Hydraulic Structures

Arturo S. Leon, PhD, PE, D.WRE

Hydraulic Structures

- **Underflow gates**
  - Underflow gates are frequently used to **control discharge** in canals or over spillway crests.
- **Spillways**
  - Used on both large and small dams to **pass flood flows**, thereby preventing overtopping and failure of the dam
- **Stilling Basins**
  - **Dissipate energy**, reduce velocity and erosion downstream
- **Culverts**
  - **Carry peak flood discharges** under roadways or other embankments to prevent embankment overflows
- It is important to estimate magnitude of **backwater upstream of the structure** for a given design discharge.
  - Head-discharge relationship for the structure

Underflow Gates

underflow gates

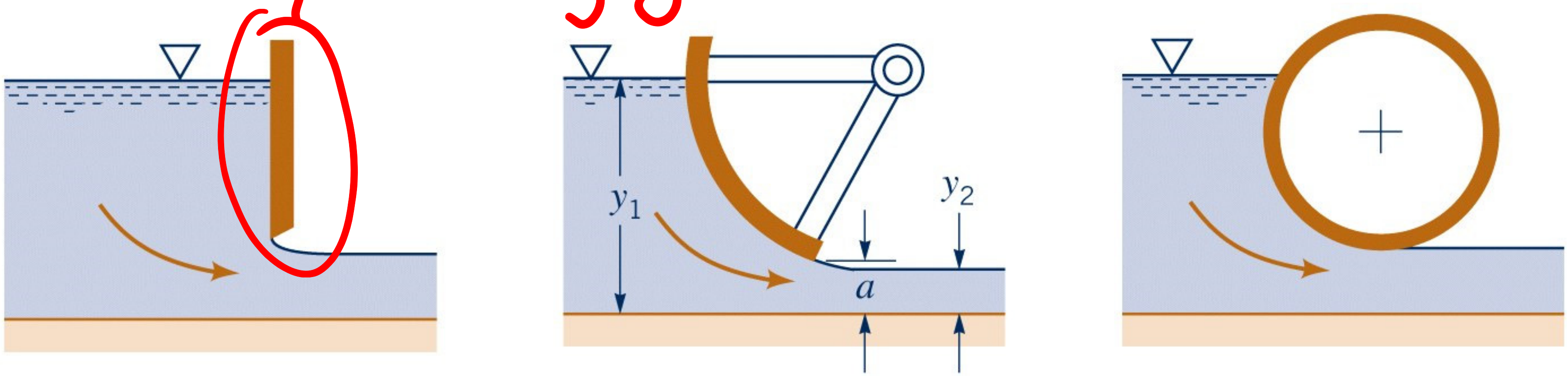
spillway

Source: https://commons.wikimedia.org/wiki/File:Benmore_(Earth)_Dam,_Spillway_Gates_-_panoramio.jpg

stilling basin

Types of Underflow Gates

Sluice gate
Sliding gate



(a) Vertical gate

(b) Radial gate

(c) Drum gate

Typically expensive (water tight, mechanically activated)

cheap (Irrigation channel)

Underflow Gates

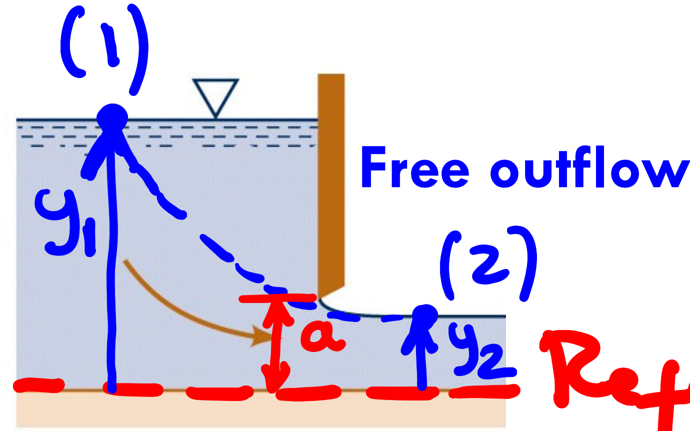
Bernoulli eq.

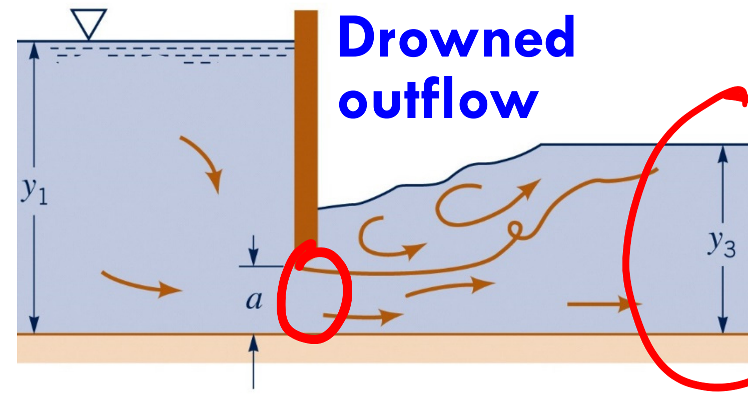
$$\frac{\overset{0}{\cancel{P_1}}}{\gamma} + \frac{\overset{\approx 0}{\cancel{V_1^2}}}{2g} + \underbrace{z_1}_{y_1} = \frac{\overset{0}{\cancel{P_2}}}{\gamma} + \frac{V_2^2}{2g} + \underbrace{z_2}_{y_2}$$

$$y_1 = \frac{V_2^2}{2g} + y_2$$

$$V_2 = \sqrt{2g(y_1 - y_2)}$$

$$Q_{\text{theoretical}} = V_2 \cdot \text{Area}_2$$





$$Q \approx a \times \text{width} \sqrt{2g(y_1 - y_2)}$$

$$q = \frac{Q}{\text{width}} \sim q = a\sqrt{2g(y_1 - y_2)}$$

$y_2 \ll y_1$

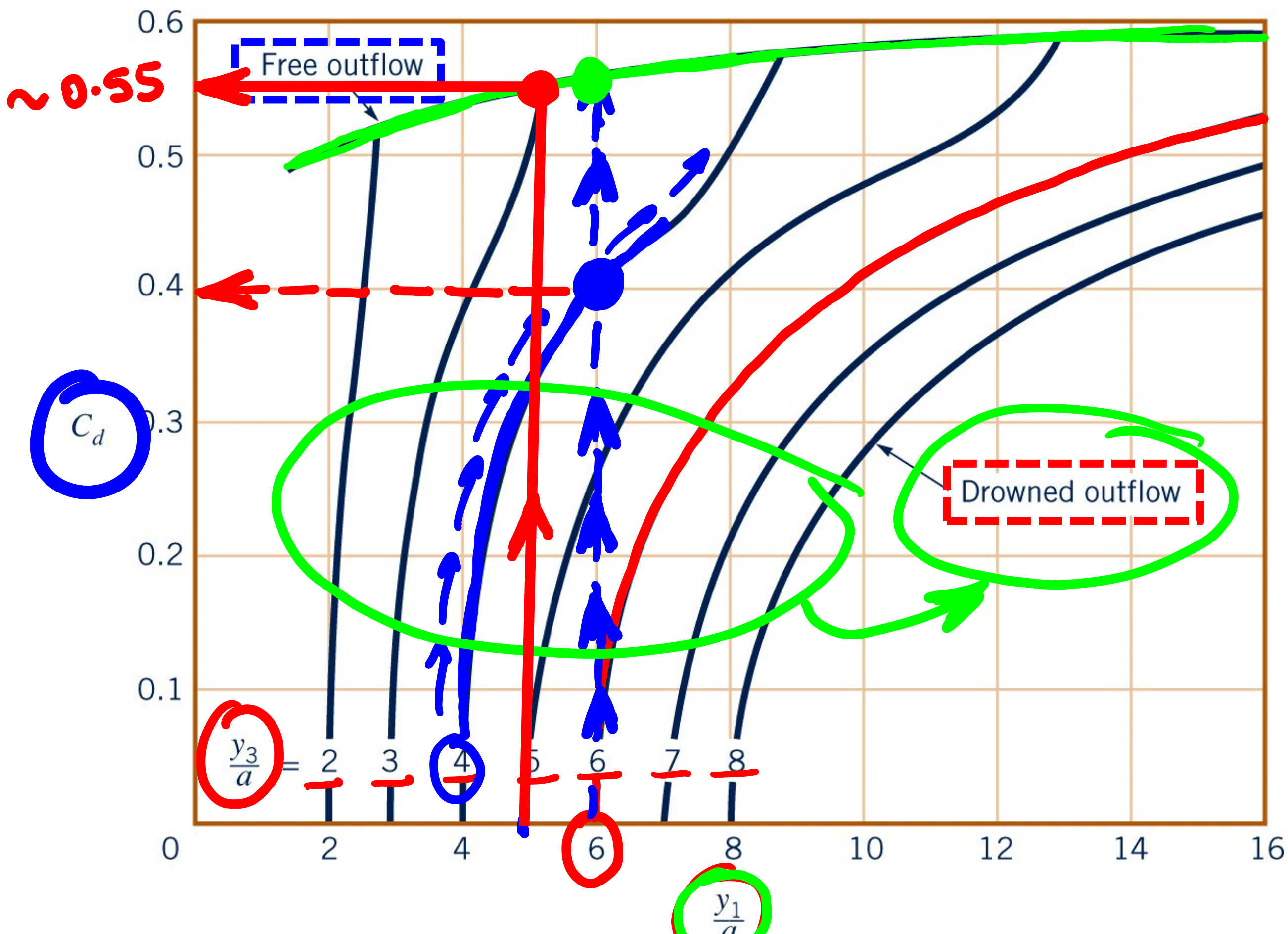
$$\boxed{q \approx a\sqrt{2gy_1}}$$

Underflow Gates

C_d : Discharge coefficient

$$q = C_d a \sqrt{2gy_1}$$

Where q is the flowrate per unit width



Typical discharge coefficient for underflow gates

$\frac{y_1}{a} = 6,$

$\frac{y_3}{a} = 4$

$C_d = ??$

$C_d = 0.4$

Example

Water flows under a sluice gate in a 60-ft-wide finished concrete channel ($n = 0.012$) as is shown in the figure below. Determine the flowrate. If the slope of the channel is 2.5 ft/200 ft, will the water depth increase or decrease downstream of the gate? Assume $C_C = y_2/a = 0.65$. Explain.

$b = 60\ ft$

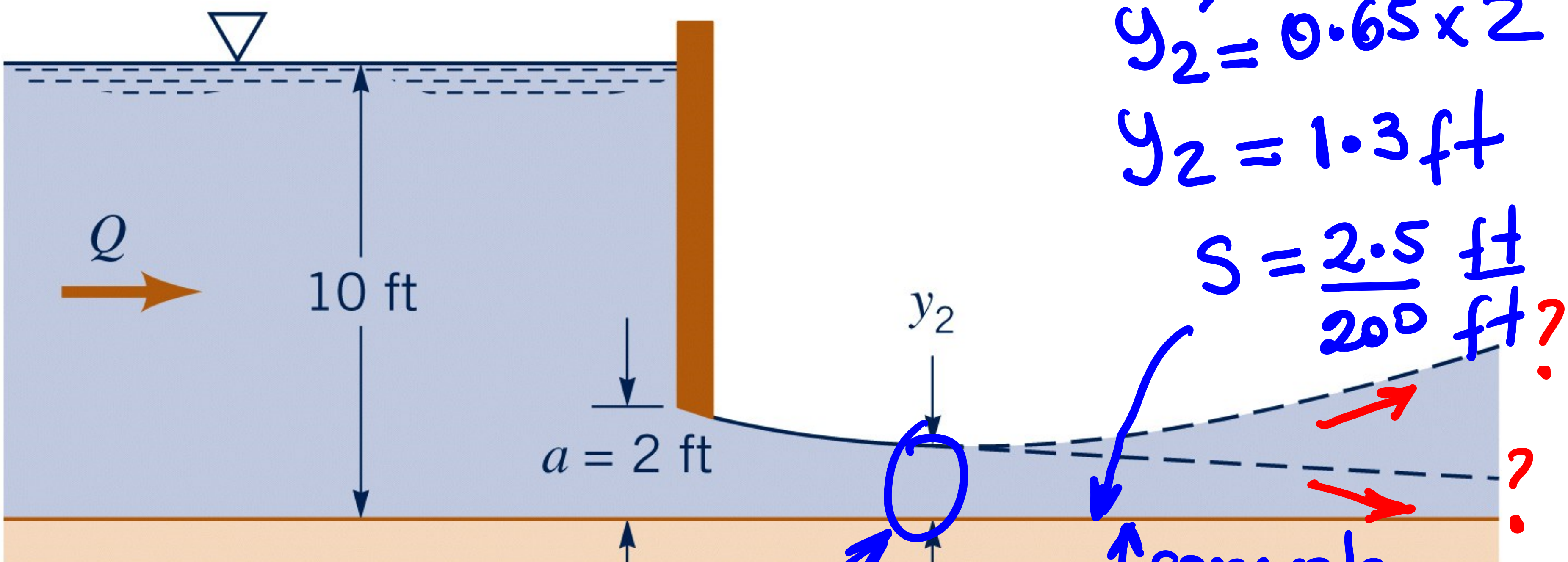
$Q = ??$

$\frac{y_2}{a} = 0.65$

$y_2 = 0.65 \times 2$

$y_2 = 1.3\ ft$

$S = \frac{2.5\ ft}{200\ ft}$



vena contracta

concrete channel
$n = 0.012$

* Flow discharge

$$q = C_d\, a\sqrt{2g y_1}$$

$a = 2\,ft$

$\frac{y_1}{a} = 5$

$y_1 = 10\,ft$

$Q = q \times b$

$C_d = 0.55$ (free outflow)

$$Q = 0.55 \times 2 \times \sqrt{2 \times 32.2 \times 10} \times 60$$

$$Q = 1670\ ft^3/s$$

* Water depth increase or decrease??

Normal depth y_0

$$Q = \frac{k}{n} A R^{2/3} S_o^{1/2}$$

$$1670 = \frac{1.49}{0.012}(60 y_0)\left(\frac{60 y_0}{60 + 2y_0}\right)^{2/3}\left(\frac{2.5}{200}\right)^{0.5}$$

$$\boxed{y_0 = 1.54\ ft}$$

$1.54\,ft > 1.3\,ft$ (water depth will increase)

Spillways

Spillways

- **Transfer large flood discharges** safely from a reservoir to the downstream river.

Types of Spillway

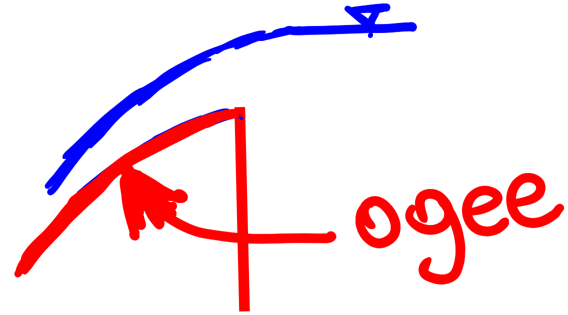
- Free overfall or straight drop Spillway
- Ogee or overflow spillway
- Chute or open channel or trough spillway
- Side-channel spillway
- Shaft or drop inlet or morning glory spillway
- Conduit or tunnel spillway
- Siphon spillway
- Labyrinth Spillway

Free overfall spillway

Source: https://twitter.com/TVAnews/status/859460297630810113/photo/2

Very common

Ogee spillway



Source: https://www.engineeringdiscoveries.net/2019/04/different-types-of-spillways.html

Chute spillway

(steep channel) supercritical flow

Source: https://yubanet.com/regional/researchers-identify-factor-behind-2017-oroville-dam-spillways-incident/

Side-channel spillway

Source: https://www.americansouthwest.net/arizona/lake_mead/overflow-channel_l.html

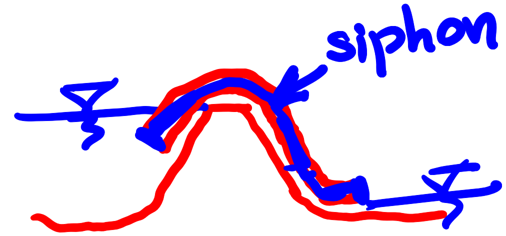
Morning Glory Spillway

Source: https://pl.pinterest.com/pin/469007748672648312/

Tunnel spillway

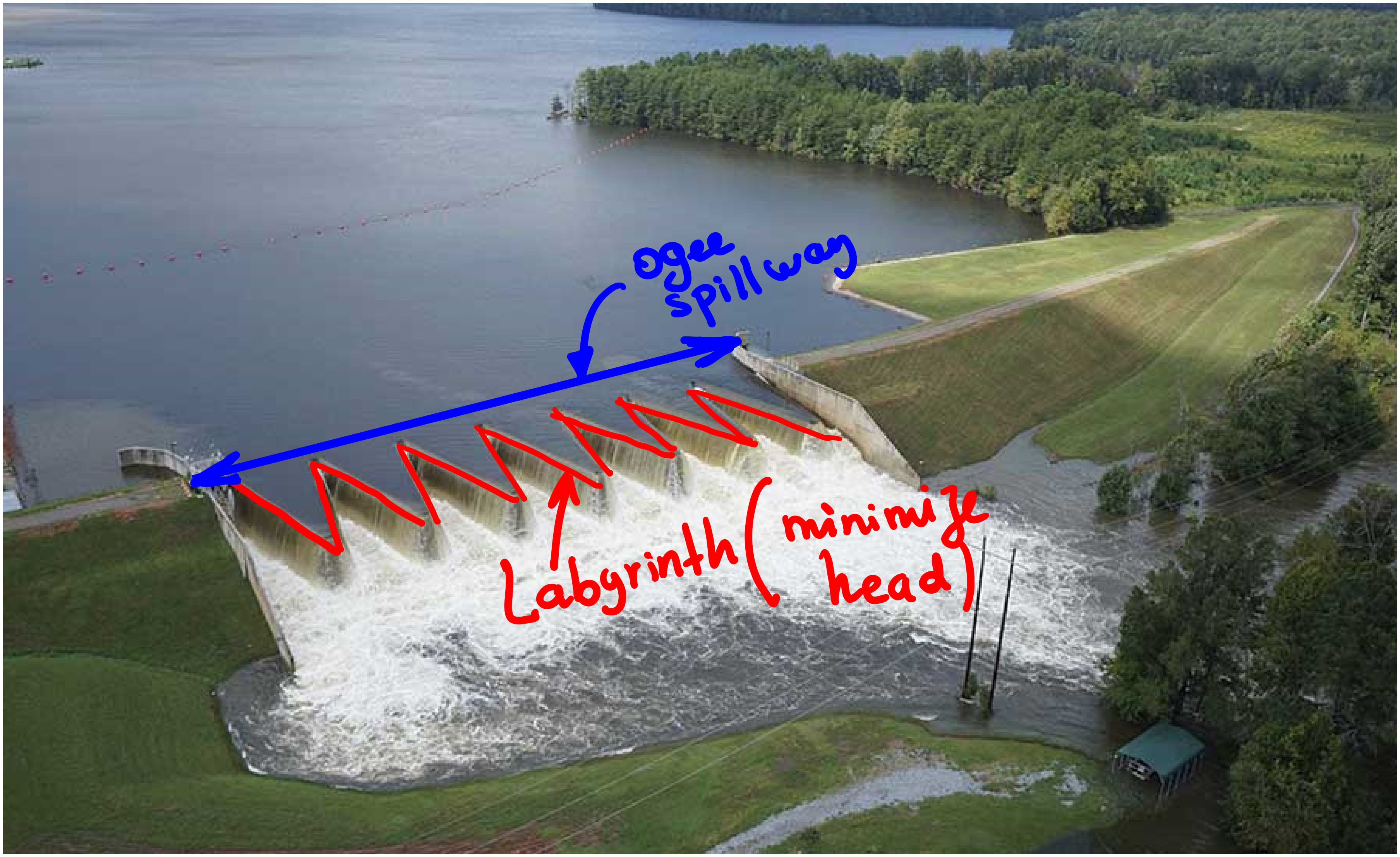
Source: https://commons.wikimedia.org/wiki/File:Kortes_Dam_Spillway_in_operation_(10742601664).jpg

Siphon spillway



Source: https://www.youtube.com/watch?app=desktop&v=emwTKOLj6c0

Labyrinth Spillway

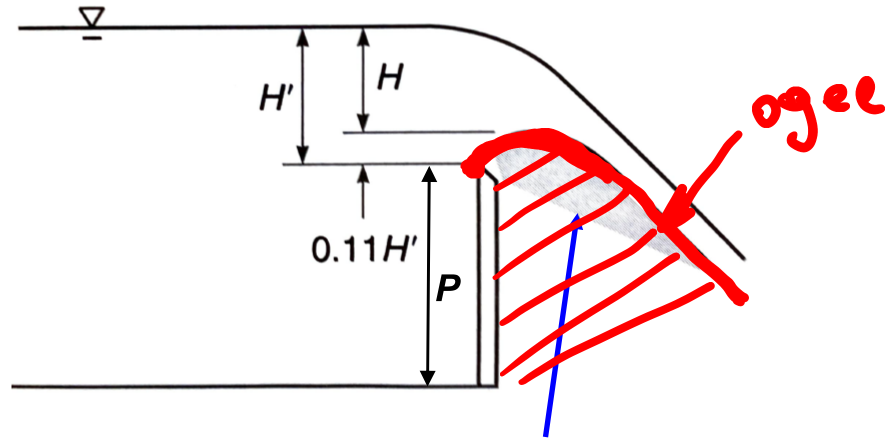


Source: https://www.enr.com/articles/45255-the-carolinas-survey-florence-damages

Ogee Spillways

- Characteristic **ogee shape is based on the shape of the underside of the nappe** coming off a ventilated, sharp-crested weir
  - to **maintain pressure on the face of the spillway near atmospheric** and well above the cavitation pressure
  - Rehbock relationship for the discharge coefficient (C_d) of a sharp-crested weir

