Introduction to Sediment Transport in Open Channel Flows

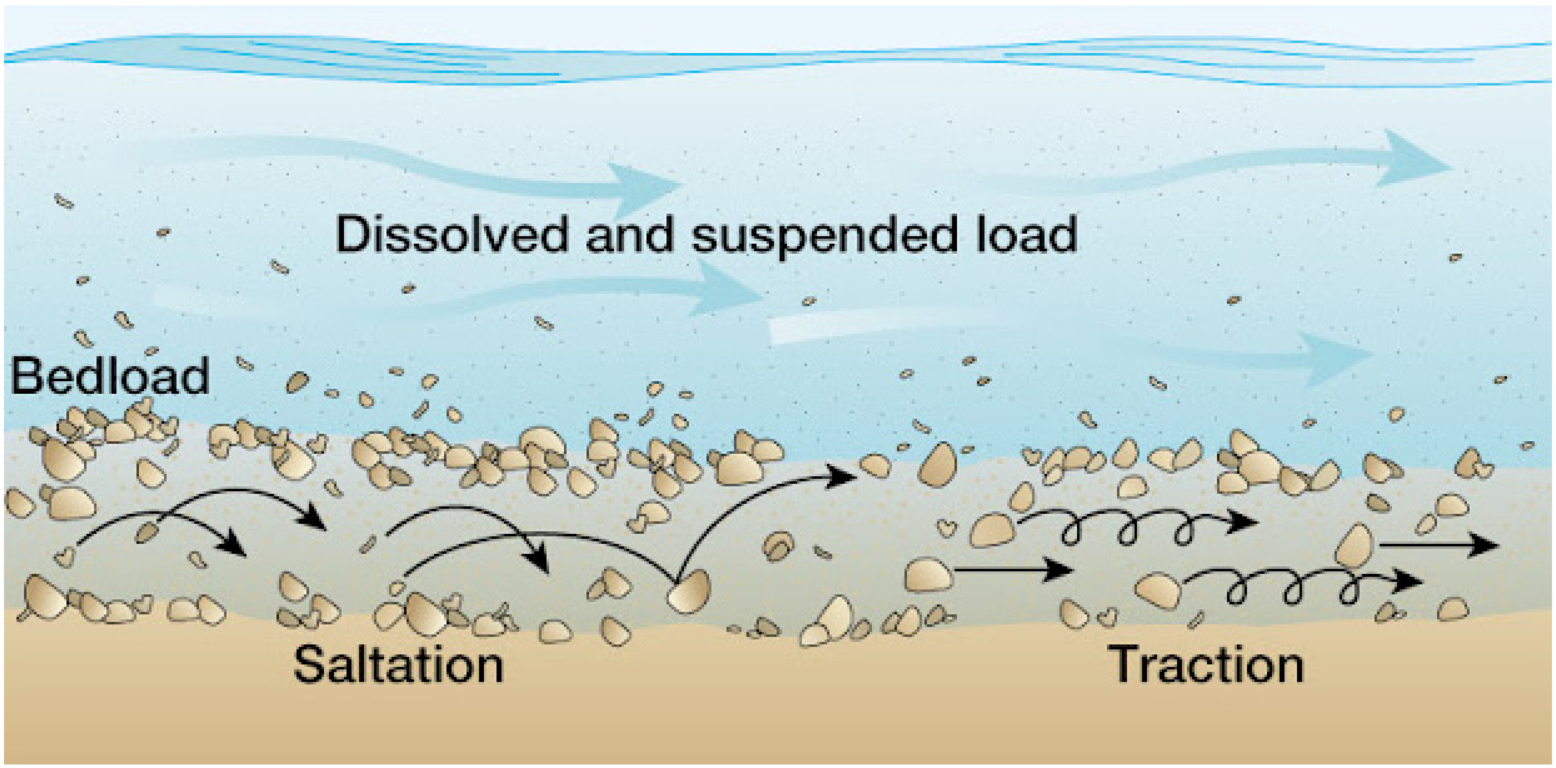

Image source: http://earthsci.org/processes/geopro/stream/stream.html

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Movies

Scour at bridge model pier

https://www.youtube.com/watch?v=48S_k6qAmsY&feature=emb_logo

Utah Flash Flood

https://www.youtube.com/watch?v=mXIr_Bgb-s0

Santa Clara Pueblo Flash Flood

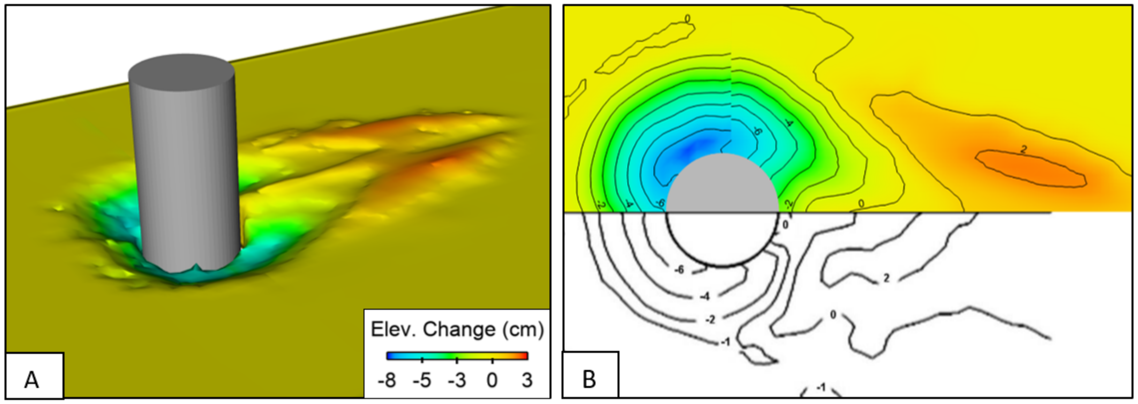
https://www.youtube.com/watch?v=nKOQzkRi4BQ

Sediment Transport

- We study sediment transport to predict the **risks of scouring of bridges**, to estimate the **siltation** of a reservoir, etc.
- Most, if not all, natural channels have **mobile** beds
- Most mobile beds are in **dynamic equilibrium**:

  on average: sediment in = sediment out

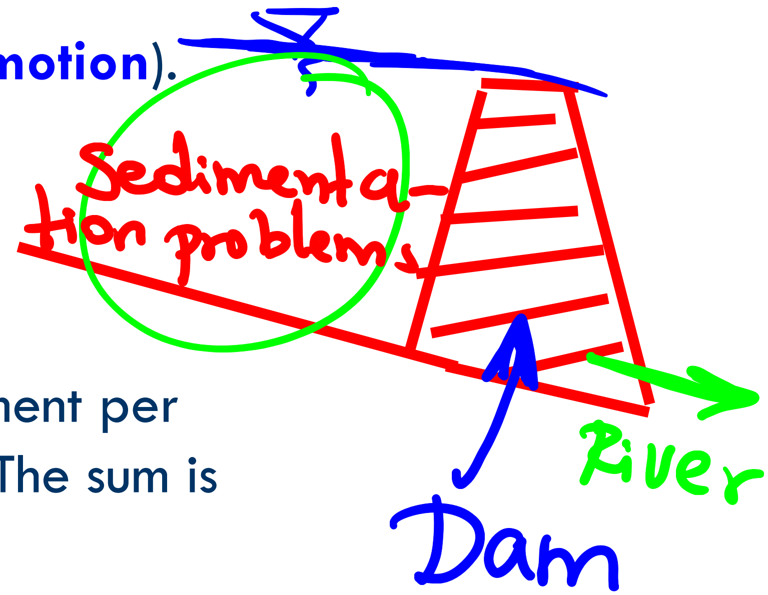
- This **dynamic** equilibrium can be **disturbed by**
  - short-term extreme events (e.g., flash floods)
  - man-made infrastructure (e.g., dams)



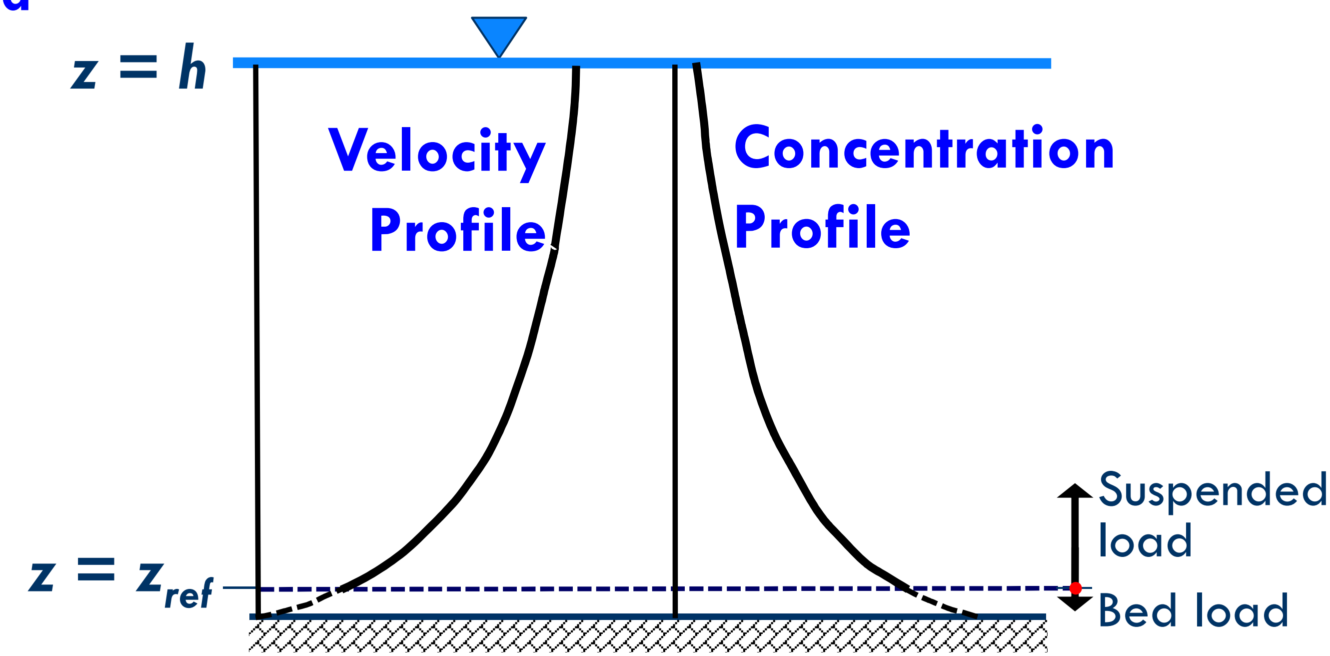
Source: https://www.flow3d.com/modeling-capabilities/sediment-transport-model/

