

Waterhammer Flows

Lecture 8, 01/29/2014

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Learning Objectives

- (1) Understand basic waterhammer flow principles
- (2) Estimate the order of magnitude of pressure surges in pipe systems due to valve closure or pump failure.

Videos for waterhammer flows

Water Hammer Illustration

<http://www.youtube.com/watch?v=X9UbcanuDk>

Water hammer control in freshwater and sewage applications

http://www.youtube.com/watch?v=nU7yZA_3xis

Column separation and its control via Surge Tank

<http://www.youtube.com/watch?v=E6NIA4LxPPw>

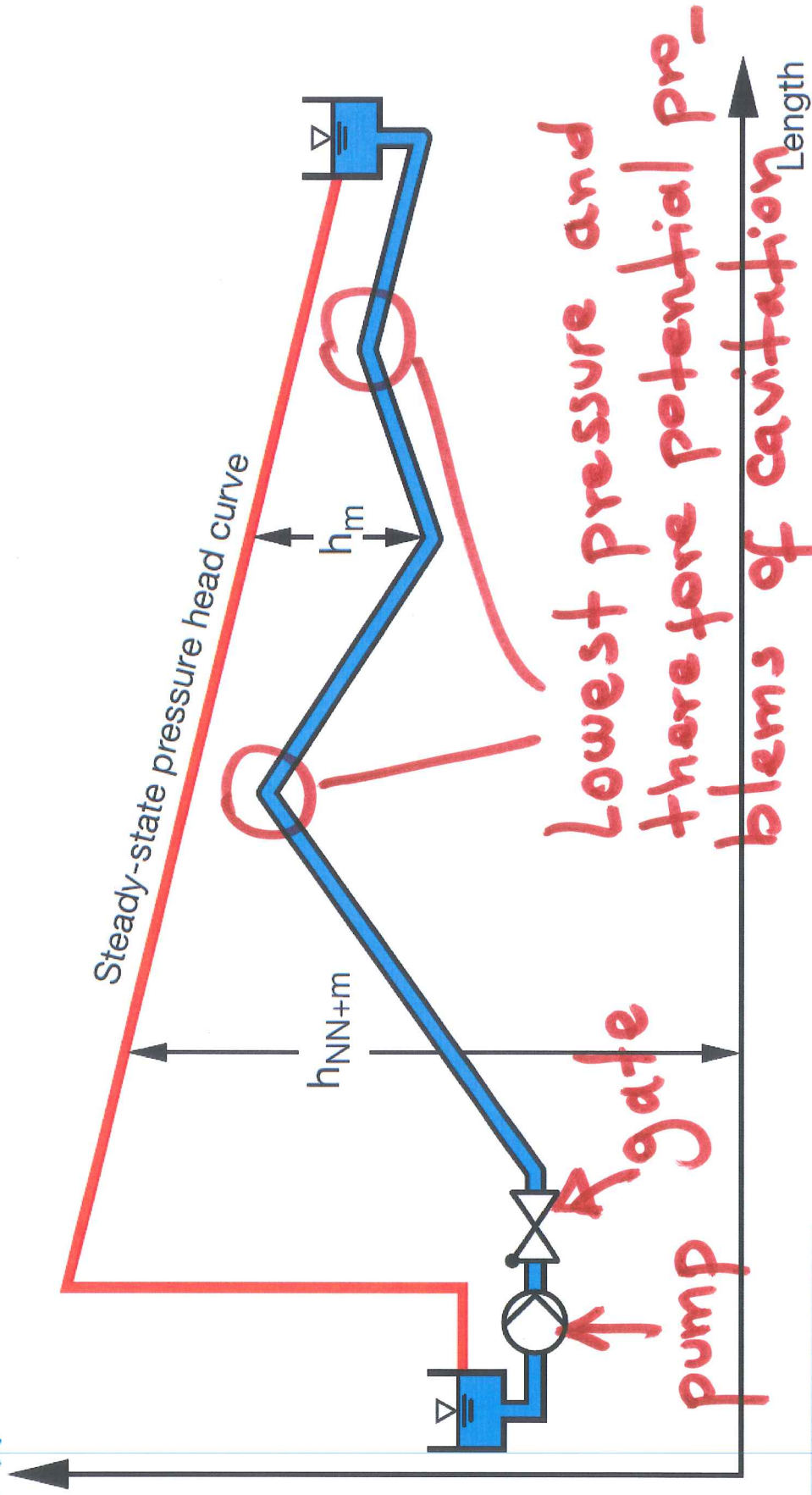
Fire Sprinkler Column Separation and Surge Suppression

<http://www.youtube.com/watch?v=0t5wubuWqVI>

Simulated transients in Chicago system (Arturo Leon, OSU)

http://www.youtube.com/watch?v=AoJg_zpvcrM

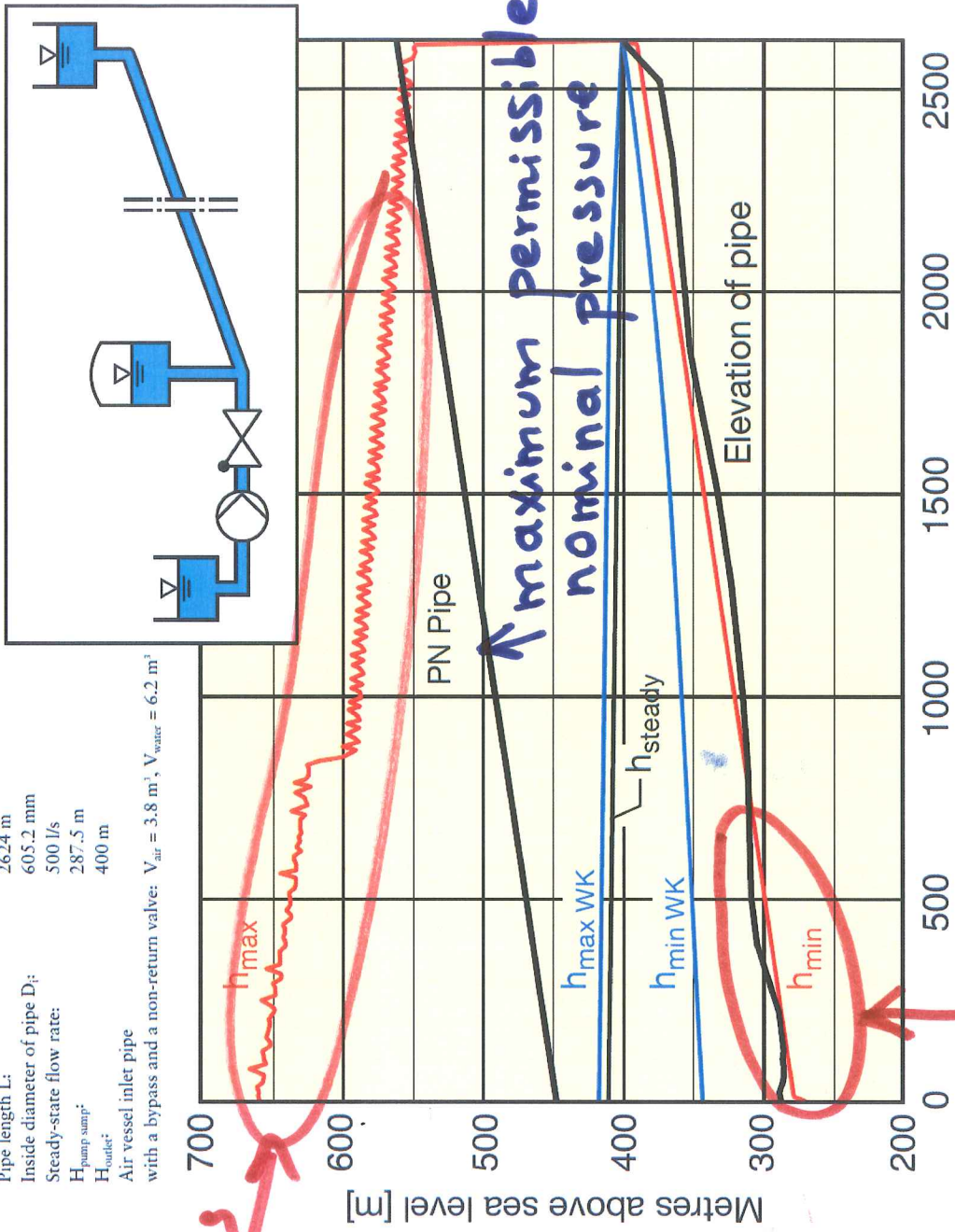
Steady and unsteady flow in a pipeline (See provided handout)