Ogee spillway and equivalent sharp-crested weir



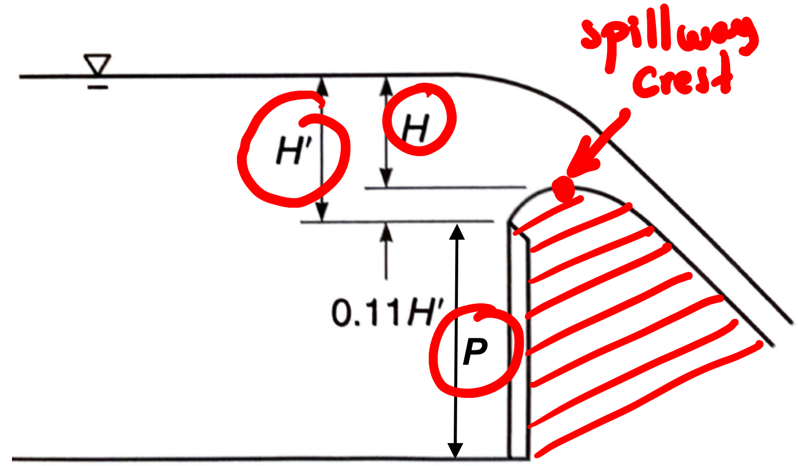
Concrete spillway crest conforming to the underside of nappe of sharp-crested weir

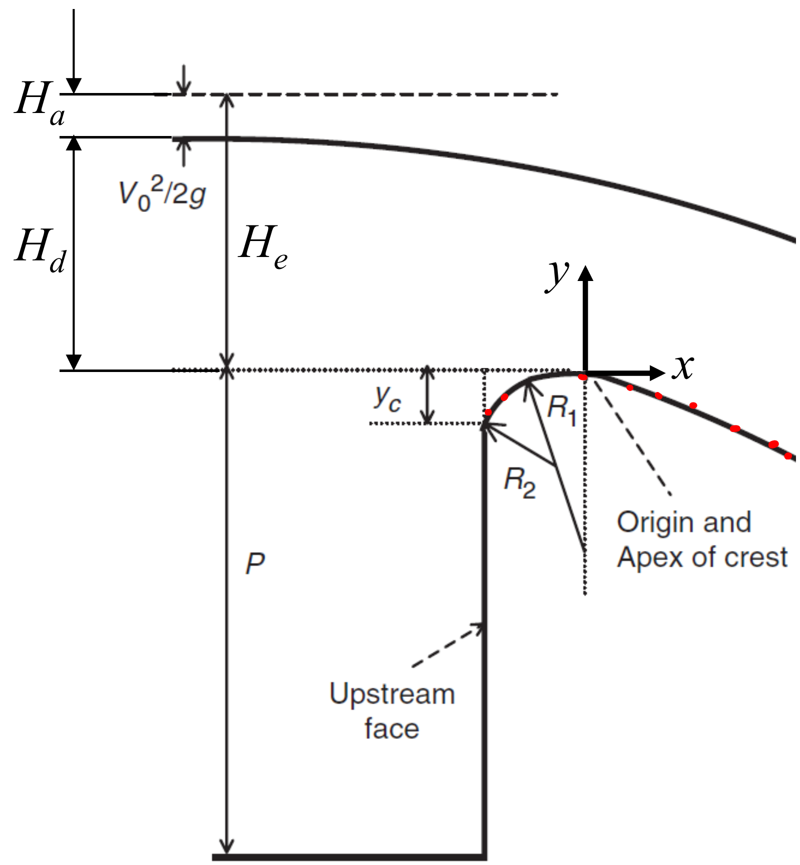
$$C_d = 0.611 + 0.08\frac{H'}{P}$$

Rehbock relationship for Cd

Very high ogee spillways

- For very high spillways, contribution of the term involving ***H′/P* becomes small**
- Discharge coefficient, ***Cd*, approaches a value of 0.611 for a head of *H′*** on a sharp-crested weir
- **In terms of the head, *H***, which is measured relative to the ogee spillway crest, ***Cd* = 0.728 because *H* = 0.89*H′***





Lower spillways

- For lower spillways, the effect of approach velocity and vertical contraction of the water surface introduce an additional geometric parameter given by H/P or its inverse.
- Design value of the discharge coefficient is valid for one specific value of head, called the **design head**, H_d
- As the **head** becomes **larger than the design head**, the **pressures on the face of the spillway become less than atmospheric** and can approach cavitation conditions.
  - Pressures are larger than atmospheric for heads less than the design head
- Risk of cavitation at heads higher than design head is counterbalanced by higher discharge coefficients
  - because of the **lower pressures on the face** of the spillway
  - spillway becomes more efficient because it **passes a higher discharge for the same head** with a larger discharge coefficient

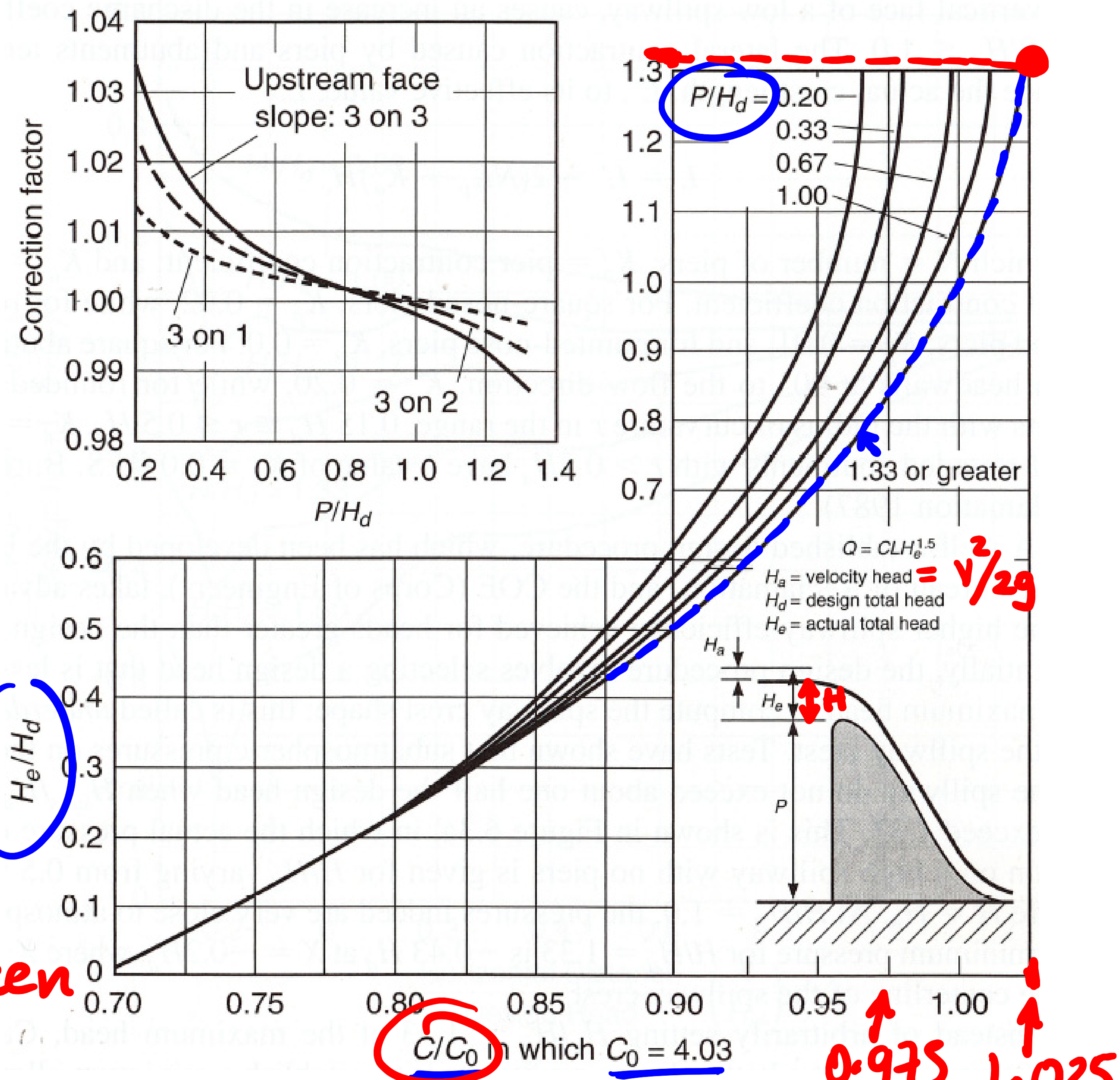
Lower spillways (Cont.)

Arbitrary head.

$$H_e = H + \frac{V^2}{2g}$$

$$H_{ed} = H_d + \left(\frac{V^2}{2g}\right)_d$$

No relation between He and Hd



Discharge coefficient for the WES standard spillway shape (Chow 1959) [C_o = 4.03, English units]

Spillways (Cont.)

$$(He)_d = Hd + \left(\frac{V^2}{2g}\right)_d$$

- In analogy with the sharp-crested weir

$$Q = CLH_e^{3/2}$$

$$He = H + \frac{V^2}{2g}$$

$$C = \frac{Q}{LH_e^{3/2}} \qquad C_d = \frac{C}{\frac{2}{3}\sqrt{2g}}$$

- The lateral contraction caused by piers and abutments tends to reduce the actual crest length, L', to its effective value, L:

$$L = L' - 2(NK_p + K_a)H_e$$

where N = number of piers; K_p = pier contraction coefficient; and K_a = abutment contraction coefficient

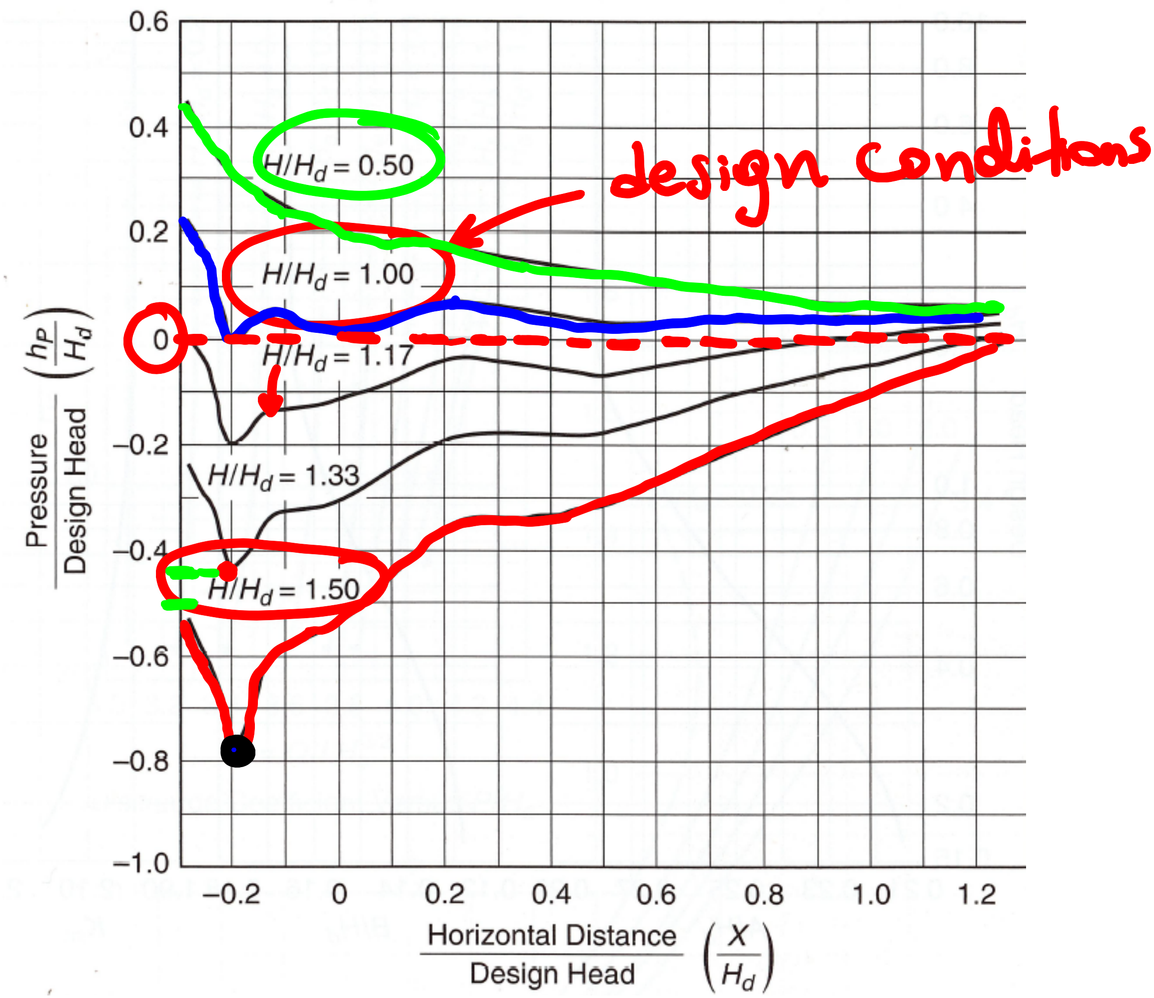
Description	K_p
Square-nosed piers	0.02
Round-nosed piers	0.01
Pointed-nosed piers	0.0

Description	K_a
Square abutments with headwalls at 90° to the flow direction	0.20
Rounded abutments with radius of curvature r in the range $0.15H_d \leq r \leq 0.5H_d$	0.10
Well-rounded abutments with $r > 0.5H_d$	0.0

USBR and USACE design method:

- Involves **selecting a design head** that is **smaller than the maximum head** to compute the spillway crest shape;
    - **Under-designing** of the spillway crest.
- Tests have shown
    - subatmospheric pressures on the face of the spillway do not exceed about one half the design head when H_{max}/H_d does not exceed **1.33**.

$\frac{H_{max}}{H_d} \leq 1{\cdot}33$



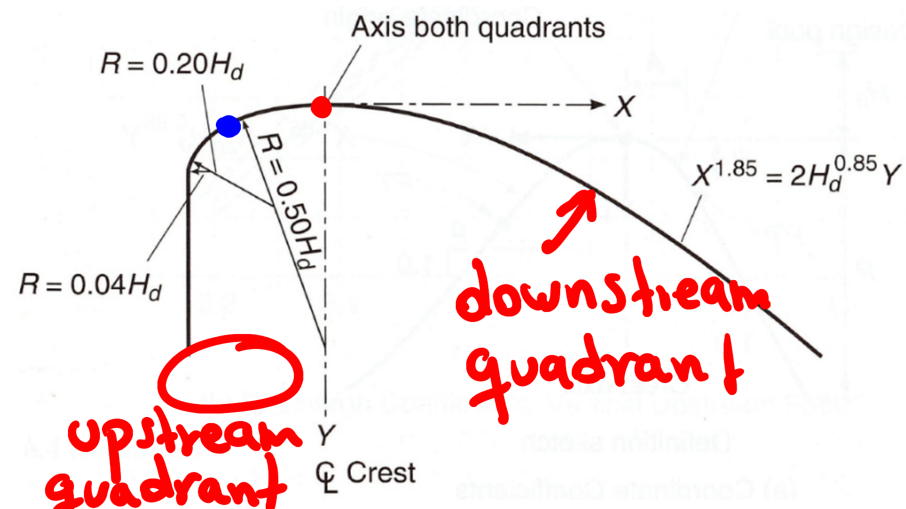
Crest pressure on WES high-overflow spillway–no piers

$He = H + V^2/2g$

USBR and USACE design method (Cont.):

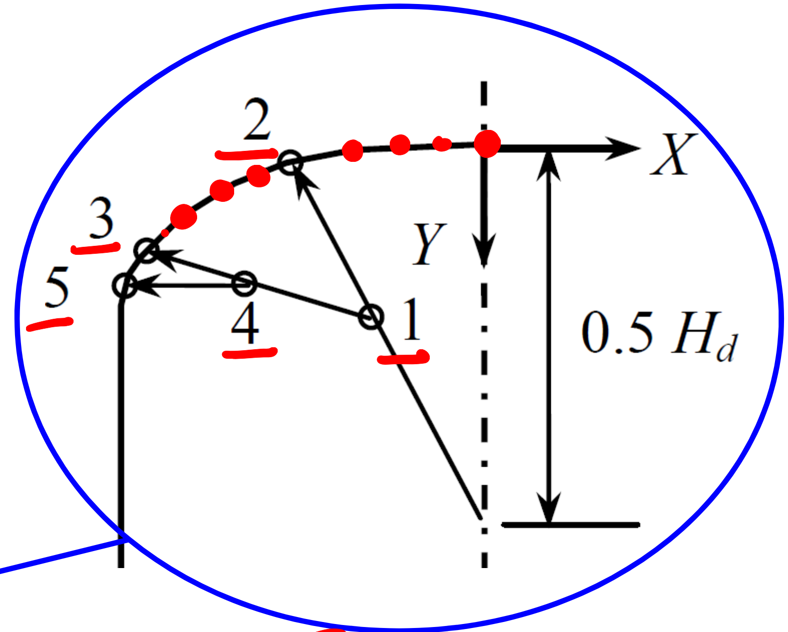
arbitrary head.

- Instead of arbitrarily setting H_e/H_d = **1.33** at the maximum head, Cassidy (1970) suggested to establish a **minimum allowable pressure on the spillway face** and then determine the design head.
- USACE recommends a conservative design procedure of **not allowing the average pressure head to fall below -15 ft to -20 ft**.
- Once the design head is determined
  - Actual shape of the spillway crest downstream of the apex, in what is called the **downstream quadrant**:
  - **Upstream quadrant** of the spillway crest is constructed from a compound circular curve
  - 0.04 Hd radius curve was added in the 1970s resulting in a slight increase in the spillway coefficient for $H/H_d > 1.0$ and $P/H_d \geq 1.33$.



Standard WES ogee spillway shape (USACE 1970, Hydraulic Design Chart 111-16)

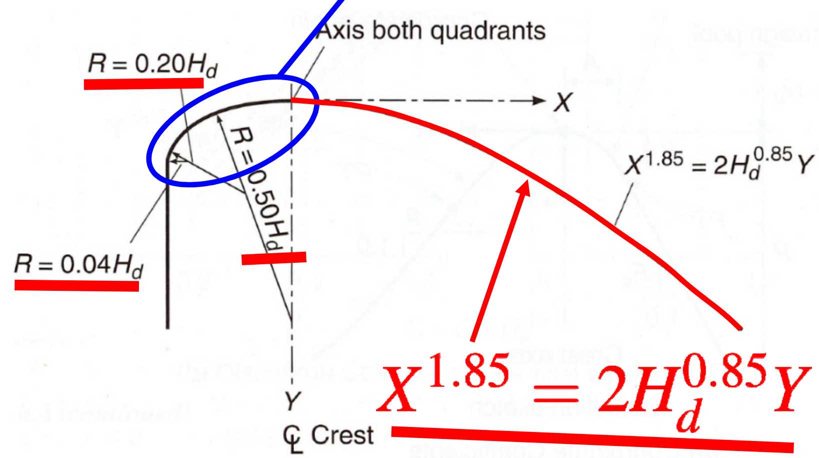
Geometry of Standard WES ogee spillway



$$(X - X_o)^2 + (Y - Y_o)^2 = R^2$$

Coordinates of the key points establishing the tricompound circular curve for the upstream quadrant in the Standard WES ogee spillway shape (USACE HDC 111-2/1, 1970):

Point No.	X/H_d	Y/H_d
1	-0.105	0.219
2	-0.175	0.0316
3	-0.276	0.1153
4	-0.2418	0.136
5	-0.2818	0.136

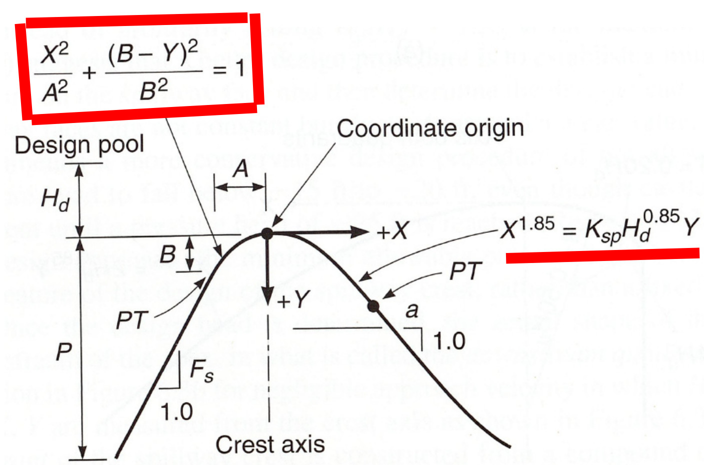


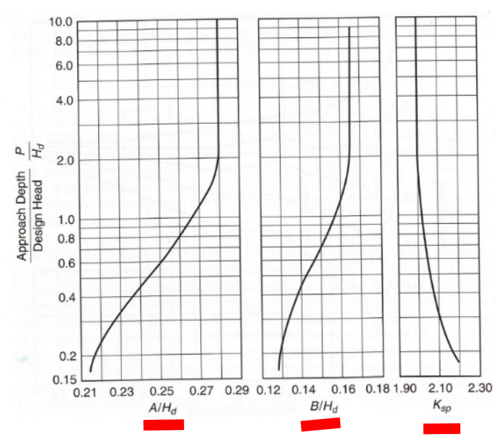
$$X^{1.85} = 2H_d^{0.85}Y$$

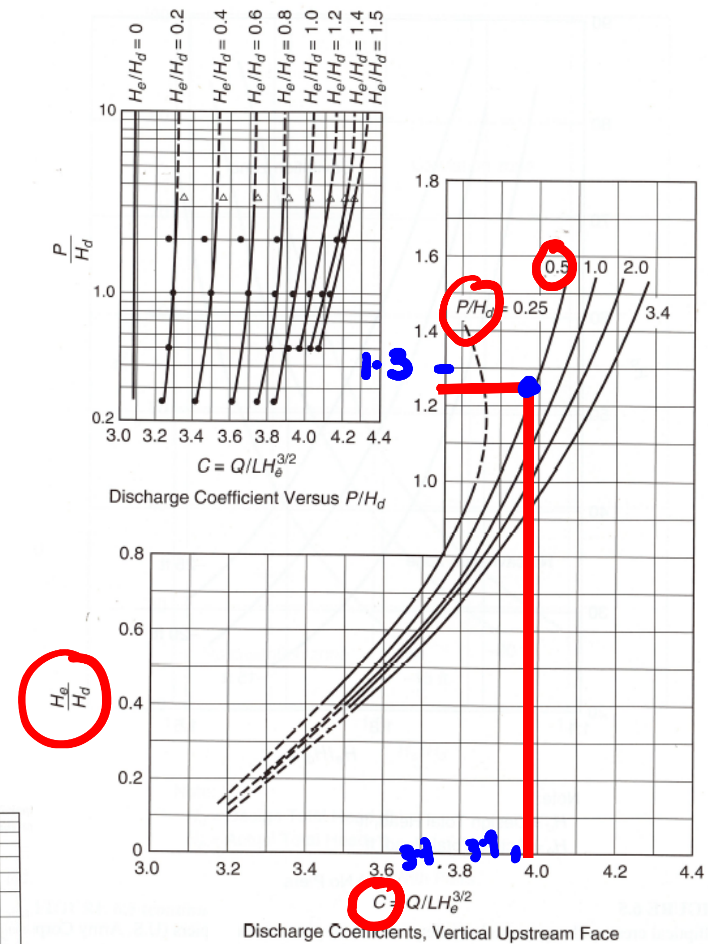
Standard WES ogee spillway shape (USACE 1970, Hydraulic Design Chart 111-16)

USBR and USACE design method (Cont.):

- For the shape of the **upstream quadrant,** instead of a compound circular curve, Reese and Maynard (1987) proposed a **quarter of an ellipse** as the figure below.
- The **discharge coefficients for the ellipse shape**, for a vertical upstream face is shown on the side figure.