Sediment Transport Fundamental Questions:

- Does sediment transport occur? (**Threshold of motion**).
- If so, then at what rate? (**Sediment load**)
- What net effect does it have on the bed? (**Scour/accretion**)



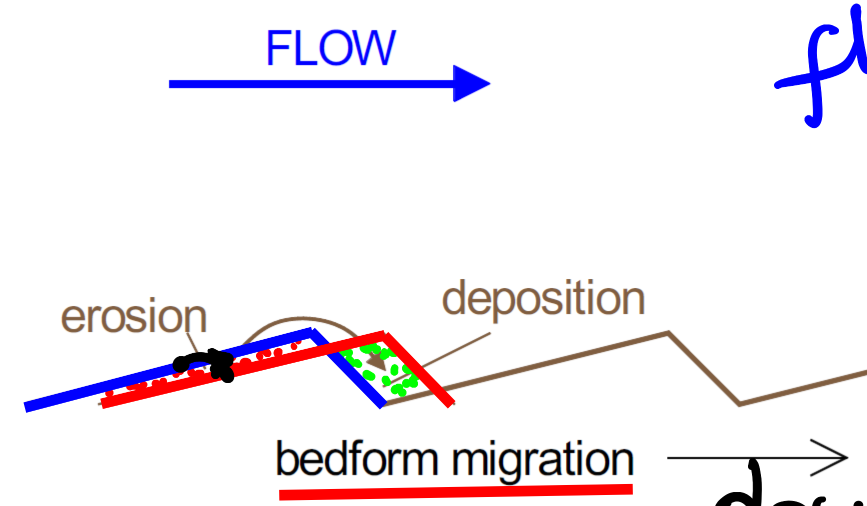
The main types of sediment load (volume of sediment per unit of time) are **Bed load and Suspended load**. The sum is **total load**



Bed form motion:

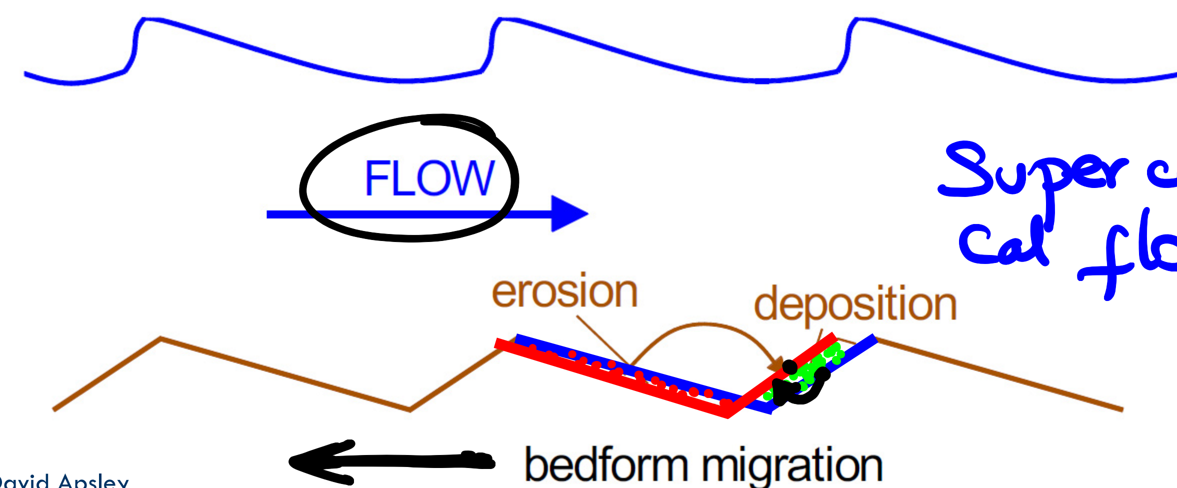
Subcritical flow

Dune



downstream

Antidune

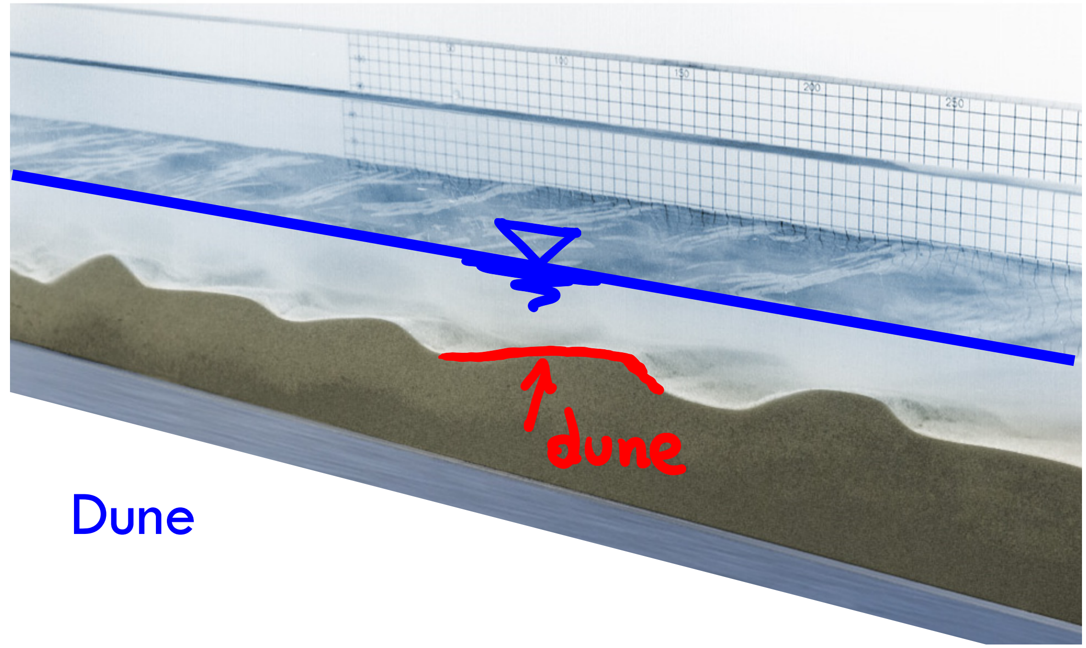


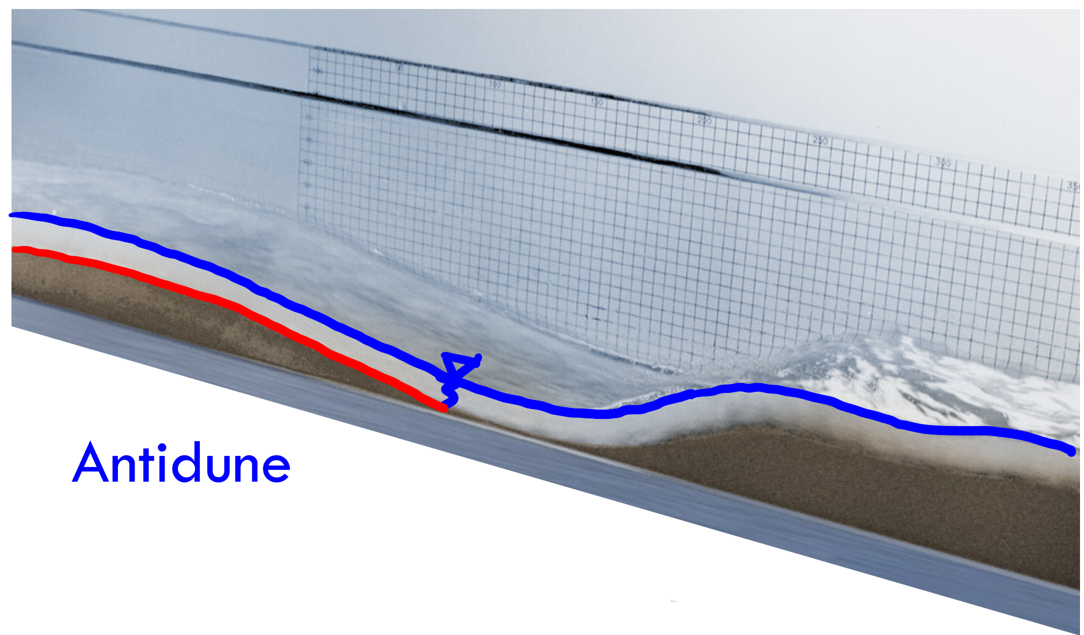
Supercritical flow

upstream

Source: Lecture notes on Hydraulic, David Apsley

Bed form motion (Cont.)





Source: https://armfield.co.uk/product/s8-mkii-sediment-transport-demonstration-channel/

Relevant Properties

$\rho_{[Quartz]} = 2{,}650 \frac{kg}{m^3}$

- **Particle**

  - Diameter, d
  - Specific gravity, $s = \rho_s/\rho$
  - Settling velocity, w_s
  - Porosity, P
  - Angle of repose, ϕ

- **Fluid**

  - Density, ρ
  - Kinematic viscosity, ν

$\nu_{(20^\circ c)} = 10^{-6}\, m^2/s$

- **Flow**

  - Bed shear stress, τ_b
  - Mean-velocity profile, $U(z)$
  - Eddy-viscosity profile, $\nu_t(z)$

$\rho_s[sand] = 2{,}650 \frac{kg}{m^3}$

$\rho[water] = 1{,}000\ kg/m^3$

$\frac{\rho_s}{\rho} = 2.65$

Inception of Motion

- Inception and magnitude of **bed-load** depends on:
  - bed shear stress τ_b
  - particle diameter d and specific gravity s

- Inception of **suspended load** depends on ratio of:
  - settling velocity w_s
  - typical turbulent velocity (friction velocity u_τ)

Particle Properties: Diameter *d*

Various types:

- Sieve diameter
- Sedimentation diameter
- Nominal diameter

Type	Diameter
Boulders	**> 256 mm**
Cobbles	64 mm – 256 mm
Gravel	2 mm – 64 mm
Sand	0.06 mm – 2 mm
Silt	0.002 mm – 0.06 mm
Clay	**< 0.002 mm** (cohesive)

In practice, there is a range of diameters (typically, lognormally distributed).