Steady-state pressure head curve of a pumping system

Steady and unsteady flow in a pipeline (Cont.)

Pipe length L: 2624 m
 Inside diameter of pipe D_i: 605.2 mm
 Steady-state flow rate: 500 l/s
 $H_{\text{pump sump}}$: 287.5 m
 H_{outlet} : 400 m
 Air vessel inlet pipe with a bypass and a non-return valve: $V_{\text{air}} = 3.8 \text{ m}^3$, $V_{\text{water}} = 6.2 \text{ m}^3$



over stress

Pressure head envelope of pressure transients following pump trip

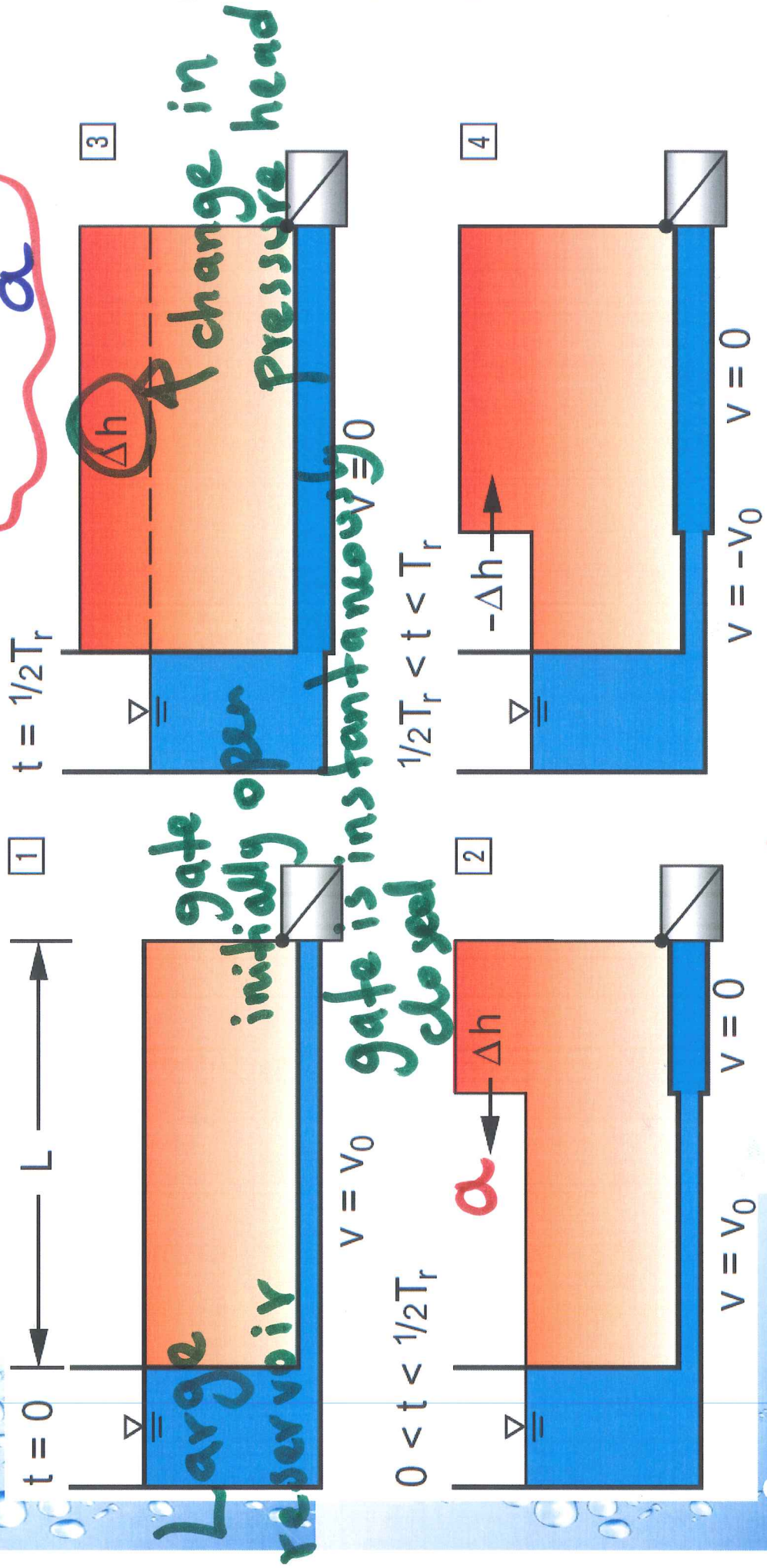
pump is turned off.

Reference: KSB Know - how

potential cavitation

Pressure and velocity waves in a single conduit, frictionless pipeline following its sudden closure