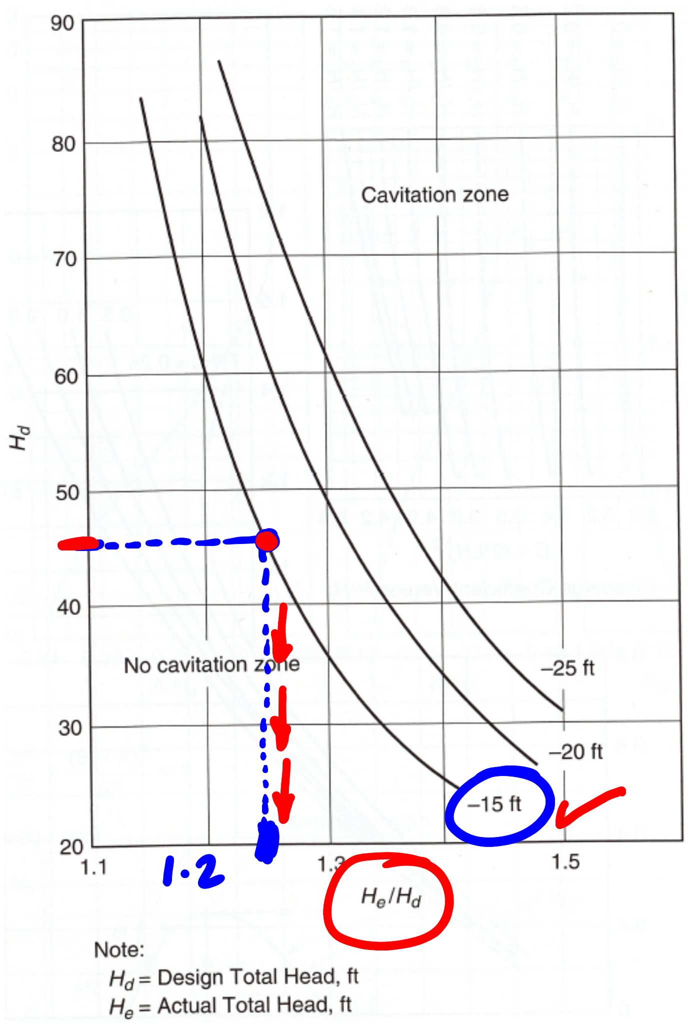




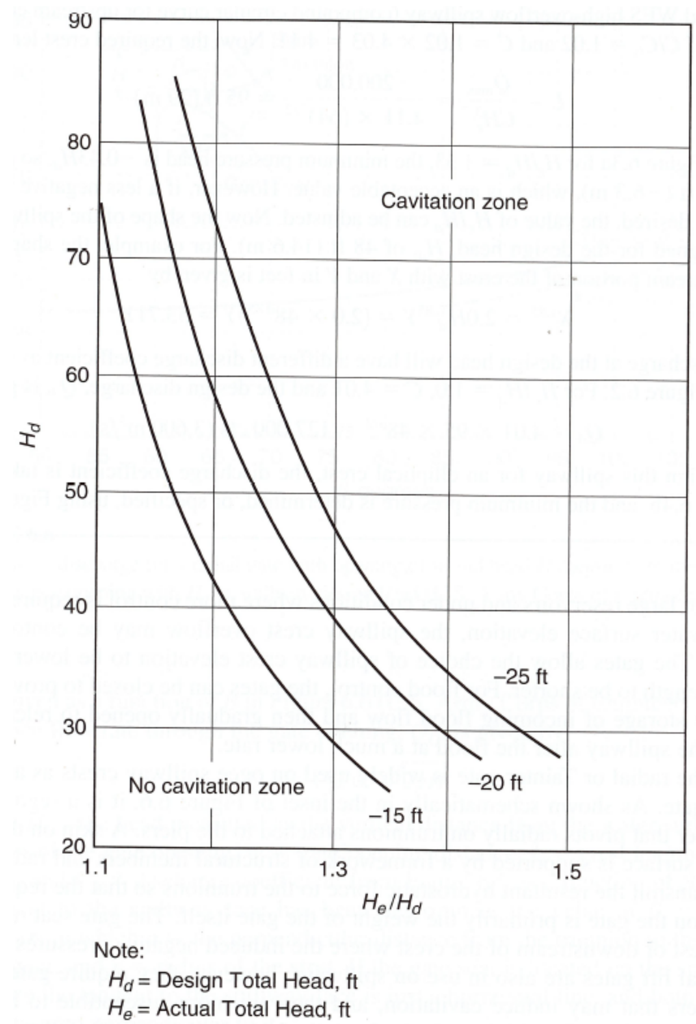
Discharge coefficients for the ellipse shape, for a vertical upstream face

Elliptical crest spillway cavitation safety curves, no piers and with piers

(USACE 1990, Hydraulic Design Chart 111-25)



No piers



With piers

Solution needs to be done in English units.

Example:

A high overflow spillway with $P/H_d > 1.33$ has a maximum discharge of 10,000 cfs with a maximum head of 20 ft. Determine the design head, spillway crest length, and the minimum pressure on the spillway. Plot the complete spillway crest shape for a compound circular curve in the upstream quadrant of the crest.

H_d ?

$L_{spillway}$?

$P_{min} = ?$

$$\frac{P}{H_d} > 1.33$$

$$Q_{max} = 10{,}000 \text{ cfs}$$

$$H_{max} = 20 \text{ ft}$$

$$\frac{H_{max}}{H_d} = 1.33 \rightarrow \frac{20}{H_d} = 1.33$$

$$\boxed{H_d = 15 \text{ ft}}$$

* Spillway crest length.

$$Q = C L H_e^{3/2} \quad \cdots (1)$$

find L for maximum discharge and head.

$$\frac{H_e}{H_d} = \frac{H_{max} + V^2/2g}{15} \approx \frac{20}{15}$$

[apron velocity is typically small compared to Hd]

$$\frac{H_e}{H_d} \approx 1.33, \quad \frac{P}{H_d} > 1.33$$

$$\Rightarrow \frac{C}{C_o} = 1.025, \quad C_o = 4.03$$

$$\rightarrow C = 1.025 \times 4.03$$

$$C = 4.13$$

$\Rightarrow$ From (1)

$$L = \frac{Q_{max}}{C H_e^{3/2}} = \frac{10{,}000}{4.13 \times 20^{3/2}} = 27.1 \text{ ft}$$

* Minimum pressure on spillway.

$\frac{H}{H_d}$ → what H I should use to get the minimum pressure??

we should use H_{max}

$$\frac{H_{max}}{H_d} = 1.33.$$ From chart,

$$\frac{h_p}{H_d} = -0.43$$

$$h_p = -0.43 \times 15 = -6.45 \text{ ft}$$

In Psi:

$$\frac{P}{\gamma} = \text{pressure head}$$

$$\frac{P}{\gamma} = -6.45 \text{ ft}$$

$$P = -6.45\,\cancel{ft}\left[62.4\,\frac{lb}{\cancel{ft^3}}\right]\left[\frac{1\,\cancel{ft^2}}{(12\,in)^2}\right]$$

$$\boxed{P = -2.795\ psi}$$

minimum pressure head obtained is -6.45 ft

The USACE recommends for the pressure head not to fall below -15 ft to -20 ft.

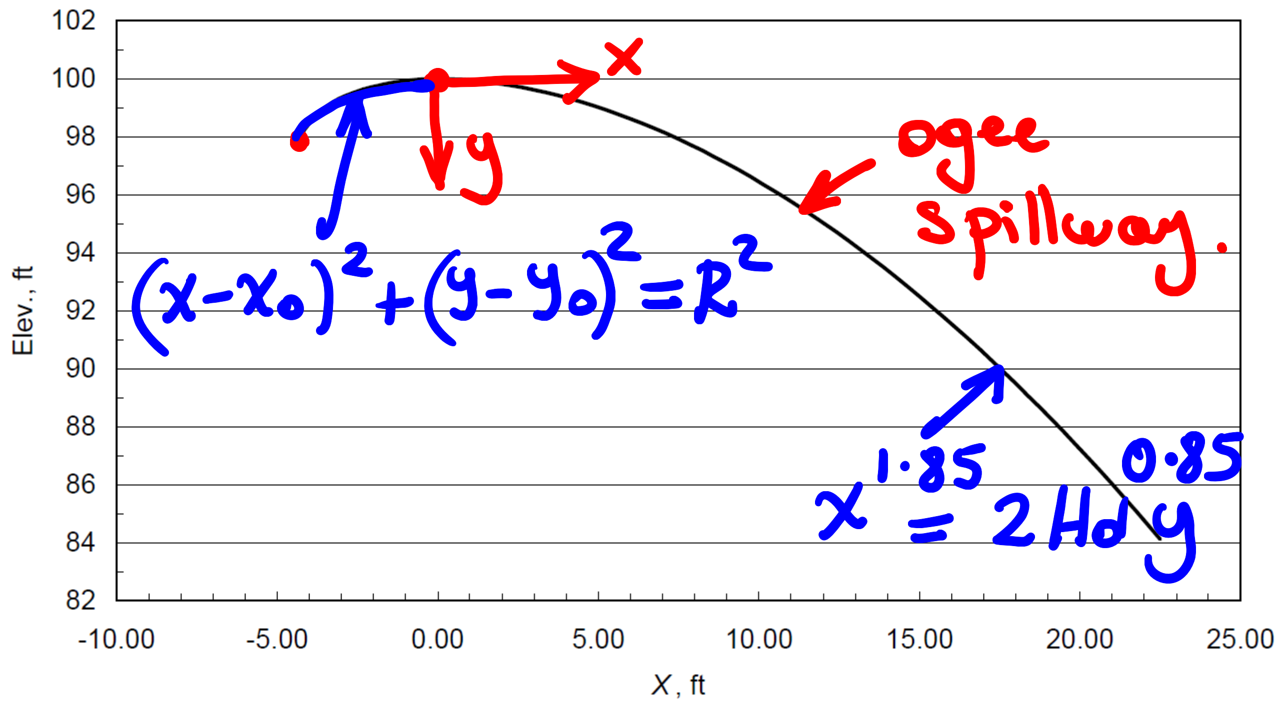
This design is OK in terms of preventing cavitation.

Plot of Ogee Spillway Shape Tricompound Circular Curve

A set of coordinates for plotting the upstream quadrant directly is given in the table below:

X/H_d	Y/H_d
0.00	0.00
-0.0500	0.0025
-0.1000	0.0101
-0.1500	0.0230
-0.1750	0.0316
-0.2000	0.0430
-0.2200	0.0553
-0.2400	0.0714
-0.2600	0.0926
-0.2760	0.1153
-0.2780	0.1190
-0.2800	0.1241
-0.2818	0.1360

Ogee Spillway Shape Tricompound Circular Curve
Design Head = 15 ft
For an arbitrary spillway crest elevation of 100 ft



We need to use English units.

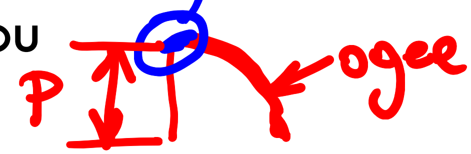
Example:

An existing ogee spillway with an elliptical crest has a crest height of 7.0 m and a crest length of 15.2 m. A minimum gage pressure of zero (atmospheric pressure) occurs at a head of 14.0 m. What maximum head and discharge would you recommend for this spillway?

elliptical

$P = 7.0\ m$

$L = 15.2\ m$



H_d should be 14 m because minimum pressure is atmospheric at this head.

$H_d = 14\ m\ (45.9\ ft)$

* From the elliptical crest spillway cavitation safety curves, for a minimum pressure head of −15 ft (USACE recommended), $\frac{H_e}{H_d} = 1.25$

$\rightarrow H_e = 1.25 \times 45.9\ ft = 57.4\ ft$

At this head, the minimum pressure is −15 ft

Maximum head = 57.4 ft

* Maximum discharge. $Q = C L H_e^{3/2}$

$L = 15.2\ m\ (49.9\ ft)$

$H_e = 57.4\ ft$

$C = ??$ $\quad \frac{H_e}{H_d} = 1.25, \quad \frac{P}{H_d} = \frac{7\cancel{m}}{14\cancel{m}} = 0.5$

$C = 3.98$ (From Ellipse chart).

$Q = 3.98(49.9)(57.4)^{3/2}$ [cfs]

$Q_{max} = 86,368$ cfs [$2,446\ m^3/s$]

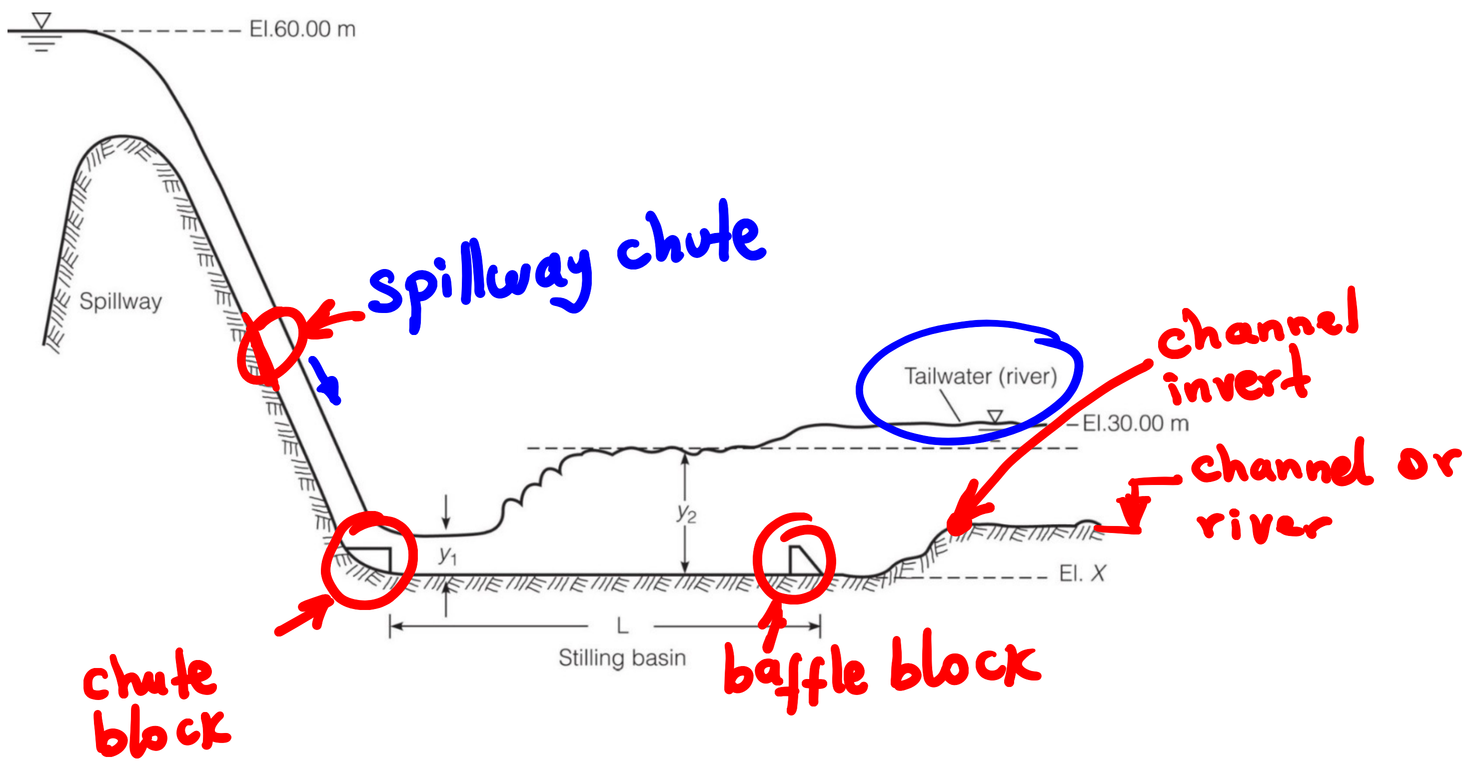
Stilling basins

Video of a stilling basin

https://www.youtube.com/watch?v=5_gbYtzr2fw

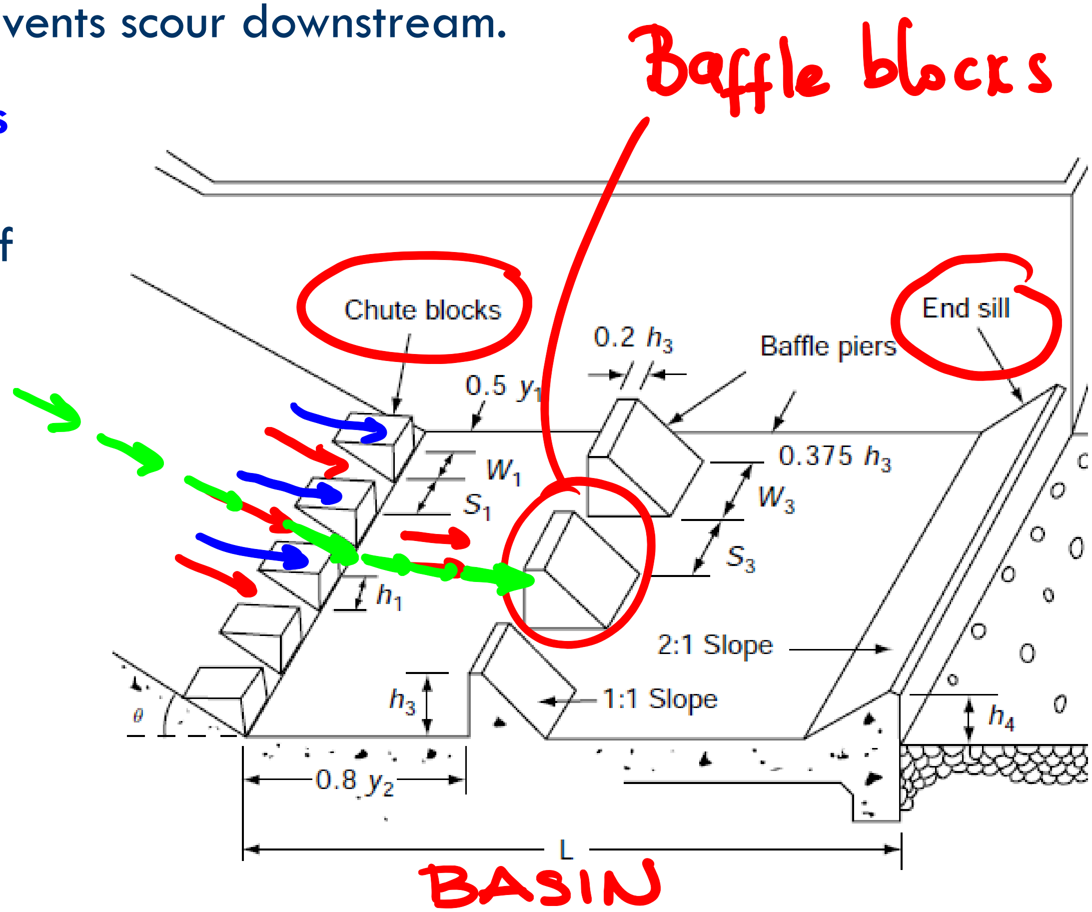
Stilling basins

- Dissipate energy, **reduce velocity and erosion downstream**
- **Control location of hydraulic jump** and its strength
- Operate correctly for a **wide range of discharges**
- Generalized designs of stilling basins have been developed by the **U.S. Bureau of Reclamation** and others.



