Florida International University
Department of Civil and Environmental Engineering
Optimization in Water Resources Engineering, Spring 2020

SIMPLIFIED APPLICATIONS IN WATER RESOURCES

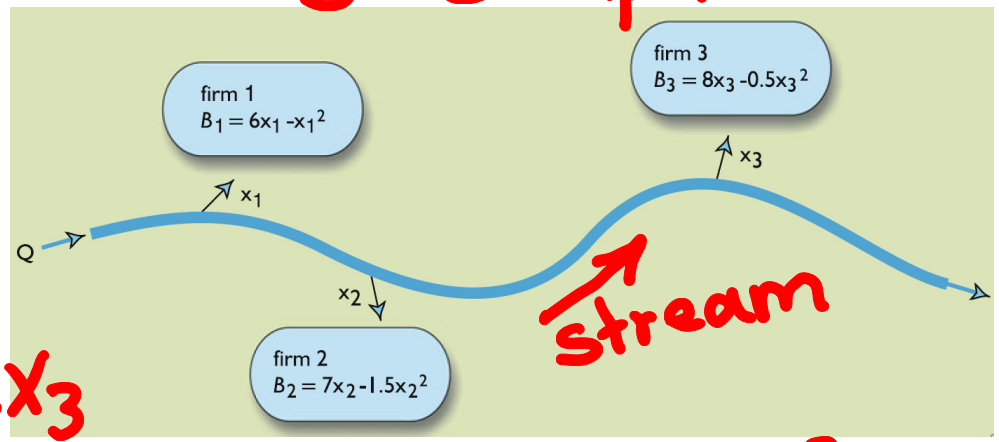
Arturo S. Leon, Ph.D., P.E., D.WRE

Water allocation to users from a stream

B = Benefit

(Adapted from Water Resources Planning and Management, Loucks and van Beek)

Consider the problem of finding the best allocations of water to the three water consuming firms shown in the figure below. The maximum allocation, x_i, to any single user "i" cannot exceed 5, and the sum of all allocations cannot exceed 6.



Decision variables: x_1, x_2, x_3

Maximize $B_1 + B_2 + B_3 = 6x_1 - x_1^2 + 7x_2 - 1.5x_2^2 + 8x_3 - 0.5x_3^2$

Subject to:

$0 \le x_i \le 5 \quad (i = 1,2,3)$

$x_1 + x_2 + x_3 \le 6$

Solution Using Excel (Using Solver Add-in)

