# Open-Channel Flows
# Lecture 11, 02/04/2014

Arturo Leon, Oregon State University

# Learning Objectives of the section of Open-Channel flows

(1) Discuss the general characteristics of open-channel flows

(2) Apply appropriate equations to analyze open-channel flows with uniform depth

(3) Calculate key properties of a hydraulic jump

(4) Determine flowrates in open-channel flow-measuring devices

# General characteristics of Open-Channel Flows

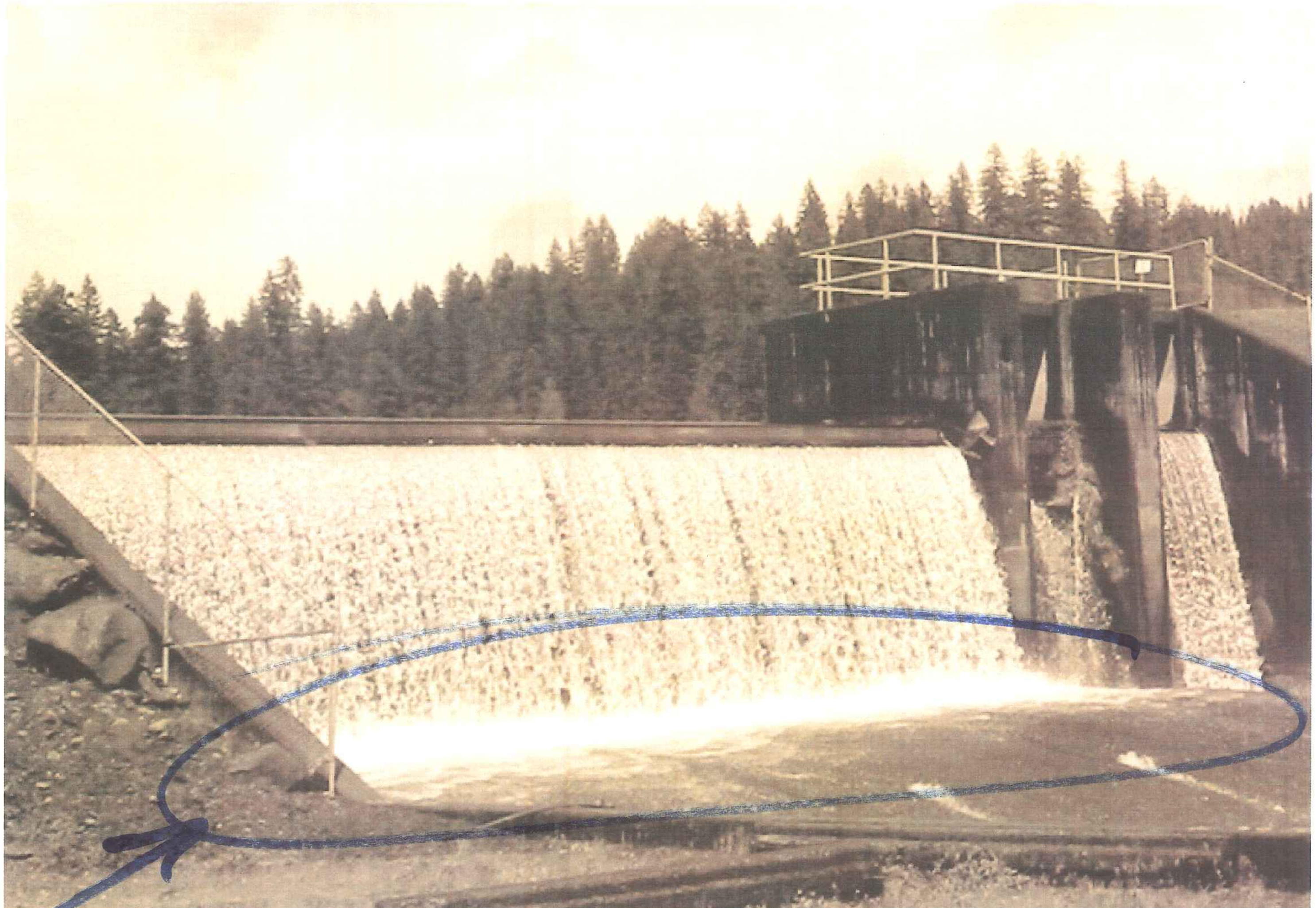
Uniform flow

Unnumbered 10 p555a


$$\frac{dy}{dx} = 0$$

$$\frac{dV}{dx} = 0$$

typically occurs in man-made channels.



$\frac{dy}{dx} \sim 1$

flow acceleration

# Rapidly varying flow (RVF)

Unnumbered 10 p555b
Photo by Marty Melchior

# Classification of open-channel flows

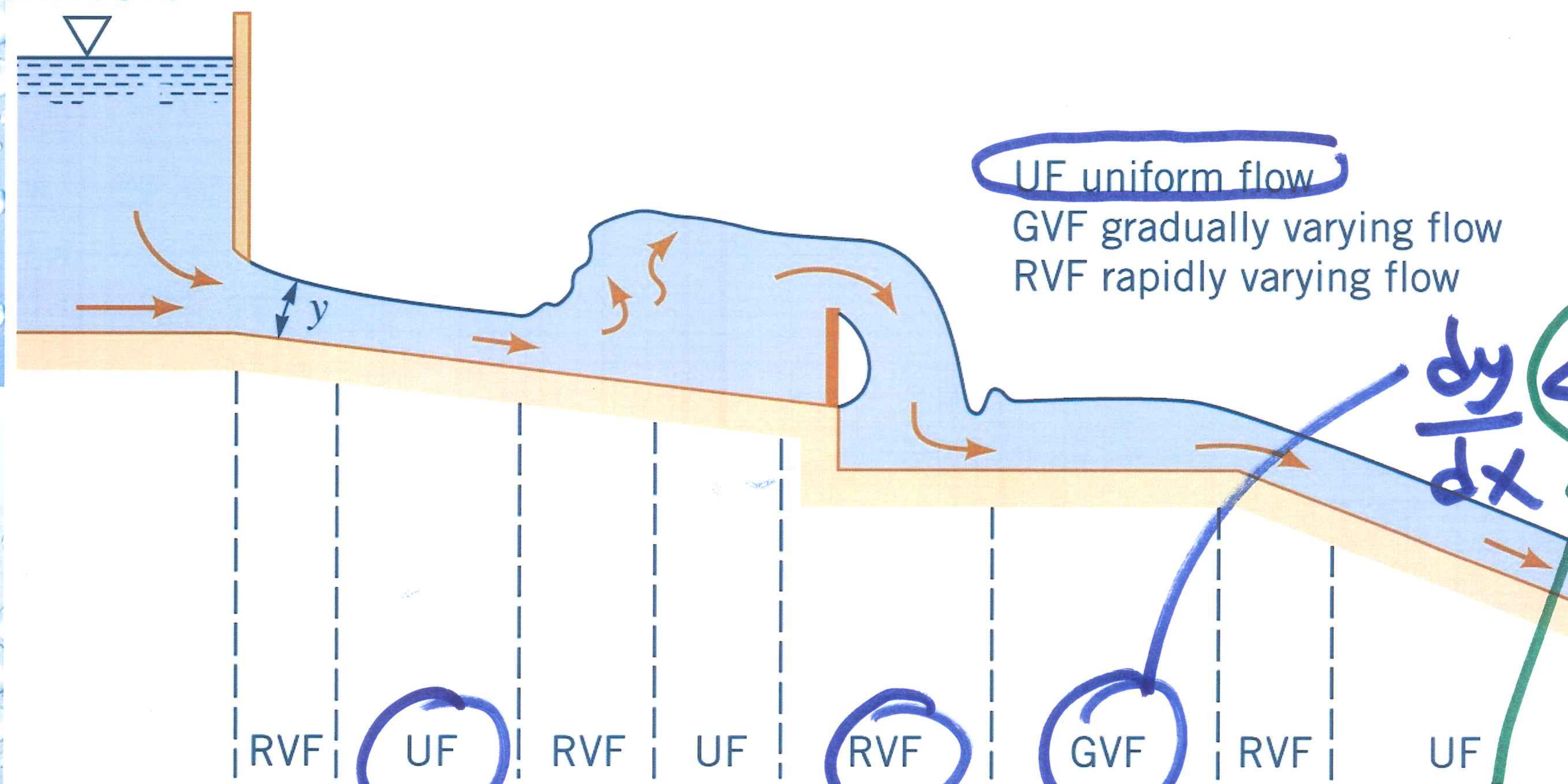



Figure 10.1


$\frac{dy}{dx} = 0$

$\frac{dy}{dx} \sim 1$

$\frac{dy}{dx} << 1$

much smaller

# Surface waves

Unnumbered 10 p556b

# Wave speed in open channel flows

For wide channels

$$\boxed{c = \sqrt{gy}}$$

y: water depth

In general:

$$c = \sqrt{\frac{gA}{T}}$$

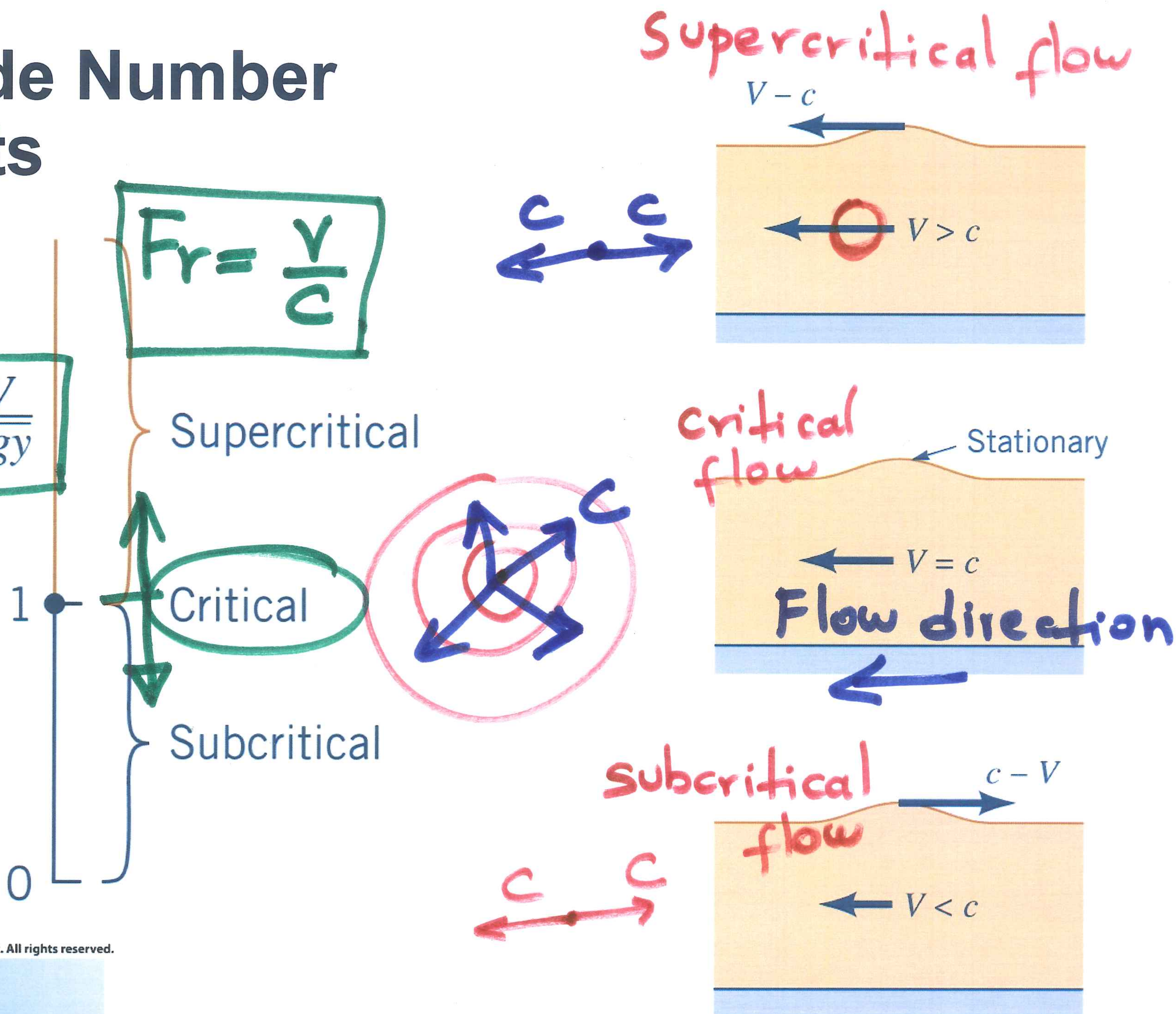
A: hydraulic area

T: surface width

# Froude Number effects

$$Fr = \frac{V}{\sqrt{gy}}$$

$$Fr = \frac{V}{c}$$



# Example of application:

In the picture below the river travels to the left and the surface wave travels upstream (to the right). The width of the river is 100 ft, the flow velocity *V* is 8ft/s, and the water depth *y* is 2ft. Is the flow subcritical, critical or supercritical?