Particle Properties: Specific Gravity s

$$s = \frac{\rho_s}{\rho}$$

Quartz-like: $\rho_s \approx 2650\ \mathrm{kg/m^3}$, $s \approx 2.65$

Anthracite: $\rho_s \approx 1500\ \mathrm{kg/m^3}$, $s \approx 1.50$

Particle Properties: Porosity P

Porosity = fraction of voids (by volume)

Typical uncompacted sediment: $P \approx 0.4$.

Particle Properties: Settling Velocity w_s

Terminal velocity in still fluid.

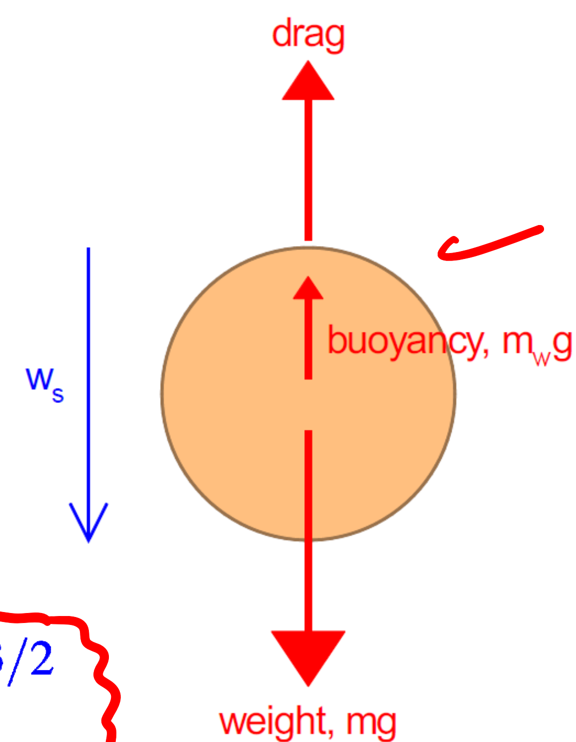
drag = weight – buoyancy

Small particles (**Stokes' Law**):

$$\boxed{w_s = \frac{1}{18}\frac{(s-1)gd^2}{\nu}}$$ (Spheres)

$$\frac{w_s d}{\nu} = \frac{1}{18}\frac{(s-1)gd^3}{\nu^2} = \frac{1}{18}d^{*3}$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$



Realistic sizes and shapes:

Cheng's formula: $$\boxed{\frac{w_s d}{\nu} = \left[\left(25 + 1.2 d^{*2}\right)^{\frac{1}{2}} - 5\right]^{3/2}}$$

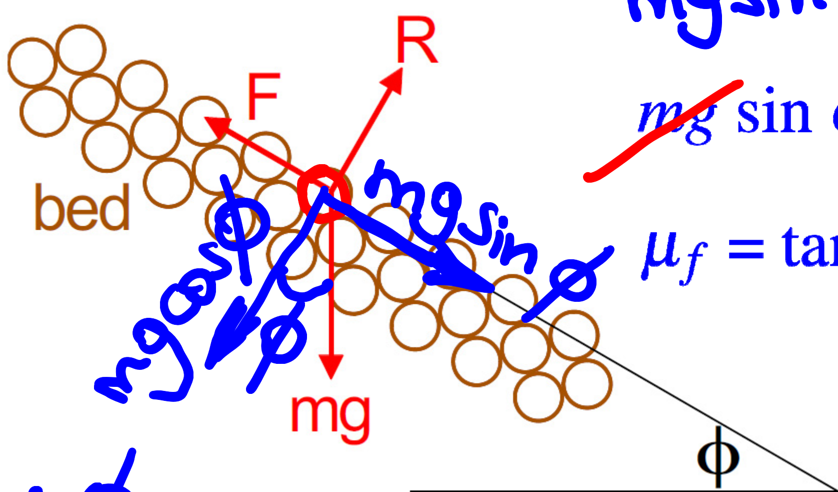
Use this formula in practice

Source: Cheng, N. S. (1997). "A simplified settling velocity formula for sediment particle." Journal of Hydraulic Engineering, ASCE, 123(2), 149-152

Particle Properties: Angle of Repose ϕ

Angle of repose ϕ = limiting angle of slope (in still fluid)

$$mg\sin\phi = \mu_f R$$



$$\cancel{mg}\sin\phi = \mu_f(\cancel{mg}\cos\phi)$$

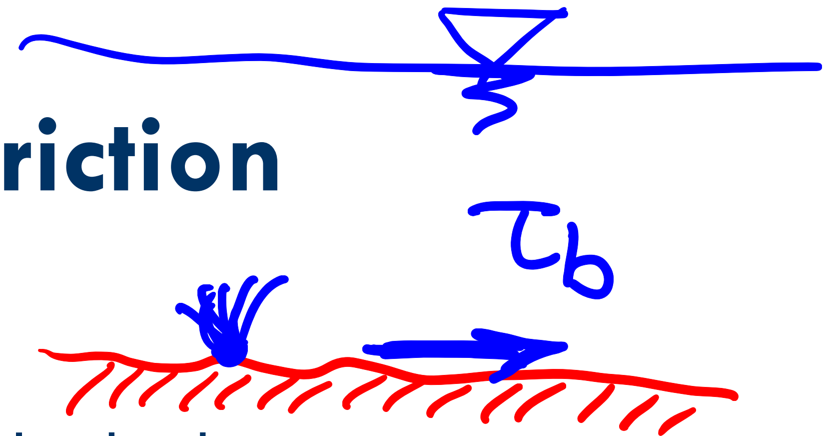
$$\mu_f = \tan\phi$$

$$R = mg\cos\phi$$

μ_f = Effective **coefficient of friction**

Can be used to estimate the effect of slopes on incipient motion

Flow Properties: Bed Friction



Bed shear stress τ_b

- Drag (per unit area) of flow on granular bed.
- Determines inception and magnitude of **bed load**.

Friction velocity $\cancel{\mu_t}$ u_τ

- Defined (on dimensional grounds) by:

friction velocity.

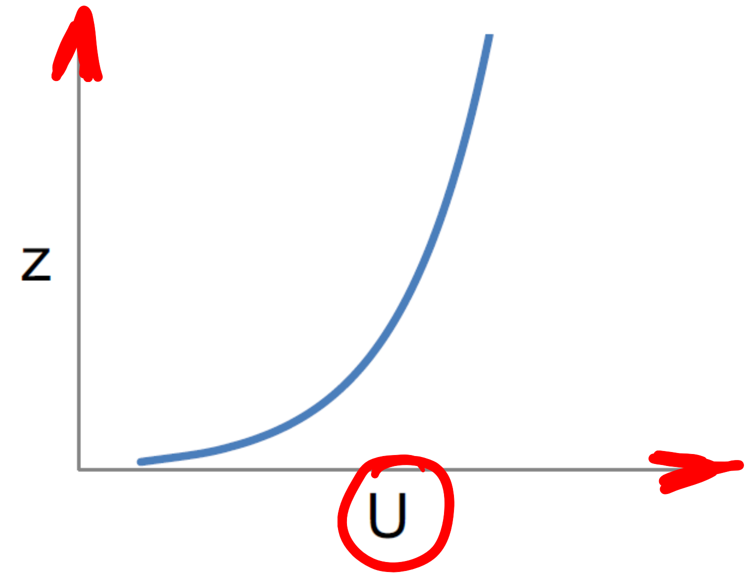
$$\boxed{\tau_b = \rho u_\tau^2} \text{ or } u_\tau = \sqrt{\tau_b/\rho}$$

- Determines inception and magnitude of **suspended load**.

Flow Properties: Mean-Velocity Profile

For a **rough** boundary:

$$U(z) = \frac{u_\tau}{\kappa} \ln\left(33\frac{z}{k_s}\right)$$



u_τ = friction velocity;

κ = von Kármán's constant (≈ 0.41);

z = distance from the bed;

k_s = roughness height (1.0 - 2.5 times particle diameter).

Flow Properties: Eddy-Viscosity Profile

A model for the effective shear stress τ in a turbulent flow:

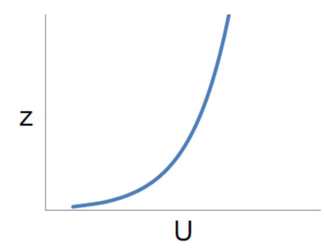
dynamic viscosity. kinematic viscosity.

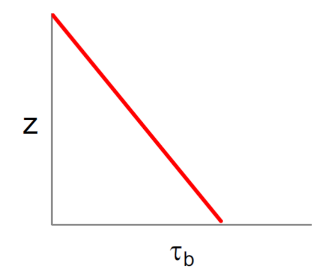
$$\tau = \mu_t \frac{dU}{dz} \quad \text{or} \quad \tau = \rho \nu_t \frac{dU}{dz}$$

μ_t and ν_t are the dynamic and kinematic **eddy viscosities**, respectiv.

At the bed ($z = 0$): $\tau = \tau_b \equiv \rho {u_\tau}^2$

At the free surface ($z = h$): $\tau = 0$

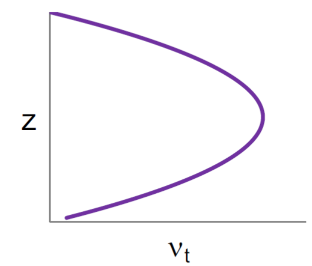




Assuming linear: $\tau = \rho u_\tau^2 (1 - z/h)$

From stress and mean-velocity profiles: $\rho u_\tau^2 (1 - \frac{z}{h}) = \rho \nu_t \frac{\mu_\tau}{\kappa z}$

$$\nu_t = \kappa u_\tau z (1 - \tfrac{z}{h})$$



Formulas For Bed Shear Stress

Normal flow: $\tau_b = \rho g R_h S$

Manning's formula: $V = \frac{k}{n} R_h^{2/3} S^{1/2}$

Strickler's formula: $n = \frac{d^{1/6}}{21.1}$

R_h = hydraulic radius in m
S = bed slope
d = is the particle diameter in m

Typical values: $n \approx 0.01$ to 0.035 m$^{-1/3}$s

Via a **friction coefficient**: $\tau_b = c_f \left(\frac{1}{2}\rho V^2\right)$

Fully-developed boundary layer (log-law): $c_f = \frac{0.34}{\left[\ln\left(\frac{12h}{k_s}\right)\right]^2}$

