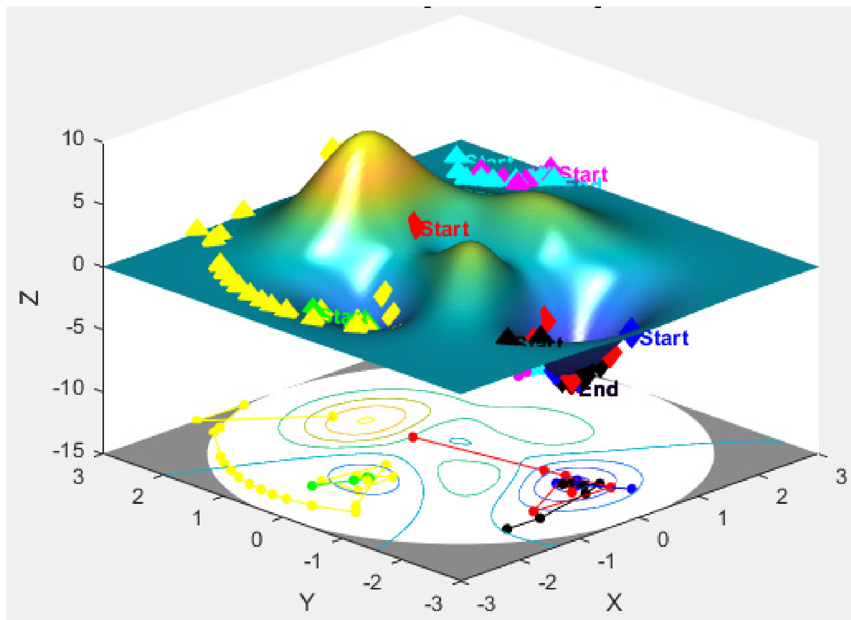
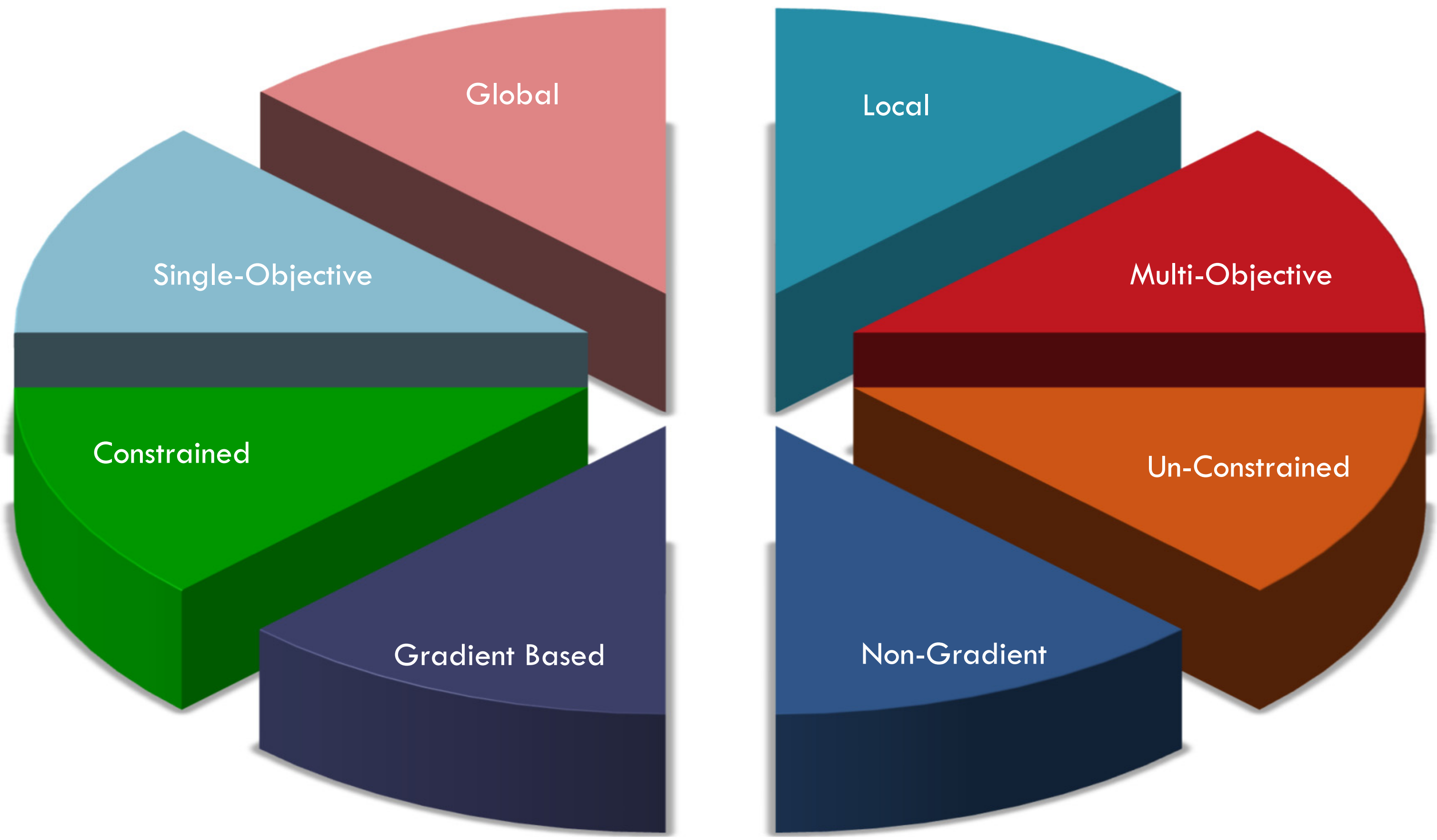
Florida International University
Department of Civil and Environmental Engineering
Optimization in Water Resources Engineering, Spring 2020

OVERVIEW OF HEURISTIC OPTIMIZATION



Arturo S. Leon, Ph.D., P.E., D.WRE

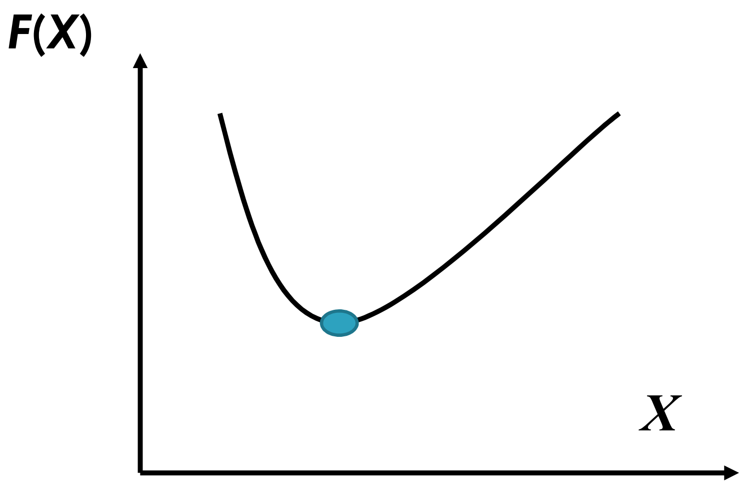
OPTIMIZATION CLASSIFICATION (Recap)

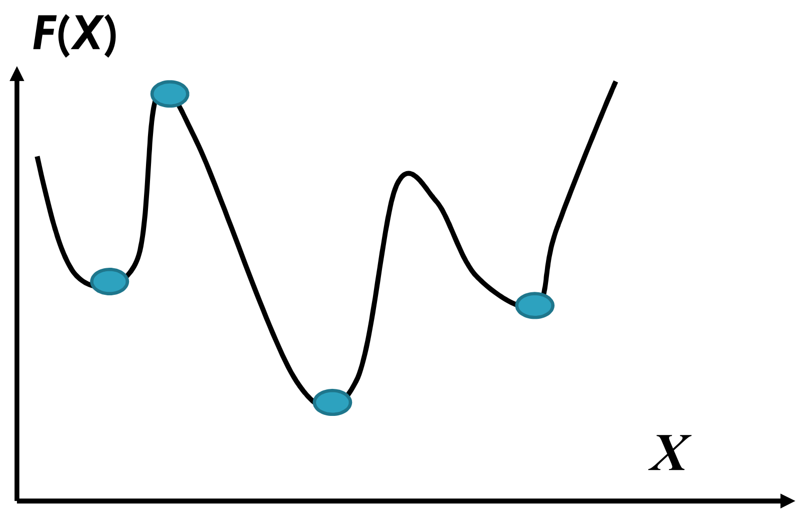


REASONS FOR Heuristic search

LIMITATIONS OF DESCENT METHODS

- Descent method procedure is efficient when the objective function *F* is *uni-modular* (one local optimum only).
- In case *F* is *multi-modular*, not easy to get out of the neighborhood of local optimum



