Frequency analysis (Cont.)

Lecture 18, 05/30/2013

Arturo Leon, Oregon State University (Spring 2013)

Example of Application

②-①

A stream channel was built to convey 200 m^3/s, which is the peak flow of the 20-yr storm-event of the watershed. What is the probability that the banks of the channel will be overtopped (in percentage):

a) at least twice in the next 20 years

b) in any year

c) exactly four times in the next 20 years

d) at least once in the next 200 years

As part of a flooding prevention program, it is planned to build levees along the stream channel to withstand the 100 yr-storm event. If the service life of the levees will be 20 years,

e) what is the reliability of the levees in %?

f) what is the probability that the levees will be overtopped at least once in the next 20 years in %?

a) $T = 20$ years, $P_{\text{at least twice}}$ $n = 20$ years?

$P = \frac{1}{T} = 0.05$

$P_{\text{at least twice}} = 1 - P_0 - P_1$

(P_0: exactly 0 times; P_1: exactly 1 time)

Probabilities

exactly 0 1 2 3 . . . 20

P_0 P_1 P_2 ... $P = 1$

$P_0 = ?$

	O	O	O	....	O
year	1	2	3	....	20
	1-P	1-P	1-P	....	1-P

$$P_0 = (1-P)^{20} = 0.358$$

$P_1 = ?$

	O	O	O	....	O
year	1	2	3	....	20

$$P_1 = C_1^{20} P^1 (1-P)^{19}$$

$$C_1^{20} = \frac{20 \times 19!}{19!\, 1!} = 20$$

$$P_1 = 20 \times 0.05 \times 0.95^{19} = 0.377$$

$$P_{at\ least\ twice} = 1 - P_0 - P_1$$
$$= 1 - 0.358 - 0.377$$
$$= 0.265 = 26.5\%$$

b) $P_{in\ any\ year} = P = 0.05 = 5\%$

c) P_4

$$C^{20}_4 P^4 (1-P)^{16} = 0.0133 = 1.33\%$$

$$C^{20}_4 = \frac{20!}{16!\,4!} = \frac{20 \times 19 \times 18 \times 17 \times 16!}{16!\,4 \times 3 \times 2 \times 1} = 4845$$

2-4

d) $P_{\text{at least once}}$ $n = 200$ years

$$P_{\text{at least once}} = 1 - P_0 = 1 - (1-P)^{200}$$

$$= 1 - 0.95^{200} = 0.99996$$

$$= 99.996\%$$

Reliability (Safety)

e) Risk (danger)

$n = 20$ years?

2-5

	O	O	O	O	.....	O
year	1	2	3	4	....	20
	1-P	1-P	1-P	1-P	...	1-P

$$\text{Reliability} = (1-P)^{20} \qquad P = \frac{1}{100} = 0.01$$

$$\text{Reliability} = (1-0.01)^{20} = 0.8179 = 81.8\%$$

f) $$P_{\text{at least once}} = 1 - P_0 = 1 - 0.8179 = 0.182 = 18.2\%$$

Example of Application

③-①

The annual precipitation for an urban area was determined to be approximately normal with a mean of 1000 mm and a standard deviation of 500 mm. Determine the probability that the annual precipitation for the urban area

a) will exceed 600 mm in the next two years.
b) will be smaller than 600 mm in all eight of the next consecutive 8 years
c) will exceed 600 mm **exactly** in years 2 and 5 of the next five years
d) will exceed 600 mm in years 2 and 5 of the next five years

$\bar{X} = 1000$ mm

$S = 500$ mm

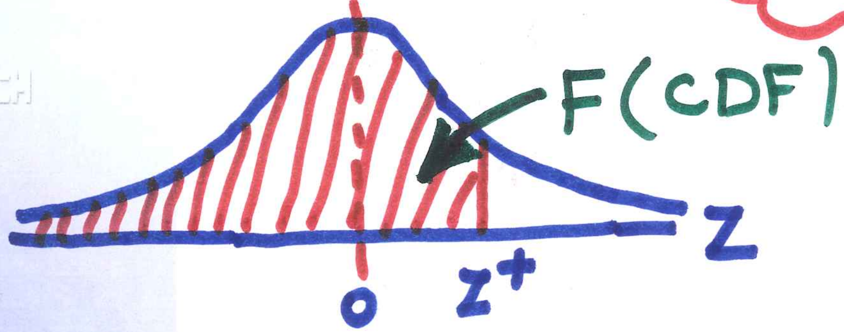
$X = \bar{X} + ZS$

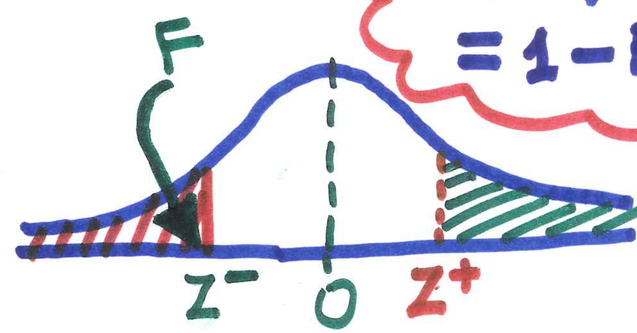
1 2 (year)

