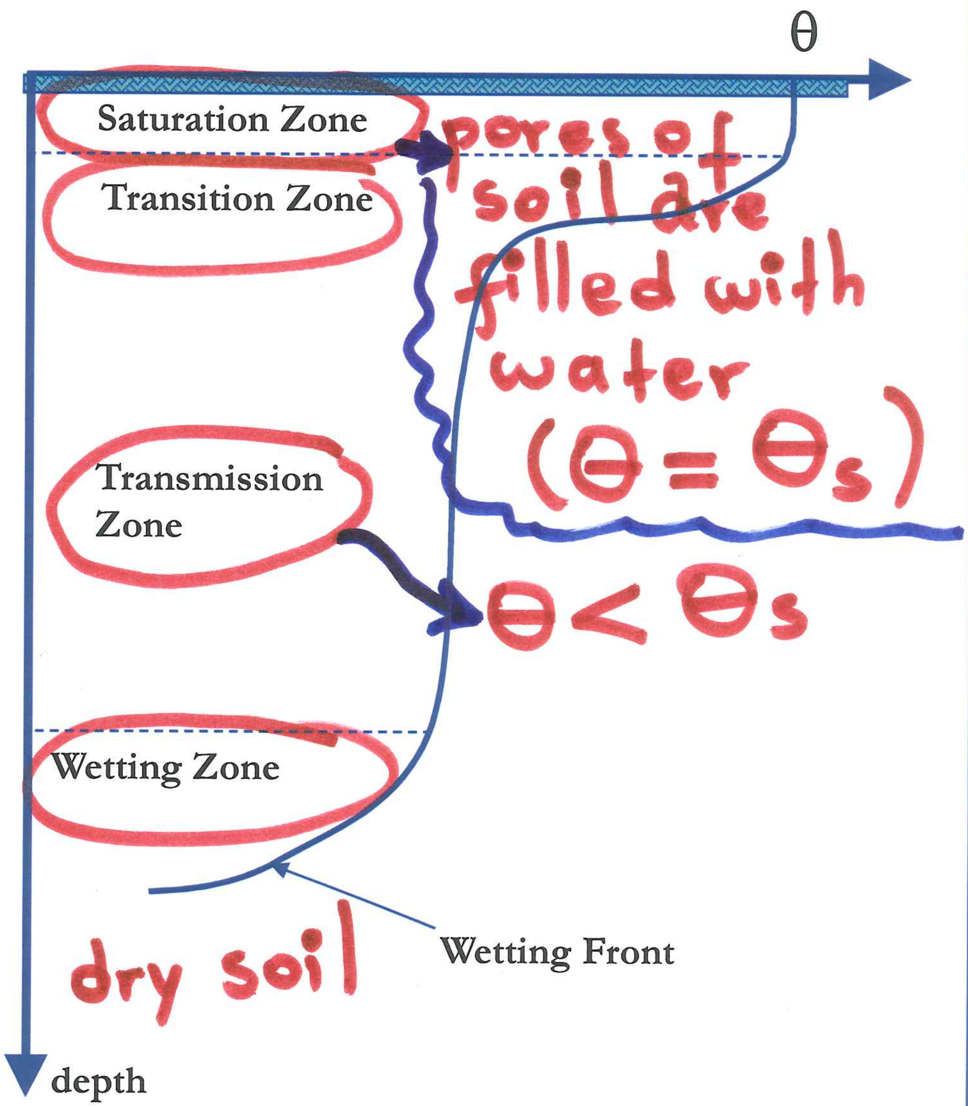
(1)

Green-Ampt Infiltration
Lecture 5, 04/16/2013

Arturo Leon, Oregon State University (Spring 2013)
Adapted from textbook and notes of Philip B. Bedient, David Maidment and Areeya Rittima

Revisiting infiltration

- Process of water penetrating from ground into soil
- Four zones
  - Saturated, transition, transmission and wetting.

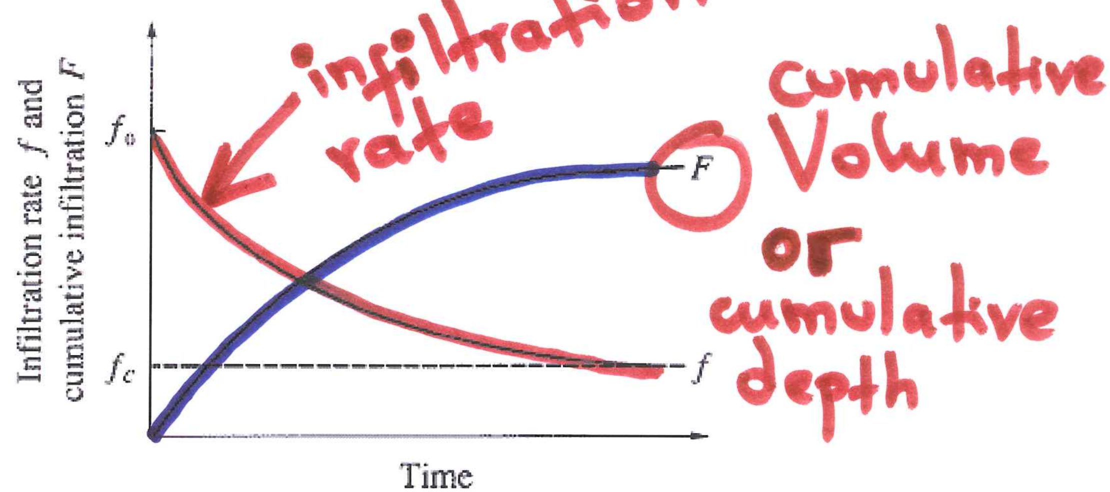


Revisiting infiltration (Cont.)

- **Infiltration rate** $f(t)$
  - Rate at which water enters the soil at the surface (in/hr or cm/hr)
- **Cumulative infiltration**
  - Accumulated depth of water infiltrating during given time period

$$F(t) = \int_0^t f(t)dt$$

$$f(t) = \frac{dF(t)}{dt}$$



Infiltration rate and cumulative infiltration.