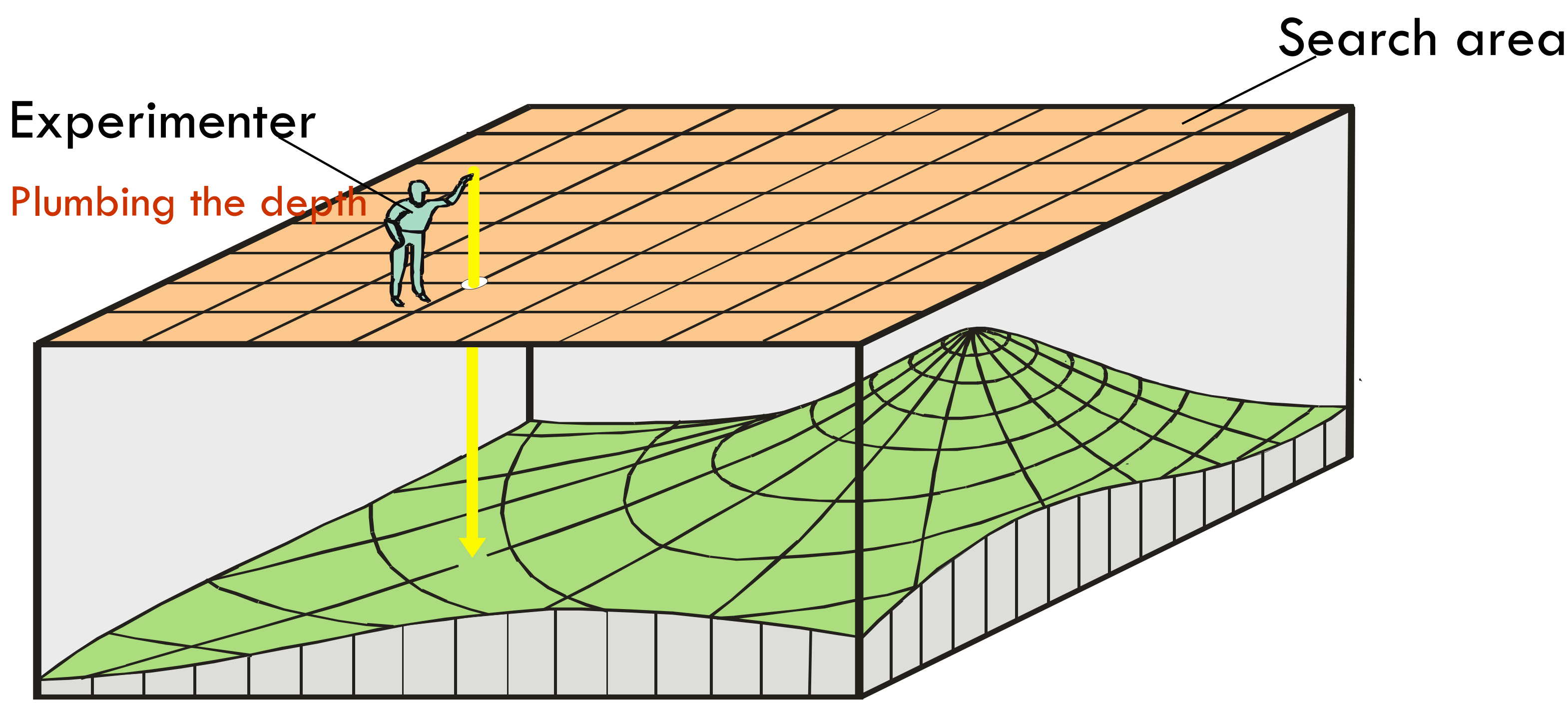


HEURISTIC METHODS – INTRODUCTION

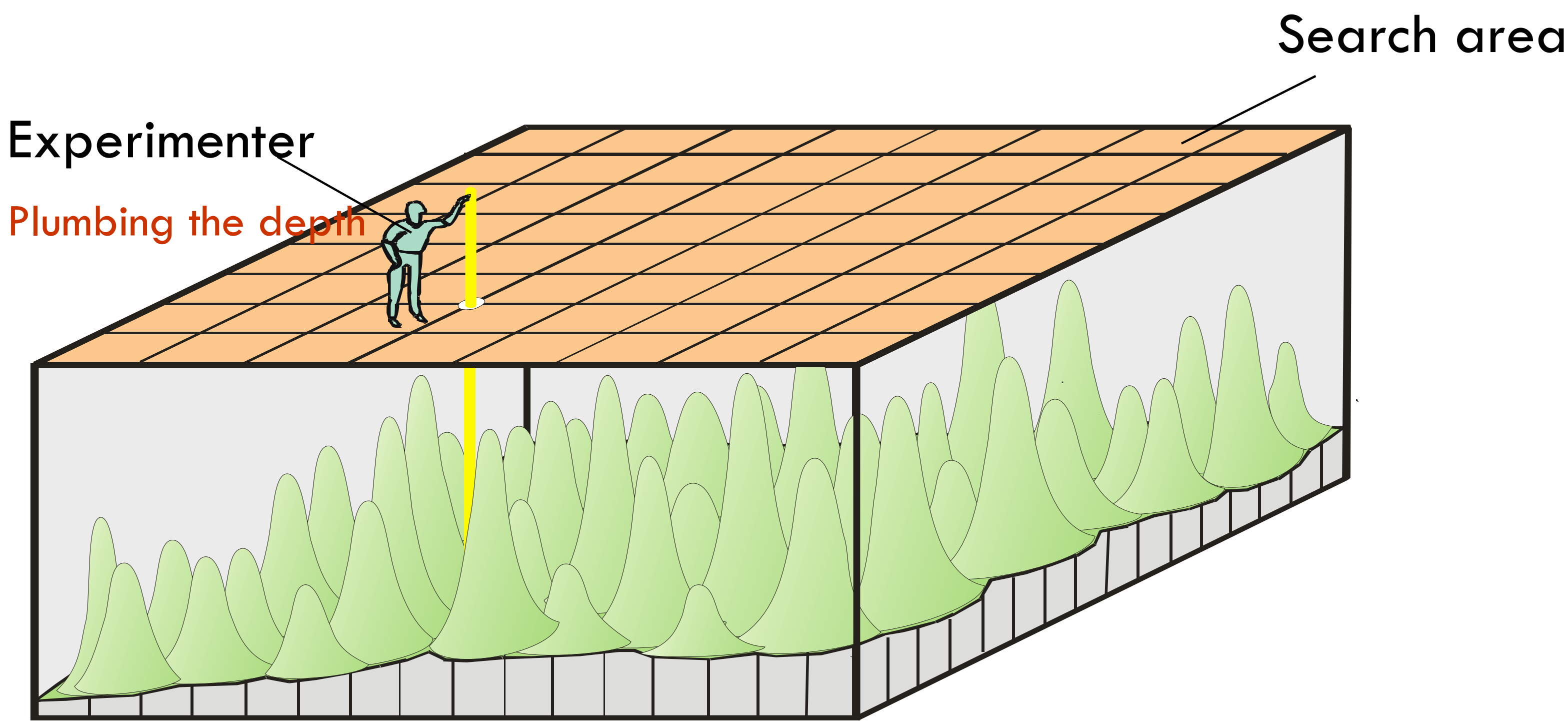
- Heuristic methods, as non-gradient methods, do not require any derivatives of the objective function in order to calculate the optimum, they are also known as black box methods.

- Heuristics are typically used to solve complex (large, nonlinear, non-convex (i.e. contain local minima)) multivariate combinatorial optimization problems that are difficult to solve.