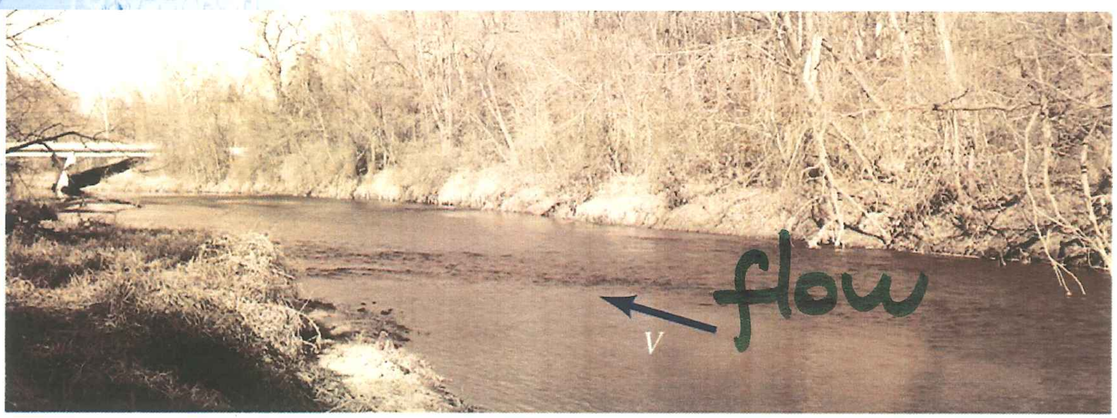
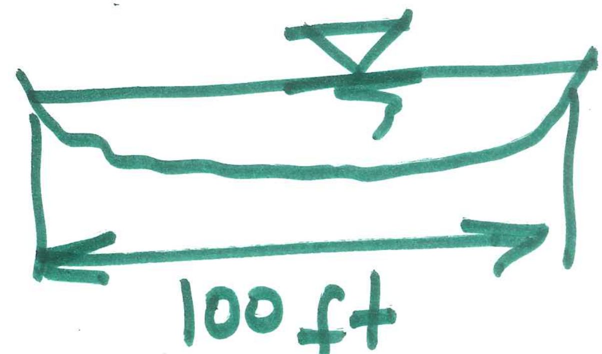



Figure E10.1a


$V = 8\,ft/s$
$y = 2\,ft$

$$Fr = \frac{V}{c} = \frac{V}{\sqrt{\frac{gA}{T}}} = \frac{8\ ft/s}{\sqrt{32.2 \times \frac{200}{100}}}$$

$$Fr = 0.997$$

$0.997 < 1$ (flow should be subcritical)

**BUT:**

$$0.8 < Fr < 1.2$$

the flow is highly unstable. In practice stay away from this range.

the flow is critical.

# Lecture 12, 02/10/2014
# Energy considerations

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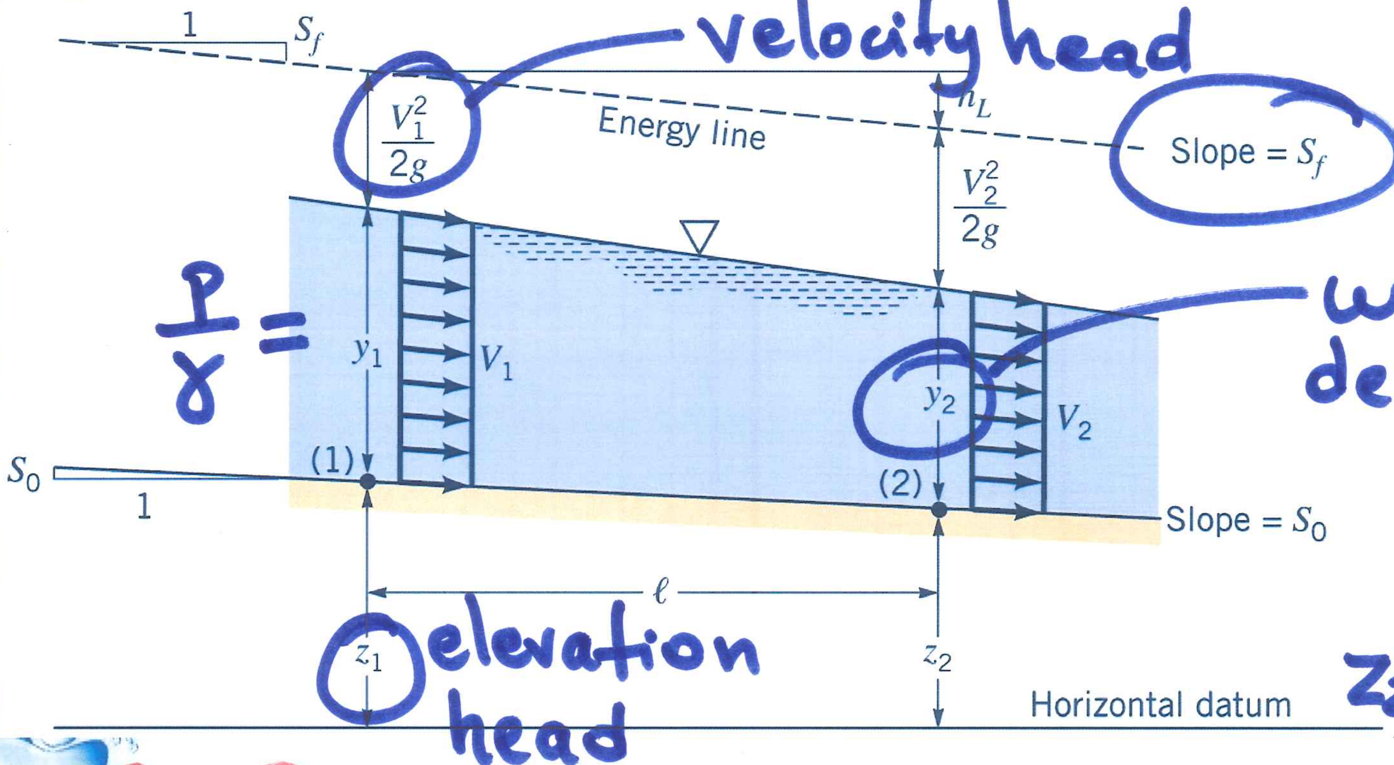
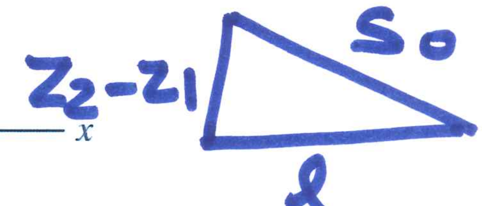
# Energy considerations

$h_L = S_f \times \ell$




$S_0 = \frac{z_2 - z_1}{\ell}$




$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\underbrace{y_1 + \frac{V_1^2}{2g}}_{E_1} + S_0\ell = \underbrace{y_2 + \frac{V_2^2}{2g}}_{E_2} + h_L$$

E : specific energy

$$\boxed{E_1 = E_2 + (S_f - S_0)\ell}$$

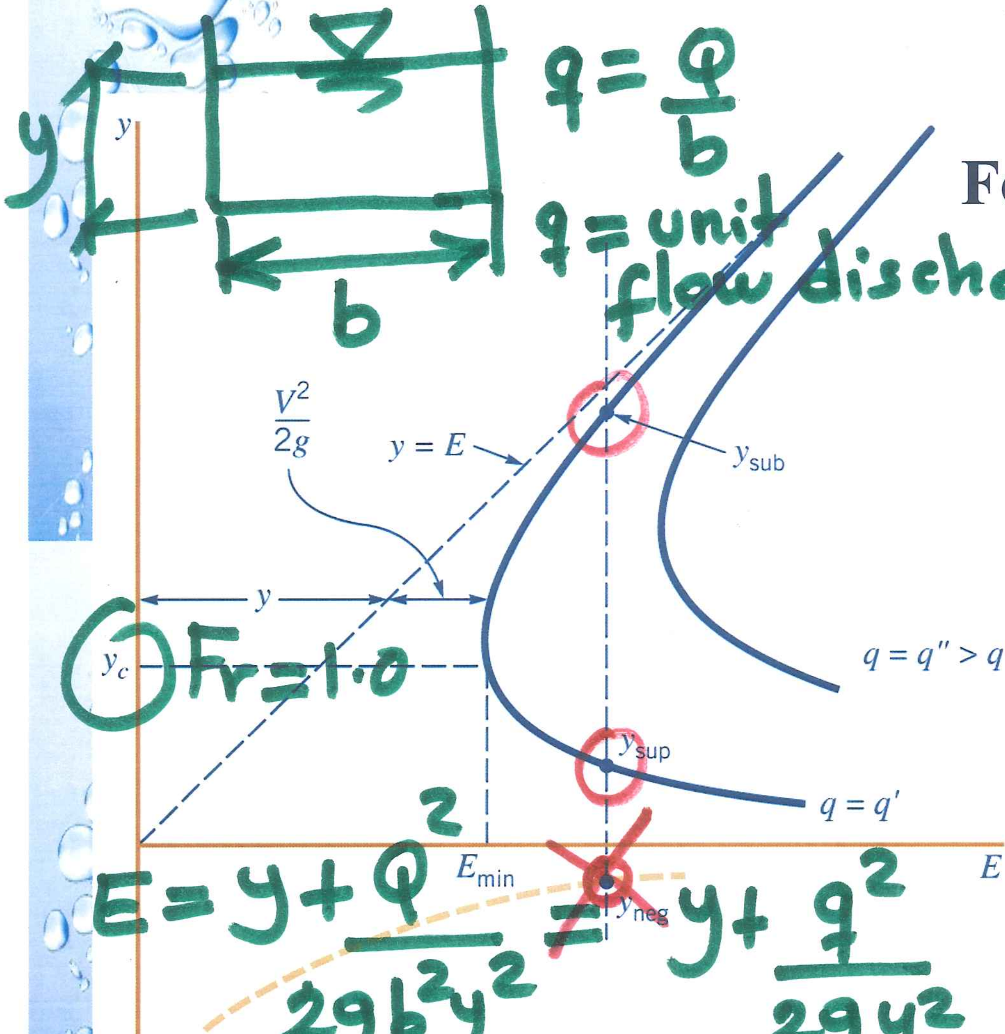
if $S_f = S_0 = 0$, $E_1 = E_2$

# Specific Energy

$$E = y + \frac{V^2}{2g}$$

**For a rectangular channel, $q = Q/b$**

$$E = y + \frac{q^2}{2gy^2}$$




$E = y + \frac{Q^2}{2gb^2y^2} = y + \frac{q^2}{2gy^2}$

Specific energy diagram

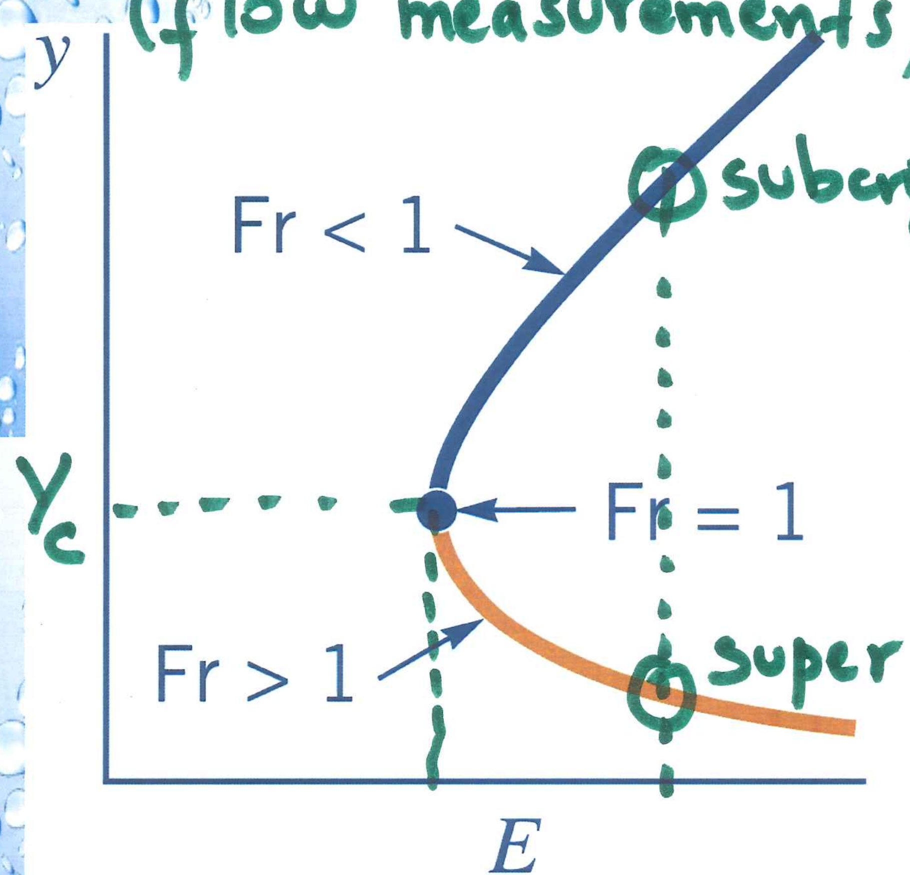
The above formula is a cubic equation with three solutions, $y_{sup}$, $y_{sub}$, and $y_{neg}$. $y_{neg}$ has no physical meaning and can be ignored. $Y_{sup}$ and $y_{sub}$ are called alternative depths.