Richard's Equation

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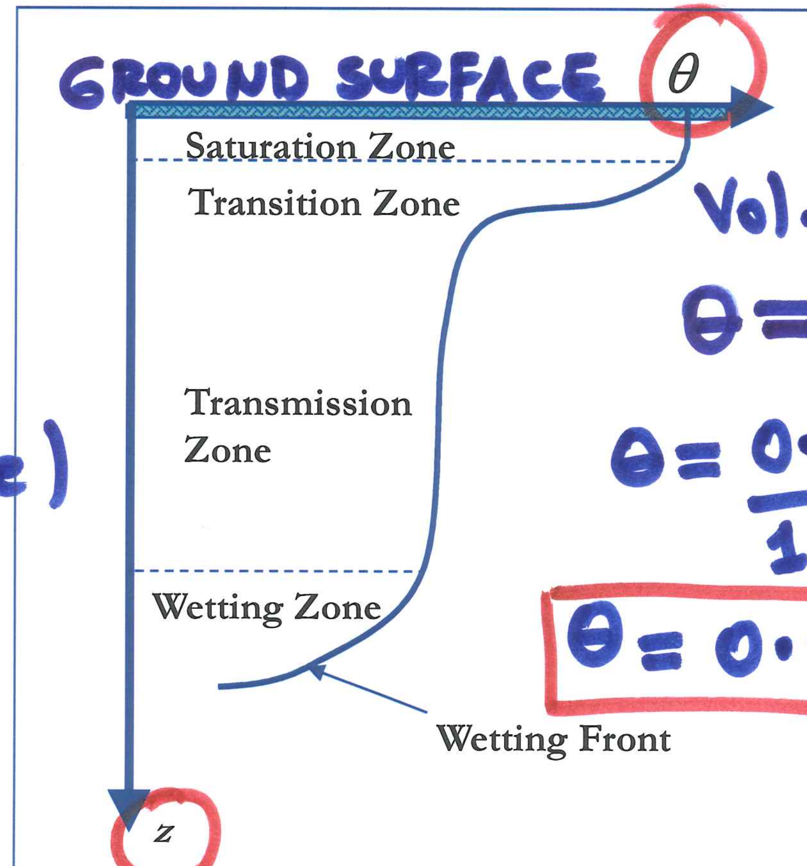
$$\frac{\partial \theta}{\partial t} = -\frac{\partial}{\partial z}\left(K(\theta)\frac{\partial \psi(\theta)}{\partial z}\right) - \frac{\partial K(\theta)}{\partial z}$$

- θ = Volumetric moisture content (**volume water/total volume**)
- z = distance below the surface
- $\psi(\theta)$ = Capillary suction (pressure) (cm of water) (Negative value)
- $K(\theta)$ = unsaturated hydraulic conductivity (cm/s)



Vol. 40% water

60% Vol. solids

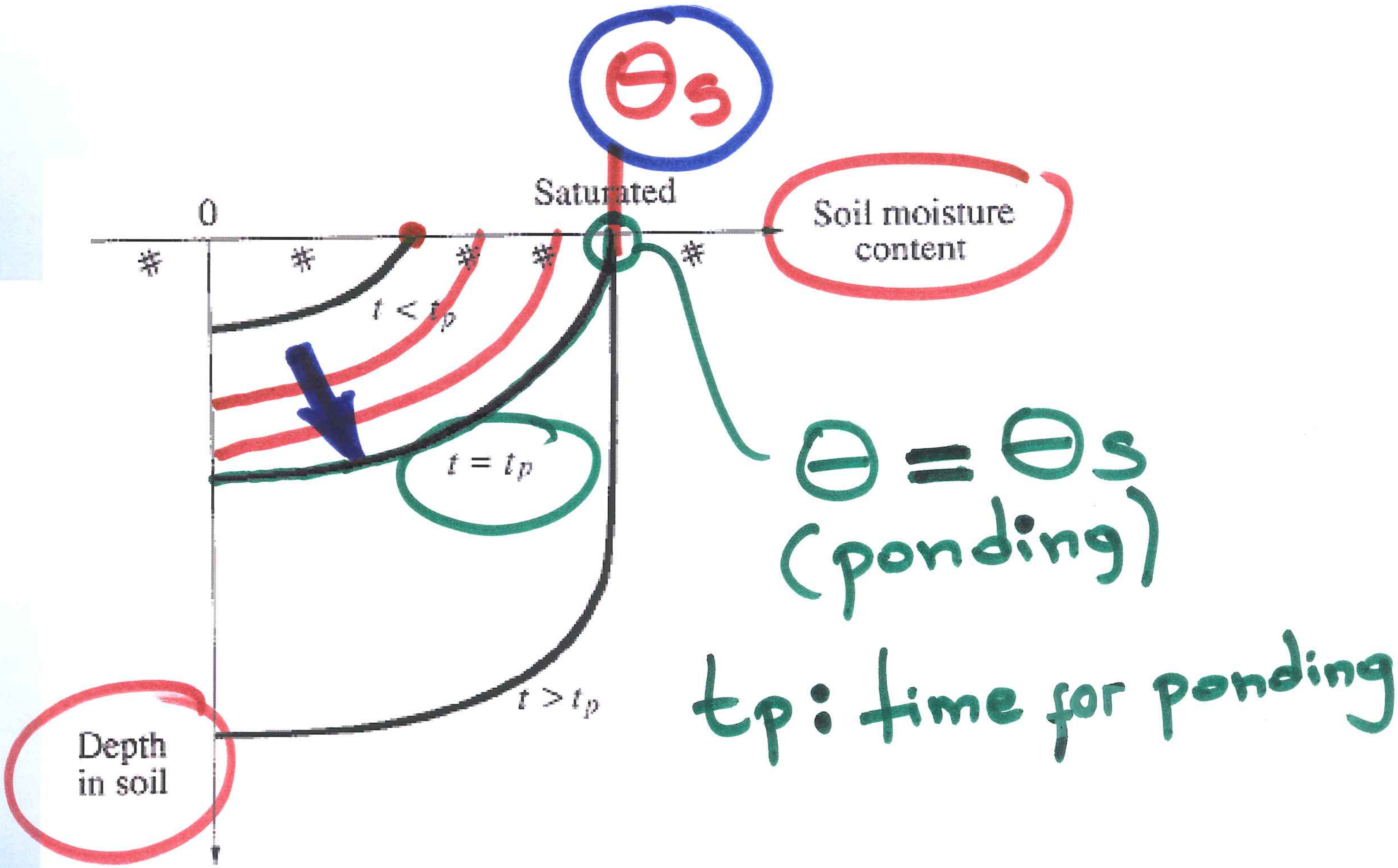
$\theta = 0.4$

$\theta = \frac{0.4 \text{ Vol}}{1 \text{ Vol}}$

$\theta = 0.4$

Ponding time (time at which $\theta = \theta_s$ (Soil surface))

Elapsed time between the time rainfall begins and the time water begins to pond on the soil surface (t_p)



Ponding Time (Saturation time)

