Introduction to Sediment Transport in Open Channel Flows

Image source: http://earthsci.org/processes/geopro/stream/stream.html

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Movies

Scour at bridge model pier
https://www.youtube.com/watch?v=48S_k6qAmsY&feature=emb_logo

Utah Flash Flood
https://www.youtube.com/watch?v=mXlr_Bgb-s0

Santa Clara Pueblo Flash Flood
https://www.youtube.com/watch?v=nKOQzkRi4BQ
Sediment Transport

- We study sediment transport to predict the risks of scouring of bridges, to estimate the siltation of a reservoir, etc.
- Most, if not all, natural channels have mobile beds
- Most mobile beds are in dynamic equilibrium:
  on average: sediment in = sediment out
- This dynamic equilibrium can be disturbed by
  - short-term extreme events (e.g., flash floods)
  - man-made infrastructure (e.g., dams)

Source: https://www.flow3d.com/modeling-capabilities/sediment-transport-model/
Sediment Transport Fundamental Questions:

- Does sediment transport occur? (Threshold of motion).
- If so, then at what rate? (Sediment load)
- What net effect does it have on the bed? (Scour/accretion)

The main types of sediment load (volume of sediment per unit of time) are Bed load and Suspended load. The sum is total load.
Bed form motion:

Subcritical flow:
- Dune
- Antidune

Super critical flow:
- Downstream

Flow:
- Bedform migration
- Erosion
- Deposition

Source: Lecture notes on Hydraulic, David Apsley
Bed form motion (Cont.)

Source: https://armfield.co.uk/product/s8-mkii-sediment-transport-demonstration-channel/
Relevant Properties

- **Particle**
  - Diameter, $d$
  - Specific gravity, $s = \frac{\rho_s}{\rho}$
  - Settling velocity, $w_s$
  - Porosity, $P$
  - Angle of repose, $\phi$

- **Fluid**
  - Density, $\rho$
  - Kinematic viscosity, $\nu$
  - Kinematic viscosity at 20°C: $\nu(20°C) = 10^{-6}$ m²/s

- **Flow**
  - Bed shear stress, $\tau_b$
  - Mean-velocity profile, $U(z)$
  - Eddy-viscosity profile, $\nu_t(z)$

$\rho_{[\text{Quartz}]} = 2,650 \frac{\text{kg}}{\text{m}^3}$

$\rho_{[\text{Sand}]} = 2,650 \frac{\text{kg}}{\text{m}^3}$

$\rho_{[\text{Water}]} = 1,000 \frac{\text{kg}}{\text{m}^3}$

$\frac{\rho_{[\text{Sand}]}}{\rho_{[\text{Water}]}} = 2.65$
Inception of Motion

- Inception and magnitude of bed-load depends on:
  - bed shear stress $\tau_b$
  - particle diameter $d$ and specific gravity $S$

- Inception of suspended load depends on ratio of:
  - settling velocity $w_s$
  - typical turbulent velocity (friction velocity $u_\tau$)
Particle Properties: Diameter $d$

Various types:
- Sieve diameter
- Sedimentation diameter
- Nominal diameter

<table>
<thead>
<tr>
<th>Type</th>
<th>Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boulders</td>
<td>$&gt; 256$ mm</td>
</tr>
<tr>
<td>Cobbles</td>
<td>$64$ mm – $256$ mm</td>
</tr>
<tr>
<td>Gravel</td>
<td>$2$ mm – $64$ mm</td>
</tr>
<tr>
<td>Sand</td>
<td>$0.06$ mm – $2$ mm</td>
</tr>
<tr>
<td>Silt</td>
<td>$0.002$ mm – $0.06$ mm</td>
</tr>
<tr>
<td>Clay</td>
<td>$&lt; 0.002$ mm (cohesive)</td>
</tr>
</tbody>
</table>

In practice, there is a range of diameters (typically, lognormally distributed).
**Particle Properties: Specific Gravity** \( s \)

\[
s = \frac{\rho_s}{\rho}
\]

- **Quartz-like**: \( \rho_s \approx 2650 \text{ kg/m}^3, s \approx 2.65 \)
- **Anthracite**: \( \rho_s \approx 1500 \text{ kg/m}^3, s \approx 1.50 \)

**Particle Properties: Porosity** \( P \)

*Porosity* = fraction of voids (by volume)

Typical uncompacted sediment: \( P \approx 0.4 \).
Particle Properties: Settling Velocity $w_s$

Terminal velocity in still fluid.

