Instructor: Arturo Leon Name SOLUTION KEY

MID-TERM EXAM

CE 412/512 Hydrology - Spring 2013

Exam is **open book** and **open notes**. For all problems, *write the equations used*, *show your calculations, include units*, and *box your answers*.

1. *(25 pts)* A watershed has an area of 150 square miles, a length to divide of 20 miles, and an average slope of 2%. The watershed is 70% good condition open space/lawn, 40% of which is soil group C and 60% is soil group A. The remaining 30% of the watershed is poor covered forest land with soil group C. Use the SCS Method to **develop** and **plot** a unit hydrograph for a 2-hr duration rainfall.

SOLUTION:

Find CNs from Table 2-1:

Land Use	Soil Group	% of Area	CN
Good condition open space/lawn	C	0.7*0.4 = 0.28	74
Good condition open space/lawn	A	0.7*0.6 = 0.42	39
Forest land with poor cover	C	0.3*1 = 0.3	77

Calculate Weighted CN: $(0.28) * 74 + (0.42) * 39 + (0.3) * 77 = 60.2 = \mathbf{60.2}$

L = length to divide $L = 20\ mi * \left(\frac{5280\ ft}{mi}\right) = 105{,}600\ ft$

y = average watershed slope (in percent) = 2%

D = rainfall duration (hr) = 2 hr

S (in inches) $S = \left(\frac{1000}{CN}\right) - 10 = \left(\frac{1000}{60.2}\right) - 10 = 6.61\ in$

t_p = lag time (hr) $t_p = \frac{L^{0.8}(S+1)^{0.7}}{1900\sqrt{y}} = \frac{(105600)^{0.8} * (6.61+1)^{0.7}}{1900\sqrt{2}} = 16.09\ hr$

T_R = time of rise (hr) $T_R = \frac{D}{2} + t_p = \left(\frac{2}{2}\right) + 16.09\ hr = 17.09\ hr$

A = watershed area (mi^2) = 150 mi^2

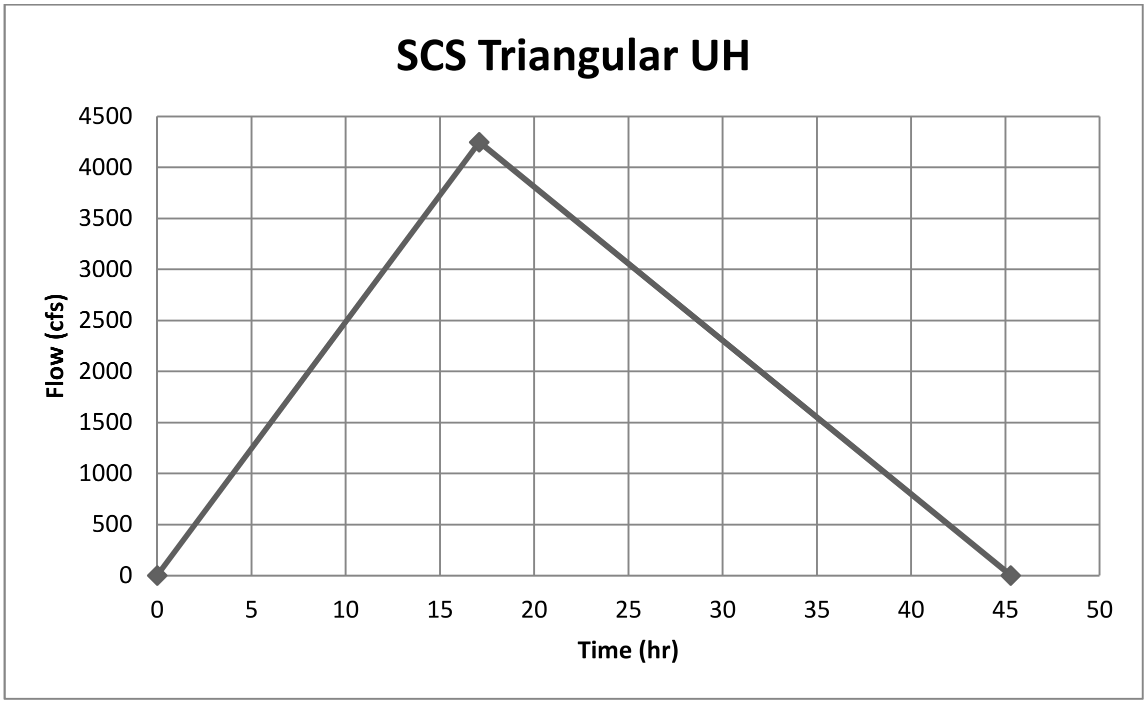
Q_P = peak flow (cfs) $Q_P = \frac{484 * A}{T_R} = \frac{(484)*(150)}{17.09} = \mathbf{4248.1\ cfs}$