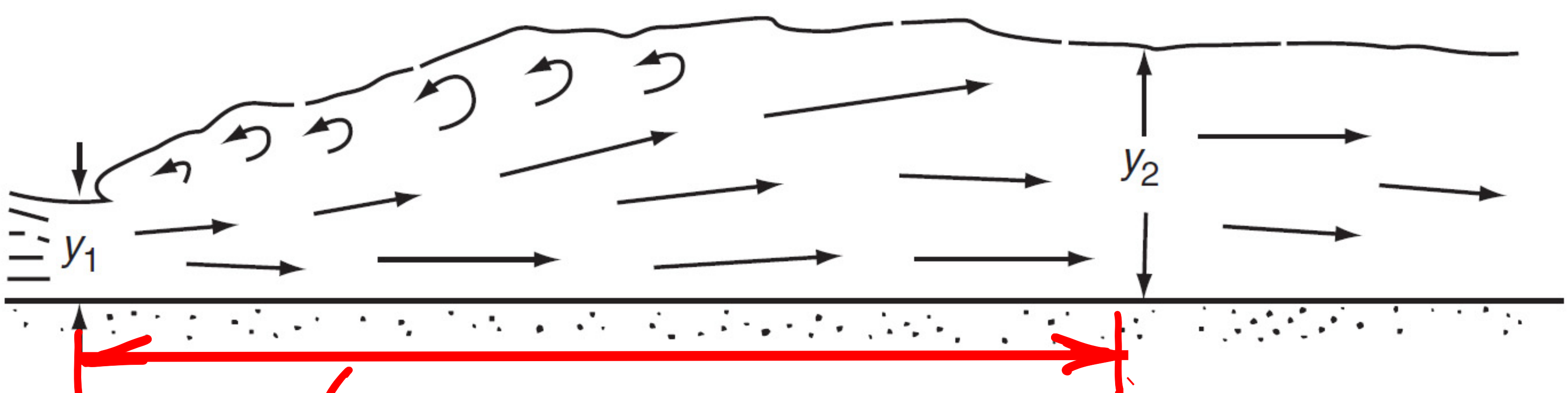
Stilling basins (Cont.)

- **Chute blocks** are placed at the **entrance of the stilling basin**. They split the incoming jet and block a portion of it to reduce the basin length and stabilize the jump.
- **End sill** is a **gradual rise at the end of the basin** to further shorten the jump and prevents scour downstream.
- **Baffle blocks** are placed **across the floor of the basin** to further dissipate energy by the impact of the high velocity jet. They are **used for only relatively low velocities of incoming flow**; otherwise, **cavitation** damage may result.



Type I stilling basin
$1.7 < F_1 < 2.5$

- For $1.7 < F_1 < 2.5$, hydraulic jump is weak
- For this range of F_1, y_2/y_1 ranges from 2 to 3.1
- This is called the Type I basin.
- No special appurtenances are needed for Type I basin

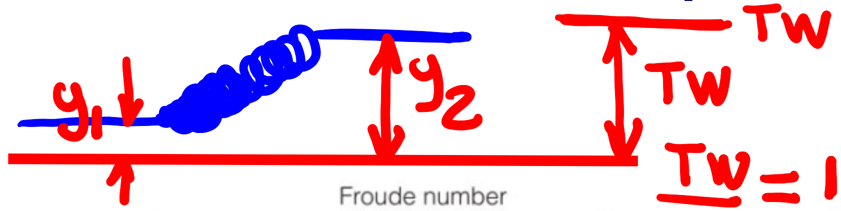


Form A: F_{r1} between 1.7 and 2.5

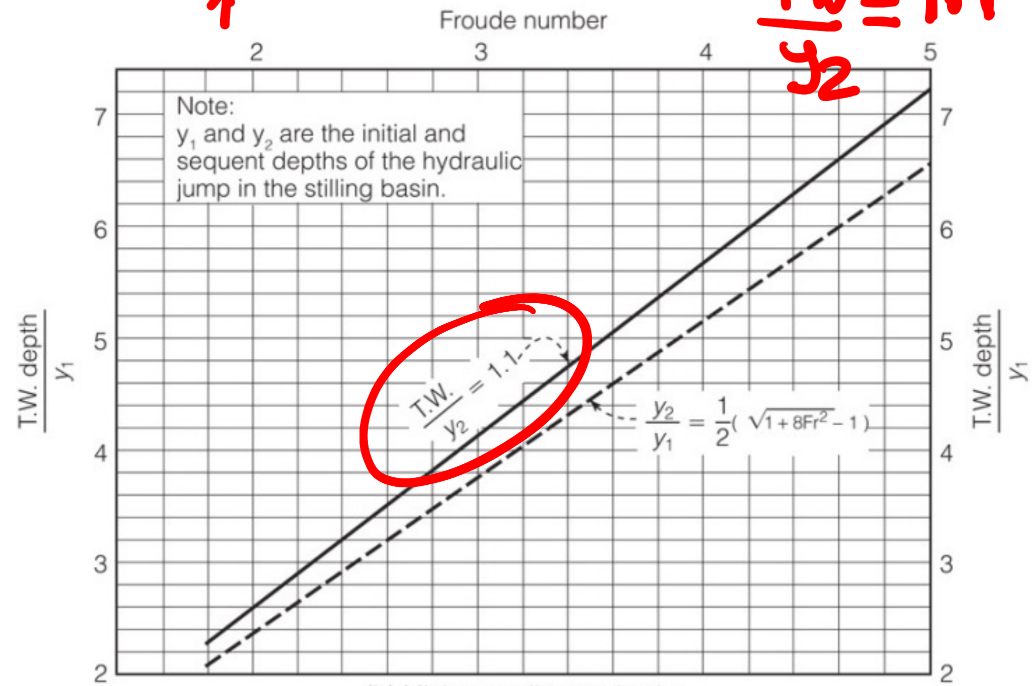
Jump length

Type IV stilling basin characteristics, $2.5 < F_1 < 4.5$

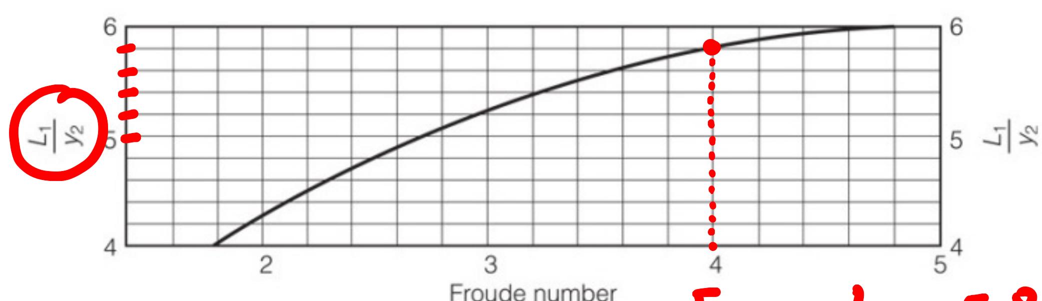
- **Recommended tailwater** depth is 10% greater than the sequent depth to help **prevent sweepout** of the jump
- Considerable wave action can remain downstream of the basin, this jump and basin are **sometimes avoided altogether** by widening the basin to increase the Froude number.



$\frac{TW}{y_2} = 1.1$

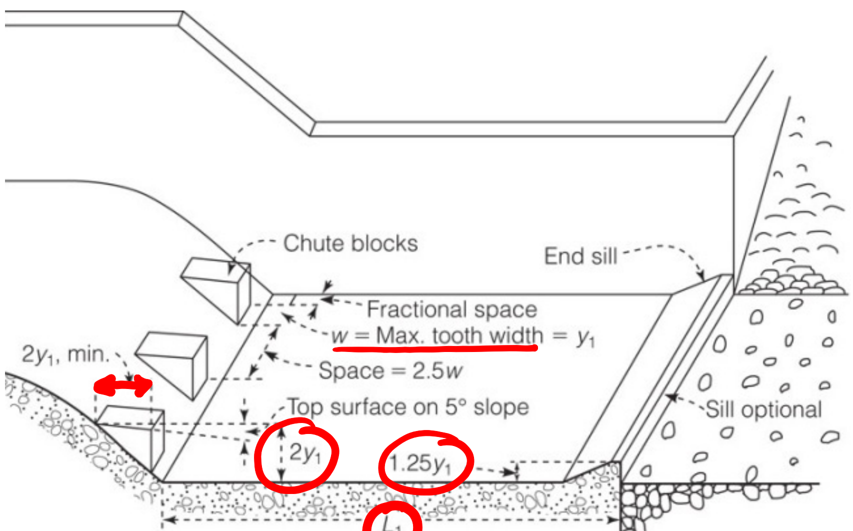


(b) Minimum tailwater depths



(c) Length of jump

$Fr = 4, \frac{L_1}{y_2} = 5.8$



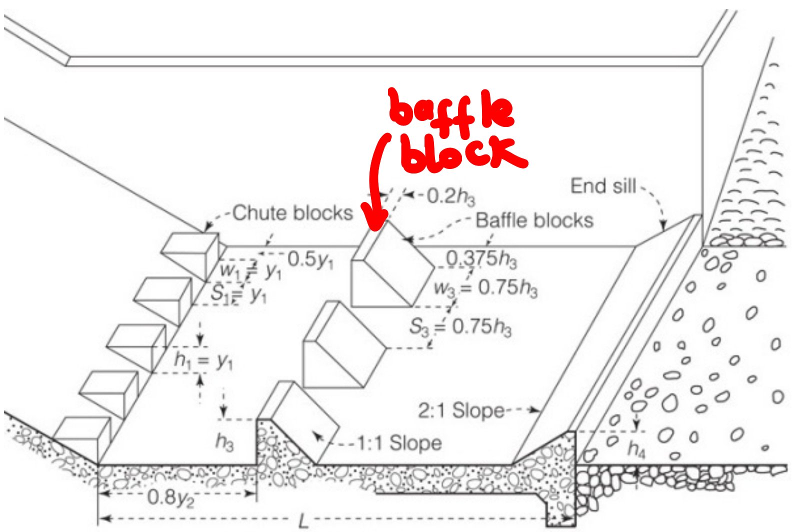
(a) Type IV basin dimensions

Source: U.S. Bureau of Reclamation (1987)

Type III stilling basin characteristics, $F_1 > 4.5$ and $V_1 \leq 18$ m/s (60 ft/s)

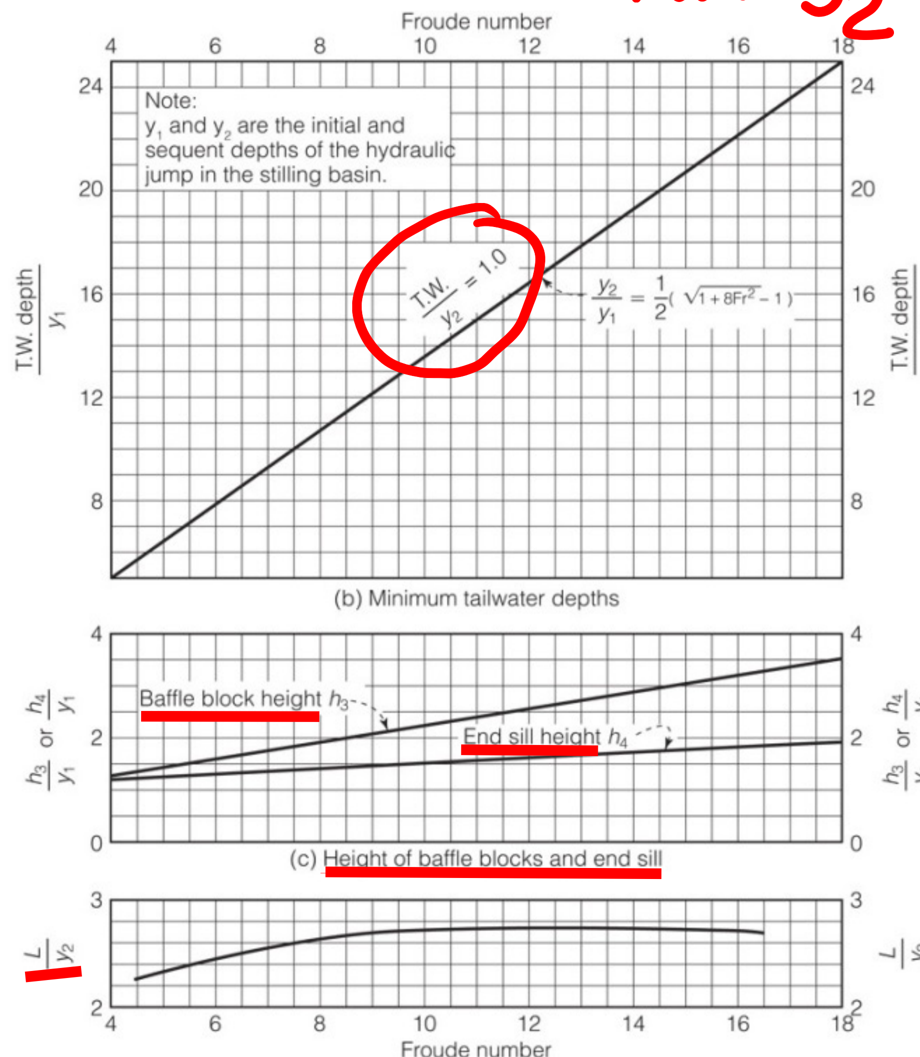
TW = y_2

- Type III basin: for incoming velocity ≤ 60ft/s
- Includes baffle blocks.



(a) Type III basin dimensions

Source: U.S. Bureau of Reclamation (1987)



(b) Minimum tailwater depths

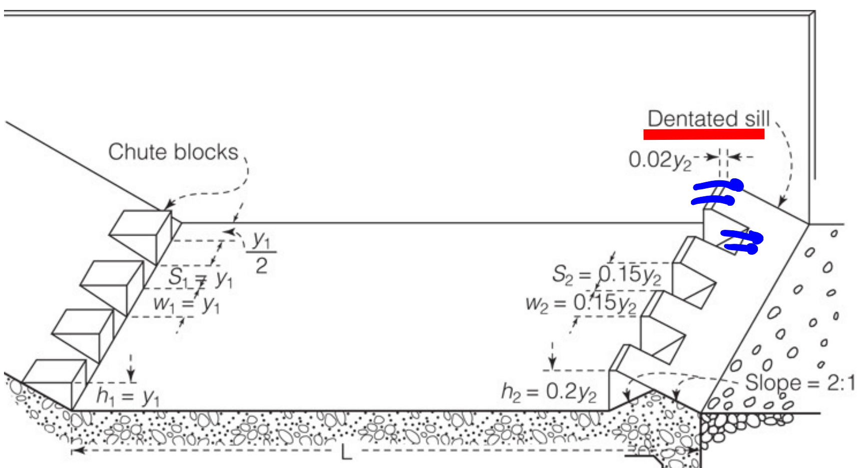
(c) Height of baffle blocks and end sill

(d) Length of jump

Type II stilling basin characteristics, $F_1 > 4.5$ and $V_1 > 18$ m/s (60 ft/s)

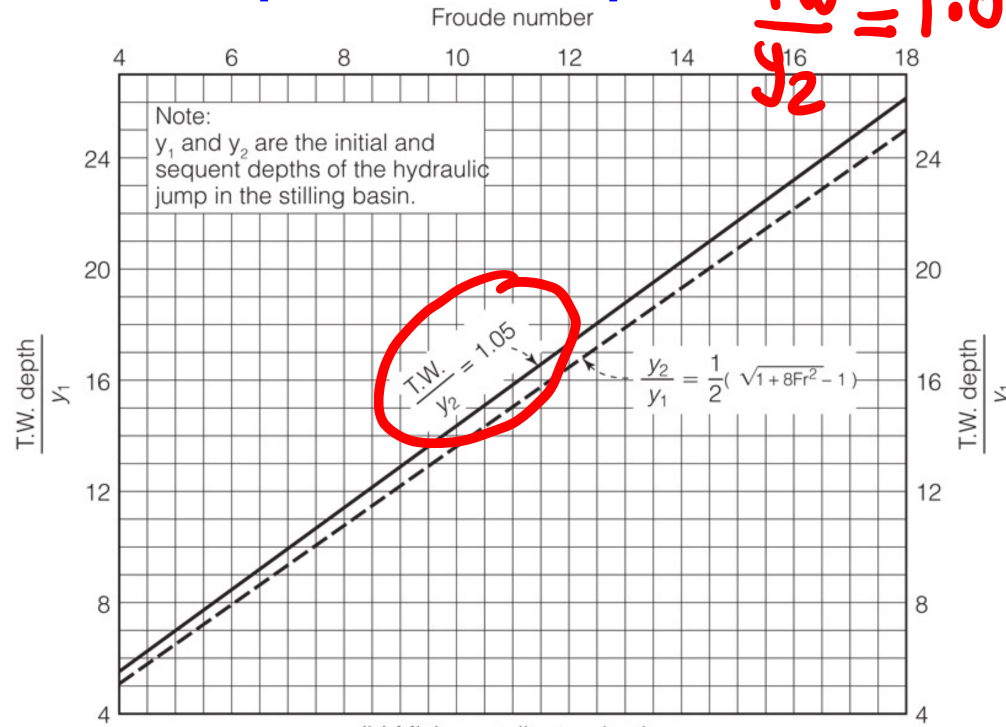
$\frac{TW}{y_2} = 1.05$

- **No baffle** blocks to **avoid cavitation**
- Dentated end sill,
- Slightly **longer than Type III** basin,
- Tailwater is recommended to be 5 percent greater than the sequent depth to help prevent sweepout.

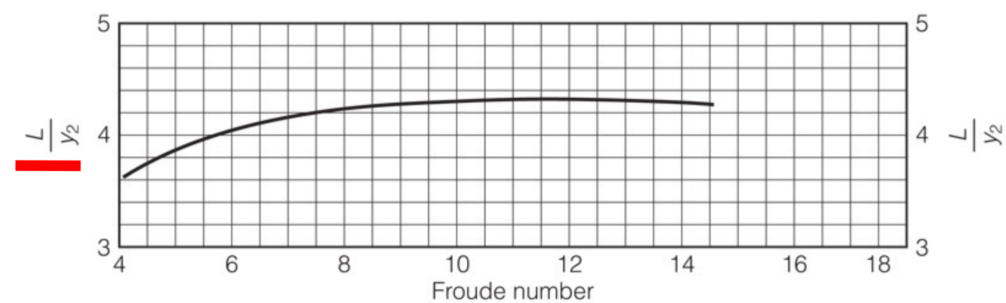


(a) Type II basin dimensions

Source: U.S. Bureau of Reclamation (1987)



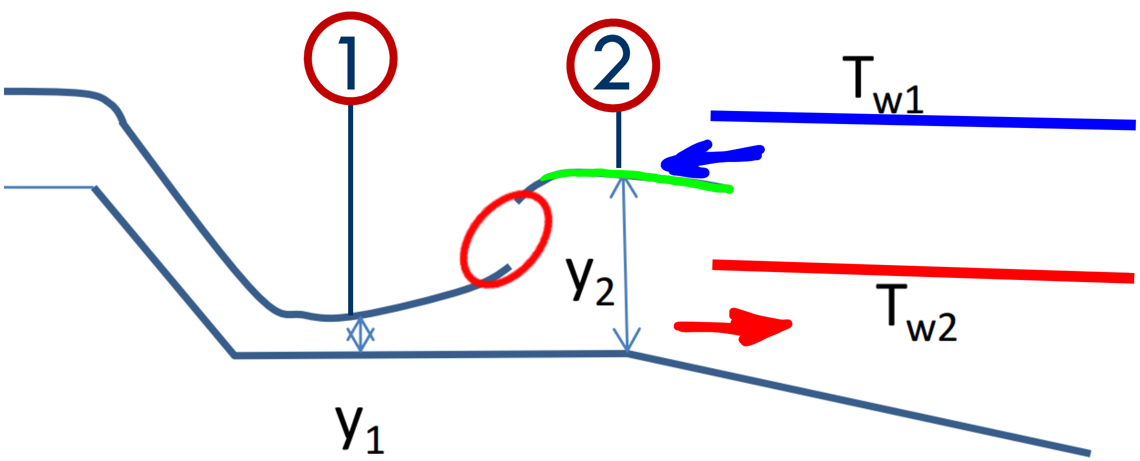
(b) Minimum tailwater depths



(c) Length of jump

Stilling basin design

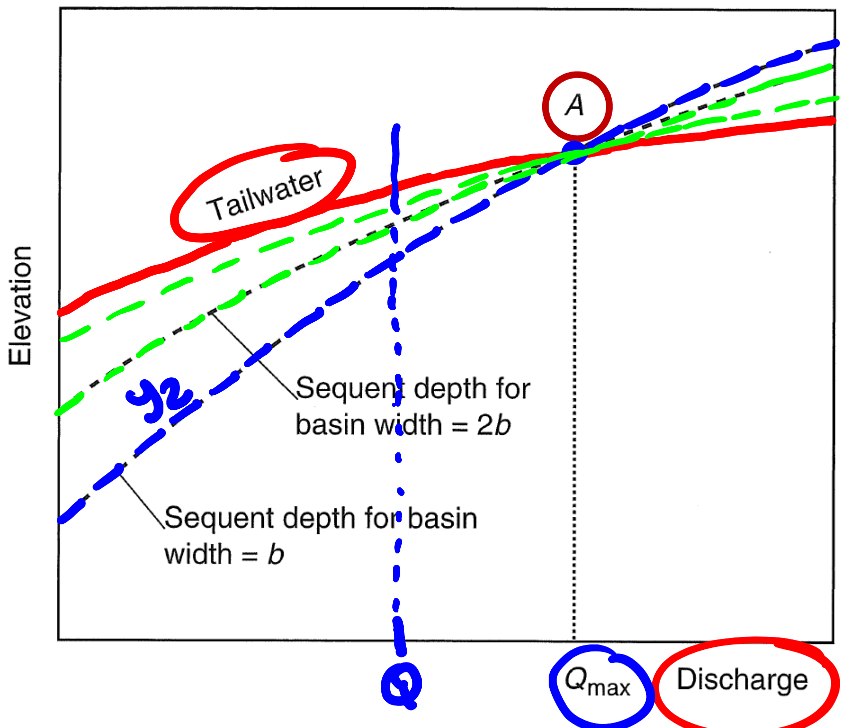
- **Main goal** of designing stilling basins: **Match** the **tailwater** and **sequent depth** curves over a range of operating discharges.
- Tailwater **lower than sequent** depth of the jump => jump may be **swept out** of the basin => no longer serves its purpose and dangerous erosion can occur downstream of the basin.
- Tailwater elevation **higher than sequent** depth => jump backs up against the spillway chute and **"drown out" or be submerged** =>no longer dissipates as much energy.