Small particles (Stokes’ Law):

$$w_s = \frac{1}{18} \frac{(s-1)gd^2}{v}$$ (Spheres)

$$\frac{w_sd}{v} = \frac{1}{18} \frac{(s-1)gd^3}{v^2} = \frac{1}{18} d^*^3$$

$$d^* = d \left[ \frac{(s-1)g}{v^2} \right]^{1/3}$$

Realistic sizes and shapes:

Cheng’s formula:

$$\frac{w_sd}{v} = \left( 25 + 1.2d^*^2 \right)^{1/2} - 5\right]^{3/2}$$

Particle Properties: Angle of Repose $\phi$

Angle of repose $\phi = \text{limiting angle of slope (in still fluid)}$

$mg \sin \phi = \mu_f R$

$mg \sin \phi = \mu_f (mg \cos \phi)$

$\mu_f = \tan \phi$

$R = mg \cos \phi$

$\mu_f = \text{Effective coefficient of friction}$

Can be used to estimate the effect of slopes on incipient motion
Flow Properties: Bed Friction

Bed shear stress $\tau_b$
- Drag (per unit area) of flow on granular bed.
- Determines inception and magnitude of bed load.

Friction velocity $u_\tau$
- Defined (on dimensional grounds) by:
  $$\tau_b = \rho u_\tau^2$$
  or
  $$u_\tau = \sqrt{\tau_b / \rho}$$
- Determines inception and magnitude of suspended load.
Flow Properties: Mean-Velocity Profile

For a rough boundary:

\[ U(z) = \frac{u_\tau}{\kappa} \ln(33 \frac{z}{k_s}) \]

\( u_\tau \) = friction velocity;
\( \kappa \) = von Kármán’s constant (≈ 0.41);
\( z \) = distance from the bed;
\( k_s \) = roughness height (1.0 - 2.5 times particle diameter).
A model for the effective shear stress $\tau$ in a turbulent flow:

$$\tau = \mu_t \frac{dU}{dz} \quad \text{or} \quad \tau = \rho \nu_t \frac{dU}{dz}$$

$\mu_t$ and $\nu_t$ are the dynamic and kinematic eddy viscosities, respectively.

**At the bed** ($z = 0$): $\tau = \tau_b \equiv \rho u_\tau^2$

**At the free surface** ($z = h$): $\tau = 0$

Assuming linear: $\tau = \rho u_\tau^2 (1 - z/h)$

From stress and mean-velocity profiles: $\rho u_\tau^2 (1 - \frac{z}{h}) = \rho \nu_t \frac{\mu_t}{\kappa z}$

$$\nu_t = \kappa u_\tau z (1 - \frac{z}{h})$$
Formulas For Bed Shear Stress

**Normal flow:** \( \tau_b = \rho g R_h S \)

Manning’s formula: \( V = \frac{k}{n} R_h^{2/3} S^{1/2} \)

Strickler’s formula: \( n = \frac{d^{1/6}}{21.1} \)

Typical values: \( n \approx 0.01 \) to 0.035 m\(^{-1/3}\)s

Via a **friction coefficient:** \( \tau_b = c_f \left( \frac{1}{2} \rho V^2 \right) \)

Fully-developed boundary layer (log-law): \( c_f = \frac{0.34}{\left[ \ln \left( \frac{12h}{k_s} \right) \right]^2} \)

Typical values of **friction coefficient** \( c_f \approx 0.003 \) to 0.01
Finding the Threshold of Motion

A mobile bed starts to move once the bed stress exceeds a critical stress $\tau_{crit}$.

$$\tau_{crit} = f\left(d^*\right)$$

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd}$$

$$d^* = d \left[ \frac{(s-1)g}{v^2} \right]^{1/3}$$

Curve fit (Soulsby, 1997):

$$\tau_{crit}^* = \frac{0.30}{1 + 1.2d^*} + 0.055 \left(1 - e^{-0.020d^*}\right)$$

Example (Adapted from Apsley, hydraulic notes):

Find the critical Shields parameter $\tau^*_{crit}$ and critical absolute stress $\tau_{crit}$ for a sand particle of diameter 1 mm in water.