$\left.\begin{matrix} Y_{sup} \\ Y_{sub} \end{matrix}\right\}$

# Critical depth

(flow measurements)

$$\boxed{E = y + \frac{q^2}{2gy^2}}$$




To determine $E_{min}$:

$$\boxed{\frac{dE}{dy} = 0 \text{ (Critical depth)}}$$

$$0 = 1 + \frac{q^2}{\cancel{2}g}(-\cancel{2})y_c^{-3}$$

$$\boxed{y_c = \sqrt[3]{\frac{q^2}{g}}}$$

**For a rectangular channel ($q = Q/b$)**

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$\boxed{q = \frac{Q}{b}}$$

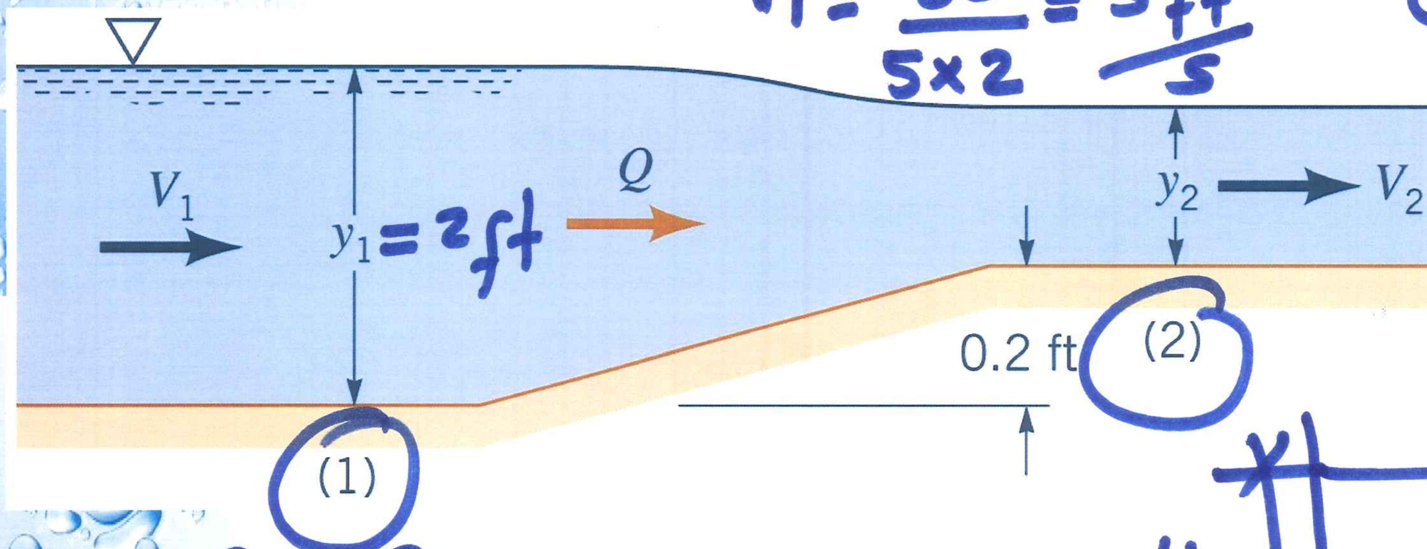
Note that the critical conditions (Fr=1) occur at the location of $E_{min}$.

# Example of application:

Water flows in a 5-ft-wide rectangular channel with a flowrate of $Q$ = 30 $ft^3/s$ and an upstream depth of $y_1$ = 2 ft as is shown in the figure below. Determine the flow depth and the surface elevation at section (2).

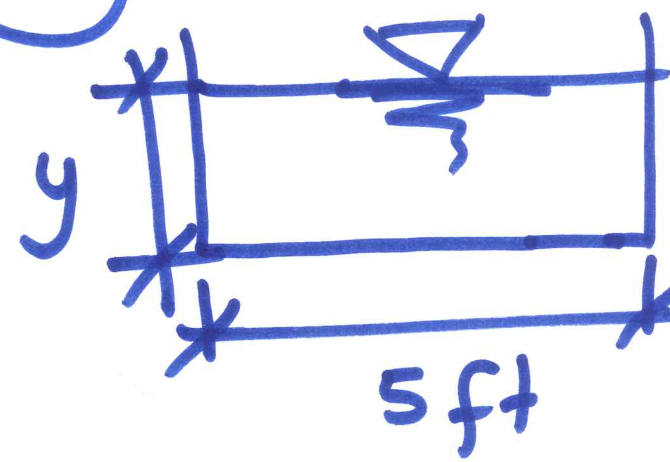
$V_1 = \frac{30}{5\times 2} = 3\ ft/s$

$Q = 30\ ft^3/s$




$y_2 = ?$

$y_2 + 0.2\ ?$




Energy (1) - (2)

$$y_1 + \frac{V_1^2}{2g} + z_1 = y_2 + \frac{V_2^2}{2g} + z_2$$

$$2 + \frac{3^2}{2\times 32.2} + 0 = y_2 + \frac{V_2^2}{2g} + 0.2 \quad (1)$$

$$Q = V_2 \, b \, y_2$$

$$30 = V_2 (5) y_2 \rightarrow \boxed{V_2 = \frac{6}{y_2}} \quad (2)$$

(2) in (1)

$$2 + \frac{9}{64.4} = y_2 + \frac{36}{29 y_2^2} + 0.2$$

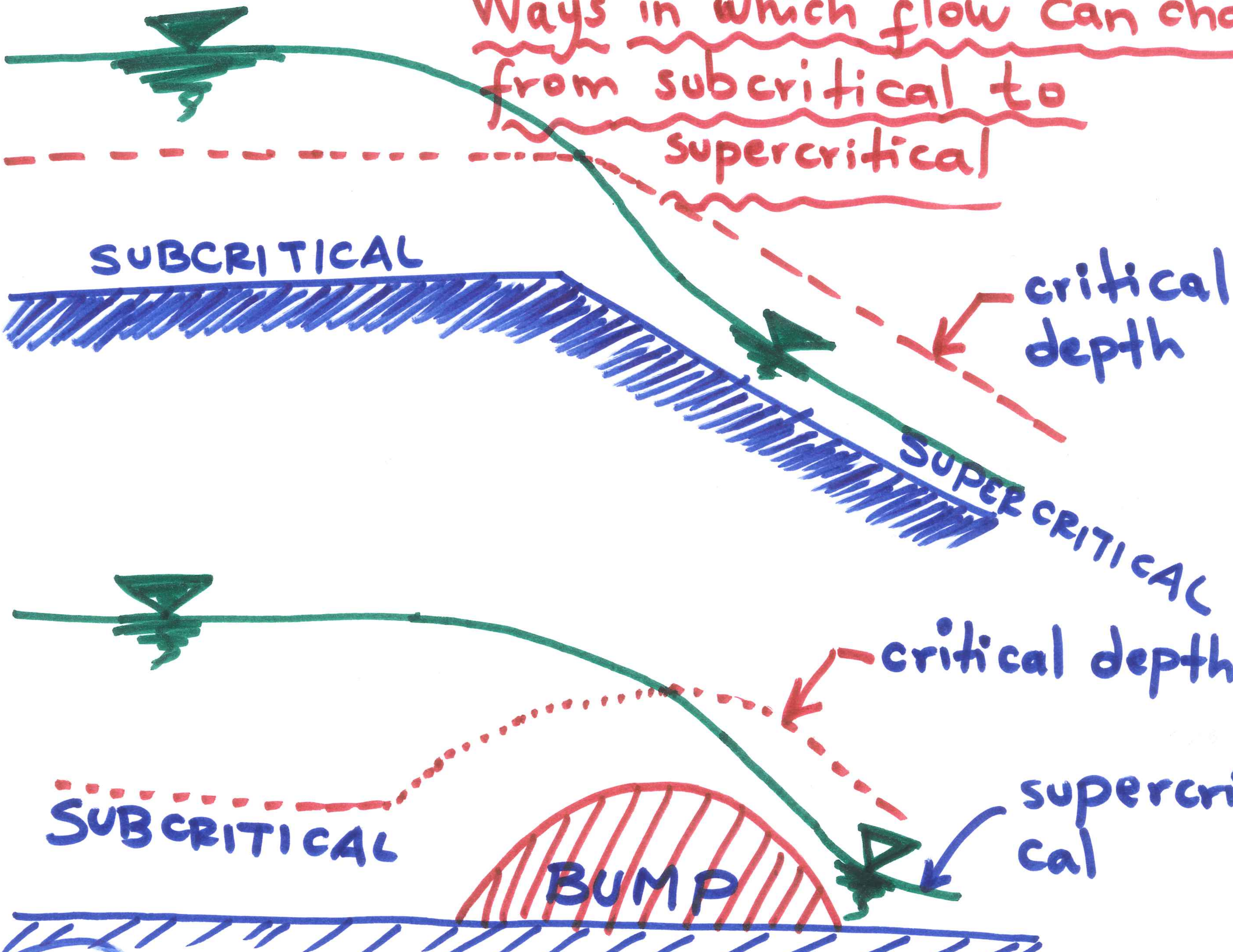
$$y_2^3 - 1.94 y_2^2 + 0.559 = 0$$

$$y_{2_1} = 1.774 \text{ ft}, \quad y_{2_2} = 0.632 \text{ ft}, \quad y_{2_3} = -0.632$$

$$Fr_1 = \frac{3}{\sqrt{32.2 \times 2}} = 0.374 < 1 \quad (\text{subcritical flow})$$

$$\boxed{y_2 = 1.774 \text{ ft}}$$ flow stays subcritical

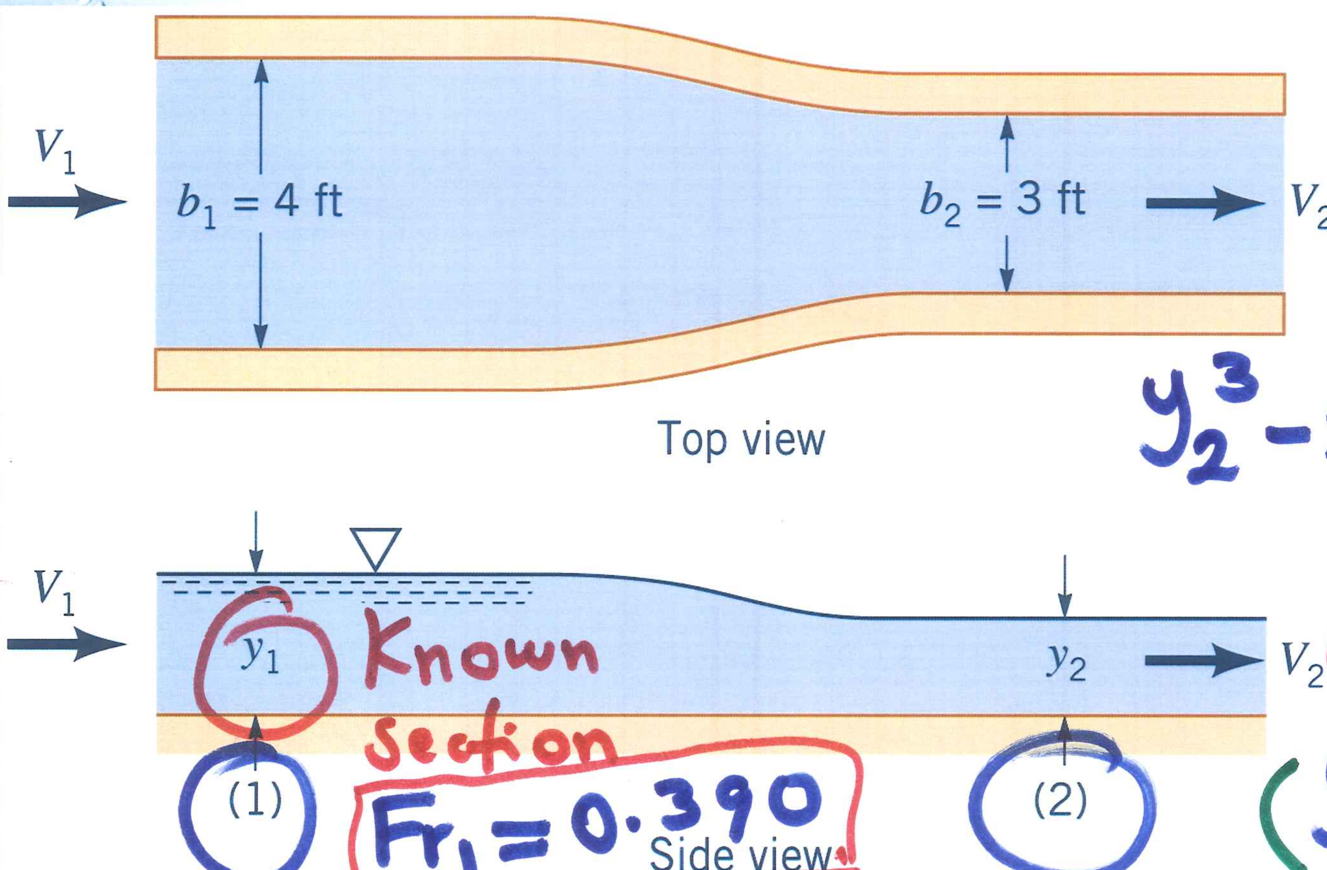
Ways in which flow can change from subcritical to supercritical




16-2

# Example of application (P10.24):

Water in a rectangular channel flows into a gradual contraction section as is indicated in Fig. P10.24. If the flowrate is $Q = 25\ ft^3/s$ and the upstream depth is $y_1 = 2$ ft, determine the downstream depth, $y_2$