- If TW > (z_2+y_2) → Jump moves upstream
- If TW < (z_2+y_2) → Jump moves downstream

Stilling basin design

- Basin floor elevation is set to **match sequent depth and tail water elevations** at the **maximum design discharge** at point A
- **Basin can be widened** to help **improve the match at lower discharges** while erring on the submerged side rather than the sweep-out side.

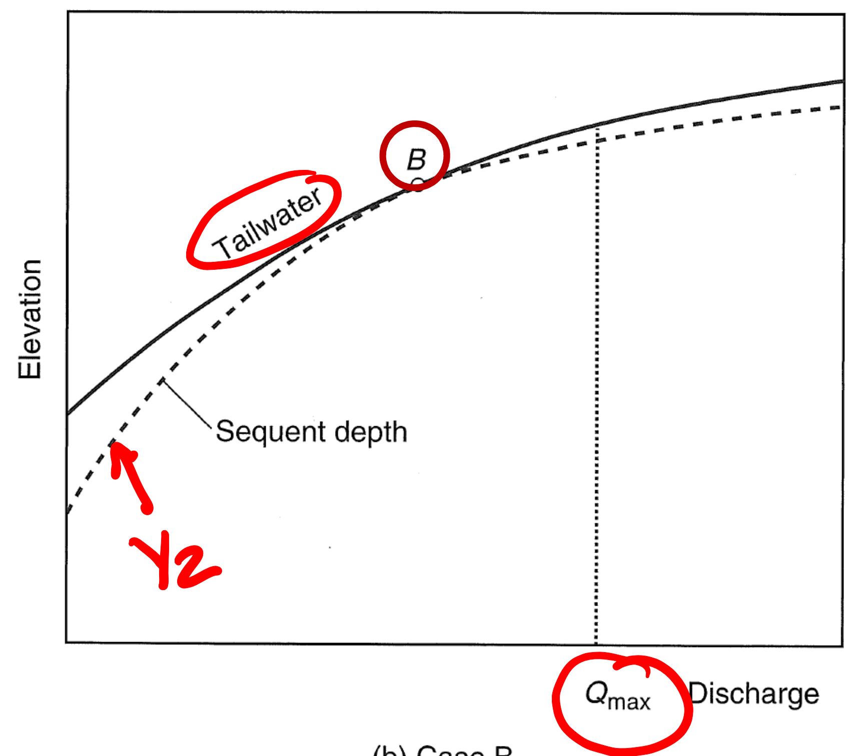


(a) Case A

y_2 = TW at max discharge. Doubling the basin width, the difference between y_2 and Tw is reduced for all discharges

Stilling basin design (Cont.)

- If the sequent depth curve is **shaped as shown in the Figure** below, the tailwater and sequent depth elevations would have to be **matched for a lower discharge than the maximum** to ensure sufficient tailwater for all discharges.
- At max discharge: $y_2 <$ Tw: This implies that at Q_{max} the jump will move upstream, towards the spillway (reduced dissipation).



(b) Case B

Example of application

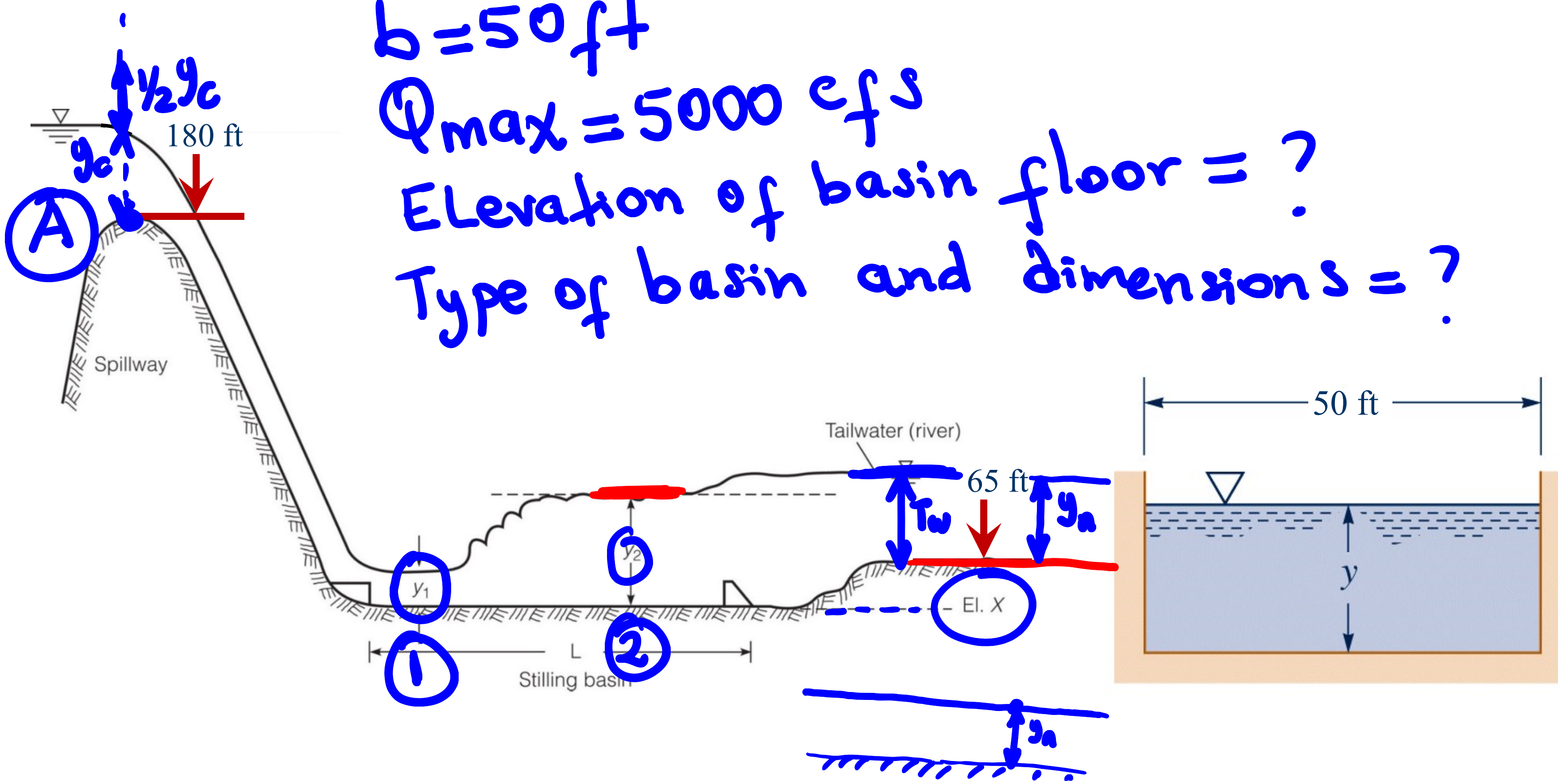
The stilling basin below at its toe has a width of 50 ft and is designed to carry a maximum discharge of 5000 cfs. Determine the elevation of the basin floor and choose the type and dimensions of the stilling basin. Neglect spillway head losses. The natural stream channel downstream of the stilling basin has a longitudinal slope of 0.001, the natural channel has a width of 50 ft, and the natural channel Manning's roughness is 0.03.

$b = 50\,ft$

$Q_{max} = 5000\ cfs$

Elevation of basin floor = ?

Type of basin and dimensions = ?



Example of application (Spreadsheet Part 1)

Steps: ① $E_A = E_1$ $\quad 180 + \frac{3}{2} y_c = x + y_1 + \frac{V_1^2}{2g}$

② Mom ①-② $\quad \frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1+8Fr_1^2}\right)$

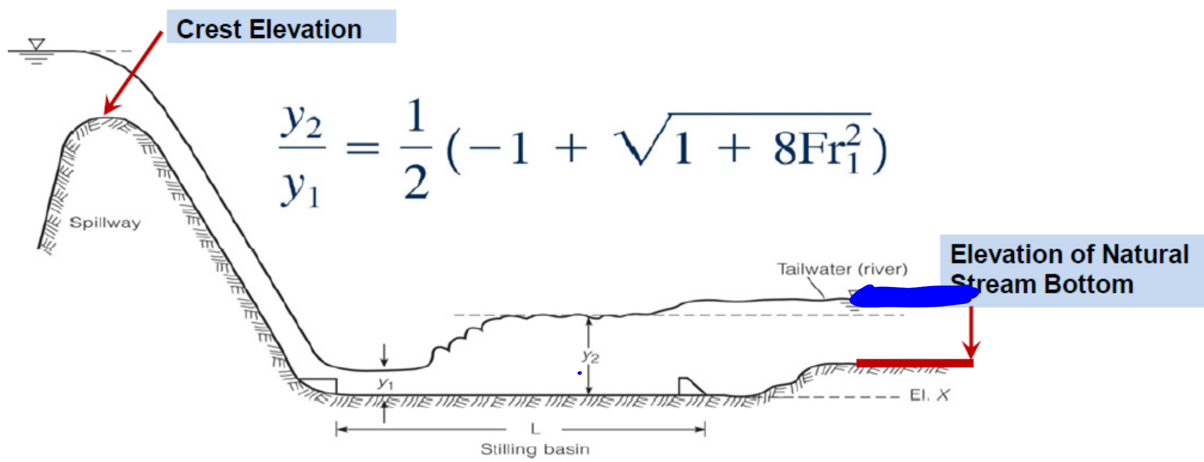
③ Tailwater can be found using normal depth

Open Channel Hydraulics, Arturo S. Leon
Analysis of a Stilling Basin

Spillway Crest elevation (ft) =	180
spillway width (ft) =	50

Here, the Manning equation is being solved to find the normal depth for a variety of discharges. In each case, values (guesses) are entered in the normal depth column (cells B13-B18), the target flow is in the A column, and the Manning equation is entered in the C column. This is then 'goalseeked' in order to get a difference of discharge equal to zero with column A. Finally, column D gives the TAILWATER elevation, by adding to the depth the elevation of the channel bottom.

Downstream Channel Slope			0.0010
Gravity (ft^2/s)			32.2
Manning n			0.03
Alpha			1.49
Elevation of Natural stream bottom (ft)			65
Target Discharge (cfs)	**Normal Depth (ft)**	**Discharge (cfs) to be 'goalseeked'**	**Tailwater Elevation (ft)**
2000	7.7738	0.000	72.774
3000	10.2025	0.000	75.202
4000	12.4252	0.000	77.425
5000	14.5179	0.000	79.518
6000	16.5194	0.000	81.519
7000	18.4531	0.000	83.453
		0.00000	



$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_1^2}\right)$$

So, we know now the WSE at the entrance of the downstream channel at our design flow of 5000 cfs.
Next, let us move to the basin floor part of the problem. Refer to the block below.

Example of application (Spreadsheet Part 2)

Finding Basin floor elevation		
1	Design Flow (cfs)	5000
2	Depth over spillway y0 = yc (ft)	6.7720
3	Energy over spillway E0 = Ec (ft)	10.1579
4	Tailwater elevation at design flow – see Manning analysis above (cell D16)	79.518
5	y1 (ft)	1.0805
6	Fr1	15.6902
7	Energy equation between spillway crest and spillway toe solved for Z (Z = Basin Floor elevation)	56.0763
8	Hydraulic jump equation in stillin basin solved for Z	56.0763
9	Difference	0.00000022
10	Basin Floor Elevation (ft)	56.07625906

$Fr_1 > 4.5$

Row 31 is the energy equation written between spillway crest and the spillway toe (supercritical flow in basin), solved for the unknown Z (Z = Basin Floor elevation). Row 32 is a combination of the hydraulic jump equation and the 'matching' equation at the downstream natural channel entrance; again solved for the unknown Z. Row 33 is the difference between these two (which is supposed to be 0). You goalseek the difference, trying to make it zero, by adjusting cell **E29 (y1)**.

Example of application (Spreadsheet Part 3)

> 60 ft/s

The final part of this problem is to plot the elevation downstream of the hydraulic jump as a function of Q (conjugate depth curve) and to superimpose this plot on top of the rating curve (Manning) for the downstream channel. We already have the data for the latter. For the former, we get it from:

A	B	C	D	E	F	G	I	J
Q (cfs)	q (ft^2 / s)	yc (ft)	Ec (ft)	y1 (ft)	Energy difference NEEDS to be ZERO	V1 (ft/s)	y2 (ft)	Tailwater Elevation (ft)
2000	40	3.7	5.5	0.439	0.00000	91.15	14.8	70.9
3000	60	4.8	7.2	0.655	0.00000	91.67	18.2	74.2
4000	80	5.8	8.8	0.868	0.00000	92.13	21.0	77.0
5000	100	6.8	10.2	1.081	0.00000	92.55	23.4	79.5
6000	120	7.6	11.5	1.291	0.00000	92.93	25.7	81.8
7000	140	8.5	12.7	1.501	0.00000	93.29	27.7	83.8
					0.00000025809			

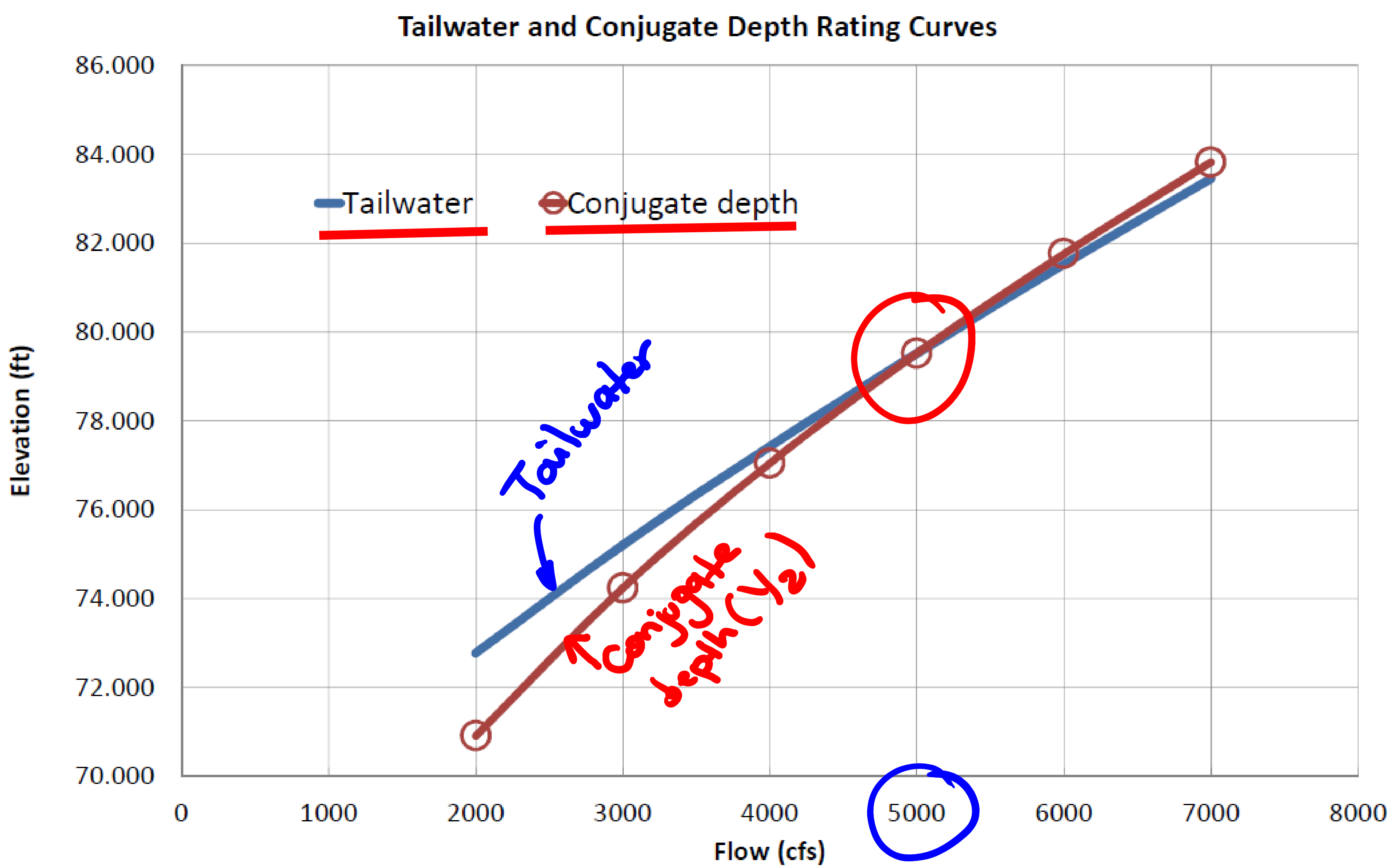
Column F is essentially the energy equation between spillway crest and the spillway toe (supercritical flow in basin). Actually, it is written as the energy at the crest minus the energy at supercritical flow. This is SUPPOSED to be zero. Use EXCEL SOLVER for this column, trying to make it zero, by adjusting y1 (Column E). So, for each flow, we find y1. We then use the hydraulic jump equation to find y2 (column I). We then add y2 to basin floor elevation to get the tailwater elevation.

Use Type II Stilling basin.

chute blocks and dentated sill are a function of y_1 (1.081 ft) and y_2 (23.4 ft). Length of jump

$Fr_1 = 15.69$, $L/y_2 = 4.3$, $L_{jump} = 100.6$ ft

Example of application (Spreadsheet Part 4)



Culverts

Examples of culverts:

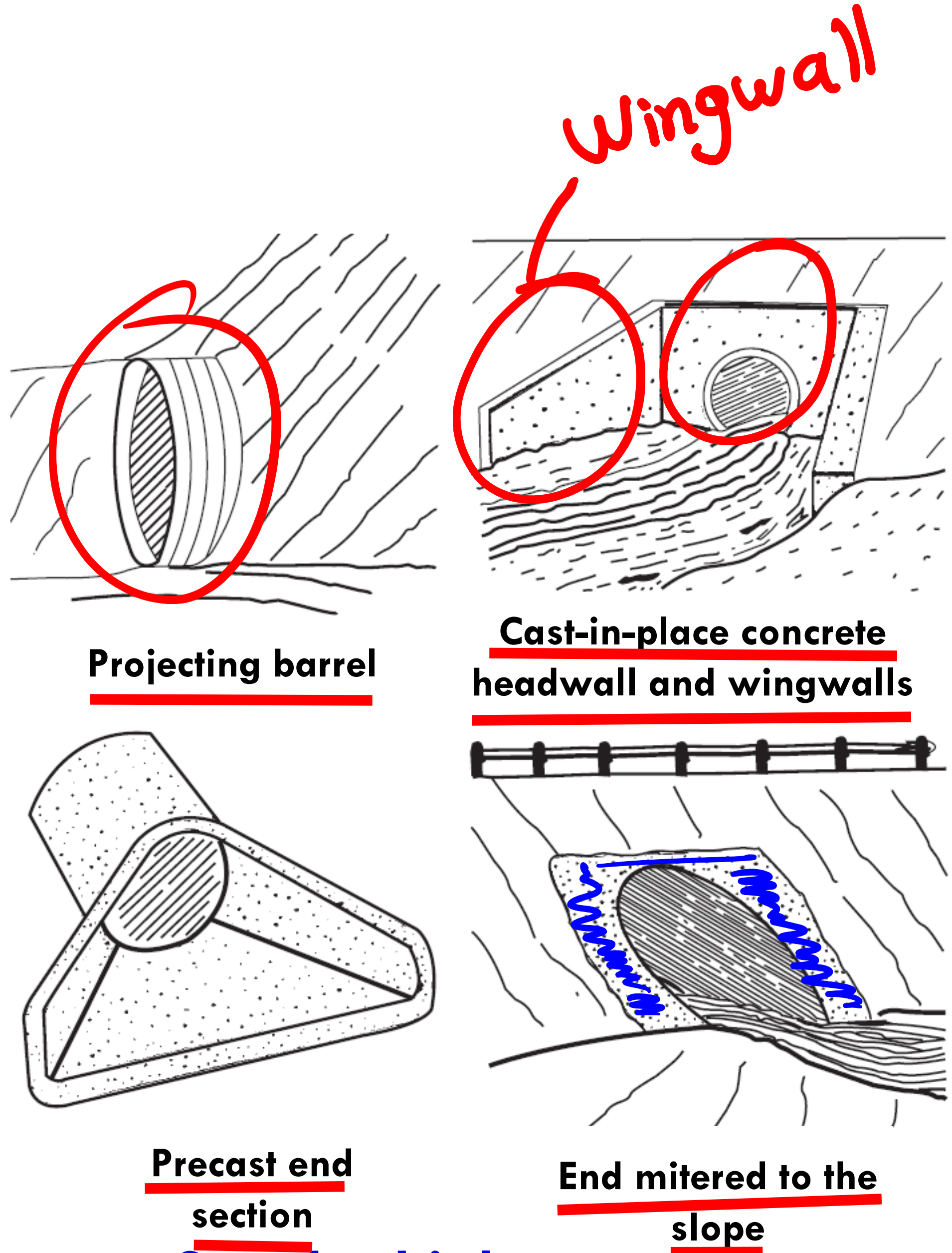
Concrete Box Culvert

Culvert with fish passage

Bottomless culvert (when erosion/ sedimentation is a problem)

Culverts

- Culverts are **short drainage conduits** that convey stormwater through highway and railway embankments.
- Culverts are also used as **outlet structures** for detention basins.
- The **inlet configuration plays an important role** in the hydraulic performance of culverts.
- The Figure on the side depicts various standard inlet types.
- **Design flow discharge** (Q)
  - Rational method, TR-55 method, etc.