$\tau^*_{crit} \quad d = 1\text{mm}$

$\tau_{crit} \quad \text{For a sand particle} \quad S = \frac{R_s}{g} = \frac{2650}{1000}$

$S = 2.65$

* $\tau^*_{crit}$ (Soulsby formula)

$\tau^*_{crit} = \frac{0.30}{1 + 1.2 d^*} + 0.055 \left(1 - e^{-0.020 d^*}\right)$

$d^* = d \left[ \frac{(S-1)g}{\nu^2} \right]^{1/3}$

$\nu = 10^{-6} \text{ m}^2/\text{s}$

$d^* = \frac{1}{1000} \left[ \frac{1.65 \times 9.81}{(10^{-6})^2} \right]^{1/3} = 25.3$
In (1)

\[ Z_{crit} = \frac{0.30}{1 + 1.2 \times 25.3} + 0.055(1 - e^{-0.02 \times 25.3}) \]

\[ Z_{crit} = 0.03141 \]

\[ Z_{bcrit} = \frac{Z_{crit} \cdot p(s-1)g \cdot d}{p(s-1)g} \]

\[ Z_{bcrit} = 0.03141 \cdot 1000 \cdot 1.65 \cdot 9.81 \cdot \frac{1}{1000} \]

\[ Z_{bcrit} = 0.508 \frac{N}{m^2} \]
Example (Adapted from Apsley, hydraulic notes):

A sluice gate is lowered into a wide channel carrying a discharge of 0.9 m$^3$/s per meter width. The bed of the channel is coarse gravel with particle diameter 60 mm and density 2650 kg/m$^3$. The critical Shields parameter is 0.056 and the bed friction coefficient is 0.01. The particles of gravel have settling velocity 1.1 m/s. Initially the bed of the channel under the sluice is horizontal and the depth of flow just upstream of the gate is 2.5 m.

(a) Show that the bed is stationary upstream of the gate.

(b) Determine the initial water depth just downstream of the gate. Show that the bed is mobile here.

(c) Assuming that the downstream water level is set by the gate and the discharge remains constant, find the final depth of scour and the final depths of flow upstream and downstream of the gate.

\[ q = \frac{Q}{b} = 0.9 \text{ m}^2/\text{s} \]
\[ d = 60 \text{mm} \left(0.06 \text{m}\right) \]
\[ p_s = 2650 \text{ kg/m}^3 \]
\[ \tau_{crit} = 0.056 \]
\[ c_f = 0.01 \]
\[ w_s = 1.1 \text{ m/s} \]
\( h_1 = 2.5 \text{ m} \)

a) Show bed upstream of gate is stationary.

Bed shear stress \( (\tau_b) \):

\[
\tau_{b1} = \frac{C_f}{2} \left( \frac{1}{2} \rho V_1^2 \right) = 0.01 \left( \frac{1}{2} \times 1000 \times 0.36^2 \right)
\]

\[
\tau_{b1} = 0.648 \text{ N/m}^2
\]

Critical Shields stress:

\[
\tau_{cr} = \frac{\tau_{bcrit}}{g(s-1)gd}
\]

\[
0.056 = \frac{\tau_{bcrit}}{1000 \times 1.65 \times 9.81 \times 0.06}
\]
\( T_{\text{crit}} = 54.39 \, \text{N/m}^2 \)

**Bed shear : critical shield stress**

0.648 \(< 54.39

Bed upstream of gate is stable (stationary)

b) \( h_2 \)?