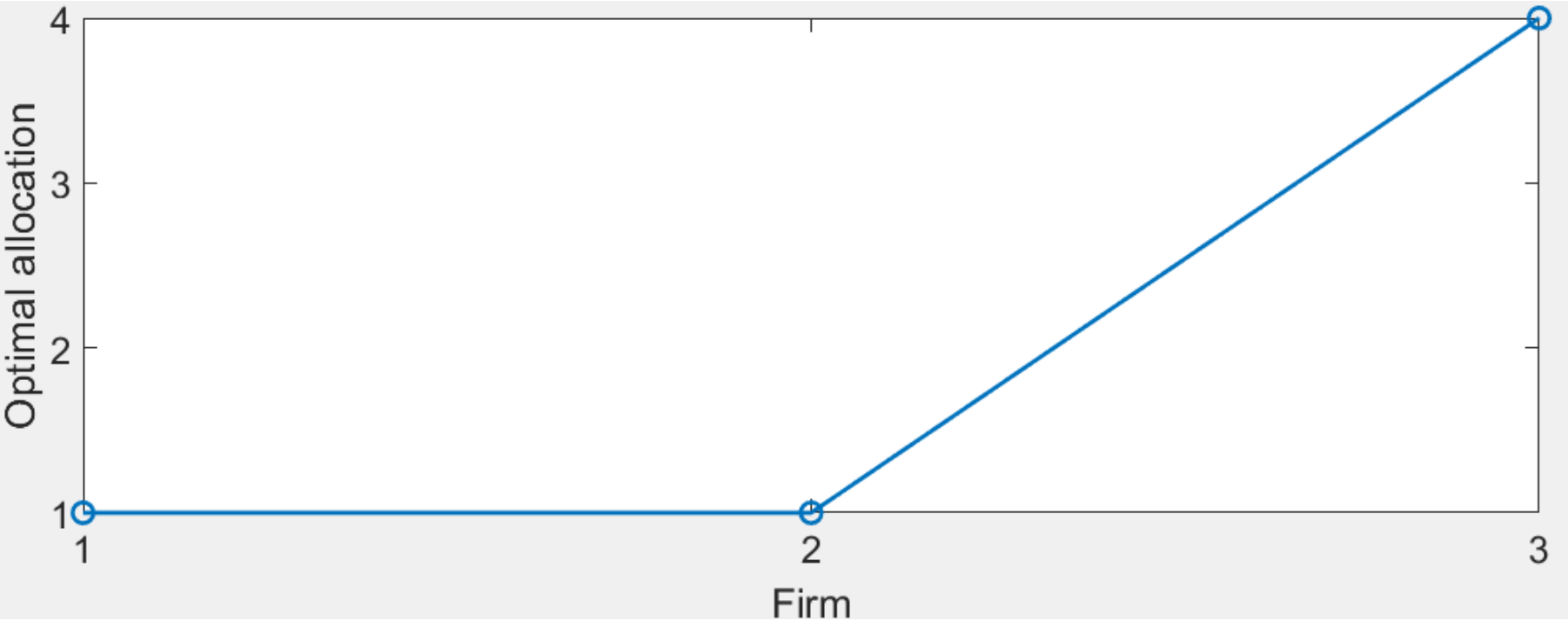
Main file: Water_Allocation.xlsx

Water Allocation

Number	1	2	3
Optimal x	1.0	1.0	4.0
Benefit	34.5		
sum of Xs	6.0		
"-xi<= 0"	-1.0	-1.0	-4.0

Solution Using MATLAB

Main file: Water_AllocationMATLAB.m



Reservoir operation

(Adapted from Water Resources Systems Analysis, Karamouz et al.)

A reservoir is constructed on a river for supplying the power load of a city located near the river and water demands of downstream agricultural lands. The water that is released for power generation can also be used for supplying other demands. The minimum required instream flow is estimated to be 1 million m^3 each month. The total capacity of the reservoir is 10 million m^3. Maximum monthly release from the reservoir is limited to 7 million m^3. The Table below shows the monthly inflows to the reservoir and benefits of power generation and water supply. The reservoir storage on January 1 is considered to be 5 million m^3. Find the optimal releases.

Decision variables : $R_1, R_2, \cdots R_{12}$

R = Release, B = Benefit

Maximize

$$B(\$) = 2.6R_1 + 2.9R_2 + 3.6R_3 +$$

Subject to:

$$3.9R_4 + 4.2R_5 + 4.2R_6 +$$
$$4.5R_7 + 4.1R_8 + 3.6R_9 +$$
$$3.1R_{10} + 2.7R_{11} + 2.5R_{12}$$

$1 \le R_i \le 7 \quad (i = 1, \cdots 12)$

$S_1 = 5$

$S_i \le 10 \quad (i = 2, \cdots 13)$

Conservation of mass

$\Sigma I - \Sigma O = \Delta S$

$I_i - R_i = S_{i+1} - S_i$

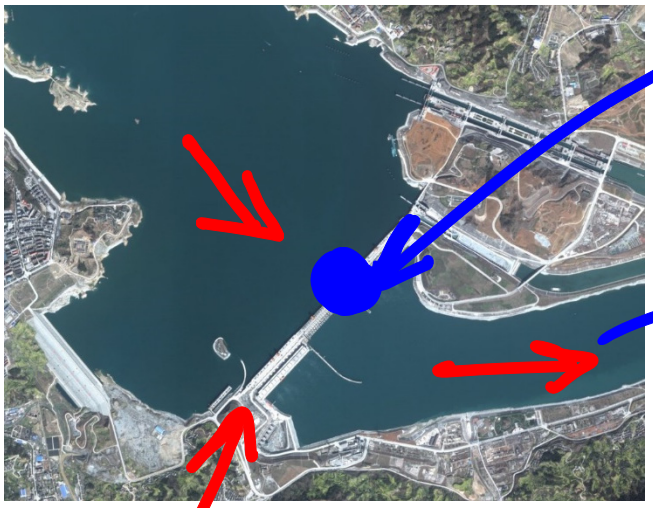
$(i = 1, 12)$

Reservoir operation (Cont.)