Standard inlet types

Normann et al., 1985)

Photos of typical standard inlet types

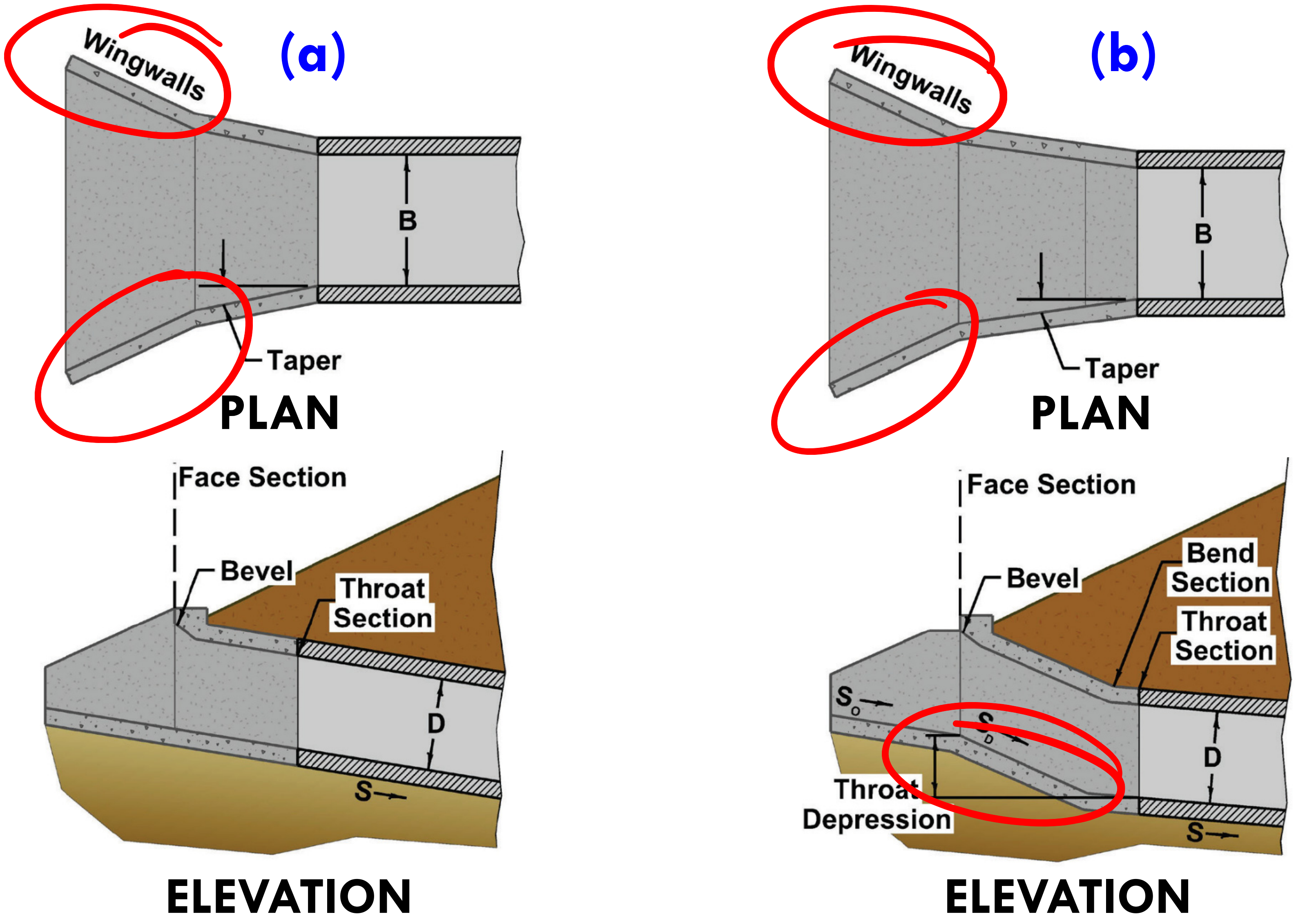
Thin Edge Projecting – The culvert barrel projects out of the embankment

Mitered entrance – The culvert barrel is cut so it is flush with the embankment slope

Square edge in headwall – The end of the culvert barrel is flush with the headwall

Groove edge projecting – A concrete pipe culvert section extends beyond the fill or headwall. **Groove end is a socket end.**

Tapered inlets



Schematic of side-tapered and slope-tapered inlets. **(a) Side-tapered inlet, (b) Slope-tapered inlet**.

Culverts (Cont.)

- Flow in a culvert can be controlled either by the **inlet** (upstream) or by the **outlet** (downstream).
- **Inlet control** occurs when the conveyance **capacity of the culvert barrel is higher than the inlet** will accept;
- Otherwise, **outlet control** flow occurs.

Factors influencing culvert performance

(FHWA, 2001)

Factor	Inlet Control	Outlet Control
Headwater elevation	X	X
Inlet area	X	X
Inlet edge configuration	X	X
Inlet shape	X	X
Barrel roughness		X
Barrel area		X
Barrel shape		X
Barrel length		X
Barrel slope		X
Tailwater elevation		X

Design Procedure

- Design is based on the selection of a **design discharge** determined from **frequency analysis**
  - **Interstate highway culverts**, for example, may be designed to carry the **100-year peak discharge**
- Sized to limit the headwater resulting from the design discharge to a specified value to **prevent overtopping** the highway embankment
- Once the design culvert size is determined, its **performance analyzed over a wide range of discharges**, including discharges that overtop the embankment
  - Plot of the complete head-discharge relation created, called the **performance curve**.
  - Helps determine whether the culvert operates under inlet or outlet control for the design discharge

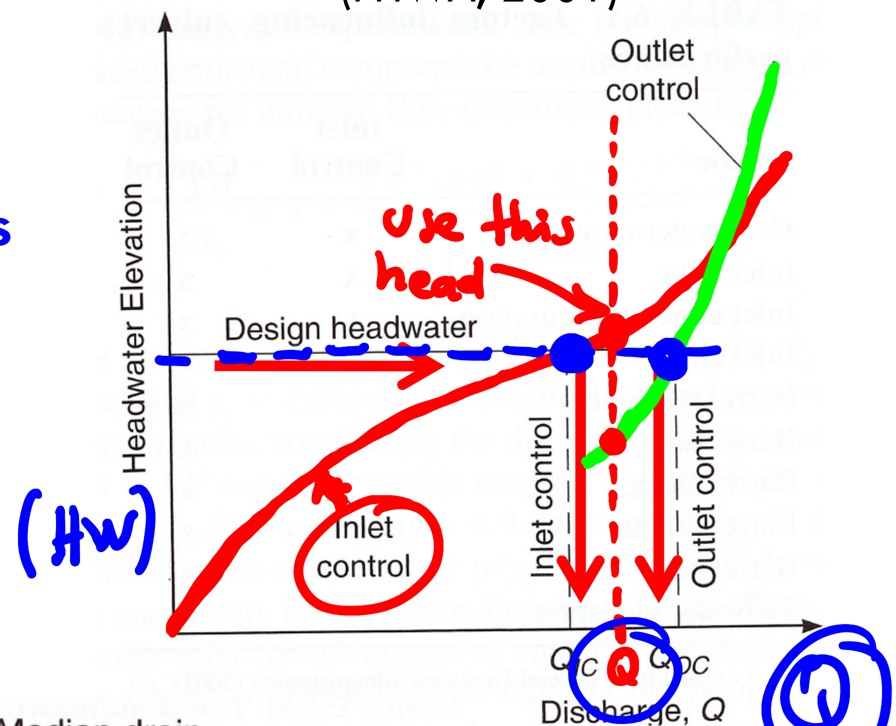
Design Procedure (Cont.)

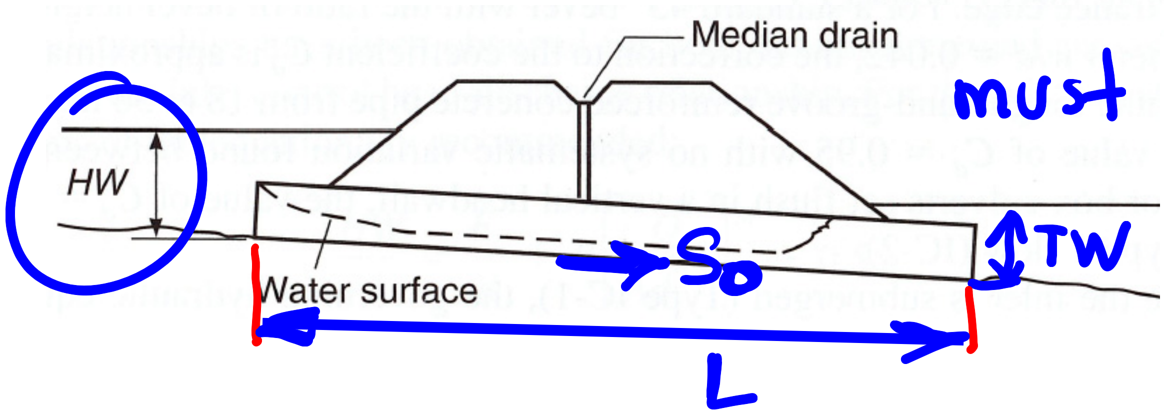
- **Higher head** resulting either from inlet or outlet control is **compared** with the **allowable headwater elevation**

Because $Q_{IC} < Q_{OC}$, **inlet capacity is less than the barrel capacity**, and the inlet controls the head-discharge relation at the design condition

- same as choosing the higher head for a given discharge

Culvert performance Curves for determination of inlet/outlet control
(FHWA, 2001)





must choose minimum of Q_{ic} and Q_{oc}

Culvert Hydraulic behavior

- Culvert may act as a weir, an orifice, or a pressurized flow
  - **Unsubmerged inlet**: culvert operates as a **weir at the inlet**, and
    - the **discharge is proportional to the head to the 3/2** power

$Q \sim H^{3/2}$

  - **Submerged inlet**: culvert is in inlet control, orifice flow occurs, and
    - **discharge is proportional to the head to the 1/2 power**.
    - when the ratio of inlet head to height of the culvert, HW/d, is in the range of 1.2 to 1.5.

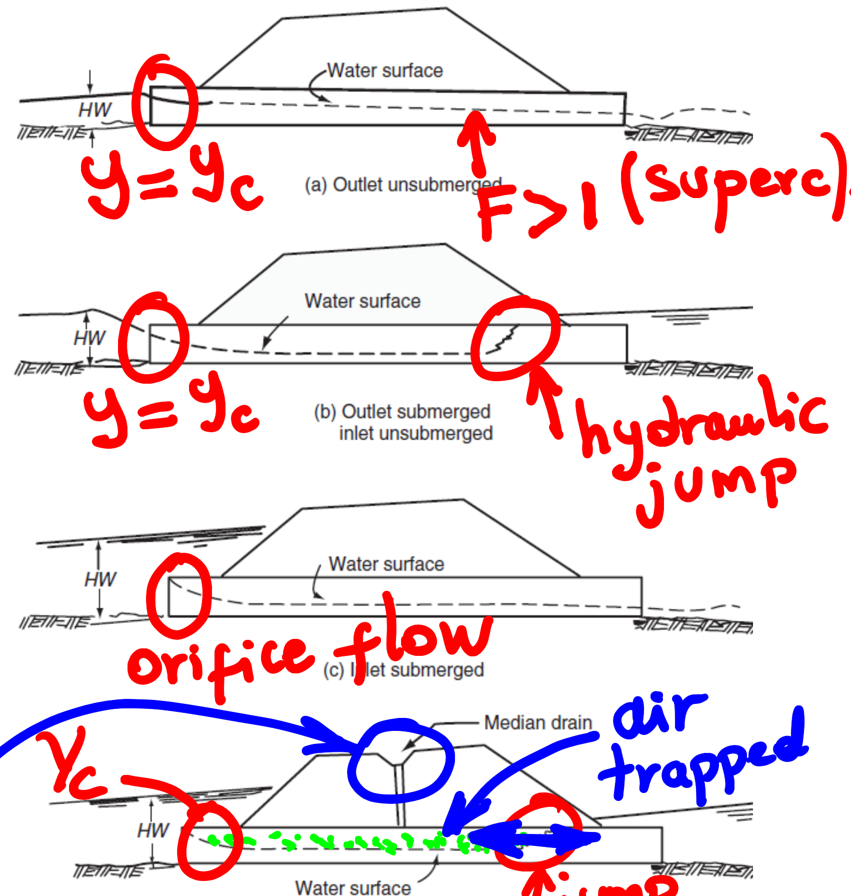
$Q \sim H^{1/2}$

  - **Pressure flow:** head-discharge relation is determined by the **effective head**, which is the **difference in total head** between the headwater and tailwater.

Types of Inlet Control

- **Fig. (a): unsubmerged inlet and outlet on steep slope.**
  - Critical depth at the inlet and the downstream flow is supercritical discharge
- **Fig. (b): submerged outlet**, forcing a **hydraulic jump in the barrel**.
- **Fig. (c): inlet submerged** and **outlet unsubmerged**.
  - Critical depth occurs just downstream of the inlet,
  - Culvert is in orifice flow
- **Fig. (d):** Both **inlet and outlet submerged**
  - **vent must be provided** to prevent an unstable flow situation, which oscillates between full flow and partly full flow.
  - **With vent in place** and hydraulic jump downstream of the culvert entrance, this remains **inlet control with orifice flow at entrance**.

Types of Inlet Control (FHWA, 2001)



Central ventilation (to prevent jump oscillation)

(Strong oscillation of jump if there is no venting)

Flow equations for inlet control (Unsubmerged Inlet):

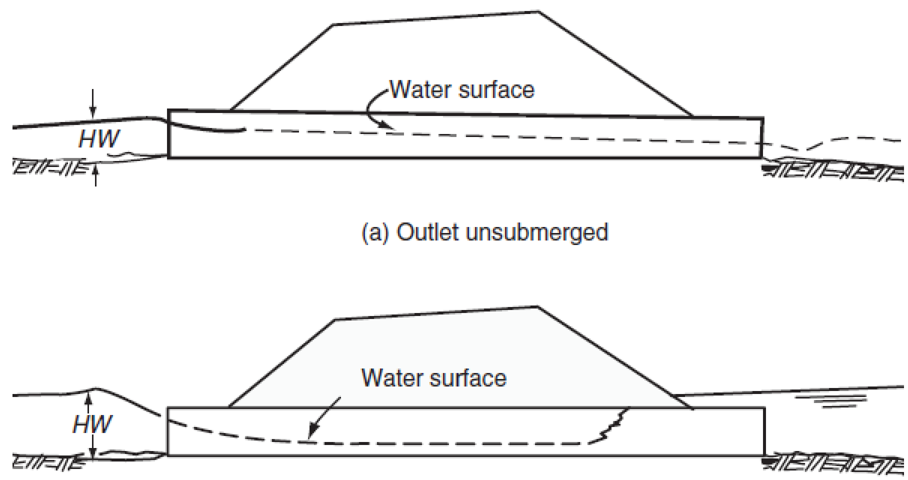
- **Inlet will be considered unsubmerged** if (FHWA, Normann et al., 1985):

$$\frac{Q}{AD^{0.5}g^{0.5}} \leq 0.62$$

where Q = discharge, *A* = cross-sectional area of the culvert, ***D* = interior height of the culvert**, and *g* = gravitational acceleration.

- Two forms of equations are available for unsubmerged inlets (**form I** and **form II**). These best-fit relationships were obtained by the National Bureau of Standards **through extensive experimental results**.

- Both **form I and form II equations are acceptable for practical purposes**, and the choice between the two is governed by the availability of the empirical coefficients for the type of the culvert being considered.



(a) Outlet unsubmerged

(b) Outlet submerged
inlet unsubmerged

Flow equations for inlet control (Unsubmerged Inlet)

- The **form I equation** is

$$\frac{HW}{D} = \frac{y_c}{D} + \frac{V_c^2}{2gD} + K_I\left(\frac{Q}{AD^{0.5}g^{0.5}}\right)^{M_I} + k_s S$$

where HW = headwater depth above the upstream invert of the culvert, y_c = critical depth, V_c = velocity at critical depth, S = culvert barrel slope, and K_I , M_I = empirical constants. The values of K_I and M_I are given in the tables below for various inlet configurations.

$k_s = 0.7$ for inlets mitered to embankment slope

$k_s = -0.5$ for inlets not mitered to embankment slope

- The **form II equation** is

$$\frac{HW}{D} = K_{II}\left(\frac{Q}{AD^{0.5}g^{0.5}}\right)^{M_{II}}$$

where K_{II} and M_{II} are empirical constants given in the table below.

Flow equations for inlet control (Submerged Inlet):

- **Inlet will be submerged if** (FHWA, Normann et al., 1985):

$$\frac{Q}{AD^{0.5}g^{0.5}} \geq 0.70$$

- The **flow equation for submerged inlets** is

$$\frac{HW}{D} = c\left(\frac{Q}{AD^{0.5}g^{0.5}}\right)^2 + Y + k_s S$$

where S = slope, c and Y are empirical constants given in the table below, and

$k_s = 0.7$ for inlets mitered to embankment slope

$k_s = -0.5$ for inlets not mitered to embankment slope

Culvert Inlet Control Flow Coefficients

(Normann et al., 1985)

Shape and material	Inlet edge description	K_I	M_I	K_{II}	M_{II}	c	Y
Circular concrete	Square edge with headwall	0.3155	2.0			1.2816	0.67
Circular concrete	Groove end with headwall	0.2512	2.0			0.9402	0.74
Circular concrete	Groove end projecting	0.1449	2.0			1.0207	0.69
Circular corrugated metal	Headwall	0.2512	2.0			1.2204	0.69
Circular corrugated metal	Mitered to slope	0.2113	1.33			1.4909	0.75
Circular corrugated metal	Projecting	0.4596	1.50			1.7807	0.54
Circular	Beveled ring, 45° bevels	0.1381	2.50			0.9660	0.74
Circular	Beveled ring, 33.7° bevels	0.1381	2.50			0.7825	0.83
Rectangular box	30–75° Wingwall flares	0.1475	1.00			1.1173	0.81
Rectangular box	90° and 15° Wingwall flares	0.2243	0.75			1.2880	0.80
Rectangular box	0° Wingwall flare	0.2243	0.75			1.3621	0.82
Corrugated metal box	90° Headwall	0.2673	2.00			1.2204	0.69
Corrugated metal box	Thick wall projecting	0.3025	1.75			1.3492	0.64
Corrugated metal box	Thin wall projecting	0.4596	1.50			1.5971	0.57
Horizontal ellipse concrete	Square edge with headwall	0.3220	2.0			1.2816	0.67
Horizontal ellipse concrete	Groove end with headwall	0.1381	2.5			0.9402	0.74
Horizontal ellipse concrete	Groove end projecting	0.1449	2.0			1.0207	0.69

Culvert Inlet Control Flow Coefficients (Cont.)

Shape and material	Inlet edge description	K_I	M_I	K_{II}	M_{II}	c	Y
Vertical ellipse concrete	Square edge with headwall	0.3220	2.0			1.2816	0.67
Vertical ellipse concrete	Groove end with headwall	0.1381	2.5			0.9402	0.74
Vertical ellipse concrete	Groove end projecting	0.3060	2.0			1.0207	0.69
Rectangular box	45° Wingwall flare $d = 0.043D$			1.623	0.667	0.9950	0.80
Rectangular box	18–33.7° Wingwall flare $d = 0.083D$			1.547	0.667	0.8018	0.83
Rectangular box	90° Headwall with 3/4″ chamfers			1.639	0.667	1.2075	0.79
Rectangular box	90° Headwall with 45° bevels			1.576	0.667	1.0111	0.82
Rectangular box	90° Headwall with 33.7° bevels			1.547	0.667	0.8114	0.865
Rectangular box	3/4″ Chamfers; 45° skewed headwall			1.662	0.667	1.2944	0.73
Rectangular box	3/4″ Chamfers; 30° skewed headwall			1.697	0.667	1.3685	0.705
Rectangular box	3/4″ Chamfers; 15° skewed headwall			1.735	0.667	1.4506	0.73
Rectangular box	45° Bevels; 10–45° skewed headwall			1.585	0.667	1.0525	0.75
Rectangular box with 3/4″ chamfers	45° Non-offset wingwall flares			1.582	0.667	1.0916	0.803
Rectangular box with 3/4″ chamfers	18.4° Non-offset wingwall flares			1.569	0.667	1.1624	0.806
Rectangular box with 3/4″ chamfers	18.4° Non-offset wingwall flares with 30° skewed barrel			1.576	0.667	1.2429	0.71

Culvert Inlet Control Flow Coefficients (Cont.)