Show that bed is mobile (Section 2)

Energy eq. 1 - 2

\[ z_1 = z_2 \]

\[ h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} \]

\[ h_1 + \frac{g^2}{2g h_1^2} = h_2 + \frac{g^2}{2g h_2^2} \]
\[ 2.5 + \frac{0.9^2}{2 \times 9.81 \times 2.5^2} = h_2 + \frac{0.9^2}{2 \times 9.81 \times h_2} \]

\[ h_2 = 0.1318 \]

Choose minimum positive water depth [supercritical flow solution]

\[ V_2 = \frac{q}{h_2} = \frac{0.9}{0.1318} = 6.83 \text{ m/s} \]

\[ \zeta b_2 = C_f \left( \frac{1}{2} \rho V_2^2 \right) \]

\[ \zeta b_2 = 0.01 \left( \frac{1}{2} \times 1000 \times 6.83^2 \right) \rightarrow \zeta b_2 = 233.2 \frac{N}{m^2} \]

\[ \zeta b_2 > \zeta \text{crit} \]

\[ 233.2 > 54.39 \]

- Bed right downstream of sluice gate is mobile
At equilibrium in (2)

\( \tau_{b2} = \tau_{b_{crit}} \)

\( \tau_{b2} = 54.39 \frac{N}{m^2} \)

\( \tau_{b2} = C_f \times \frac{1}{2} \rho V_2^2 \)

54.39 = 0.0 \times \frac{1}{2} \times 1000 V_2^2 \rightarrow V_2 = 3.298 m/s

\( V_2 = \frac{9}{h_2} \rightarrow h_2 = \frac{9}{3.298} \rightarrow h_2 = 0.273 m \)
* Height of scour:

\[ h_s = 0.273 - 0.1318 = 0.141 \text{ m} \]

* Froude number at (2):

\[ Fr_2 = \frac{V_2}{\sqrt{g h_2}} = \frac{3.298}{\sqrt{9.81 \times 0.273}} = 2.02 \text{ (supercritical flow)} \]

* \( h_1 \)

Apply new energy equation to find \( h_1 \) and \( V_1 \)

\[ Z_2 + h_1 + \frac{V_1^2}{2g} = Z_2 + h_2 + \frac{V_2^2}{2g} \]

\[ 0.141 + h_1 + \frac{g^2}{2gh_1^2} = 0 + 0.273 + \frac{g^2}{2gh_2^2} \]
\[
0.141 + h_1 + \frac{0.9^2}{2 \times 9.81 h_1^2} = 0.273 + \frac{0.9^2}{2 \times 9.81 (0.223^2)}
\]

\[
h_1 + \frac{0.9^2}{19.62 h_1^2} = 0.6861
\]

\[
h_1 = 0.549 \text{ m}
\]

Choose maximum positive value (subcritical solution)

\[
V_1 = \frac{0.9}{0.549} = 1.64 \text{ m/s}
\]

\[
F_{r_1} = \frac{1.64}{\sqrt{9.81 \times 0.549}}
\]

\[
F_{r_1} = 0.7
\]

*We assumed that section 1 does not have erosion. Let's verify this.*
\[ T_{b1} = C_f \times \frac{1}{2} PV_i^2 \]
\[ T_{b1} = 0.01 \times \frac{1}{2} \times 1000 \times 1.64^2 \]
\[ T_{b1} = 13.4 \, \frac{N}{m^2} \quad (< 54.39 \, \frac{N}{m^2}) \]

*No erosion at section 1, therefore our assumption is correct.*
Example (Adapted from Apsley, hydraulic notes):

A river of width 12 m and slope 0.003, carrying a maximum discharge of 200 m$^3$/s, is to be stabilized by using an armour layer of stones of density 2650 kg/m$^3$. Assuming a critical Shields parameter of 0.056, estimate the minimum size of stone that should be used and the corresponding river depth at maximum discharge.

\[ b = 12 \text{ m} \]
\[ Q = 200 \text{ m}^3/\text{s} \]
\[ d_{\text{min}} = ?? \]

\[ \rho_s = 2650 \text{ kg/m}^3 \]
\[ \tau^*_{\text{crit}} = 0.056 \]
\[ h = ? \text{ [max. discharge]} \]

\[ \tau^*_{\text{crit}} = \frac{\tau_b \text{ crit}}{\rho (s-1)g d} \]

* Bed shear stress for incipient motion.