$$T_r = \frac{2L}{a}$$

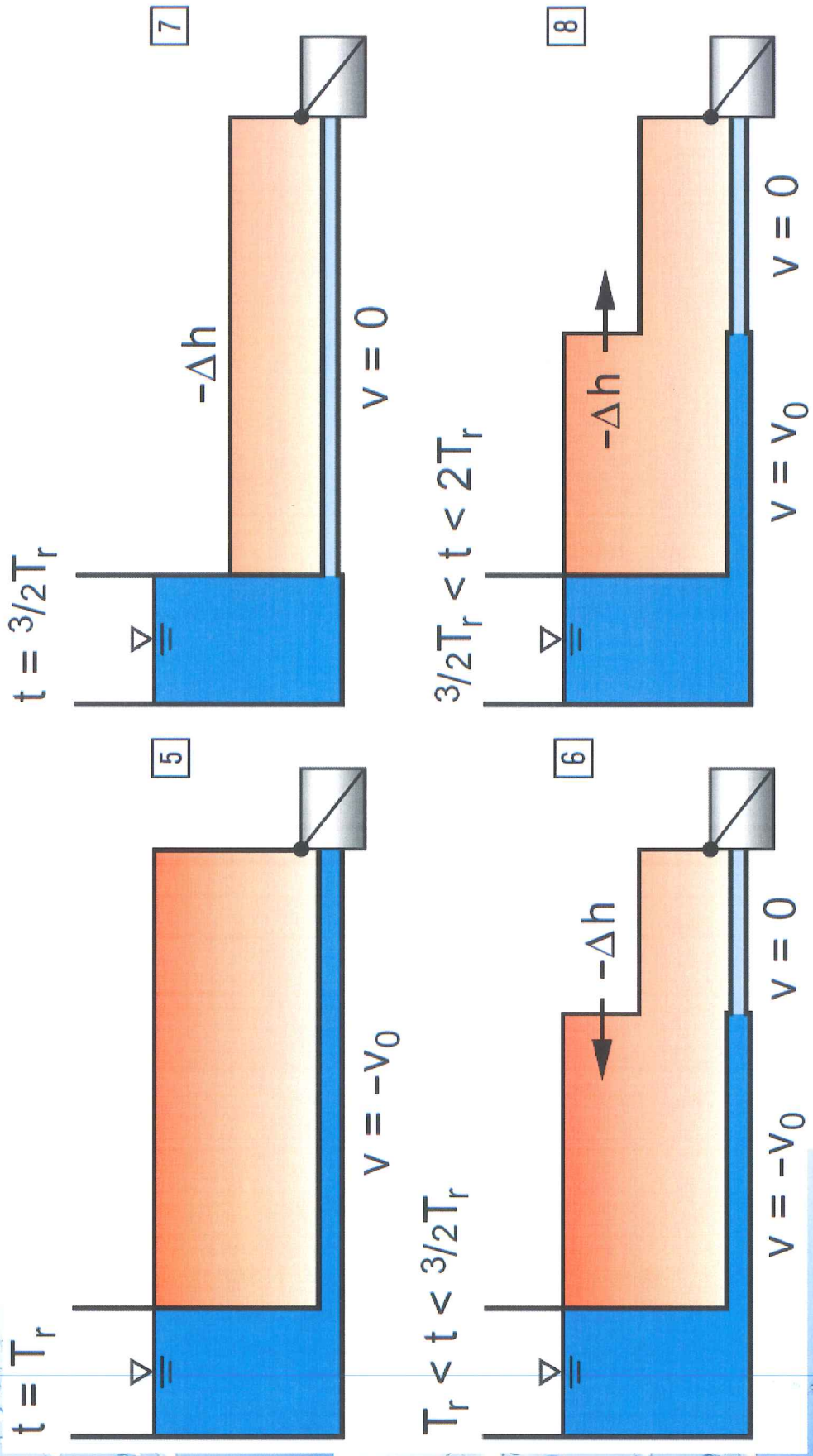


$a =$ waterhammer wave speed
 pressure wave speed

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Reference: KSB Know - how

Pressure and velocity waves in a single conduit, frictionless pipeline following its sudden closure (Cont.)

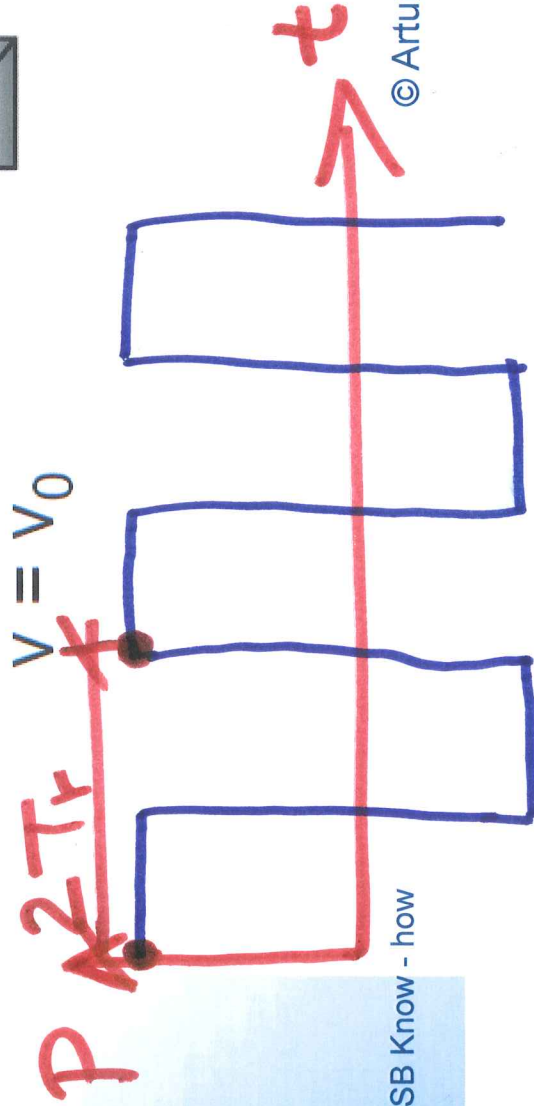
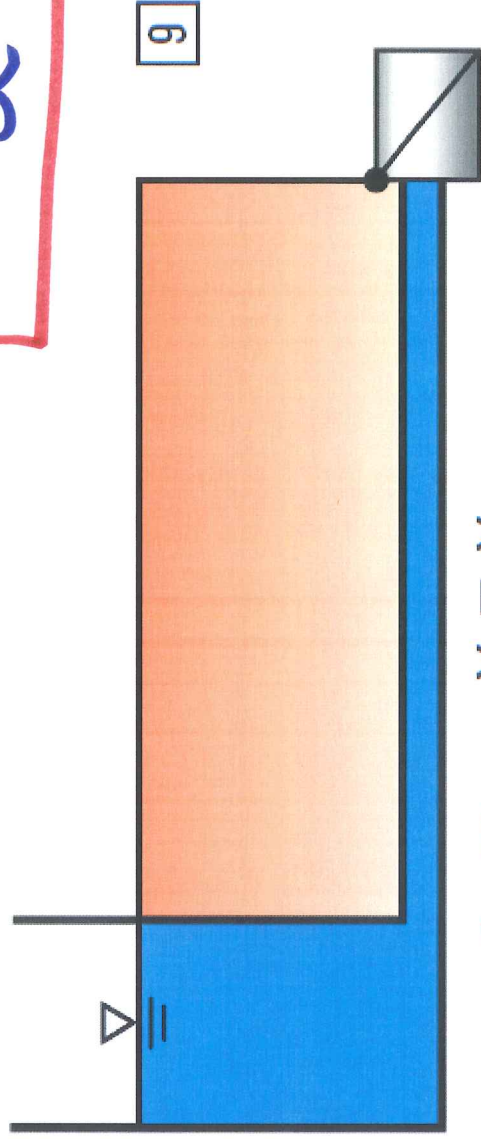


Pressure and velocity waves in a single conduit, frictionless pipeline following its sudden closure (Cont.)

