Waterhammer Flows
Lecture 8, 01/29/2014

Arturo Leon, Oregon State University
Learning Objectives

(1) Understand basic waterhammer flow principles
(2) Estimate the order of magnitude of pressure surges in pipe systems due to valve closure or pump failure.
Videos for waterhammer flows

Water Hammer Illustration
http://www.youtube.com/watch?v=X9UbzcanuDk

Water hammer control in freshwater and sewage applications
http://www.youtube.com/watch?v=nU7yZA_3xjs

Column separation and its control via Surge Tank
http://www.youtube.com/watch?v=E6NIA4LxPPw

Fire Sprinkler Column Separation and Surge Suppression
http://www.youtube.com/watch?v=0t5wubuWqVI

Simulated transients in Chicago system (Arturo Leon, OSU)
http://www.youtube.com/watch?v=AoJg_zpvcrM

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Steady and unsteady flow in a pipeline (See provided handout)

Steady-state pressure head curve

$h_{NN+m}$

$L_{m}$

Lowest pressure and therefore potential problems of cavitation

Steady-state pressure head curve of a pumping system

Reference: KSB Know-how

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Steady and unsteady flow in a pipeline (Cont.)

Pressure head envelope of pressure transients following pump trip

Pump is turned off.

Reference: KSB Know - how

Maximum permissible nominal pressure

Potential cavitation
Pressure and velocity waves in a single conduit, frictionless pipeline following its sudden closure

\[ T_r = \frac{2L}{a} \]

1. \( t = 0 \)  
   - Large reservoir
   - \( v = v_0 \)

2. \( 0 < t < \frac{1}{2}T_r \)  
   - Gate initially open
   - \( \Delta h \)

3. \( t = \frac{1}{2}T_r \)  
   - Change in pressure head
   - \( \Delta h \)

4. \( \frac{1}{2}T_r < t < T_r \)  
   - Gate is instantaneously closed
   - \( v = 0 \)

Reference: KSB Know-how

\( a = \text{waterhammer wave speed} \)

\( \text{pressure wave speed} \)
Pressure and velocity waves in a single conduit, frictionless pipeline following its sudden closure (Cont.)

$t = T_r$

$v = -v_0$

$T_r < t < 3/2 T_r$

$v = -v_0$

$3/2 T_r < t < 2 T_r$

$v = v_0$

Reference: KSB Know-how

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Pressure and velocity waves in a single conduit, frictionless pipeline following its sudden closure (Cont.)

\[ t = 2T_r \]

\[ T = \frac{4L}{a} \]

Reference: KSB Know - how

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Negative pressure wave

The negative pressure wave can not become lower than the vapour pressure of the fluid. *If the pressure falls below the vapour pressure, a vapour bubble is formed.* The water column separates from the valve. *When the pressure increases again the bubble collapses.* This phenomenon is called cavitation.

Source: W. Kinzelbach et al.
Time trace of pressure fluctuations

Pressure-time diagram showing cyclic nature of pressure pulses with decay due to friction
Measures against pressure surges

Impulse Turbine

\[ \frac{d^2 h}{dt^2} + \frac{d}{dt} \frac{dh}{dt} + d_2 = 0 \]

\[ h = A \sin \omega t \]

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Measures against pressure surges (Cont.)

Reaction Turbine

- Upstream Reservoir
- Low Pressure Tunnel
- Penstock
- Spherical Valve
- Bypass Line
- Generator
- Surge Tank
- Electrical Transmission Line
- Downstream Reservoir
- Turbine
- Power House
- Tailrace

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Measures against pressure surges (Cont.)
Measures against pressure surges (Cont.)

Surge vessels

Air valves
The Joukowsky equation

\[ \Delta h_{\text{Jou}} = \frac{a}{g} \cdot \Delta v \]

\[ \Delta v = 15 - 0 \text{ m/s} \]

\[ \Delta h = \frac{1500 \text{ m/s}}{9.8 \text{ m/s}^2} \times 15 \frac{\text{m}}{\text{s}} \]

\[ \Delta h \approx 2,250 \text{ m} \]

\[ \Delta h_{\text{Jou}} = \text{Pressure head change (m or ft)} \]

\[ \Delta v = \text{Flow velocity change (m/s or ft/s)} \]

\[ a = \text{Wave propagation velocity through the fluid in the pipeline (m/s or ft/s)} \]

\[ g = \text{Acceleration due to gravity} \]
Example:

In the pipeline system depicted below, (1) determine the maximum and minimum pressure head that can be produced due to instantaneous valve closure if the valve is located at the downstream end of the pipeline. (2) Is cavitation a problem for this pipeline? Assume that the water temperature is 20°C, the pipe diameter is 0.3 m, and the pressure wave celerity (waterhammer wave speed) is 200 m/s.

Solution

\[ \frac{v^2}{2g} = 0.9 \text{ m} \]

\[ v = 4.2 \text{ m/s} \]

\[ \frac{(P)}{8} \text{ max} = \frac{(P)}{8} \text{ A} + \Delta h \]

\[ \frac{(P)}{8} \text{ min} = \frac{(P)}{8} \text{ B} - \Delta h \]

\[ \Delta h = \frac{200 \times 4.2}{9.8} \]

\[ \Delta h = 85.7 \text{ m} \]
\[
\left(\frac{P}{g}\right)_{\text{max}} = 325.7 \text{ m}
\]

\[
\left(\frac{P}{g}\right)_{\text{min}} = 64.3 \text{ m}
\]

There is no cavitation.