⑥

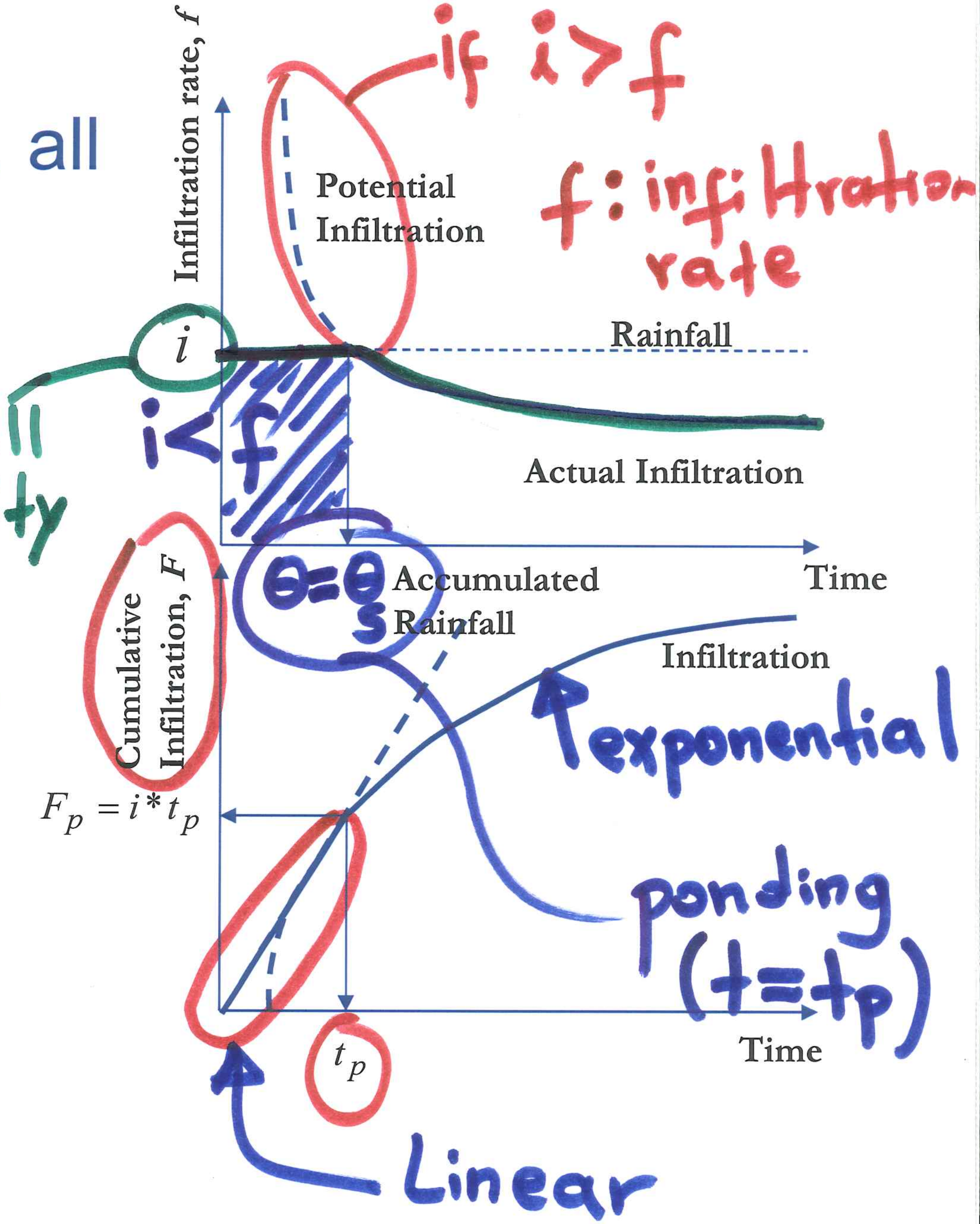
- Up to the time of ponding, all rainfall has infiltrated (i = rainfall rate)

