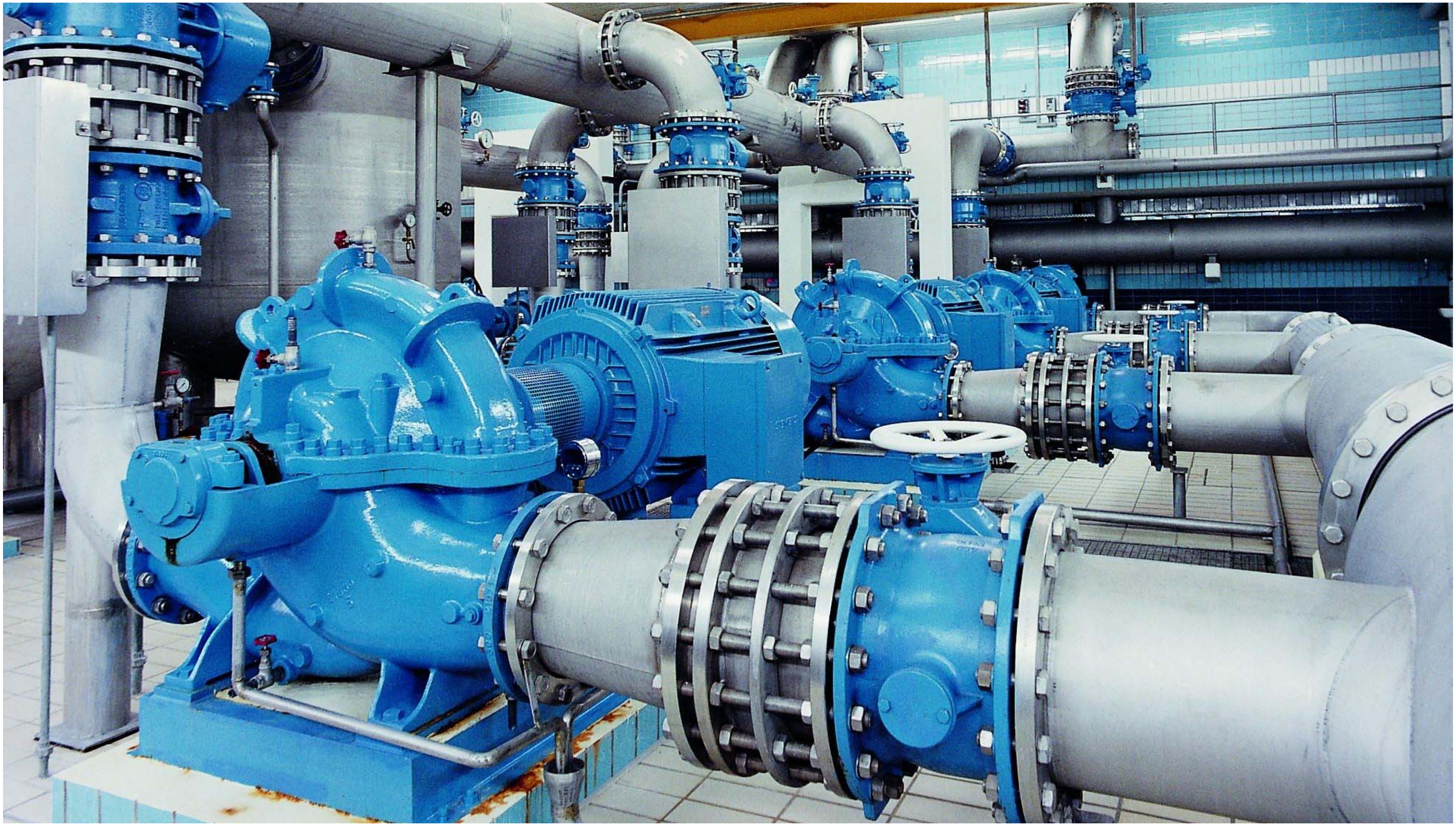
Water hammer Flows



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Videos of water hammer flows