Typical values of **friction coefficient** $c_f \approx 0.003$ to 0.01

Shields

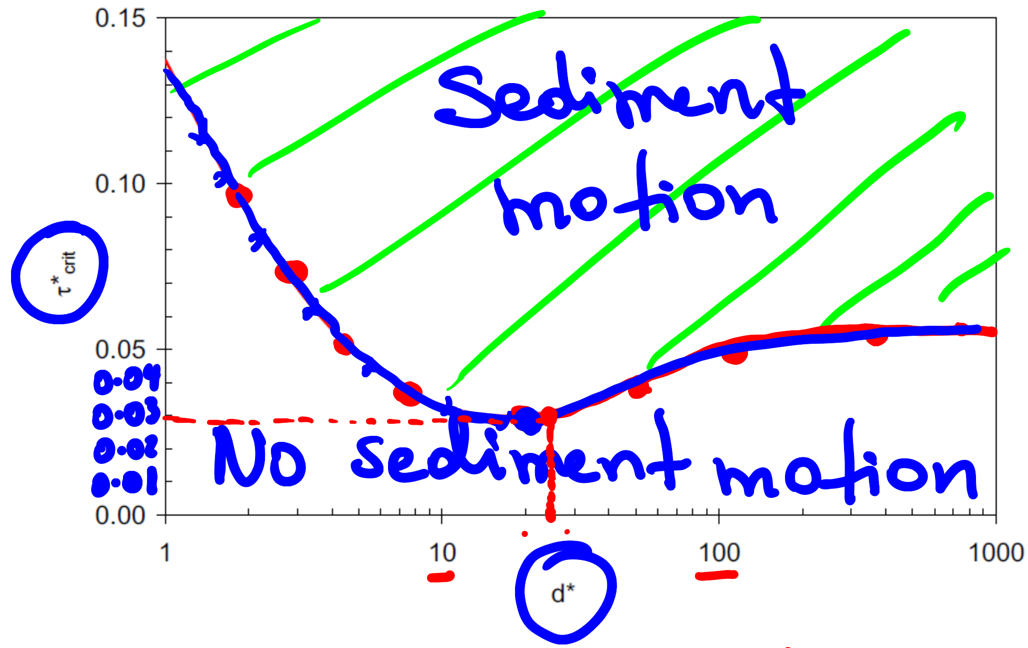
Finding the Threshold of Motion

A mobile bed **starts to move** once the bed stress exceeds a **critical stress** τ_{crit}.

$$\tau^*_{crit} = f(d^*)$$

$$\boxed{\tau^* = \frac{\tau_b}{\rho(s-1)gd}}$$

$$\boxed{d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}}$$



(*) = dimensionless

Curve fit (Soulsby, 1997):

$$\tau^*_{crit} = \frac{0.30}{1+1.2d^*} + 0.055\left(1 - e^{-0.020d^*}\right)$$

Soulsby, R., 1997, "Dynamics of Marine Sands", Thomas Telford.

Example (Adapted from Apsley, hydraulic notes):

Find the critical Shields parameter τ^*_{crit} and critical absolute stress τ_{crit} for a sand particle of diameter 1 mm in water.

τ^*_{crit} ?

τ_{crit} ?

$d = 1\,mm$

For a sand particle $s = \frac{\rho_s}{\rho} = \frac{2650}{1000}$

$S = 2.65$

* τ^*_{crit} (Soulsby formula)

$$\tau^*_{crit} = \frac{0.30}{1+1.2d^*} + 0.055\left(1 - e^{-0.020d^*}\right) \quad \ldots (1)$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

$\nu = 10^{-6}\ m^2/s$

$$d^* = \frac{1}{1000}\left[\frac{1.65 \times 9.81}{(10^{-6})^2}\right]^{1/3} = 25.3$$

In 1

$$\tau^*_{crit} = \frac{0.30}{1 + 1.2 \times 25.3} + 0.055\left(1 - e^{-0.02 \times 25.3}\right)$$

$$\boxed{\tau^*_{crit} = 0.03141}$$

Critical Shieds Parameter

* τ_{crit}

$$\tau^*_{crit} = \frac{\tau b_{crit}}{\rho(s-1)gd}$$

$$\tau b_{crit} = \tau^*_{crit}\, \rho(s-1)gd$$

$$\tau b_{crit} = 0.03141 * 1000(1.65) * 9.81 * \frac{1}{1000}$$

$$\boxed{\tau b_{crit} = 0.508 \ \frac{N}{m^2}}$$

Example (Adapted from Apsley, hydraulic notes):

A sluice gate is lowered into a wide channel carrying a discharge of 0.9 m^3/s per meter width. The bed of the channel is coarse gravel with particle diameter 60 mm and density 2650 kg/m^3. The critical Shields parameter is 0.056 and the bed friction coefficient is 0.01. The particles of gravel have settling velocity 1.1 m/s. Initially the bed of the channel under the sluice is horizontal and the depth of flow just upstream of the gate is 2.5 m.

(a) Show that the bed is stationary upstream of the gate.

(b) Determine the initial water depth just downstream of the gate. Show that the bed is mobile here.

(c) Assuming that the downstream water level is set by the gate and the discharge remains constant, find the final depth of scour and the final depths of flow upstream and downstream of the gate.

$$q = \left(\frac{Q}{b}\right) = 0.9\ \frac{m^2}{s}$$

$$d = 60mm\ (0.06m)$$

$$\rho_s = 2650\ \frac{kg}{m^3}$$

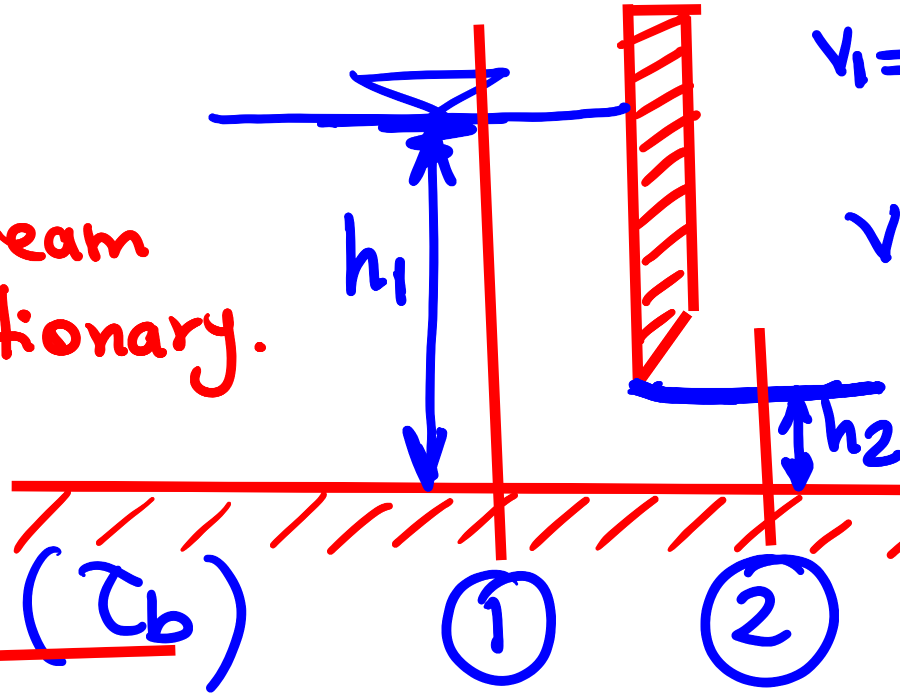
$$\tau^*_{crit} = 0.056$$

$$c_f = 0.01$$

$$w_s = 1.1\ m/s$$

$h_1 = 2.5\,m$

a) show bed upstream of gate is stationary.



$V_1 = \frac{Q}{A} = \frac{q \times \cancel{b_1}}{h_1 \cancel{b_1}}$

$V_1 = \frac{q}{h_1} = \frac{0.9}{2.5}$

$V_1 = 0.36\,\frac{m}{s}$

Bed shear stress (τ_b)

$$\tau_{b_1} = c_f\left(\frac{1}{2}\rho V_1^2\right) = 0.01\left(\frac{1}{2} \times 1000 \times 0.36^2\right)$$

$$\boxed{\tau_{b_1} = 0.648\ \frac{N}{m^2}}$$

Critical Shields stress

$$\tau^*_{crit} = \frac{\tau_{b\,crit}}{\rho(s-1)gd} \rightarrow 0.056 = \frac{\tau_{b\,crit}}{1000(1.65)9.81 \times 0.06}$$

$$\boxed{\tau_{b_{crit}} = 54.39 \ N/m^2}$$

Bed shear   critical shield stress

$$0.648 < 54.39$$

Bed upstream of gate
is stable (stationary)

b) h_2 ?

show that bed is mobile (section 2)

Energy eq. 1 - 2 $z_1 = z_2$

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g}$$

$$h_1 + \frac{q^2}{2gh_1^2} = h_2 + \frac{q^2}{2gh_2^2}$$

$$2.5 + \frac{0.9^2}{2\times 9.81\times 2.5^2} = h_2 + \frac{0.9^2}{2\times 9.81\times h_2^2}$$

$$\boxed{h_2 = 0.1318}$$

choose minimum positive water depth [supercritical flow solution]

$$V_2 = \frac{q}{h_2} = \frac{0.9}{0.1318} = 6.83 \text{ m/s}$$

$$\tau_{b_2} = c_f \left(\frac{1}{2}\rho V_2^2\right)$$

$$\tau_{b_2} = 0.01 \left(\frac{1}{2}\times 1000 \times 6.83^2\right) \rightarrow \tau_{b_2} = 233.2 \frac{N}{m^2}$$