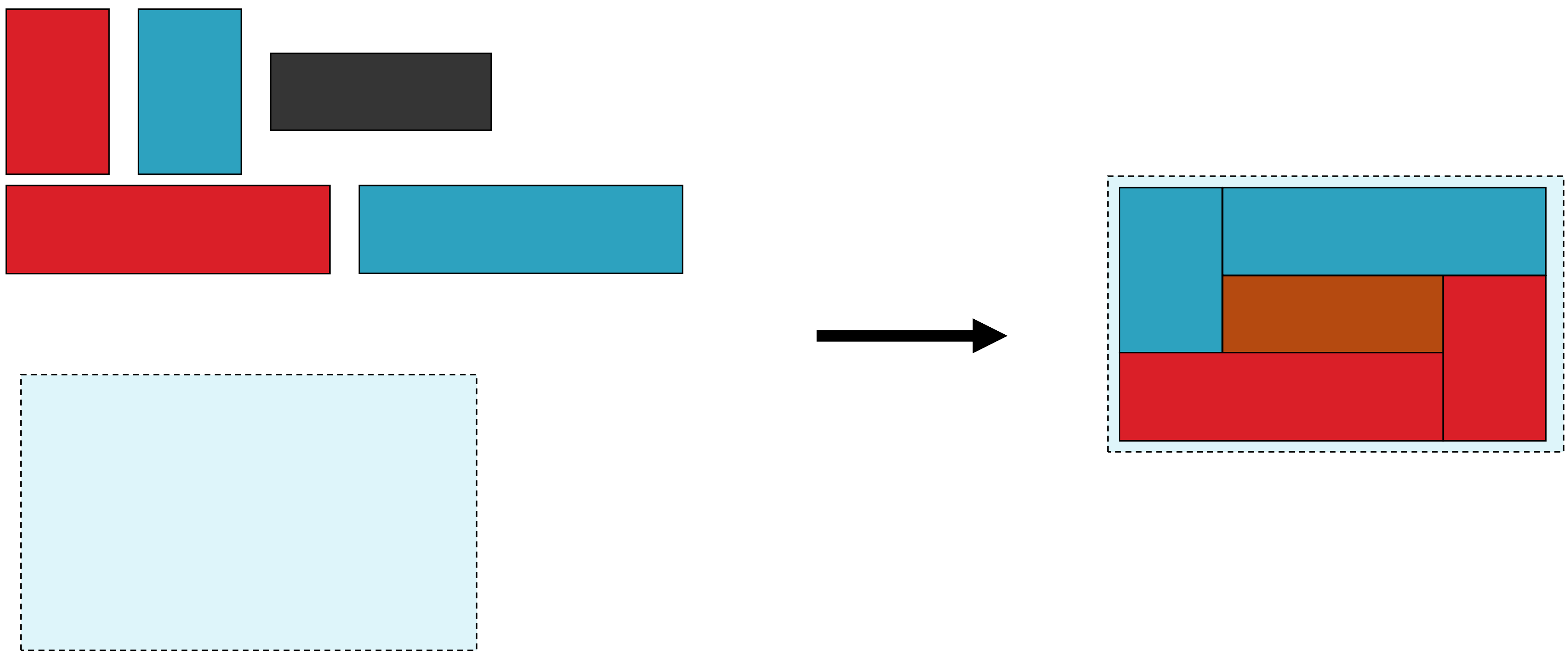
THE SEARCH FOR THE OPTIMUM IN HEURISTIC METHODS (smooth functions)



THE SEARCH FOR THE OPTIMUM IN HEURISTIC METHODS (Non-smooth functions)

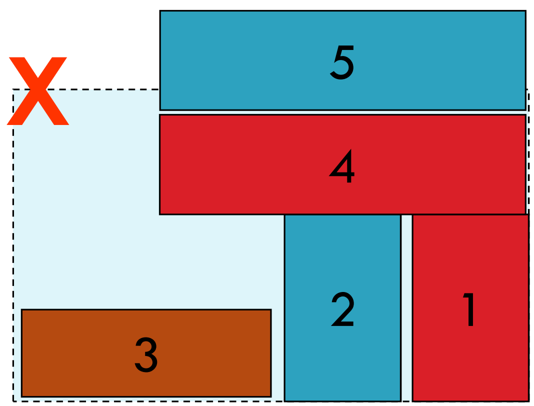


2-D PACKAGING – AN EXAMPLE

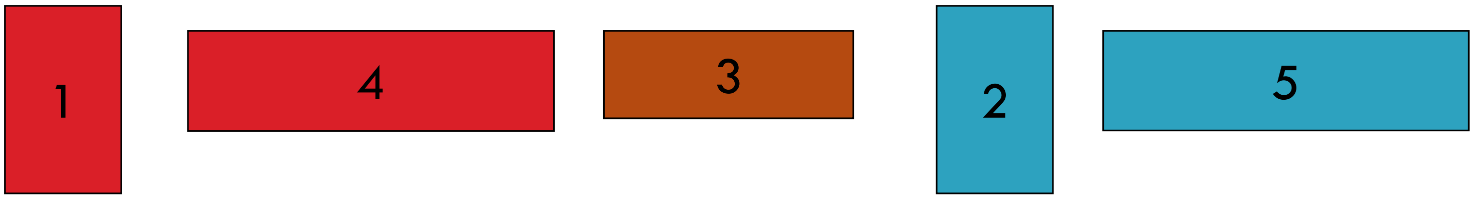


2-D PACKAGING – AN EXAMPLE

Sequence = 1,2,3,4,5



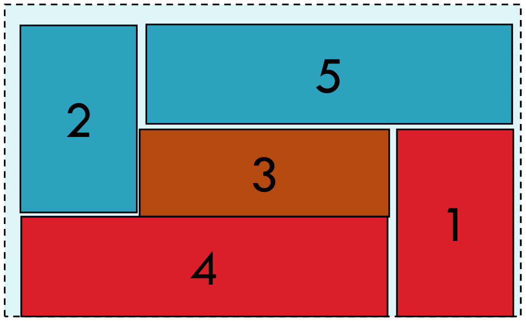
2-D PACKAGING – AN EXAMPLE



Sequence = 1, 4, 3, 2, 5

2-D PACKAGING – AN EXAMPLE

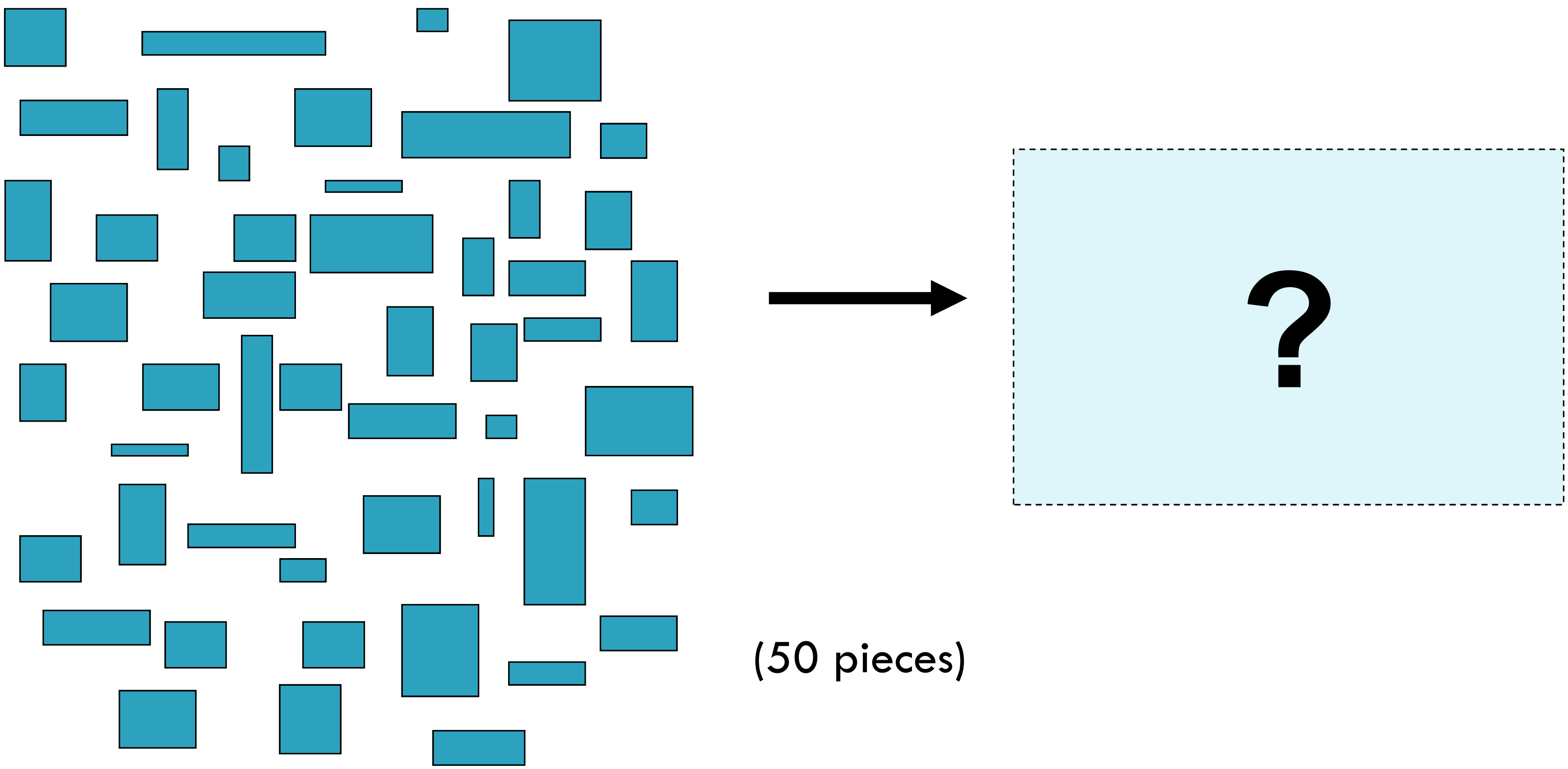
Sequence = 1, 4, 3, 2, 5



THAT WAS EASY!

