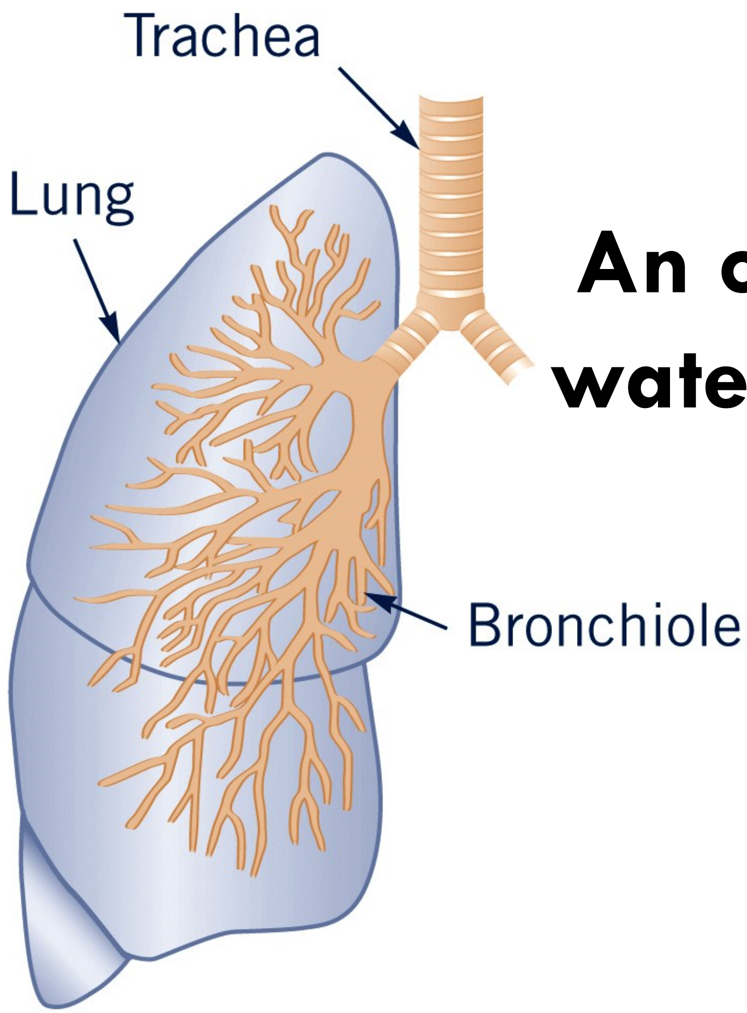
Florida International University
Optimization in Water Resources Engineering
Spring 2020



An overview of optimization in water resources engineering and other applications

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Videos of optimization

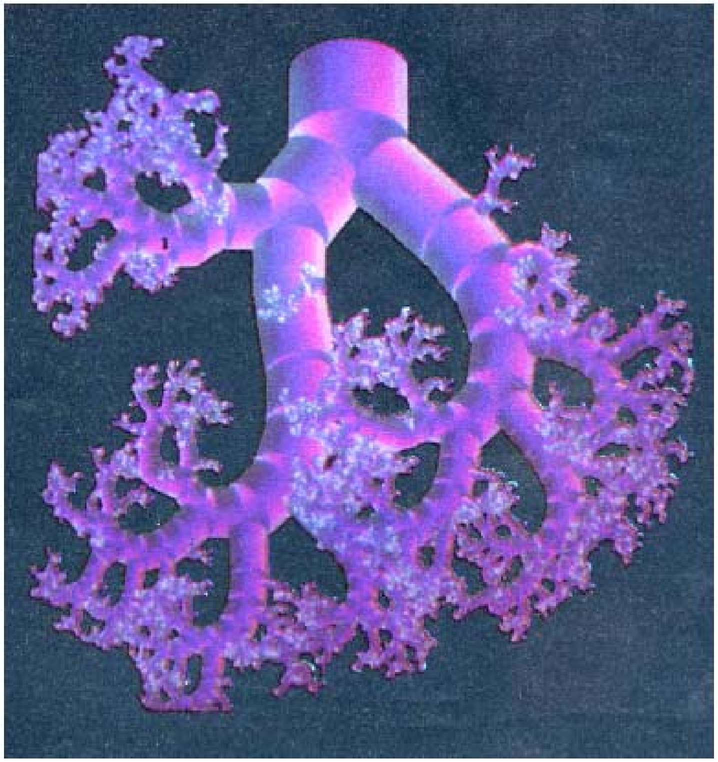
Genetic Algorithm. Learning to walk - OpenAI Gym

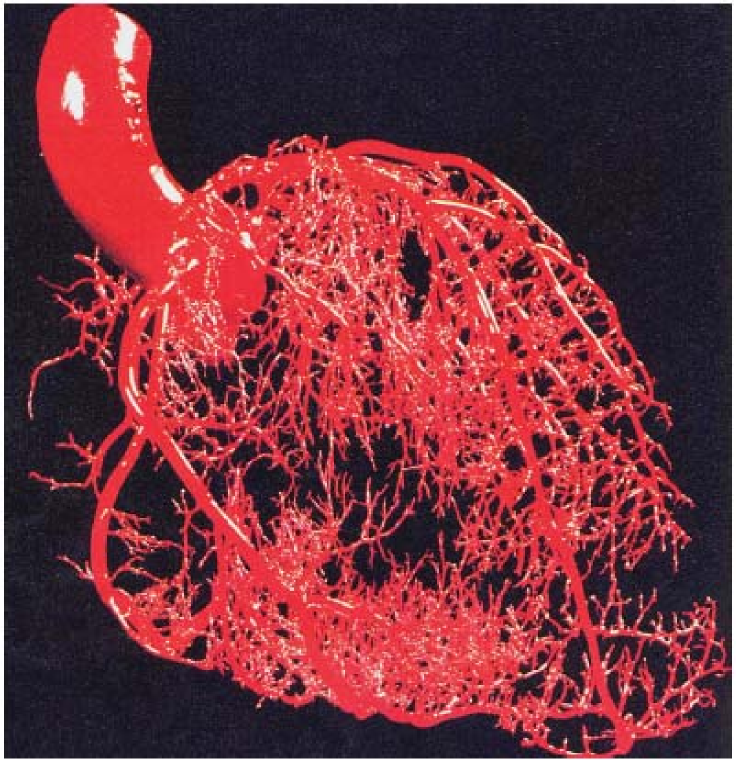
https://www.youtube.com/watch?v=uwz8JzrEwWY

Genetic algorithms - evolution of a 2D car in Unity

https://www.youtube.com/watch?v=FKbarpAlBkw

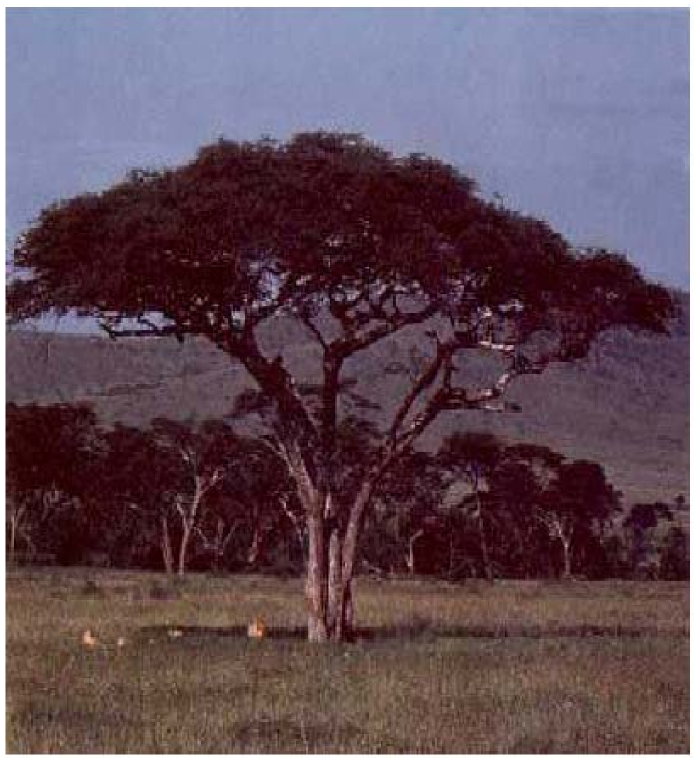
Optimization in nature



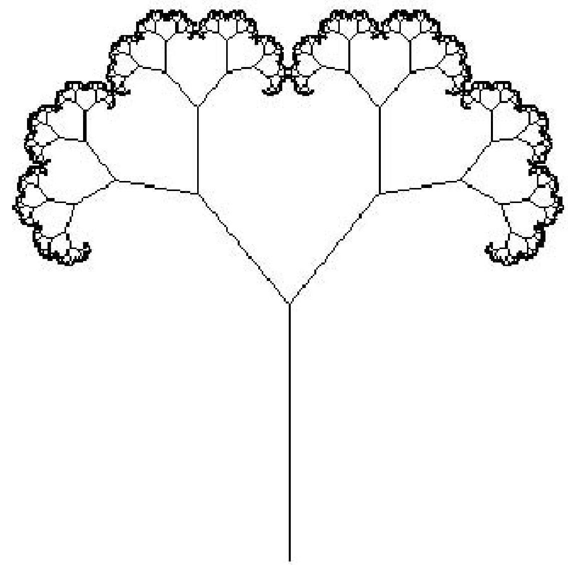


Source: P. Perona, IHW –ETH

Optimization in nature



Source: P. Perona, IHW –ETH



Optimization in rivers

Braided river, Denali National Park, Alaska

Delta (Minimize energy)