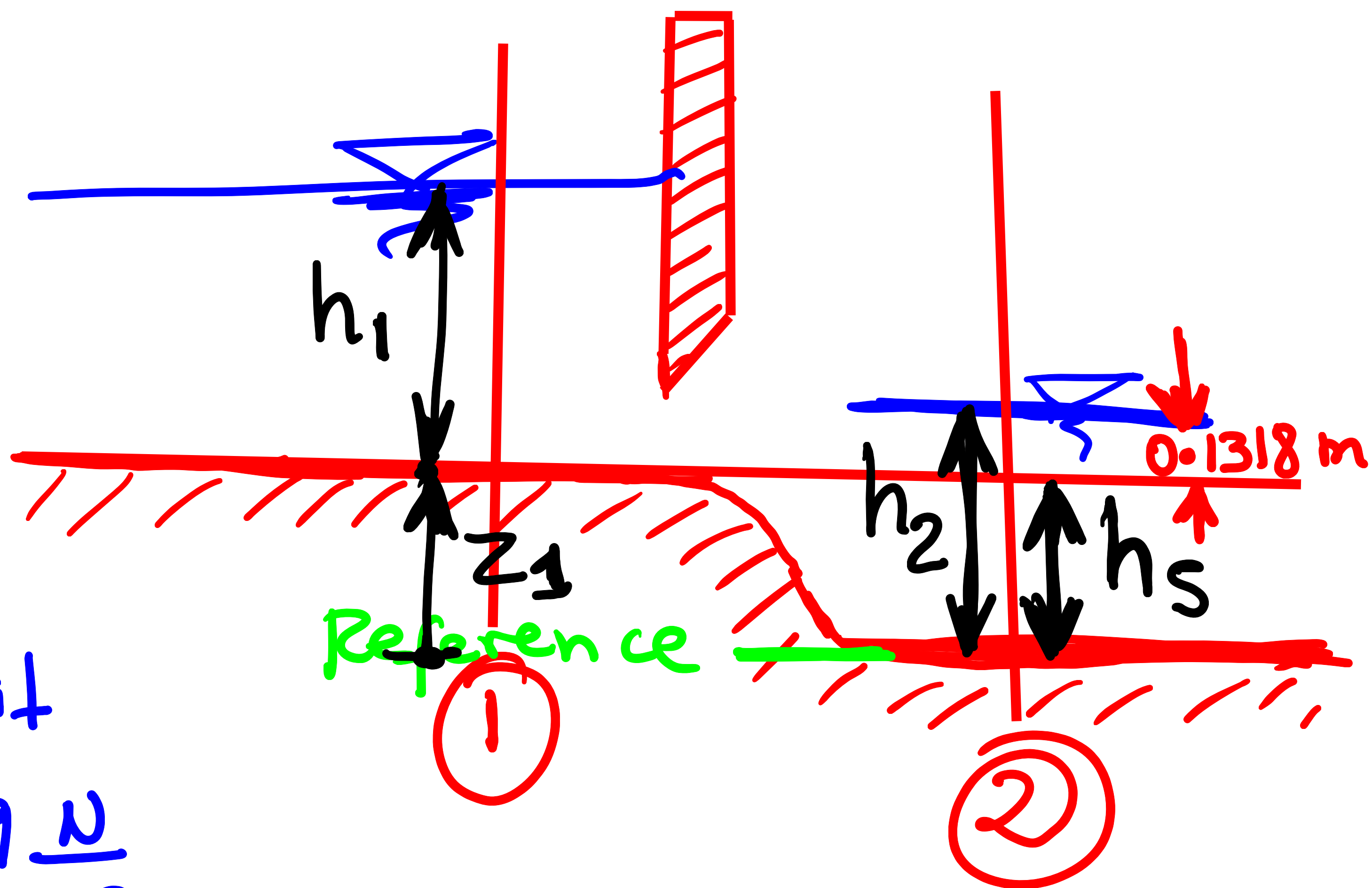
$$\tau_{b_2} > \tau_{crit}$$

$$233.2 > 54.39$$

$\therefore$ Bed right downstream of sluice gate is <u>mobile</u>

c) h_s ?

h_1, h_2 ?



At equilibrium in 2

$$\tau_{b_2} = \tau_{b\,crit}$$

$$\tau_{b_2} = 54.39\ \frac{N}{m^2}$$

$$\tau_{b_2} = C_f \times \frac{1}{2} \rho V_2^2$$

$$54.39 = 0.0 \times \frac{1}{2} \times 1000\, V_2^2 \rightarrow V_2 = 3.298\ m/s$$

$$V_2 = \frac{q}{h_2} \rightarrow h_2 = \frac{0.9}{3.298} \rightarrow \boxed{h_2 = 0.273\ m}$$

* height of scour :

$$h_s = 0.273 - 0.1318 = 0.141\ m$$

* Froude number at 2

$$Fr_2 = \frac{V_2}{\sqrt{g y_2}} = \frac{3.298}{\sqrt{9.81 \times 0.273}} = 2.02$$

(supercritical flow)

* h_1

Apply new energy equation to find h_1 and V_1

$$\underbrace{Z_1}_{h_s} + h_1 + \frac{V_1^2}{2g} = Z_2 + h_2 + \frac{V_2^2}{2g}$$

$$0.141 + h_1 + \frac{q^2}{2g h_1^2} = 0 + 0.273 + \frac{q^2}{2g h_2^2}$$

$$0.141 + h_1 + \frac{0.9^2}{2\times 9.81 h_1^2} = 0.273 + \frac{0.9^2}{2\times 9.81\,(0.273^2)}$$

$$h_1 + \frac{0.9^2}{19.62\, h_1^2} = 0.6861$$

$$\boxed{h_1 = 0.549\,m}$$

choose maximum positive value (subcritical solution)

$$V_1 = \frac{0.9}{0.549} = 1.64\ m/s$$

$$Fr_1 = \frac{1.64}{\sqrt{9.81\times 0.549}}$$

$$Fr_1 = 0.7$$

* We assumed that section 1 doesn't have erosion. Let's veryfy this.

$$\tau_{b_1} = c_f \times \frac{1}{2} \rho V_1^2$$

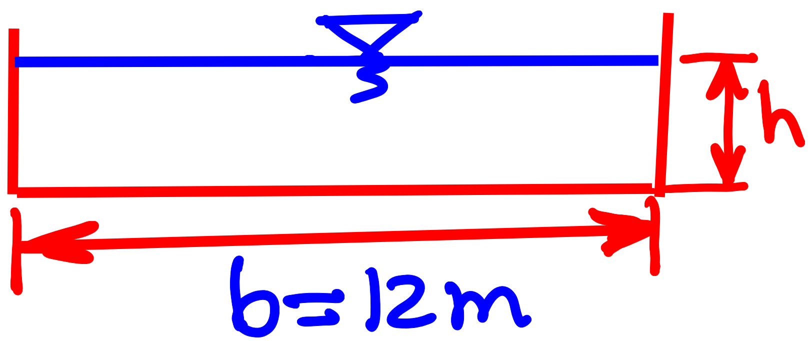
$$\tau_{b_1} = 0.01 \times \frac{1}{2} \times 1000 \times 1.64^2$$

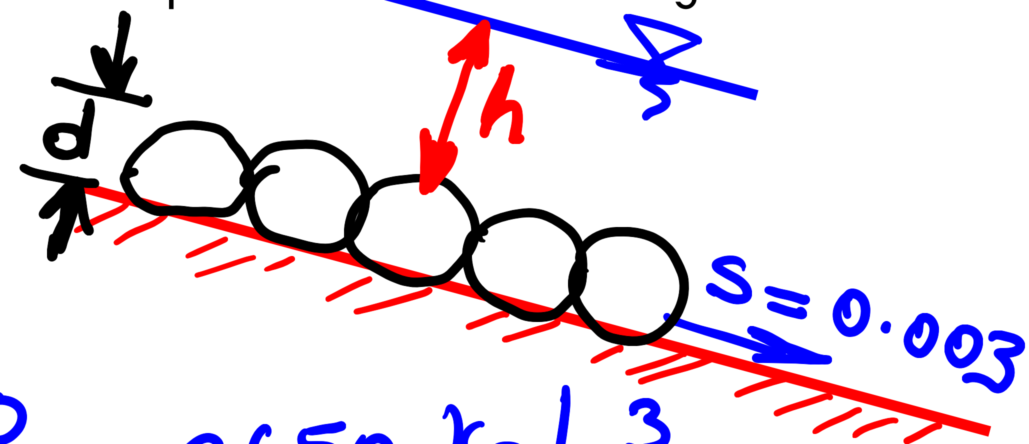
$$\tau_{b_1} = 13.4 \ \frac{N}{m^2} \quad \left(< 54.39 \ \frac{N}{m^2} \right)$$

* No erosion at section 1, therefore our assumption is correct.

Example (Adapted from Apsley, hydraulic notes):

A river of width 12 m and slope 0.003, carrying a maximum discharge of 200 m^3/s, is to be stabilized by using an armour layer of stones of density 2650 kg/m^3. Assuming a critical Shields parameter of 0.056, estimate the minimum size of stone that should be used and the corresponding river depth at maximum discharge.





$Q = 200\ m^3/s$

$d_{min} = ??$

$\rho_s = 2650\ kg/m^3$

$\tau^*_{crit} = 0.056$

$h = ?$ [max. discharge].

* Bed shear stress for incipient motion.

$$\tau^*_{crit} = \frac{\tau_{b\,crit}}{\rho (s-1) g d}$$

For min. diameter

$$\tau_b = \tau_{b\,crit}$$

$$0.056 = \frac{\tau_{b\,crit}}{1000(1.65)\times 9.81\times d}$$

because we are estimating d_{min}

$$\boxed{\tau_{b\,crit} = 906.4\, d}$$

$\tau_b = \tau_{b\,crit}$

* For normal flow

$\tau_b = \rho g R_h S$, $\quad Q = \frac{k}{n} A R^{2/3} S^{1/2}$, $\quad n = \frac{d^{1/6}}{21.1}$

$$906.4\, d = 1000 \times 9.81 \times \left(\frac{12h}{12+2h}\right) \times 0.003$$

$$R = \frac{A}{P} = \frac{bh}{b+2h}$$

$$R = \frac{12h}{12+2h}$$

$$\frac{1}{R} = \frac{12+2h}{2h} = \frac{0.032469}{d} \quad \dots (1)$$

Rearranging.

