Momentum Principles

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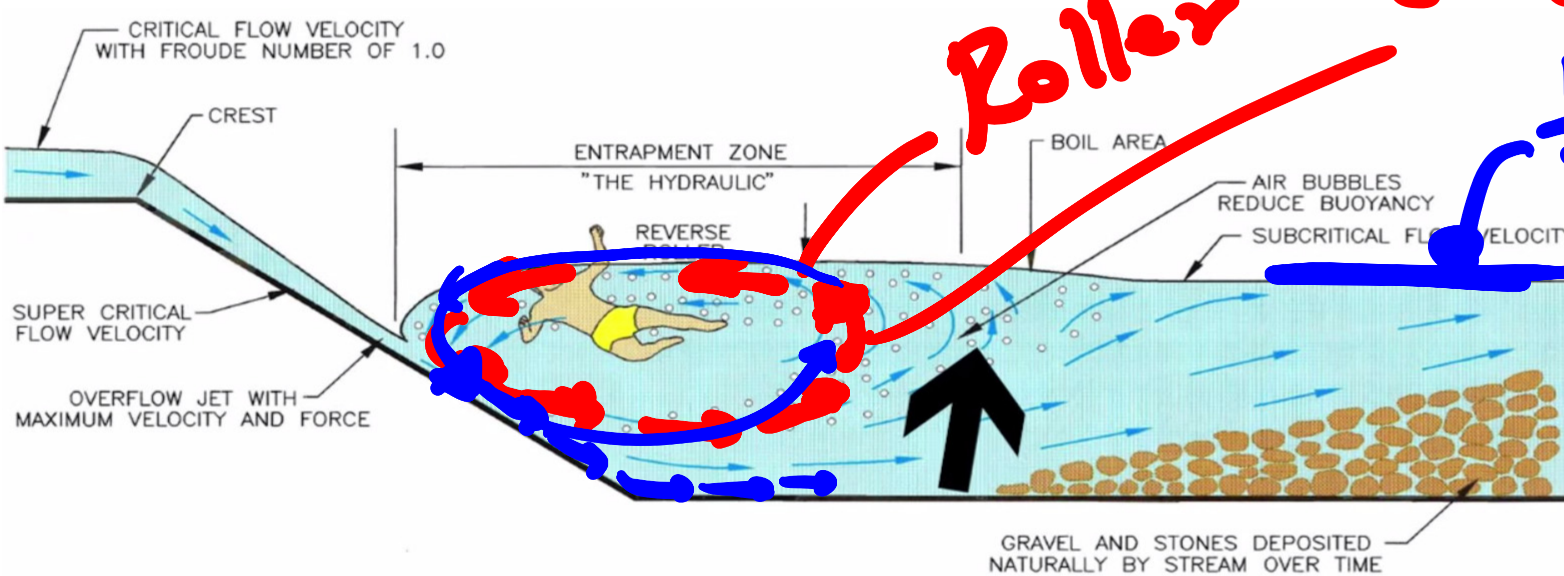
Momentum Equation

- Open channel flow with complex internal flow patterns can have high **energy loss**, the nature of which is **difficult to estimate**.
- In cases of **complex internal flow patterns**, the **energy equation cannot be applied** to relate flow parameters. In these cases, the momentum equation with suitable assumptions is recommended.
- For instance, the application of the momentum equation to **hydraulic jumps** yields meaningful results.

Hydraulic Jump

- Occurs when a **supercritical** flow **meets** a **subcritical** flow.
- Jump consists of a steep change in the water-surface elevation with a **reverse flow roller** on the major part.
- **Roller entrains** considerable quantity of **air** and the surface has white, frothy and choppy appearance.

Large DO



See Video: https://www.youtube.com/watch?v=XsYgODmmiAM

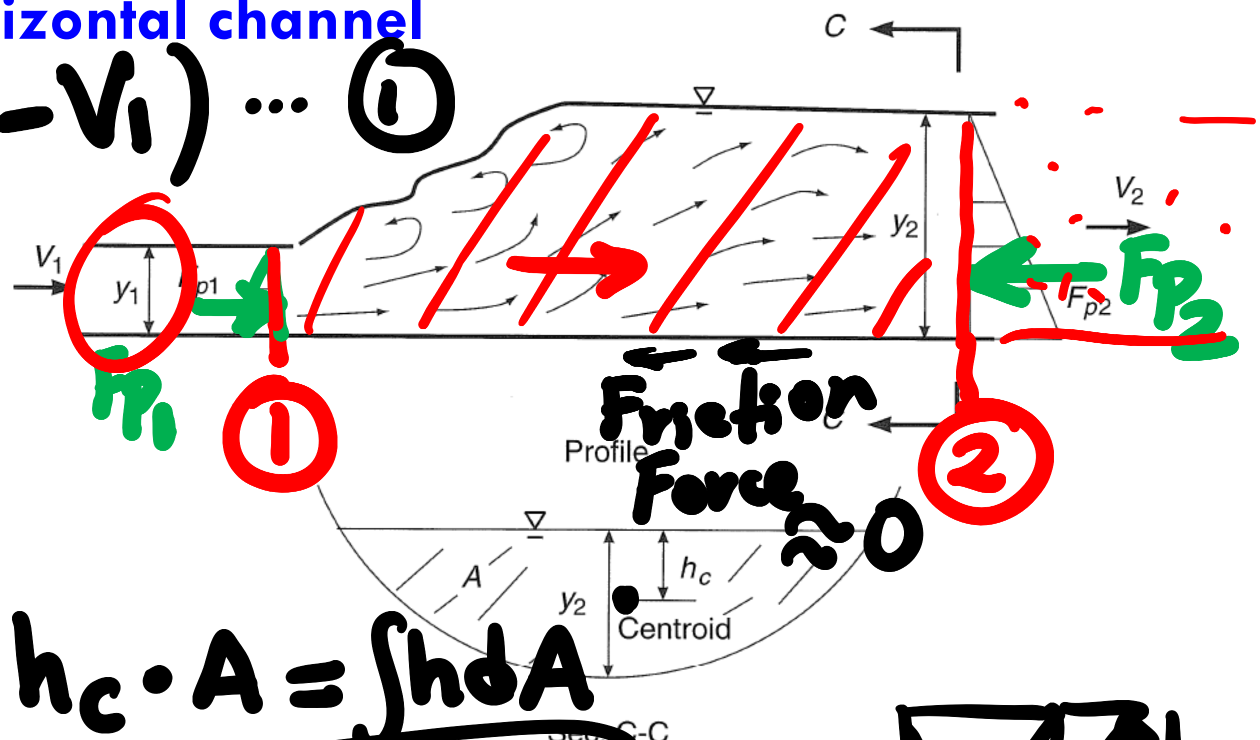
$$\Sigma F = \dot{m}(V_2 - V_1)$$

$$\dot{m} = \rho A V = \rho Q \rightarrow x$$

Momentum function

For steady flow and horizontal channel

$$F_{p_1} - F_{p_2} = \rho Q (V_2 - V_1) \quad \cdots \text{(1)}$$



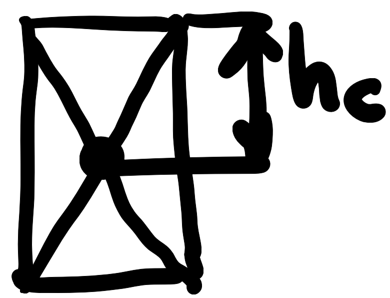
$$F_p = \gamma h_c \cdot A$$

$$Q = A \cdot V$$

$$V = \frac{Q}{A}$$

$$h_c \cdot A = \int h\,dA$$

$$\boxed{h_c = \frac{\int h\,dA}{A}}$$



Where: h_c = centroid of the area A, F_p = pressure force at the control surface

Momentum function (Cont.)

$\gamma = \rho \cdot g$

In (1)

$$\cancel{\gamma} h_{c_1} A_1 - \cancel{\gamma} h_{c_2} A_2 = \cancel{\rho} Q \left(\frac{Q}{A_2} - \frac{Q}{A_1} \right) \frac{\cancel{g}}{g}$$

$$\boxed{A_1 h_{c_1} + \frac{Q^2}{g A_1} = A_2 h_{c_2} + \frac{Q^2}{g A_2}}$$

$$M = A \cdot h_c + \frac{Q^2}{g A}$$

M: Momentum function

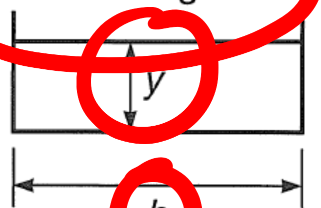
$$\boxed{M_1 = M_2}$$

Momentum function (M) for various channels

$$M = Ah_c + \frac{Q^2}{gA}$$

$(M_1 = M_2)$

Rectangular

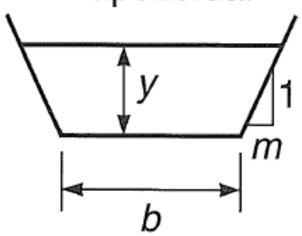


$by^2/2 + Q^2/(gby)$

$M = by \cdot \frac{y}{2} + \frac{Q^2}{gby}$

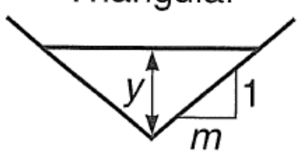
$$\boxed{M = \frac{by^2}{2} + \frac{Q^2}{gby}}$$