a) $X = 600$ mm

$600 = 1000 + Z(500)$

$Z = -0.8$



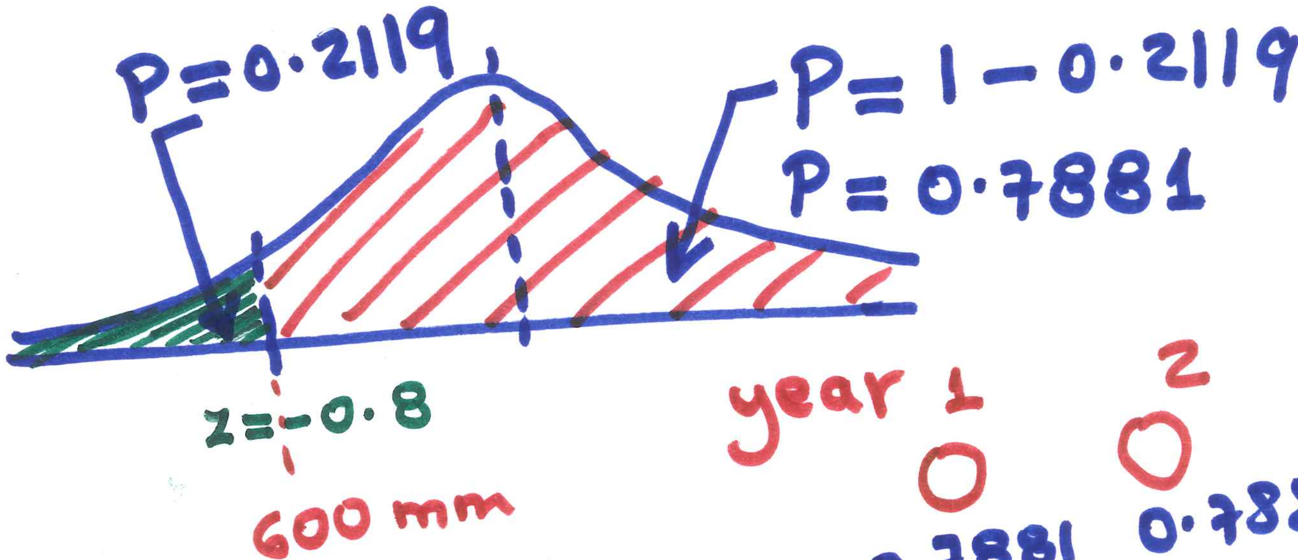


$F(z^-) = 1 - F(z^+)$

$z^{+} = 0.8, \quad F(z = 0.8) = 0.7881$

(3-2)

$F(z^{-}) = 1 - 0.7881 = \underline{0.2119}$ ✓



year 1 2

0.7881 0.7881

$P = (0.7881)^2 = 0.621 = 62.1\%$

b) $P(< 600mm)$

year 1 2 3 8

$P = 0.2119$

0.2119 0.2119 0.2119 . . . 0.2119

$P = 0.2119^8 = 0.00000408 = \underline{0.0004\%}$ ✓

3-3

c)

	O	O ✓	O	O	O ✓
years	1	2	3	4	5
	0.2119	0.7881	0.2119	0.2119	0.7881

$$P = 0.2119^3 \, 0.7881^2 = 0.0059 = 0.59\%$$

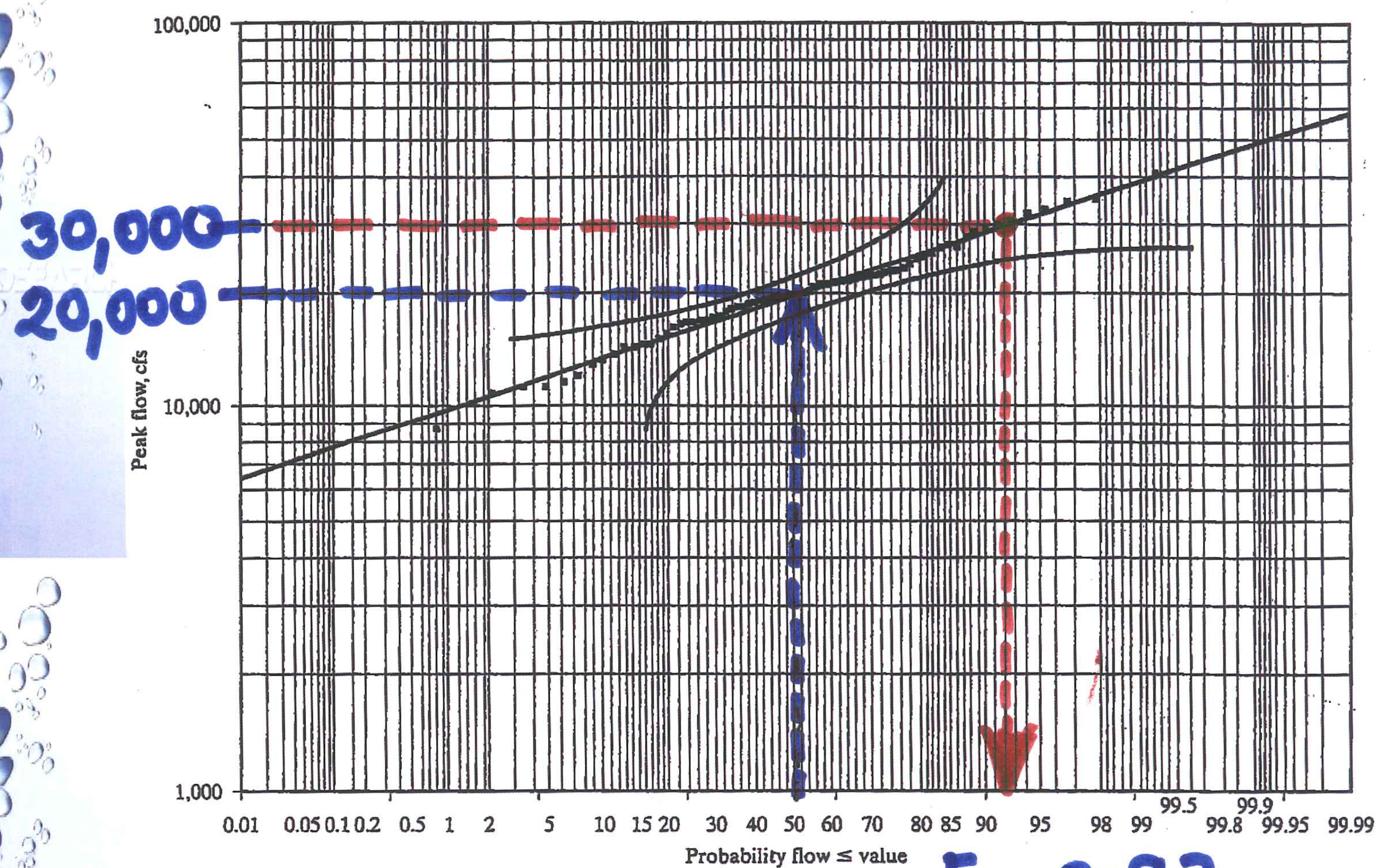
d)