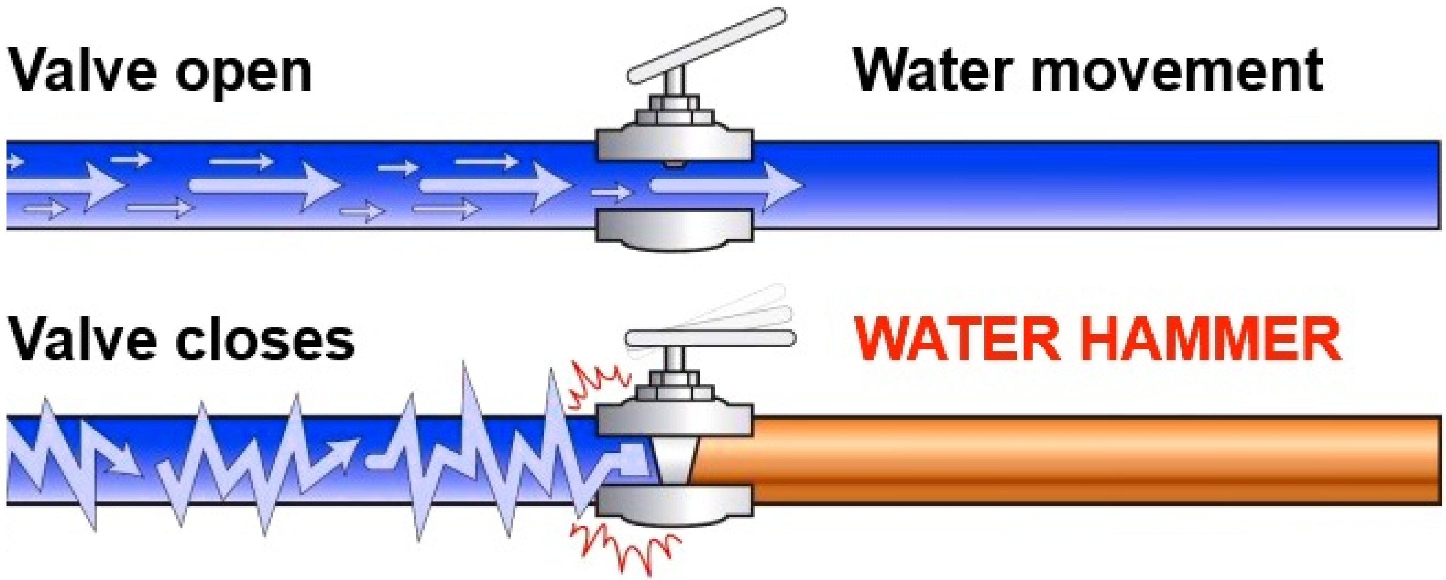
What is Water Hammer?

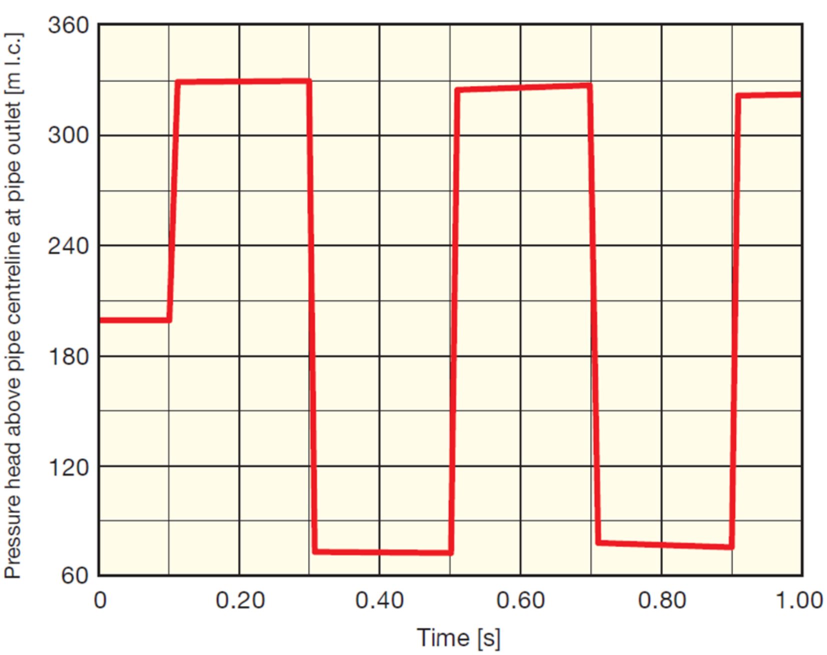
https://www.youtube.com/watch?v=6ydsAIHWVNM