Source: http://www.flickr.com/photos/deadlyphoto/2457635320/

Optimization in rivers

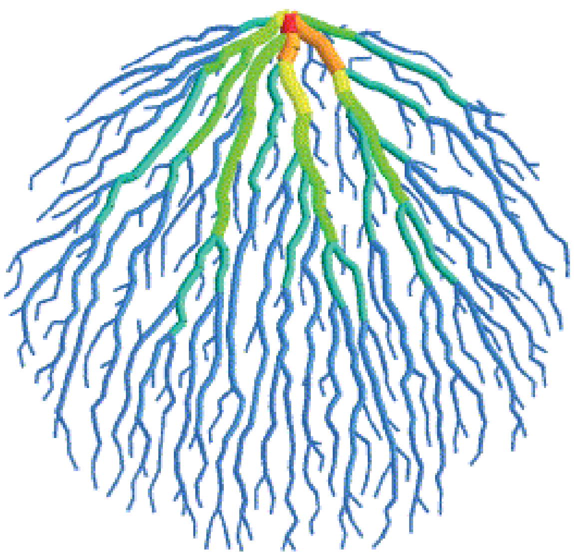
A meandering river (The meandering Tigre River, Argentina)

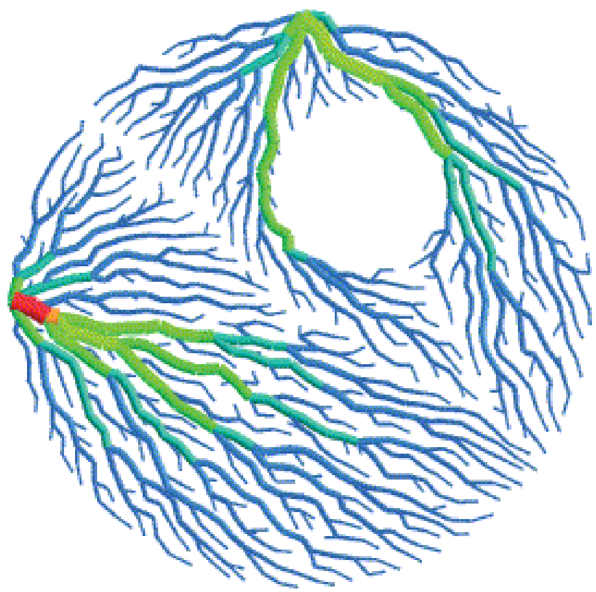
Minimize energy

Source: http://www.scienceclarified.com/landforms/Ocean-Basins-to-Volcanoes/Stream-and-River.html

Optimization of Flow networks

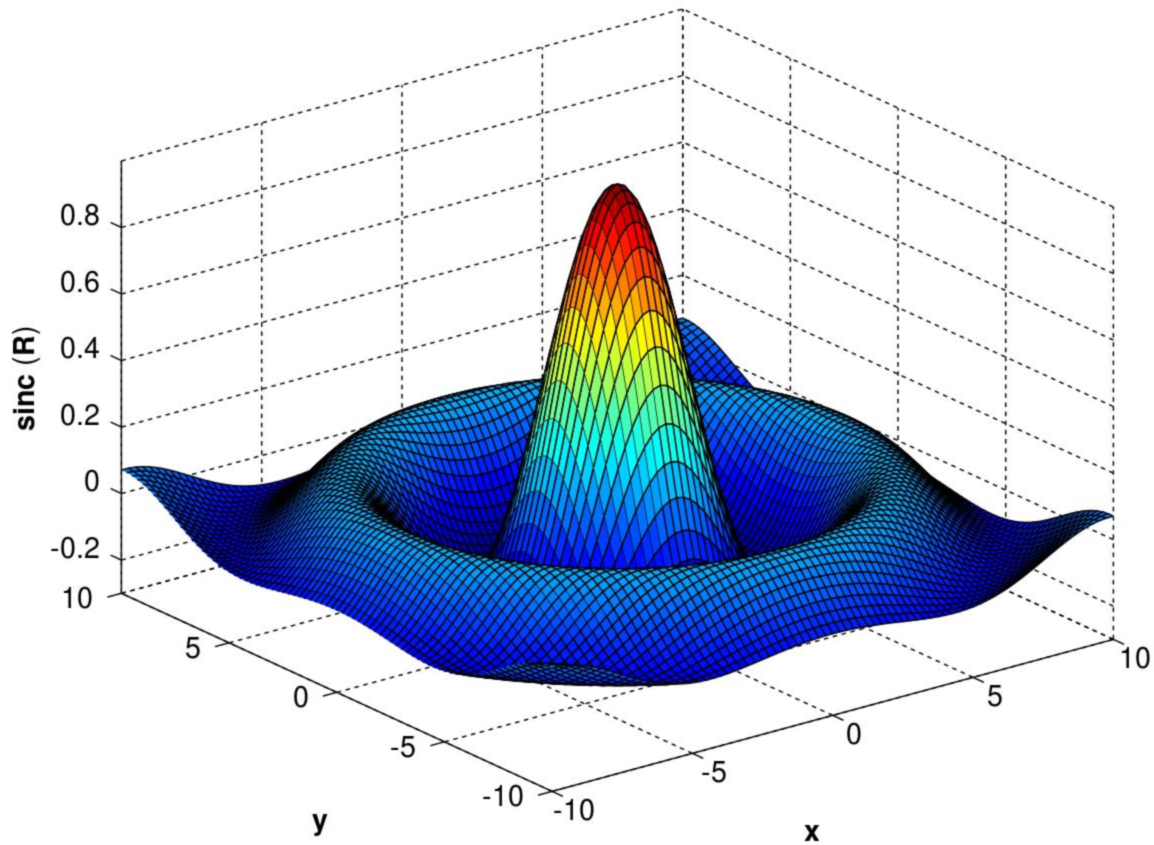
- Flow network optimization (Klarbring et al, 2003)
  Ground structure approach
  Minimize dissipation / pressure drop



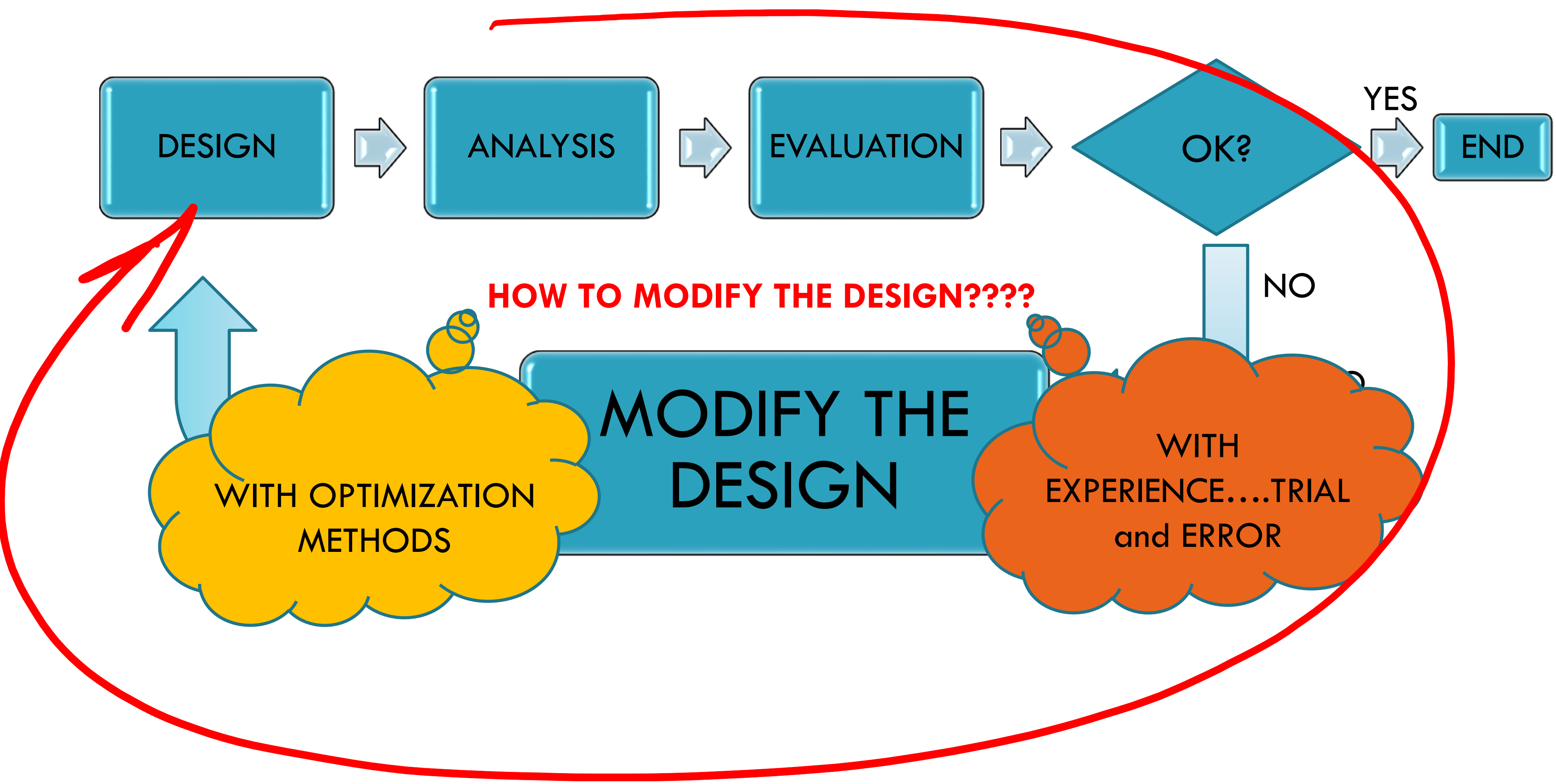


WHAT IS OPTIMIZATION?