Monthly Inflows to the Reservoir and Benefits of Power Generation and Water Supply

Month	Inflow (million m^3)	Benefits of Power Generation ($\$10^3$/million m^3)	Benefits of Water Supply ($\$10^3$/million m^3)
January	2	1.6	1.0
February	2	1.7	1.2
March	3	1.8	1.8
April	4	1.9	2.0
May	3	2.0	2.2
June	2	2.0	2.2
July	2	2.0	2.5
August	1	1.9	2.2
September	2	1.8	1.8
October	3	1.7	1.4
November	3	1.6	1.1
December	2	1.5	1.0

Sketch of Reservoir system



Hydropower production

water released for hydropower (This will be used for other purposes).

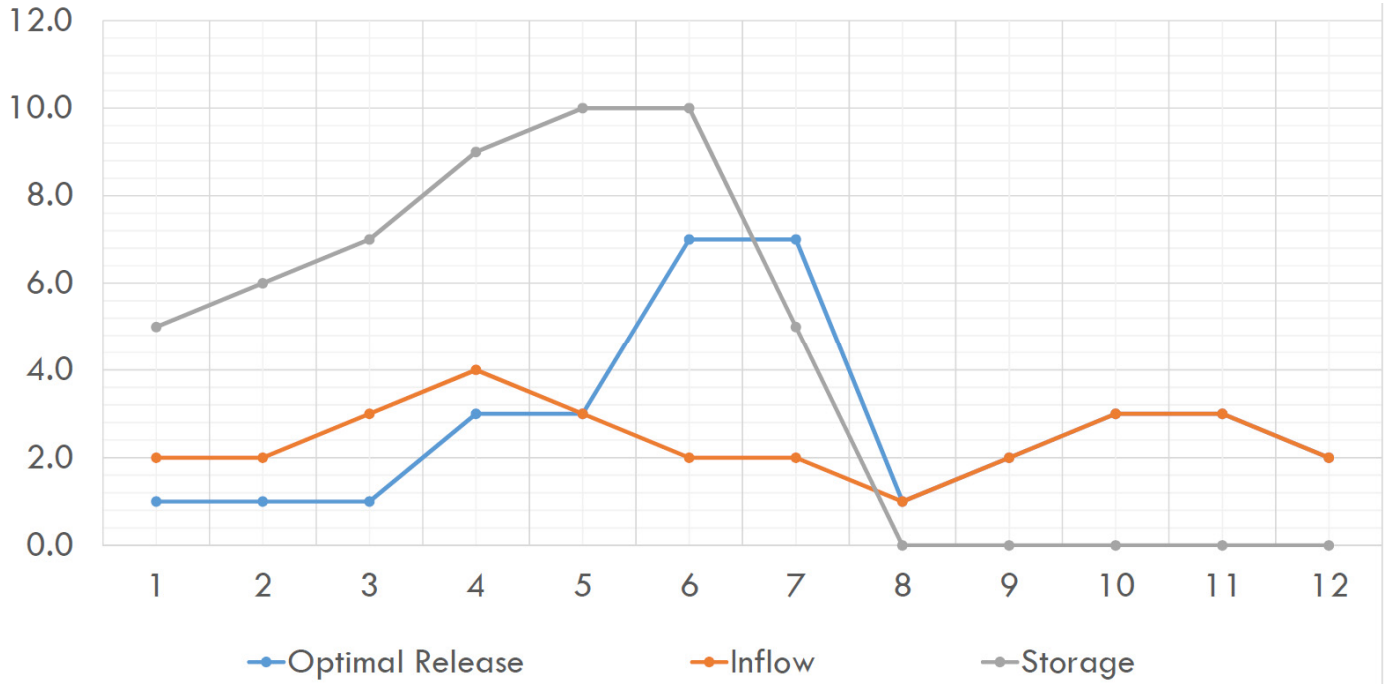
Dam

Thus, we only need 12 decision variables (flow releases $R_1, R_2, \ldots R_{12}$)

Solution Using Excel (Using Solver Add-in)