- For **5 pieces** there are **5 x 4 x 3 x 2 x 1** different orderings for the placement
- = 120 combinations
- Piece of cake for a computer to try each combination in (nearly) no time at all

How do we pack all of these?



(50 pieces)

THAT'S NOT SO EASY!

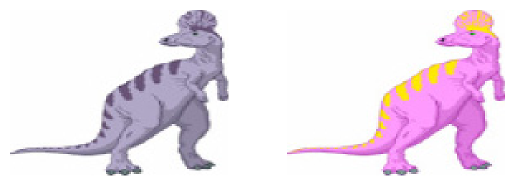
- 50 pieces means 50 x 49 x…..2 x 1 different orderings
- =
  30414093201713378043612608166064768844377641568960512000000000000
- That's quite a lot really. If a computer could evaluate 1000 orderings per second it'd still take approximately:
- 964424568801159882154128873860501295166718720476931506 years! (and that's without being allowed to rotate the pieces)

OVERVIEW OF MOST COMMON HERISTIC METHODS

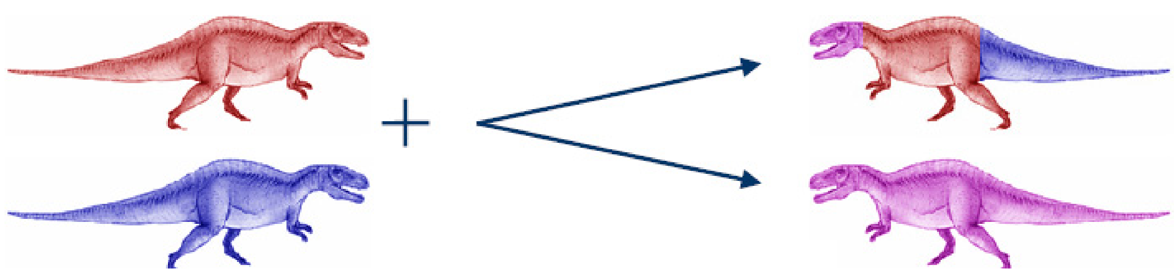
GENETIC ALGORITHM (Holland –1975)

Inspired by genetics and natural selection – survival of the fittest

- Organisms produce a number of offspring similar to themselves but can have variations due to:
  - **Mutations** (random changes)



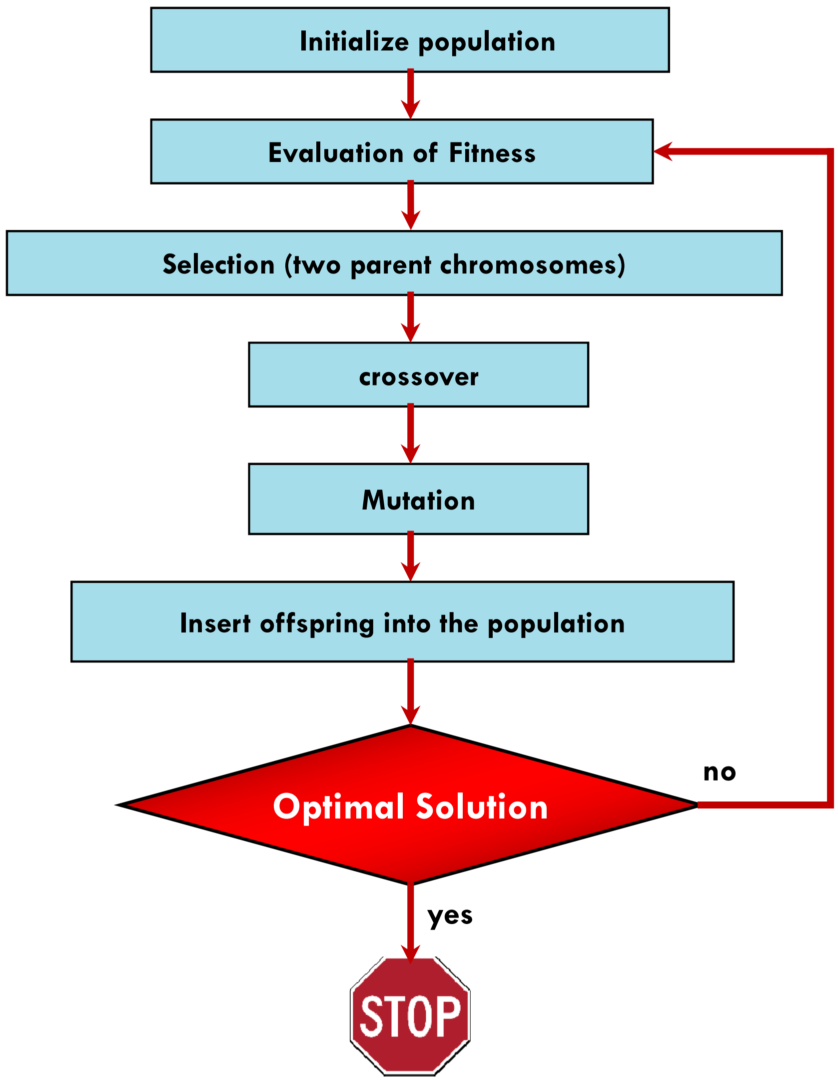
  - **Sexual reproduction** (offspring have combinations of features inherited from each parent)



GENETIC ALGORITHM (Cont.)

- GAs were invented to mimic some of the processes observed in natural and biological evolution

- Provide efficient, effective techniques for optimization and machine learning applications



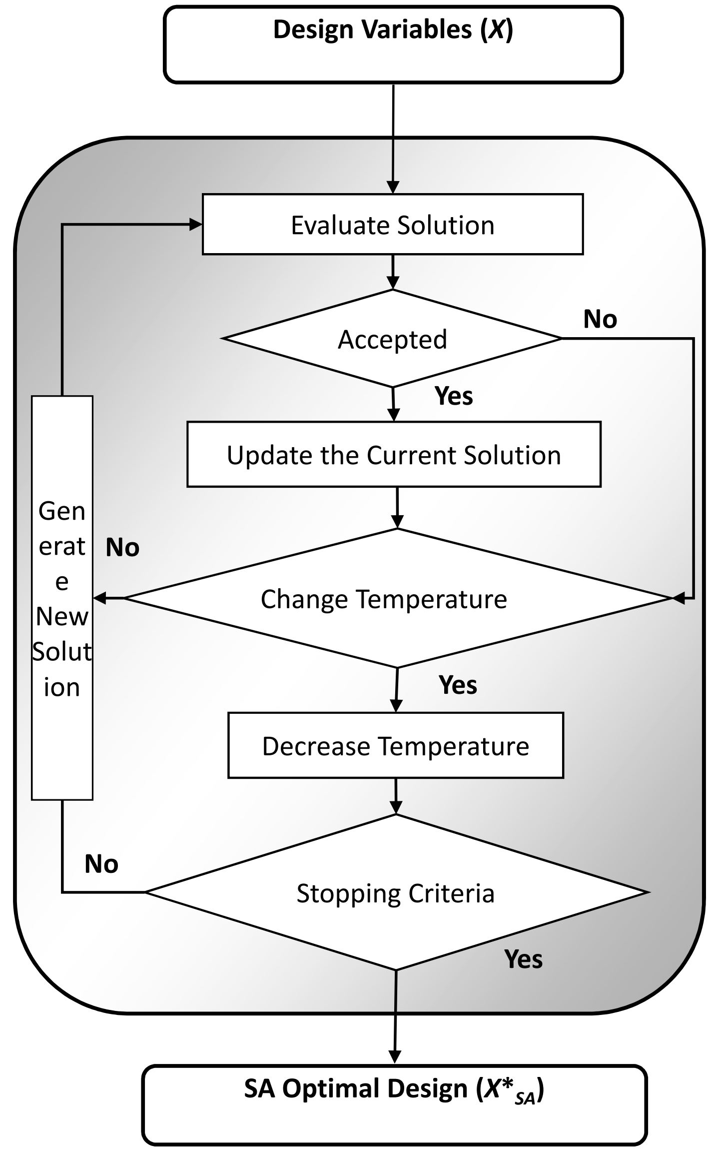
SIMULATED ANNEALING (Kirkpatrick –1983)

- Simulated annealing is based on an analog of cooling the material in a heat bath – a process known as annealing. A solid material is heated in a heat bath until it melts, then cooling it down slowly until it crystallizes into a solid state (low-energy state).