$$h = \frac{30.8\, d}{1-5.133\, d} \quad \dots (2)$$

* Manning's formula $Q = \frac{k}{n} A R^{2/3} S^{1/2}$

From ①

$R = 30.8\,d$

$$200 = \frac{1}{n}(12h)(30.8\,d)^{2/3} \times 0.003^{0.5} \quad \ldots ③$$

$h = h(d)$ ✓

$$n = \frac{d^{1/6}}{21.1}$$

In ③

$$200 = \frac{21.1}{d^{1/6}}\left(\frac{12 \times 30.8\,d}{1 - 5.133\,d}\right)(30.8\,d)^{2/3} \times 0.003^{0.5}$$

$$200 = \frac{4198\,d^{3/2}}{1 - 5.133\,d}$$

Solving by iteration

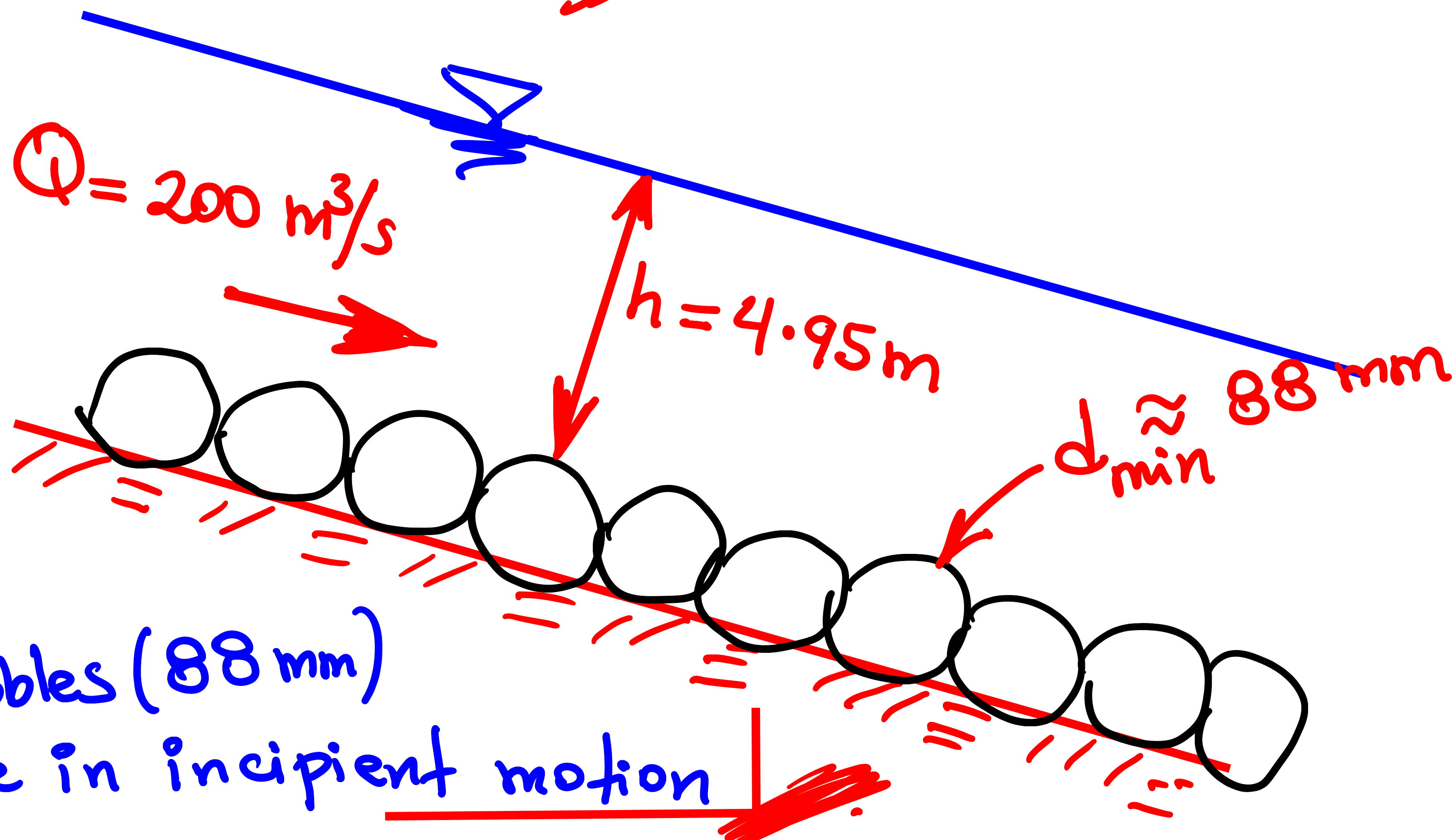
$d = 0.08802\ m$

$$\boxed{d_{min} = 88\ mm}$$

≈ 3.5 inches

In ② $h = \dfrac{30.8 \times 0.08802}{1 - 5.133 \times 0.08802}$

$$\boxed{h = 4.95\ m}$$



* Cobbles (88 mm) are in incipient motion

Inception of Motion in Normal Flow

Assume:

coarse sediment: $\frac{\tau_b}{(\rho_s - \rho) g d} > 0.056$

normal flow: $\tau_b = \rho g R_h S$

The bed will be mobile if $\frac{R_h S}{(\rho_s/\rho - 1} > 0.056\, d$

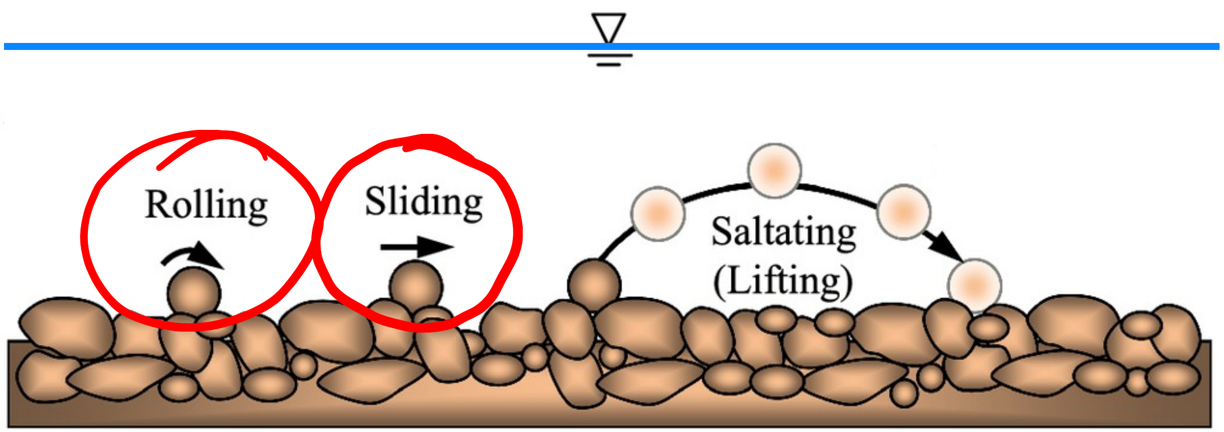
For sand ($\rho_s/\rho = 2.65$), the bed will be mobile if $d < 10.8 R_h S$

Bed Load

- **Bed load** consists of particles **sliding**, **rolling or saltating**, but remaining essentially in **contact with the bed**

- It is the **dominant form of sediment transport for larger particles** (settling velocity too large for suspension)

- The **bed-load flux** q_b is the volume of non-suspended sediment crossing unit width of bed per unit time.



Dimensionless Groups

$$\boxed{q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}}$$

dimensionless **bed-load flux**

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd}$$

dimensionless **bed shear stress**
(Shields parameter)

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

dimensionless **particle diameter**

Bed-Load Formulas

Reference	Formula
Meyer-Peter and Müller (1948)	$q^* = 8(\tau^* - \tau^*_{crit})^{3/2}$
Nielsen (1992)	$q^* = 12(\tau^* - \tau^*_{crit})\sqrt{\tau^*}$
Van Rijn (1984)	$q^* = \frac{0.053}{d^{*0.3}}\left(\frac{\tau^*}{\tau^*_{crit}} - 1\right)^{2.1}$
Einstein-Brown(Brown, 1950)	$q^* = \begin{cases} \frac{Ke^{\frac{-0.391}{\tau^*}}}{0.465} & \text{if } \tau^* < 0.182 \\ 40K\tau^{*3} & \text{if } \tau^* \geq 0.182 \end{cases}$ $K = \sqrt{\frac{2}{3} + \frac{36}{d^{*3}}} - \sqrt{\frac{36}{d^{*3}}}$
Yalin (1963)	$q^* = 0.635r\sqrt{\tau^*}\left[1 - \frac{1}{\sigma r}\ln(1+\sigma r)\right]$ $r = \frac{\tau^*}{\tau^*_{crit}} - 1, \sigma = 2.45\frac{\sqrt{\tau^*_{crit}}}{S^{0.4}}$

Reference: Apsley, D.D.; Stansby, P. Bed-load sediment transport on large slopes: Model formulation and implementation within a RANS solver. *J. Hydraul. Eng.* **2008**, *134*, 1440–1451.

Calculating Bed Load

bed load flux

$$q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$

dimensionless

$$q^* = f(\tau^*, d^*)$$

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd}$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3}$$

To find bed-load flux:

- from particle and fluid properties, find d^* ✓
- from formula or graph, find τ^*_{crit} ✓
- from flow hydraulics, find τ_b ✓ and hence τ^* ✓
- if $\tau^* > \tau^*_{crit}$ (or $\tau > \tau_{crit}$), find q^* by chosen model ✓
- Find bed-load flux per unit width, q_b ✓
- Multiply by channel width to get bed-load flux, Q_b