✓ **period**

$$T = \frac{4L}{\alpha}$$

$$t = 2T_r$$



Lecture 9, 01/31/2014

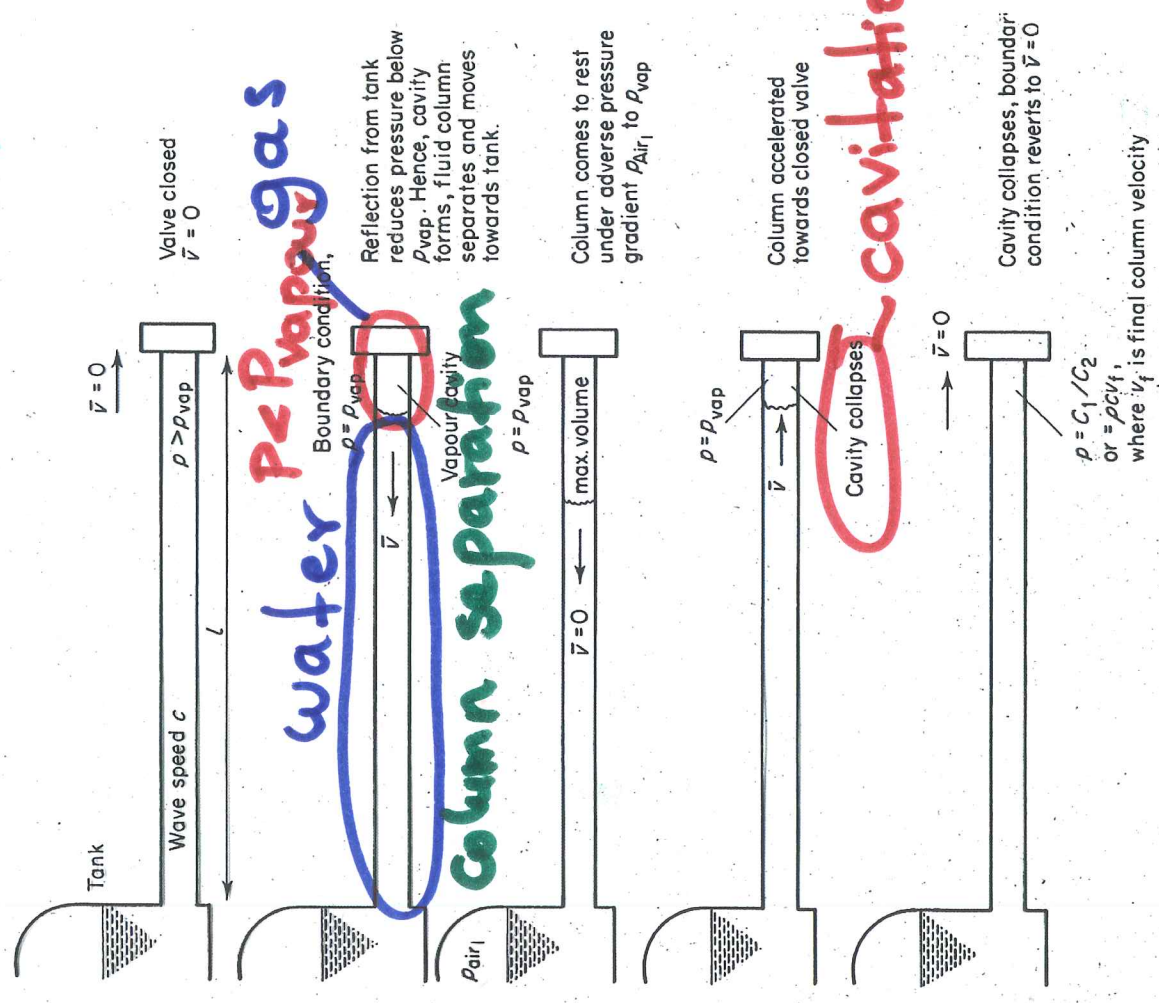
Negative pressure wave

The negative pressure wave can not become lower than the vapour pressure of the fluid.

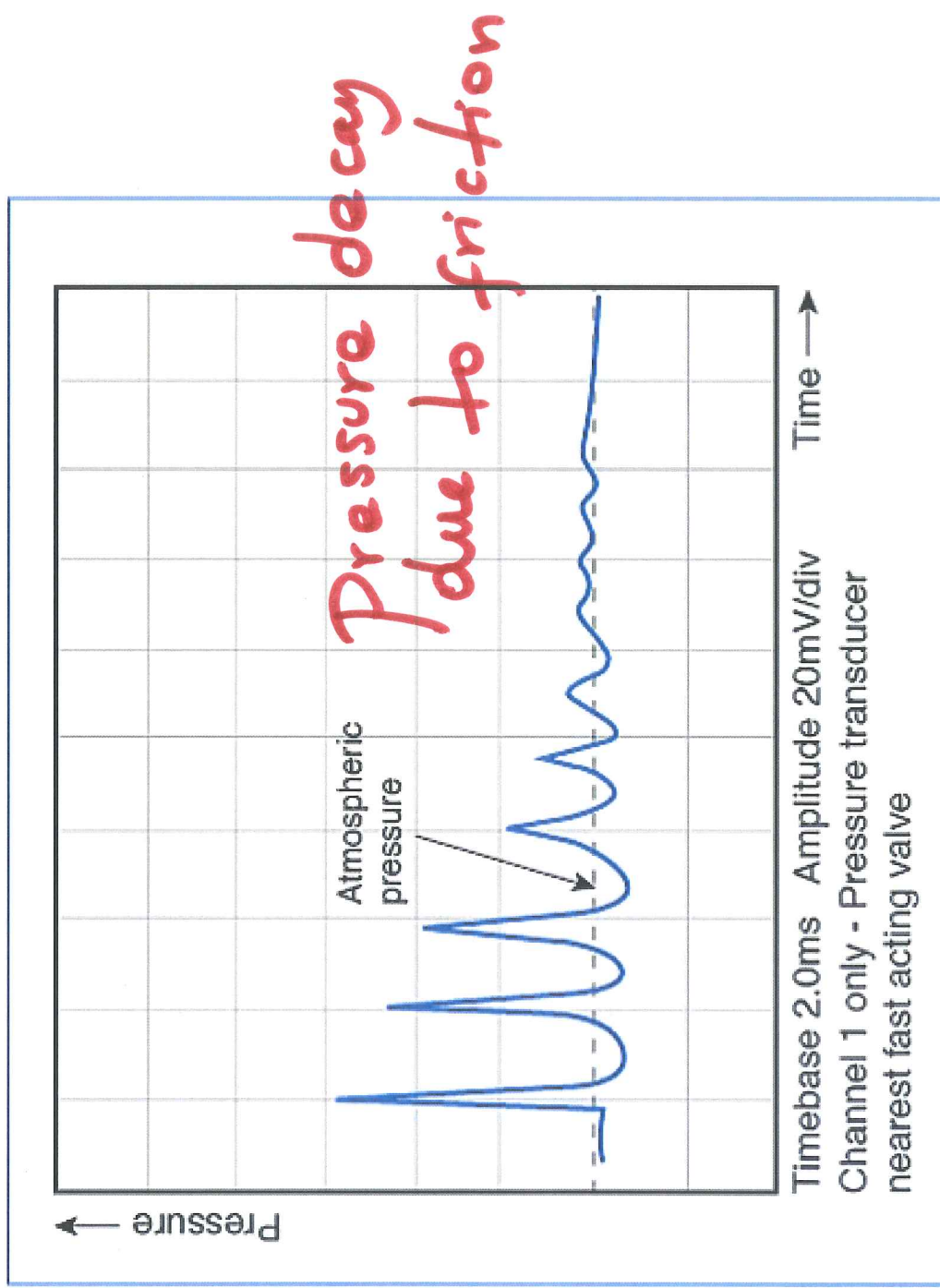
If the pressure falls below the vapour pressure, a vapour bubble is formed. The water column separates from the valve.

When the pressure increases again the bubble collapses.

This phenomenon is called cavitation.