B = time to fall (hr) $Vol = (150\ mi^2)\left(\frac{5280\ ft}{mi}\right)^2\left(\frac{ac}{43{,}560\ ft^2}\right) * (1\ in) = 96{,}000\ ac - in$

$Vol = 96{,}000\ ac - in \approx 96{,}000\ cfs - hr$

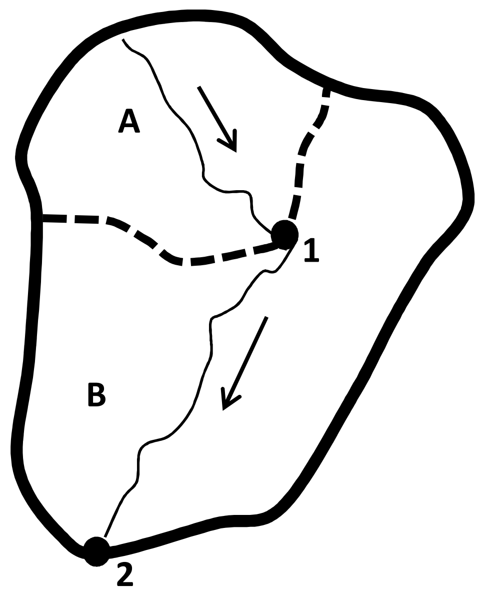
$Vol = 96{,}000\ cfs - hr = \frac{Q_P * T_R}{2} + \frac{Q_P * B}{2} = \frac{4248.1*17.09}{2} + \frac{4248.1*B}{2}$

$B = \mathbf{28.11\ hr}$


SCS Triangular UH
Flow (cfs)
4500
4000
3500
3000
2500
2000
1500
1000
500
0
0
5
10
15
20
25
30
35
40
45
50
Time (hr)

2. *(25 pts)* A small watershed is made up of an upper area (sub-basin A) and a lower park area (sub-basin B), as pictured below. The upper sub-basin (A) has recently been developed. The 0.5-hr unit hydrographs for each sub-basin are given in the table below and the travel time between point 1 and 2 is 1 hr. Determine the storm hydrograph at the outlet (point 2) for the rainfall intensity *i* and infiltration *f* given in the table below.

Time (hr)	**0-0.5**	**0.5-1**	**1-1.5**
***i* (cm/hr)**	6	10	7
f_A (cm/hr)	1	0.5	0.5
f_B (cm/hr)	2	1	0.5



0.5-hr Unit Hydrographs for sub-basins A and B

Sub-basin A - developed	
Time (hr)	**Q (m3/s)**
0	0
0.5	5
1	13
1.5	23
2	15
2.5	10
3	5
3.5	0

Sub-basin B - park	
Time (hr)	**Q (m3/s)**
0	0
0.5	12
1	28
1.5	20
2	6
2.5	0
3	
3.5	

SOLUTION:

Calculate P_1, P_2, and P_3 for each sub-basin:

	Sub-basin A			Sub-basin B		
	0-0.5 hr	**0.5-1 hr**	**1-1.5 hr**	**0-0.5 hr**	**0.5-1 hr**	**1-1.5 hr**
***i* (cm/hr)**	6	10	7	6	10	7
***f* (cm/hr)**	1	0.5	0.5	2	1	0.5
Net rainfall intensity (cm/hr) (= *i* -*f*)	5	9.5	6.5	4	9	6.5
Net rainfall depth, P (cm) (=Rain Intensity*0.5hr)	**2.5**	**4.75**	**3.25**	**2**	**4.5**	**3.25**