	O	O ✓	O	O	O ✓
years	1	2	3	4	5
	1	0.7881	1	1	0.7881

$$P = 0.7881^2 \times 1^3 = 0.62 = 62\%$$

Example of Application

(4) (1)



F=0.93

The lognormal plot of the Siletz river flows is depicted above

a) What is the return period of the 30,000 cfs flow?

b) What is the 2-yr flow?

c) What is the probability that peak flows larger than or equal to 30,000 cfs will occur at least twice in the next 100 years?

4-2

a) $T(Q = 30{,}000\ cfs) = ?$

$$T = \frac{1}{1-F} = \frac{1}{1-0.93} = 14.3 \text{ years}$$

$$P = \frac{1}{T} = 0.07$$

b) $T = 2$ years, $P = \frac{1}{2} = 0.5$

$$F = 0.5$$

$$Q_{T=2} = 20{,}000\ cfs$$

4-3

c) $P_{at\ least\ twice}$? $P_{any\ year} = 0.07$

$n = 100$ years

$$P = 1 - P_0 - P_1 = 1 - (1-P)^{100} - C_1^{100} P^1 (1-P)^{99}$$

$$= 1 - 0.93^{100} - 100 \times 0.07 \times 0.93^{99}$$

$$= 0.99398 = 99.4\%$$

$$C_1^{100} = \frac{100!}{99!\,1!} = \frac{100 \times 99!}{99!\,1!} = \frac{100}{1} = 100$$

In-class Assignment 6

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Student Name:

The statistics for the Siletz River are presented in the following table. Assuming that the Siletz River data are lognormally distributed, find the following:

	Original Data (cfs)	Log_{10} Data (log cfs)
Mean	20,796	4.29217
Standard Deviation	7386	0.1527
Station Skewness	1.341	−0.3510
Weighted Skewness		−0.2773

a) Peak flow of the 50-yr flood
b) Probability that a flood will be less than or equal to 30,000 cfs
c) Return period of the 30,000 cfs flood
d) Probability that a flood will exceed 23,000 cfs in all five of the next consecutive 5 years

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CE 412/512 Final Exam Review

Arturo Leon, Oregon State University (Spring 2013)

