y_{u_i}$  = water depth at upstream end of reach *j*;

 $z_{d_i}$  = channel bottom elevation at downstream end of reach *j*;

 $z_{u_i}$  = channel bottom elevation at upstream end of reach *j*;

 $\Delta t = \text{time step;}$ 

#### **Subscripts**

d = downstream end index;

i = node index;

j = river reach index;

u = upstream end index;

### Superscripts

n =discrete-time index;

# List of Figures

1	Example of a Hydraulic Performance Graph (HPG)	31
2	Example of a Volumetric Performance Graph (VPG)	32
3	Example of a Rating Performance Graph (RPG).	33
4	Schematic of a river cross-section	34
5	Flow chart of the OSU Unsteady Routing model	35
6	Schematic of a river reach	36
7	Typical instability problems during construction of HPGs	37
8	Schematic of an in-line structure and a junction node	38
9	Schematic of first interpolation case	39
10	Schematic of second interpolation case	40
11	Schematic of a simple network system	41
12	Plan view of HEC-RAS looped river system	42
13	Inflow hydrograph at node 1 for the slow and fast flood-wave cases	43
14	Rating curve boundary at node 26	44
15	Flow hydrograph at downstream end of reaches 4 and 18 for case 1. $% \left( {{{\rm{A}}_{\rm{B}}}} \right)$ .	45
16	Stage hydrographs at downstream end of reaches 4 and 18 for case 1	46
17	Difference in flow discharges at downstream end of reaches 9 and 18	
	for case 1	47
18	Difference in water stages at downstream end of reaches 9 and 18 for	
	case 1	48
19	Difference in conservation of volume at downstream end of reaches 9	
	and 18 for case 1	49
20	Flow hydrographs at downstream end of reaches 4 and 18 for case 2	50
21	Stage hydrographs at downstream end of reaches 4 and 18 for case 2	51

22	Difference in flow discharges at downstream end of reaches 9 and 18					
	for case 2	52				
23	Difference in water stages at downstream end of reaches 9 and 18 for					
	case 2	53				
24	Difference in conservation of volume at downstream end of reaches 9					
	and 18 for case 2	54				
25	Flow discharge vs. water stage at downstream end of reach 18 for case 2.	55				
26	Numerical accuracy of OSU Unsteady Routing model due to time dis-					
	cretization for the upstream end of reach 14 and downstream end of					
	reach 23	56				



FIG. 1. Example of a Hydraulic Performance Graph (HPG).



FIG. 2. Example of a Volumetric Performance Graph (VPG).



FIG. 3. Example of a Rating Performance Graph (RPG).



FIG. 4. Schematic of a river cross-section



FIG. 5. Flow chart of the OSU Unsteady Routing model.



FIG. 6. Schematic of a river reach



FIG. 7. Typical instability problems during construction of HPGs



(b) Schematic of a junction node

## FIG. 8. Schematic of an in-line structure and a junction node



FIG. 9. Schematic of first interpolation case



FIG. 10. Schematic of second interpolation case



FIG. 11. Schematic of a simple network system



FIG. 12. Plan view of HEC-RAS looped river system.



FIG. 13. Inflow hydrograph at node 1 for the slow and fast flood-wave cases.



FIG. 14. Rating curve boundary at node 26



FIG. 15. Flow hydrograph at downstream end of reaches 4 and 18 for case 1.



FIG. 16. Stage hydrographs at downstream end of reaches 4 and 18 for case 1.



FIG. 17. Difference in flow discharges at downstream end of reaches 9 and 18 for case 1.



FIG. 18. Difference in water stages at downstream end of reaches 9 and 18 for case 1.



FIG. 19. Difference in conservation of volume at downstream end of reaches 9 and 18 for case 1.



FIG. 20. Flow hydrographs at downstream end of reaches 4 and 18 for case 2.



FIG. 21. Stage hydrographs at downstream end of reaches 4 and 18 for case 2.



FIG. 22. Difference in flow discharges at downstream end of reaches 9 and 18 for case 2.



FIG. 23. Difference in water stages at downstream end of reaches 9 and 18 for case 2.



FIG. 24. Difference in conservation of volume at downstream end of reaches 9 and 18 for case 2.



FIG. 25. Flow discharge vs. water stage at downstream end of reach 18 for case 2.



FIG. 26. Numerical accuracy of OSU Unsteady Routing model due to time discretization for the upstream end of reach 14 and downstream end of reach 23

Tuble 1. Geometric characteristics of fiver federics									
Reach ID	Length (m)	<i>z</i> <sup><i>u</i></sup> (m)	$z_d$ (m)	Slope (m/m)					
1	35.05	6.2332	6.1631	0.002000					
2	35.05	6.1631	6.0930	0.002000					
3	38.10	6.0930	6.0198	0.001920					
4	32.00	6.0198	5.9497	0.002190					
5	24.38	5.9497	5.8887	0.002500					
6	39.62	5.8887	5.8339	0.001385					
7	39.62	5.8339	5.7790	0.001385					
8	45.72	5.7790	5.7241	0.001200					
9	44.20	5.7241	5.6693	0.001241					
10	32.00	5.6693	5.6144	0.001714					
11	36.58	5.6144	5.5596	0.001500					
12	39.62	5.5596	5.5047	0.001385					
13	25.91	5.5047	5.4712	0.001294					
14	21.34	5.9497	5.8979	0.002429					
15	51.82	5.8979	5.8491	0.000941					
16	54.86	5.8491	5.7760	0.001333					
17	48.16	5.7760	5.7638	0.000253					
18	56.39	5.7638	5.7028	0.001081					
19	56.39	5.7028	5.6510	0.000919					
20	54.86	5.6510	5.5839	0.001222					
21	57.00	5.5839	5.5535	0.000535					
22	57.91	5.5535	5.5047	0.000842					
23	21.34	5.5047	5.4712	0.001571					
24	47.24	5.4712	5.4315	0.000839					
25	42.67	5.4315	5.4132	0.000429					
26	51.82	5.4132	5.3950	0.000353					

Table 1. Geometric characteristics of river reaches

DescriptionOSU Unsteady Routing Time (s)HEC-RAS Time (s)Case 1 (slow flood-wave) $218.8 (\overline{\Delta t} = 9.77 \text{ s})$  $752.7 (\Delta t = 10 \text{ s})$ Case 2 (fast flood-wave) $7.3 (\overline{\Delta t} = 10.27 \text{ s})$  $52.6 (\Delta t = 10 \text{ s})$ 

Table 2. Comparison of CPU times for the simulation of cases 1 and 2.