For min. diameter
\[ \tau_b = \tau_b \text{ crit} \]
\[ 0.056 = \frac{T_{bcrit}}{1000 \times (1.65) \times 9.81 \times d} \]

\[ T_{bcrit} = 906.4 \text{~d} \]

*For normal flow*

\[ T_b = T_{bcrit} \]

\[ T_b = \frac{p_g R_h S}{n}, \quad Q = \frac{k}{n} A R S \]

\[ 906.4 \text{~d} = 1000 \times 9.81 \times \left( \frac{12h}{12+2h} \right) \times 0.003 \]

\[ \frac{1}{R} = \frac{12+2h}{2h} = 0.032469 \]

\[ h = \frac{30.8d}{1-5.133d} \]

Rearranging:

\[ R = \frac{A}{P} = \frac{bh}{b+2h} \]

\[ R = \frac{12h}{12+2h} \]
Manning's formula: \( Q = \frac{k}{n} A R^{2/3} S^{1/2} \)

From 1:
\( R = 30.8d \)

\[
200 = \frac{1}{n} (12h)(30.8d)^{2/3} \times 0.003
\]
\( n = \frac{d}{21.1} \)

In 3:
\[
200 = \frac{4198}{d^{3/2}} \frac{d^{1/6}}{1 - 5.133d}
\]

Solving by iteration:
\( d = 0.08802 \text{ m} \)
\( d_{\text{min}} = 88 \text{ mm} \approx 3.5 \text{ inches} \)
In (2) \[ h = \frac{30.8 \times 0.08802}{1 - 5.133 \times 0.08802} \]

\[ h = 4.95 \text{ m} \]

\[ Q = 200 \text{ m}^3/\text{s} \]

\[ d_{\text{min}} \approx 88 \text{ mm} \]

*Cobbles (88 mm) are in incipient motion*
Inception of Motion in Normal Flow

Assume:

**coarse sediment:** \[
\tau_b \left( \frac{(\rho_s - \rho)gd}{(\rho_s - \rho)gd} \right) > 0.056
\]

**normal flow:** \[
\tau_b = \rho g R_h S
\]

The bed will be mobile if \[
\frac{R_h S}{(\rho_s/\rho - 1)} > 0.056 d
\]

For sand (\(\rho_s/\rho = 2.65\)), the **bed will be mobile if** \(d < 10.8 R_h S\)
Bed Load

- **Bed load** consists of particles sliding, rolling or saltating, but remaining essentially in contact with the bed.

- It is the dominant form of sediment transport for larger particles (settling velocity too large for suspension).

- The **bed-load flux** $q_b$ is the volume of non-suspended sediment crossing unit width of bed per unit time.
Dimensionless Groups

\[ q^* = \frac{q_b}{\sqrt{(s-1)gd^3}} \]

dimensionless bed-load flux

\[ \tau^* = \frac{\tau_b}{\rho(s-1)gd} \]

dimensionless bed shear stress
(Shields parameter)

\[ d^* = d \left[ \frac{(s-1)g}{\nu^2} \right]^{1/3} \]