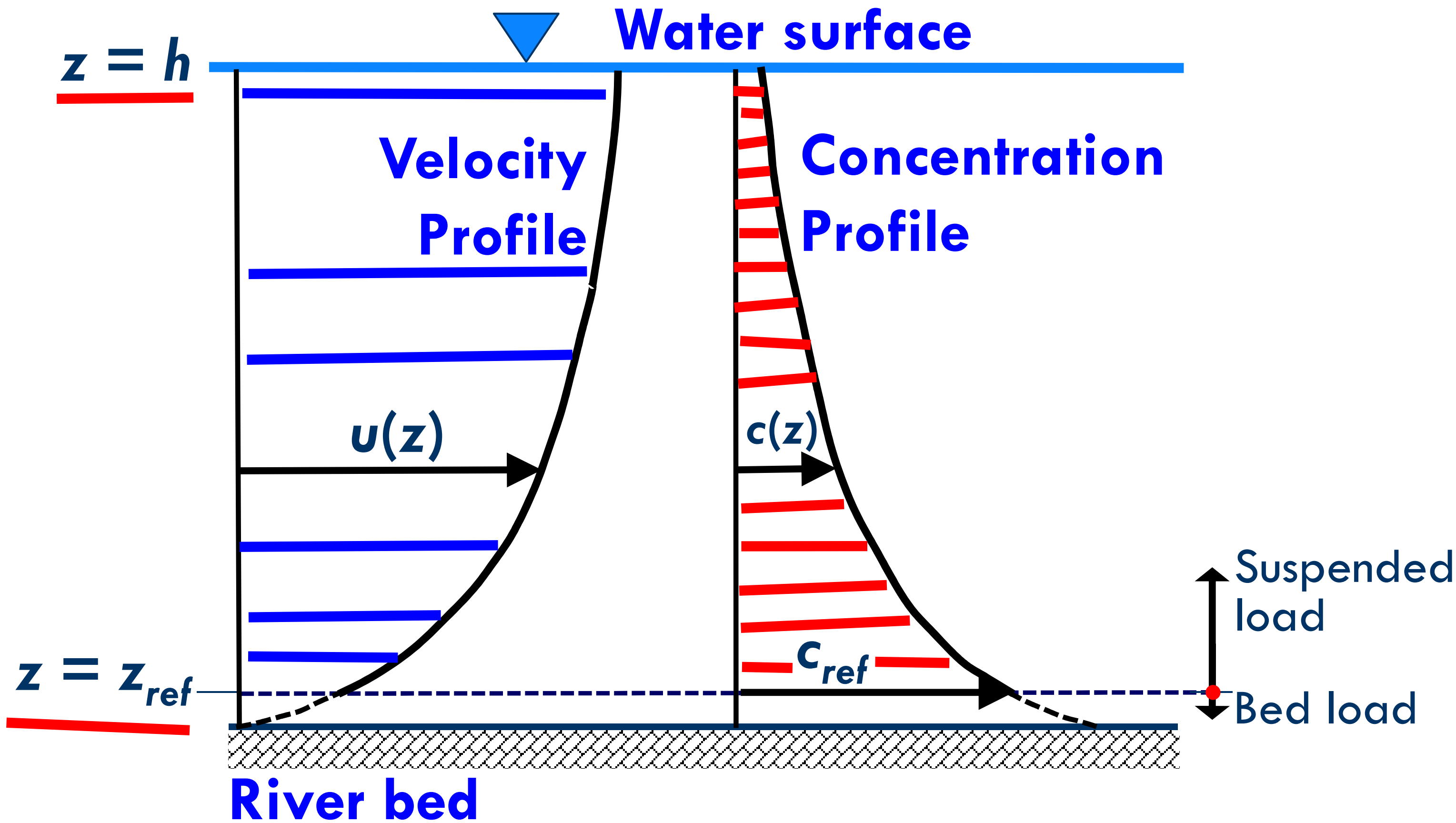
Suspended Load

- **Suspended load** consists of **finer particles carried in suspension** by turbulent fluid flow.
- Significant **suspended load** occurs if **turbulent velocity fluctuations are larger than the settling velocity**. A typical turbulent velocity fluctuation is of the order of the friction velocity u_τ. Thus, **suspended load will occur if**

$$\frac{u_\tau}{w_s} > 1$$

- For **coarser sediment**, suspended load does not occur and all **sediment motion is bed load**.

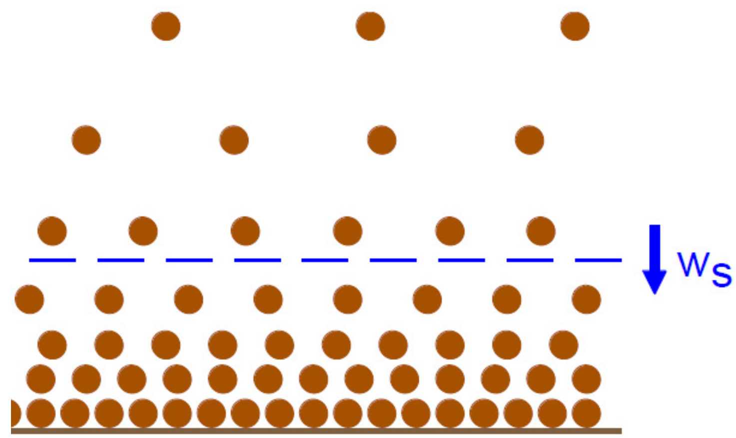
Suspended Load (Cont.)



- In practice, the **separation between what constitutes bed load and suspended load is fuzzy**

Concentration

Sediment **Concentration** C is the volume of sediment per total volume of material (fluid + sediment)



- Sediment settles, so **concentrations are larger near the bed.**
- **Upward-moving eddies** tend to **carry more sediment** than downward-moving ones.
- This leads to a **net upward diffusion of material.**
- Equilibrium when **downward settling = upward diffusion**.

Concentration Profile

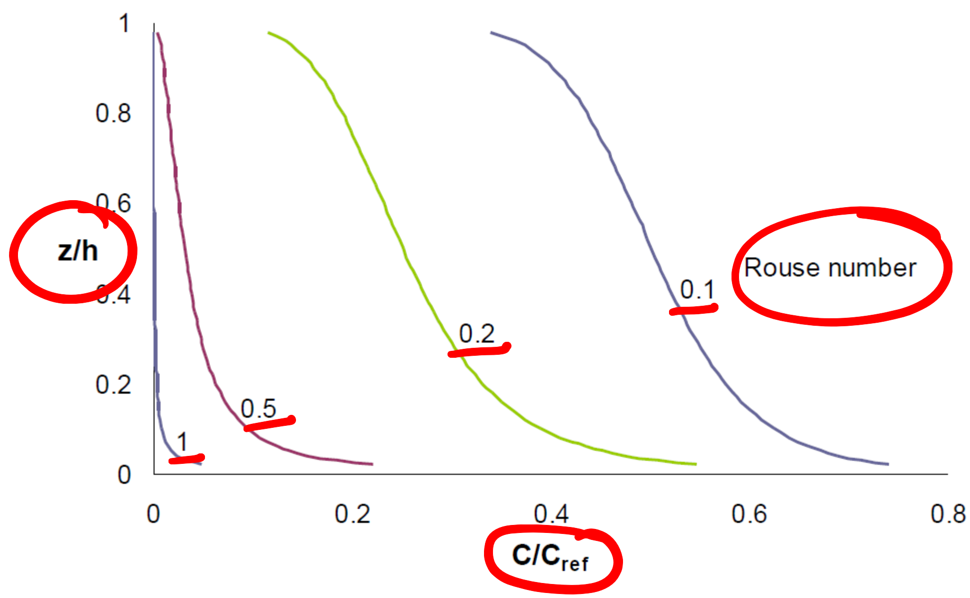
Rouse profile: $\dfrac{C}{C_{ref}} = \left(\dfrac{h/z-1}{h/z_{ref}-1}\right)^{\frac{w_s}{\kappa u_\tau}}$

Rouse number: $\dfrac{w_s}{\kappa u_\tau}$

w_s =settling velocity of the particle

u_τ =friction velocity of the flow, $u_\tau = \sqrt{\tau_b/\rho}$

κ =von Kármán's constant(≈0.41)



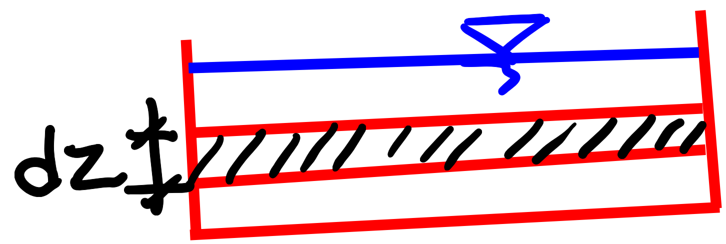
Calculation of Suspended Load

Volume flow rate of water: $u\ \mathrm{d}A$

per unit span: $u\ \mathrm{d}z$ (through depth $\mathrm{d}z$)

Volume flux of sediment = concentration × volume flux of water

$= Cu\ \mathrm{d}z$



Suspended load:

$$\boxed{q_s = \int_{z_{ref}}^{h} C u(z)\, \mathrm{d}z}$$

Load

$$u(z) = \frac{u_\tau}{\kappa} \ln\left(33 \frac{z}{k_s}\right)$$

$$\boxed{\frac{C}{C_{ref}} = \left(\frac{h/z - 1}{h/z_{ref} - 1}\right)^{\frac{w_s}{\kappa u_\tau}}}$$

Concentration

Calculation of Suspended Load (cont.)

It is necessary to specify C_{ref} at some depth z_{ref}, typically at a height representative of the bed load. The **formula of Van Rijn** (see, Chanson 2004):

$$\boxed{C_{ref} = \min\left[\frac{0.117}{d^*}\left(\frac{\tau^*}{\tau^*_{crit}} - 1\right), 0.65\right]}$$

$$\boxed{\frac{z_{ref}}{d} = 0.3 d^{*0.7}\left(\frac{\tau^*}{\tau^*_{crit}} - 1\right)^{1/2}}$$

Reference: Chanson H. (2004). The Hydraulics of Open Channel Flow : An Introduction. Butterworth-Heinemann, Oxford, UK, 2nd Edition.

Example (Adapted from Apsley, hydraulic notes):

A wide channel of slope 1:800 has a gravel bed with d_{50}=3 mm. The discharge is 4 m^3/s per meter width. The density of the gravel is 2650 kg/m^3.

(a) Estimate Manning's n using Strickler's formula.

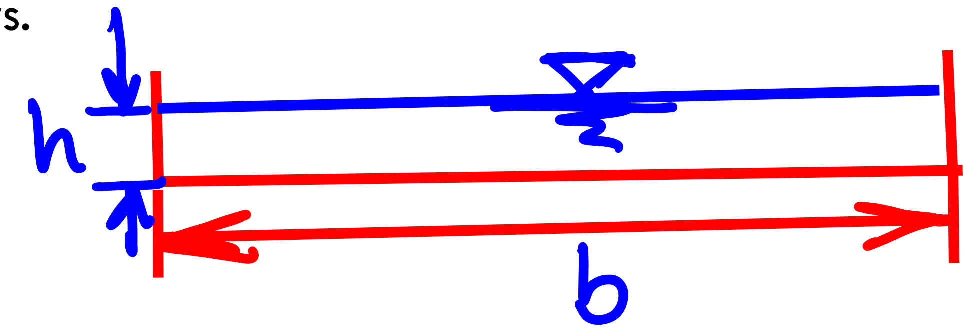
(b) Find the depth of flow; (assume normal flow).

(c) Find the bed shear stress.

(d) Show that the bed is mobile and calculate the bed-load flux (per meter width) using (i) Meyer-Peter and Müller; (ii) Van Rijn formulas.

(e) Determine whether suspended load occurs.

Wide channel $(b \gg h)$



$S = \frac{1}{800} = 0.00125$

$d_{50} = 3\,mm = \frac{3}{1000}\,m$

$q = 4\,\frac{m^2}{s}$

$R = \frac{A}{P} = \frac{bh}{b+2h}$ ≈ small compared to "b".