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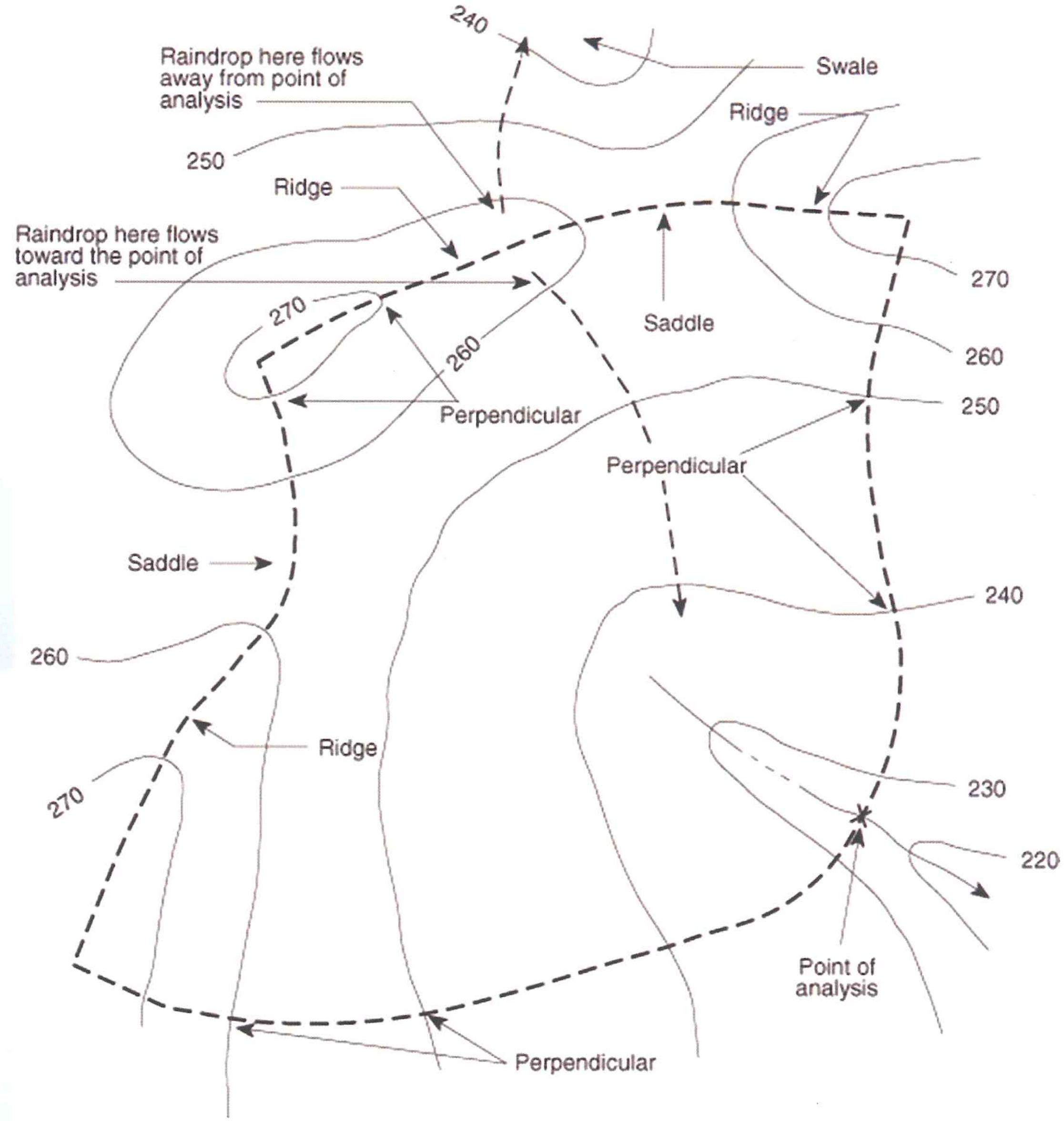
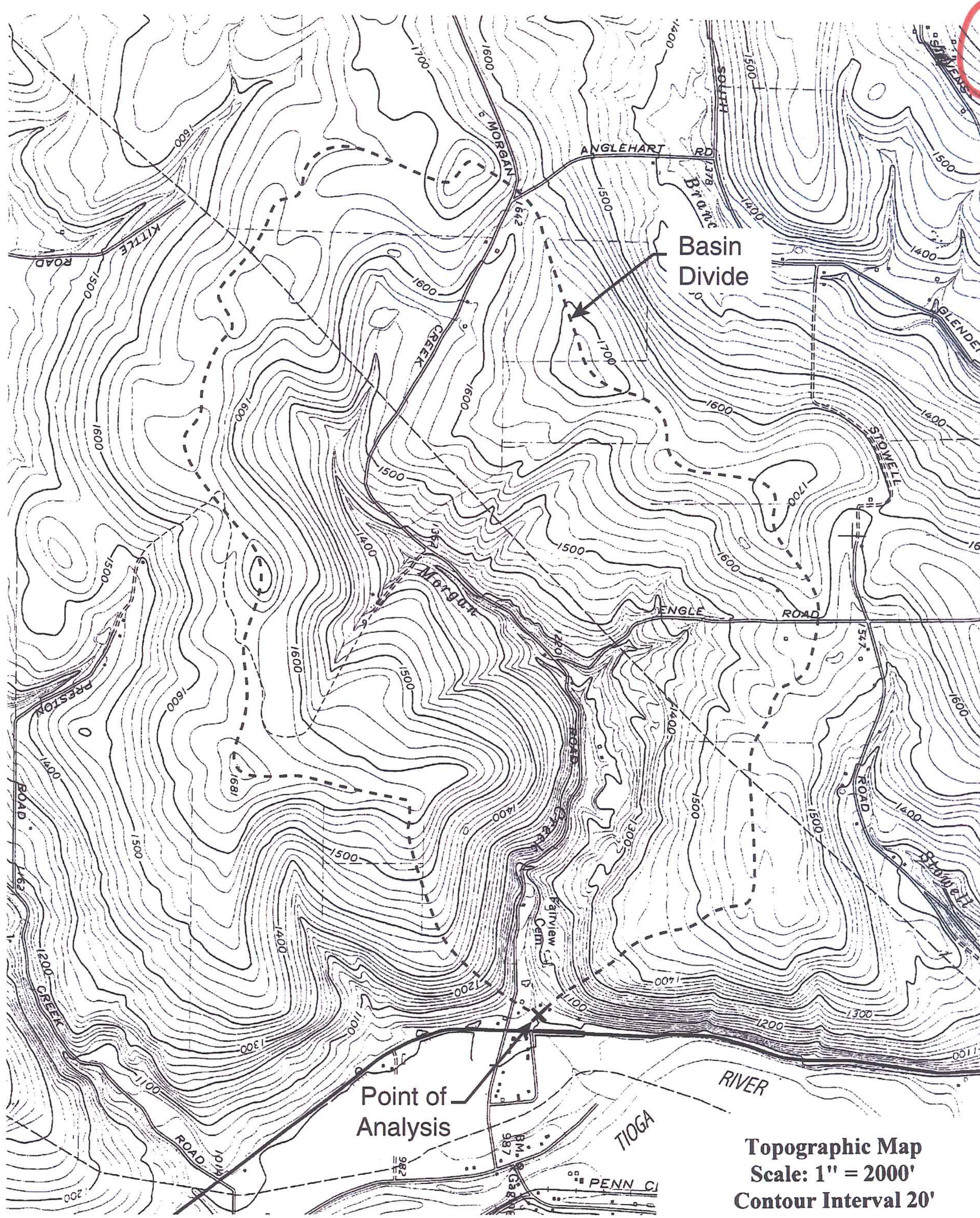


Principles used in delineating a drainage basin

8

1) Delineate the watershed at the "Point of Analysis"



Topographic Map
Scale: 1" = 2000'
Contour Interval 20'

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2) A class A pan is maintained near a small lake to determine daily evaporation (see table). The level in the pan is observed at the end of everyday. Water is added if the level falls *near* 7 in. For each day the difference in height level is calculated between the current and previous day, and the precipitation value is from the current day. Determine the daily lake evaporation if the pan coefficient is 0.70.