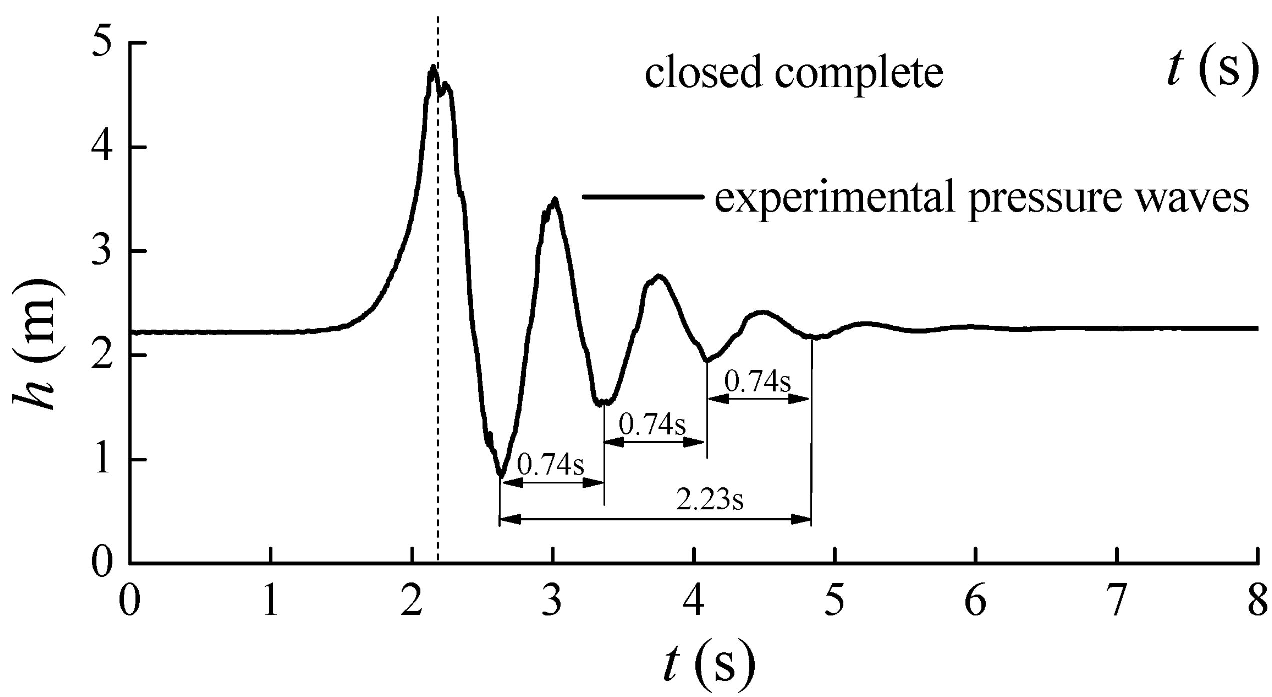
Surge suppression tank

https://www.youtube.com/watch?v=lvKDw-IE2gw

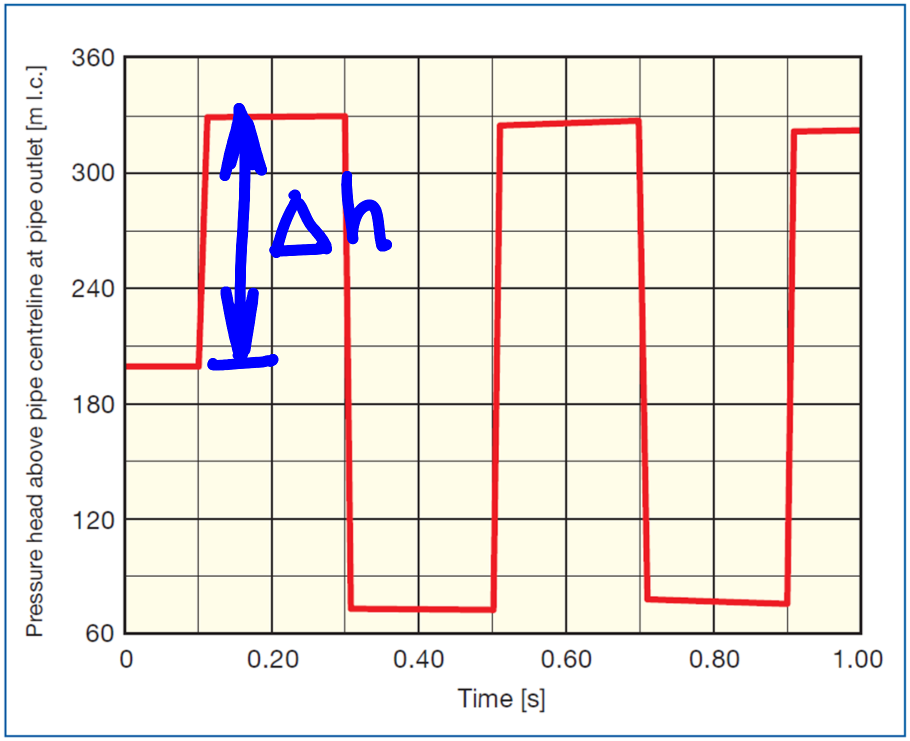
Pressure Transients due to Gate Closure







The Joukowsky equation



$$\Delta h = \frac{c}{g}\Delta V$$

$V_0 = 10$ m/s

$V_f = 0$ m/s

$c \sim 1000$ m/s

$$\Delta h = \frac{1000}{9.8} \times 10$$

~ 1000 m ✓

Δh = Pressure head change (m or ft)

ΔV = Flow velocity change (m/s of ft/s)

c = Wave propagation velocity through the fluid in the pipe (m/s or ft/s)