Trapezoidal



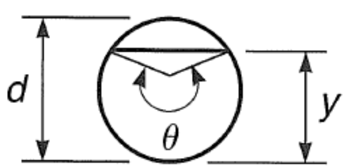
$by^2/2 + my^3/3 + Q^2/[gy(b + my)]$

Triangular



$my^3/3 + Q^2/(gmy^2)$

Circular†

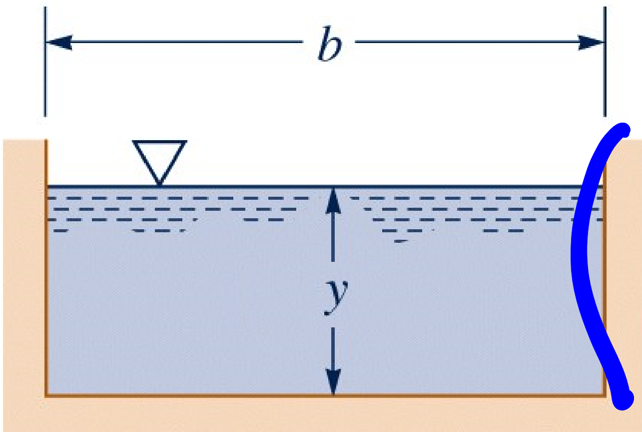


$[3\sin(\theta/2) - \sin^3(\theta/2) - 3(\theta/2)\cos(\theta/2)]\, d^3/24 + Q^2/[gd^2(\theta - \sin\theta)/8]$

† $\theta = 2\cos^{-1}[1 - 2(y/d)]$

Hydraulic Jump in a rectangular channel

Hydraulic jump in a horizontal, rectangular channel



$M_1 = M_2$

$$A_1 h_{c1} + \frac{Q^2}{gA_1} = A_2 h_{c2} + \frac{Q^2}{gA_2}$$

$$\left(b\frac{y_1^2}{2} + \frac{Q^2}{gby_1} = b\frac{y_2^2}{2} + \frac{Q^2}{gby_2} \right) / b$$

$$q = \frac{Q}{b}$$

$$\frac{y_1^2}{2} + \frac{Q^2}{b^2} \cdot \frac{1}{gy_1} = \frac{y_2^2}{2} + \frac{Q^2}{b^2} \cdot \frac{1}{gy_2}$$

$$\left[\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{q^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_1}\right) \right] / \frac{y_1^2}{2}$$

Hydraulic Jump in a rectangular channel (Cont.)

$$1-\left(\frac{y_2}{y_1}\right)^2 = \frac{2q^2}{g}\left[\frac{1}{y_2 y_1^2} - \frac{1}{y_1^3}\right] \quad \ldots\ldots (1)$$

$$*\ F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{Q}{b y_1 \sqrt{g y_1}} = \frac{q}{y_1\sqrt{g y_1}}$$

$$F_1^2 = \frac{q^2}{g y_1^3} \quad \ldots\ldots (2)$$

Hydraulic Jump in a rectangular channel (Cont.)

F_1^2

In 1 $\quad 1-\left(\frac{y_2}{y_1}\right)^2 = \frac{2q^2}{gy_1^3}\left[\frac{y_1}{y_2} - 1\right] \quad \frac{y_2}{y_1} = a$

$$1 - a^2 = 2F^2\left(\frac{1}{a} - 1\right)$$

$$(\cancel{1-a})(1+a) = 2F^2 \frac{(\cancel{1-a})}{a}$$

$$a^2 + a - 2F_1^2 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Upstream flow is known

Belanger momentum equation

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8F_1^2}\right)}$$

Hydraulic Jump in a rectangular channel (Cont.)

In terms of F_2 (Subcritical Froude number)

$$\boxed{\frac{y_1}{y_2} = \frac{1}{2}\left(-1+\sqrt{1+8F_2^2}\right)}$$

get y_1 (downstream flow is known)

Jump Losses:

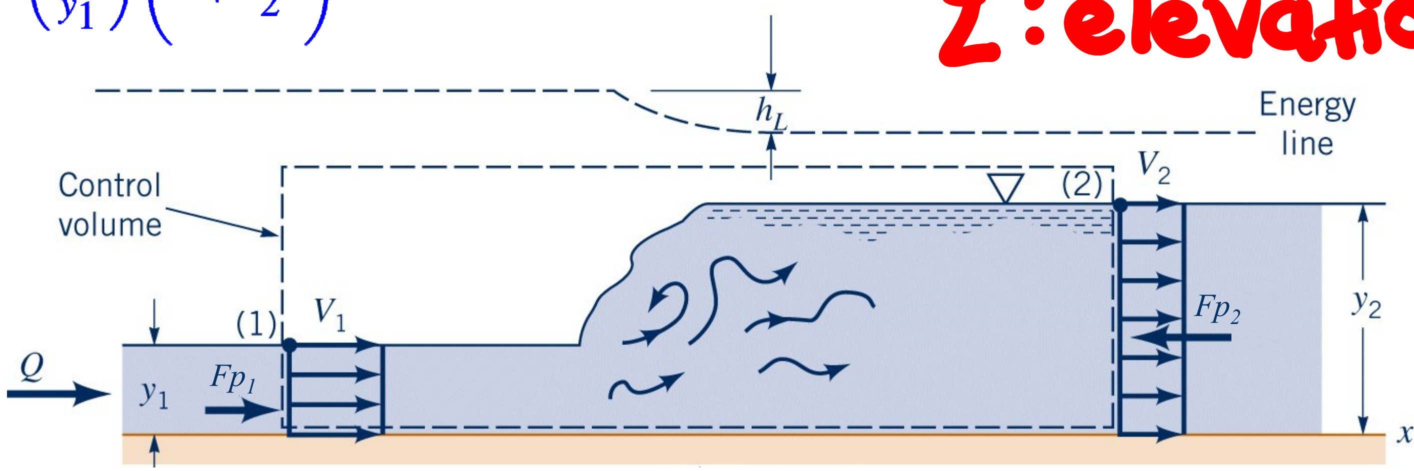
$$E_L = \frac{(y_2-y_1)^3}{4y_1y_2}$$

$$\frac{E_L}{E_1} = \frac{\left(\frac{y_2}{y_1}-1\right)^3}{4\left(\frac{y_2}{y_1}\right)\left(1+\frac{F_1^2}{2}\right)}$$

where: $E_L = E_1 - E_2$ (Head loss)

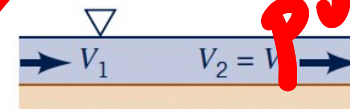
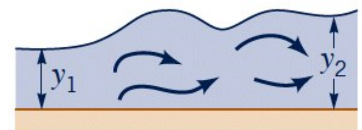
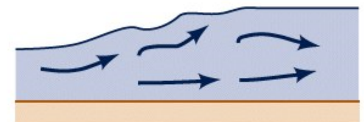
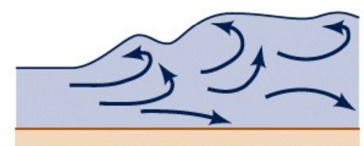
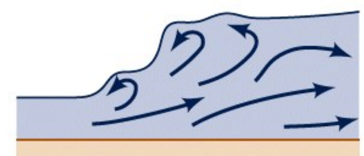
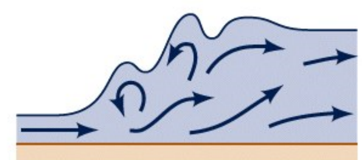
$$E = y + \frac{V^2}{2g} + Z$$

Z : elevation head



Classification of Hydraulic Jumps

for recreational purposes

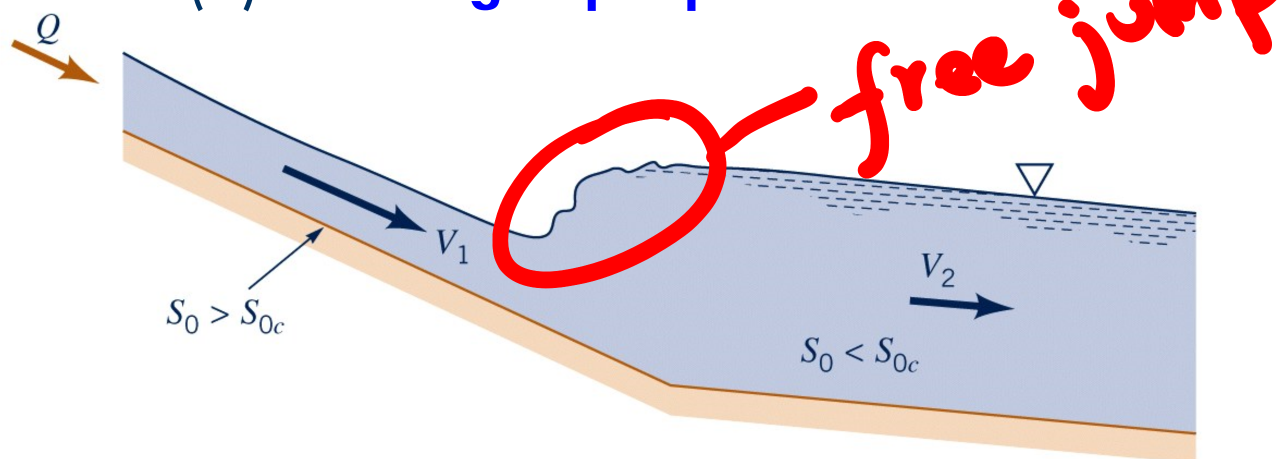
Fr_1	y_2/y_1	Classification	Sketch
<1	1	Jump impossible	
1 to 1.7	1 to 2.0	Standing wave or undulant jump	
1.7 to 2.5	2.0 to 3.1	Weak jump	
2.5 to 4.5	3.1 to 5.9	Oscillating jump	
$\boxed{4.5 \text{ to } 9.0}$	5.9 to 12	Stable, well-balanced steady jump; insensitive to downstream conditions	
>9.0	>12	Rough, somewhat intermittent strong jump	

For Energy dissipators.