Day	Rainfall (in.)	Water Level (in.)
1	0	8.00
2	0.23	7.92
3	0.56	7.87
4	0.05	7.85
5	0.01	7.76
6	0	7.58
7	0.02	7.43
8	0.01	7.32
9	0	7.25
10	0	7.19
11	0	7.08*
12	0.01	7.91
13	0	7.86
14	0.02	7.8

*Refilled at this point to 8 inches

3) Elk Lake has a surface area of 500 ac. Over a 10-day period, the average inflow was 27,648 in^3/s and the average outflow was 20,736 in^3/s. The total precipitation was measured as 1.8 inches and the total evaporation loss was 1.1 inches. Over the 10-day period, a +2 inch storage change (or increase in lake level) was recorded. Estimate the amount of seepage (in inches) over the 10-day period for the lake. (Note: 1 ac = 43,560 ft^2 = 6,272,640 in^2).

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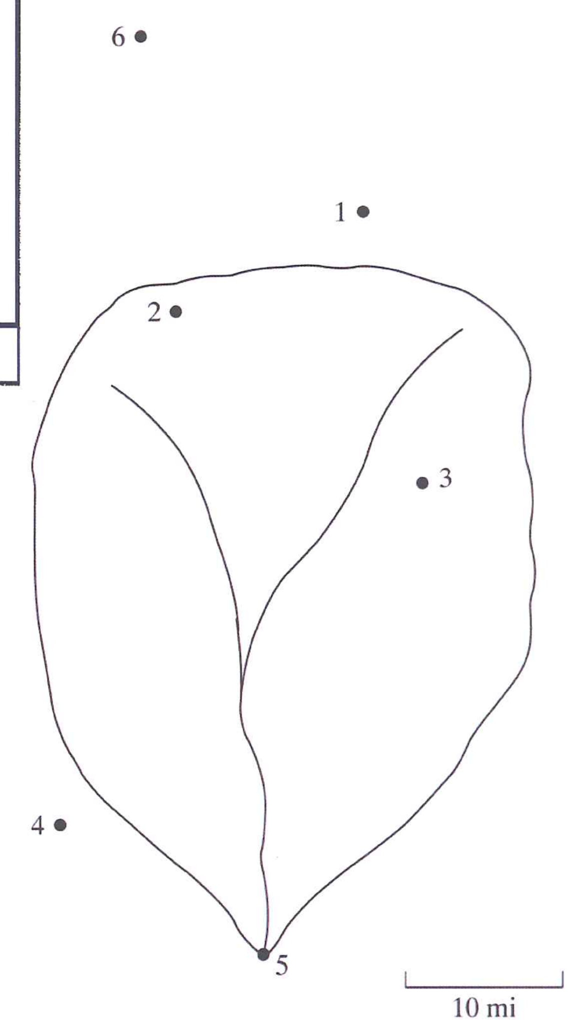
4) Find the mean areal rainfall using the Thiessen Polygon Method

Gage	Area (sq mi)	Area (%)	Rainfall (cm)	Weighted Rainfall (cm)
1			5.5	
2			4.5	
3			4	
4			6.2	
5			7	
6			2.1	
SUM		100		



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5) A soil has the following soil properties for use in the Green-Ampt equation:

$K_s = 1.5$ in/hr $\qquad \theta_s = 0.523$

$\psi = -9.8$ in $\qquad \theta_i = 0.308$

$i = 8.2$ in/hr

Using the Green-Ampt infiltration method:

A. Calculate the initial moisture deficit, M_d.
B. Find the volume of water that will infiltrate before saturation is reached, F_s.
C. Find the time to saturation.
D. Plot the infiltration rate, *f*, vs. the infiltration volume, F.

Green-Ampt Equations:

$$F_S = \frac{\psi M_d}{(1 - i/K_S)}$$

$$f = K_s\left(1 - \frac{M_d \psi}{F}\right)$$

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6) A small polluted watershed has an area of 2 mi^2 and a weighted curve number (CN) of 65. Due to environmental concerns, all runoff produced at this watershed is captured in a reservoir and then pumped to a treatment plant. Local regulations for this watershed state that all hydraulic structures will be designed to withstand the 24-hr storm event for a return period of 100 years. Hydrological studies for this watershed show that the 24-hr accumulated total storm rainfall (gross rainfall) for a return period of 100 years is 10 inches. If this accumulated total storm rainfall is projected to increase in 5% due to global warming scenarios, determine the extra storage in ft^3 that would be required to capture the extra runoff associated to the to global warming scenarios (5% increase in accumulated total storm rainfall). Use the SCS method and assume that the Initial Abstraction (I_a) is equal to 0.3 times the potential abstraction (S).

