Instructor: Arturo Leon Name SOLUTION KEY

QUIZ 2

CE 412/512 Hydrology - Spring 2013

Quiz is closed book and closed notes. For all problems, **write the equations used**, **show your calculations**, **include units**, and **box your answer**.

1. *(30 pts)* The initial rate of infiltration of a watershed is estimated as 2.1 in/hr, the final capacity is 0.2 in/hr, and the time constant, k, is 0.4 hr^{-1}. Use Horton's Equation to find:

   A. The infiltration capacity at t = 2 hr and t = 6 hr; and
   B. The total volume of infiltration over the 6-hr period.

Horton's Equation: $f = f_c + (f_0 - f_c)e^{-kt}$

$$F = f_c t + \frac{(f_0 - f_c)}{-k}(e^{-kt})$$

Should have been: $F = f_c t + \frac{(f_0 - f_c)}{k}(1 - e^{-kt})$

SOLUTION

A. $f = f_c + (f_0 - f_c)e^{-kt}$ *(15 pts)*

$$f = 0.2\frac{in}{hr} + (2.1 - 0.2)\frac{in}{hr} * e^{-0.4t}$$

$$f = 0.2\frac{in}{hr} + 1.9(e^{-0.4t})\frac{in}{hr}$$

At t = 2 hr:

$$f = 0.2\frac{in}{hr} + 1.9(e^{-0.4*2})\frac{in}{hr} = \boxed{1.05\frac{in}{hr}}$$

At t = 6 hr:

$$f = 0.2\frac{in}{hr} + 1.9(e^{-0.4*6})\frac{in}{hr} = \boxed{0.37\frac{in}{hr}}$$

B. $Volume = \int f\,dt = \int 0.2 + 1.9(e^{-0.4*t})dt$ *(15 pts)*

$$Volume = [0.2t + \left(\frac{1.9}{-0.4}\right)(e^{-0.4*t})]_0^6$$

$Volume = 5.52\ in$ Wrong equation (no points deducted for this solution)

$$Volume = \int f\,dt = \int 0.2 + 1.9(e^{-0.4*t})dt$$

$$Volume = [0.2t + \left(\frac{1.9}{0.4}\right)(1 - e^{-0.4*t})]_0^6$$

$Volume = \boxed{5.52\ in}$ Correct equation/answer

NOTE: The infiltration volume (F) is calculated as the integral of the infiltration (f) equation, and it must be solved from time t = 0 to 6 hr. Simply plugging in t=6 hr into the F equation is not complete.

Due to confusion with the provided equation for F, full credit was given for the following solution as well (even though it is incomplete):

$$F = f_c t + \frac{(f_0 - f_c)}{-k}(e^{-kt}) = \left(0.2\frac{in}{hr}\right) * (6\ hr) + \frac{\frac{(2.1 - .02)in}{hr}}{-\frac{0.4}{hr}}\left(e^{-\frac{0.4}{hr}*6\ hr}\right) = \boxed{0.769\ in}$$

2. *(20 pts)* Use the rainfall data below to determine the ϕ index for the watershed if the runoff depth was 6.6 in.

Time (hr)	Rainfall (in/hr)
0-1	1.1
1-3	1.8
3-5	2.6
5-8	1.3

SOLUTION

Method 1:

Total rainfall = $1hr\left(\frac{1.1in}{hr}\right) + 2\ hr\left(1.8\frac{in}{hr}\right) + 2\ hr\left(2.6\frac{in}{hr}\right) + 3\ hr\left(\frac{1.3in}{hr}\right) = 13.8\ in$

Runoff = 6.6 in

Infiltration = 13.8 – 6.6 = 7.2 in

ϕ * 8 hr = 7.2 in

ϕ = 0.9 in/hr

Method 2:

$$1(1.1-\phi) + 2(1.8-\phi) + 2(2.6-\phi) + 3(1.3-\phi) = 6.6\ in.$$

By trial and error:

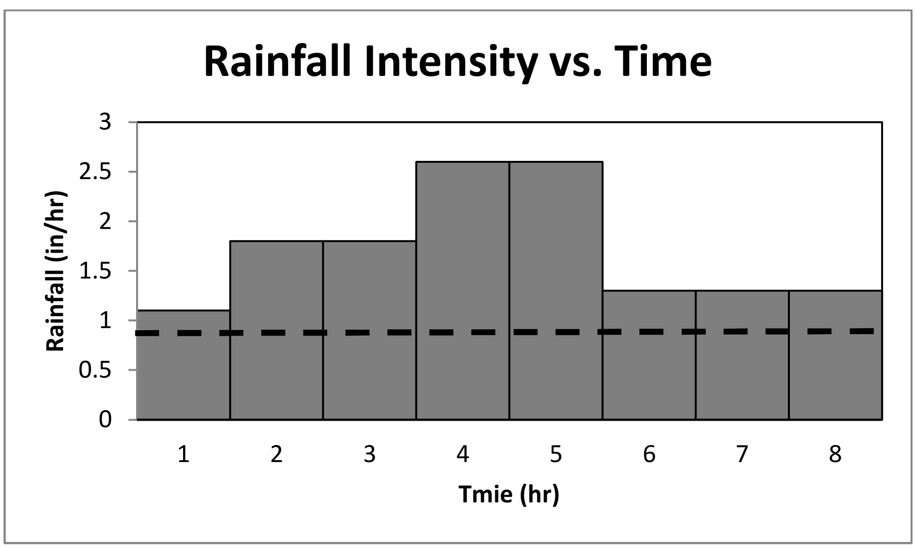
Try ϕ = 1

$$1(1.1-1) + 2(1.8-1) + 2(2.6-1) + 3(1.3-1) = 5.8\ in. \quad \neq 6.6\ in.$$

Try ϕ = 0.9

$$1(1.1-0.9) + 2(1.8-0.9) + 2(2.6-0.9) + 3(1.3-0.9) = 6.6\ in.$$

Therefore **ϕ = 0.9 in/hr**



3. *(50 pts)* A soil has the following soil properties for use in the Green-Ampt equation:

K_s = 1.5 in/hr θ_s = 0.523

ψ = -9.8 in θ_i = 0.308

i = 8.2 in/hr

Using the Green-Ampt infiltration method:

A. Calculate the initial moisture deficit, M_d.

B. Find the volume of water that will infiltrate before saturation is reached, F_s.

C. Find the time to saturation.

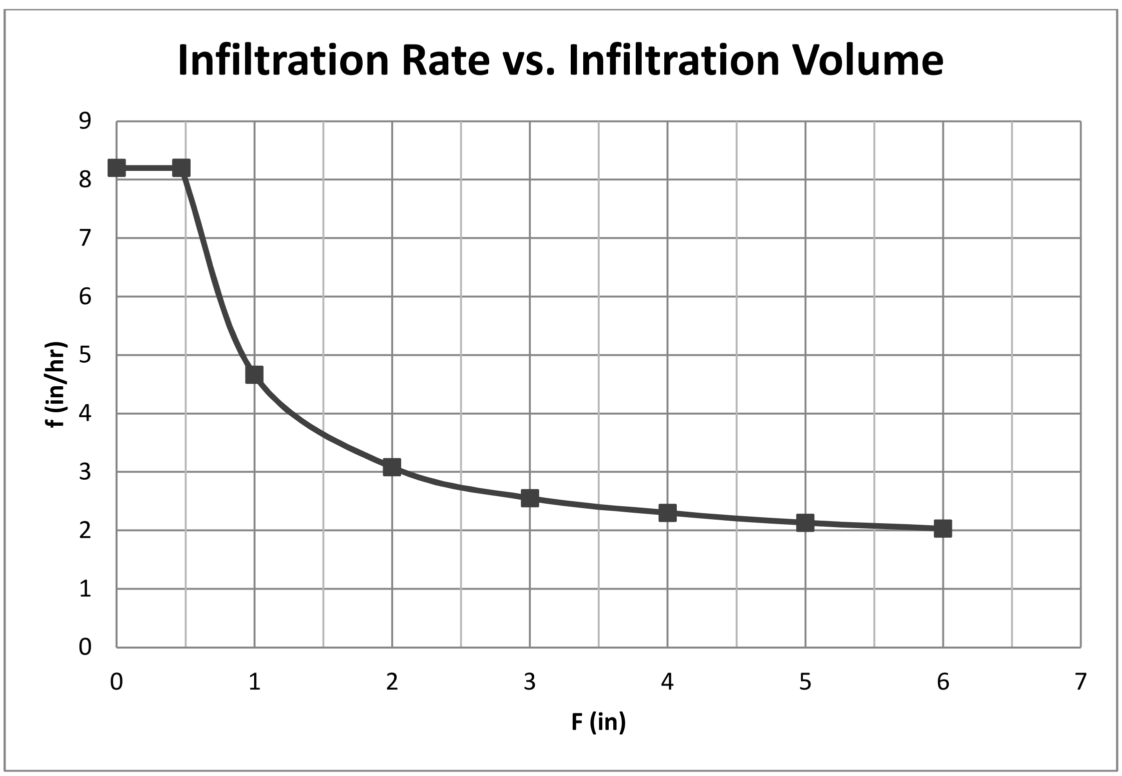
D. Plot the infiltration rate, *f*, vs. the infiltration volume, F, on the plot provided.

Green-Ampt Equations:

$$F_S = \frac{\psi M_d}{(1 - {}^{i}/_{K_S})}$$

$$f = K_s\left(1 - \frac{M_d \psi}{F}\right)$$

You may use the back of this page to show your calculations and solutions.



SOLUTION

A. $M_d = \theta_s - \theta_i$ *(10 pts)*

$M_d = 0.523 - 0.308 = \boxed{0.215}$

B. $F_s = \frac{\psi M_d}{(1 - i/K_s)}$ *(10 pts)*

$$F_s = \frac{(-9.8\ in)(0.215)}{(1 - \frac{8.2\frac{in}{hr}}{1.5\frac{in}{hr}})}$$

$$\boxed{F_s = \ 0.47\ in}$$

C. $Time\ to\ saturation = \frac{F_s}{i} = \frac{0.47\ in}{8.2\frac{in}{hr}} = \boxed{0.057\ hr}$ *(10 pts)*

D. $f = K_s\left(1 - \frac{M_d \psi}{F}\right)$ *(20 pts)*

$$f = 1.5\frac{in}{hr}\left(1 - \frac{(0.215)(-9.8\ in)}{F}\right)$$

$$f = 1.5\left(1 + \frac{2.107}{F}\right)$$

F (in)	f (in/hr)
0	8.2
0.47	8.2
1	4.66
2	3.08
3	2.55
4	2.30
5	2.13
6	2.03

See final graph on previous page.