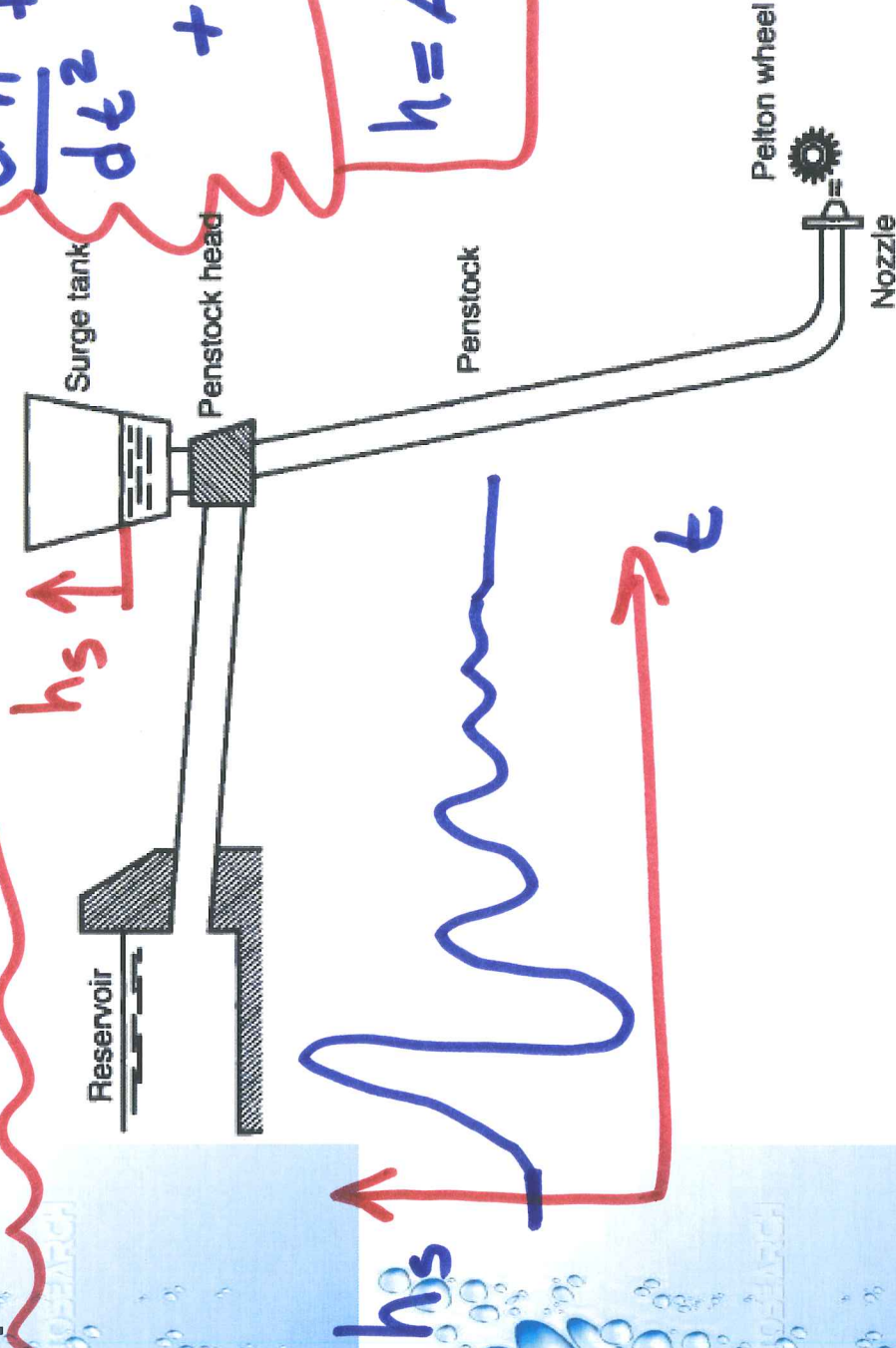
Time trace of pressure fluctuations



Pressure-time diagram showing cyclic nature of pressure pulses with decay due to friction

Measures against pressure surges

Impulse Turbine

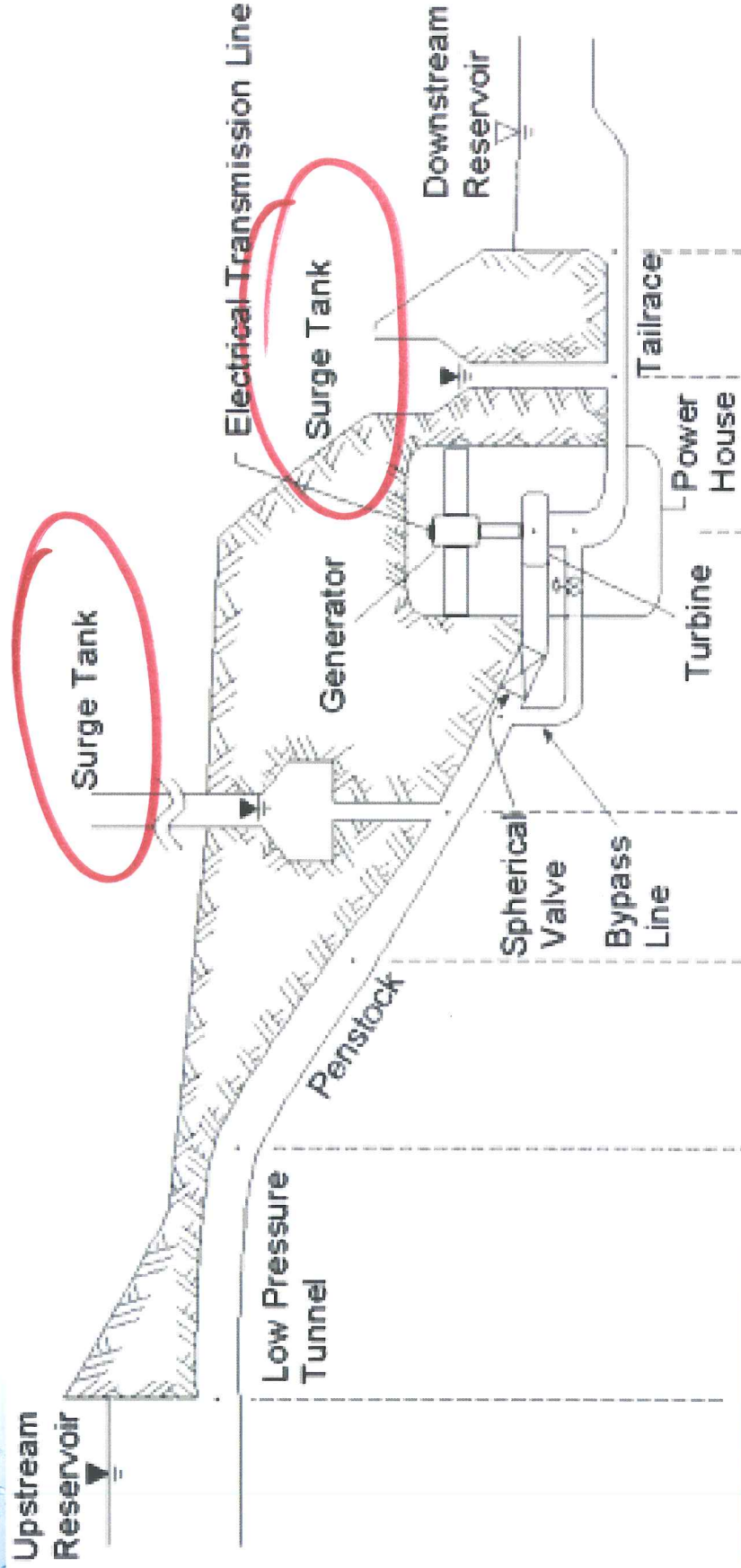


$$\frac{d^2h}{dt^2} + \alpha_1 \frac{dh}{dt} + \alpha_2 = 0$$

$$h = A \sin \omega t$$

Measures against pressure surges (Cont.)

Reaction Turbine



Measures against pressure surges (Cont.)