- **Definition:** A ***heuristic*** technique that mathematically mirrors the ***cooling*** of a set of atoms to a state of minimum energy.

SIMULATED ANNEALING

- SA is a ***general solution*** method that is ***easily*** applicable to a large number of problems.
- ***"Tuning"*** of the parameters (initial temp, decrement of temp, stop criterion) is relatively easy.
- Generally the quality of the results of SA is good.
- SA can leave an optimal solution and not find it again (so try to remember the best solution found so far).



PARTICLE SWARM OPTIMIZATION

(Eberhart Kennedy -1995)

- A robust stochastic optimization technique inspired by the social behavior of swarms of insects or flocks of birds –maximize "food".
- Apply the concept of social interaction to problem solving.
- Developed in 1995 by James Kennedy (social-psychologist) and Russell Eberhart (electrical engineer).

PARTICLE SWARM OPTIMIZATION

- In PSO individuals strive to improve themselves and often achieve this by observing and imitating their neighbors.

- Each PSO individual has the ability to remember.

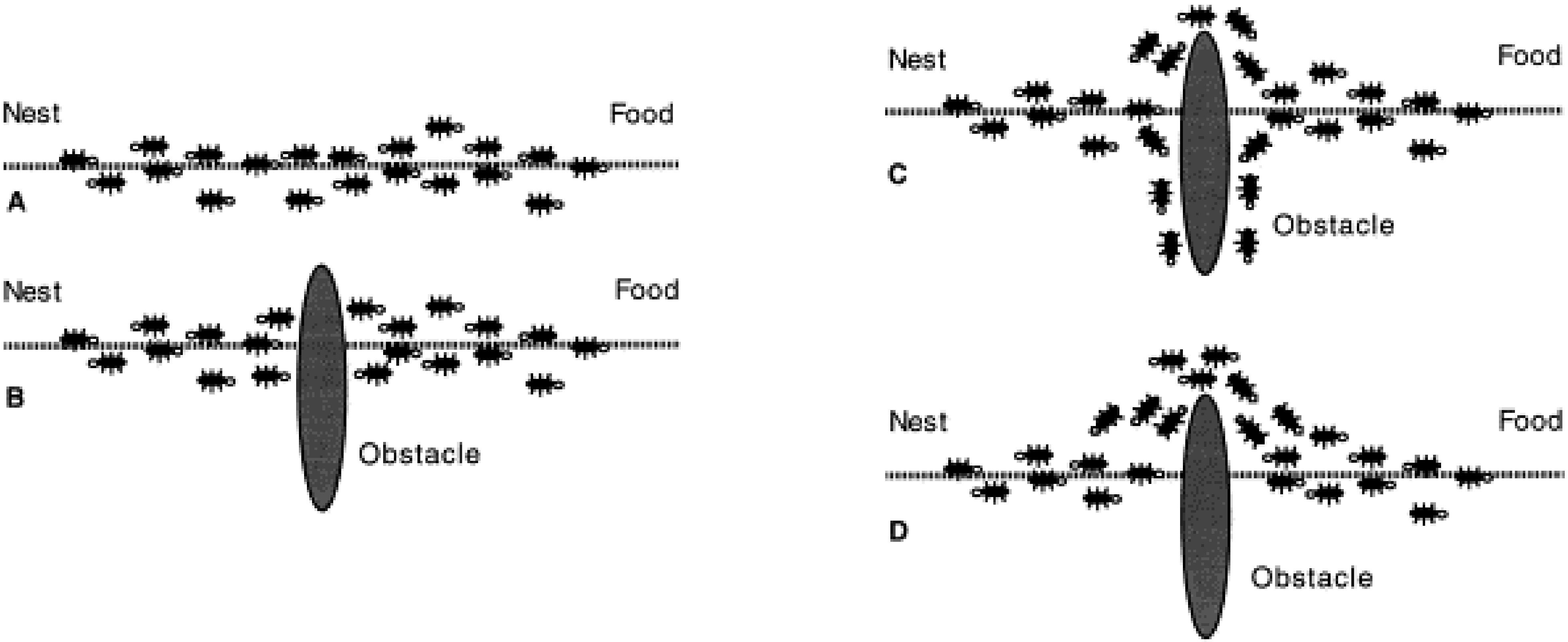
- PSO has simple algorithms and low overhead.
  - Making it more popular in some circumstances than Genetic/ Evolutionary Algorithms
  - Has only one operation calculation:
    - Velocity: a vector of numbers that are added to the position coordinates to move an individual

- Inspiration from Flock of birds, Schools of fish.

ANT COLONY OPTIMIZATION

- Ant Algorithms
  - Inspired by observation of real ants
- Ant Colony Optimization (ACO)
  - Inspiration from ant colonies' foraging behavior (actions of the colony finding food)
- Pheromone trail for stigmergic communication
- Sequence of moves to find shortest paths

NATURAL BEHAVIOUR OF ANTS



ANT COLONY OPTIMIZATION (Cont.)

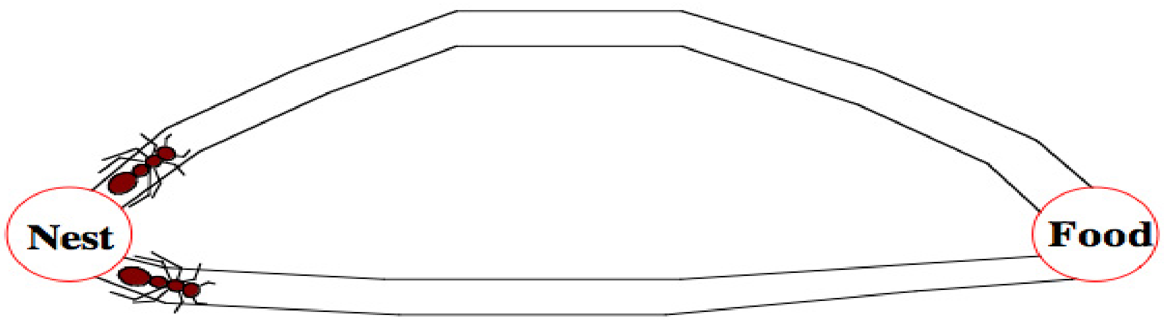
- Inspired by foraging behavior of ants.

- Ants find shortest path to food source from nest.

- Ants deposit pheromone along traveled path which is used by other ants to follow the trail.
  - This kind of indirect communication via the local environment is called stigmergy.

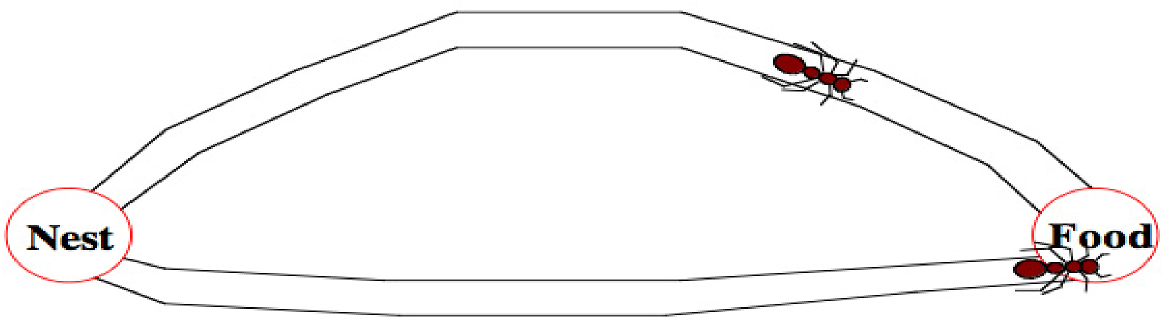
- Has adaptability, robustness and redundancy.