- "Making things better"
- "Generating more profit"
- "Determining the best"
- "Do more with less"

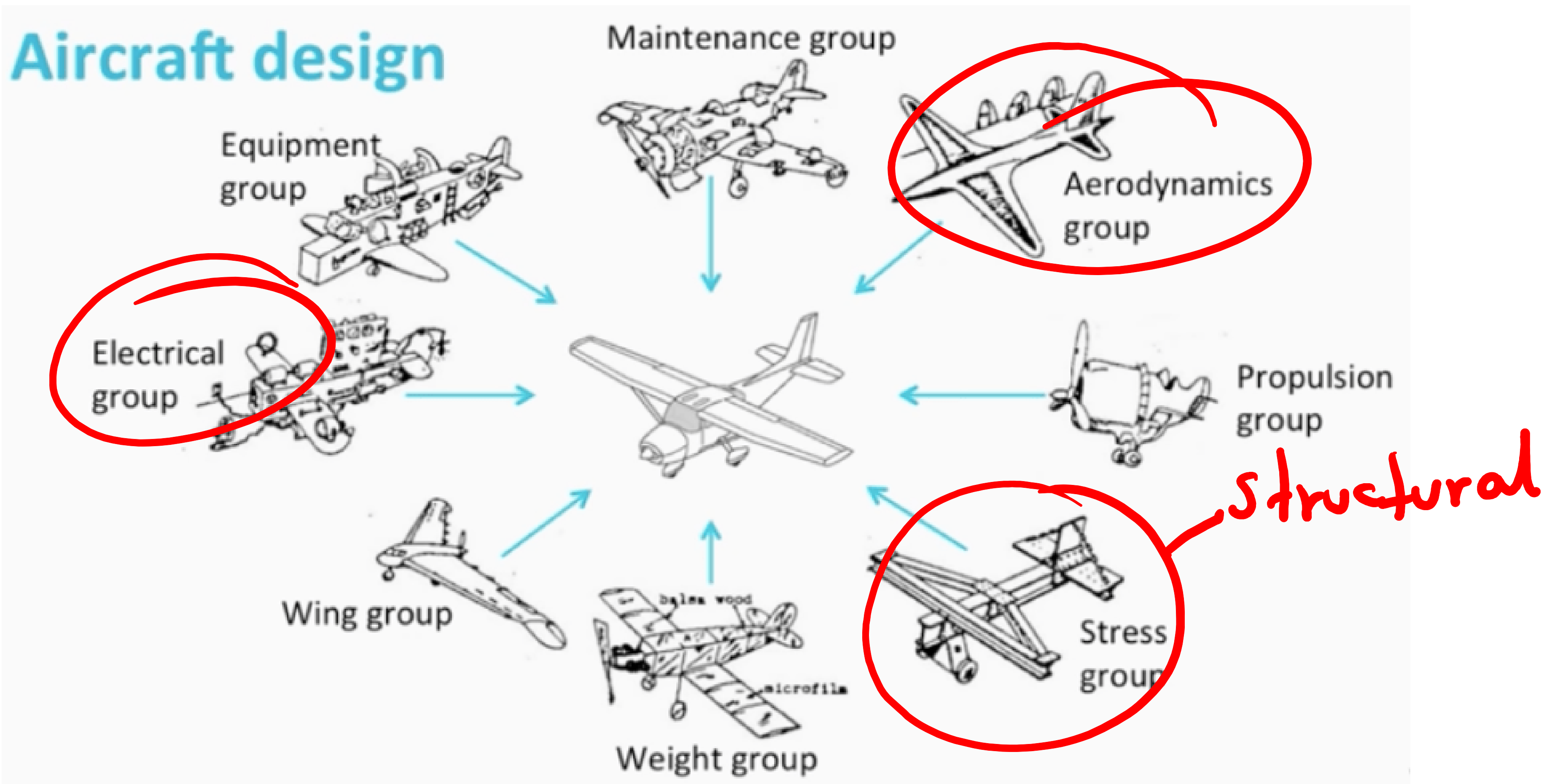


Why OPTIMIZATION?



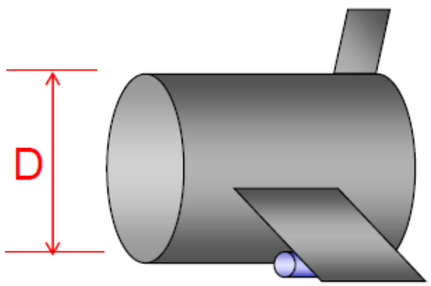
Why OPTIMIZATION? (Cont.)

Is there one aircraft which is the fastest, most efficient, quietest, most inexpensive, most light weight ?

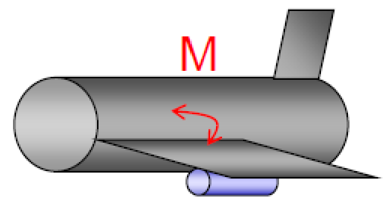


https://b-reddy.org/how-much-will-your-weight-matter-when-going-to-mars-spacex-style/

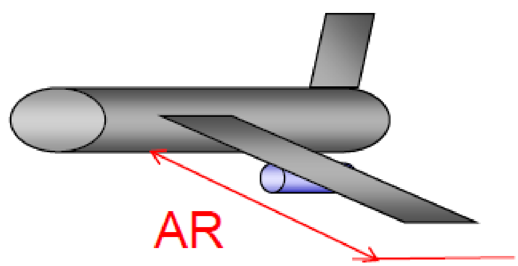
Why OPTIMIZATION? (Cont.)



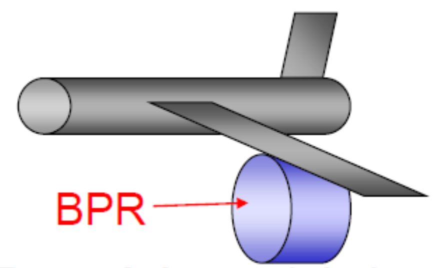
Marketing: maximize passenger volume
→ Cabin diameter



Structures: minimize structural mass
→ Wing-root moment



Aero: maximize L/D
→ Aspect Ratio



Propulsion: minimize specific fuel consumption (SFC)→ Bypass Ratio

HISTORICAL PERSPECTIVE

Lagrange (1750):	**Constrained minimization**
Cauchy (1847):	Steepest descent
Dantzig (1947):	Simplex method (LP)
Kuhn, Tucker (1951):	**Optimality conditions**
Karmakar (1984):	Interior point method (LP)
Bendsoe, Kikuchi (1988):	Topology optimization