Known section

$\boxed{Fr_1 = 0.390}$ → subcritical flow

$$y_2^3 - 2.15 y_2^2 + 1.077 = 0$$

sub

$(y_{2_1} = 1.828\ ft)$

super

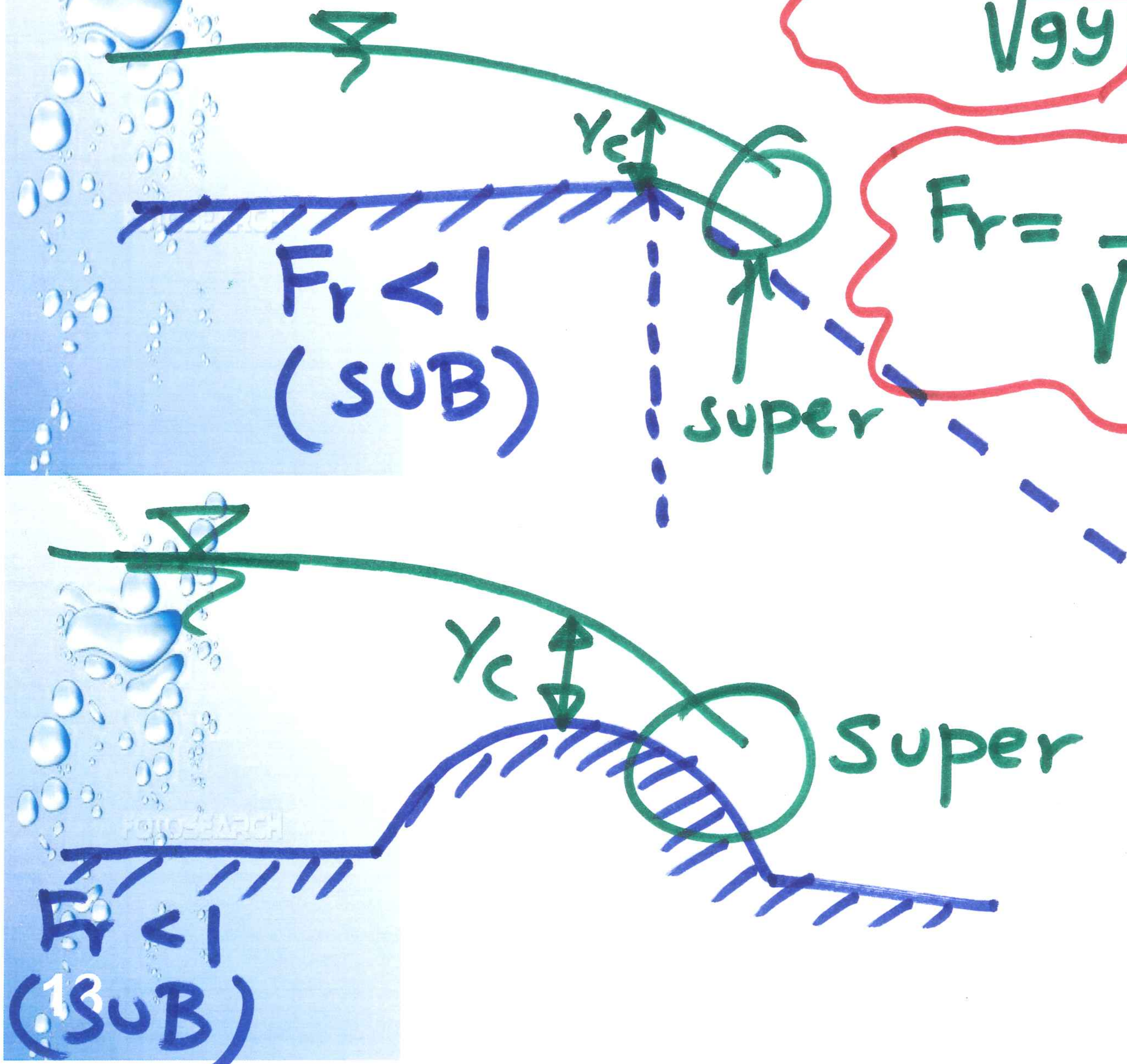
$(y_{2_2} = 0.946\ ft)$

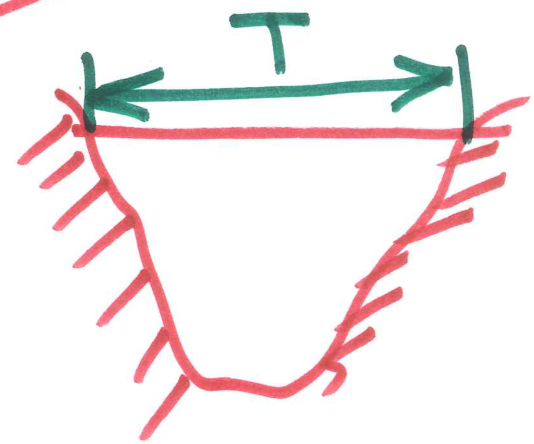
$\times\ y_{2_3} = -0.623\ ft$

$$F_r = \frac{V}{\sqrt{gy}}$$

Rectangular channels or wide channels

$$F_r = \frac{V}{\sqrt{g\frac{A}{T}}}$$



# Lecture 13, 02/14/2014
# Uniform depth channel Flow

$$\frac{dy}{dx} = 0$$

$$\frac{dv}{dx} = 0$$

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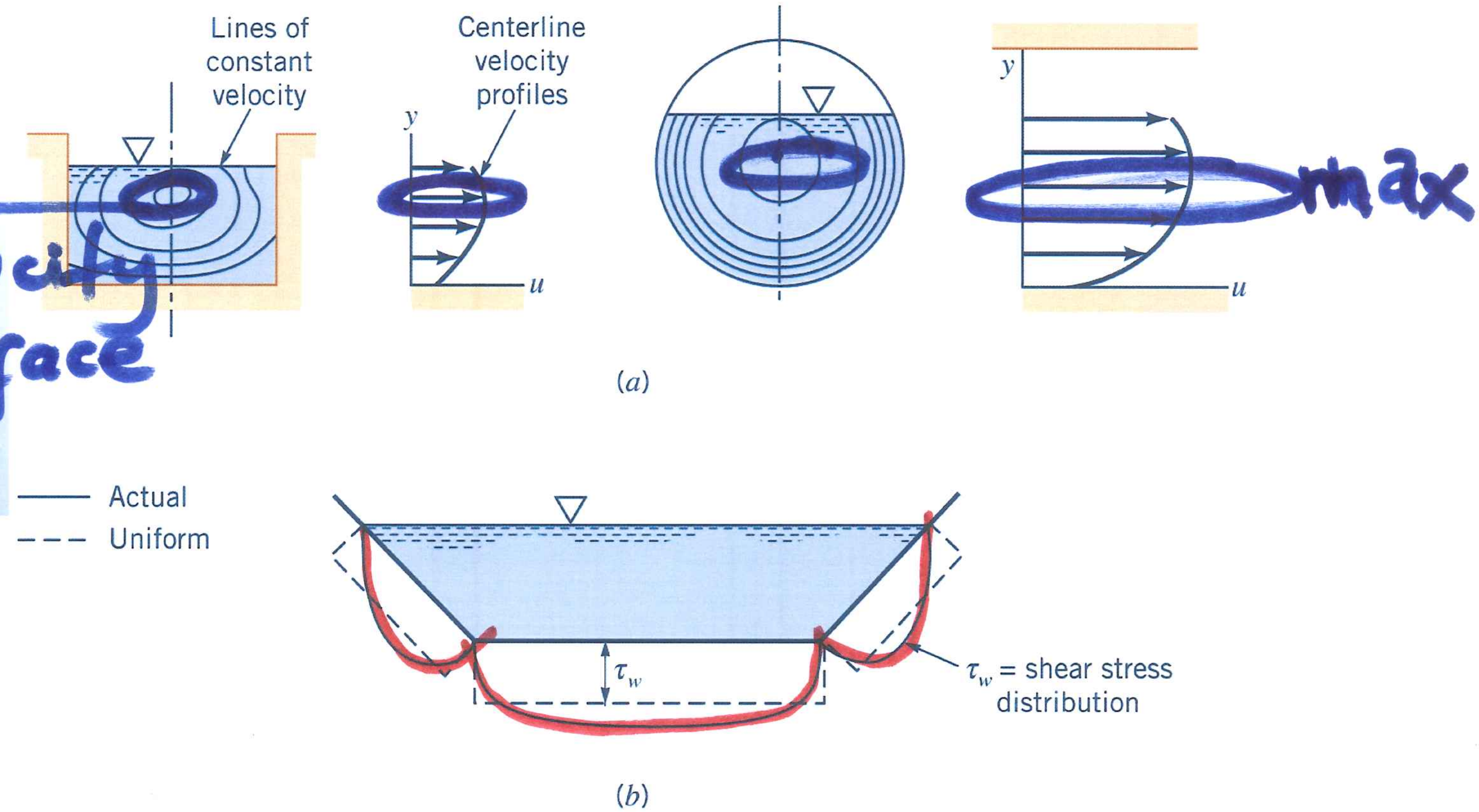
# Uniform depth channel Flow



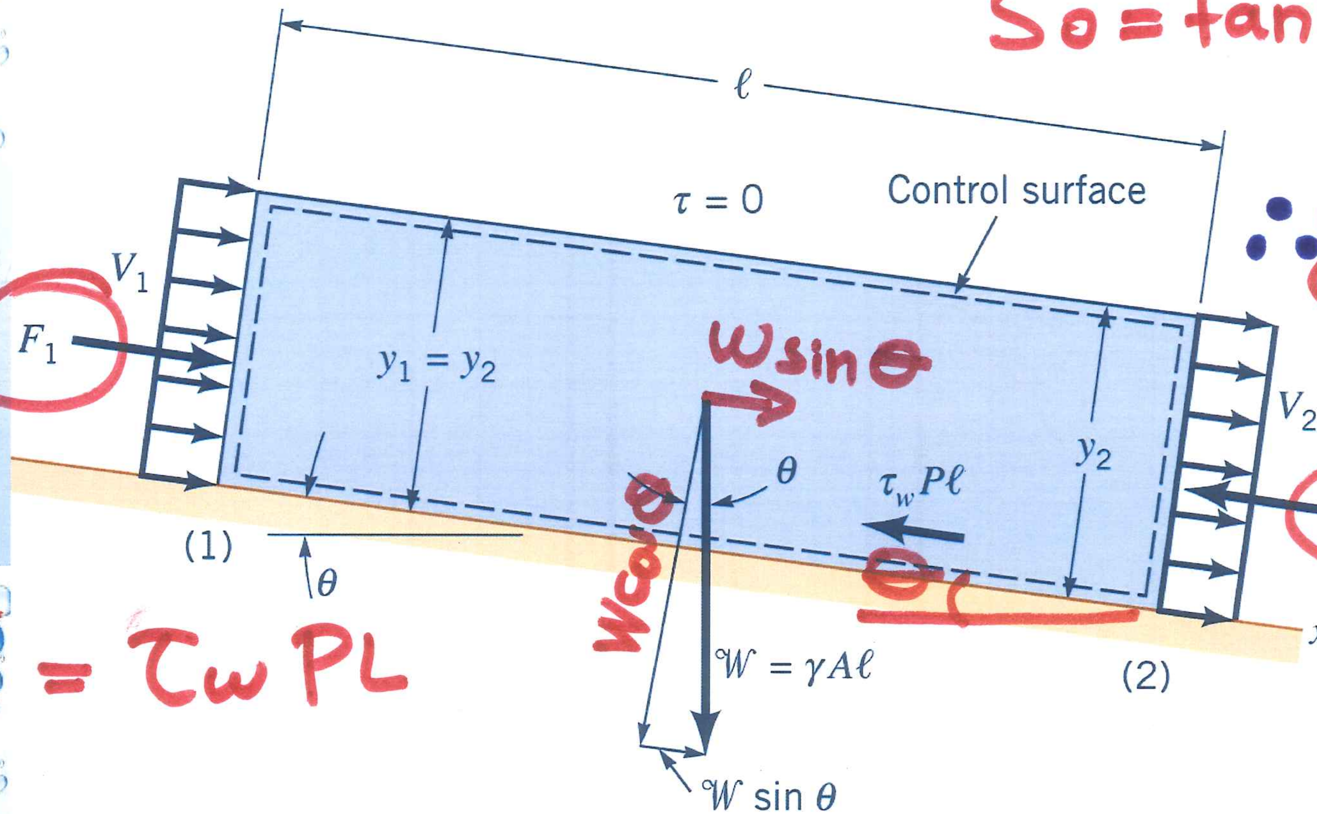

Typical velocity and shear stress distributions in an open channel: (a) velocity distribution throughout the cross section. (b) shear stress distribution on the wetted perimeter.

# The Chezy & Manning Equation

$\theta$ is small

$S_o = \tan\theta = \frac{\sin\theta}{\cancel{\cos\theta}}$, $\cos\theta \approx 1$

$\therefore \boxed{S_o = \sin\theta}$




Control volume for uniform flow in an open channel.

Shear force = $\tau_w P L$

Pressure force $F_1 = F_2$

$\gamma A \cancel{L} S_o = \tau_w P \cancel{L}$

$$\tau_w = \frac{\gamma A \ell S_0}{P\ell} = \gamma R_h S_0$$

$\tau_w = \gamma \underbrace{\frac{A}{P}}_{R} S_o \rightarrow \boxed{\tau_w = \gamma R_h S_o}$

$S_o$ = slope

$\gamma$ = specific weight

# The Chezy & Manning Equation

Chezy equation was developed in 1768 by A. Chezy (1718-1798), a French engineer who designed a canal for the Paris water supply.

$$V = C\sqrt{R_h S_0}$$ **Chezy equation**

used mostly in Europe

Robust (discharge is not significantly reduced due to change in roughness at channel perimeter).

Where $C$ is the Chezy coefficient

It is relatively robust

# The Chezy & Manning Equation

In 1889, R. Manning (1816-1897), an Irish engineer, developed a similar equation to what is shown below.

$$V = \frac{\kappa}{n} R_h^{2/3} S_0^{1/2}$$

**Manning equation**

used in US and most of the countries

not robust (sensitive to changes in roughness at channel perimeter).