Shape and material	Inlet edge description	K_I	M_I	K_{II}	M_{II}	c	Y
Rectangular box with top bevels	45° Wingwall flares – offset			1.582	0.667	0.9724	0.835
Rectangular box with top bevels	33.7° Wingwall flares – offset			1.576	0.667	0.8144	0.881
Rectangular box with top bevels	18.4° Wingwall flares –offset			1.569	0.667	0.7309	0.887
Circular	Smooth tapered inlet throat			1.699	0.667	0.6311	0.89
Circular	Rough tapered inlet throat			1.652	0.667	0.9306	0.90
Rectangular	Tapered inlet throat			1.512	0.667	0.5764	0.97
Rectangular concrete	Side tapered – less favorable edges			1.783	0.667	1.5005	0.85
Rectangular concrete	Side tapered – more favorable edges			1.783	0.667	1.2172	0.87
Rectangular concrete	Slope tapered – less favorable edges			1.592	0.667	1.5005	0.65
Rectangular concrete	Slope tapered – more favorable edges			1.592	0.667	1.2172	0.71

$c = 0.6311$

$Y = 0.89$

Transition from unsubmerged to submerged condition for inlet control:

- A **transition from unsubmerged to submerged** condition occurs for

$$0.62 < \frac{Q}{AD^{0.5}g^{0.5}} < 0.70$$

- A **linear interpolation** between the submerged and unsubmerged inlet equations can be used for the transition zone.

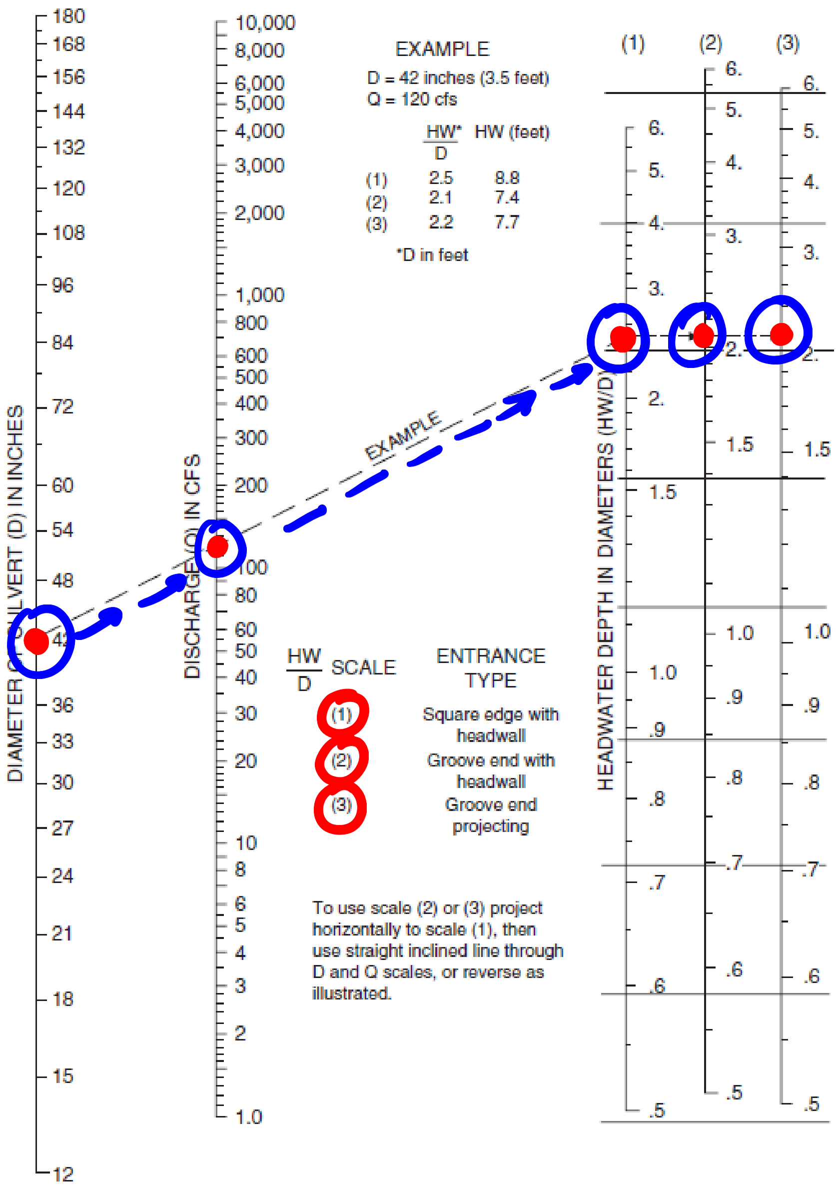
Federal Highway Administration

- There are **several nomographs by the FHWA** (Normann et al., 1985) **for quick calculations** of culvert flow. The two figures below are included as examples of concrete pipe culverts and box culverts, respectively.

HW

Headwater depth for concrete pipe culverts with inlet control

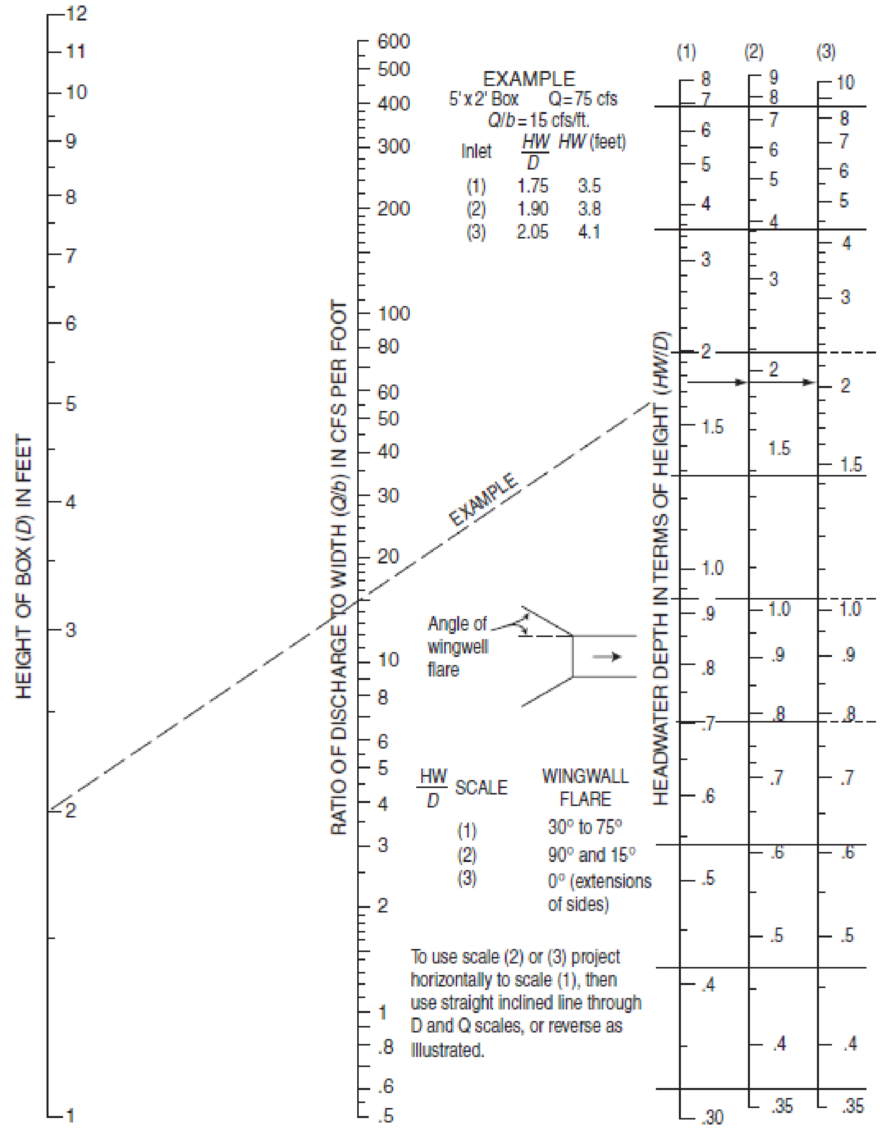
(After Normann et al., 1985)



HW

Headwater depth for box culverts with inlet control

(After Normann et al., 1985)

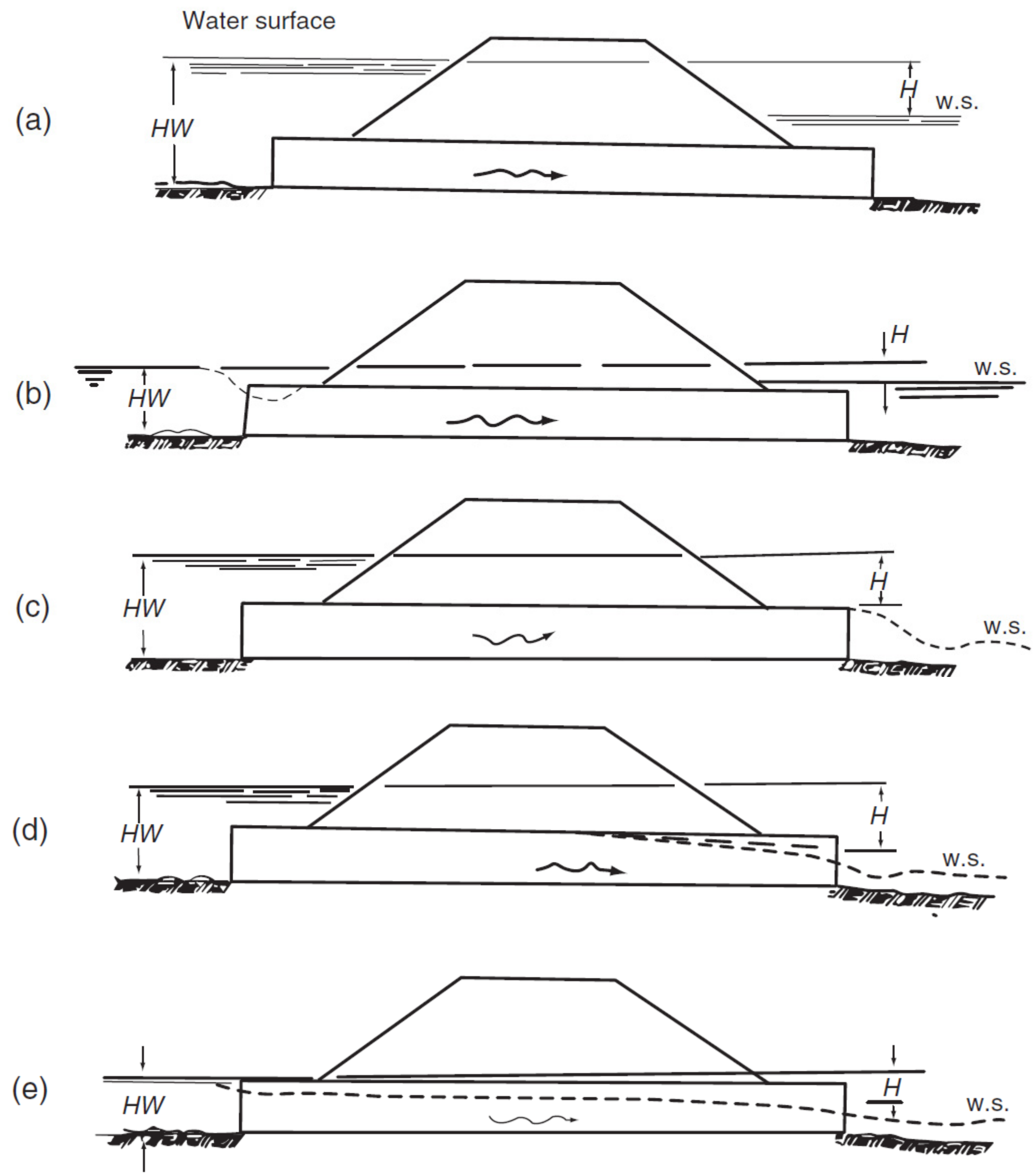


Outlet control flow:

- A culvert may **flow full** or **partially full** under outlet control conditions.
- **Several** outlet control flow **types** are depicted on the side figure.
- **Conditions a, d, and e** shown in this figure are **most common**.

Types of outlet control flow

(Normann et al., 1985)

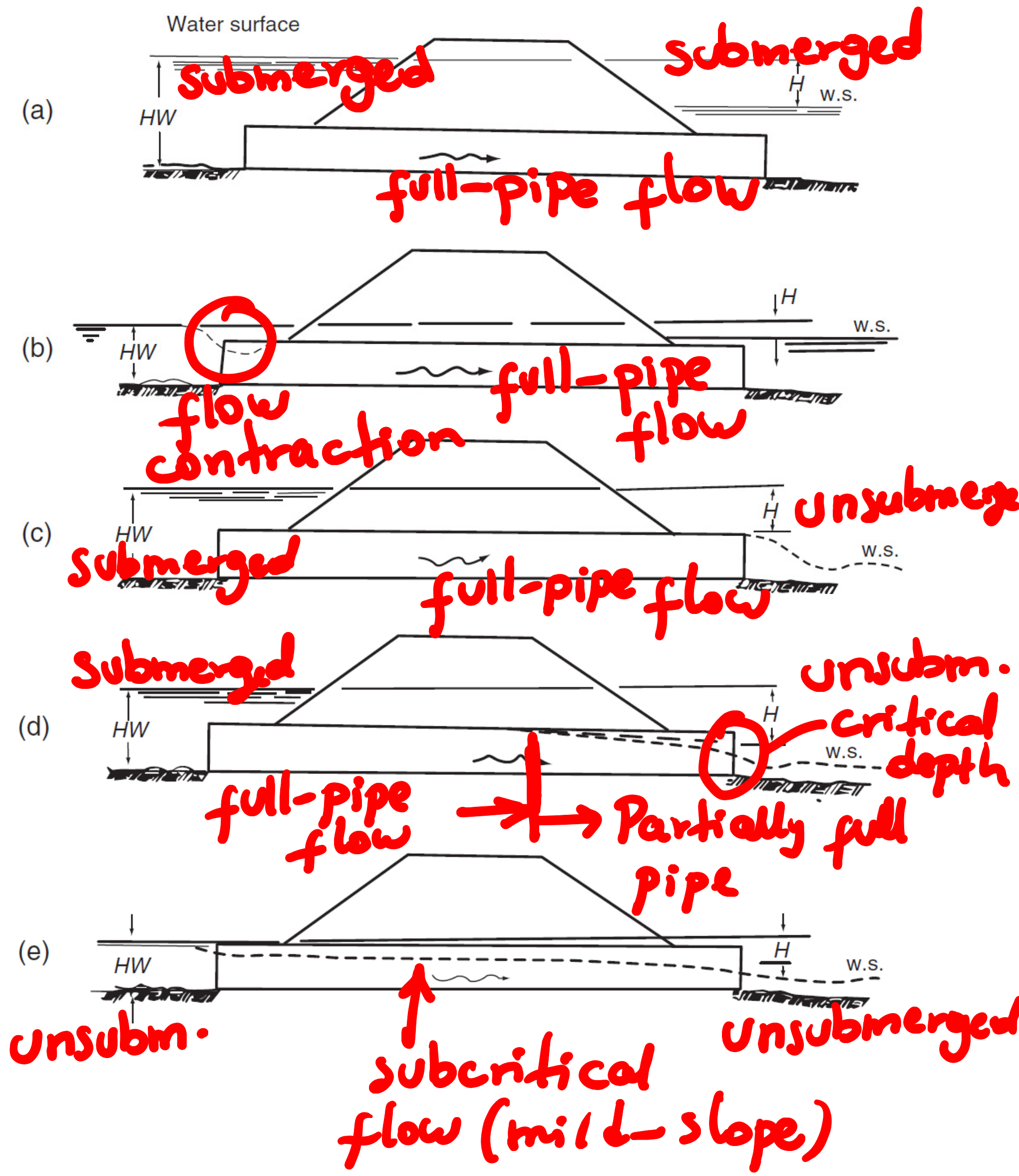


Outlet control flow (Cont.):

- **Fig. (a)**: classic full-pipe flow, in which **pressurized flow** occurs throughout the barrel.
- **Fig. (b)**: the **outlet is submerged** but the **inlet is unsubmerged** for low values of headwater because of the flow contraction at the inlet.
- **Fig. (c)**: **outlet is unsubmerged**, but the culvert still flows full due to a **high headwater**.
- **Fig. (d)**: **outlet not only is unsubmerged**, the **barrel flows partly full near the outlet** and passes through critical depth there.
- **Fig. (e)**: **both the inlet and outlet are unsubmerged** and we have open channel flow that is subcritical on a mild slope

Types of outlet control flow

(Normann et al., 1985)



Outlet control (Full-flow conditions):

Energy equation

$$HW = TW - SL + \left(1 + k_e + \frac{2gn^2L}{k_n^2 R^{4/3}}\right)\frac{Q^2}{2gA^2}$$

where TW = tailwater depth measured from the downstream invert of the culvert, S = culvert slope, L = culvert length, g = gravitational acceleration, n = Manning roughness factor (see table below), R = hydraulic radius, A = cross-sectional area, k_e = entrance loss coefficient given in the table below and,

$$k_n = 1.0 \quad \text{m}^{1/3}/\text{s (SI units)}$$

$$k_n = 1.49 \quad \text{ft}^{1/3}/\text{s (English units)}$$

- The two figures below present **nomographs** for **full flow in concrete pipe culverts** and **concrete box culverts**, respectively. Full-flow nomographs for other types of culverts are also available in the literature (Normann et al., 1985).

Recommended Manning's *n* values for selected conduits

(FHWA, 2001)

Type of conduit	Wall and joint description	Manning's *n*
Concrete pipe	Good joints, smooth walls	0.011–0.013
	Good joints, rough walls	0.014–0.016
	Poor joints, rough walls	0.016–0.017
Concrete box	Good joints, smooth finished walls	0.012–0.015
	Poor joints, rough, unfinished walls	0.014–0.018
Corrugated metal pipes and boxes, annular corrugations	$2\frac{2}{3}$ by $\frac{1}{2}$ in. corrugations	0.027–0.022
	6 by 1 in. corrugations	0.025–0.022
	5 by 1 in. corrugations	0.026–0.025
	3 by 1 in. corrugations	0.028–0.027
	6 by 2 in. structural plate	0.035–0.033
	9 by $2\frac{1}{2}$ in. structural plate	0.037–0.033
Corrugated metal pipes, helical corrugations, full circular flow	$2\frac{2}{3}$ by $\frac{1}{2}$ in. corrugations, 24 in. plate width	0.012–0.024
Spiral rib metal pipe	$\frac{3}{4}$ by $\frac{3}{4}$ in. recesses at 12 in. spacing, good joints	0.012–0.013

Entrance Loss Coefficients k_e full or partial full entrance head loss

(After Normann et al., 1985)

TABLE 6.5 Entrance loss coefficients: Outlet control, full or partly full entrance head loss, where $H_L = K_e\left(\frac{V^2}{2g}\right)$

Type of structure and design of entrance	Coefficient K_e
Pipe, concrete	
Projecting from fill, socket end (groove end)	0.2
Projecting from fill, square cut end	0.5
Headwall or headwall and wingwalls	
Socket end of pipe (groove end)	0.2
Square edge	0.5
Rounded (radius = $\frac{1}{12}$ d)	0.2
Mitered to conform to fill slope	0.7
End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Pipe, or pipe arch, corrugated metal	
Projecting from fill (no headwall)	0.9
Headwall or headwall and wingwalls, square edge	0.5
Mitered to conform to fill slope, paved or unpaved slope	0.7
End section conforming to fill slope	0.5
Beveled edges, 33.7° or 45° bevels	0.2
Side- or slope-tapered inlet	0.2
Box, reinforced concrete	
Headwall parallel to embankment (no wingwalls)	
Square edged on three edges	0.5
Rounded on three edges to radius of $\frac{1}{12}$ barrel dimension, or beveled edges on three sides	0.2
Wingwalls at 30°–75° to barrel	
Square edged at crown	0.4
Crown edge rounded to radius of $\frac{1}{12}$ barrel dimension, or beveled top edge	0.2
Wingwall at 10°–25° to barrel	
Square edged at crown	0.5
Wingwalls parallel (extension of sides)	
Square edged at crown	0.7
Side- or slope-tapered inlet	0.2

Outlet control (Partly full-flow conditions):

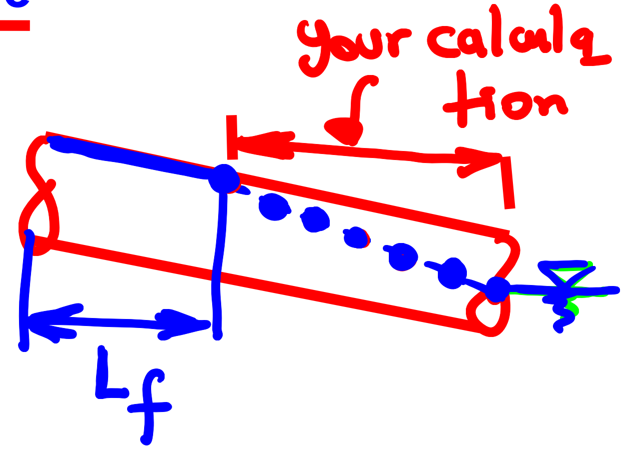
(Case d)

- For partly full flow in culverts controlled by the outlet, an **accurate relationship** between discharge and headwater elevation can be obtained by using the **gradually-varied flow (GVF) calculations**.

