CE 544 Open Channel Hydraulics - Solution
Mid-term, Winter 2015

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- ✓ **You will have 50 minutes to complete the exam. The exam is closed book, closed notes and open mind.**
- ✓ **The procedure will be graded. Please justify your answers**

1. (25 points) A trapezoidal channel is to be designed to carry a discharge of 75 m^3/s at **maximum hydraulic efficiency**. The side slopes of the channel are 2H:1V and the Manning's roughness *n* is 0.030.
   a. If the maximum allowable velocity in the channel is 1.75*m/s*, what should be the dimensions of the channel (bottom width and height)?
   b. What should be the longitudinal slope of the channel if the flow is uniform?

a)

$$A = Q/V = 75/1.75 = 42.857 m^2$$

$$b = 2y\left(\sqrt{1+z^2} - z\right) = 0.472y$$

$$A = by + zy^2 = 0.472y^2 + 2y^2 = 2.472y^2$$

$$A = 42.86 = 2.472y^2$$

$$y^2 = 17.336 \rightarrow y = 4.16 m \Rightarrow b = 0.472y = 1.96m$$

b)

$$P = b + 2y\sqrt{1+z^2} = 1.96 + 2(4.16)\sqrt{1+2^2} = 20.56m$$

$$R = A/P = 42.86/20.56 = 2.08 m$$

$$V = 1.75 = 1/0.030 (2.08)^{2/3} S_o^{1/2}$$

$$S_o = 0.00104 m/m$$

$$Q = \frac{k}{n} A R^{2/3} S^{1/2} = C \frac{A^{5/3}}{P^{2/3}} \qquad A = by + zy^2$$

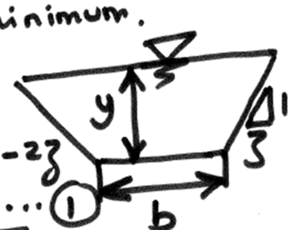
Hydraulic efficiency: $\quad P = b + 2y\sqrt{1+z^2}$

① for a given A, Q_{max} } they are equivalent and give the same answer
② for a given Q, min A.

In our case: Q is given so A needs to be minimum $\rightarrow \left(\frac{dA}{dy} = 0\right)$

$$P^{2/3} Q = C A^{5/3}$$

if Q is constant, A will be minimum if P is minimum.



$A_{min}\left(\frac{dA}{dy} = 0\right) \rightarrow$

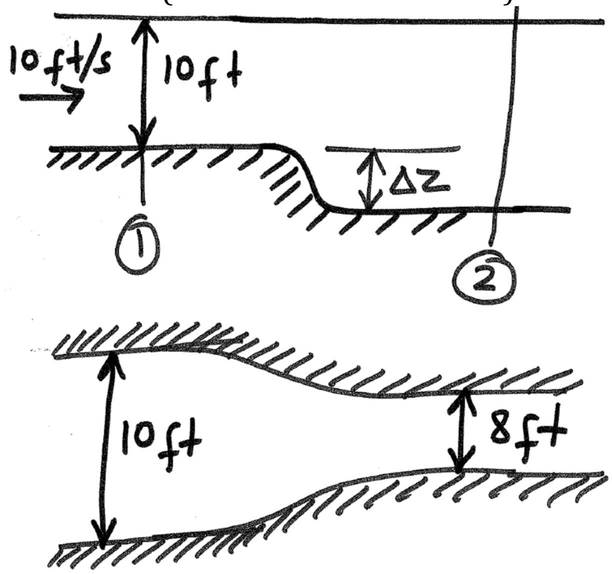
$$b + y\frac{db}{dy} + 2zy = 0 \rightarrow \frac{db}{dy} = \frac{-b}{y} - 2z \quad \ldots ①$$

$$P_{min}\left(\frac{dP}{dy} = 0\right) \rightarrow \frac{db}{dy} + 2\sqrt{1+z^2} = 0 \quad \ldots ②$$

① in ② $\Rightarrow -\frac{b}{y} - 2z + 2\sqrt{1+z^2} = 0$

$$\boxed{\frac{b}{y} = 0.47}$$

2. (25 points) A 10-ft wide rectangular channel is flowing at a depth of 10-ft with a velocity of 10 ft/s. If the channel has a smooth contraction in width from 10 ft to 8 ft, how much should the channel bottom drop to maintain a constant water surface elevation through the transition? (Head loss coefficient = 0)



②

$q_1 = 100 \frac{ft^2}{s}$, $y_c = 6.77\ ft$

$E_1 = 11.55\ ft$

$Q_1 = 100 \times 10 = 1000\ ft^3/s$

$Q_1 = Q_2$

$1000 = q_2 b_2 = q_2 \times 8$

$\boxed{q_2 = 125\ ft^2/s}$

$y_{c_2} = \left[\frac{q_2^2}{g}\right]^{1/3} = \left(\frac{125^2}{32.2}\right)^{1/3}$