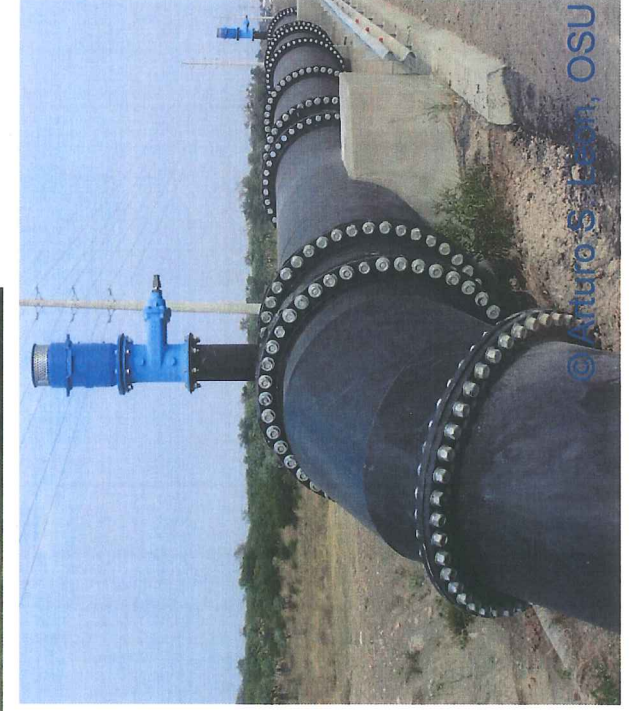


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Measures against pressure surges (Cont.)

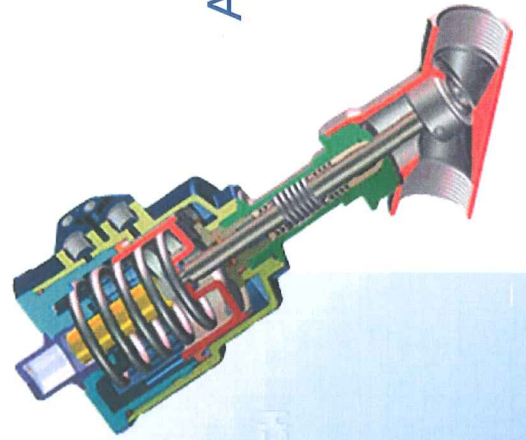


Surge vessels



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Air valves



The Joukowski equation

$$\Delta v = 15 - 0$$

max

$$\Delta h = \frac{1500 \text{ m/s}}{9.8 \text{ m/s}^2} \times 15 \frac{\text{m}}{\text{s}}$$

$$\Delta h \approx 2,250 \text{ m}$$

Large pressure increase.

$$\Delta h_{\text{Jou}} = \frac{a}{g} \cdot \Delta v$$

Δh_{Jou} = Pressure head change (m or ft)

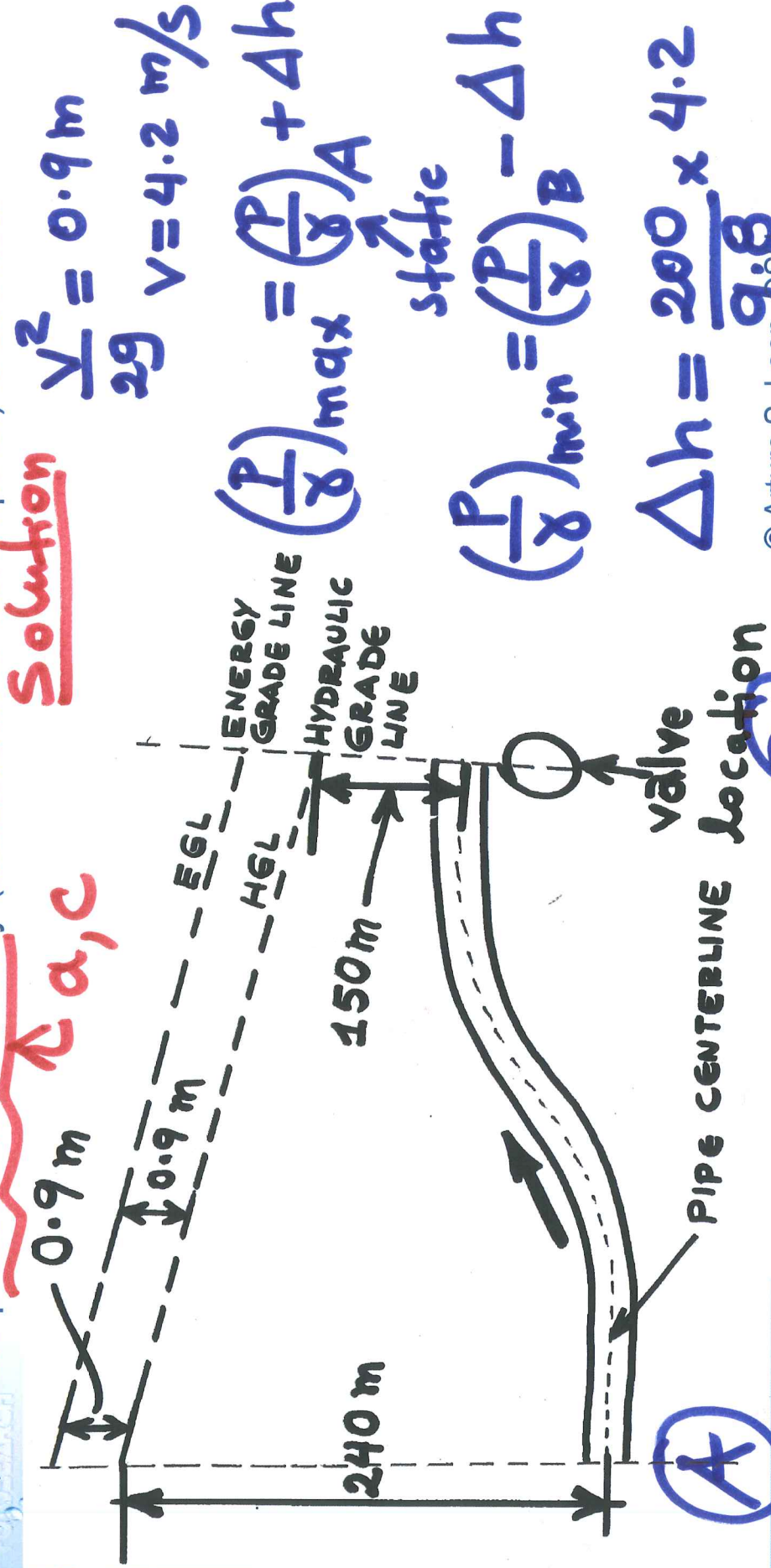
Δv = Flow velocity change (m/s or ft/s)

a = Wave propagation velocity through the fluid in the pipeline (m/s or ft/s)

g = Acceleration due to gravity

Example:

In the pipeline system depicted below, (1) determine the maximum and minimum pressure head that can be produced due to instantaneous valve closure if the valve is located at the downstream end of the pipeline. (2) Is cavitation a problem for this pipeline? Assume that the water temperature is 20°C, the pipe diameter is 0.3 m, and the pressure wave celerity (waterhammer wave speed) is 200 m/s.



Solution

$$\frac{V^2}{2g} = 0.9 \text{ m}$$

$$V = 4.2 \text{ m/s}$$

$$\left(\frac{P}{\gamma}\right)_{\max} = \left(\frac{P}{\gamma}\right)_A + \Delta h$$

static

$$\left(\frac{P}{\gamma}\right)_{\min} = \left(\frac{P}{\gamma}\right)_B - \Delta h$$

$$\Delta h = \frac{200}{9.8} \times 4.2$$

$$\Delta h = 85.7 \text{ m}$$

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$$\left(\frac{P}{\gamma}\right)_{\max} = \underline{325.7 \text{ m}}$$

$$\left(\frac{P}{\gamma}\right)_{\min} = \underline{64.3 \text{ m}}$$

There is no cavitation.