- In the GVF calculations, the downstream depth is set equal to the higher of the tailwater depth (TW) and the critical depth (y_c). If the calculated water surface profile intersects the top of the barrel, full-flow equations are used between that point and the upstream end of the culvert. **The head loss (h_{Lf}) for the full-flow segment is calculated as**

head loss

$$h_{L_f} = \left(1 + k_e + \frac{2gn^2 L_f}{k_n^2 R^{4/3}}\right)\frac{Q^2}{2gA^2}$$



where L_f = length of the full-flow segment.

Outlet control (Partly full-flow conditions): Flow conditions (d) and (e)

- To **avoid tedious gradually-varied flow calculations**, the FHWA (Normann et al., 1985) developed an approximate method for partly-full outlet control. In this method, the headwater elevation is calculated using

$$HW = H_D - SL + \left(1 + k_e + \frac{2gn^2L}{k_n^2R^{4/3}}\right)\frac{Q^2}{2gA^2}$$

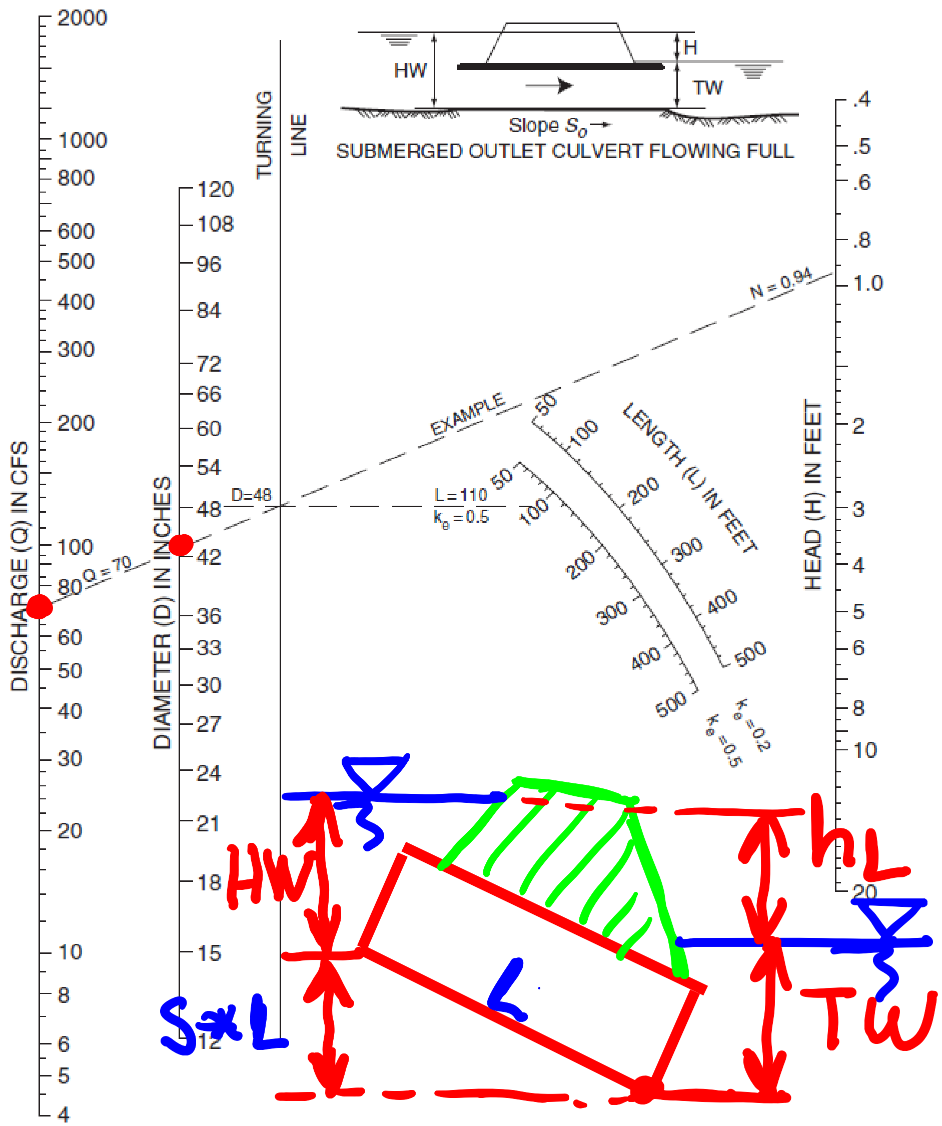
in which R and A are calculated assuming the culvert is full. Also, **H_D is set equal to the tailwater depth, TW, if $TW > (y_c + D)/2$** where y_c = critical depth and D = interior height of the culvert. **Otherwise, $H_D = (y_c + D)/2$.**

- The equation above is deemed satisfactory when the **culvert flows full over at least part of its length**. The approximate method becomes less accurate if free-surface flow occurs over the entire length of the culvert, in which case the results are **acceptable only if HW > 0.75D**. For lower headwater elevations, gradually-varied flow calculations are required.

hL (Not Headwater)

Head loss in concrete pipe culverts flowing full with n = 0.012

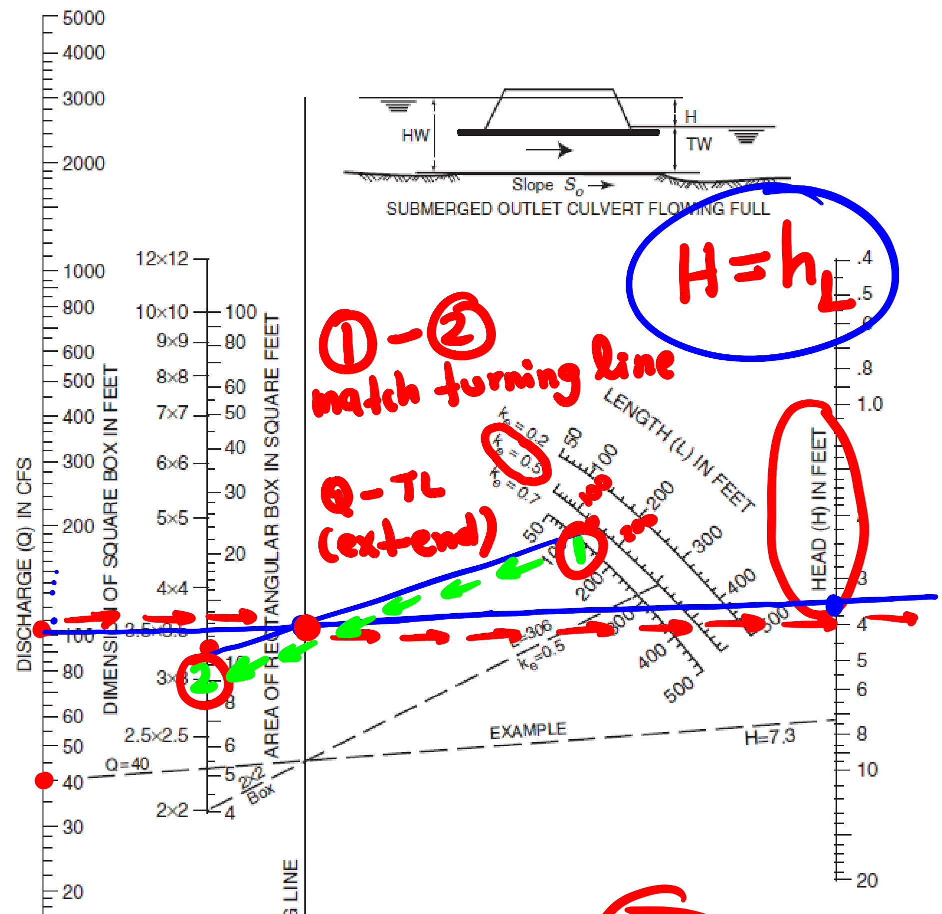
(After Normann et al., 1985)



hL

Head loss in concrete box culverts flowing full with n = 0.012

(After Normann et al., 1985)



$HW + SL = h_L + TW$

$HW = h_L + TW - SL$

Example:

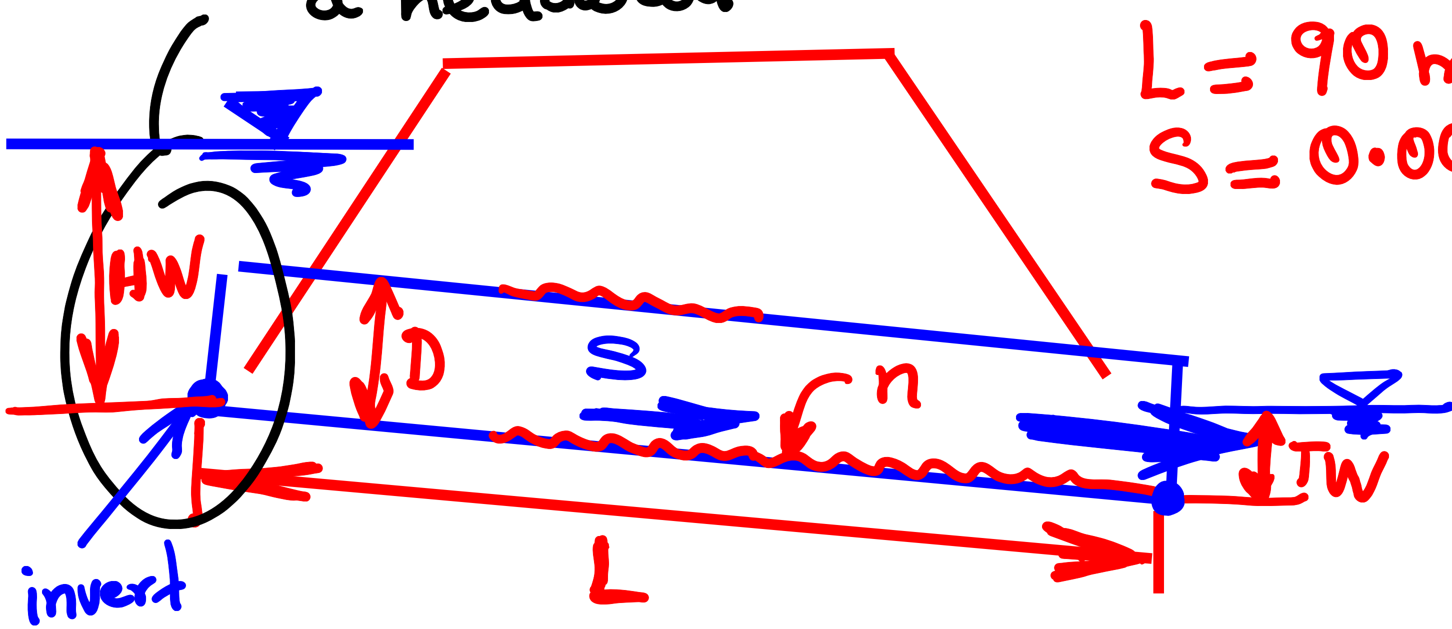
A 0.91 m diameter concrete pipe culvert (n = 0.012) has a length of 90 m and a slope of 0.0067. The entrance has a square edge in a headwall. At the design discharge of 1.2 m^3/s, the tailwater is 0.45 m above the outlet invert. Determine the head on the culvert at the design discharge.

Square edge in a headwall

$D = 0.91\,m$
$n = 0.012$
$L = 90\,m$
$S = 0.0067$

$Q_d = 1.2\,m^3/s$
$TW = 0.45\,m$

$HW = ??$



* Assume inlet control

Submerged or Unsubmerged ?

$$\frac{Q}{AD^{0.5} g^{0.5}}$$

$$= \frac{1.2}{\frac{\pi \times 0.91^2}{4} \times 0.91^{0.5} * 9.8^{0.5}}$$

$$= 0.62$$ [Unsubmerged].

* Form I or Form II ?

→ With Form II

$$\frac{HW}{D} = K_{II} \left(\frac{Q}{AD^{0.5} g^{0.5}} \right)^{M_{II}}$$

$$\frac{HW}{D} = K_{II} \times 0.62^{M_{II}}$$

$K_{II} =$
$M_{II} =$ } Couldn't find appropriate values for desired inlet type

→ With Form I

$$\frac{HW}{D} = \frac{y_c}{D} + \frac{V_c^2}{2g} + K_I\left(\frac{Q}{AD^{0.5}g^{0.5}}\right)^{M_I} + K_s \cdot S$$

y_c (When $F=1$) $= 0.65$ m

$A_c\ [f(y_c)] = 0.49\ m^2$

$$V_c = \frac{Q}{A_c} = \frac{1.2}{0.49} = 2.45\ m/s$$

$K_I = 0.3155$, $M_I = 2.0$, $K_s = -0.5$

$\frac{HW}{D} = 1.168 \rightarrow \boxed{HW = 1.06\ m}$ For inlet control.

Let's calculate the normal depth to check if flow in culvert is supercritical.

$$Q = \frac{k}{n} A R^{2/3} S^{1/2} \longrightarrow y_n = 0.57\,m$$

$$y_n \qquad\qquad y_c$$
$$0.57\,m < 0.65\,m$$

(Flow is supercritical)

* **Let's assume outlet control**

Full-flow or Partly-full flow condition?

Tw is low $(0.45\,m < 0.91\,m)$. Also, the inlet is unsubmerged.

Partly-full flow condition

$$HW = H_D - SL + \left(1 + K_e + \frac{2 g n^2 L_f}{k_n^2 R^{4/3}}\right)\frac{Q^2}{2gA^2} \quad ..(*)$$

$$H_D = \begin{cases} TW & \text{if } TW > \frac{y_c + D}{2} \\ \frac{y_c + D}{2} \end{cases}$$

$TW = 0.45\,m$, $\frac{y_c + D}{2} = \frac{0.65 + 0.91}{2} = 0.78\,m$

$H_D = 0.78\,m$

$k_e = 0.50$

$A = \frac{\pi D^2}{4} = 0.65$

$R = \frac{A}{P} = \frac{\cancel{\pi} D^{\cancel{2}}}{4 \cancel{\pi} \cancel{D}} = \frac{D}{4} = 0.2275$

In (*)

$HW = 0.76\,m$ [Partly-full as assumed].

* Culvert flow has INLET control.

$HW = 1.06\,m$

$F > 1.0$

Example:

A 100-ft long circular culvert has a diameter $D = 4$ ft and a bottom slope $S = 0.02$. The culvert has a smooth tapered inlet throat, not mitered to the embankment slope. Determine the headwater depth when the culvert carries 120 cfs under inlet control conditions.

$Q = 120$ cfs
$L = 100$ ft
$D = 4$ ft
$S = 0.02$

$HW = ?$

INLET CONTROL

* Submerged or Unsubmerged ?

$$\frac{Q}{AD^{0.5}g^{0.5}}$$

$$\frac{120}{\frac{\pi \times 4^2}{4} \times 4^{0.5} \times 32.2^{0.5}} = 0.84$$

$0.84 > 0.70$

(INLET IS SUBMERGED)

$$\therefore \frac{HW}{D} = c\left[\frac{Q}{AD^{0.5}g^{0.5}}\right]^2 + Y + k_s * S$$

$k_s = -0.5$ (inlet not mittered)

$c = 0.6311$

$Y = 0.89$

$$\frac{HW}{D} = 0.6311(0.84)^2 + 0.89 + (-0.5) \times 0.02$$

$$\frac{HW}{D} = 1.33$$

$$\boxed{HW = 5.32 \text{ ft}}$$

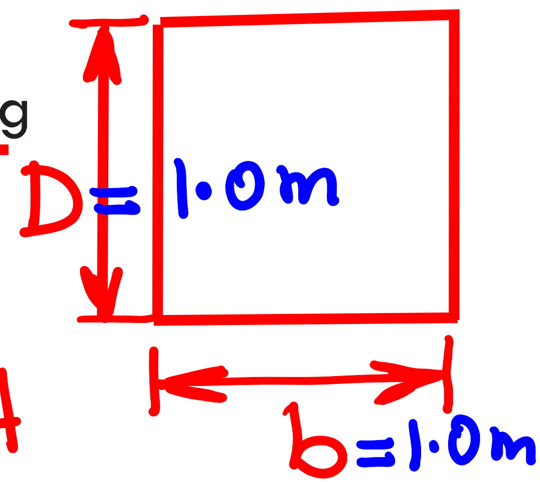
Example:

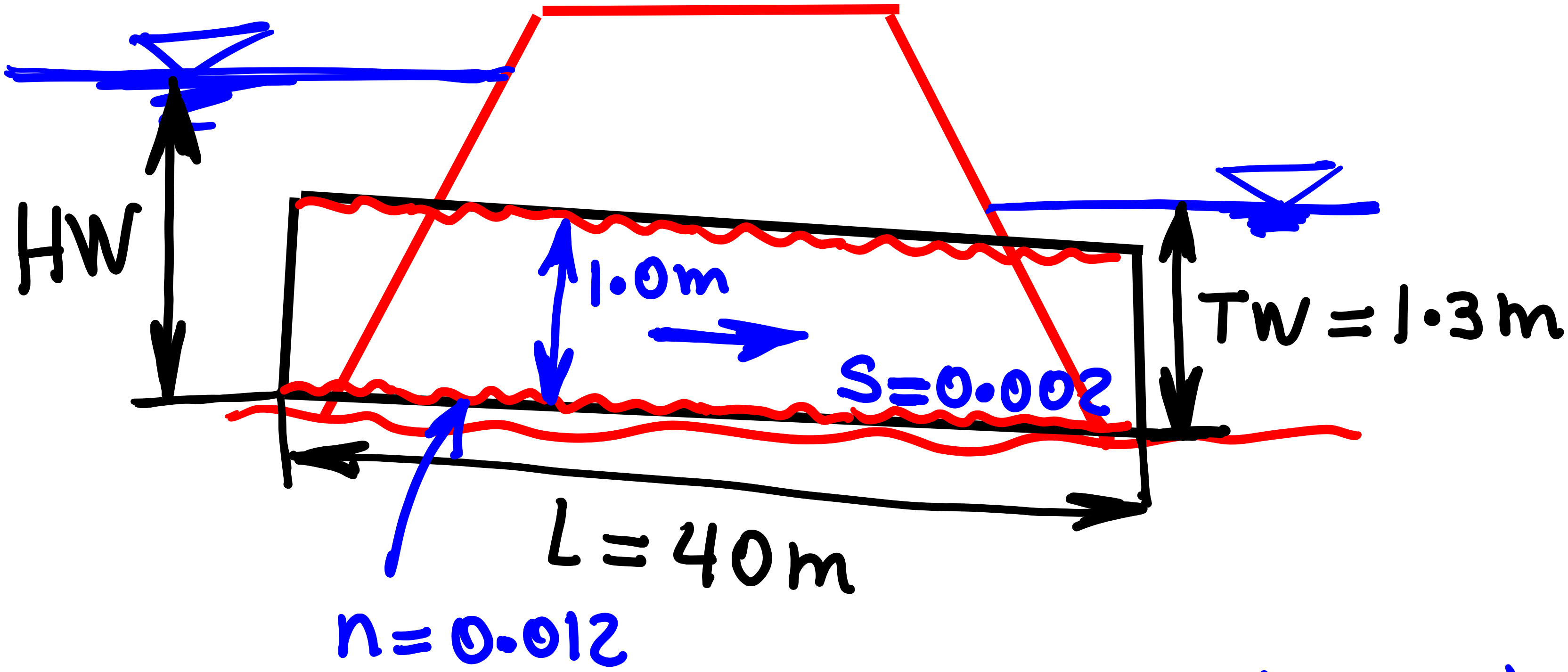
A reinforced concrete rectangular box culvert has the following properties: D = 1.0 m, b = 1.0 m, L = 40 m, n = 0.012 and S = 0.002. The inlet is square-edged on three edges and has a headwall parallel to the embankment, and the outlet is submerged with TW = 1.3 m. Determine the headwater depth, HW, when the culvert is flowing full at Q = 3.0 m^3/s

$L = 40\,m$
$D = 1.0\,m$
$b = 1.0\,m$
$n = 0.012$
$S = 0.002$

$TW = 1.3\,m$
$HW = ??$
$Q = 3\,m^3/s$

Outlet Control





*Because culvert is flowing full, we have outlet control.

$$HW = TW - SL + \left(1 + K_e + \frac{2gn^2L}{k_n^2 R^{4/3}}\right)\frac{Q^2}{2gA^2}$$

$K_e = 0.5$ (square edged on three edges)

$$HW = 1.3 - 0.002\times 40 + \left(1+0.5+\frac{2\times 9.81\times 0.012^2\times 40}{1^2\times 0.25^{4/3}}\right)\frac{3^2}{2\times 9.81\times 1^2}$$

$$R = \frac{A}{P} = \frac{1}{1+1+1+1} = 0.25$$

$$\boxed{HW = 2.24\ m}$$

* Using Monograph (FHWA)

$$HW = TW + h_L - SL$$

Called "H" in monograph

$$HW = 1.3 + 3.8\times \underline{0.3048} - 0.002\times 40$$

to convert to meters

$$\underline{HW = 2.38\ m}$$