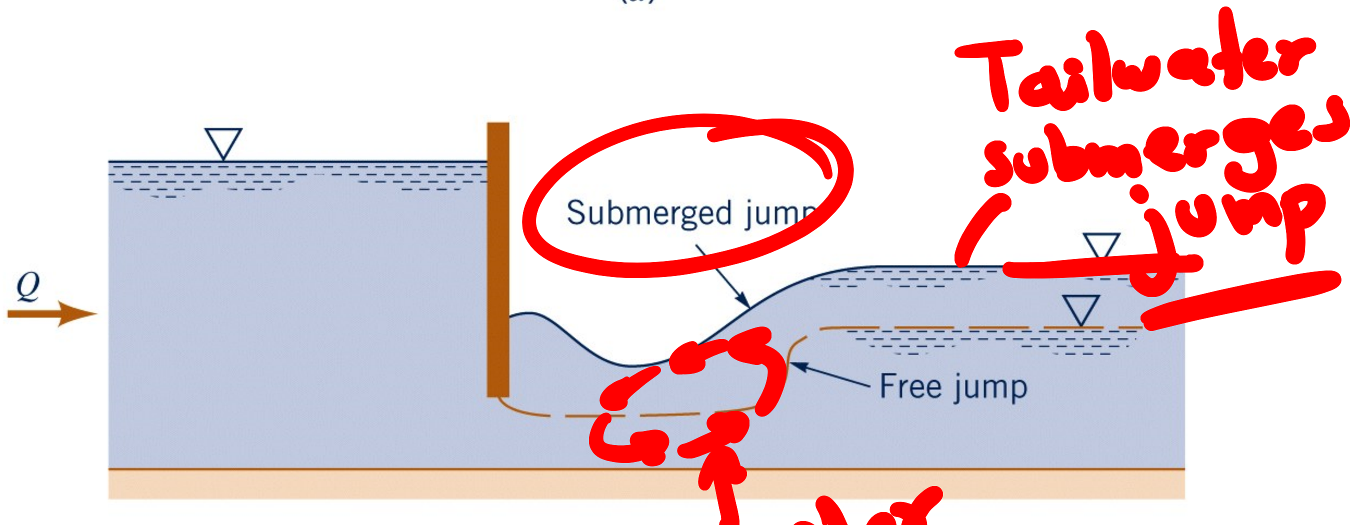
Source: Munson, Young and Okiishi's Fundamentals of Fluid Mechanics, 8th Edition

Hydraulic Jump Variations

(a) jump caused by a **change in channel slope**,
(b) **submerged jump**



(a)



(b)

Example:

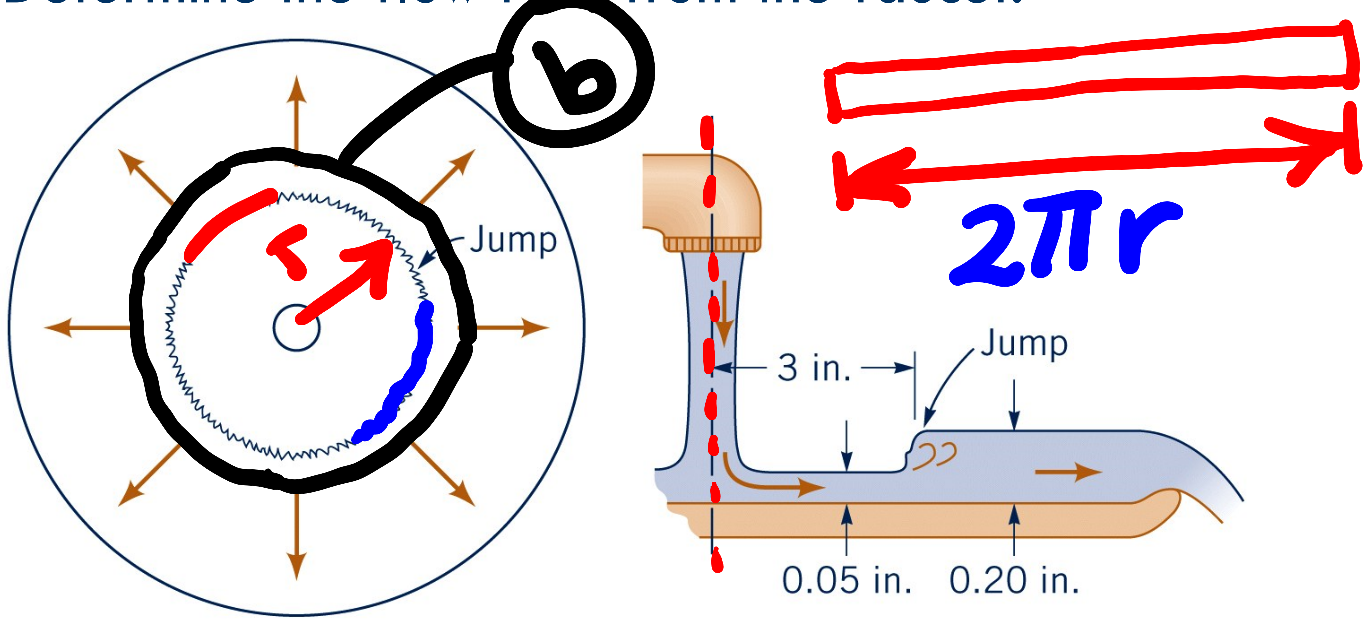
Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in the figure below. Consider a situation where a jump forms 3.0 in from the center of the plate with depths upstream and downstream of the jump of 0.05 in and 0.20 in, respectively. Determine the flow rate from the faucet.

$Q = ?$

$y_1 = 0.05$ in

$y_2 = 0.20$ in

(English units)



$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1+8Fr_1^2}\right)$$

$$\frac{0.2}{0.05} = \frac{1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right)$$

$$\boxed{Fr_1 = 3.16}$$

$$Fr = \frac{V}{\sqrt{gy}}$$

$$3.16 = \frac{V_1}{\sqrt{32.2 \times \frac{0.05}{12}}} \rightsquigarrow V_1 = 1.16\, \frac{ft}{s}$$

$$* \quad Q = A_1 V_1 = \underline{b} y_1 V_1$$

$$b = 2\pi\left(\frac{3}{12}\right)$$

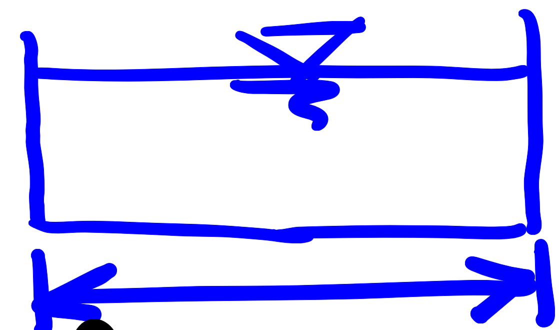
$$Q = 2\pi\left(\frac{3}{12}\right) \times \frac{0.05}{12} \times 1.16$$

$$\boxed{Q = 0.00759 \, ft^3/s}$$

Example:

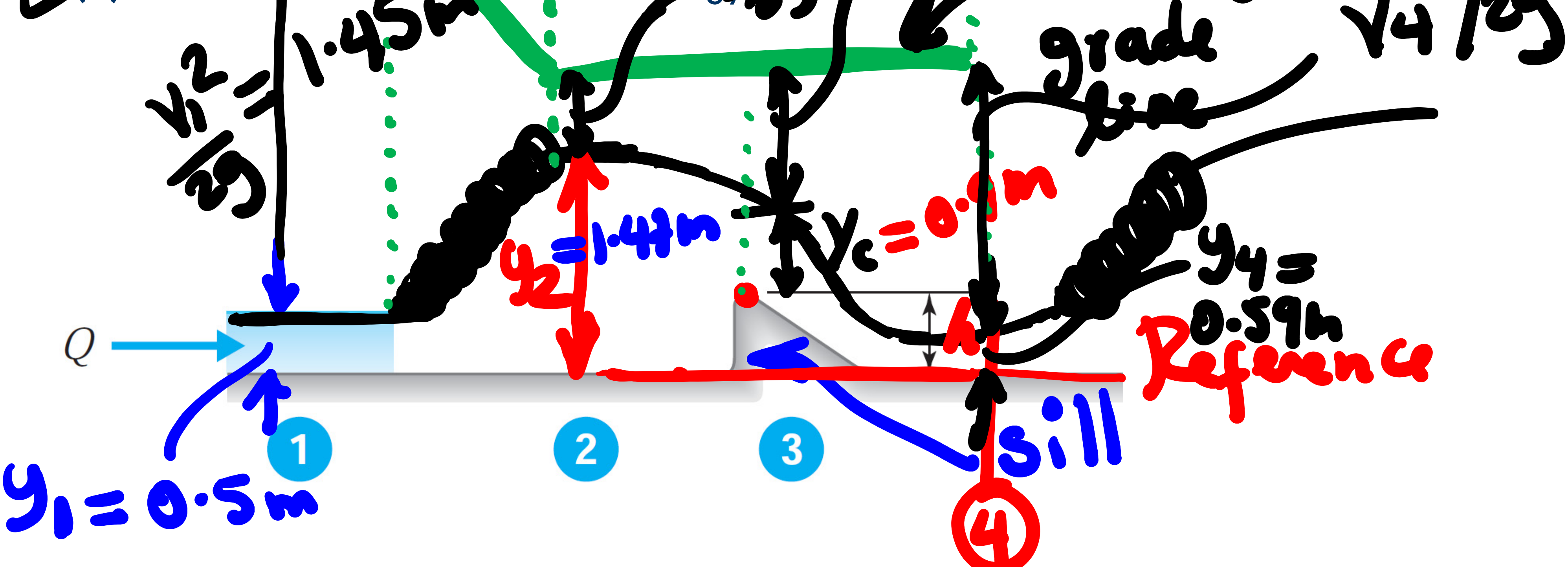
Water enters a reach of a rectangular channel where $y_1 = 0.5$ m, $b = 7.5$ m, and $Q = 20$ m^3/s. It is desired that a hydraulic jump occur upstream (location 2) of the sill and on the sill critical conditions exist (location 3). Other than across the jump, losses can be neglected. Determine the following:

(a) Depths at locations 2 and 3
(b) Required height of the sill, h
(c) Resultant force acting on the sill
(d) Sketch the water surface and energy grade line.



b = 7.5 m

Q = 20 m³/s



a) y_2, y_3.