FORAGING BEHAVIOR OF ANTS



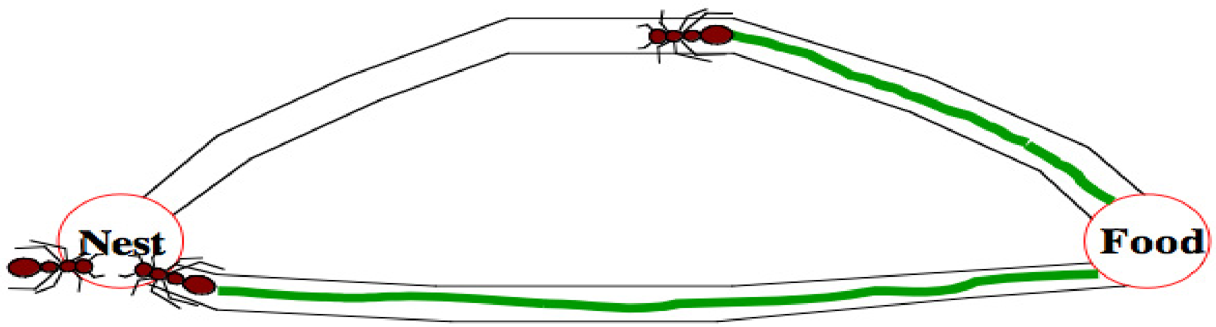
- Two ants start with equal probability of going on either path.

FORAGING BEHAVIOR OF ANTS



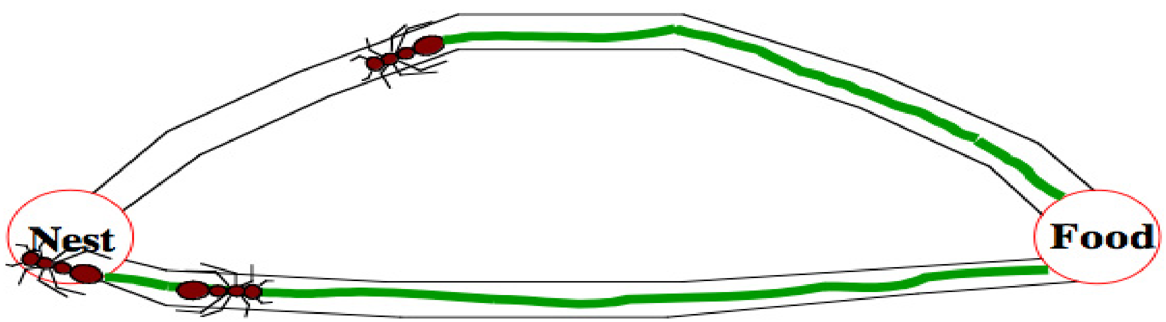
- The ant on shorter path will arrive earlier to the food.

FORAGING BEHAVIOR OF ANTS (Cont.)



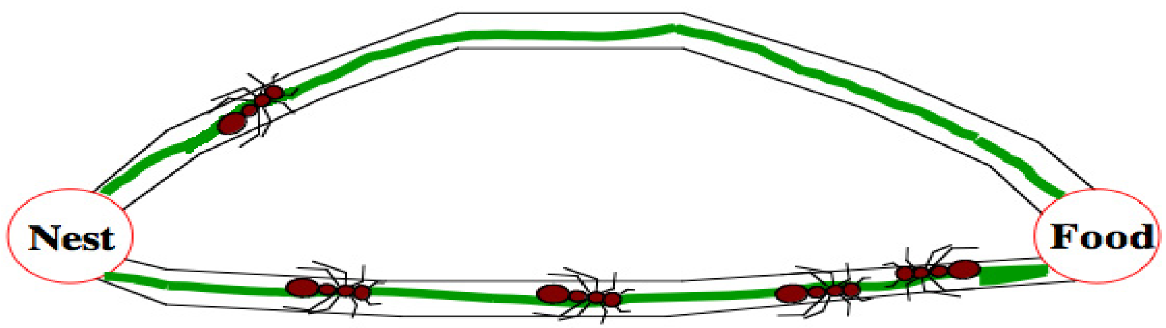
- The density of pheromone on the shorter path is higher because of 2 passes by the ant (as compared to 1 by the other).

FORAGING BEHAVIOR OF ANTS (Cont.)



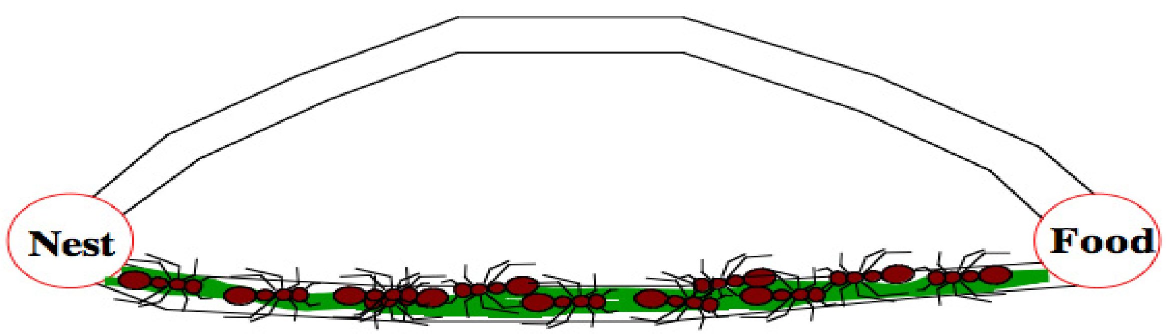
- The next ant takes the shorter route.

FORAGING BEHAVIOR OF ANTS (Cont.)



- Over many iterations, more ants begin using the path with higher pheromone, thereby further reinforcing it.

FORAGING BEHAVIOR OF ANTS (Cont.)



- After some time, the shorter path is almost exclusively used.

TABU SEARCH

- Attributed to Glover (1990)
- Search by avoiding points in the design space that were previously visited (TABU) – KEEP MEMORY
- Accept a new "poorer" solution if it avoids a solution that was already investigated – MAXIMIZE NEW INFORMATION
- Intent is to avoid local minima
- Record all previous moves in a "running list" = MEMORY

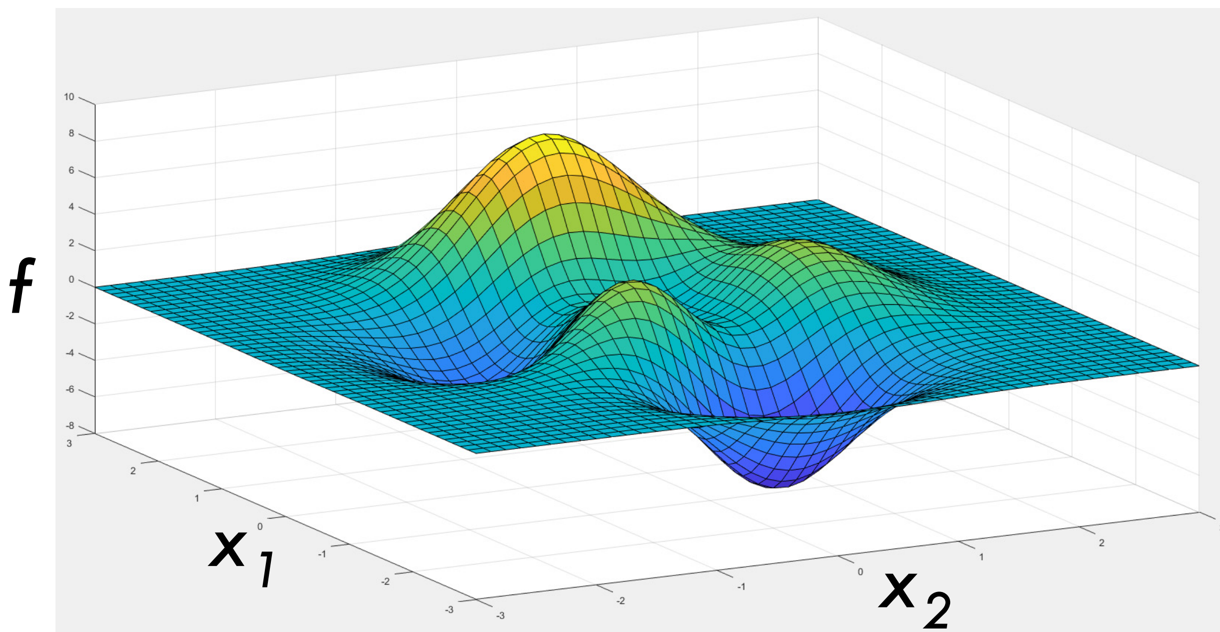
TABU SEARCH (Cont.)