In theory, cavitation occurs when

$$\left(\frac{P}{\gamma}\right)_{\min} \text{ is below } \sim -10.1 \text{ m}$$

In practice, cavitation occurs when

$$\left(\frac{P}{\gamma}\right)_{\min} \text{ is below about } -6 \text{ or } -7 \text{ m (presence of gases in fluid).}$$

Lecture 10, 02/03/2014

Analysis of water hammer phenomenon due to gradual and sudden valve closure

The pressure rise due to water hammer depends upon:

- (a) The velocity of the flow of water in pipe,
- (b) The length of pipe,
- (c) Time taken to close the valve,
- (d) Elastic properties of the material of the pipe.

The following cases of water hammer will be considered:

- Gradual closure of valve,
- Sudden closure of valve when pipe is rigid, and
- Sudden closure of valve when pipe is elastic.

• The time required for the pressure wave to travel from the valve to the reservoir and back to the valve is:



$$t = \frac{2L}{C}$$

Where:

L = length of the pipe (m)

C = speed of pressure wave, celerity (m/s)

(a)

• If the valve time of closure is t_c , then

➤ If $t_c > \frac{2L}{C}$ the closure is considered gradual

➤ If $t_c \leq \frac{2L}{C}$ the closure is considered sudden

time of closure

water hammer wave speed

The speed of pressure wave "C" depends on :

- the pipe wall material.
- the properties of the fluid.
- the anchorage method of the pipe.

$$C = \sqrt{\frac{E_b}{\rho}}$$

if the pipe is rigid

$$C = \sqrt{\frac{E_c}{\rho}}$$

if the pipe is elastic

General formula

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{Dk}{E_p e}$$

and

derive

Where:

- C = celerity of pressure wave due to water hammer.
- ρ = water density (1000 kg/m^3).
- E_b = bulk modulus of water ($2.1 \times 10^9 \text{ N/m}^2$).
- E_c = effective bulk modulus of water in elastic pipe.
- E_p = Modulus of elasticity of the pipe material.
- e = thickness of pipe wall.
- D = diameter of pipe.
- k = factor depends on the anchorage method:

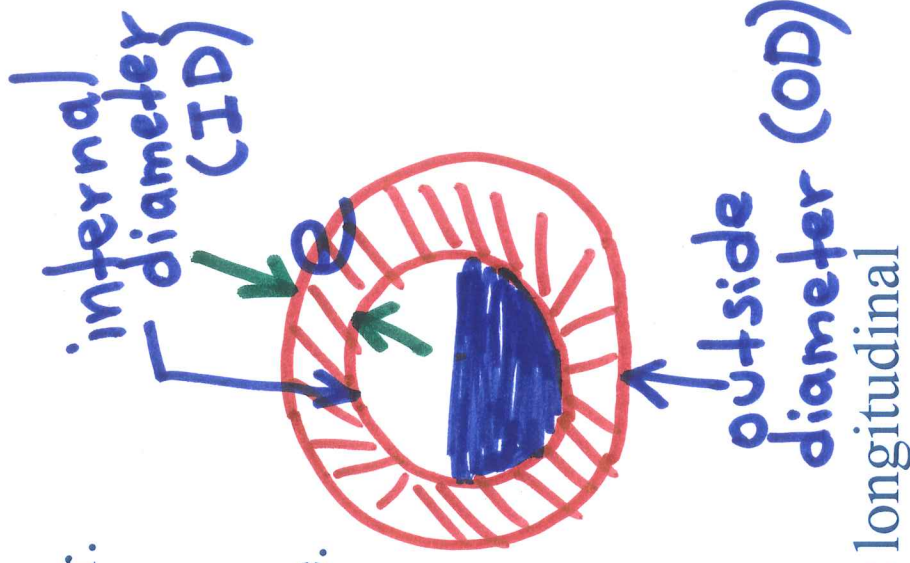
$$k = \left(\frac{5}{4} - \varepsilon \right) \text{ for pipes free to move longitudinally,}$$

$$k = (1 - \varepsilon^2) \text{ movement}$$

$$k = (1 - 0.5\varepsilon) \text{ for pipes with expansion joints.}$$

- where ε = poisson's ratio of the pipe material (0.25 - 0.35).

$\varepsilon = 0.25$ for common pipe materials.



Modulus of elasticity of the pipe material (E_p)

Pipe Material	E_p (N/m ²)	E_p (psi)
Aluminum	$7.0 \cdot 10^{10}$	$10 \cdot 10^6$
Brass, Bronze	$9.0 \cdot 10^{10}$	$13 \cdot 10^6$
Cast-iron, gray	$1.1 \cdot 10^{11}$	$16 \cdot 10^6$
Cast-iron, malleable	$1.6 \cdot 10^{11}$	$23 \cdot 10^6$
Concrete, reinforced	$1.6 \cdot 10^{11}$	$25 \cdot 10^6$
Glass	$7.0 \cdot 10^{10}$	$10 \cdot 10^6$
Lead	<u>$3.1 \cdot 10^8$</u>	$4.5 \cdot 10^4$
Lucite	<u>$2.8 \cdot 10^8$</u>	$4 \cdot 10^4$
Copper	$9.7 \cdot 10^{10}$	$14 \cdot 10^6$
Rubber, vulcanized	$1.4 \cdot 10^{10}$	$2 \cdot 10^6$
Steel	$1.9 \cdot 10^{11}$	$28 \cdot 10^6$

rigid

elastic

Maximum pressure created by water hammer

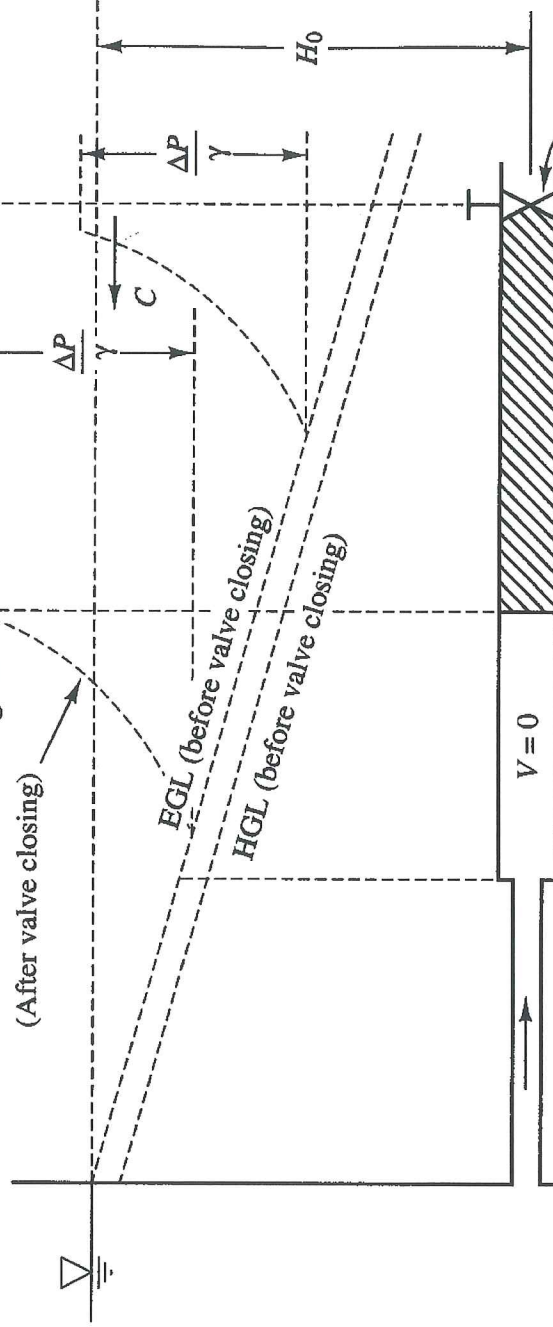
The total pressure experienced by the pipe is