$$f = i$$

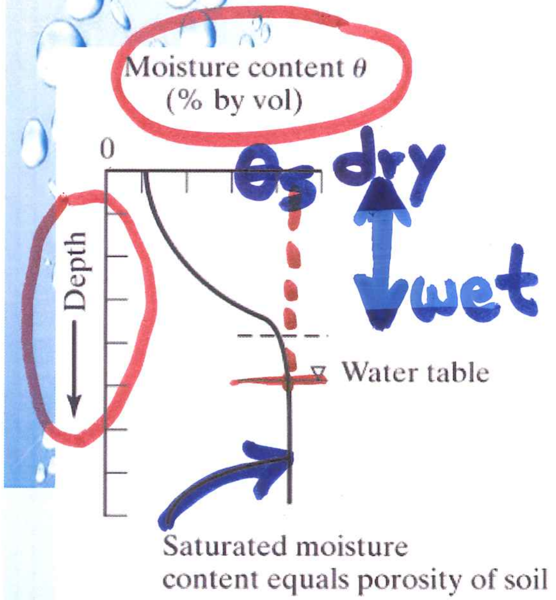
$$F = i * t_p = i * t_s$$

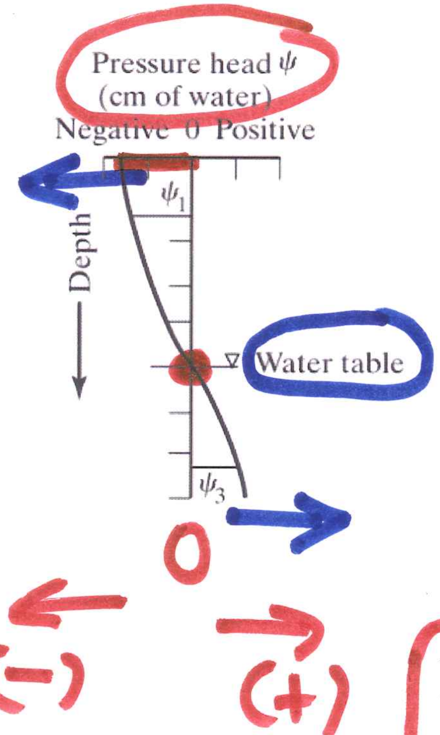
t_p = Ponding time

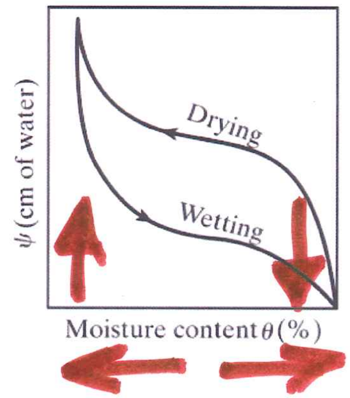
t_s = Saturation time

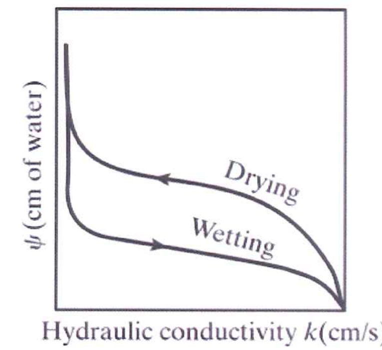


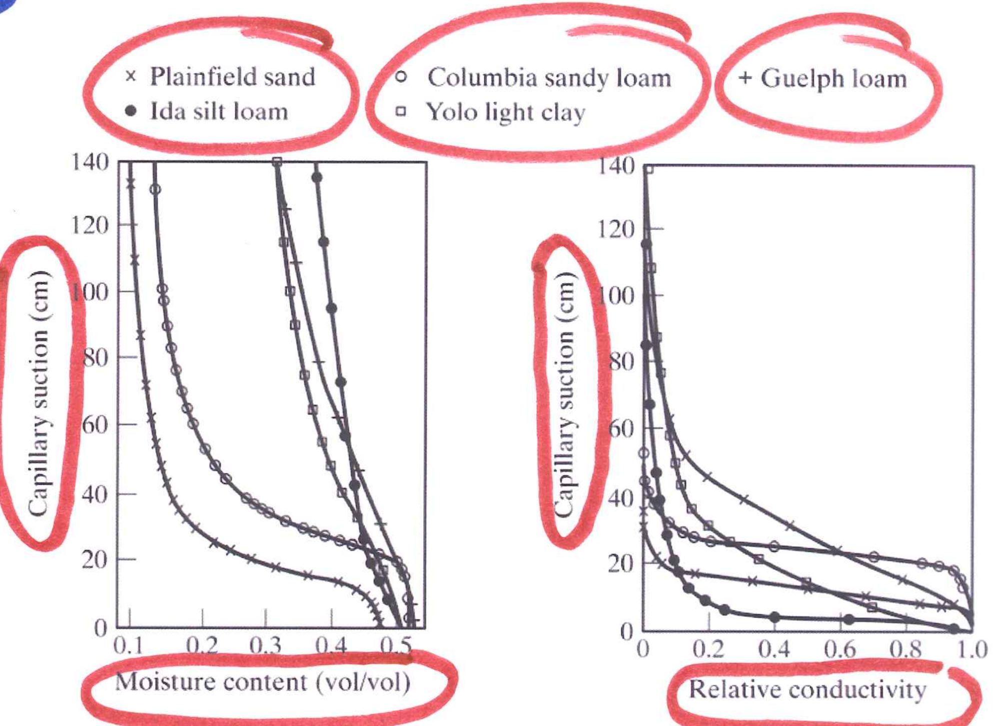
Typical relationships in the unsaturated zone











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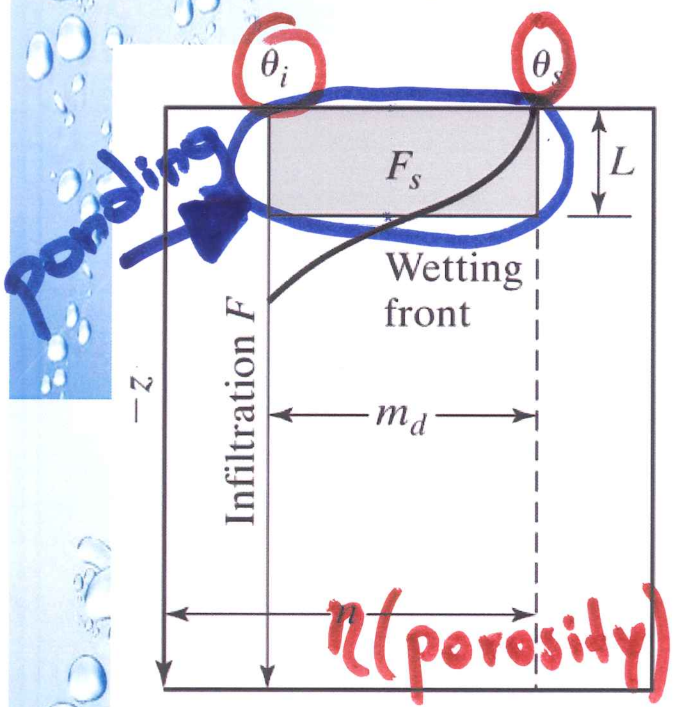
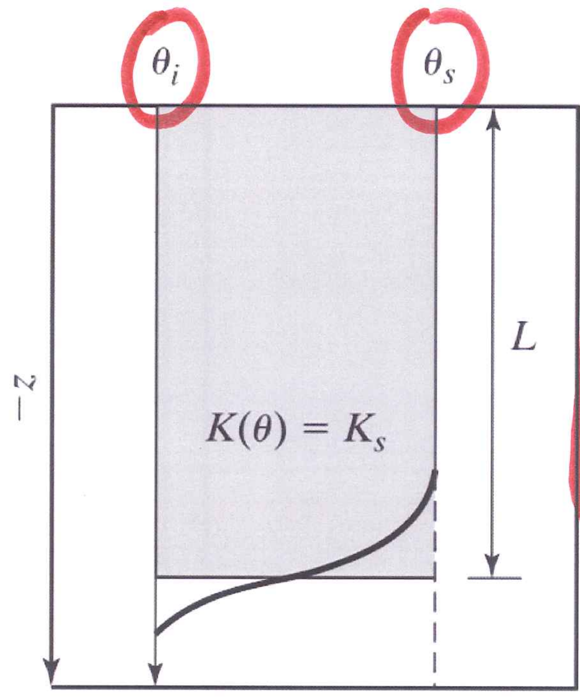
Moisture and infiltration relationships:

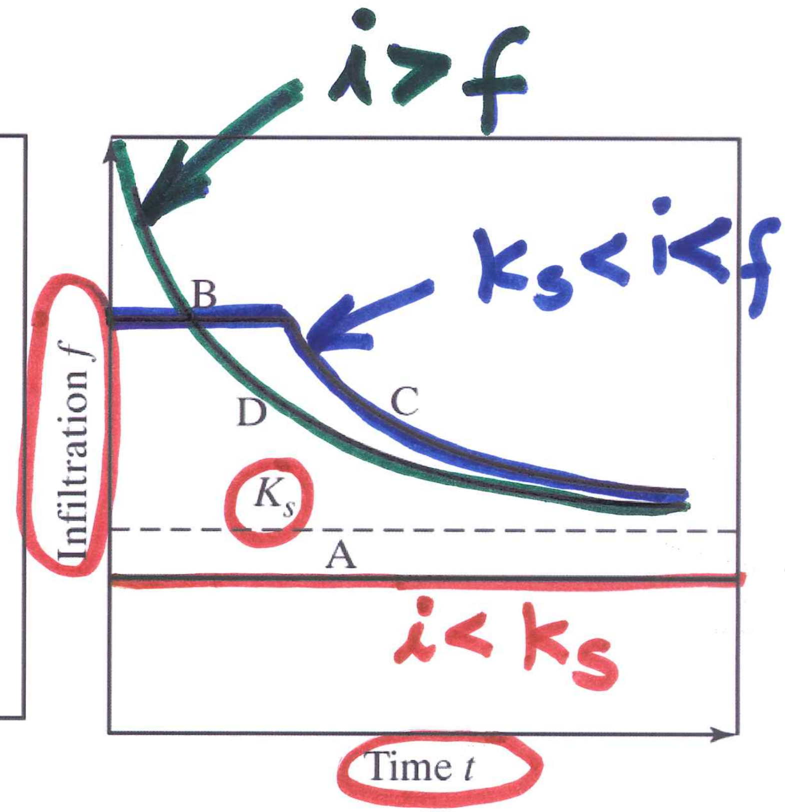
(a) Moisture profile at moment of surface saturation. (b) Moisture profile at a later time. (c) Infiltration behavior under different rainfall.