P_A = [2.5, 4.75, 3.25] cm

P_B = [2, 4.5, 3.25] cm

Calculate hydrograph for each sub-basin:

Sub-basin A					
Time (hr)	**Q (m3/s)**	**$Q*P_1 = Q*2.5$**	**$Q*P_2 = Q*4.75$**	**$Q*P_3 = Q*3.25$**	**Q_A = SUM**
0	0	0			**0**
0.5	5	12.5	0		**12.5**
1	13	32.5	23.75	0	**56.25**
1.5	23	57.5	61.75	16.25	**135.5**
2	15	37.5	109.25	42.25	**189**
2.5	10	25	71.25	74.75	**171**
3	5	12.5	47.5	48.75	**108.75**
3.5	0	0	23.75	32.5	**56.25**
4			0	16.25	**16.25**
4.5				0	**0**

Sub-basin B					
Time (hr)	**Q (m3/s)**	**$Q*P_1 = Q*2$ cm**	**$Q*P_2 = Q*4.5$ cm**	**$Q*P_3 = Q*3.25$ cm**	**Q_B = SUM**
0	0	0			**0**
0.5	12	24	0		**24**
1	28	56	54	0	**110**
1.5	20	40	126	39	**205**
2	6	12	90	91	**193**
2.5	0	0	27	65	**92**
3			0	19.5	**19.5**
3.5				0	**0**

Combine the hydrographs for A and B with the lag time of 1 hr:

Sub-basin A and B			
Time (hr)	**Q_B**	**Q_A**	**SUM**
0	0		**0**
0.5	24		**24**
1	110	0	**110**
1.5	205	12.5	**217.5**
2	193	56.25	**249.25**
2.5	92	135.5	**227.5**
3	19.5	189	**208.5**
3.5	0	171	**171**
4		108.75	**108.75**
4.5		56.25	**56.25**
5		16.25	**16.25**
5.5		0	**0**

3. *(25 pts)* The 10 minute unit hydrograph (UH) for a watershed is presented below. You have been asked to size a stormwater treatment facility to capture the runoff from the watershed at the outlet. If rain falls over the watershed with a constant intensity of 4 in/hr for a week, find the maximum flow discharge (cfs) that can be measured at the outlet of the watershed. Assume a constant infiltration rate of 1.5 in/hr and baseflow rate of 5 cfs.

Time (min)	**0**	**10**	**20**	**30**	**40**	**50**	**60**
Q (cfs)	0	5	11	17	12	6	0

SOLUTION:

Note that because the rainfall duration is very long (i.e., one week) and the problem asks for the maximum flow discharge only, it is not necessary to shift the developed S-curve and subtract from the original one. You still can do this but the results will be the same. The reason is that in the S-curve after about one hour (1 hour << 1 week), the flow discharge is constant and this maximum flow discharge in the original S-curve is not affected at all by the shifted S-curve.

t (min)	Q (cfs)	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	SUM
0	0							0
10	5	0						5
20	11	5	0					16
30	17	11	5	0				33
40	12	17	11	5	0			45
50	6	12	17	11	5	0		51
60	0	6	12	17	11	5	0	51
70		0	6	12	17	11	5	51
80			0	6	12	17	11	46
90				0	6	12	17	35
100					0	6	12	18
110						0	6	6
120							0	0
130								0

10-min UH means that the precipitation is 1 in for the 10 minutes, therefore the intensity is

$$\frac{1\ in}{\left(\frac{10}{60}\ hr\right)} = 6\frac{in}{hr}$$

$$Q_{max} = Q_{sum} * \left[\frac{(i-f)}{i_{UH}}\right] + Q_{baseflow}$$

$$Q_{max} = 51\ cfs * \left[\frac{\left(4\frac{in}{hr} - 1.5\frac{in}{hr}\right)}{6\frac{in}{hr}}\right] + 5\ cfs = \mathbf{26.25\ cfs}$$