(14)

7) Design Storm: 100-yr, 6-hr Design Storm for Corvallis, OR

Development of 6-hour dimensionless cumulative design storms for Corvallis, Oregon

(1) Return Period (years)	(2) Duration (hr)	(3) Intensity (in/hr)	(4) Depth (in)	(5) Incremental Depth (in)	(6) Design storm (in)	(7) Cumulative Design storm (in)	(1) Dimensionless Cumulative Design storm
100	1						
	2						
	3						
	4						
	5						
	6						

8) A storm event produced a rainfall pattern (total rainfall) of 5cm/h for the first 10 min, 10 cm/hr in the second 10 min, and 5 cm/hr in the next 10 min as shown below. The watershed is divided into two subbasins (see watershed below) with the unit hydrographs given in the table below. Subbasin "B" had a loss rate of 2.5 cm/hr for the first 10 min and 1.0 cm/hr thereafter. Subbasin "C" had a loss rate of 1.0 cm/hr for the first 10 min and 0 cm/hr thereafter. Determine the flow hydrograph at the outlet (point 2) if the lag time between point 1 and 2 is 20 minutes. Give your answer for the flow hydrograph in units of m^3/s for the flow discharge and in minutes for the time.

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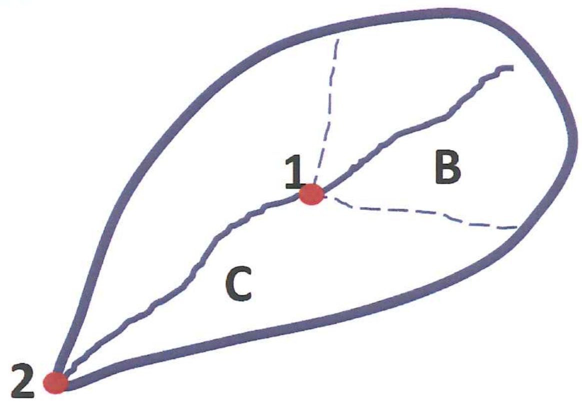

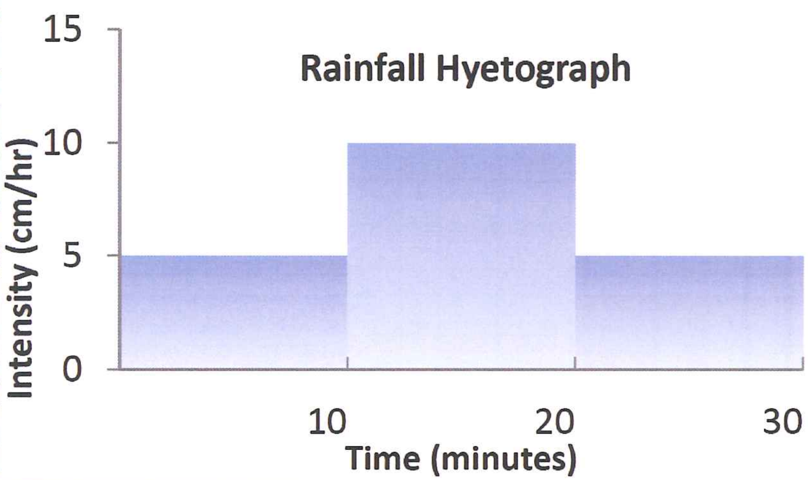

10-minute unit hydrographs for subbasins "B" and "C"

Subbasin B		Subbasin C	
Time (min)	$Q(m^3/s)$	Time (min)	$Q(m^3/s)$
0	0	0	0
10	5	10	16.7
20	10	20	33.4
30	15	30	50.0
40	20	40	33.4
50	25	50	16.7
60	20	60	0
70	15		
80	10		
90	5		
100	0		

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9) The 7.5 minute Unit Hydrograph (UH) for a parking lot in Corvallis (Oregon) is presented below. If rain falls over this parking lot with a constant intensity of 2.5 in/hr during an entire week, find the maximum flow discharge (*cfs*) that can be measured at the outlet of the parking lot. Assume a constant infiltration rate of 1.0 in/hr and a baseflow rate of zero.

t (hr)	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875
Q (cfs)	0	0	0.25	0.50	0.75	0.50	0.25	0

(17)

10) Given the S-curve below (developed from a 1-hr unit hydrograph), find the 2-hr unit hydrograph.

Time (hr)	S-curve (cfs)
0	0
1	55
2	145
3	260
4	335
5	365
6	385
7	385
8	385

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11) A reservoir has a storage-discharge relationship of

$$S = kQ,$$

Where k = 1.21 hr. The inflow hydrograph for a storm event is given in the table below. Determine the flow hydrograph at the outlet of the reservoir using a Δt=1 hr and assuming that the reservoir is initially empty. **<u>Provide only the first 5 ordinates of the flow hydrograph</u>.**