Reservoir Operation

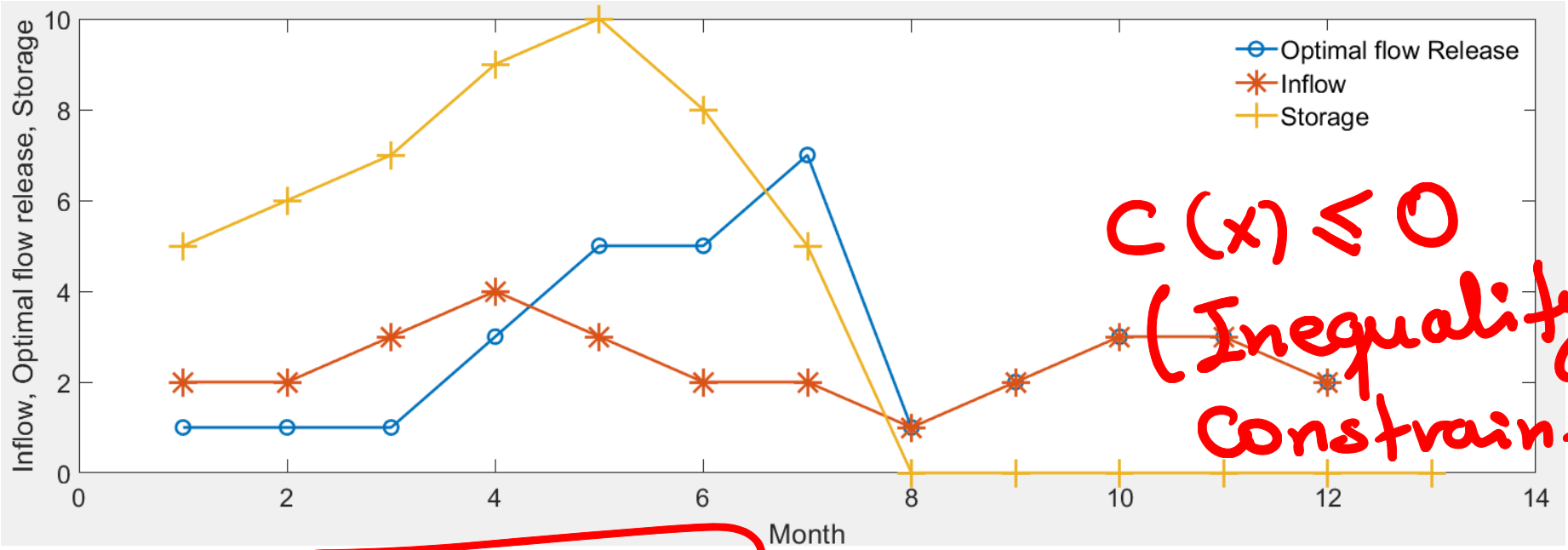
Number	1	2	3	4	5	6	7	8	9	10	11	12	13
Inflow	2.0	2.0	3.0	4.0	3.0	2.0	2.0	1.0	2.0	3.0	3.0	2.0	
Optimal Release	1.0	1.0	1.0	3.0	3.0	7.0	7.0	1.0	2.0	3.0	3.0	2.0	
Storage	5.0	6.0	7.0	9.0	10.0	10.0	5.0	0.0	0.0	0.0	0.0	0.0	0.0
Benefit	128.0												



Solution Using MATLAB

Main file: Reservoir_operationMATLAB.m

MATLAB
Ceq [Equality Constraint]
Ceq(x) = 0



C(x) ≤ 0
(Inequality Constraint)

$R_i \leq 7$ $\quad x_i - 7 \leq 0$

$R_i \geq 1$ $\quad x_i - 1 \geq 0 \rightarrow -(x_i - 1) \leq 0$

$1 - x_i \leq 0$

C(x) ≤ 0
(MATLAB)

Water supply

A river supplies water to an industrial complex and agricultural lands located downstream of the complex. The average monthly industrial and irrigation demands and the monthly river flows in a dry year are as follows (numbers are in million m^3):

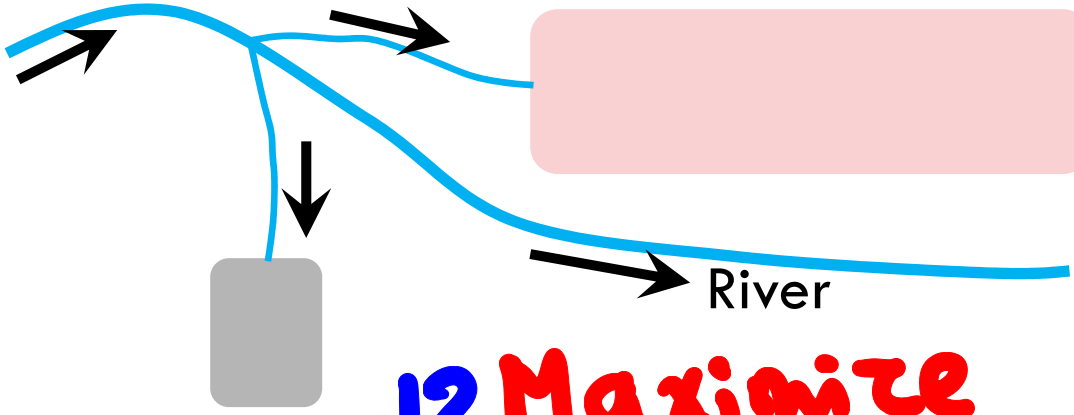
	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Industrial	8	8.5	8.5	9	9	9.5	9.5	10	9.5	9	8.5	8.5
Irrigation	0	0	20	50	55	70	70	65	20	10	0	0
Monthly flows	25	27	40	48	60	42	25	17	14	18	20	25