(a)



(b)



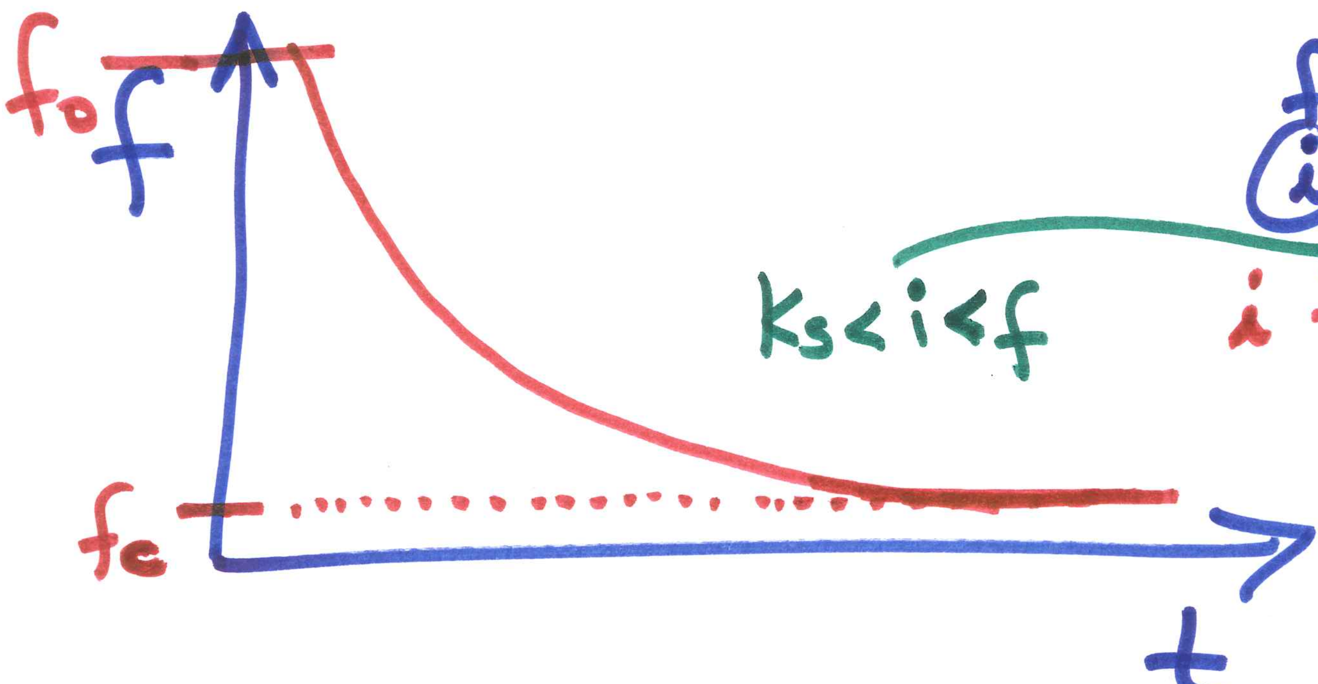
(c)

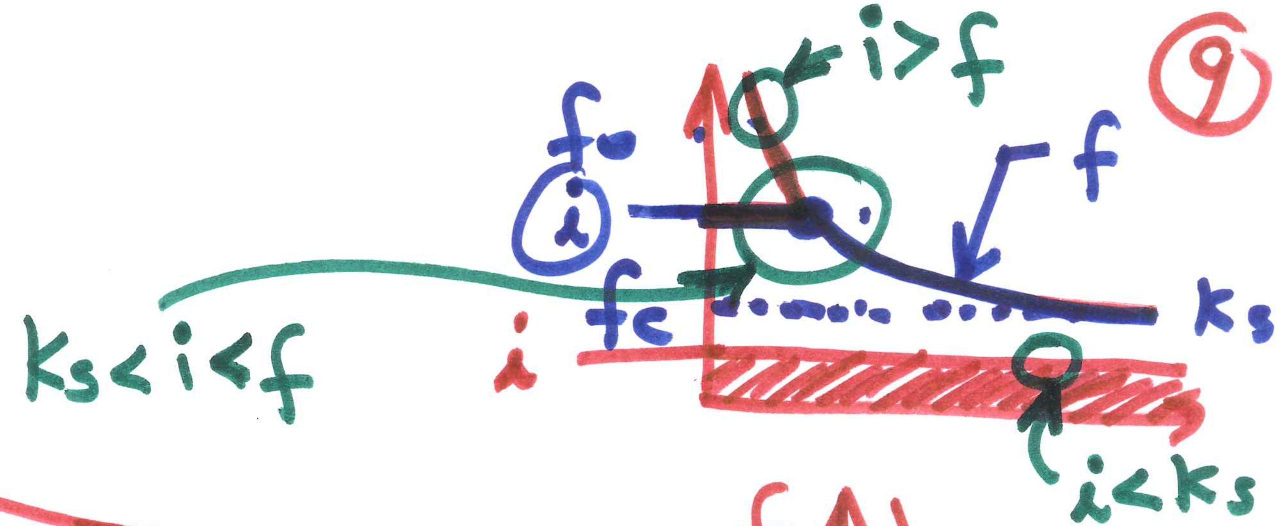
Θ_i = initial moisture

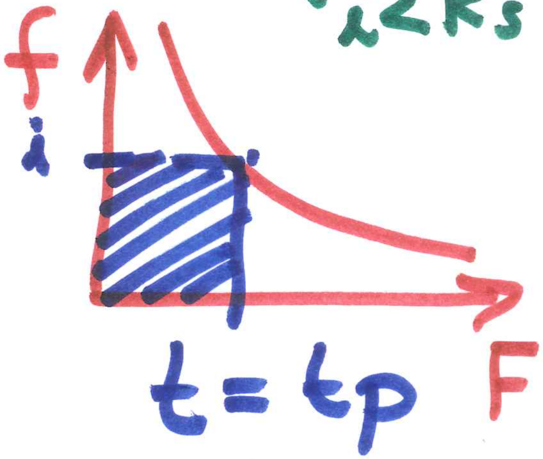
Θ_s = final moisture

η = porosity, F_s = Cumulative infiltration for saturation

K_s = Saturated hydraulic conductivity.