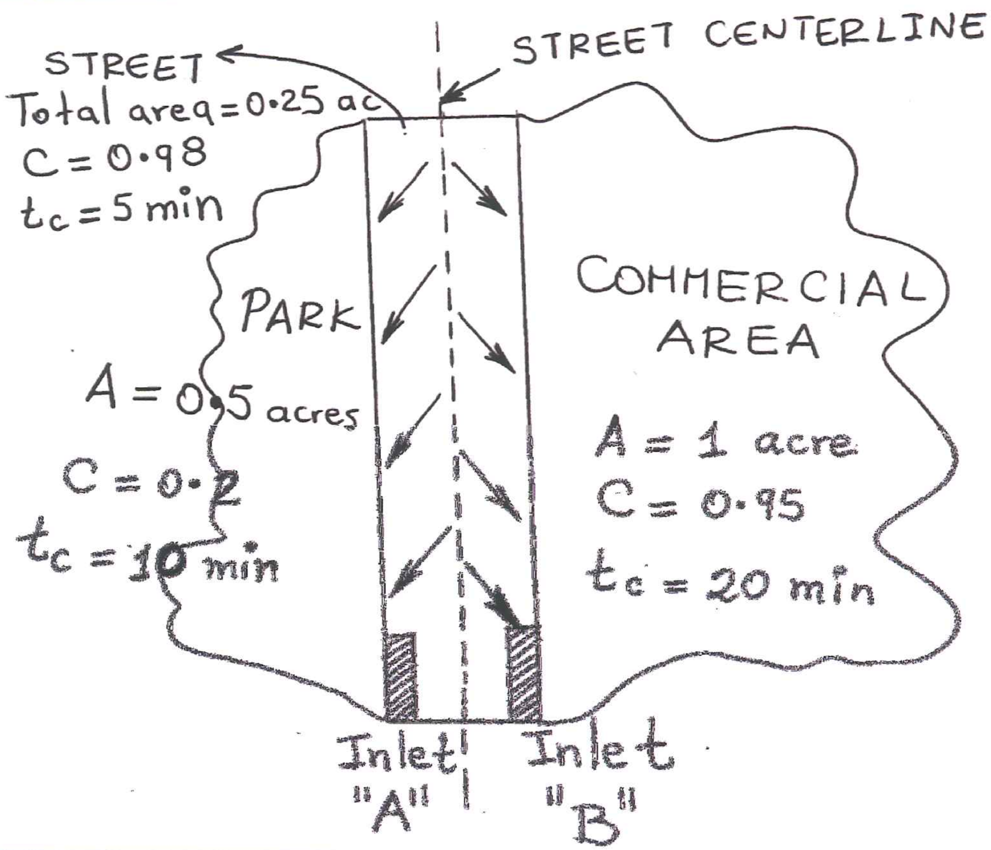
Time (hr)	Inflow (m^3/s)
0	0
1	100
2	200
3	400
4	300
5	200
6	100
7	50
8	0

19

12) A basin is to be developed into a 0.5 acre park (C = 0.2) and 1 acre of a commercial area (C = 0.95). An asphalt street (C = 0.98) having a total area of 0.25 acres separates the future park and commercial area as sketched below. The times of concentration (t_c) for the park, commercial area and street are 10, 20 and 5 minutes, respectively. If the local IDF curve is given by the following relation

$$i = \frac{a}{b + t_r}$$

where i = rainfall intensity (in/hr), t_r = duration (min) and "a" and "b" are constants equal to 160 and 18, respectively for a return period of 5-years. What should be the 5-year design peak flow at inlet "A" (see sketch below). Assume that half of the flow of the street drains to each inlet.



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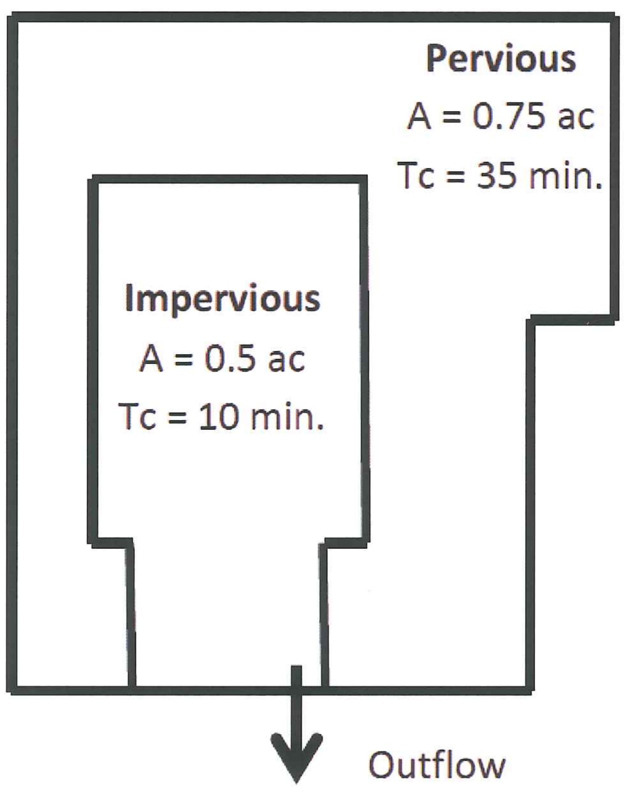
13) A subdivision is planned to be constructed in Corvallis, OR, and each individual plot (e.g, each house) can be represented as shown in the figure below. The plot consists of 0.5 ac of impervious area (C=0.9), and 0.75 ac of pervious area (C=0.15). The inlet time for flow to travel from the upper end of the catchment (upper end of pervious area) is 35 min. and the inlet time for the impervious area is only 10 min. Using the tabulated intensity-duration-frequency (IDF) information, what should be the design peak flow at the plot outlet?

Rational Method: $\boldsymbol{Q = CiA}$, where Q is the peak discharge in cfs, C is the runoff coefficient (unitless), *i* is the rainfall intensity in inches/hour, and A is the drainage area in acres.



IDF Information for Corvallis, OR

Duration (min)	Intensity (in./hr.)
5	2.65
10	2.05
15	1.75
20	1.5
25	1.35
30	1.2
35	1.1
40	1
45	0.92

14) Given the following frequency data, **develop and plot** the relative frequency histogram and the cumulative frequency histogram on the graphs provided on the following page. 21

a. What is the probability that the stream flow will be between 2000-3000 cfs?

b. What is the probability that the stream flow will be less than or equal to 3000 cfs?

c. What is the probability that the stream flow will be greater than 5000 cfs?