4. *(25 pts)* A reservoir has a storage-discharge relationship of

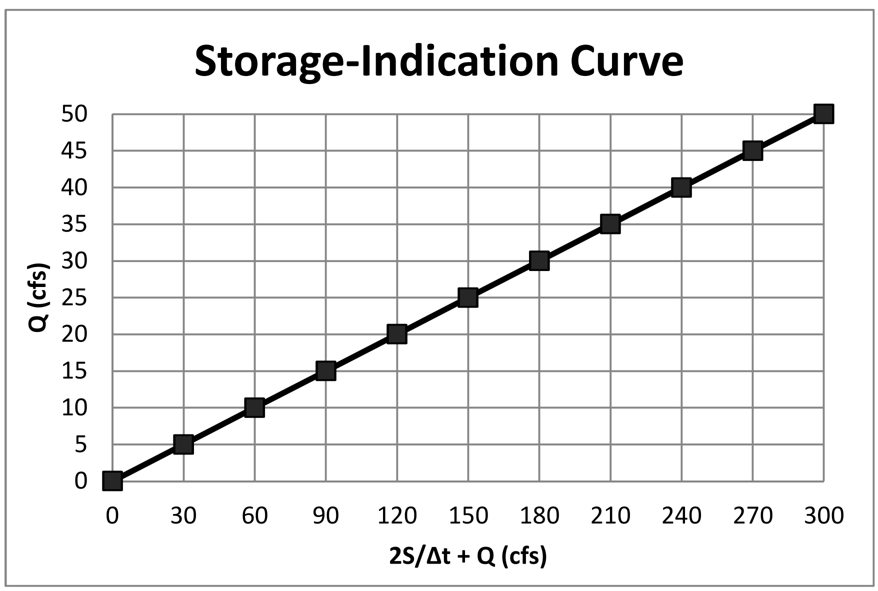
$$S = kQ,$$

where k = 2.5 hr. The inflow hydrograph for a storm event is given in the table below. Determine the flow hydrograph at the outlet of the reservoir using Δt = 1 hr and assume that $Q_0 = 0$ and $\underline{S_0 = 9{,}000\ ft^3}$. ***Provide the first 5 ordinates of the flow hydrograph.***

Time (hr)	Inflow (cfs)
0	0
1	15
2	30
3	40
4	35
5	20
6	5
7	0

SOLUTION:

Outflow (Q) (cfs)	**Storage (S)** (cfs-hr)	**2S/Δt + Q** (cfs)
0	0	0
5	12.5	30
10	25	60
15	37.5	90
20	50	120
25	62.5	150
30	75	180
35	87.5	210
40	100	240
45	112.5	270
50	125	300



Initial conditions: $\frac{2S_n}{\Delta t} - Q_n = \frac{2*(9000 ft^3)}{3600\ s} - 0 = 5\ cfs$

$$\frac{2S_{n+1}}{\Delta t} + Q_{n+1} = (I_n + I_{n+1}) + \left(\frac{2S_n}{\Delta t} - Q_n\right)$$

Find Q_{n+1} from Storage-Indication Curve or from storage-discharge relationship

$$\frac{2S_n}{\Delta t} - Q_n = \left(\frac{2S_n}{\Delta t} + Q_n\right) - 2Q_n$$

Time (hr)	**I_{n+1}** (cfs)	**$I_n + I_{n+1}$** (cfs)	**$2S_n/\Delta t$ - Q_n** (cfs)	**$2S_{n+1}/\Delta t + Q_{n+1}$** (cfs)	**Q_{n+1}** (cfs)
0	0	0	5	5	**0**
1	15	15	5	20	**3.333**
2	30	45	13.333	58.333	**9.722**
3	40	70	38.889	108.889	**18.148**
4	35	75	72.593	147.593	**24.599**
5	20	55	98.395	153.395	**25.566**
6	5	25	102.263	127.263	**21.211**
7	0	5	84.842	89.842	**14.974**
8	0	0	59.895	59.895	**9.982**