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right)$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}}$$

$$V_1 = \frac{20}{(7.5\times 0.5)}$$

$$V_1 = 5.33 \text{ m/s}$$

$$Fr_1 = \frac{5.33}{\sqrt{9.8\times 0.5}}$$

$$Fr_1 = 2.41$$

$$\frac{y_2}{0.5} = \frac{1}{2}\left(-1+\sqrt{1+8*2.41^2}\right)$$

$$\boxed{y_2 = 1.47 \text{ m}}$$

* $y_3 = y_c$

$$y_c = \sqrt[3]{\frac{q^2}{g}}$$

$$q = \frac{Q}{b}$$

$$y_c = \sqrt[3]{\frac{\left(\frac{20}{7.5}\right)^2}{9.8}} = 0.90\ m$$

$$V_2 = \frac{Q}{A_2}$$

$$V_2 = 1.81\ m/s$$

$$V_3 = Q/A_3$$

$$V_3 = 2.96\ m/s$$

$$\boxed{y_3 = 0.90\ m}$$

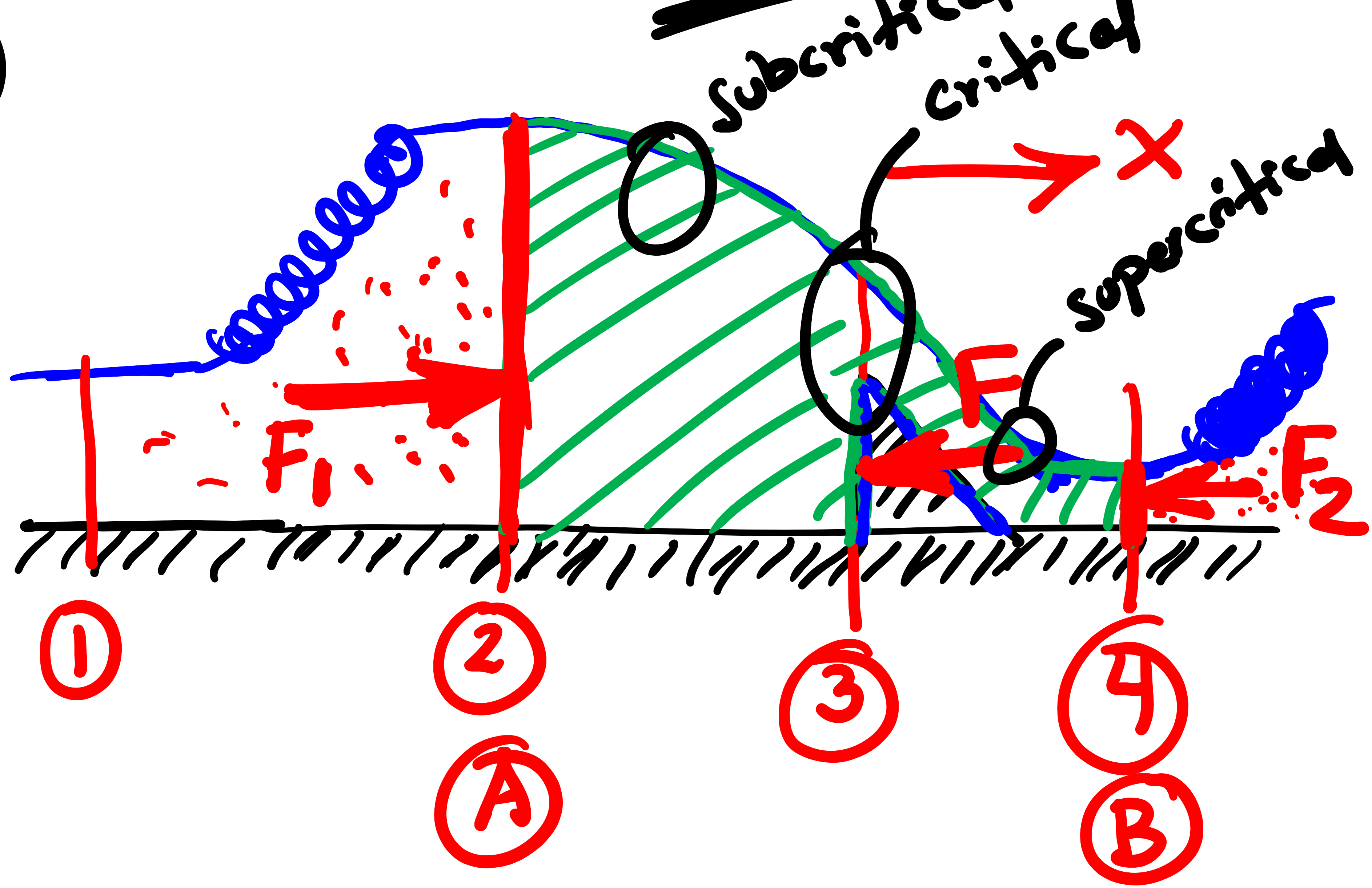
b) $h = ?$ $\quad E_2 = E_3$

$$y_2 + \frac{V_2^2}{2g} + \underbrace{z_2}_{0} = y_3 + \frac{V_3^2}{2g} + \underbrace{z_3}_{h}$$

$$1.47 + \frac{1.81^2}{2\times 9.8} = 0.9 + \frac{2.96^2}{2\times 9.8} + h$$

(c)

$\boxed{h = 0.29\ m}$



$$\Sigma F = \dot{m}(V_B - V_A)$$

$$V_B \to V_4 \qquad V_A \to V_2$$

$$F_2 - F_4 - F = \rho Q (V_4 - V_2) \quad \dots (*)$$

$$F_2 = \gamma A_2 y_{c_2} = 1000 \times 9.8 * (7.5 * 1.47) * \frac{1.47}{2}$$

$$F_4 = \gamma A_4 y_{c4} \rightarrow \underline{y_4 \text{ is unknown}}.$$

$$E_4 = E_3 = E_2 \rightarrow E_2 = E_4$$

$$y_2 + \frac{V_2^2}{2g} + \cancel{Z_2}^{\,0} = y_4 + \frac{V_4^2}{2g} + \cancel{Z_4}^{\,0}$$

$$1.47 + \frac{1.81^2}{2\times 9.8} = y_4 + \frac{20^2}{2\times 9.8\,(7.5\,y_4)^2}$$

$$\boxed{y_4 = 0.59\,m}$$

3 roots [2 (+), 1 (−)]

choose the smaller positive value [flow is supercritical]

F_4 ✓

$\dot{m} = \rho A_2 V_2 =$ ✓

In (*) $F = 1.26 * 10^4\,N$

Surges

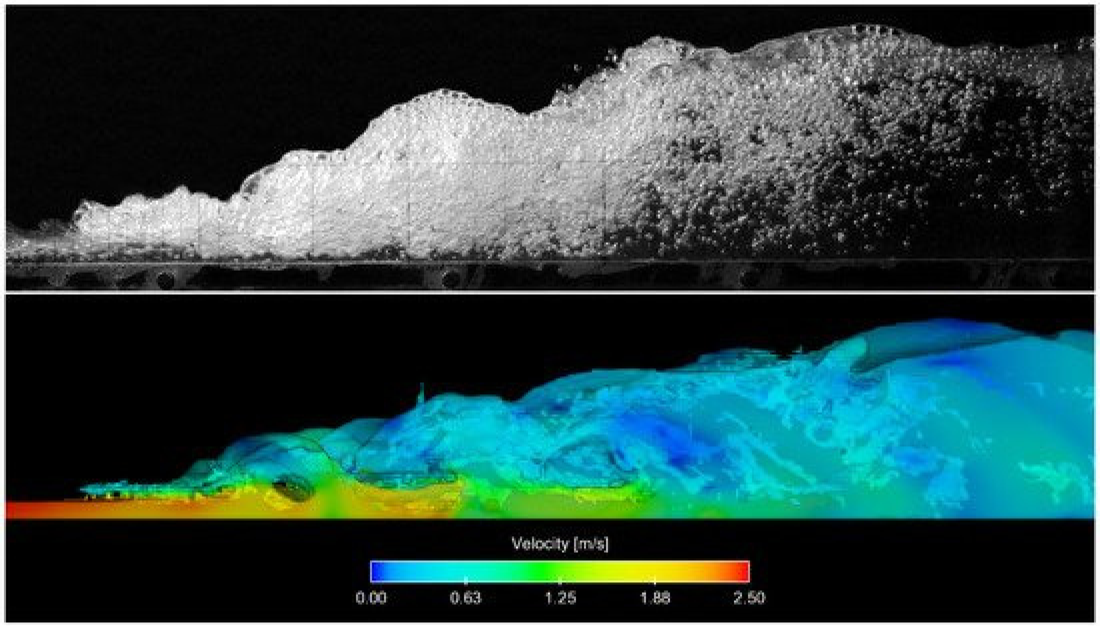
A tidal bore in Morecambe Bay, the United Kingdom

~~**Video of a tidal bore in China**~~

https://www.youtube.com/watch?v=axAxtsyHreQ

Surges

- Surges belong in a discussion of **unsteady flows.**
- However, they can be **transformed into a steady flow problem** by superimposing a surge velocity to make the surge stationary.
- For an observer moving at the speed of the surge, this becomes a **steady**-flow formation of a **hydraulic jump**