OPTIMIZATION

$$\min \mathbf{J}(\mathbf{x},\mathbf{p})$$

Objective Function

$$\text{s.t.} \quad \mathbf{g}(\mathbf{x},\mathbf{p}) \le 0$$

Inequality Constraints

$$\mathbf{h}(\mathbf{x},\mathbf{p}) = 0$$

Equality Constraints

$$x_{i,LB} \le x_i \le x_{i,UB}$$

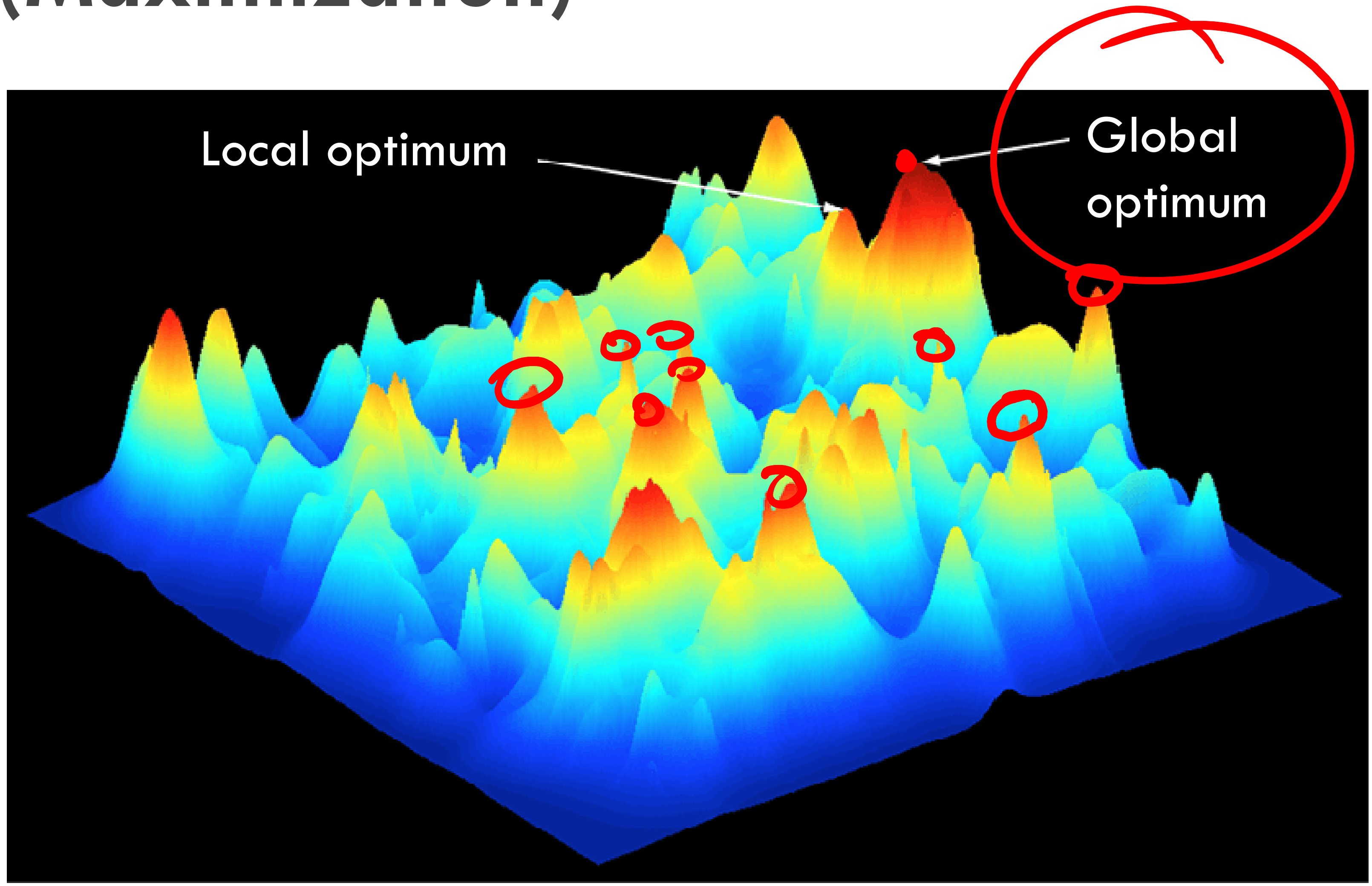
Bounds

$$\text{where} \quad \mathbf{J} = \begin{bmatrix} J_1(\mathbf{x}) & \cdots & J_z(\mathbf{x}) \end{bmatrix}^T$$

$$\mathbf{x} = \begin{bmatrix} x_1 & \cdots & x_i & \cdots & x_n \end{bmatrix}^T$$

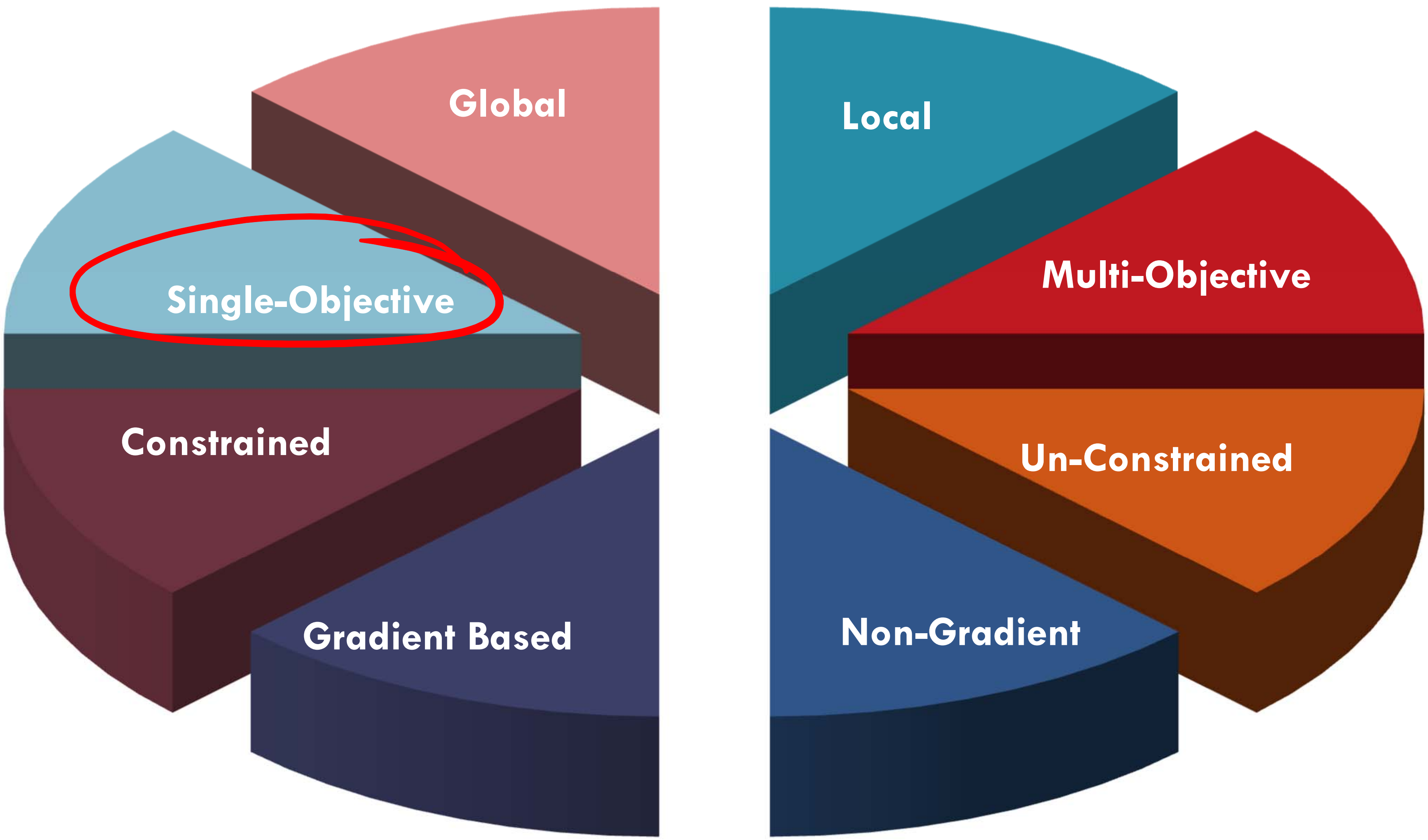
Design Variables

LOCAL AND GLOBAL OPTIMA (Maximization)