dimensionless particle diameter
# Bed-Load Formulas

<table>
<thead>
<tr>
<th>Reference</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meyer-Peter and Müller (1948)</td>
<td>( q^* = 8(\tau^* - \tau_{crit}^*)^{3/2} )</td>
</tr>
<tr>
<td>Nielsen (1992)</td>
<td>( q^* = 12(\tau^* - \tau_{crit}^<em>)\sqrt{\tau^</em>} )</td>
</tr>
<tr>
<td>Van Rijn (1984)</td>
<td>( q^* = \frac{0.053}{d^{<em>0.3}} \left( \frac{\tau^</em>}{\tau_{crit}^*} - 1 \right)^{2.1} )</td>
</tr>
<tr>
<td>Einstein-Brown (Brown, 1950)</td>
<td>( q^* = \begin{cases} \frac{Ke^{-0.391}}{0.465 \tau^<em>} &amp; \text{if } \tau^</em> &lt; 0.182 \ 40K \tau^* &amp; \text{if } \tau^* \geq 0.182 \end{cases} )</td>
</tr>
<tr>
<td></td>
<td>( K = \sqrt{\frac{2}{3} + \frac{36}{d^{*3}}} - \sqrt{\frac{36}{d^{*3}}} )</td>
</tr>
<tr>
<td>Yalin (1963)</td>
<td>( q^* = 0.635r\sqrt{\tau^*} \left[ 1 - \frac{1}{\sigma r} \ln(1 + \sigma r) \right] )</td>
</tr>
<tr>
<td></td>
<td>( r = \frac{\tau^<em>}{\tau_{crit}^</em>} - 1, \sigma = 2.45 \frac{\sqrt{\tau_{crit}^*}}{S^{0.4}} )</td>
</tr>
</tbody>
</table>

Calculating Bed Load

\[ q^* = f(\tau^*, d^*) \]

\[ \tau^* = \frac{\tau_b}{\rho(s-1)gd} \]

\[ d^* = d \left[ \frac{(s-1)g}{v^2} \right]^{1/3} \]

**To find bed-load flux:**
- from particle and fluid properties, find \( d^* \)
- from formula or graph, find \( \tau^*_{\text{crit}} \)
- from flow hydraulics, find \( \tau_b \) and hence \( \tau^* \)
- if \( \tau^* > \tau^*_{\text{crit}} \) (or \( \tau > \tau_{\text{crit}} \)), find \( q^* \) by chosen model
- Find bed-load flux per unit width, \( q_b \)
- Multiply by channel width to get bed-load flux, \( Q_b \)
Suspended Load

- **Suspended load** consists of **finer particles carried in suspension** by turbulent fluid flow.

- Significant **suspended load** occurs if **turbulent velocity fluctuations are larger than the settling velocity**. A typical turbulent velocity fluctuation is of the order of the friction velocity \( u_\tau \). Thus, **suspended load will occur if**

\[
\frac{u_\tau}{w_s} > 1
\]

- For **coarser sediment**, suspended load does not occur and all **sediment motion is bed load**.
In practice, the separation between what constitutes bed load and suspended load is fuzzy.
Concentration

Sediment Concentration $C$ is the volume of sediment per total volume of material (fluid + sediment)

- Sediment settles, so concentrations are larger near the bed.
- Upward-moving eddies tend to carry more sediment than downward-moving ones.
- This leads to a net upward diffusion of material.
- Equilibrium when downward settling = upward diffusion.
Concentration Profile

Rouse profile:

\[
\frac{C}{C_{ref}} = \left( \frac{h/z - 1}{h/z_{ref} - 1} \right)^{\frac{w_s}{\kappa u_\tau}}
\]

Rouse number: \( \frac{w_s}{\kappa u_\tau} \)

\( w_s \) = settling velocity of the particle
\( u_\tau \) = friction velocity of the flow, \( u_\tau = \sqrt{\tau_b / \rho} \)
\( \kappa \) = von Kármán’s constant (≈0.41)
Calculation of Suspended Load

Volume flow rate of water: \( u \, dA \)
per unit span: \( u \, dz \) (through depth \( dz \))

Volume flux of sediment = concentration \( \times \) volume flux of water
= \( Cu \, dz \)

Suspended load:

\[
q_s = \int_{z_{ref}}^{h} Cu(z) \, dz
\]

\[
u(z) = \frac{u_x}{K} \ln(33 \frac{z}{k_s})
\]

\[
\frac{C}{C_{ref}} = \left( \frac{h/z-1}{h/z_{ref}-1} \right) \frac{w_s}{Ku_x}
\]
Calculation of Suspended Load (cont.)