f_c = final capacity ($k_s = f_c$)

i = rainfall intensity

① $i < k_s$

② $k_s < i < f$

Green – Ampt Infiltration

L = Depth to Wetting Front

θ_i = Initial Soil Moisture

Ψ = Suction head

$$F(t) = L(\eta - \theta_i) = L\Delta\theta$$

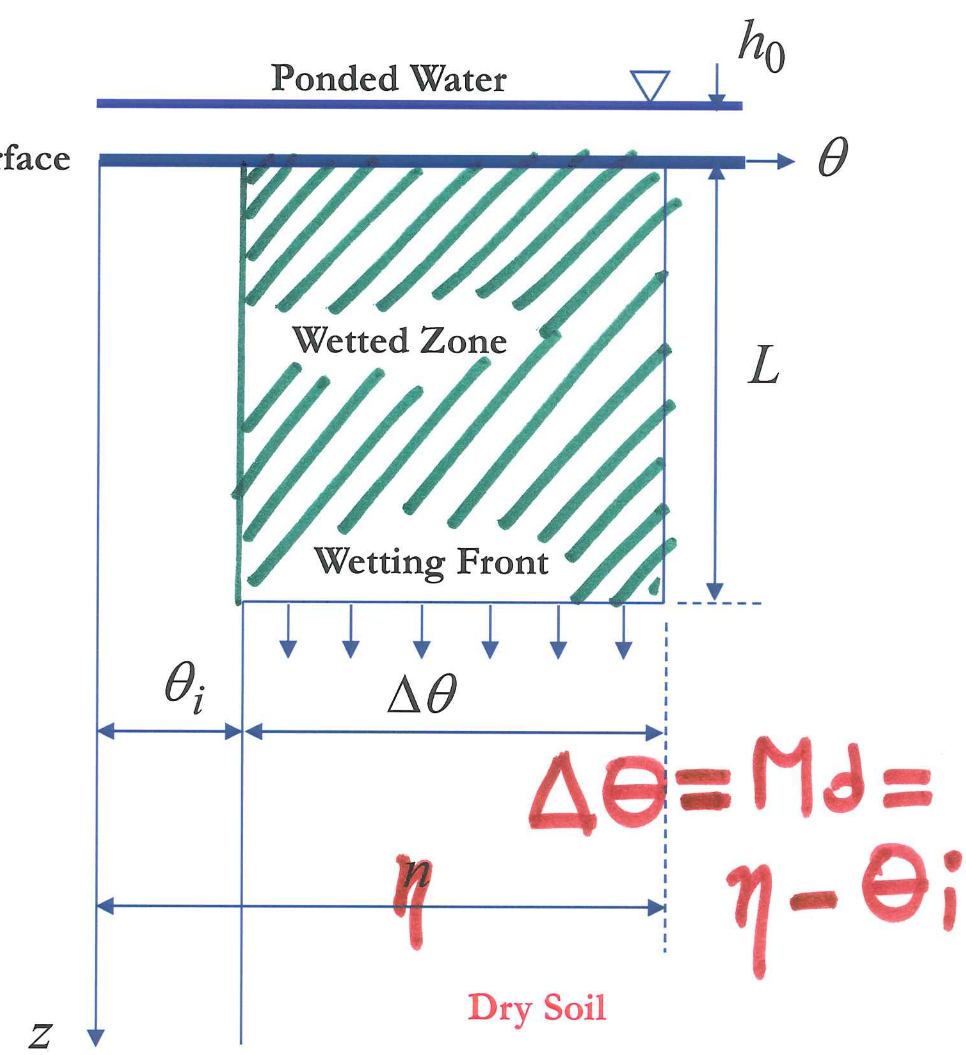
$$f = \frac{dF}{dt} = \Delta\theta \frac{dL}{dt}$$

$$q_z = -K\frac{\partial h}{\partial z} = -f$$

Darcy's Law

$$h = \Psi + z$$

$$f = K\frac{\partial \psi}{\partial z} + K$$



Green – Ampt Infiltration (Cont.)

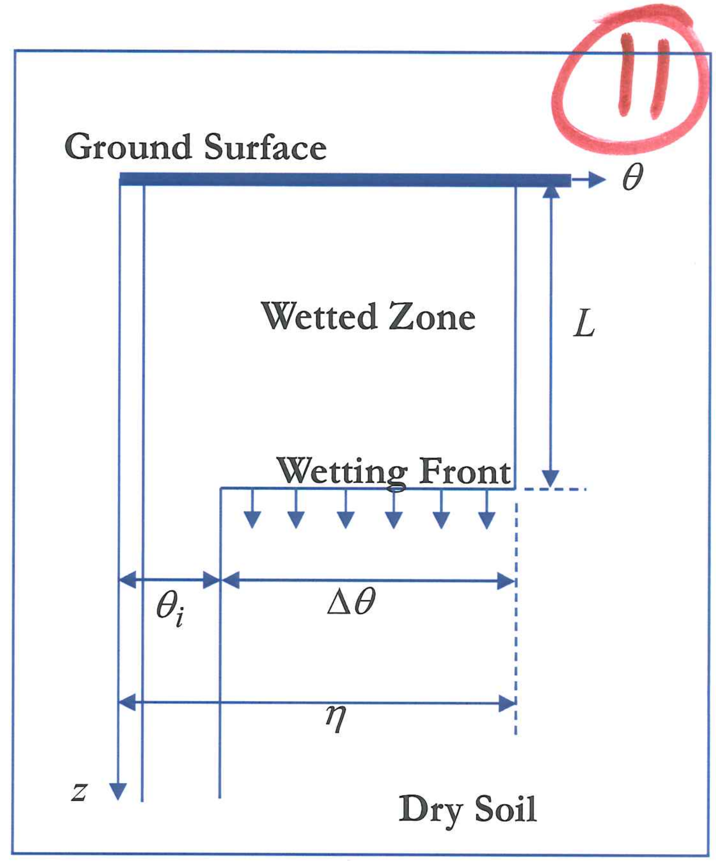
$$f = K\frac{\partial \psi}{\partial z} + K$$

- Apply finite difference to the derivative, between
  - Ground surface $z = 0, \psi = 0$
  - Wetting front $z = L, \psi = \psi_f$

$$f = K\frac{\partial \psi}{\partial z} + K = K\frac{\Delta \psi}{\Delta z} + K = K\frac{\psi_f - 0}{L - 0} + K$$

$$F(t) = L\Delta\theta$$

$$L = \frac{F}{\Delta\theta}$$



$$f = K\frac{\partial \Psi}{\partial z} + K$$

$$f = K_s\left(1 - \frac{M_d \psi}{F}\right)$$

$$M_d = \Delta\theta$$

Eq. of Green-Ampt

Soil Parameters

$\eta \approx \theta_s$ $\Delta\theta = Md = \eta - \theta_i$ (12) K_s

Table 2–4 Green–Ampt Infiltration Parameters for Various Soil Texture Classes

Soil Class	Porosity η	Effective Porosity θ_e	Wetting Front Suction Head ψ (-cm)	Hydraulic Conductivity K (cm/hr)
Sand	0.437	0.417	4.95	11.78
	0.374–0.500	0.354–0.480	0.97–25.36	
Loamy sand	0.437	0.401	6.13	2.99
	0.363–0.506	0.329–0.473	1.35–27.94	
Sandy loam	0.453	0.412	11.01	1.09
	0.351–0.555	0.283–0.541	2.67–45.47	
Loam	0.463	0.434	8.89	0.34
	0.375–0.551	0.334–0.534	1.33–59.38	
Silt loam	0.501	0.486	16.68	0.65
	0.420–0.582	0.394–0.578	2.92–95.39	
Sandy clay loam	0.398	0.330	21.85	0.15
	0.332–0.464	0.235–0.425	4.42–108.0	
Clay loam	0.464	0.309	20.88	0.10
	0.409–0.519	0.279–0.501	4.79–91.10	
Silty clay loam	0.471	0.432	27.30	0.10
	0.418–0.524	0.347–0.517	5.67–131.50	
Sandy clay	0.430	0.321	23.90	0.06
	0.370–0.490	0.207–0.435	4.08–140.2	
Silty clay	0.479	0.423	29.22	0.05
	0.425–0.533	0.334–0.512	6.13–139.4	
Clay	0.475	0.385	31.63	0.03
	0.427–0.523	0.269–0.501	6.39–156.5	

Source: Rawls et al., 1983.