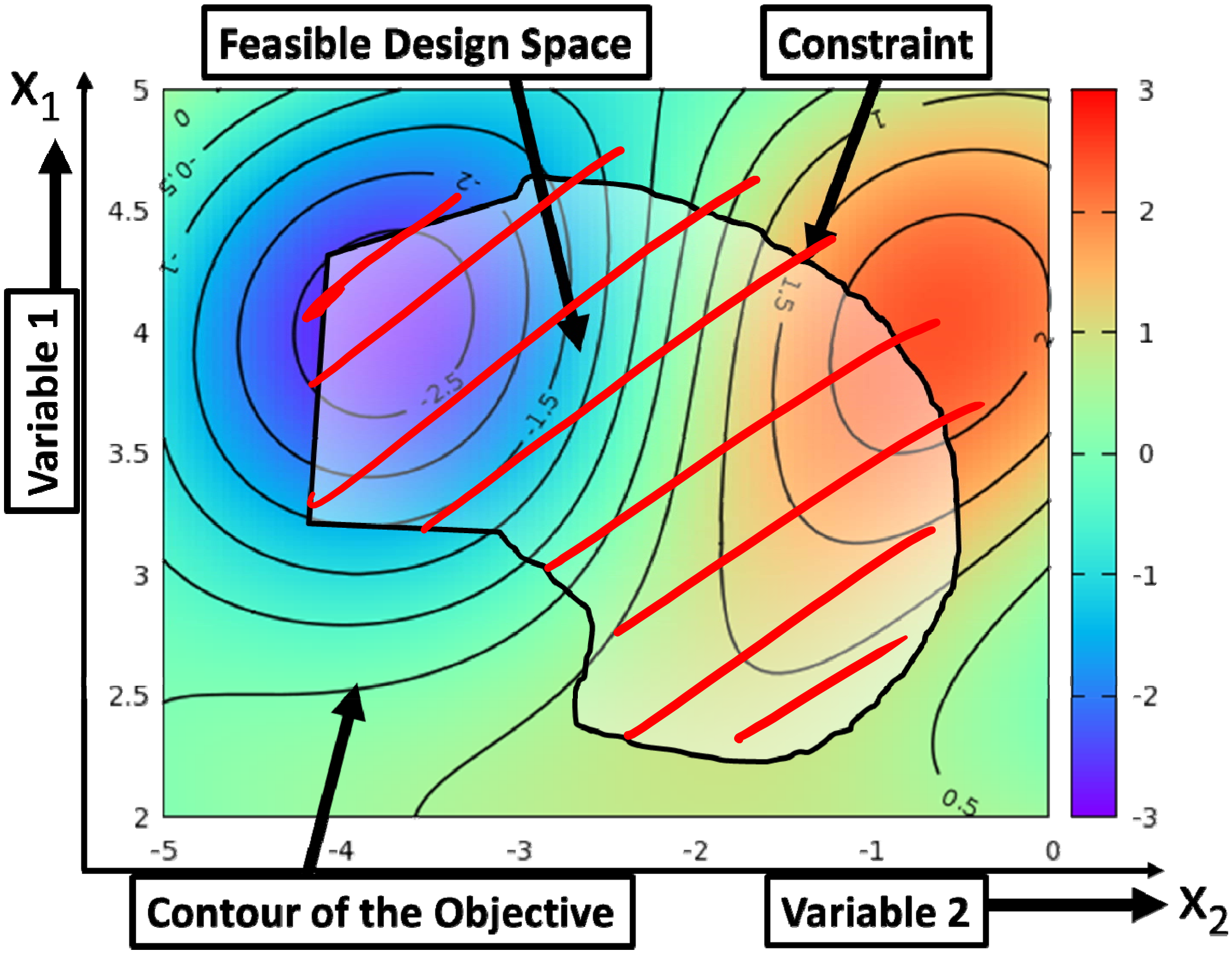


https://kevinbinz.com/2015/08/05/an-introduction-to-natural-selection/

Optimization Problems



Constrained Optimization



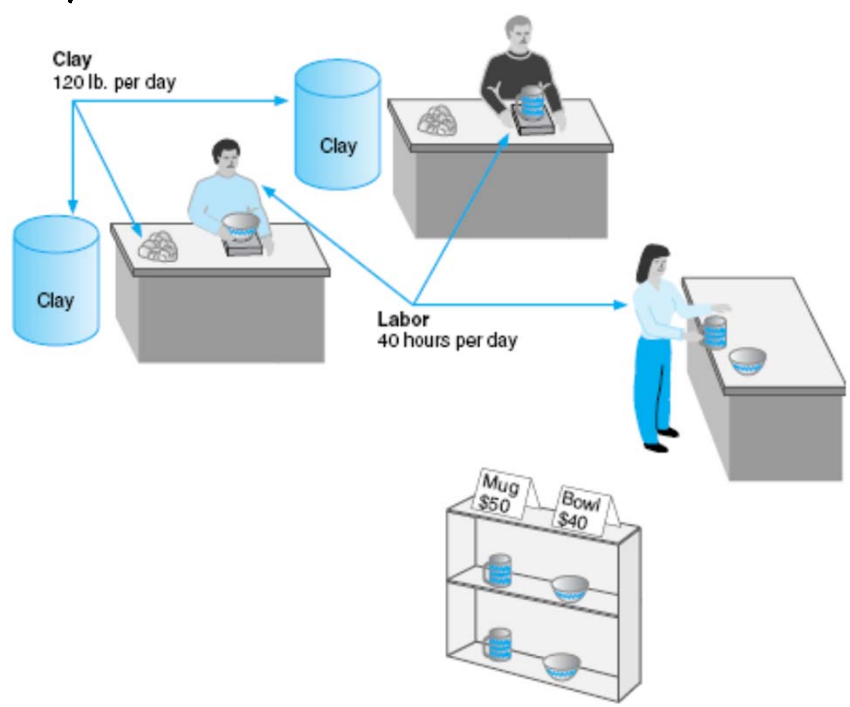
https://designinformaticslab.github.io/productdesign_tutorial/2016/11/20/ansys.html

LINEAR PROGRAMMING

Maximization Problem

- Example The Beaver Creek Pottery Company produces bowls and mugs. The two primary resources used are special pottery clay and skilled labour. The two products have the following resource requirements for production and profit per item produced (that is, the model parameters).

Product	Resource Requirements		
	Labor (hr/unit)	Clay (lb/unit)	Profit ($/unit)
Bowl	1	4	40
Mug	2	3	50



- Resource available: 40 hours of labour per day and 120 pounds of clay per day. How many bowls and mugs should be produced to maximizing profits give these labour resources?

LP Model Formulation
A Maximization Example

- Product mix problem - Beaver Creek Pottery Company
- How many bowls and mugs should be produced to maximize profits given labor and materials constraints?
- Product resource requirements and unit profit:

	Resource Requirements		
Product	Labor (hr/unit)	Clay (lb/unit)	Profit ($/unit)
Bowl	1	4	40
Mug	2	3	50

A Maximization Example (Cont.)

Let xi be denoted as xi product to be produced,
and
i = 1, 2
or

Let $x1$ be numbers of product 1 to be produced   bowls

and $x2$ be numbers of product 2 to be produced   mugs

Decision variables

Maximize $Z = 40x_1 + 50x_2$ (\$)

Objective function

subject to

$1x_1 + 2x_2 \le 40$ hours of Labor

$4x_1 + 3x_2 \le 120$ pounds of clay

$x_1, x_2 \geq 0$

Constraints

A Maximization Example (Cont.)

Step 1: define decision variables

Let x_1=number of bowls to produce/day

x_2= number of mugs to produce/day

Step 2: define the objective function

maximize Z = \$ $40x_1 + 50x_2$

where Z= profit per day

Step 3: state all the resource constraints

$1x_1 + 2x_2 \leq 40$ hours of labor (resource constraint 1)

$4x_1 + 3x_2 \leq 120$ pounds of clay (resource constraint 2)

Step 4: define non-negativity constraints

$x_1 \geq 0, \quad x_2 \geq 0$

Complete Linear Programming Model:

Maximize $Z = 40x_1 + 50x_2$

Subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$

A FEASIBLE SOLUTION

- A **feasible solution** does not violate any of the constraints:

Example

$x_1 = 5$ bowls

$x_2 = 10$ mugs

$Z = 40(5) + 50(10) = 700$

Labor constraint check:

$1(5) + 2(10) = 25 < 40$

within constraint

Clay constraint check:

$4(5) + 3(10) = 70 < 120$

within constraint

An INFEASIBLE SOLUTION

- An **infeasible solution** violates at least one of the constraints:

Example $x_1 = 10$ bowls
$x_2 = 20$ mugs
$Z = \$1400$

Labor constraint check:

$1(10) + 2(20) = 50 < 40$ (NO)