It is necessary to specify $C_{ref}$ at some depth $z_{ref}$, typically at a height representative of the bed load. The formula of Van Rijn (see, Chanson 2004):

\[
C_{ref} = \min \left[ \frac{0.117}{d^*} \left( \frac{\tau^*}{\tau_{crit}^*} - 1 \right), 0.65 \right]
\]

\[
\frac{z_{ref}}{d} = 0.3d^{*0.7} \left( \frac{\tau^*}{\tau_{crit}^*} - 1 \right)^{1/2}
\]

Example (Adapted from Apsley, hydraulic notes):

A wide channel of slope 1:800 has a gravel bed with \( d_{50} = 3 \text{ mm} \). The discharge is 4 \( \text{ m}^3/\text{s} \) per meter width. The density of the gravel is 2650 \( \text{ kg/m}^3 \).

(a) Estimate Manning’s \( n \) using Strickler’s formula.

(b) Find the depth of flow; (assume normal flow).

(c) Find the bed shear stress.

(d) Show that the bed is mobile and calculate the bed-load flux (per meter width) using (i) Meyer-Peter and Müller; (ii) Van Rijn formulas.

(e) Determine whether suspended load occurs.

\[
\begin{align*}
\text{Wide channel (} b \gg h \text{)} \\
S &= \frac{1}{800} = 0.00125 \\
d_{50} &= 3 \text{ mm} = \frac{3}{1000} \text{ m} \\
q &= 4 \frac{\text{ m}^2}{\text{s}} \\
R &= \frac{A}{P} = \frac{bh}{b+2h} \\
R &\approx \frac{bh}{b} = h
\end{align*}
\]
\( S_s = 2650 \text{ kg/m}^3 \)

a) Manning's \( n \) (Strickler's formula)

\[
 n = \frac{d^{1/6}}{21.1^{1/3}} = 0.018 \text{ m/s}
\]

b) Flow depth (normal flow)

\[
 Q = \frac{K}{n} AR S^{2/3} R^{1/2}
\]

\[
 q/b = \frac{1}{(h/h_s)^{2/3} S^{1/2}}
\]

\[
 4 = \frac{1}{0.018} h \times 0.00125^{1/2}
\]
c) Bed shear stress ($C_b$)

$$C_b = \rho g \rho h S = 1000 \times 9.81 \times 1.532 \times 0.00125$$

$$C_b = 18.79 \text{ N/m}^2$$

d) Show bed is mobile?

Bed load flux using Meyer-Peter, Van Rijn.

Bed will be mobile if $C_b > C_{b, crit}$

By Soulsby:

$$C_{b, crit} = \frac{0.30}{1 + 1.2d^*} + 0.055 \left( 1 - e^{-0.020d^*} \right)$$
\[ d^* = d \left[ \frac{(S-1)g}{\gamma^2} \right]^{1/3} = \frac{3}{1000} \left[ \frac{1.65 \times 9.81}{(10^{-6})^2} \right]^{1/3} \]

\[ d^* = 75.89 \]

\[ \text{In (1)} \]

\[ \gamma^* = \frac{\gamma \text{crit}}{g(S-1)g \ell} \]

\[ 0.0462 = \frac{\gamma \text{crit}}{1000(1.65) \times 9.81 \times \frac{3}{1000}} \]

\[ \gamma \text{crit} = 2.24 \text{ N/m}^2 \]

\[ \frac{\gamma}{18.79} > \frac{\gamma \text{crit}}{2.24} \quad \text{[bed is mobile]} \]
\( q^* = 8 (z^* - z_{\text{crit}})^{3/2} \)

\( z^* = \frac{2b}{g(5-1)gd} = \frac{18.79}{1000(1.65 \times 9.81 \times \frac{3}{1000}} \)

\( z^* = 0.3869 \)

\( q^* = 8 (0.3869 - 0.0462)^{3/2} = 1.591 \)

\[ q^* = \frac{q_b}{\sqrt{(5-1)gd^3}} \]

\[ q_b = q^* \sqrt{(5-1)gd^3} \]

\[ q_b = 1.591 \sqrt{1.65 \times 9.81 \times \left(\frac{3}{1000}\right)^3} \]
\[ q_b = 1.052 \times 10^{-3} \frac{m^2}{S} \quad \text{(per meter width)} \]