$\boxed{y_{c_2} = 7.86\ ft}$

Energy equation between ① and ②

$E_1 = y_2 + \frac{q_2^2}{2g y_2^2} - \Delta z$

$y_2 - \Delta z = 10\ ft$

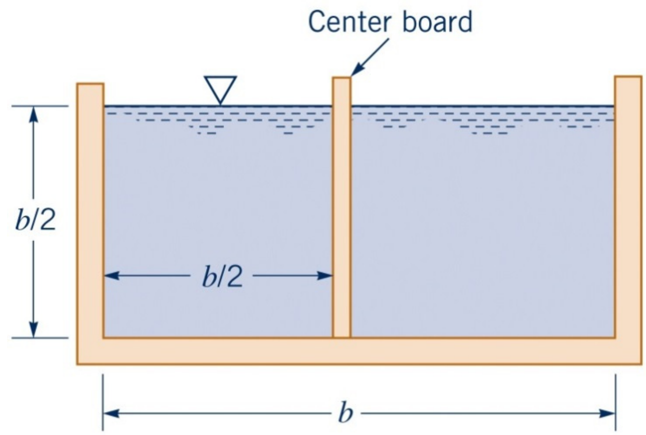
$11.55 = 10\ ft + \frac{125^2}{2 \times 32.2\, y_2^2}$

$1.55 = \frac{242.624}{y_2^2}$

$y_2 = 12.51\ ft$

$\Delta z = y_2 - 10 = 2.51\ ft$

3. (25 points) By what percent is the flowrate reduced in the rectangular channel shown in figure below because of the addition of the thin center board? All surfaces are of the same material.



$$Q = \frac{k}{n} A R_h^{2/3} S_o^{1/2}$$

Without the center board:

$$A = b\left(\frac{b}{2}\right) = \frac{b^2}{2} \quad ; \quad R_h = \frac{A}{P} = \frac{\frac{b^2}{2}}{2b} = \frac{b}{4}$$

$$\Rightarrow Q_{without\,center\text{-}board} = \frac{k}{n}\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3} S_o^{1/2} \qquad \text{Eq. (1)}$$

With the center board:

$$Q_{with\text{-}center\text{-}board} = 2 Q_2 \quad , \text{where } A_2 = \left(\frac{b}{2}\right)^2$$

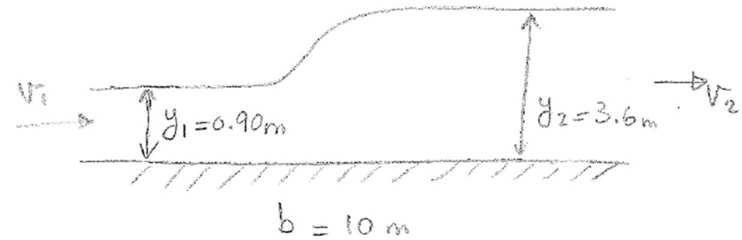
$$R_{h_2} = \frac{A_2}{P_2} = \frac{\left(\frac{b}{2}\right)^2}{3\left(\frac{b}{2}\right)} = \frac{b}{6}$$

$$\Rightarrow Q_{with\text{-}center\text{-}board} = 2\left[\frac{k}{n}\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3} S_o^{1/2}\right] \qquad \text{Eq. (2)}$$

Devide Eq. (2) by Eq. (1) to obtain $\frac{Q_{with\_centerboard}}{Q_{without\_centerboard}} = \frac{2\left(\frac{b}{2}\right)^2\left(\frac{b}{6}\right)^{2/3}}{\left(\frac{b^2}{2}\right)\left(\frac{b}{4}\right)^{2/3}} = 0.763$

100 − 76.3 = **23.7% reduction**

4. (25 points) A hydraulic jump at the base of a spillway of a dam is such that the depths upstream and downstream of the jump are 0.90 and 3.6 m, respectively. If the spillway is 10 m wide, what is the flowrate over the spillway?



$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

$$\Rightarrow \frac{3.6\,m}{0.9\,m} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right) \Rightarrow Fr_1 = 3.16$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} \Rightarrow V_1 = 3.16 \times \sqrt{9.81\left(\frac{m}{s^2}\right) \times 0.9(m)} \Rightarrow \boxed{V_1 = 9.39\ \frac{m}{s}}$$

$$\Rightarrow Q_1 = A_1 V_1 = b y_1 V_1 = (10m)(0.9m)(9.39\,m/s)$$

$$\Rightarrow \boxed{Q_1 = 84.5\ \frac{m^3}{s}}$$