violates constraint

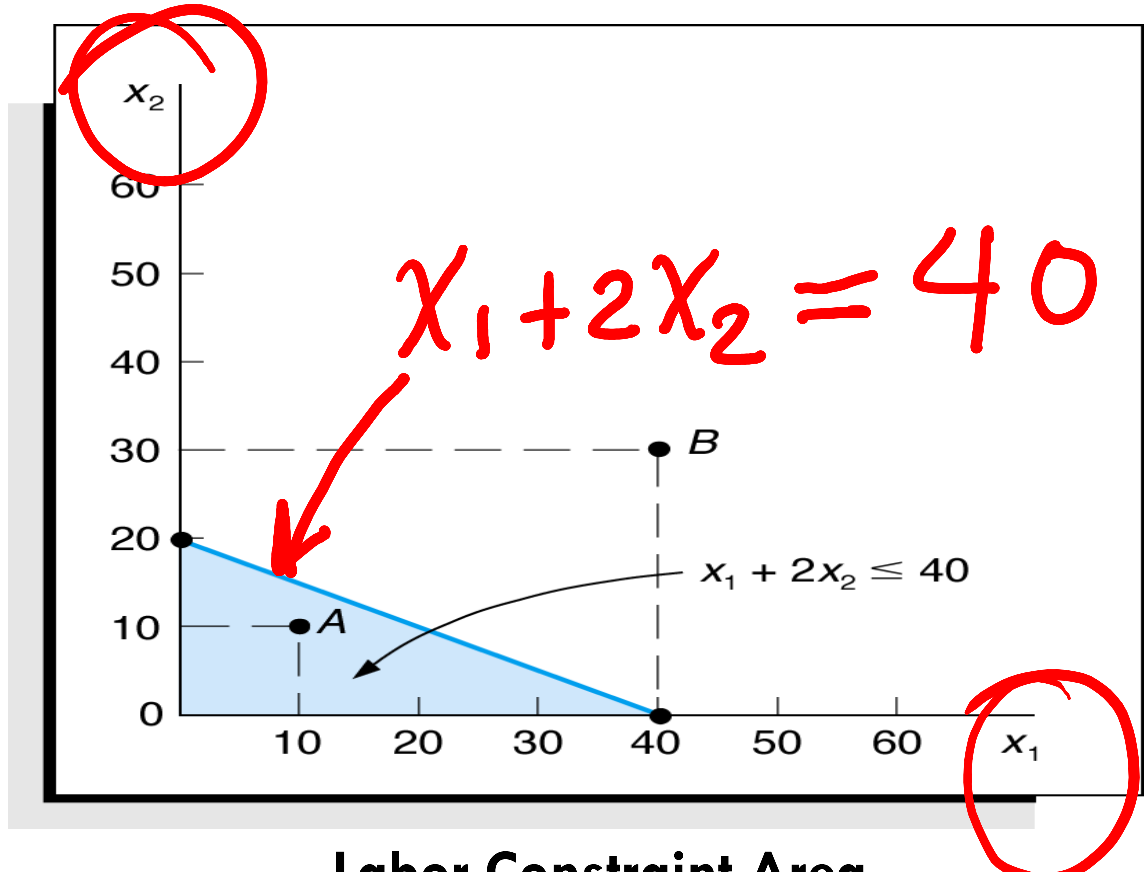
Graphical Solution of Maximization

Labor Constraint Area

Maximize $Z = \$40x_1 + \$50x_2$

subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$

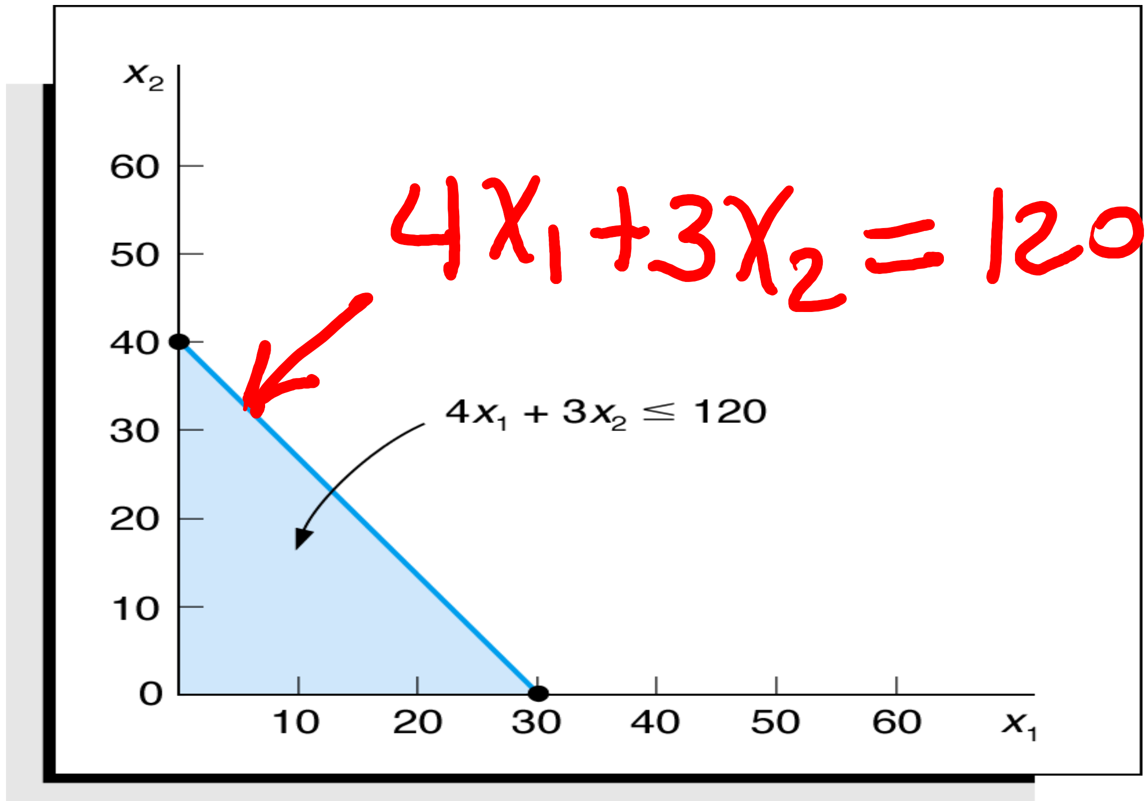


Labor Constraint Area

Clay Constraint Area

Maximize $Z = \$40x_1 + \$50x_2$
subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$

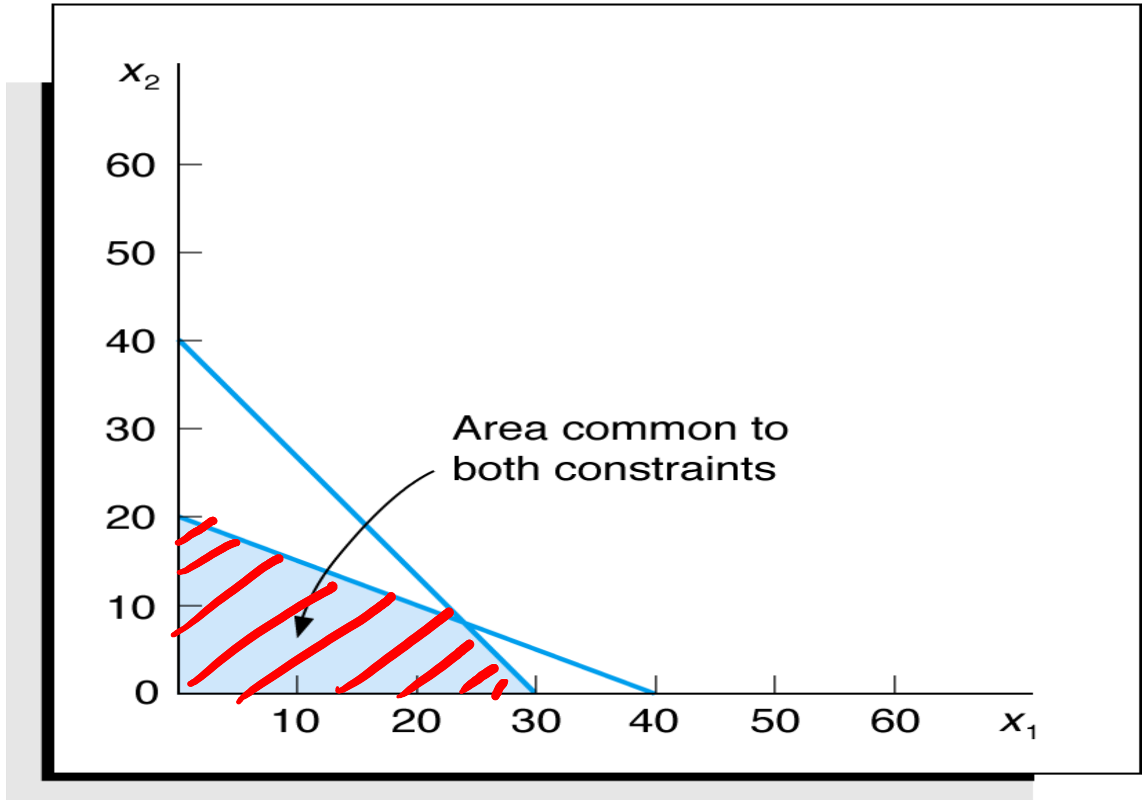


Clay Constraint Area

Graph of Both Model Constraints

Maximize $Z = \$40x_1 + \$50x_2$
subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$