Where $\kappa = 1$ if SI units are used, $\kappa = 1.49$ if English units are used.

$n$ is the Manning resistance coefficient. Depeds on surface material of the channel. It has the units of $s/m^{1/3}$ or $s/ft^{1/3}$

# Value of the Manning Coefficient

■ Table 10.1

Values of the Manning Coefficient, $n$ (Ref. 6)

$y > 10 K_s$  $y$ = water depth

$K_s$, roughness

| Wetted Perimeter | $n$ | Wetted Perimeter | $n$ |
|---|---|---|---|
| A. *Natural channels* | | D. *Artificially lined channels* | |
| Clean and straight | 0.030 | Glass | 0.010 |
| Sluggish with deep pools | 0.040 | Brass | 0.011 |
| Major rivers | 0.035 | Steel, smooth | 0.012 |
| B. *Floodplains* | | Steel, painted | 0.014 |
| Pasture, farmland | 0.035 | Steel, riveted | 0.015 |
| Light brush | 0.050 | Cast iron | 0.013 |
| Heavy brush | 0.075 | Concrete, finished | 0.012 |
| Trees | 0.15 | Concrete, unfinished | 0.014 |
| C. *Excavated earth channels* | | Planed wood | 0.012 |
| Clean | 0.022 | Clay tile | 0.014 |
| Gravelly | 0.025 | Brickwork | 0.015 |
| Weedy | 0.030 | Asphalt | 0.016 |
| Stony, cobbles | 0.035 | Corrugated metal | 0.022 |
| | | Rubble masonry | 0.025 |

# The best hydraulic cross section

The best hydraulic cross section is defined as the section of minimum area ($A$) for a given flowrate $Q$, slope, $S_o$, and the roughness coefficient, $n$.

$R_h = \frac{A}{P}$

$A = \text{constant}$, $Q = \max$

$$Q = \frac{\kappa}{n} A R_h^{2/3} S_0^{1/2}$$

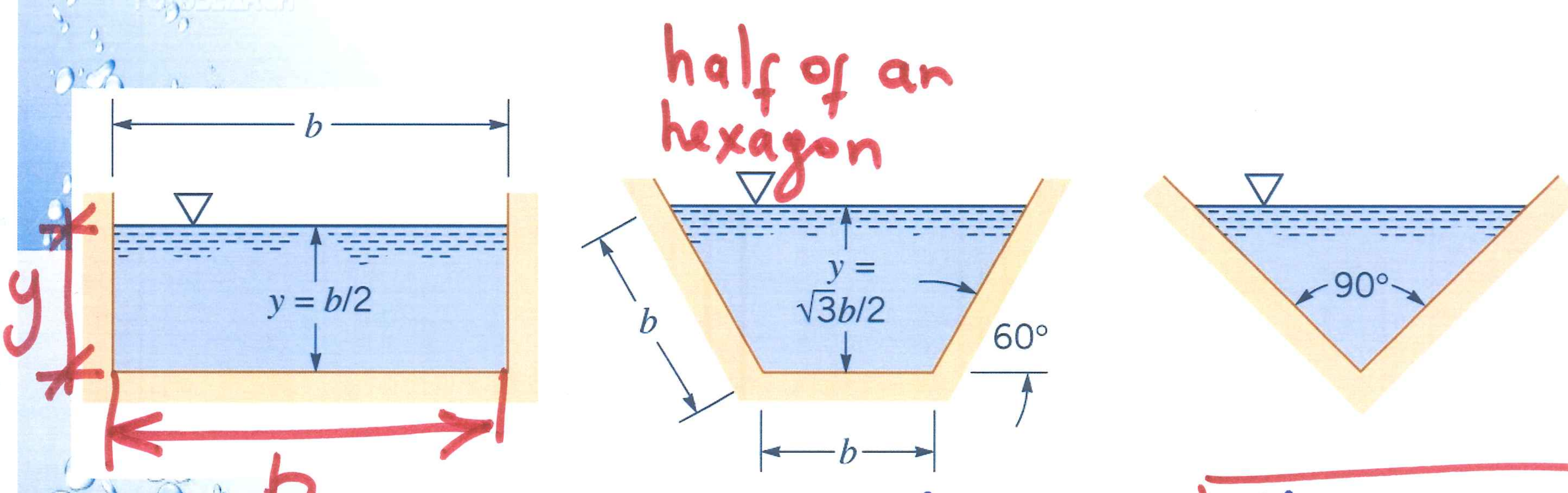
$$Q = C_1 A R_h^{2/3}$$

$$Q = C_1 \frac{A^{5/3}}{P^{2/3}}$$

$Q_{max}$ if $P_{min}$

A channel with minimum $A$ is one with a minimum $P$

# The best hydraulic cross-section for various shapes




$A = by$ (constant)

$P = b + 2y$ (minimize P)

$b + y(-2) = 0$

$\boxed{b = 2y}$

$\frac{dP}{dy} = 0 \quad \boxed{\frac{db}{dy} + 2 = 0}$ ①

$\frac{dA}{dy} = 0, \quad \boxed{b + y\frac{db}{dy} = 0}$ ②

# Example of application (P10.42):

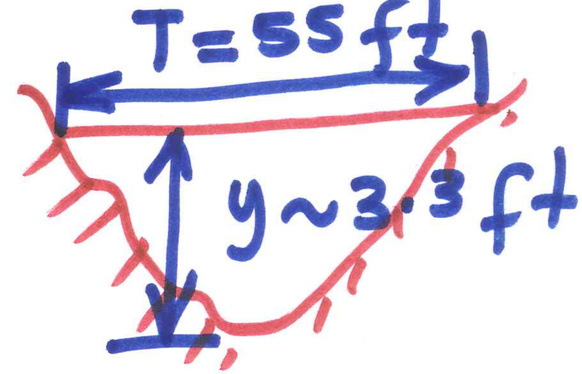
The following data are obtained for a particular reach of the Provo River in Utah: $A$ = 183 ft$^2$, free-surface width = 55 ft, average depth = 3.3 ft, $R_h$ = 3.32 ft, $V$ = 6.56 ft/s, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine (a) the average shear stress on the wetted perimeter, (b) the Manning coefficient, $n$, and (c) the Froude number of the flow.

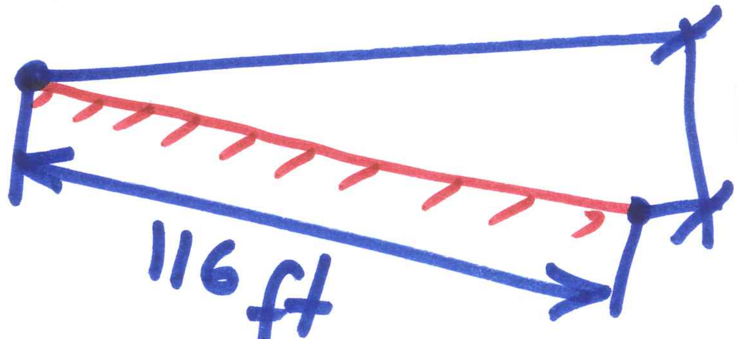
$A = 183\ ft^2$

$R_h = 3.32\ ft$

$V = 6.56\ ft/s$

$L = 116\ ft$




$S_o = \frac{1.04}{116}$

$S_o = 0.00897$

a) $\tau_w = \gamma R_h S_o$

$\tau_w = 62.4 \frac{lb}{ft^3} \times 3.32\ ft \times 0.00897$

$$\tau_w = 1.80 \ \frac{lb}{ft^2}$$

b) $$V = \frac{k}{n} \times R_h^{2/3} S_o^{1/2}$$

$k = 1.49$ (English)

$k = 1.0$ (SI)

$$n = \frac{1.49 \times R_h^{2/3} S_o^{1/2}}{V} = \frac{1.49 \times 3.32^{2/3} \times 0.00897}{6.56}$$

$$n = 0.0469$$

c) $$F_r = \frac{V}{\sqrt{g\frac{A}{T}}} = \frac{6.56}{\sqrt{32.2 \times \frac{183}{55}}} = 0.63$$

Subcritical flow

# Example of application (P10.61):

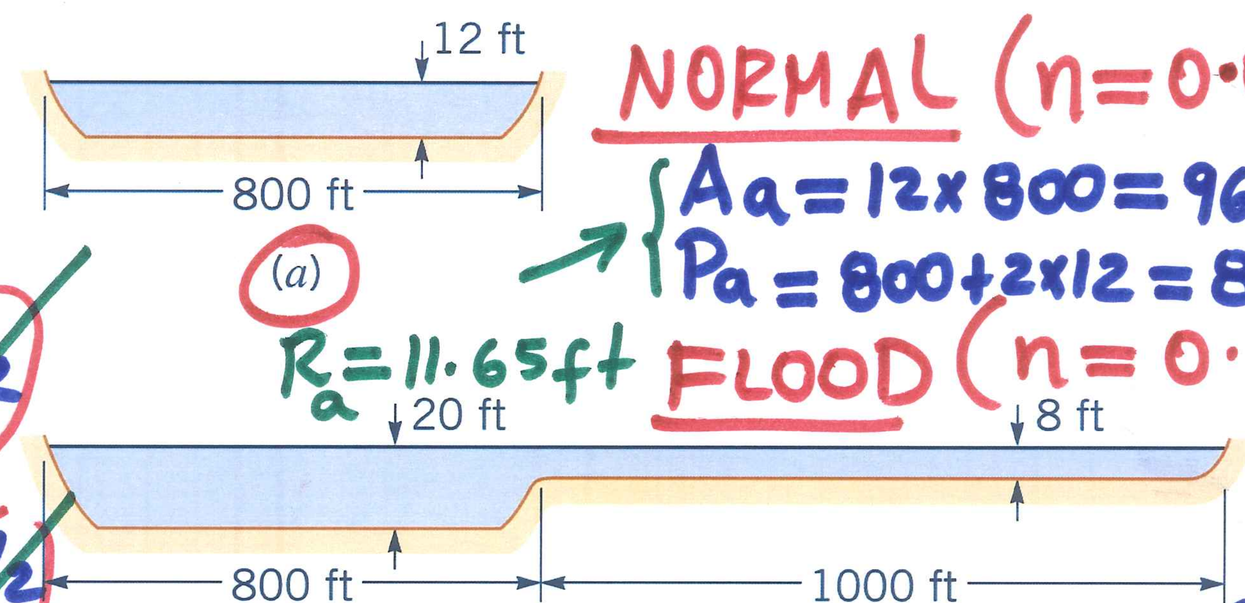
At a given location, under normal conditions a river flows with a Manning coefficient of 0.030, and a cross section as indicated in Fig. P10.61a. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of tress and brush in the floodplain) and a cross section as shown in Fig. P10.61b. Determine the ratio of the flowrate during flood conditions to that during normal conditions.

$\frac{Q_b}{Q_a} = ?$

$Q_a = \frac{k}{n} A_a R_a^{2/3} \cancel{S_o^{1/2}}$

$Q_b = \frac{k}{n} A_b R_b^{2/3} \cancel{S_o^{1/2}}$

these change




NORMAL $(n = 0.03)$

$A_a = 12 \times 800 = 9600\,ft^2$

$P_a = 800 + 2 \times 12 = 824\,ft$

$R_a = 11.65\,ft$

FLOOD $(n = 0.04)$

$A_b = 24{,}000\,ft^2$

$P_b = 20 + 800 + 20 + 1000 = 1840\,ft$

$R_b = 13.04\,ft$

$$\frac{Q_b}{Q_a} = \frac{n_a}{n_b} \frac{A_b R_b^{2/3}}{A_a R_a^{2/3}} = \frac{0.03}{0.04} \times \frac{24{,}000}{9{,}600} \times \frac{13.04^{2/3}}{11.65^{2/3}}$$

$$\boxed{\frac{Q_b}{Q_a} = 2.02}$$

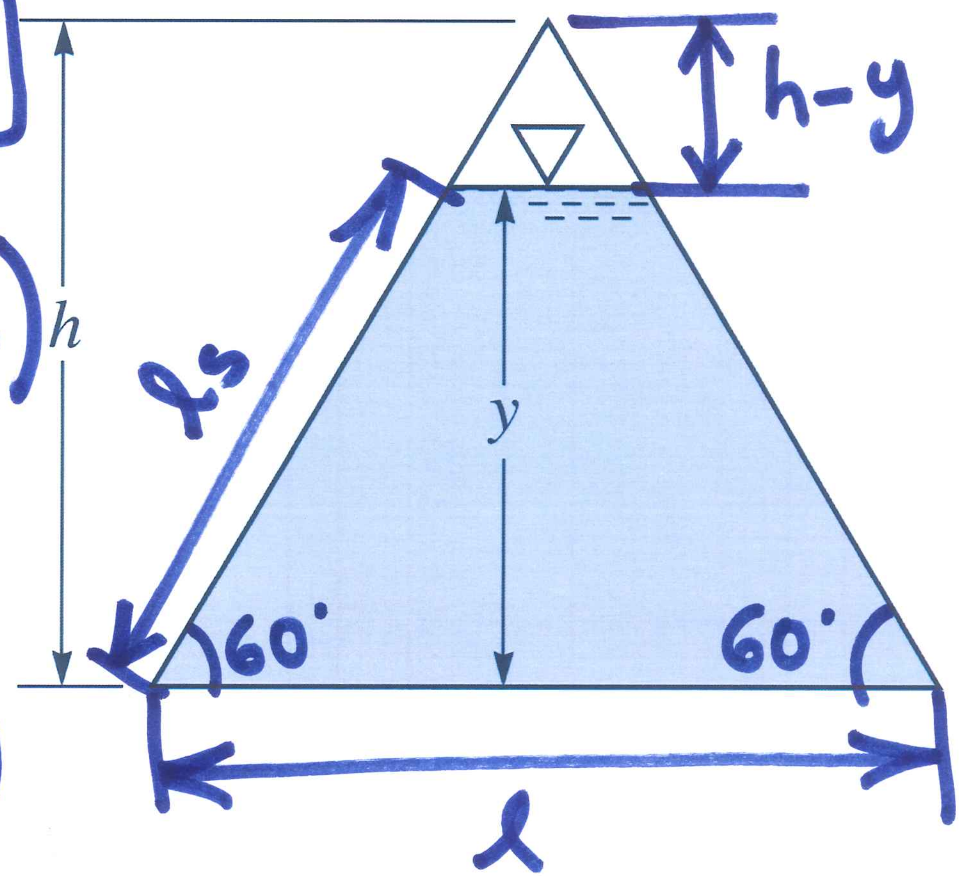
# Example of application (P10.79):