Stream Flow (cfs)	Frequency
0-1000	0
1000-2000	2
2000-3000	9
3000-4000	17
4000-5000	13
5000-6000	8
6000-7000	5
7000-8000	1
TOTAL	55

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15) A stream channel can carry $100 m^3/s$, which is the peak flow of the 20-yr storm event of a watershed. What is the probability that the banks of the channel will be overtopped (In percentage):

a) at least twice in the next 20 years
b) in any year
c) exactly four times in the next 20 years
d) at least once in the next 200 years

As part of a flooding prevention program, it is planned to build levees along the above stream channel to withstand the 100 yr- storm event. If the lifetime of the levees will be 20 years,

e) what is the reliability of the levees in %?
f) what is the probability that the levees will be overtopped at least once in the next 20 years in %?

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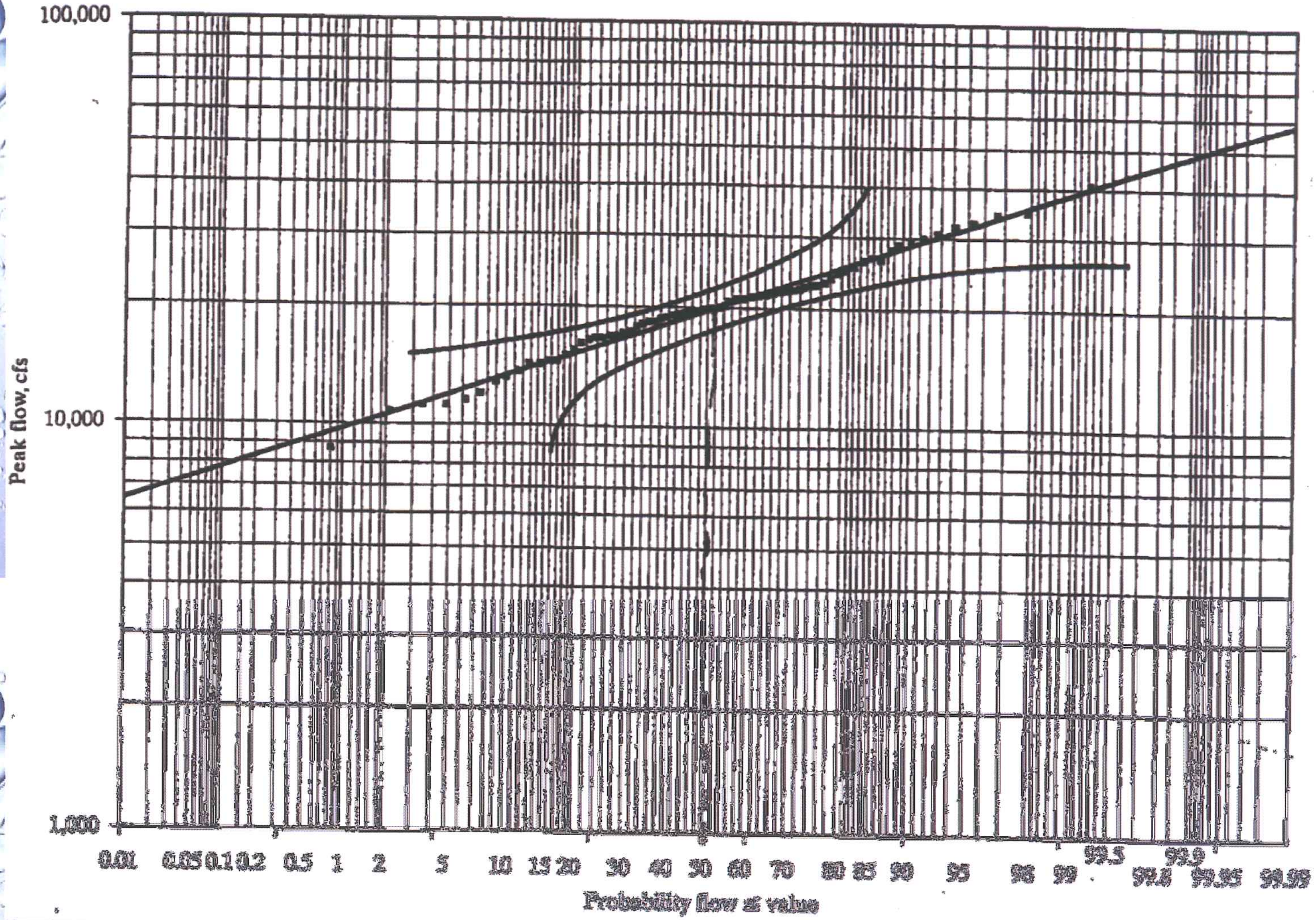
16) The lognormal plot of the Siletz river flows is depicted below.



a) What is the return period of a flow of 50,000 cfs?
b) What is the 500-yr flow?
c) What is the probability that the peak flow will be greater than or equal to 50,000 cfs in %?
d) What is the probability that at least one 50,000 cfs peak flow will occur in the next 5 years in %?

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17) The statistics for the Siletz River are presented in the following table. Assuming that the Siletz River data are lognormally distributed, find the following:

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a) Peak flow of the 50-yr flood

b) Probability that a flood will be less than or equal to 30,000 cfs

c) Return period of the 30,000 cfs flood

d) Probability that a flood will exceed 23,000 cfs in all five of the next consecutive 5 years