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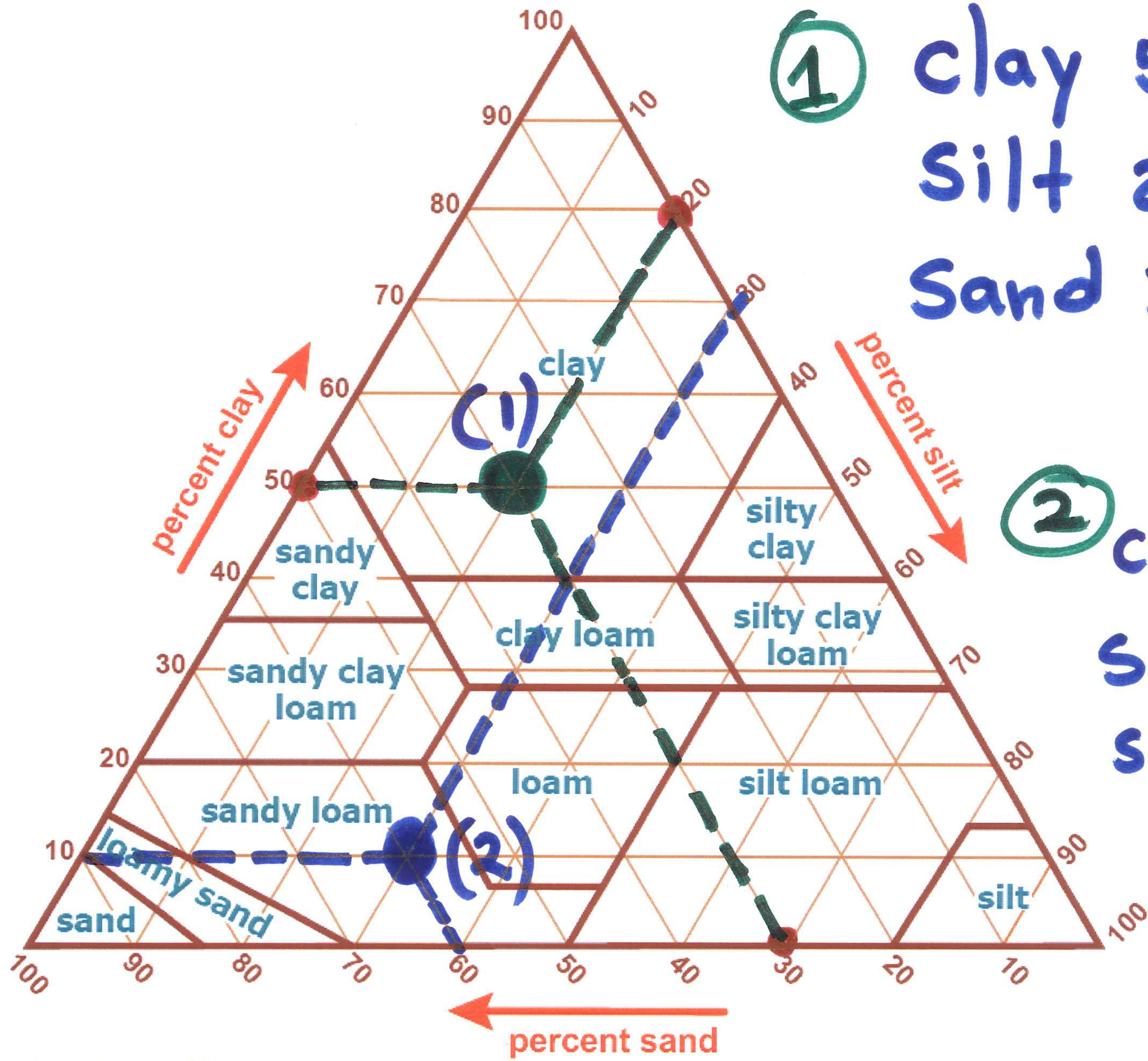
Soil Parameters (Cont.)



(1) clay 50%
Silt 20%
Sand 30%

(2) clay 10%
Sand 60%
silt 30%

Source: http://www.oneplan.org/Water/soil-triangle.asp

(14)

Green-Ampt Infiltration: Problem 2.35

A sandy loam has an initial moisture content of 0.18, hydraulic conductivity of 7.8 mm/hr, and average capillary suction of 100 mm. Rain falls at 2.9 cm/hr, and the final moisture content is measured to be 0.45. When does surface saturation occur? Plot the infiltration rate vs. the infiltration volume, using the Green and Ampt method of infiltration.

$\theta_i = 0.18$, $\theta_f = 0.45$

$M_d = \theta_f - \theta_i = 0.27$

$K_s = 0.78 \frac{cm}{hr}$, $\psi = -10$ cm

$i = 2.9$ cm/hr

$$F_s = \frac{\psi M_d}{(1 - i/K_s)} = \frac{-10 \times 0.27}{1 - \frac{2.9}{0.78}} = 0.993 \text{ cm}$$

$$t_s = F_s/i = \frac{0.993 \text{ cm}}{2.9 \text{ cm/hr}} = 0.34 \text{ hr}$$

Saturation occurs at $t = 0.34$ hr.

$$f = K_s(1 - M_d\psi/F)$$

→ This equation is used for computing the rate of infiltration after surface saturation.

Green-Ampt Infiltration: Problem 2.35 (Cont.)