Water flows in a channel with an equilateral triangle cross section as shown in Fig. P10.79. For a given Manning coefficient, *n*, and channel slope, determine the depth that gives the maximum flowrate.

$$A = \frac{1}{\tan 60^\circ}\left[h^2 - (h-y)^2\right]$$

$$P = 2\left(\frac{h}{\tan 60^\circ} + \frac{y}{\sin 60^\circ}\right)$$

$$\therefore R_h = \frac{2hy - y^2}{2\left(h + \frac{y}{\cos 60^\circ}\right)}$$



Then :

$$Q = \frac{k}{n} \frac{1}{\tan 60^\circ} (2hy - y^2) \left[ \frac{2hy - y^2}{2\left(h + \frac{y}{\cos 60^\circ}\right)} \right]^{2/3} S_o^{1/2}$$

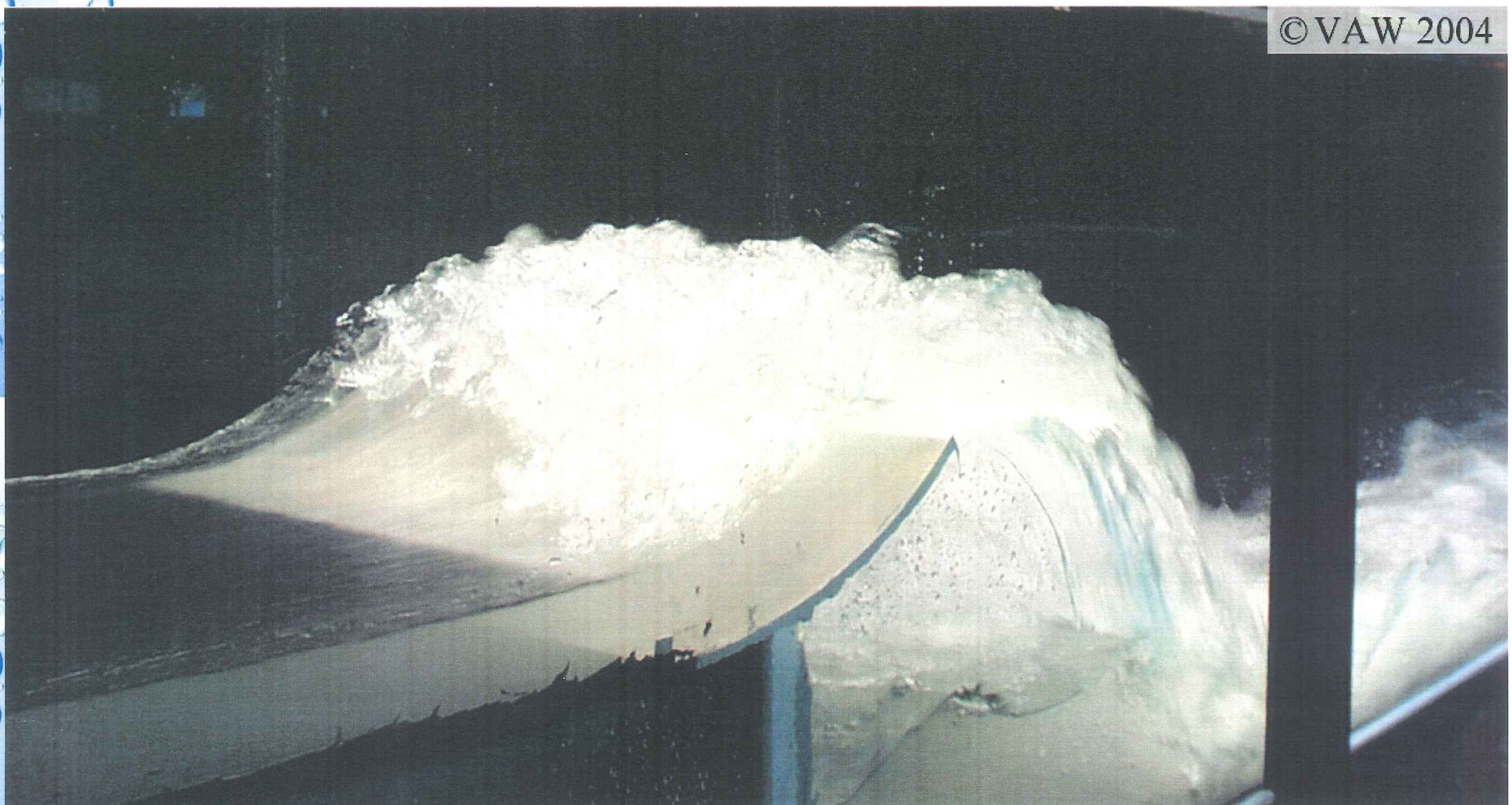
For max. flow rate $\frac{dQ}{dy} = 0$

This gives : $8\left(\frac{y}{h}\right)^2 - \left(\frac{y}{h}\right) - 5 = 0$

$$\therefore \frac{y}{h} = \frac{1 \pm \sqrt{1 + 4(8)(5)}}{16} \Rightarrow \boxed{\frac{y}{h} = -0.73} \quad \boxed{\frac{y}{h} = 0.86}$$

Negative root has no physical meaning. Thus, $\boxed{y = 0.86\, h}$

# Lecture 14, 02/19/2014
# Rapidly Varied Flow

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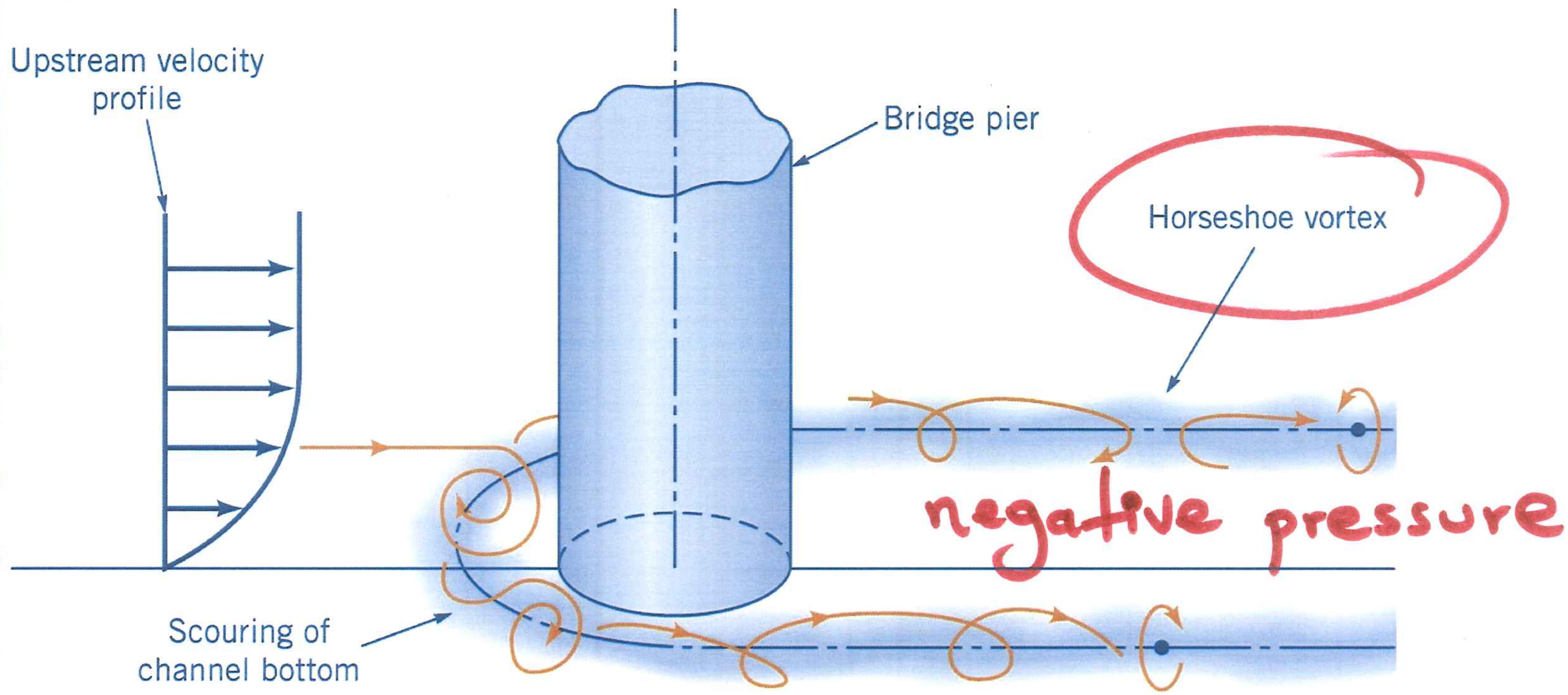
# Example of Rapidly Varied Flow

The scouring of a river bed in the neighborhood of a bridge pier

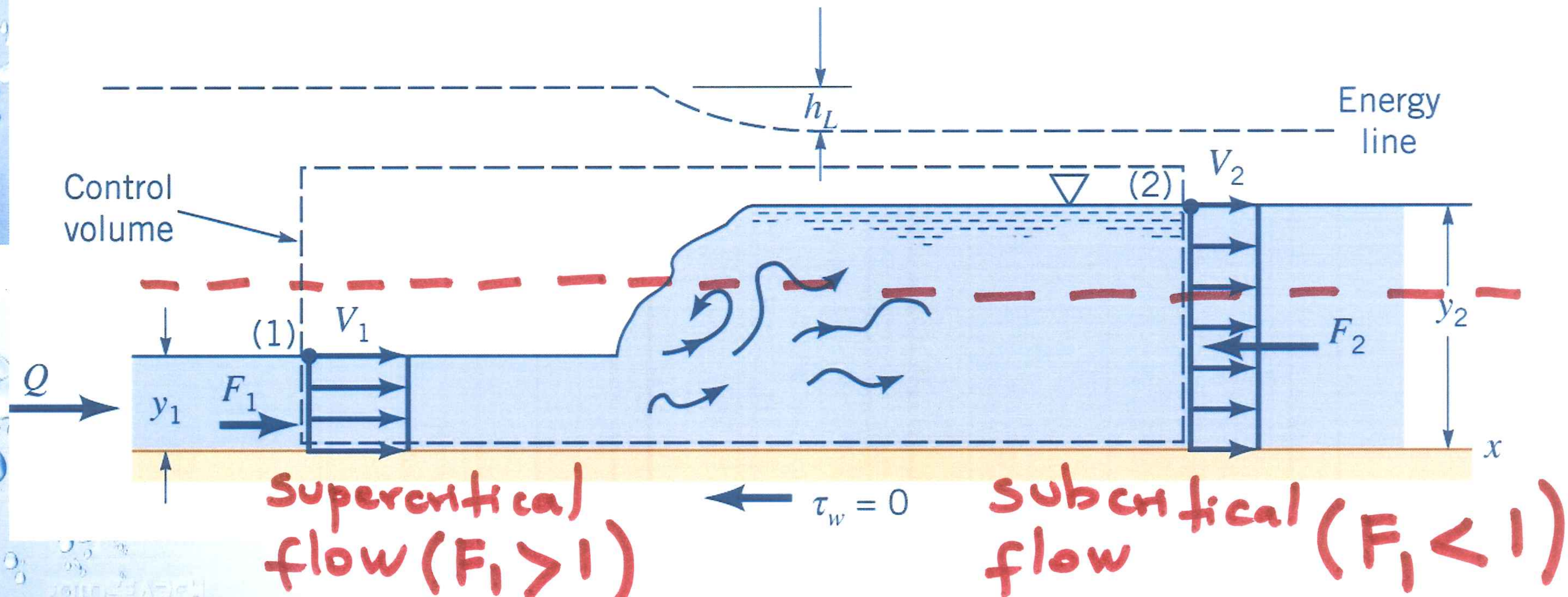



complex three-dimensional flow structure around a bridge pier

# The Hydraulic Jump

# The Hydraulic Jump

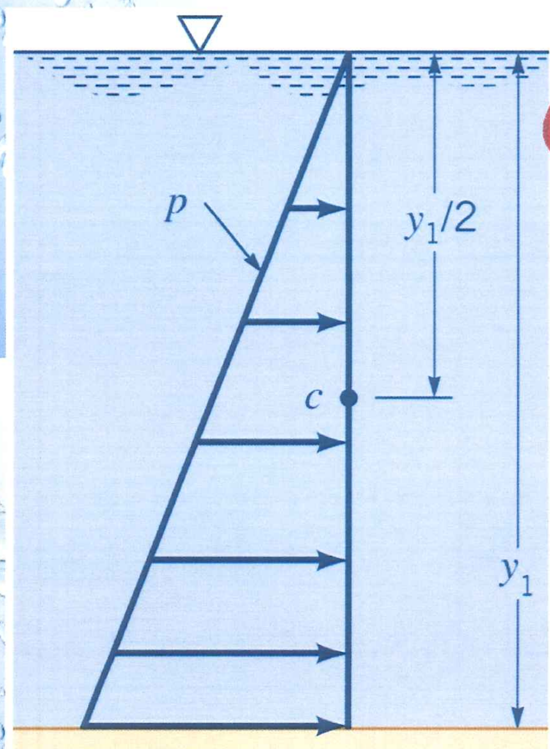
Hydraulic jump in a horizontal, rectangular channel