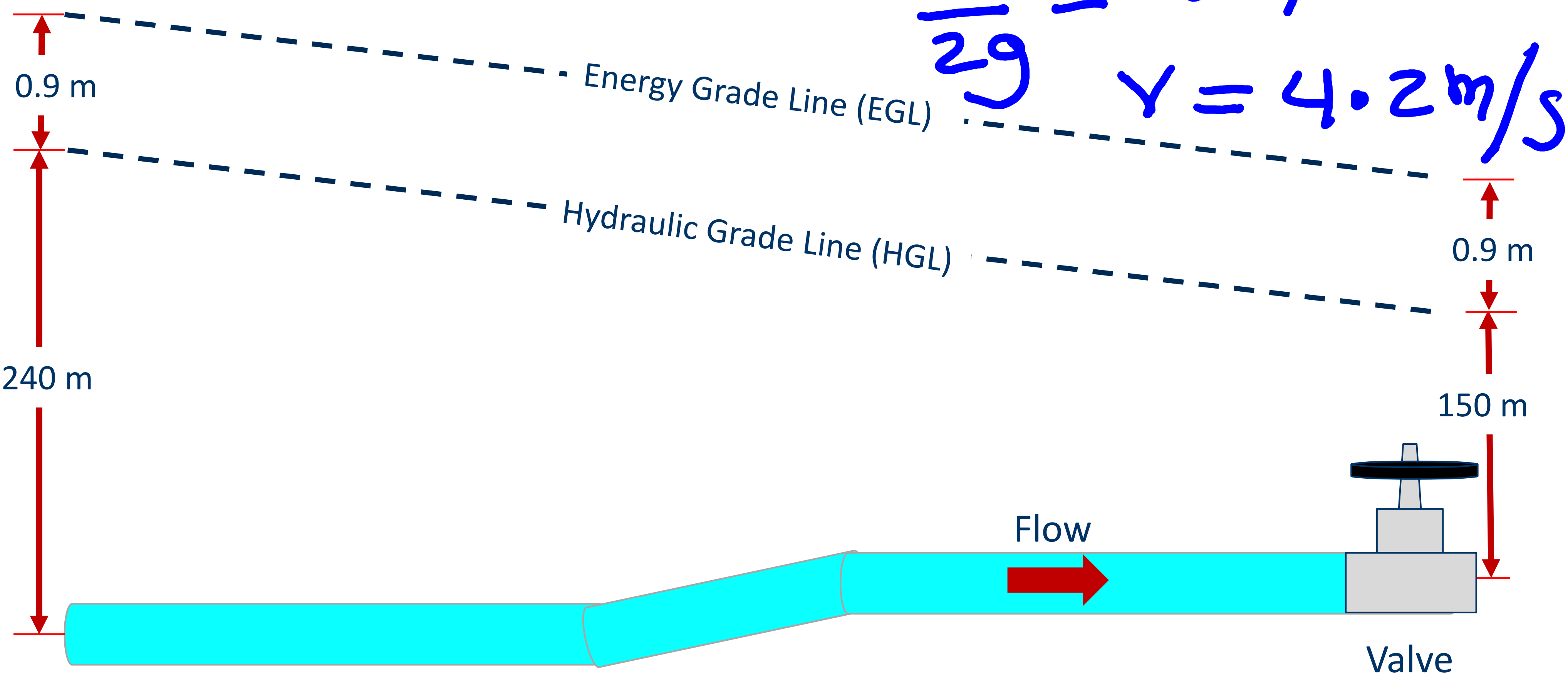
g = Acceleration due to gravity

Example:

In the pipeline system depicted below, **(1)** determine the **maximum** and **minimum** pressure head that can be produced due to instantaneous valve closure if the valve is located at the downstream end of the pipeline. **(2)** Is cavitation a problem for this pipeline? Assume that the water temperature is 20°C, the pipe diameter is 0.3 m, and the pressure wave celerity (waterhammer wave speed) is 200 m/s.

$$\frac{v^2}{2g} = 0.9$$

$$v = 4.2 \text{ m/s}$$



a) $\left(\frac{P}{\gamma}\right)_{max}$, $\left(\frac{P}{\gamma}\right)_{min}$

$$\Delta h = \frac{c}{g} \Delta V$$

$$\left(\frac{P}{\gamma}\right)_{max} = 240 + \Delta h$$

$$\left(\frac{P}{\gamma}\right)_{min} = 150 - \Delta h$$

$$\Delta h = \frac{200 \times 4.2}{9.8}$$

$$\Delta h = 85.7 \, m$$

$$\left(\frac{P}{\gamma}\right)_{max} = 325.7 \, m$$

$$\left(\frac{P}{\gamma}\right)_{min} = 64.3 \, m$$

Cavitation

In theory occurs when $\left(\frac{P}{\gamma}\right)$ is below $-10.1 \, m$

In practice ~ -6 or $-7 \, m$ (gage pressure)

Analysis of water hammer phenomenon due to gradual and sudden valve closure

The pressure rise due to water hammer depends upon:

(a) The velocity of the flow of water in pipe,

(b) The length of pipe,

(c) Time taken to close the valve,

(d) Elastic properties of the material of the pipe.

The following cases of water hammer will be considered:

- Gradual closure of valve,
- Sudden closure of valve when pipe is rigid, and
- Sudden closure of valve when pipe is elastic.

- The time required for the pressure wave to travel from the valve to the reservoir and back to the valve is:

$$t = \frac{2L}{C}$$

Where:

L = length of the pipe (m)

C = speed of pressure wave, celerity (m/s)

- If the valve time of closure is t_c, then

  - If $t_c > \frac{2L}{C}$ the closure is considered **gradual**

  - If $t_c \leq \frac{2L}{C}$ the closure is considered **sudden**

The speed of pressure wave *"C"* depends on :

- the pipe wall material.
- the properties of the fluid.
- the anchorage method of the pipe.