$R \approx \frac{\cancel{b}h}{\cancel{b}} = h$

$\rho_s = 2650\ kg/m^3$

a) Manning's n (Strickler's formula)

$$n = \frac{d^{1/6}}{21.1} = 0.018\ m^{-1/3}\,s$$

$$q = \frac{Q}{b}$$

b) Flow depth (normal flow)

$$Q = \frac{k}{n} A R^{2/3} S^{1/2}$$

$$q\cancel{b} = \frac{1}{n} \cancel{b} h \cdot h^{2/3} S^{1/2}$$

$$4 = \frac{1}{0.018} h^{5/3} \times 0.00125^{1/2}$$

$$\boxed{h = 1.532\ m}$$

c) Bed shear stress (τ_b)

$$\tau_b = \rho g R_h S = 1000 \times 9.81 \times 1.532 \times 0.00125$$

$$\boxed{\tau_b = 18.79\ \frac{N}{m^2}}$$

d) Show bed is mobile?

Bed load flux using Meyer-Peter, Van Rijn.

Bed will be mobile if $\tau_b > \tau_{b\,crit}$

By Soulsby:

$$\tau^*_{crit} = \frac{0.30}{1+1.2d^*} + 0.055\left(1 - e^{-0.020 d_*}\right) \quad \ldots (1)$$

$$d^* = d\left[\frac{(s-1)g}{\nu^2}\right]^{1/3} = \frac{3}{1000}\left[\frac{1.65\times 9.81}{(10^{-6})^2}\right]^{1/3}$$

$$d^* = 75.89$$

In 1

$$\boxed{\tau^*_{crit} = 0.0462}$$

$$\tau^*_{crit} = \frac{\tau b_{crit}}{\rho(s-1)g\,d}$$

$$0.0462 = \frac{\tau b_{crit}}{1000(1.65)\times 9.81\times \frac{3}{1000}}$$

$$\tau b_{crit} = 2.24\ \frac{N}{m^2}$$

$$\frac{\tau b}{18.79} > \frac{\tau b_{crit}}{2.24}$$ [bed is mobile]

* Bed-load Meyer-Peter and Müller.

$$q^* = 8\left(\tau^* - \tau^*_{crit}\right)^{3/2}$$

$$\tau^* = \frac{\tau_b}{\rho(s-1)gd} = \frac{18.79}{1000(1.65)\times 9.81\times \frac{3}{1000}}$$

$$\tau^* = 0.3869$$

$$q^* = 8\left(0.3869 - 0.0462\right)^{3/2} = 1.591$$

$$\rightarrow q^* = \frac{q_b}{\sqrt{(s-1)gd^3}}$$

$$q_b = q^* \sqrt{(s-1)gd^3}$$

$$q_b = 1.591\sqrt{1.65\times 9.81\times\left(\frac{3}{1000}\right)^3}$$

$q_b = 1.052 \times 10^{-3}\ \frac{m^2}{s}$ (per meter width)

For a river of 100 m in one day.

Vol. sediments : $100\, q_b \times 24 \times 3600\, s$

Vol. (1 day) $\approx$ 9000 m^3

* Bed load with Van Rijn

$$q^* = \frac{0.053}{(d^*)^{0.3}} \left(\frac{\tau^*}{\tau^*_{crit}} - 1 \right)^{2.1} = 0.9604$$

$$\therefore\ q_b = q^* \sqrt{(s-1) g d^3} = 6.35 \times 10^{-4}\ m^2/s$$

e) Suspended load will occur if $\frac{U_\tau}{W_s} > 1$

* Fall velocity W_s

Cheng's formula for W_s

$$\frac{W_s d}{\nu} = \left[\left(25 + 1.2 d^{*2}\right)^{1/2} - 5\right]^{3/2}$$

$$\frac{W_s (3/1000)}{10^{-6}} = \left[\left(25 + 1.2 \times 75.89^2\right)^{1/2} - 5\right]^{3/2}$$

$$\boxed{W_s = 0.2309 \text{ m/s}}$$

* U_τ

$$U_\tau = \sqrt{\frac{\tau_b}{\rho}} = \sqrt{\frac{18.79}{1000}} = 0.137 \text{ m/s}$$

$$\frac{U_\tau}{W_s} = \frac{0.137}{0.2309} = 0.59$$

* No significant suspended load will occur.

Example (Adapted from Apsley, hydraulic notes):

The vertical profile of mean velocity U in a rough-walled turbulent flow is:

$$U(z) = \frac{u_\tau}{\kappa} \ln\left(33\frac{z}{k_s}\right)$$

Use numerical integration to calculate the suspended-load sediment flux in a channel using the following data:

channel width: $b = 5$ m;

flow depth: $h = 1.5$ m;

friction velocity: $u_\tau = 0.2$ m/s;

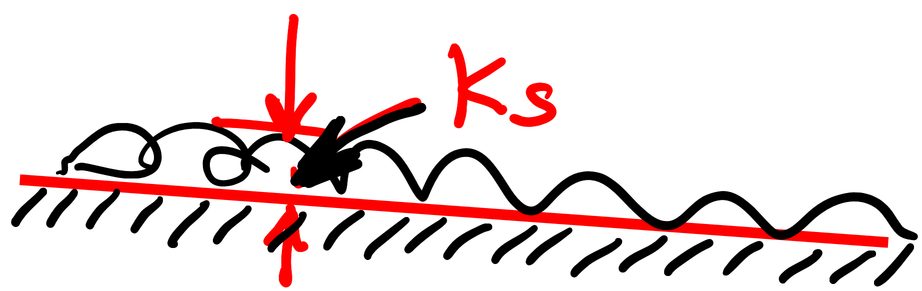
settling velocity: $w_s = 0.03$ m/s;

roughness length: $k_s = 0.001$ m;

reference concentration: $c_{ref} = 0.65$;

reference height: $z_{ref} = 0.001$ m.

κ is a universal constant with value 0.41



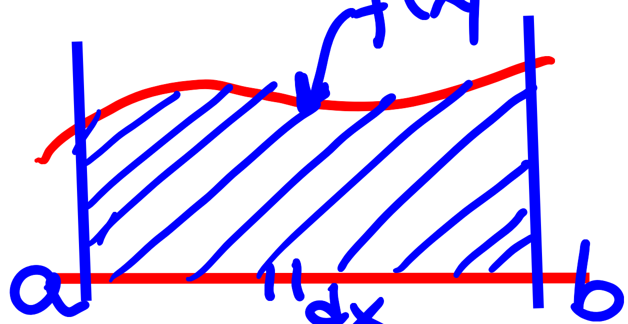
$$q_s = \int_{z_{ref}}^{h} c\, u(z)\, dz \quad , \quad Q_s = b\, q_s$$

$$Q_s = 5\int_{0.001}^{1.5} C_{ref}\left[\frac{h/z - 1}{h/z_{ref} - 1}\right]^{\frac{w_s}{\kappa u_*}} \cdot \frac{u_*}{\kappa}\ln\left(33\,\frac{z}{k_s}\right) dz$$

$$Q_s = 5\int_{0.001}^{1.5} \underbrace{0.65\left[\frac{\frac{1.5}{z} - 1}{\frac{1.5}{0.001} - 1}\right]^{\frac{0.03}{0.41\times 0.2}} \times \frac{0.2}{0.41}\ln\left(33\,\frac{z}{0.001}\right)}_{f} dz$$

We can integrate this using many methods.
We can also use MATLAB or other tools.

$$\int_a^b f(x)\, dx$$

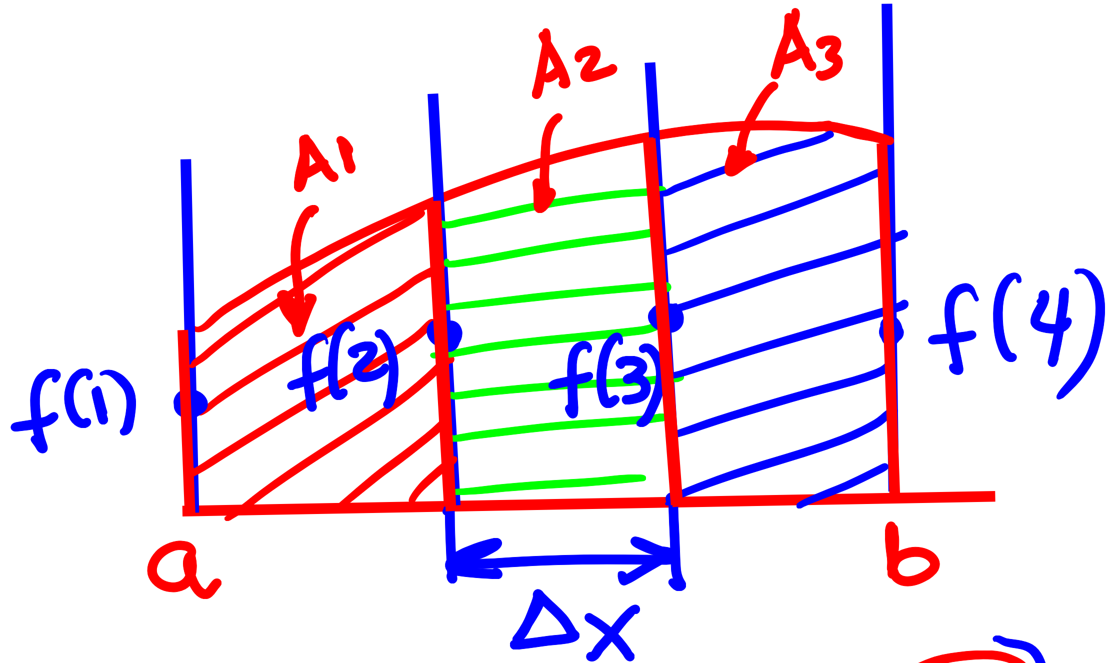


$$\Delta x = \frac{b-a}{N}$$

$$N = 3$$

$$\Delta x = \frac{1.5 - 0.001}{3}$$

(Δz)



$$\text{Area} = \left[\frac{f(1)+f(2)}{2} + \frac{f(2)+f(3)}{2} + \frac{f(3)+f(4)}{2}\right]\Delta x$$

In general: $$\text{Area} = \frac{\Delta x}{2}\left[f(x_1) + f(x_{N+1}) + 2\sum_{k=2}^{N} f(x_k)\right]$$

z	f
0.001	1.1089
0.5007	0.2731
1.0004	0.1762
1.50	0.000

$\Delta x = 0.4997$

Δz

$$Q_s = 5 \times \frac{0.4997}{2} \times \left[1.1089 + 0.000 + 2(0.2731 + 0.1762) \right]$$

$$\boxed{Q_s = 2.508 \ m^3/s}$$