Assume that the flow at sections (1) and (2) is nearly uniform, steady, and one-dimensional

# The Hydraulic Jump

Hydraulic jump in a horizontal, rectangular channel




$$\frac{y_1^2}{2} - \frac{y_2^2}{2} = \frac{V_1 y_1}{g}(V_2 - V_1)$$

*x*-momentum equation

$$y_1 b V_1 = y_2 b V_2 = Q$$

Conservation of mass

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + h_L$$

Energy equation

The head loss is due to the violent turbulent mixing and dissipation

# The Hydraulic Jump

$y_2$: downstream depth

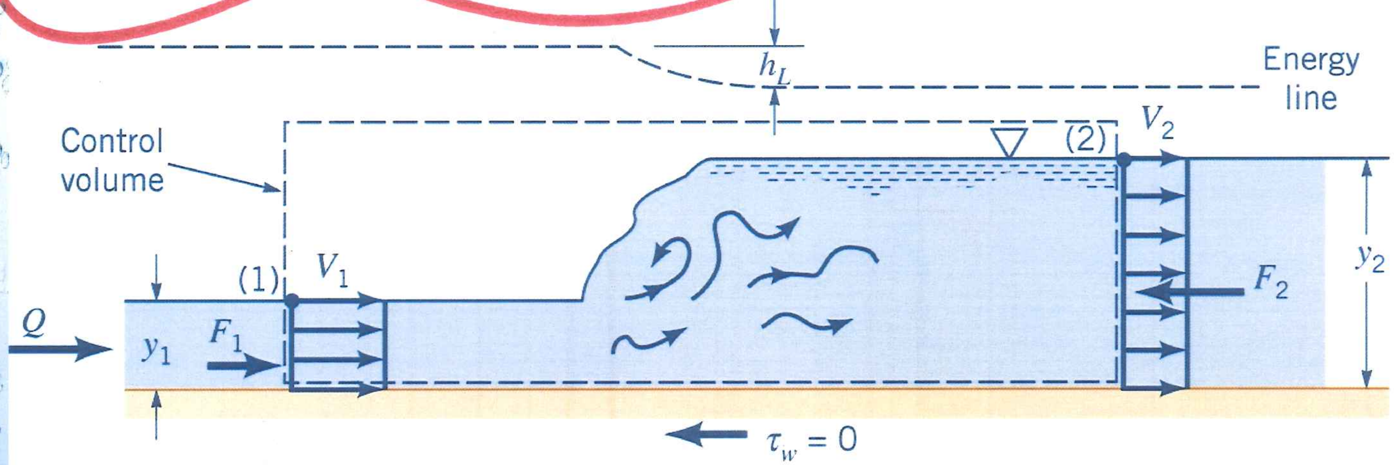
$y_1$: upstream depth

$Fr_1$: upstream Froude number

$h_L$: head loss

$$\frac{y_2}{y_1} = \frac{1}{2}\left(-1 + \sqrt{1 + 8\mathrm{Fr}_1^2}\right)$$

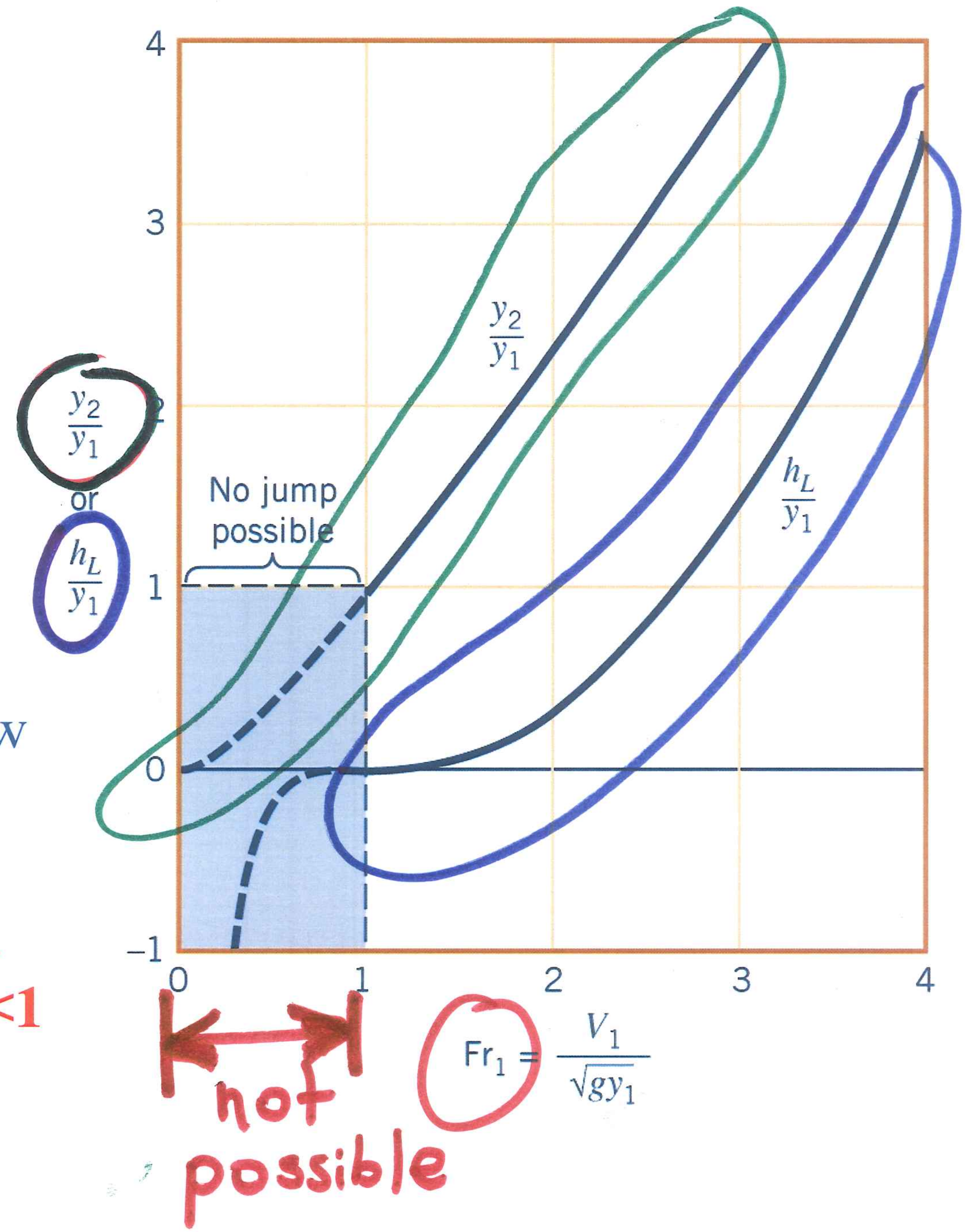
$$\frac{h_L}{y_1} = 1 - \frac{y_2}{y_1} + \frac{\mathrm{Fr}_1^2}{2}\left[1 - \left(\frac{y_1}{y_2}\right)^2\right]$$



# The Hydraulic Jump

The head loss is negative if $Fr_1<1$ (Violate the second law of thermodynamics)

**Not possible to produce a hydraulic jump with $Fr_1<1$**

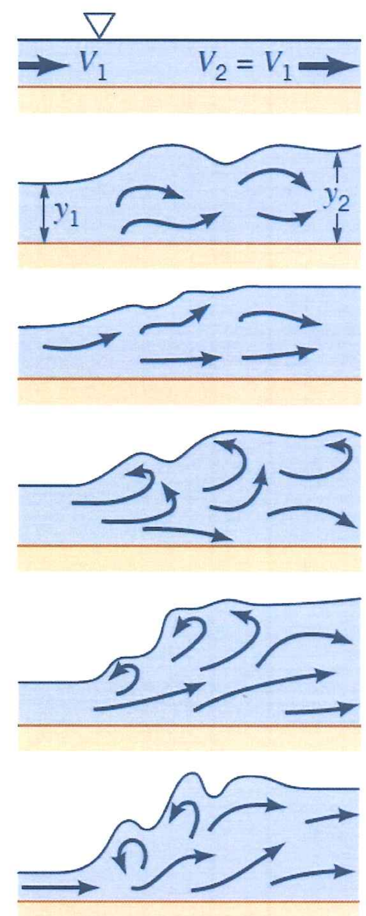


# Classification of Hydraulic Jumps

**Table 10.2**

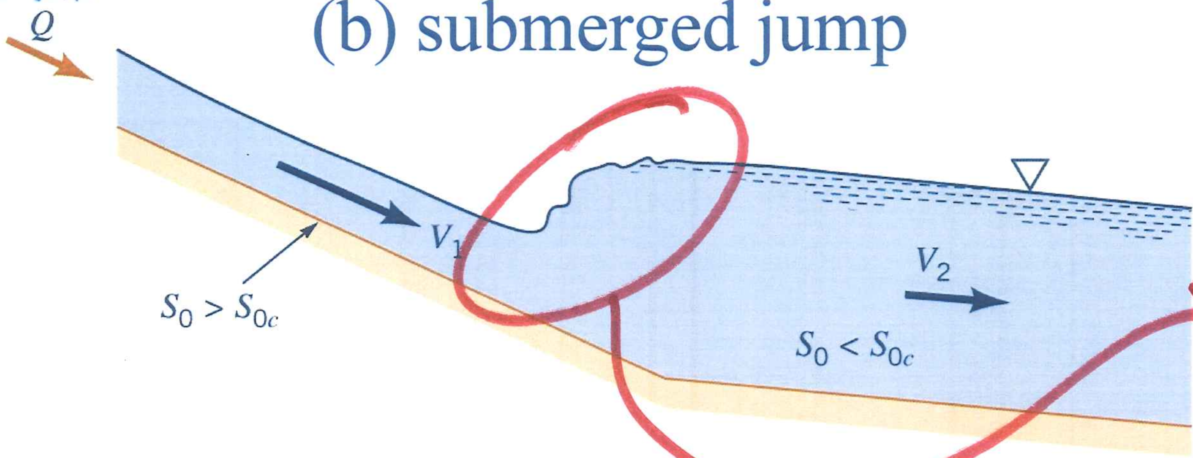
**Classification of Hydraulic Jumps (Ref. 12)**

| $Fr_1$ | $y_2/y_1$ | Classification | Sketch |
|---|---|---|---|
| <1 | 1 | Jump impossible | |
| 1 to 1.7 | 1 to 2.0 | Standing wave or undulant jump (stationary wave) | |
| 1.7 to 2.5 | 2.0 to 3.1 | Weak jump | |
| 2.5 to 4.5 | 3.1 to 5.9 | Oscillating jump | |
| 4.5 to 9.0 | 5.9 to 12 | Stable, well-balanced steady jump; insensitive to downstream conditions | |
| >9.0 | >12 | Rough, somewhat intermittent strong jump | |



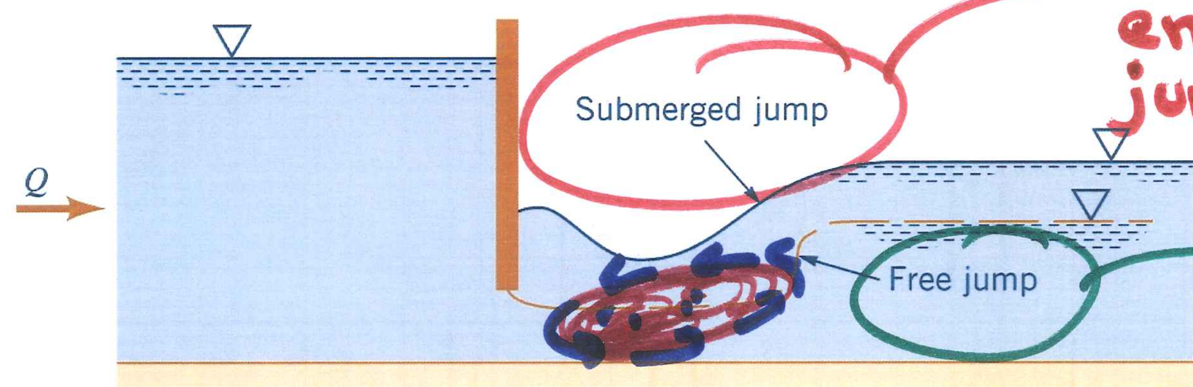
# Hydraulic Jump Variations

(a) jump caused by a change in channel slope,
(b) submerged jump




(a)

free hydraulic jump




(b)

tailwater is high enough to submerge jump

tailwater doesn't submerge jump

# Example of application (P10.97):

Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in Fig. 10.97. Consider a situation where a jump forms 3.0 in from the center of the plate with depths upstream and downstream of the jump of 0.05 in and 0.20 in, respectively. Determine the flow rate from the faucet.