Source: https://www.mdpi.com/2073-4441/11/1/28

Surges in rectangular channels

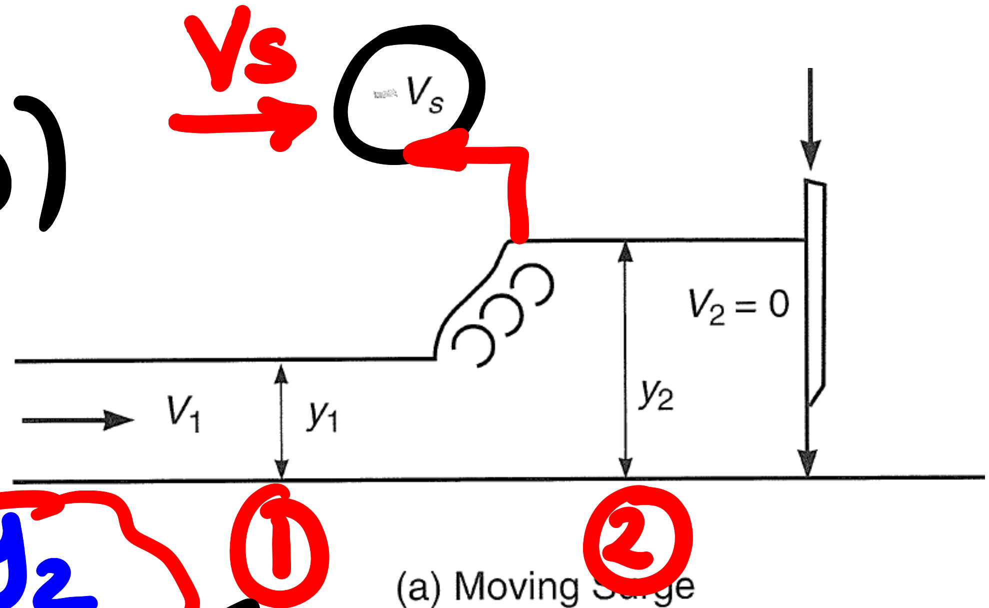
Continuity

Stationary jump: $(V_s = 0)$

$$Q_1 = Q_2 = A \cdot V$$

$$\cancel{b}\, y_1 V_1 = \cancel{b}\, y_2 V_2$$

$$\boxed{V_1 y_1 = V_2 y_2}$$

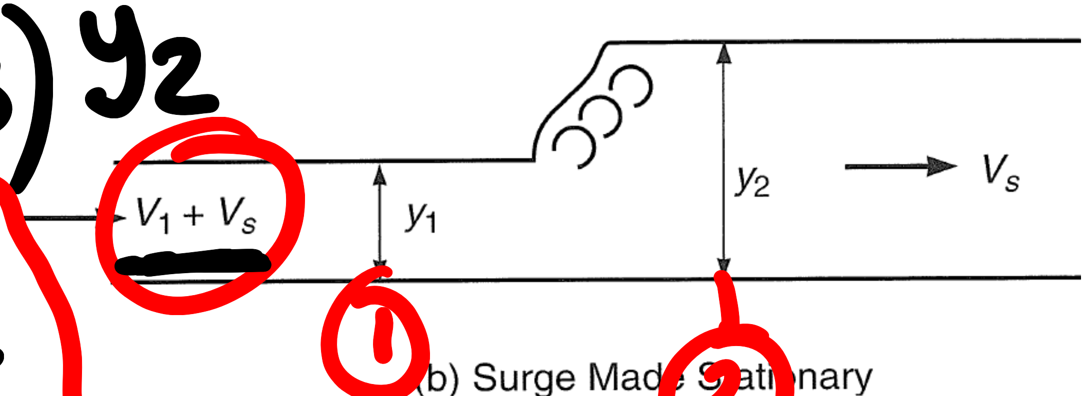


(a) Moving Surge

Moving jump:

$$(V_1 + V_s)\, y_1 = (V_2 + V_s)\, y_2$$

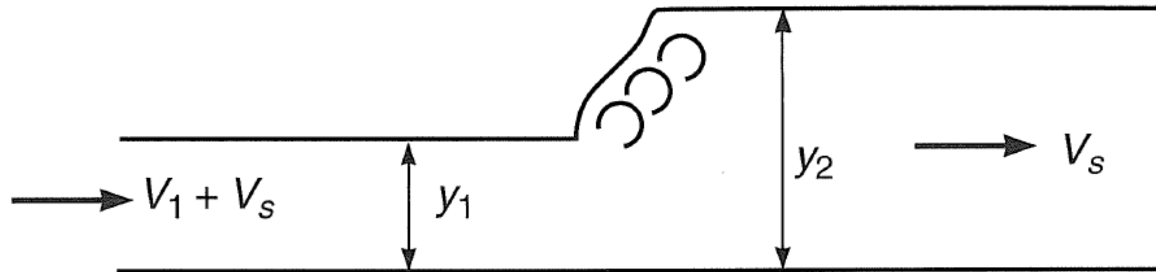
$$\boxed{V_s = \frac{V_1 y_1 - V_2 y_2}{y_2 - y_1}}$$



(b) Surge Made Stationary

Surges (cont.)

$$q = \frac{Q}{b} = \frac{V\cdot \cancel{b}\cdot y}{\cancel{b}} = V\cdot y$$



Momentum

Stationary jump:

$$\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{q^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_1}\right) = \frac{V_1^2 y_1^2}{g}\left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

Moving jump:

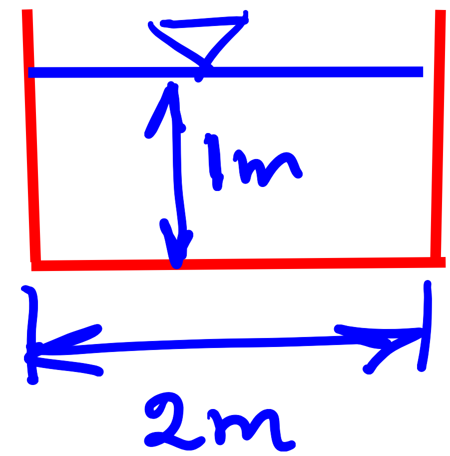
$$\frac{(y_1+y_2)\cancel{(y_1-y_2)}}{2} = \frac{(V_1+V_s)^2 y_1^2}{g}\left(\frac{\cancel{y_1-y_2}}{y_2 y_1}\right)$$

$$\frac{(V_1+V_s)^2}{g y_1} = \frac{1}{2}\left[\frac{y_2}{y_1} + \left(\frac{y_2}{y_1}\right)^2\right]$$

$$\boxed{\frac{(V_1+V_s)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1+\frac{y_2}{y_1}\right)}$$

Example of Application:

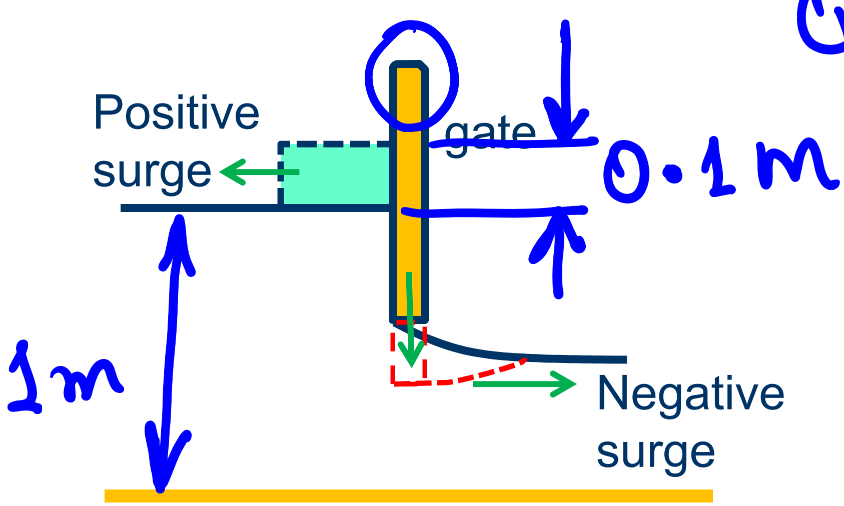
A 2m wide rectangular channel carries a discharge of 1 m^3/s at a flow depth of 1m. A sluice gate located in the channel is suddenly lowered and it is desired to produce a 0.1m high surge upstream of the gate. Find the velocity of the surge and the flow velocity at a section after the surge has passed. Assume a frictionless and horizontal channel.

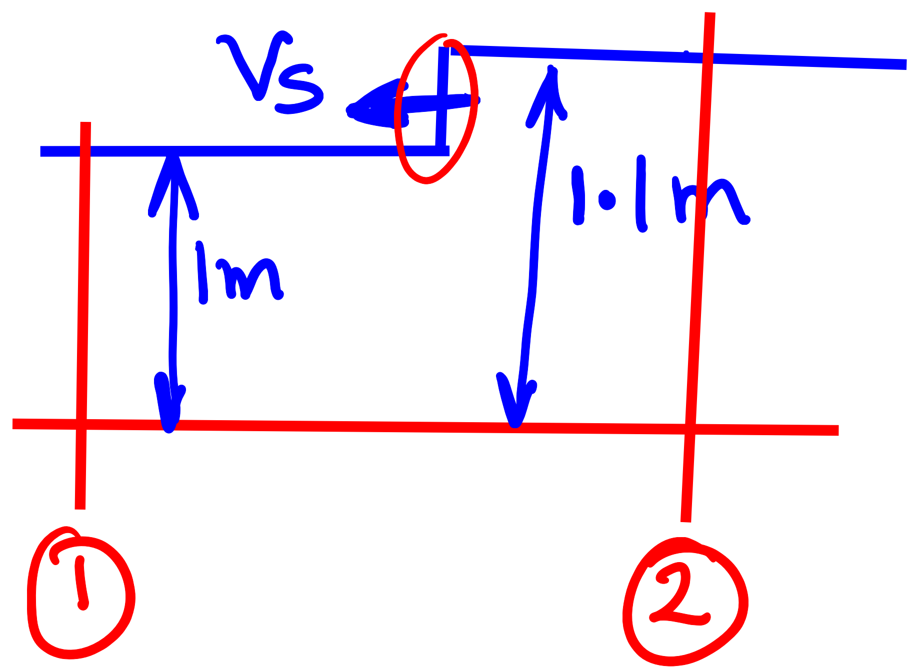


$V_s = ??$

$V_2 = ??$

$Q = 1\ m^3/s$





$$V_1 = \frac{Q}{A_1} = \frac{1}{2\times 1} = 0.5\ m/s$$

$y_1 = 1m$

$y_2 = 1.1m$

* Momentum:

$$\frac{(V_1+V_s)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1+\frac{y_2}{y_1}\right)$$

$$\frac{(0.5+V_s)^2}{9.8\times 1} = \frac{1}{2}\left(\frac{1.1}{1}\right)\left(1+\frac{1.1}{1}\right)$$

$$\boxed{V_s = 2.86\ m/s}$$

* Continuity:

$$(V_1+V_s)y_1 = (V_2+V_s)y_2$$

$$(0.5+2.86)\times 1.0 = (V_2+2.86)\times 1.1$$

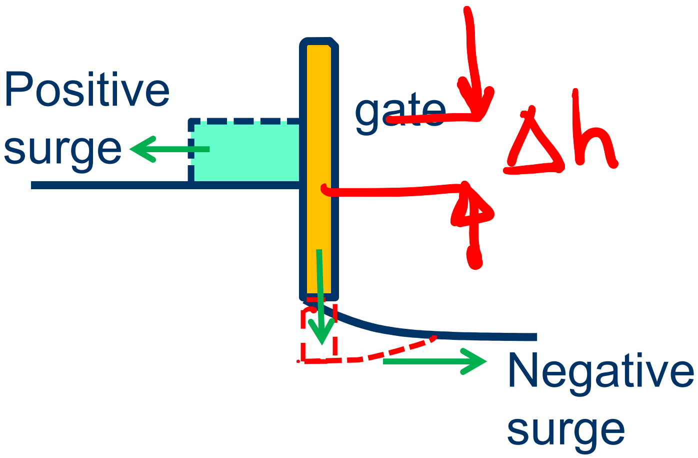
$$\boxed{V_2 = 0.19\ m/s}$$

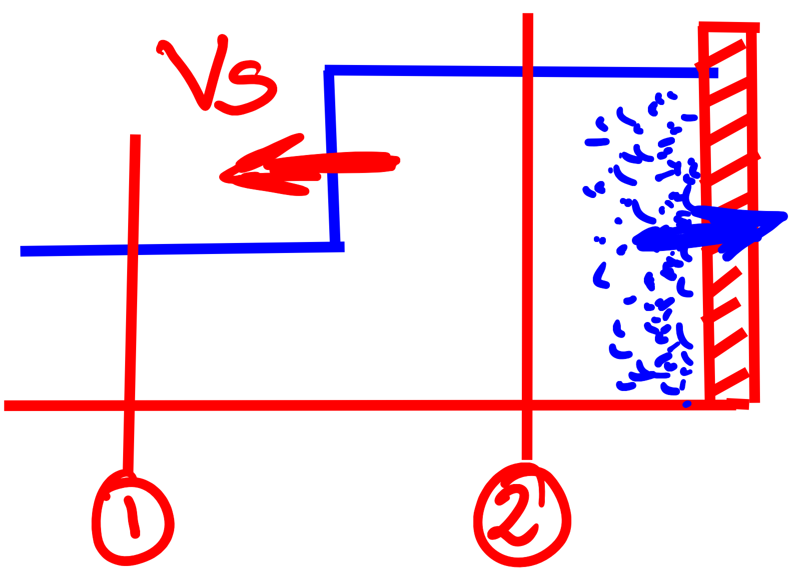
Example of Application:

What is the velocity of the surge and surge height in previous example if the gate is completely closed?

$V_s = ??$

$\Delta h = ??$





$\Delta h > 0.1\,m$??

$y_1 = 1.0\,m$

$V_2 = 0\,m/s$

$V_1 = 0.5\,m/s$

* Momentum: $\frac{(V_1+V_s)^2}{g y_1} = \frac{1}{2}\frac{y_2}{y_1}\left(1+\frac{y_2}{y_1}\right)$

$$\frac{(0.5+V_s)^2}{9.8\times 1} = \frac{1}{2}\frac{y_2}{1.0}\left(1+\frac{y_2}{1.0}\right) \quad \ldots\ldots (1)$$

* Continuity: $(V_1+V_s)\,y_1 = (V_2+V_s)\,y_2$

$$(0.5+V_s)\times 1.0 = (0+V_s)\,y_2 \quad \ldots (2)$$

$$\hookrightarrow y_2 = \frac{0.5+V_s}{V_s} \checkmark$$

In (1)

$$\frac{(0.5+V_s)^2}{9.8} = \frac{1}{2}\left(\frac{0.5+V_s}{V_s}\right)\left[1+\frac{0.5+V_s}{V_s}\right] \rightarrow V_s = 3.02\,\frac{m}{s}$$

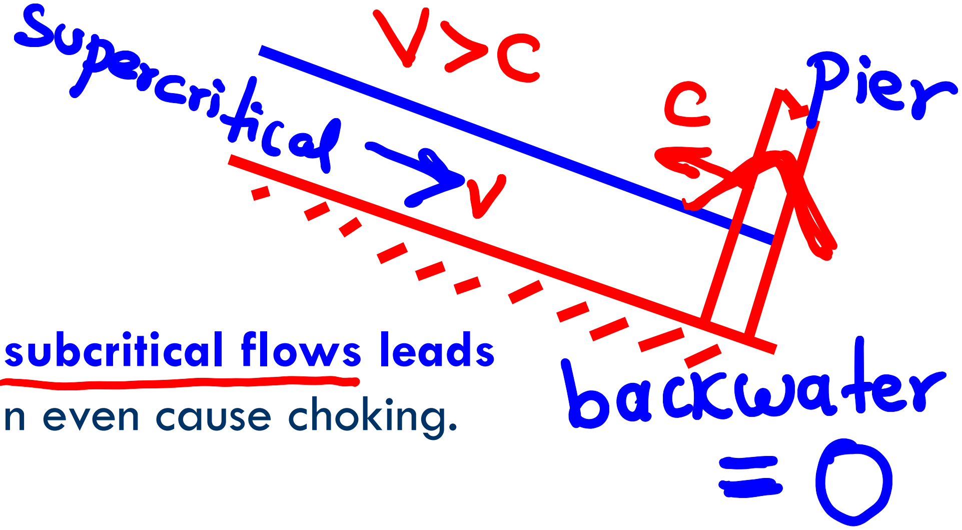
* $y_2 = \frac{0.5+3.02}{3.02} = 1.17\,m$

$\Delta h = y_2 - y_1 = 1.17 - 1.0\,m = 0.17\,m$

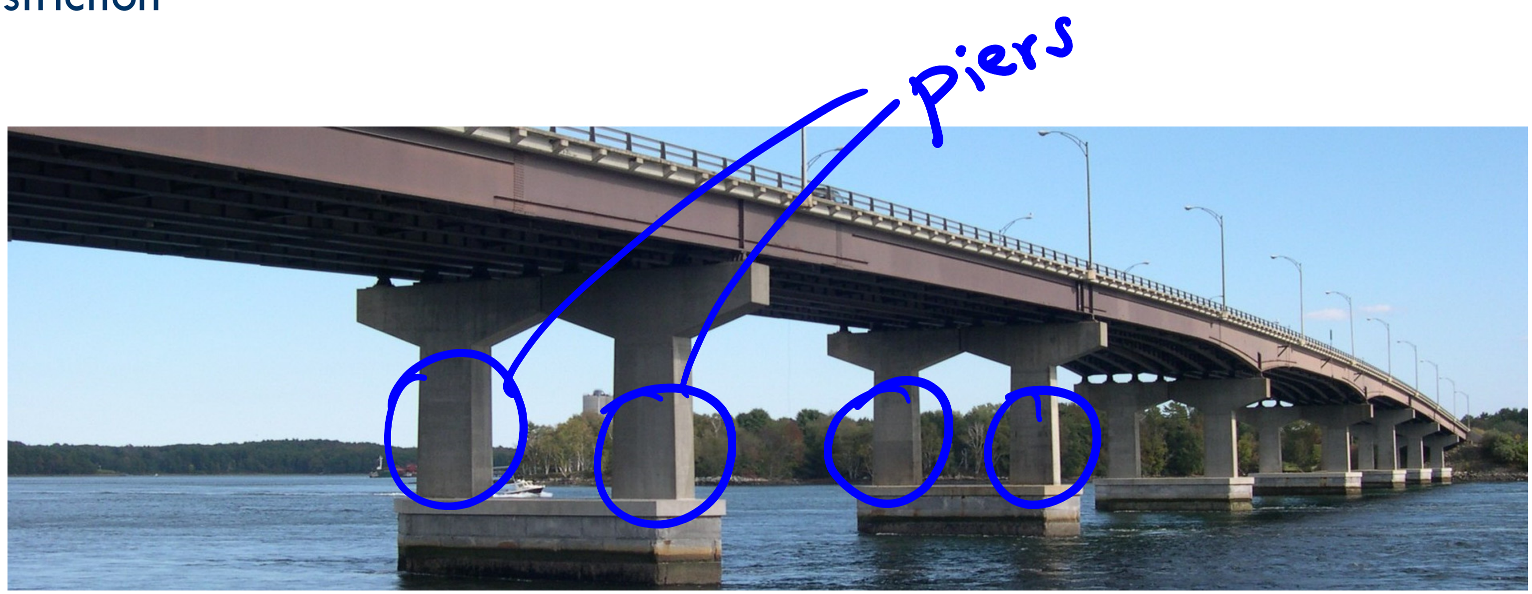
Bridge piers

Source: https://commons.wikimedia.org/wiki/File:Bridge_Piers_P2110004_US_27_Central_Ave.JPG

Bridge piers

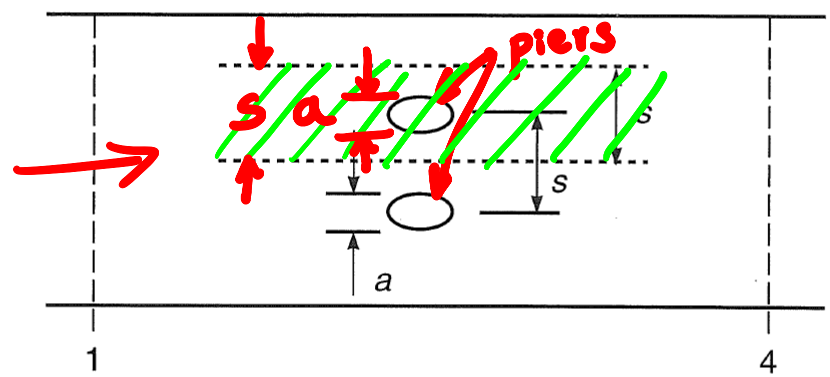


- Obstruction caused by bridge piers in **subcritical flows leads to backwater effects upstream** and can even cause choking.
- **Two types** of flow.
- **Type I flow:** The depth decreases when passing through the constriction with the **flow remaining subcritical**.
- **Type II flow: choking occurs** with critical depth existing in the constriction