Show Excel spreadsheet

Green-Ampt Infiltration Method

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Rainfall intensity (cm/hr)	2.90
Tp (Time of ponding in minutes) =	20.55
Ks (cm/hr) =	0.780
Fponding (cm)=	0.993
psi (cm) =	-10.00
Md (moisture difference) =	0.270

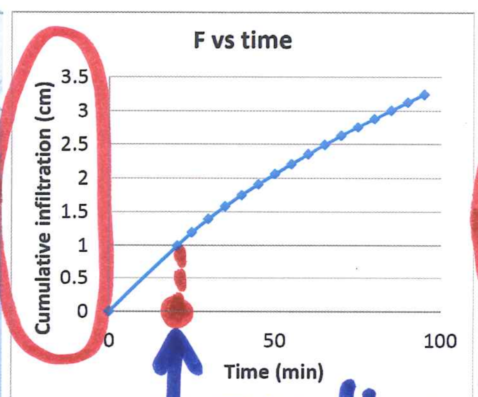
Enter data only in yellow fields

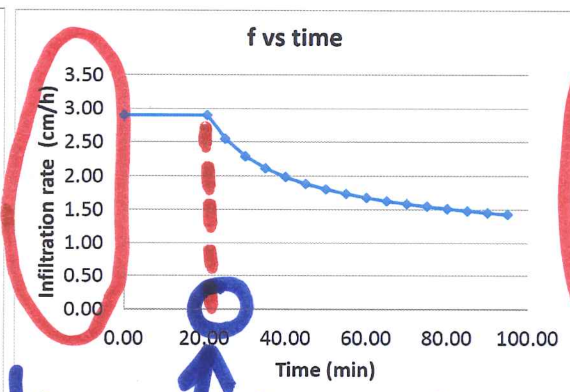
Residuals (must be zero)	Time (min)	F (cm)
Don't modify this row	0	0
Don't modify this row	20.55	0.993
0.0000000000	25.00	1.194
0.0000000000	30.00	1.395
0.0000000000	35.00	1.578
0.0000000000	40.00	1.748
0.0000000000	45.00	1.909
0.0000246620	50.00	2.063
0.0000000000	55.00	2.210
0.0000000000	60.00	2.352
0.0000000000	65.00	2.489
0.0000000000	70.00	2.623
0.0000000000	75.00	2.753
0.0000000000	80.00	2.881
0.0000000000	85.00	3.005
0.0000000000	90.00	3.128
0.0000000000	95.00	3.248
0.0000246621		

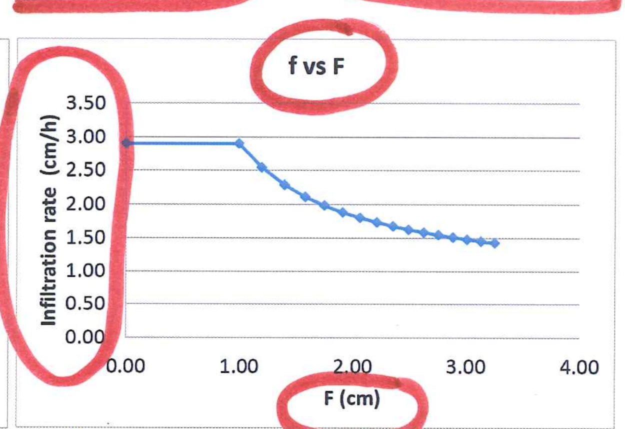
time	f(cm/s)
0.00	2.900000
20.55	2.900000
25.00	2.543765
30.00	2.289990
35.00	2.114732
40.00	1.984527
45.00	1.882971
50.00	1.800984
55.00	1.732969
60.00	1.675450
65.00	1.625987
70.00	1.582878
75.00	1.544885
80.00	1.511082
85.00	1.480762
90.00	1.453374
95.00	1.428479

F (cm)	f(cm/s)
0.00	2.900000
0.99	2.900000
1.19	2.543765
1.39	2.289990
1.58	2.114732
1.75	1.984527
1.91	1.882971
2.06	1.800984
2.21	1.732969
2.35	1.675450
2.49	1.625987
2.62	1.582878
2.75	1.544885
2.88	1.511082
3.01	1.480762
3.13	1.453374
3.25	1.428479

This plot is requested in problem 2.35







Green-Ampt Infiltration: Example 2-13

Silt loam has the following soil properties for use in the Green–Ampt equation:

$$K_s = 1.81 \times 10^{-4}\ \text{cm/sec},$$
$$\theta_s = 0.523,$$
$$\psi = -17.0\ \text{cm water}.$$

For an initial moisture content of $\theta_i = 0.3$, what is the ponding depth (F_s) and ponding time (t_s) based on this rainfall intensity? What will be the infiltration depth 20 minutes after the beginning of the rainfall? (Assume that the rainfall has a constant intensity.)

$$i = 6K_s \text{ for 10 minutes.}$$

$$F_s = \frac{\psi M_d}{(1 - i/K_s)} = \frac{-17(0.223)}{1 - \frac{6K_s}{K_s}} = 0.76\ \text{cm}$$

$$t_s = F_s/i = \frac{0.76}{6K_s} = 11.7\ \text{min}$$

Equation for F after ponding.

$$K_s(t - t_p) = F - F_p + \psi M_d \ln\left[\frac{M_d\psi - F}{M_d\psi - F_p}\right] \quad (1)$$

$$f = K_s(1 - M_d\psi/F) \quad (2)$$

Eq. for "f" after ponding. "F" is computed using Eq. (1).

Green-Ampt Infiltration: Example 2-13 (Cont.)

Show Excel spreadsheet

Green-Ampt Infiltration Method

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Rainfall intensity (cm/hr)	3.91
Tp (Time of ponding in minutes) =	11.64
Ks (cm/hr) =	0.652
Fponding (cm)=	0.758
psi (cm) =	-17.00
Md (moisture difference) =	0.223

Enter data only in yellow fields

Residuals (must be zero)	Time (min)	F (cm)	time	f(cm/s)	F (cm)	f(cm/s)
Don't modify this row	0	0	0.00	3.909600	0.00	3.909600
Don't modify this row	11.64	0.758	11.64	3.909600	0.76	3.909600
0.0000000196	15.00	0.956	15.00	3.234384	0.96	3.234384
0.0000000000	20.00	1.202	20.00	2.707252	1.20	2.707252
0.0000000000	25.00	1.413	25.00	2.399203	1.41	2.399203
0.0000000000	30.00	1.604	30.00	2.191379	1.60	2.191379
0.0000000000	35.00	1.780	35.00	2.039172	1.78	2.039172
0.0000000000	40.00	1.945	40.00	1.921580	1.95	1.921580
0.0000000000	45.00	2.101	45.00	1.827250	2.10	1.827250
0.0000000000	50.00	2.250	50.00	1.749435	2.25	1.749435
0.0000000000	55.00	2.393	55.00	1.683842	2.39	1.683842
0.0000000000	60.00	2.531	60.00	1.627592	2.53	1.627592
0.0000000000	65.00	2.665	65.00	1.578674	2.66	1.578674
0.0000000000	70.00	2.794	70.00	1.535634	2.79	1.535634
0.0000000000	75.00	2.921	75.00	1.497390	2.92	1.497390
0.0000000000	80.00	3.044	80.00	1.463122	3.04	1.463122
0.0000000000	85.00	3.165	85.00	1.432191	3.16	1.432191
0.0000000196						

Residuals