In theory, cavitation occurs when \( \left(\frac{P}{g}\right)_{\text{min}} \) is below \( \sim -10.1 \text{ m} \).

In practice, cavitation occurs when \( \left(\frac{P}{g}\right)_{\text{min}} \) is below about \(-6 \text{ or } -7 \text{ m} \) (presence of gases in fluid).
Analysis of water hammer phenomenon due to gradual and sudden valve closure.

The pressure rise due to water hammer depends upon:

(a) The velocity of the flow of water in pipe,
(b) The length of pipe,
(c) Time taken to close the valve,
(d) Elastic properties of the material of the pipe.

The following cases of water hammer will be considered:

- Gradual closure of valve,
- Sudden closure of valve when pipe is rigid,
- Sudden closure of valve when pipe is elastic.
The time required for the pressure wave to travel from the valve to the reservoir and back to the valve is:

\[ t = \frac{2L}{C} \]

Where:
- \( L \) = length of the pipe (m)
- \( C \) = speed of pressure wave, celerity (m/s)

If the valve time of closure is \( t_c \), then

- If \( t_c > \frac{2L}{C} \) the closure is considered gradual
- If \( t_c \leq \frac{2L}{C} \) the closure is considered sudden

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The speed of pressure wave "C" depends on:

- the pipe wall material.
- the properties of the fluid.
- the anchorage method of the pipe.

\[ C = \sqrt{\frac{E_b}{\rho}} \]  
if the pipe is rigid

\[ C = \sqrt{\frac{E_c}{\rho}} \]  
if the pipe is elastic

\[ \frac{1}{E_c} = \frac{1}{E_b} + \frac{Dk}{E_pe} \]  
and

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Where:
- $C =$ celerity of pressure wave due to water hammer.
- $\rho =$ water density (1000 kg/m$^3$).
- $E_b =$ bulk modulus of water (2.1 x 10$^9$ N/m$^2$).
- $E_c =$ effective bulk modulus of water in elastic pipe.
- $E_p =$ Modulus of elasticity of the pipe material.
- $\varepsilon =$ thickness of pipe wall.
- $D =$ diameter of pipe.
- $k =$ factor depends on the anchorage method:
  - $K = \left(\frac{5}{4} - \varepsilon\right)$ for pipes free to move longitudinally,
  - $K = (1 - \varepsilon^2)$ for pipes anchored at both ends against longitudinal movement
  - $K = (1 - 0.5\varepsilon)$ for pipes with expansion joints.
- where $\varepsilon =$ poisson’s ratio of the pipe material (0.25 - 0.35).
  - $\varepsilon = 0.25$ for common pipe materials.
## Modulus of elasticity of the pipe material ($E_p$)

<table>
<thead>
<tr>
<th>Pipe Material</th>
<th>$E_p$ (N/m$^2$)</th>
<th>$E_p$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>$7.0 \cdot 10^{10}$</td>
<td>$10 \cdot 10^6$</td>
</tr>
<tr>
<td>Brass, Bronze</td>
<td>$9.0 \cdot 10^{10}$</td>
<td>$13 \cdot 10^6$</td>
</tr>
<tr>
<td>Cast-iron, gray</td>
<td>$1.1 \cdot 10^{11}$</td>
<td>$16 \cdot 10^6$</td>
</tr>
<tr>
<td>Cast-iron, malleable</td>
<td>$1.6 \cdot 10^{11}$</td>
<td>$23 \cdot 10^6$</td>
</tr>
<tr>
<td>Concrete, reinforced</td>
<td>$1.6 \cdot 10^{11}$</td>
<td>$25 \cdot 10^6$</td>
</tr>
<tr>
<td>Glass</td>
<td>$7.0 \cdot 10^{10}$</td>
<td>$10 \cdot 10^6$</td>
</tr>
<tr>
<td>Lead</td>
<td>$3.1 \cdot 10^{8}$</td>
<td>$4.5 \cdot 10^4$</td>
</tr>
<tr>
<td><strong>Lucite</strong></td>
<td>$2.8 \cdot 10^{8}$</td>
<td>$4 \cdot 10^4$</td>
</tr>
<tr>
<td>Copper</td>
<td>$9.7 \cdot 10^{10}$</td>
<td>$14 \cdot 10^6$</td>
</tr>
<tr>
<td>Rubber, vulcanized</td>
<td>$1.4 \cdot 10^{10}$</td>
<td>$2 \cdot 10^6$</td>
</tr>
<tr>
<td>Steel</td>
<td>$1.9 \cdot 10^{11}$</td>
<td>$28 \cdot 10^6$</td>
</tr>
</tbody>
</table>
Maximum pressure created by water hammer

The total pressure experienced by the pipe is

$$P = \Delta P + P_0$$

Pressure wave celerity

(Water hammer pressure head)

(After valve closing)

EGL (before valve closing)

HGL (before valve closing)

$$P = -\Delta P + P_0$$

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Case 1: Gradual Closure of Valve

- If the time of closure \( t_c > \frac{2L}{C} \), then the closure is said to be gradual and the increased pressure is

\[
\Delta P = \frac{\rho L V_0}{t}
\]

where,

- \( V_0 \) = initial velocity of water flowing in the pipe before pipe closure
- \( t \) = time of closure.
- \( L \) = length of pipe.
- \( \rho \) = water density.