$$P = \Delta P + P_0$$

$$P = -\Delta P + P_0$$

Pressure wave celerity
maximum pressure

minimum pressure



Case 1: Gradual Closure of Valve

- If the time of closure $t_c > \frac{2L}{C}$, then the closure is said to be gradual and the increased pressure is

$$\Delta P = \frac{\rho L V_0}{t}$$

P : Pressure (Psi, N/m²)
 H : Pressure head (ft, m)

where,

- V_0 = initial velocity of water flowing in the pipe before pipe closure
- t_c = time of closure.
- L = length of pipe.
- ρ = water density.
- The pressure head caused by the water hammer is

$$P = \gamma \cdot H$$

$$\Delta H = \frac{\Delta P}{\gamma} = \frac{\rho L V_0}{\rho g t} = \frac{L V_0}{g t}$$

Another method for gradual closure of valve ($t > 2 L/C$)

The maximum water hammer calculated by the Allievi formula is

$$\Delta P = P_o \left(\frac{N}{2} + \sqrt{\frac{N^2}{4} + N} \right)$$

Where P_o is the steady-state pressure in the pipe, and

$$N = \left(\frac{\rho L V_o}{P_o t} \right)$$

Case 2: Sudden closure of valve when pipe is rigid

- If time of closure $t_c \leq \frac{2L}{C}$, then the closure is said to be *Sudden*.
- The pressure head due caused by the water hammer is

$$\Delta P = \rho C V_0 \rightarrow a \frac{V_0}{g} = \frac{a V_0}{g}$$

$$C = \sqrt{\frac{E_b}{\rho}}$$

- But for rigid pipe

$$\Delta H = \frac{V_0}{g} \sqrt{\frac{E_b}{\rho}}$$

so:

$$\Delta P = V_0 \sqrt{E_b \rho}$$

Case 3: Sudden closure of valve when pipe is elastic

- If time of closure $t_c \leq \frac{2L}{C}$, then the closure is said to be *Sudden*.
- The pressure head caused by the water hammer is

$$\Delta P = \rho C V_0$$

$$\Delta H = \frac{C V_0}{g}$$

- But for elastic pipe $C = \sqrt{\frac{E_c}{\rho}}$ so:

$$\Delta H = \frac{V_0}{g} \sqrt{\frac{1}{\rho \left(\frac{1}{E_b} + \frac{DK}{E_p e} \right)}}$$

$$\Delta P = V_0 \sqrt{\frac{\rho}{\left(\frac{1}{E_b} + \frac{DK}{E_p e} \right)}}$$

Applying the above formulas we can determine the maximum and minimum pressures and pressure heads.

The total pressure at any point (e.g., point M) in the pipe after closure (water hammer) is

$$P_M = P_{M, \text{before closure}} \pm \Delta P$$

↳ $P < P_{\text{vap}}$ (cavitation)

$$H_M = H_{M, \text{before closure}} \pm \Delta H$$

↳ $H < \sim -6\text{m}$ (cavitation)
-7m

↑ practice

Pressure

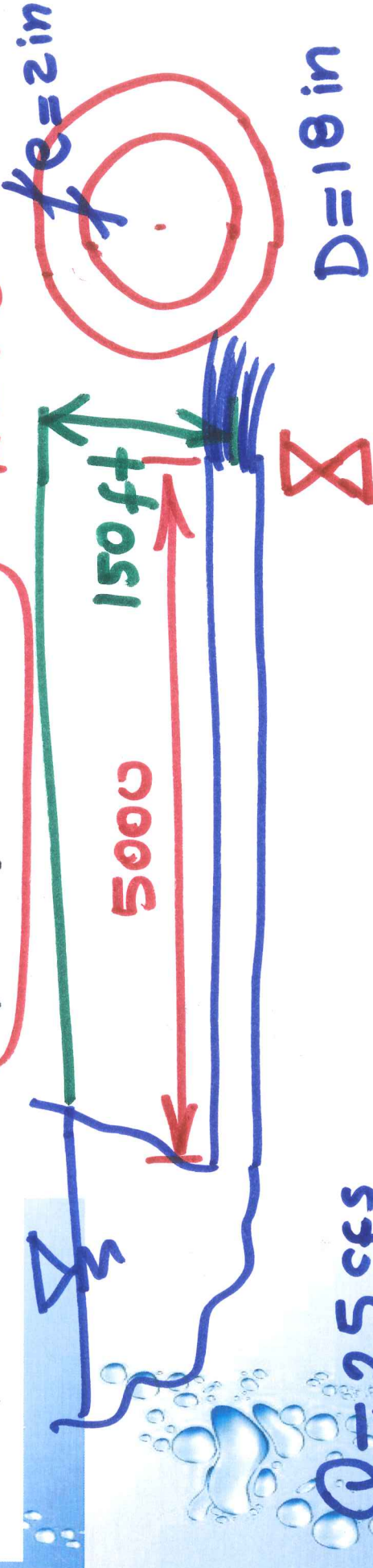
and

Pressure head

Example

$$V = \frac{Q}{A} = \frac{25}{\frac{\pi}{4}(1.5)^2} = 14.1 \frac{\text{ft}}{\text{s}}$$

A steel pipe 5000 ft long laid on a uniform slope has an 18-in. diameter and a 2-in. wall thickness. The pipe carries water from a reservoir and discharges it into the air at an elevation 150 ft below the reservoir free surface. A valve installed at the downstream end of the pipe allows a flow rate of 25 cfs. If the valve is completely closed in 1.4 sec, calculate the maximum water hammer pressure at the valve. Neglect longitudinal stresses. $k = 1.0$



$$Q = 25 \text{ cfs}$$

$$t_c = 1.4 \text{ s}$$

$$P_{\text{max}} = P_s + \Delta P$$

$$E_P = 28 \times 10^6 \text{ psi} \quad (\text{steel})$$

$$(E_b = 3.0 \times 10^5 \text{ psi})$$

$$L = 5000 \text{ ft}$$

$$D = 18 \text{ in} = 1.5 \text{ ft}, \quad Q = 25 \text{ cfs}, \quad t_c = 1.4 \text{ s}$$

$$P_{\max} = P_s + \Delta P = 65.03 \text{ Psi} + \Delta P$$

$$P = 8\#$$

854 Psi

$$P_{\max} = 919 \text{ Psi}$$

$k=1.0$

$$\frac{1}{E_c} = \frac{1}{E_b} + \frac{Dk}{E_P \cdot e} = \frac{1}{3.0 \times 10^5} + \frac{18 \text{ in}}{28 \times 10^6 \times 2 \text{ in}}$$

$$E_c = 2.74 \times 10^5 \text{ psi}$$

$$C = \sqrt{\frac{E_c}{\rho}}$$

$$C = \sqrt{\frac{2.74 \times 10^5 (144)}{1.94}} = 4510 \text{ ft/s}$$

$$t = \frac{2L}{C} = \frac{2 \times 5000}{4510} = 2.22 \text{ s}$$

$$2.22 > 1.4 \text{ (sudden)}$$

$$\Delta P = 9\% C = 1.94 \times 14.1 \times 4510 = 1.23 \times 10^5 \frac{\text{lb}}{\text{ft}^2} (854 \text{ psi})$$