For a river of 100 m in one day:

\[ \text{Vol. sediments: } 100 q_b \times 24 \times 3600 S \]

\[ \text{Vol. (1 day)} \approx 9000 \text{ m}^3 \]

* Bed load with Van Rijn

\[ q^* = \frac{0.053}{(d^*)^{0.3}} \left( \frac{Z^*}{Z_{crit}^*} - 1 \right)^{2.1} \]

\[ q^* = 0.9604 \]

\[ q_b = q^* \sqrt{(s-1)g} \approx 6.35 \times 10^{-4} \text{ m}^2/\text{S} \]
\( e \) Suspended load will occur if \( \frac{U_2}{W_s} > 1 \)

\* Fall velocity \( W_s \)

Cheng's formula for \( W_s \)

\[
\frac{W_s d}{\gamma} = \left[ \left(25 + 1.2 \left( \frac{d}{1000} \right)^2 \right)^{1/2} - 5 \right]^{3/2}
\]

\[
\frac{W_s (3/1000)}{10^{-6}} = \left[ \left(25 + 1.2 \times 75.89^2 \right)^{1/2} - 5 \right]^{3/2}
\]

\( W_s = 0.2309 \text{ m/s} \)

\* \( U_2 \)

\[
U_2 = \sqrt{\frac{2b}{\phi}} = \sqrt{\frac{18.79}{1000}} = 0.137 \text{ m/s}
\]

\[
\frac{U_2}{W_s} = \frac{0.137}{0.2309} = 0.59
\]
No significant suspended load will occur.
Example (Adapted from Apsley, hydraulic notes):

The vertical profile of mean velocity $U$ in a rough-walled turbulent flow is:

$$U(z) = \frac{u_\tau}{\kappa} \ln\left(33 \frac{z}{k_s}\right)$$

Use numerical integration to calculate the suspended-load sediment flux in a channel using the following data:

channel width: $b = 5$ m;
flow depth: $h = 1.5$ m;
friction velocity: $u_\tau = 0.2$ m/s;
settling velocity: $w_s = 0.03$ m/s;
roughness length: $k_s = 0.001$ m;
reference concentration: $c_{ref} = 0.65$;
reference height: $z_{ref} = 0.001$ m.

$k_s$ is a universal constant with value 0.41.
\[ q_s = \int_{z_{ref}}^{h} C u(z) \, dz \quad , \quad Q_s = b \cdot q_s \]

\[ Q_s = 5 \int_{0.001}^{1.5} C_{ref} \left[ \frac{h/\bar{z} - 1}{h/\bar{z}_{ref} - 1} \right] \frac{W_s}{kU_e} \cdot \frac{\bar{u}_z}{k} \ln \left( \frac{33 \, Z}{k_s} \right) \, dz \]

We can integrate this using many methods, we can also use MATLAB or other tools.

\[ \int_{a}^{b} f(x) \, dx \]
\[ \Delta x = \frac{b-a}{N} \]

\[ N = 3 \]

\[ \Delta x = \frac{1.5 - 0.001}{3} \]

\[ (\Delta z) = \frac{3}{3} \]

Area = \left[ \frac{f(1) + f(2)}{2} + \frac{f(2) + f(3)}{2} + \frac{f(3) + f(4)}{2} \right] \Delta x

In general: \[ \text{Area} = \frac{\Delta x}{2} \left[ f(x_1) + f(x_N) + 2 \sum_{k=2}^{N-1} f(x_k) \right] \]
<table>
<thead>
<tr>
<th>( z )</th>
<th>( f )</th>
<th>( \Delta f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>1.1089</td>
<td>0.1087</td>
</tr>
<tr>
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<tr>
<td>1.50</td>
<td>0.0000</td>
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</tr>
</tbody>
</table>

\[ \Delta x = 0.4997 \]

\[ Q_s = 5 \times \frac{0.4997}{2} \times \left[ \frac{1.1089 + 0.000 + 0.2731 + 0.1762}{2} \right] \]

\[ Q_s = 2.508 \text{ m}^3/\text{s} \]