- The pressure head caused by the water hammer is

\[
\Delta H = \frac{\Delta P}{\gamma} = \frac{\rho L V_0}{\rho g t} = \frac{L V_0}{g t}
\]
Another method for gradual closure of valve \((t > 2 \, L/C)\)

The maximum water hammer calculated by the Allievi formula is

\[
\Delta P = P_o \left( \frac{N}{2} + \sqrt{\frac{N^2}{4} + N} \right)
\]

Where \(P_o\) is the steady-state pressure in the pipe, and

\[
N = \left( \frac{\rho L V_o}{P_o t} \right)
\]
Case 2: Sudden closure of valve when pipe is rigid

- If time of closure $t_c \leq \frac{2L}{C}$, then the closure is said to be *Sudden*.
- The pressure head due caused by the water hammer is

\[
\Delta P = \rho CV_0
\]

\[
\Delta H = \frac{CV_0}{g}
\]

\[
C = \sqrt{\frac{E_b}{\rho}}
\]

\[
\text{so:} \quad \Delta H = \frac{V_0}{g} \sqrt{\frac{E_b}{\rho}}
\]

\[
\Delta P = V_0 \sqrt{E_b \rho}
\]

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Case 3: Sudden closure of valve when pipe is elastic

- If time of closure $t_c \leq \frac{2L}{C}$, then the closure is said to be *Sudden*.
- The pressure head caused by the water hammer is
  \[ \Delta P = \rho CV_0 \]
  \[ \Delta H = \frac{CV_0}{g} \]

- But for elastic pipe
  \[ C = \sqrt{\frac{E_c}{\rho}} \]
  so:
  \[ \Delta H = \frac{V_0}{g} \sqrt{\frac{1}{\rho \left( \frac{1}{E_b} + \frac{DK}{E_p e} \right)}} \]

\[ \Delta P = V_0 \sqrt{\frac{\rho}{\left( \frac{1}{E_b} + \frac{DK}{E_p e} \right)}} \]
Applying the above formulas we can determine the maximum and minimum pressures and pressure heads.

The total pressure at any point (e.g., point M) in the pipe after closure (water hammer) is

\[ P_M = P_{M, \text{before closure}} \pm \Delta P \]

\[ H_M = H_{M, \text{before closure}} \pm \Delta H \]

\[ \text{if } P < P_{\text{vap}} \text{ (cavitation)} \]

\[ \text{if } H < -6 \text{m (cavitation)} \]

\[ \text{min } H = -7 \text{m} \]

\[ \uparrow \text{ practice} \]
Example

A steel pipe 5000 ft long laid on a uniform slope has an 18-in. diameter and a 2-in. wall thickness. The pipe carries water from a reservoir and discharges it into the air at an elevation 150 ft below the reservoir free surface. A valve installed at the downstream end of the pipe allows a flow rate of 25 cfs. If the valve is completely closed in 1.4 sec, calculate the maximum water hammer pressure at the valve. Neglect longitudinal stresses.

\[ V = \frac{Q}{A} = \frac{25}{\frac{\pi}{4} (1.5)^2} = 14.1 \text{ ft} \]

\[ K = 1.0 \]

\[ t_c = 1.4 \text{ sec} \]

\[ P_{max} = P_s + \Delta P \]

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\[ E_p = 28 \times 10^6 \text{ psi}, \quad L = 5000 \text{ ft} \]

\[ P = 18 \text{ in} = 1.5 \text{ ft}, \quad Q = 25 \text{ cfs}, \quad t_c = 1.4 \text{ s} \]

\[ P_{\text{max}} = P_s + \Delta P \]

\[ P_{\text{max}} = 919 \text{ Psi} \]

\[ \frac{1}{E_c} = \frac{1}{E_p} + \frac{Dk}{E_p} \]

\[ E_c = \frac{1}{3.0 \times 10^5} + \frac{18 \text{ in}}{28 \times 10^6 \times 2 \text{ in}} \]

\[ E_c = 2.74 \times 10^5 \text{ psi} \]

\[ C = \sqrt{\frac{E_c}{P}} = \sqrt{\frac{2.74 \times 10^5}{1.94}} = 4510 \text{ ft/s} \]

\[ t = \frac{2L}{C} = \frac{2 \times 5000}{4510} = 2.22 \text{ s} \]

\[ \Delta P = PV_0 C = 1.94 \times 14.1 \times 4510 = 1.23 \times 10^5 \frac{\text{lb}}{\text{ft}^2} \]

\[ P = 8 \text{ ft} \]

\[ E_b = 3.0 \times 10^5 \text{ psi} \]