- Record recent, now forbidden, moves in “tabu” list.
- First “diversification” then “intensification”.
- Applied to combinatorial optimization problems.
- Neighbor Search
- Sequential
- Adaptive

THE TWO-DIMENSIONAL SURFPACK EXAMPLE (MATLAB "PEAKS" FUNCTION)

Matlab Folder: Global Search Demo

$$f(x_1, x_2) = 3(1-x_1)^2 e^{\left(-x_1^2-(x_2+1)^2\right)} - 10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right) e^{\left(-x_1^2-x_2^2\right)} - \left(\frac{1}{3}\right) e^{\left(-x_2^2-(x_1+1)^2\right)}$$



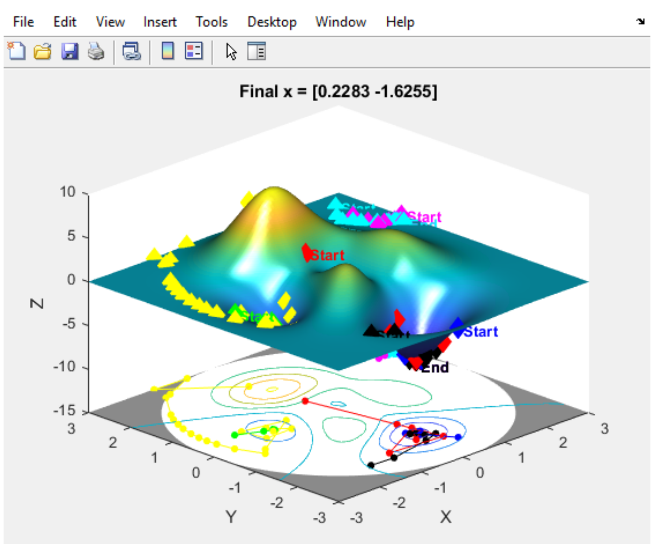
THE TWO-DIMENSIONAL SURFPACK EXAMPLE (CONT.)

Objective function: Minimize $f(x_1,x_2)$.

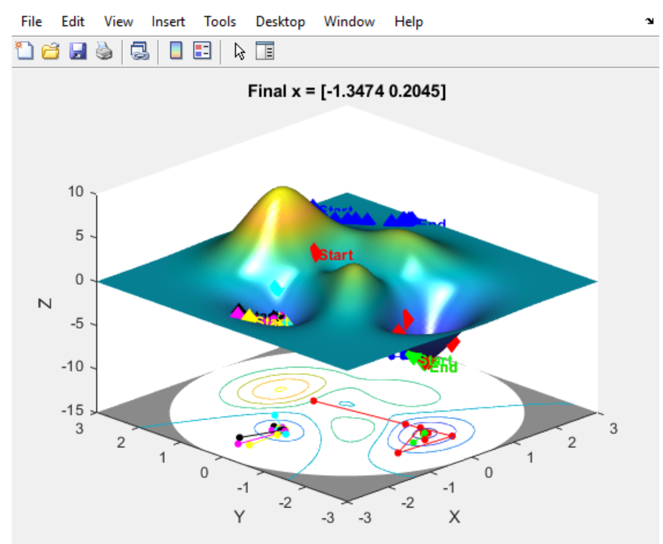
Constraints: $x_1^2+x_2^2 \leq 9$

Name of MATLAB File:
GlobalOptimization_MAIN_FILE.m

Multistart Method with fmincon

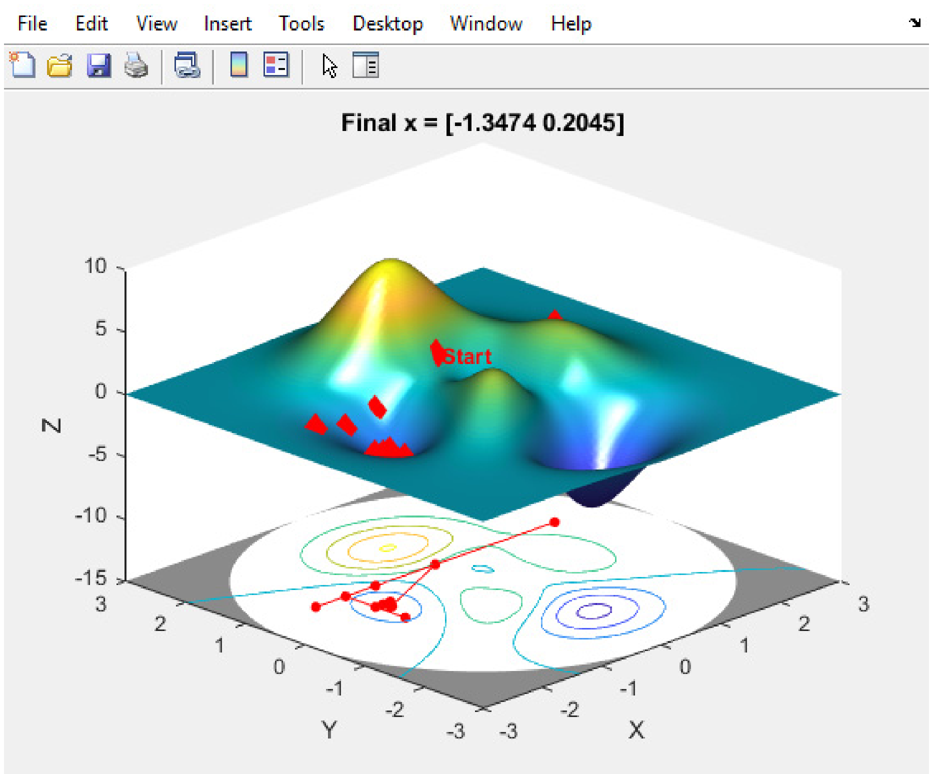


GlobalSearch Method with fmincon

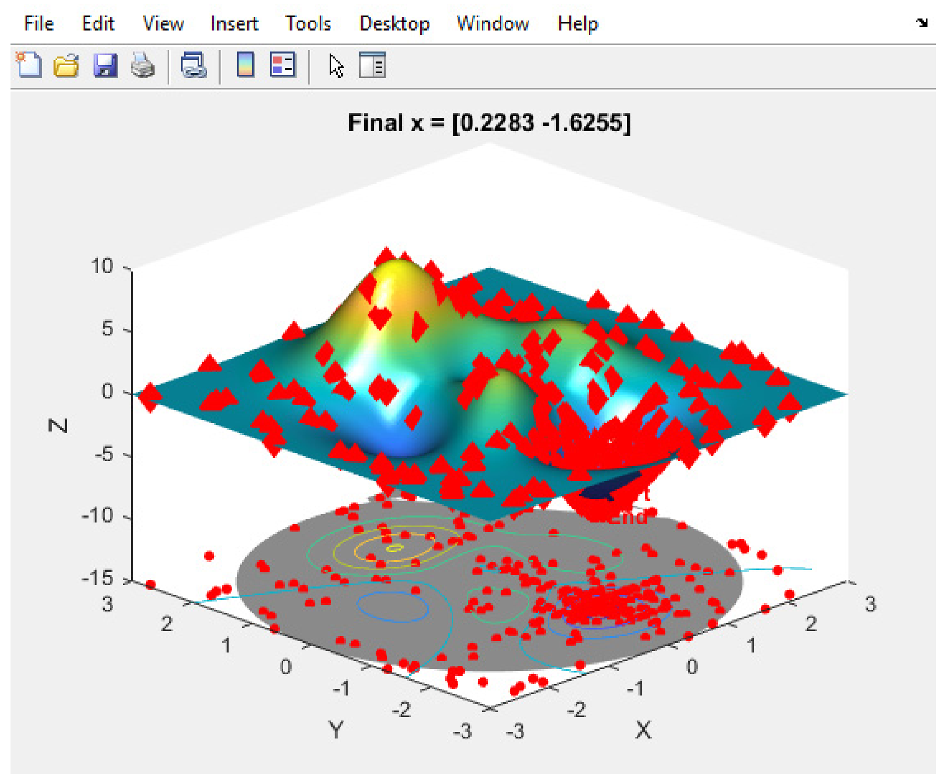


THE TWO-DIMENSIONAL SURFPACK EXAMPLE (CONT.)