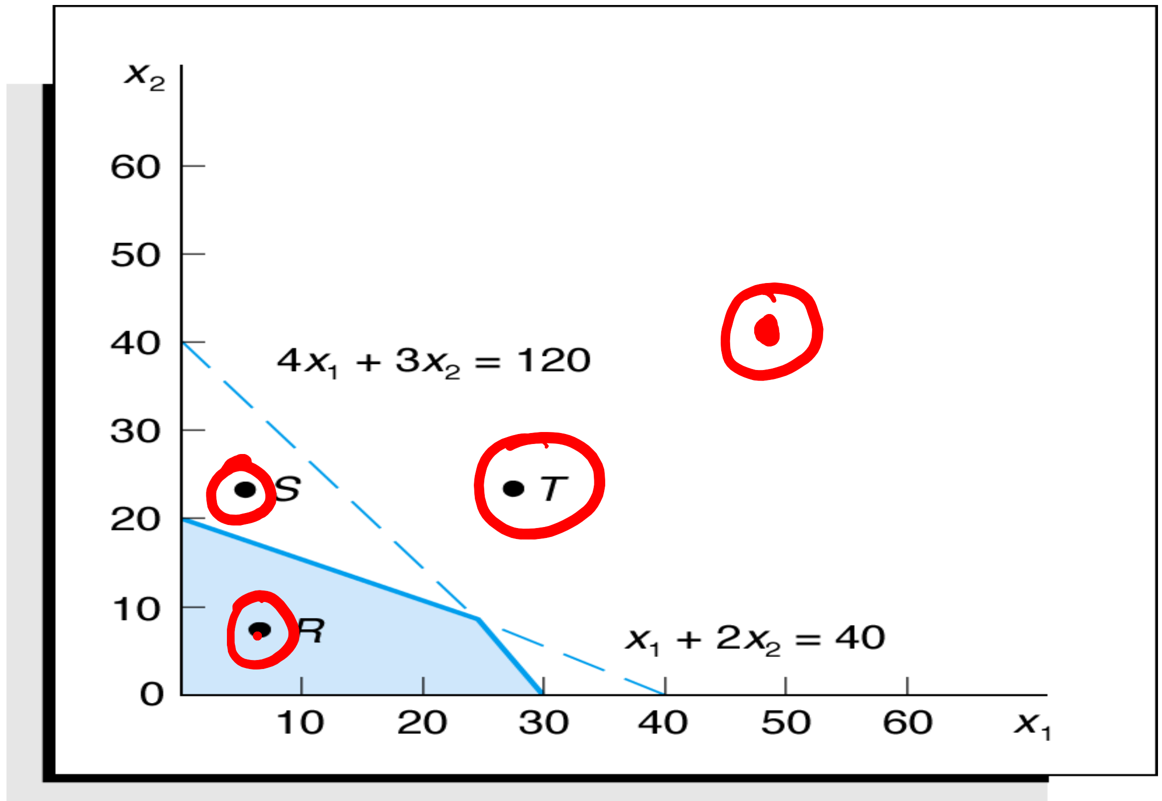
Graph of Both Model Constraints

Feasible Solution Area

Maximize $Z = \$40x_1 + \$50x_2$

subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$



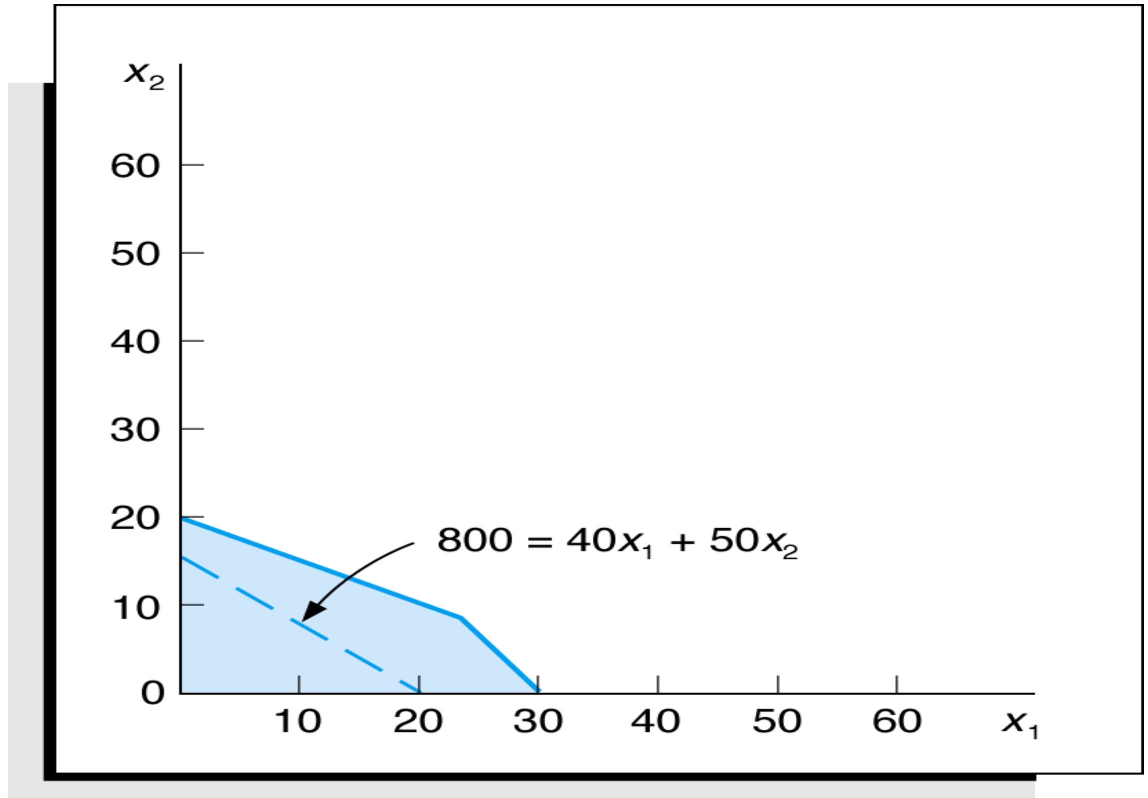
Feasible Solution Area

Objective Function Solution = $800

Maximize $Z = \$40x_1 + \$50x_2$

subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$

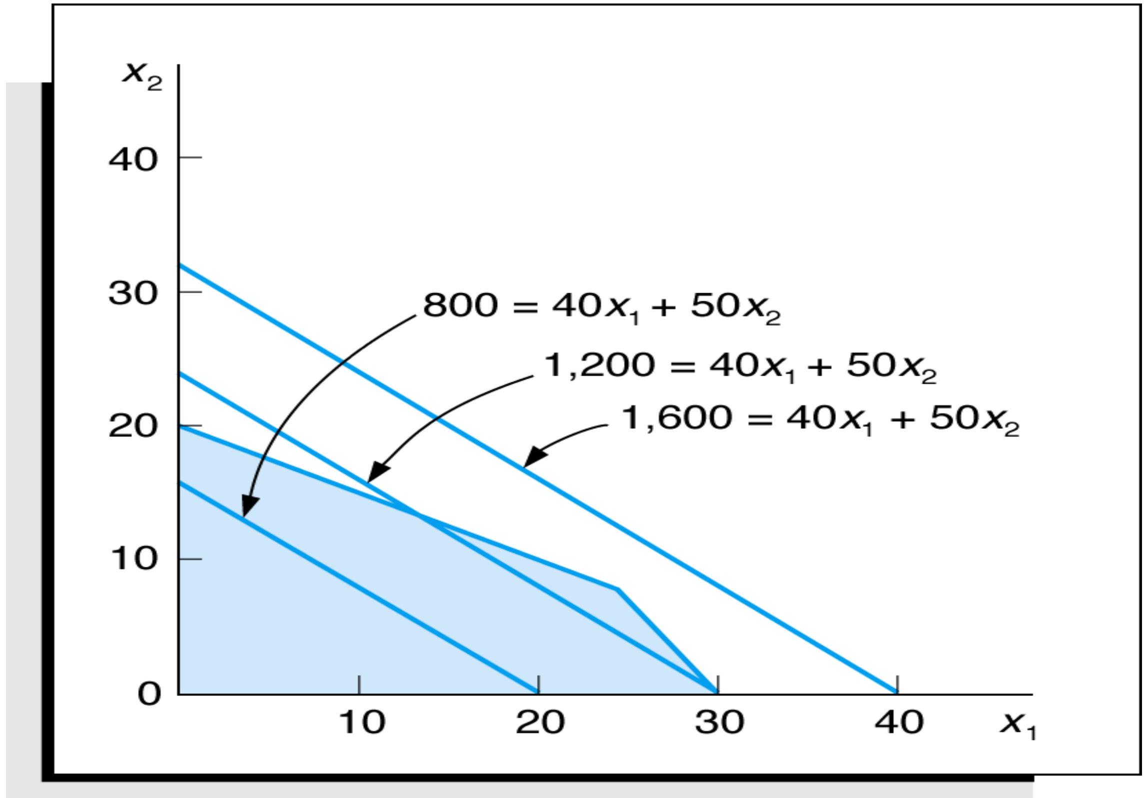


Objection Function Line for Z = $800

Alternative Objective Function Solution Lines

Maximize $Z = \$40x_1 + \$50x_2$
subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$



Alternative Objective Function Lines

Identification of Optimal Solution

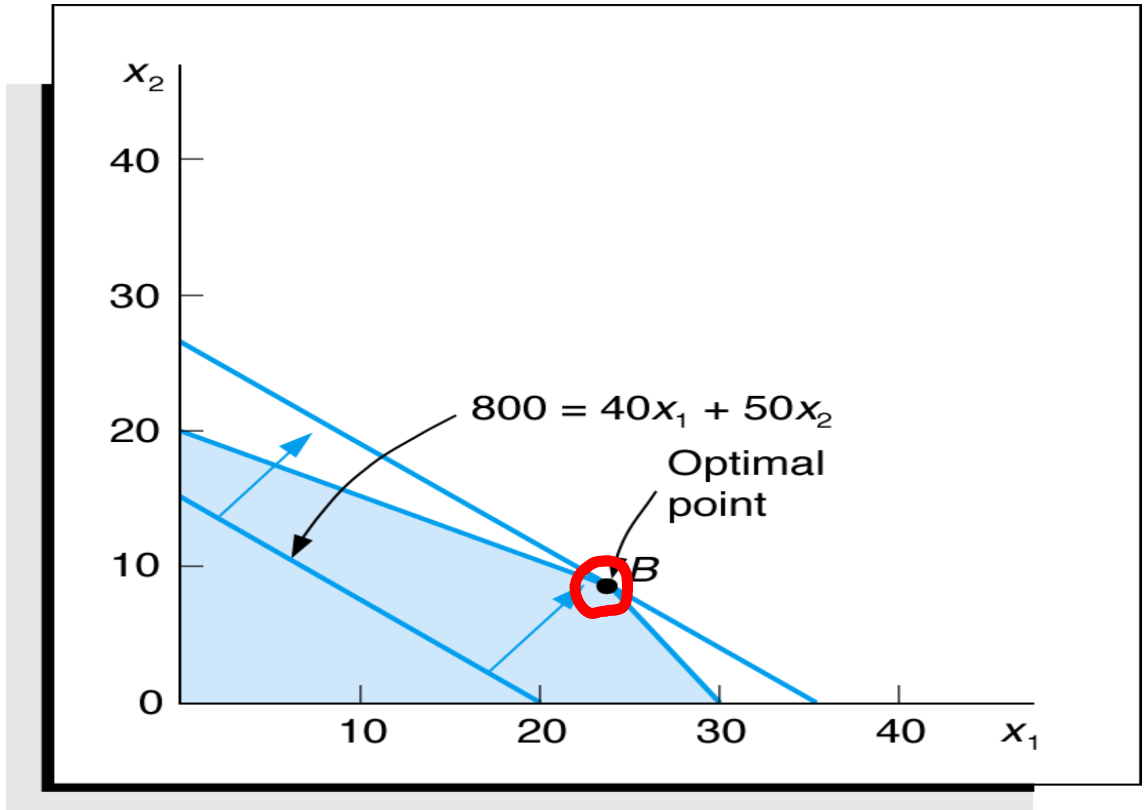
Maximize $Z = \$40x_1 + \$50x_2$

subject to:

$$1x_1 + 2x_2 \leq 40$$

$$4x_1 + 3x_2 \leq 120$$

$$x_1, x_2 \geq 0$$



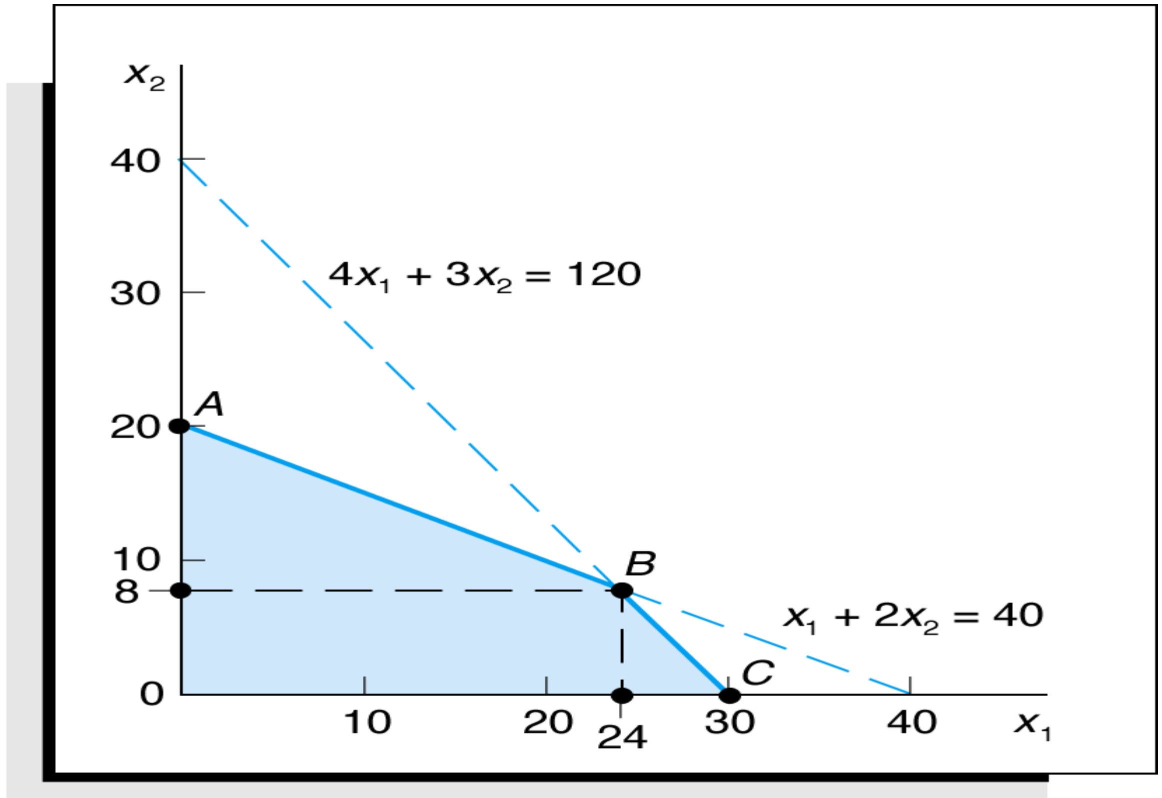
Identification of Optimal Solution

Optimal Solution Coordinates

Maximize $Z = \$40x_1 + \$50x_2$

subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$

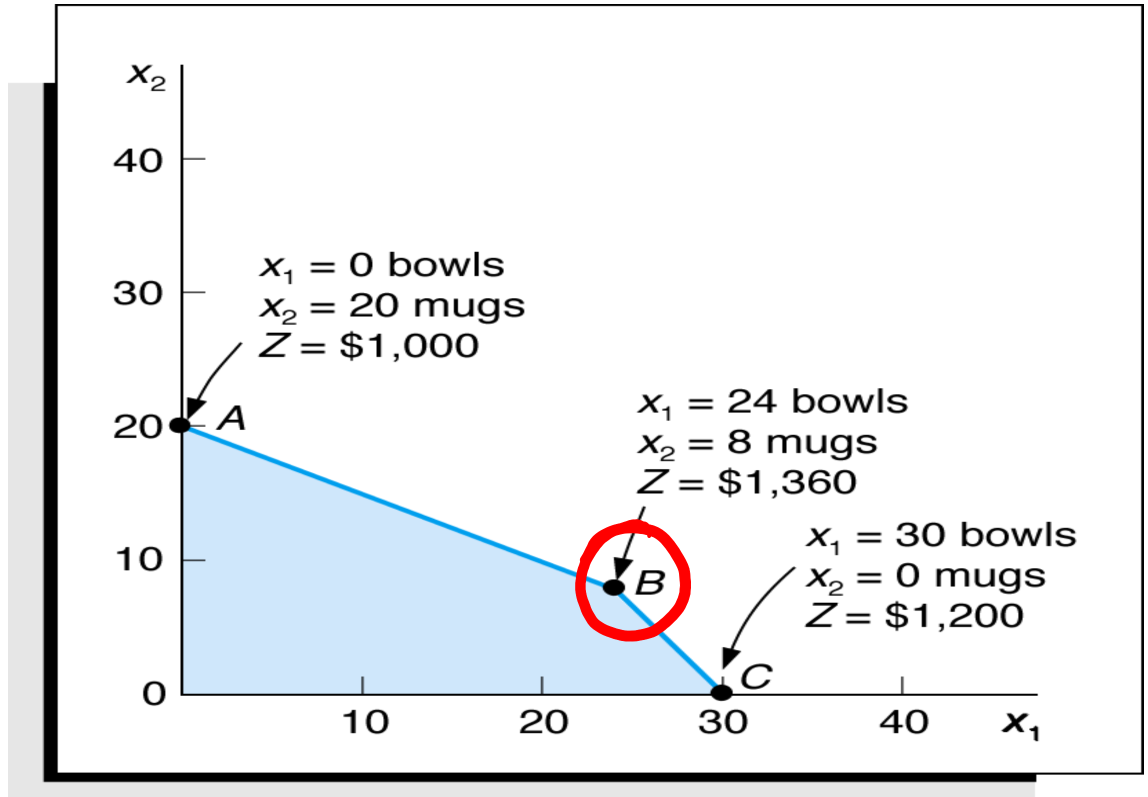


Optimal Solution Coordinates

Extreme (Corner) Point Solutions

Maximize $Z = \$40x_1 + \$50x_2$
subject to:

$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$



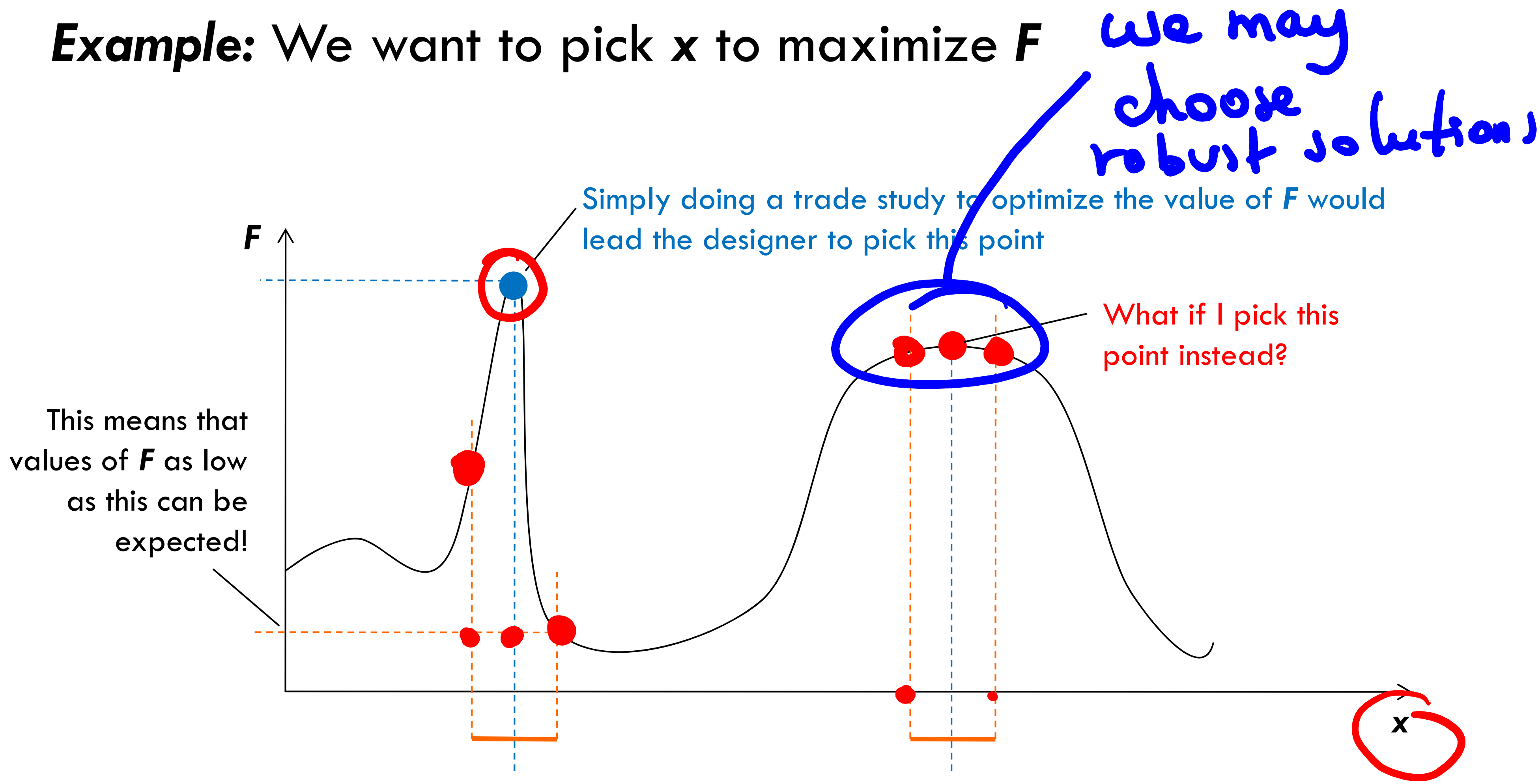
Solutions at All Corner Points

Solution with Excel (Show in class)

Product mix problem - Beaver Creek Pottery		
Number	1	2
Profit	40	50
Optimal x	24	8
Benefit	1360.0	
Constraints:		
$1x1 + 2x2 \leq 40$	0	
$4x1 + 3x2 \leq 120$	0	
$x1, x2 \geq 0$		

ROBUST DESIGN

Example: We want to pick ***x*** to maximize ***F***



Robust design: a design whose performance is insensitive to variations.