- $C=\sqrt{\dfrac{E_b}{\rho}}$ if the pipe is rigid
- $C=\sqrt{\dfrac{E_c}{\rho}}$ if the pipe is elastic **(General Formula)**

where $$\boxed{\frac{1}{E_c}=\frac{1}{E_b}+\frac{Dk}{E_p e}}$$

Where:

- C = celerity of pressure wave due to water hammer.
- ρ = water density (1000 kg/m^3).
- E_b = bulk modulus of water (2.1 x 10^9 N/m^2). (3.0 x 10^5 psi)
- E_c = effective bulk modulus of water in elastic pipe.
- E_p = Modulus of elasticity of the pipe material.
- e = thickness of pipe wall.
- D = diameter of pipe.
- k = factor depends on the anchorage method:

= $(\frac{5}{4} - \varepsilon)$ for pipes free to move longitudinally,

= $(1 - \varepsilon^2)$ for pipes anchored at both ends

= $(1 - 0.5\varepsilon)$ for pipes with expansion joints.

- where ε = Poisson's ratio of the pipe material (0.25 - 0.35).

  ε = 0.25 for common pipe materials.

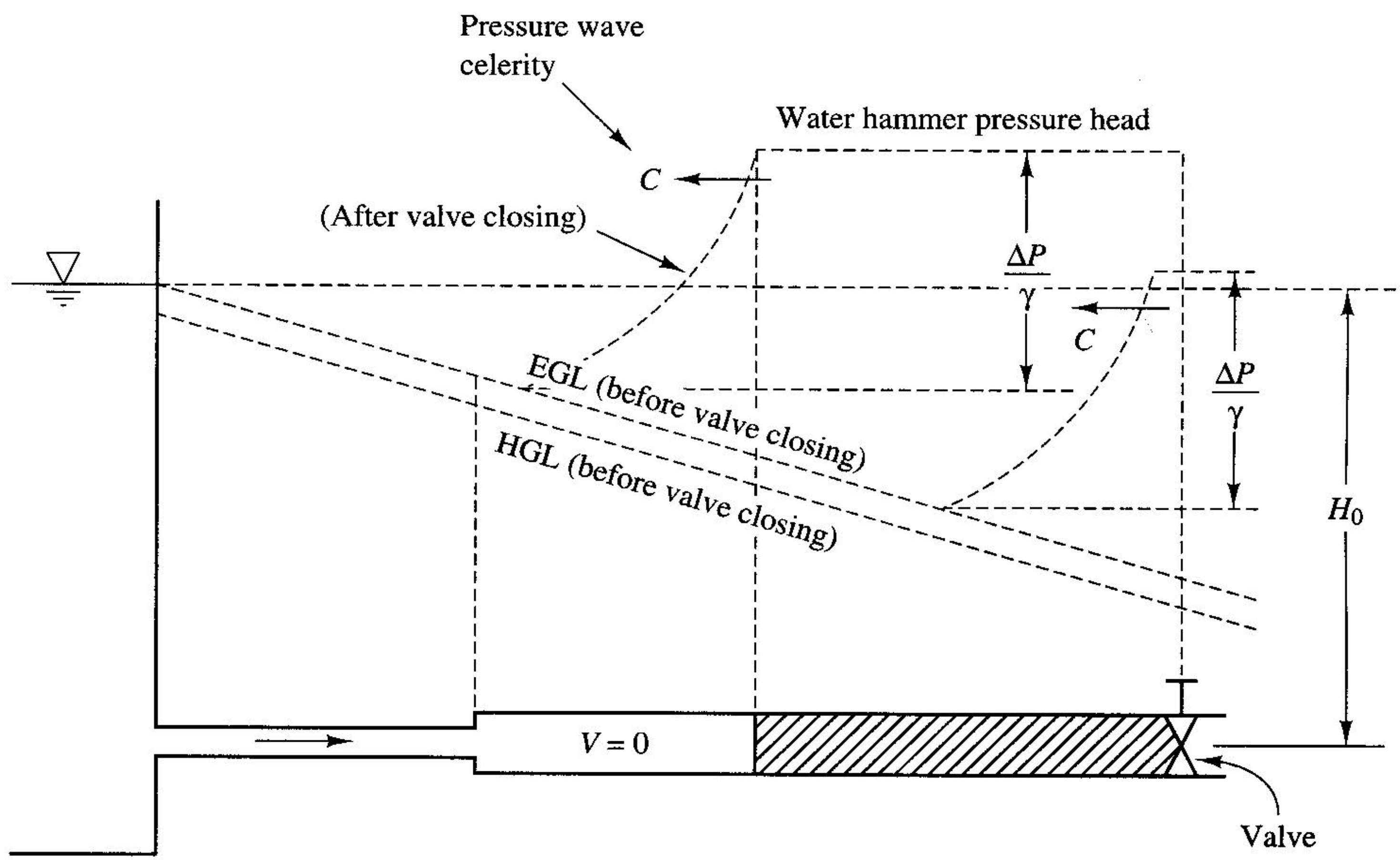
Modulus of elasticity of the pipe material (E_p)