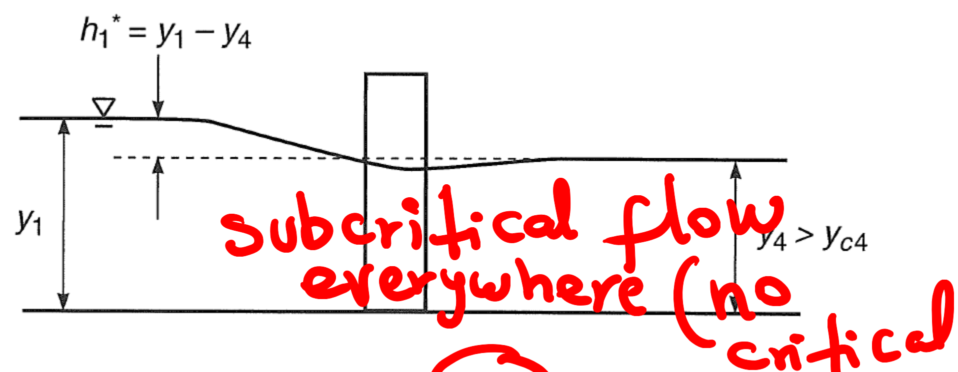


Source: https://engineeringmaster.in/2017/04/05/how-bridges-are-built-over-water/

Bridge piers

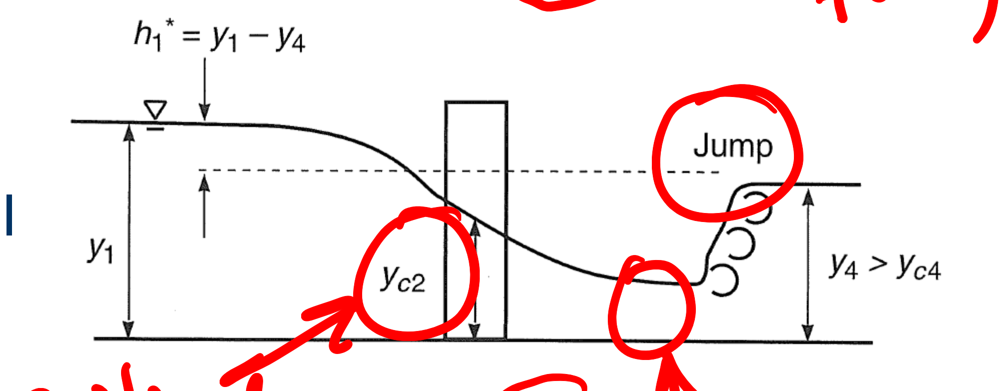


Type I Flow: Subcritical approach flow **without choking**



(b) Profile of Type I Flow

Type II Flow: Subcritical approach flow **with choking** and supercritical flow downstream of pier



(c) Profile of Type II Flow

Bridge piers (Cont.)

Type I Flow

$$M_1 = M_4 + \frac{D}{\gamma}$$

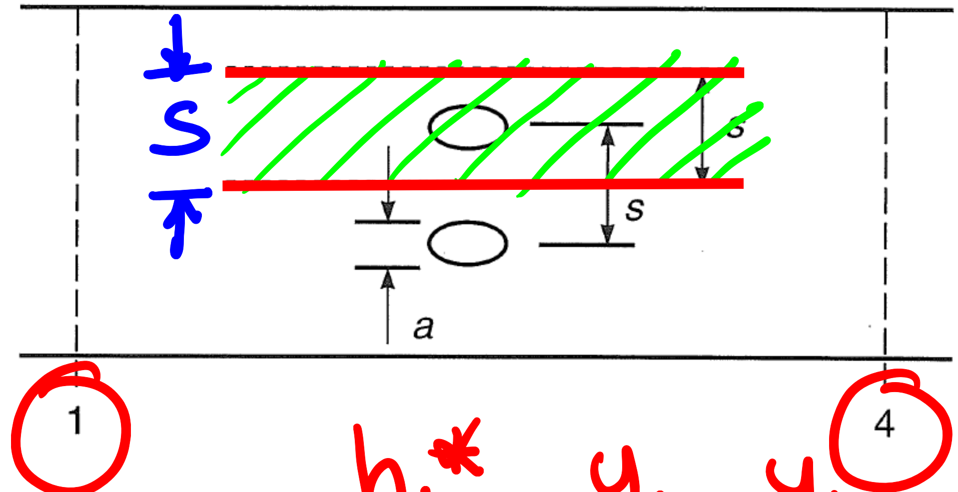
$$M = A h_c + \frac{Q^2}{gA}$$

$$\boxed{D = C_D \rho A_p \frac{V_1^2}{2}}$$

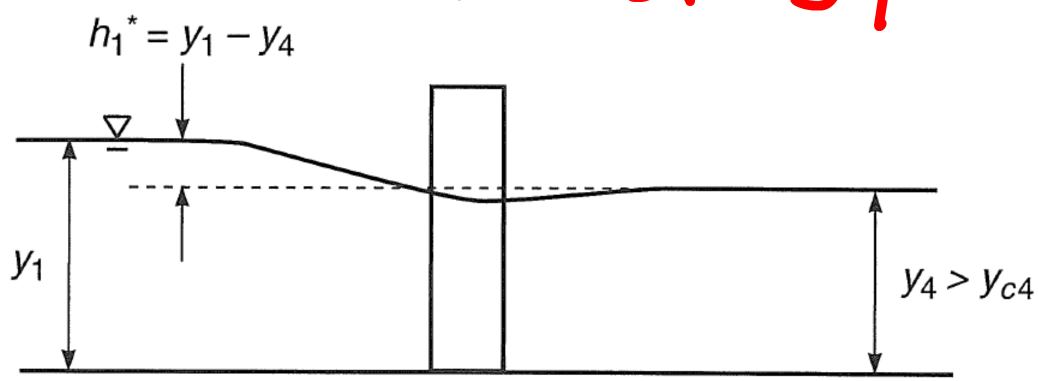
C_D = Drag coefficient

A_p = Frontal area of pier

$h_1{*}$ = Change in depth or **backwater**



$h_1^* = y_1 - y_4$



(b) Profile of Type I Flow

Bridge piers

s: spacement

$$M_1 = M_4 + \frac{D}{\gamma}$$

$$\boxed{b = s}$$

For a rectangular channel:

$$\frac{M = \dfrac{by^2}{2} + \dfrac{Q^2}{gby}}{s}$$

$$\frac{M_1}{s} = \frac{M_4}{s} + \frac{D}{\gamma s}$$

$$q = \frac{Q}{s}$$

Ap

$$\frac{y_1^2}{2} + \frac{Q^2}{g b^2 y_1} = \frac{y_4^2}{2} + \frac{Q^2}{g b^2 y_4} + \frac{C_D (a y_1) V_1^2}{2gs}$$

$$\frac{y_1^2}{2} + \frac{q^2}{g y_1} = \frac{y_4^2}{2} + \frac{q^2}{g y_4} + \frac{C_D a y_1 V_1^2}{2gs} \quad \ldots (1)$$

$$R = \frac{C_D a}{S}, \qquad h_1^* = y_1 - y_4 \text{ (backwater height)}.$$

$$\lambda = \frac{y_1 - y_4}{y_4} = \frac{y_1}{y_4} - 1$$

$$\lambda + 1 = \frac{y_1}{y_4}, \quad \lambda + 2 = \frac{y_1}{y_4} + 1$$

In (1)

$$\frac{y_1^2 - y_4^2}{2} + \frac{q^2}{g y_1} - \frac{q^2}{g y_4} = \frac{R y_1 V_1^2}{2g}$$

$$\frac{y_1^2 - y_4^2}{2} = \frac{y_4^2}{2}\left[\left(\frac{y_1}{y_4}\right)^2 - 1\right] = \frac{y_4^2}{2}\underbrace{\left(\frac{y_1}{y_4} - 1\right)}_{\lambda}\underbrace{\left(\frac{y_1}{y_4} + 1\right)}_{\lambda+2}$$

$$\frac{y_1^2 - y_4^2}{2} = \frac{y_4^2}{2}\lambda(\lambda+2) \quad (*)$$

$$Q_1 = Q_4$$

$$V_1 y_1 \cancel{s} = V_4 y_4 \cancel{s}$$

$$q = \frac{Q}{s}$$

$$q = \frac{V_4 y_4 \cancel{s}}{\cancel{s}}$$

$$\boxed{q = V_4 y_4}$$

$$* \quad \frac{q^2}{g y_1} - \frac{q^2}{g y_4} = \frac{V_4^2 y_4^2}{g y_4}\left[\frac{y_4}{y_1} - 1\right]$$

$$Fr_4^2$$

$$Fr_4 = \frac{V_4}{\sqrt{g y_4}}$$

$$Fr_4^2 = \frac{V_4^2}{g y_4}$$

$$\frac{1}{\lambda+1}$$

$$= Fr_4^2\, y_4^2\left(\frac{y_4}{y_1} - 1\right) = Fr_4^2\, y_4^2\left(\frac{1}{\lambda+1} - 1\right)$$

$$= Fr_4^2\, y_4^2\left(\frac{-\lambda}{\lambda+1}\right)$$

$$* \quad \frac{C_D\, a\, y_1 V_1^2}{2gS} = \frac{R\, y_1 V_1^2}{2g} = \frac{R\cancel{y_1}}{2g}\,\frac{V_4^2\, y_4^2}{y_1^{\cancel{2}}}$$

$$= \frac{V_4^2\, y_4^2}{y_1 \times y_4}\left(\frac{R}{2g}\right) y_4 = \left(\frac{V_4^2}{g\, y_4}\right)\left(\frac{y_4^2}{y_1}\right)\left(\frac{R}{2}\right) y_4$$

$$= \frac{Fr_4^2}{2}\, y_4^2 \left(\frac{y_4}{y_1}\right) R = \frac{Fr_4^2\, y_4^2}{2}\left(\frac{1}{\lambda+1}\right) R$$