The price of water for industrial and irrigation uses is \$90,000 and \$10,000/million m^3, respectively. (a) Formulate the problem for optimizing the water allocation for this river. (b) Solve the optimization problem.

I_i : water allocated for industrial use at month "i"

A_i : water allocated for agricultural use at month "i"

Sketch of river system



R : River

D_I: Demand for industrial

D_A: Demand for agricultural

$$\text{Maximize} \sum_{i=1}^{12} (90{,}000\, I_i + 10{,}000\, A_i)$$

Subject to:

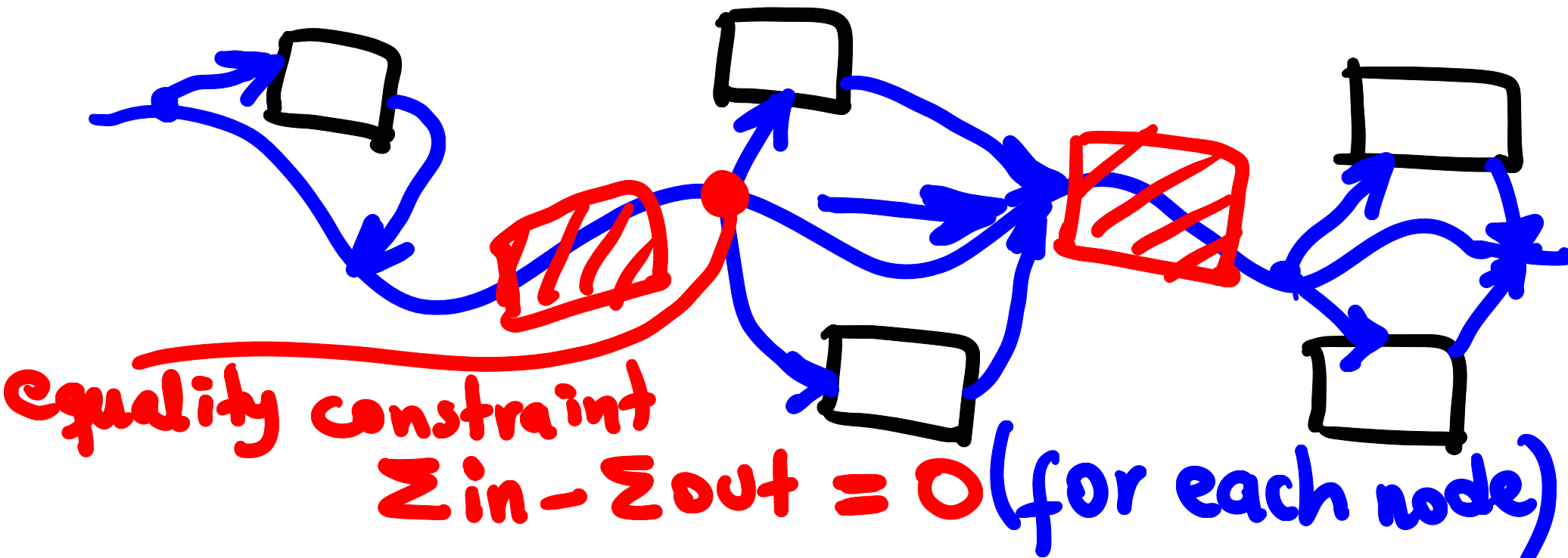
Total allocated monthly flow $\leq$ River monthly flow

$$I_i + A_i \leq R_i \quad (i = 1, 2, \ldots 12)$$

$$I_i \leq DI_i \quad (i = 1, 2, \ldots 12)$$

$$A_i \leq DA_i \quad (i = 1, 2, \ldots 12)$$

For multi-location allocations:

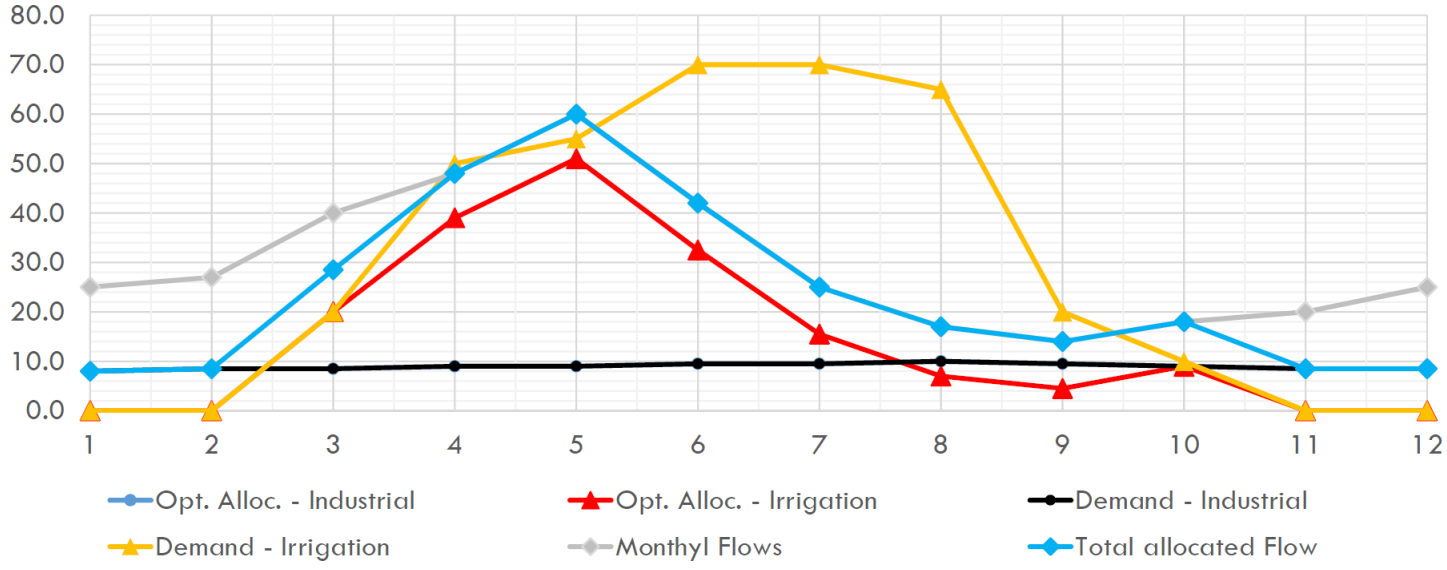


equality constraint

$$\Sigma in - \Sigma out = 0 \quad \text{(for each node)}$$

Solution Using Excel (Using Solver Add-in)

Number	1	2	3	4	5	6	7	8	9	10	11	12
Demand - Industrial	**8.0**	**8.5**	**8.5**	**9.0**	**9.0**	**9.5**	**9.5**	**10.0**	**9.5**	**9.0**	**8.5**	**8.5**
Demand - Irrigation	**0.0**	**0.0**	**20.0**	**50.0**	**55.0**	**70.0**	**70.0**	**65.0**	**20.0**	**10.0**	**0.0**	**0.0**
Monthyl Flows	**25.0**	**27.0**	**40.0**	**48.0**	**60.0**	**42.0**	**25.0**	**17.0**	**14.0**	**18.0**	**20.0**	**25.0**
Opt. Alloc. - Industrial	8.0	8.5	8.5	9.0	9.0	9.5	9.5	10.0	9.5	9.0	8.5	8.5
Opt. Alloc. - Irrigation	0.0	0.0	20.0	39.0	51.0	32.5	15.5	7.0	4.5	9.0	0.0	0.0
Total allocated Flow	8.0	8.5	28.5	48.0	60.0	42.0	25.0	17.0	14.0	18.0	8.5	8.5
Price	7.200E+05	7.650E+05	9.650E+05	1.200E+06	1.320E+06	1.180E+06	1.010E+06	9.700E+05	9.000E+05	9.000E+05	7.650E+05	7.650E+05
Total Price	1.146E+07											



Solution Using MATLAB

Write a MATLAB solver from scratch or modify any of the previous supplied codes.