Pipe Material	E_p (N/m^2)	E_p (psi)
Cast-iron, gray	$1.1x10^{11}$	$16x10^6$
Cast-iron, malleable	$1.6x10^{11}$	$23x10^6$
Concrete, reinforced	$1.6x10^{11}$	$25x10^6$
Copper	$9.7x10^{10}$	$14x10^6$
Steel	$1.9x10^{11}$	$28x10^6$

Maximum pressure created by water hammer

The total pressure experienced by the pipe is

$$P = \Delta P + P_o$$



Case 1: Gradual Closure of Valve

- If the time of closure $t_c > \dfrac{2L}{C}$, then the closure is said to be gradual and the increased pressure is

$$\Delta P = \frac{\rho L V_0}{t}$$

where,

- V_0 = initial velocity of water flowing in the pipe before pipe closure
- t = time of closure.
- L = length of pipe.
- ρ = water density.
- The pressure head caused by the water hammer is

$$\Delta H = \frac{\Delta P}{\gamma} = \frac{\rho L V_0}{\rho g t} = \frac{L V_0}{g t}$$

Another method for gradual closure of valve (*t* > 2 *L*/*C*)

The maximum water hammer calculated by the Allievi formula is

$$\Delta P = P_o\left(\frac{N}{2} + \sqrt{\frac{N^2}{4} + N}\right)$$

Where *P*o is the steady-state pressure in the pipe, and

$$N = \left(\frac{\rho L V_o}{P_o t}\right)$$

Case 2: Sudden closure of valve when pipe is rigid

- If time of closure $t_c \leq \frac{2L}{C}$, then the closure is said to be *Sudden*.
- The pressure head due caused by the water hammer is

$$\Delta P = \rho C V_0$$

$$\Delta H = \frac{C V_0}{g}$$

- But for rigid pipe $C = \sqrt{\frac{E_b}{\rho}}$ so, $\Delta H = \frac{V_0}{g}\sqrt{\frac{E_b}{\rho}}$

$$\Delta P = V_0 \sqrt{E_b \rho}$$

Case 3: Sudden closure of valve when pipe is elastic

- If time of closure $t_c \leq \frac{2L}{C}$, then the closure is said to be *Sudden*.
- The pressure head caused by the water hammer is

$$\Delta P = \rho C V_0 \qquad \boxed{\Delta H = \frac{C V_0}{g}}$$

- But for elastic pipe $C = \sqrt{\frac{E_c}{\rho}}$ so,

$$\boxed{\Delta P = V_0 \sqrt{\frac{\rho}{\left(\frac{1}{E_b} + \frac{DK}{E_p e}\right)}}} \qquad \boxed{\Delta H = \frac{V_0}{g} \sqrt{\frac{1}{\rho\left(\frac{1}{E_b} + \frac{DK}{E_p e}\right)}}}$$

Applying the above formulas we can determine the maximum and minimum pressures and pressure heads.

The **total pressure** and **pressure head** at any point in the pipe after closure (water hammer) are given by:

$$P_M = P_{M,before\,closure} \pm \Delta P$$

$$H_M = H_{M,before\,closure} \pm \Delta H$$

Example:

A steel pipe 5,000 ft long laid on a uniform slope has an 18-in. diameter and a 2-in. wall thickness. The pipe carries water from a reservoir and discharges it into the air at an elevation 150 ft below the reservoir free surface. A valve installed at the downstream end of the pipe permits a flow rate of 25 cfs. If the valve is completely closed in 1.4 sec, calculate the maximum water hammer pressure at the valve. Assume the longitudinal stresses in the pipeline are negligible.