* Substituting :

$$\cancel{\frac{y_4^2}{2}}\,\lambda(\lambda+2) + Fr_4^2\,\cancel{\frac{y_4^2}{2}}\left(\frac{-\lambda}{\lambda+1}\right) = \frac{Fr_4^2}{\cancel{2}}\,\cancel{\frac{y_4^2}{2}}\,\frac{R}{(\lambda+1)}$$

$$\lambda(\lambda+2) + 2\underline{Fr_4^2}\left(\frac{-\lambda}{\lambda+1}\right) = \underline{Fr_4^2}\,\frac{R}{\lambda+1}$$

$$\lambda(\lambda+2) = \frac{Fr_4^2}{\lambda+1}\left[R + 2\lambda\right]$$

$$R = \frac{C_D a}{S}$$

$$Fr_4^2 = \frac{\lambda(\lambda+1)(\lambda+2)}{R+2\lambda}$$

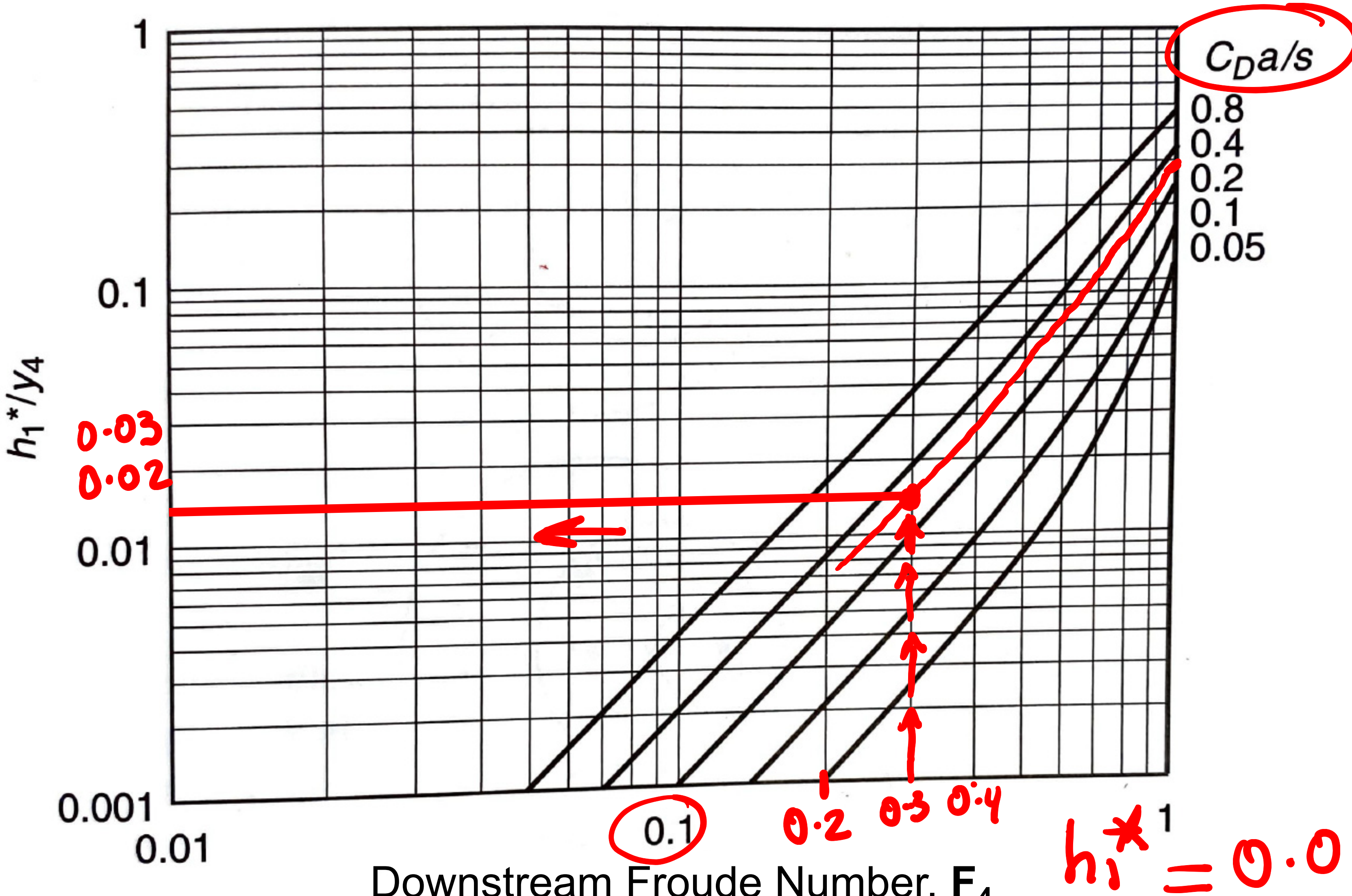
$$\lambda = \frac{h_1^*}{y_4}$$

$$h_1^* = y_1 - y_4$$

$$F_4^2 = \frac{\lambda(\lambda+1)(\lambda+2)}{\frac{C_D a}{s}+2\lambda}$$

Solution for backwater caused by bridge piers in Type I flow

$$F_4^2 = \frac{\lambda(\lambda+1)(\lambda+2)}{\frac{C_D a}{s}+2\lambda}$$



$$\frac{h_1^*}{y_4} = 0.015$$

$h_1^* = 0.06\text{m}$

Example of application

For a river flow between bridge piers 3 m in diameter with a spacing of 20 m, determine the backwater using the momentum method if the downstream depth is 4.0 m and the downstream velocity is 1.9 m/s. Assume a drag coefficient of 2.0 for the bridge piers.

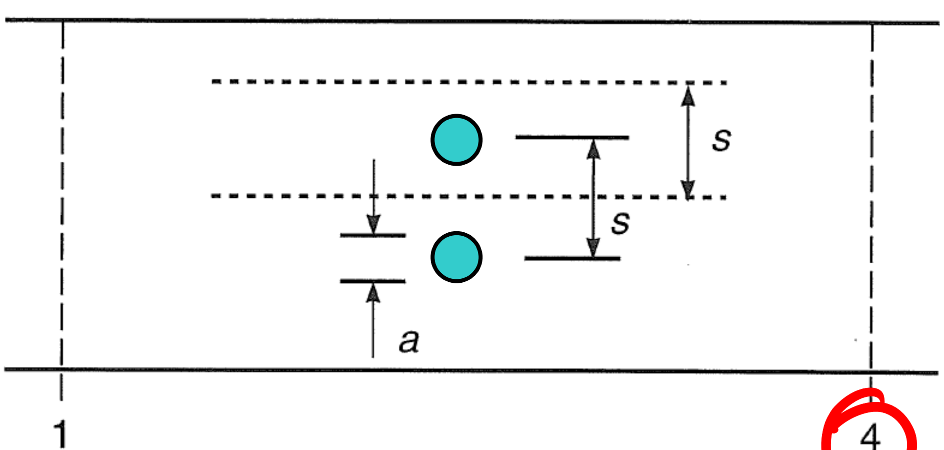
$a = 3\,m$

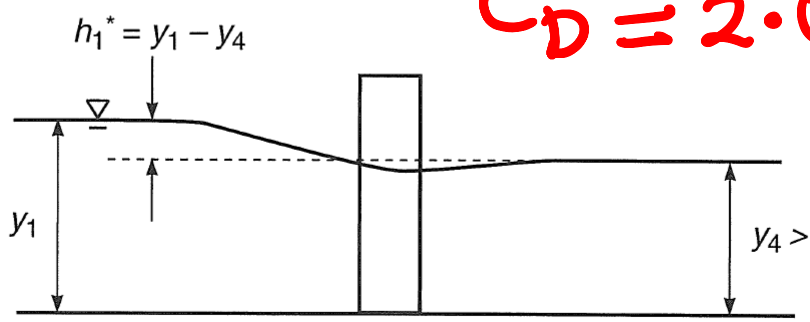
$s = 20\,m$

$y_4 = 4\,m$

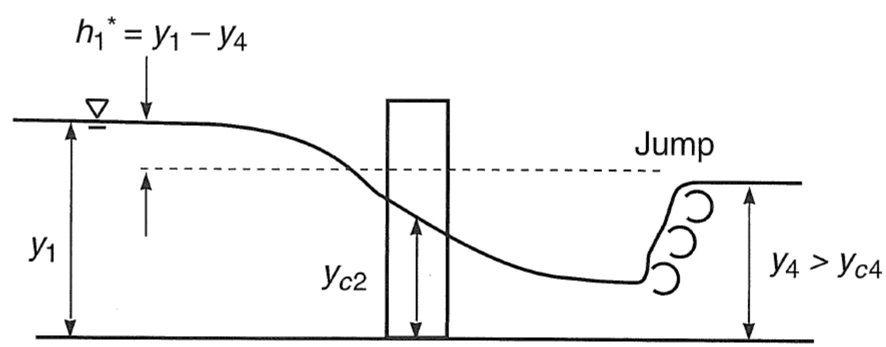
$V_4 = 1.9\,m/s$

$C_D = 2.0$





(b) Profile of Type I Flow



(c) Profile of Type II Flow

$$h_1^* = y_1 - y_4 = ??$$

$$Fr_4^2 = \frac{\lambda(\lambda+1)(\lambda+2)}{C_D \frac{a}{s} + 2\lambda} \quad \ldots (1)$$

$$* \ \frac{C_D a}{S} = 2.0 \times \frac{3}{20} = 0.3$$

$$* \ F_{r_4} = \frac{V_4}{\sqrt{g y_4}} = \frac{1.9}{\sqrt{9.8 \times 4}} = 0.303$$

In (1)

$$(0.303)^2 = \frac{\lambda(\lambda+1)(\lambda+2)}{0.3 + 2\lambda} \longrightarrow \boxed{\lambda = 0.0148}$$

$$\lambda = \frac{h_1^*}{y_4} = 0.0148 \longrightarrow h_1^* = 0.0148 \times 4$$

$$h_1^* = 0.059\,m \approx 6\ cm$$

(backwater)