Pattern Search Solver



Genetic Algorithm Solver



COMPARISON OF FUNCTION EVALUATIONS AND CPU TIME

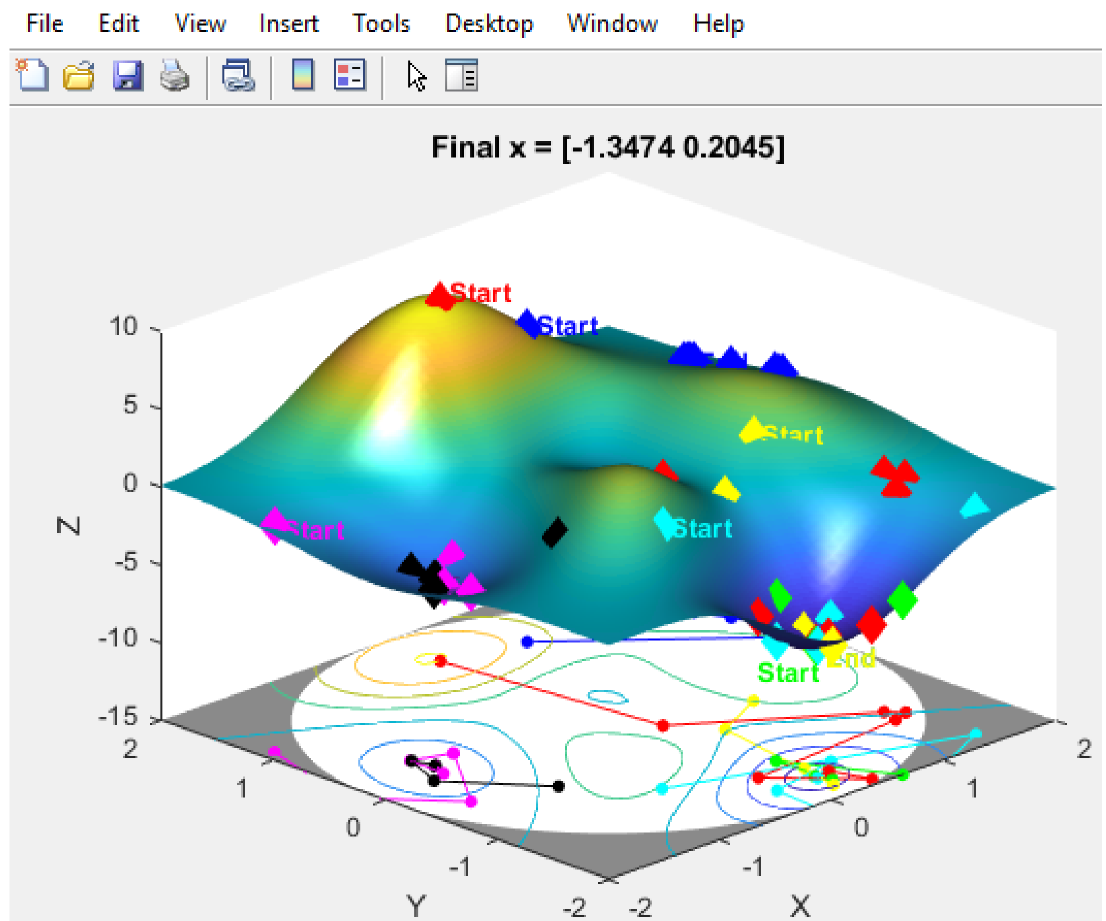
Name of MATLAB File: FUNCTION_EVALU_CPU_comparison.m

```
Results from different Global Optimization solvers (default settings)

Solver                  Fcn Calls   Time(s)
---------------------------------------------------------------
MultiStart                  502         0.227032
GlobalSearch                2378        0.267756
Pattern Search              272         0.041423
Simulated Annealing         1603        0.210671
Genetic Algorithm           13050       0.620040
```

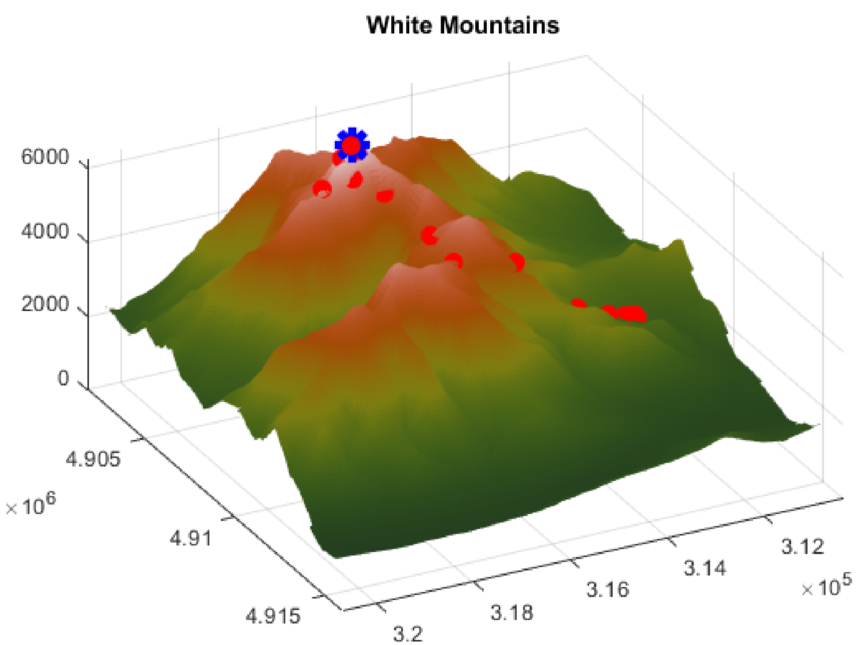
HANDS-ON DEMONSTRATION

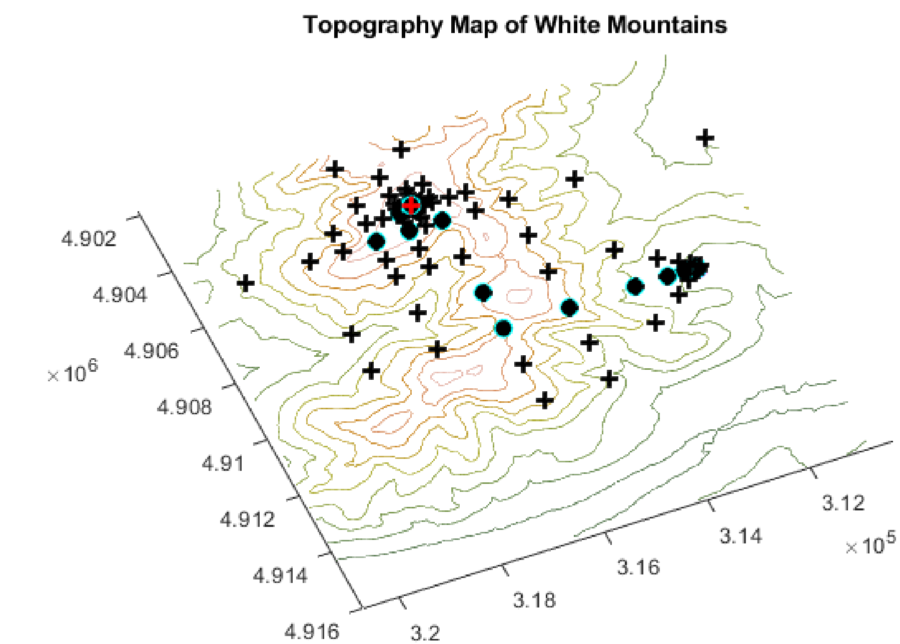
1. Change initial point
2. Change constraint function (**peaksCon.m**) (from circle with radius 3 to circle with radius 2)
3. Other changes?



DEMO: TOPOGRAPHY MAP OF WHITE MOUNTAINS

In this demo, we will find the highest point in the topography of the White Mountains in Washington using the **pattern search method**. The topography data is provided by the United States Geological Survey (USGS) 7.5-minute Digital Elevation Model (DEM) in Spatial Data Transfer Standard (SDTS) format for the Mt. Washington quadrangle (http://edc.usgs.gov/products/elevation/dem.html). (x,y) are co-ordinates of a point in easting and northing units, respectively and Z is the elevation in feet.





DEMO: TOPOGRAPHY MAP OF WHITE MOUNTAINS (CONT.)

Name of MATLAB Folder: Pattern_search_Washington_Demo
Name of MATLAB main File: MAINFile.m

Demo tasks:

1. Change initial starting point
2. Change optimization and plot options
3. Evaluate sensitivity of solutions
4. Write a while loop for evaluating 10 random initial starting points
5. Write a while loop for evaluating 10 'InitialMeshSize' and 'MeshTolerance' (Use MATLAB Random function randi([Xmin Xmax]))