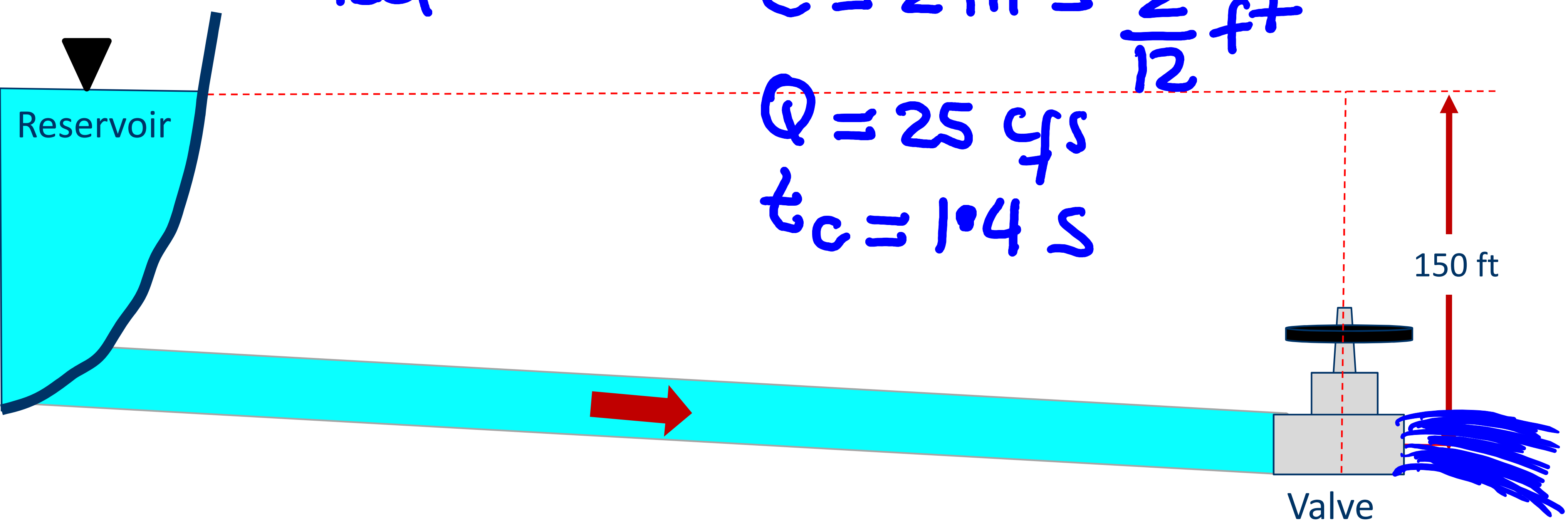
$L = 5000\,ft$ steel

$D = 18\,in = 1.5\,ft$

$e = 2\,in = \frac{2}{12}\,ft$

$Q = 25\,cfs$

$t_c = 1.4\,s$



* maximum waterhammer pressure ΔP

find C (∴ find Ec)

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{DK}{E_p e}$$

$E_b = 3.0 \times 10^5$ psi , $E_p = 28 \times 10^6$ psi

$k = 1.0 \left(\frac{5}{4} - 0.25\right)$, $e = 2\text{in} \left(\frac{2}{12}\text{ft}\right)$

$$\frac{1}{E_c} = \frac{1}{3.0 \times 10^5} + \frac{18\,\text{in} \times 1.0}{(28 \times 10^6)\,2\,\text{in}}$$

$E_c = 2.74 \times 10^5$ psi ∴ $C = \sqrt{\frac{E_c}{\rho}}$

$$C = \sqrt{\frac{2.74 \times 10^5 (144)}{1.94}} = 4510 \text{ ft/s}$$

$$t = \frac{2L}{c} = \frac{2 \times 5000}{4510} = 2.22 \text{ seconds}$$

$1.45s < 2.22s$ [sudden closure]

* $$V_0 = \frac{Q}{A} = \frac{25}{\frac{\pi \times 1.5^2}{4}} = 14.1 \text{ ft/s}.$$

$$\Delta p = \rho V_0 C = 1.94 \times 14.1 \times 4510$$

$$= 1.23 \times 10^5 \frac{lb}{ft^2}$$

$$= 854 \text{ psi}$$