$Q = ??$

$y_1 = 0.05 \text{ in}$

$y_2 = 0.20 \text{ in}$

$$\frac{0.20}{0.05} = \frac{1}{2}\left(-1+\sqrt{1+8Fr_1^2}\right)$$

$$\boxed{Fr_1 = 3.16}$$

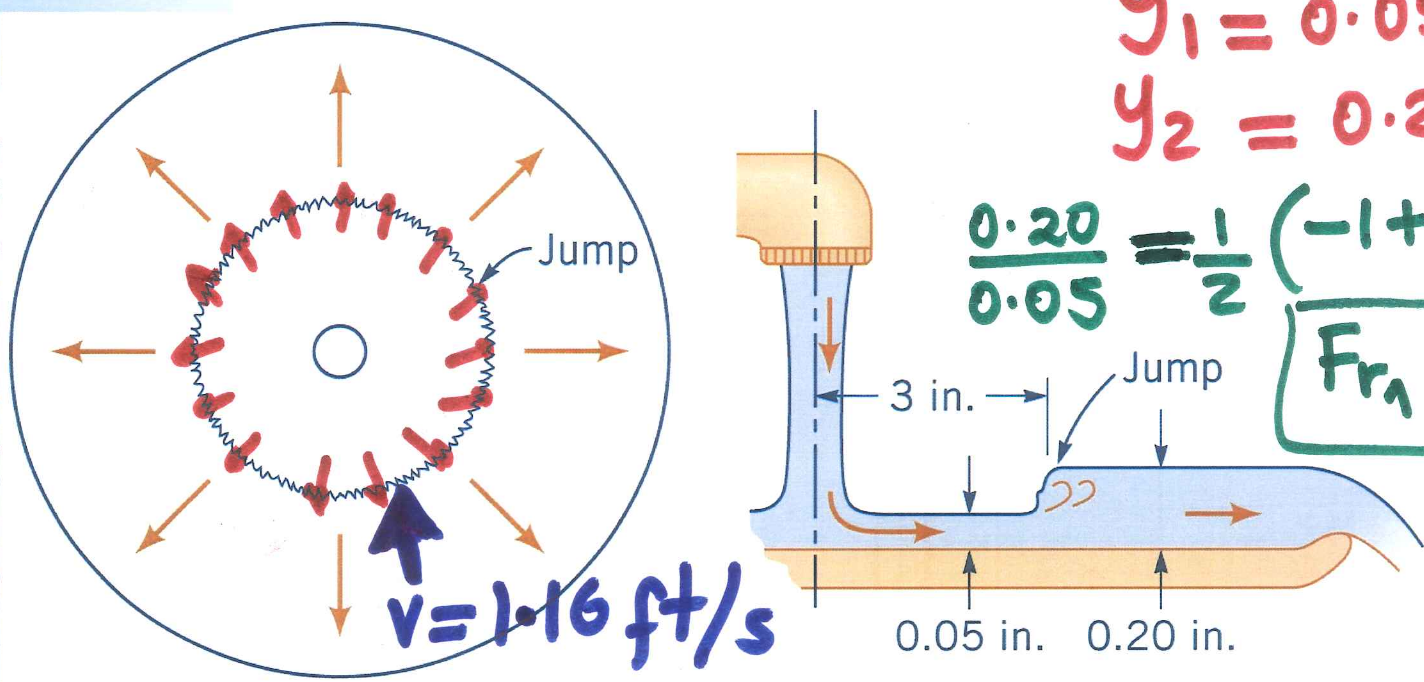



Figure P10.97

$$Fr_1 = \frac{V}{\sqrt{g y_1}} = 3.16 \rightarrow \boxed{V_1 = 1.16\ ft/s}$$

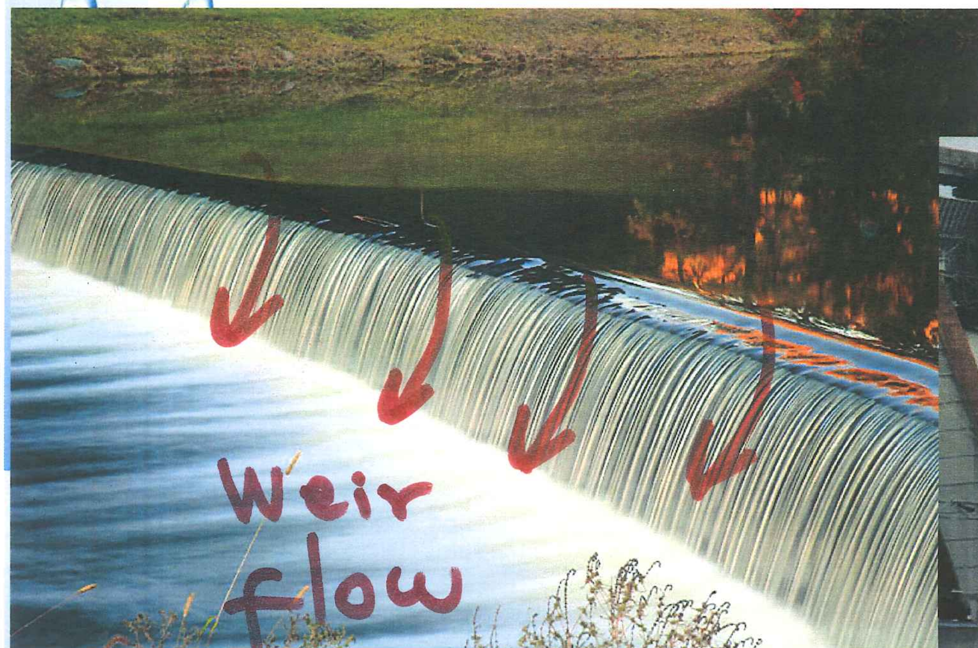
$$Q = V \cdot A$$

$$P = 2\pi R$$

$$Q = 1.16 \times 2\pi\left(\frac{3}{12}\right) \times \left(\frac{0.05}{12}\right) \quad A = P \times y_1$$

$$Q = 0.00759\ \frac{ft^3}{s}$$

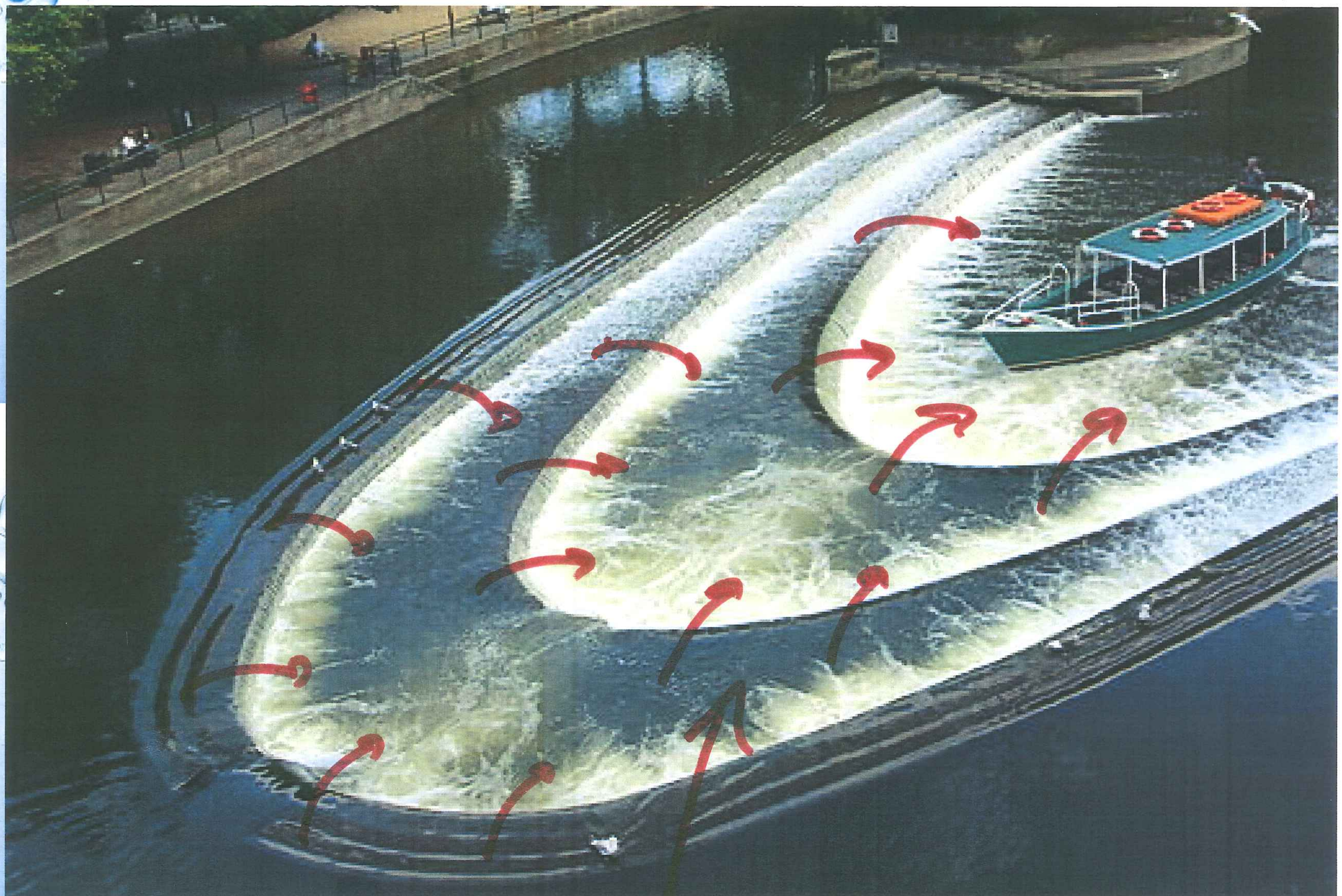
# Lecture 15, 02/21/2014
# Weirs and Gates





Arturo Leon, Oregon State University

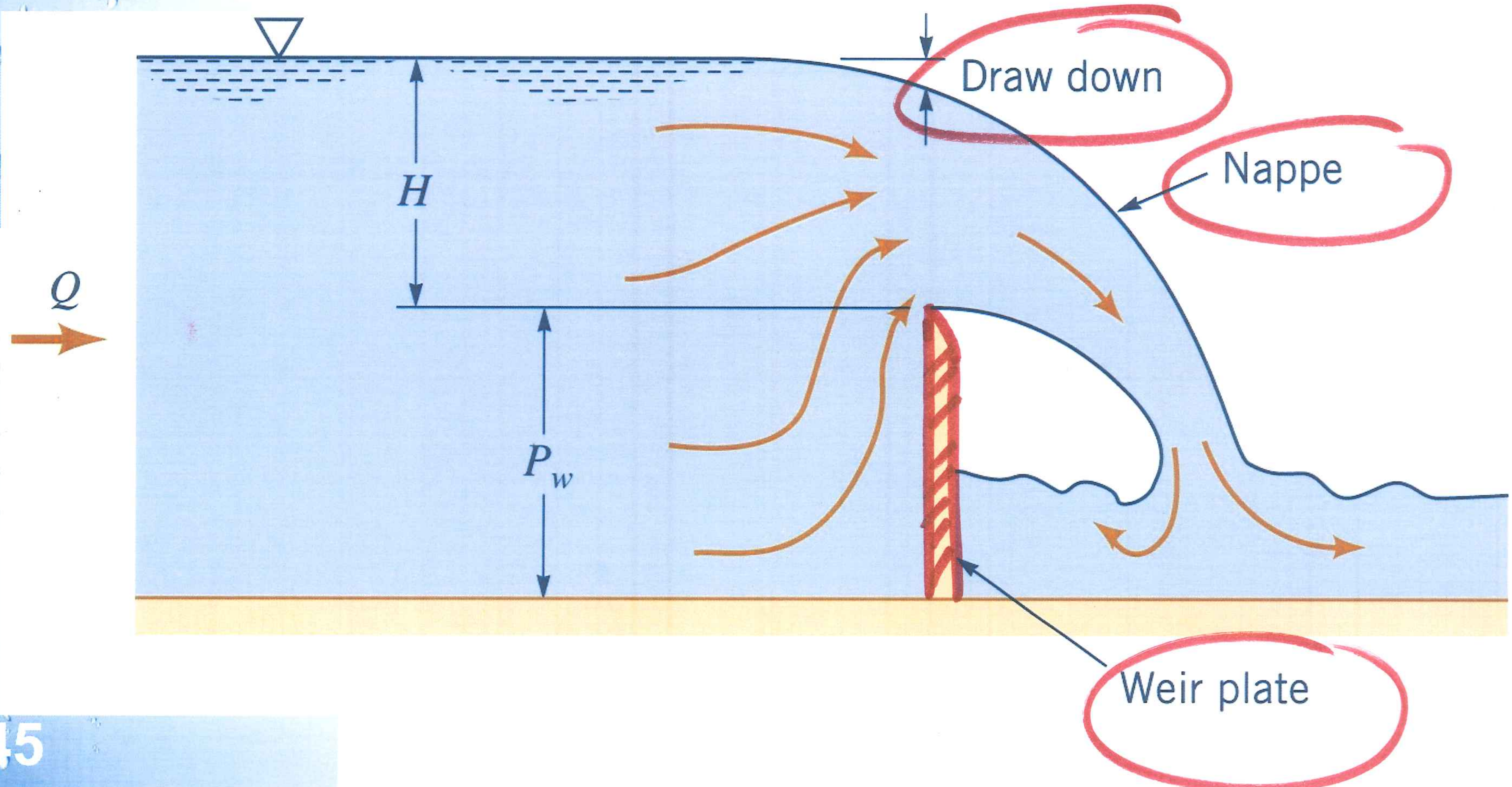
# Weirs



weir flows

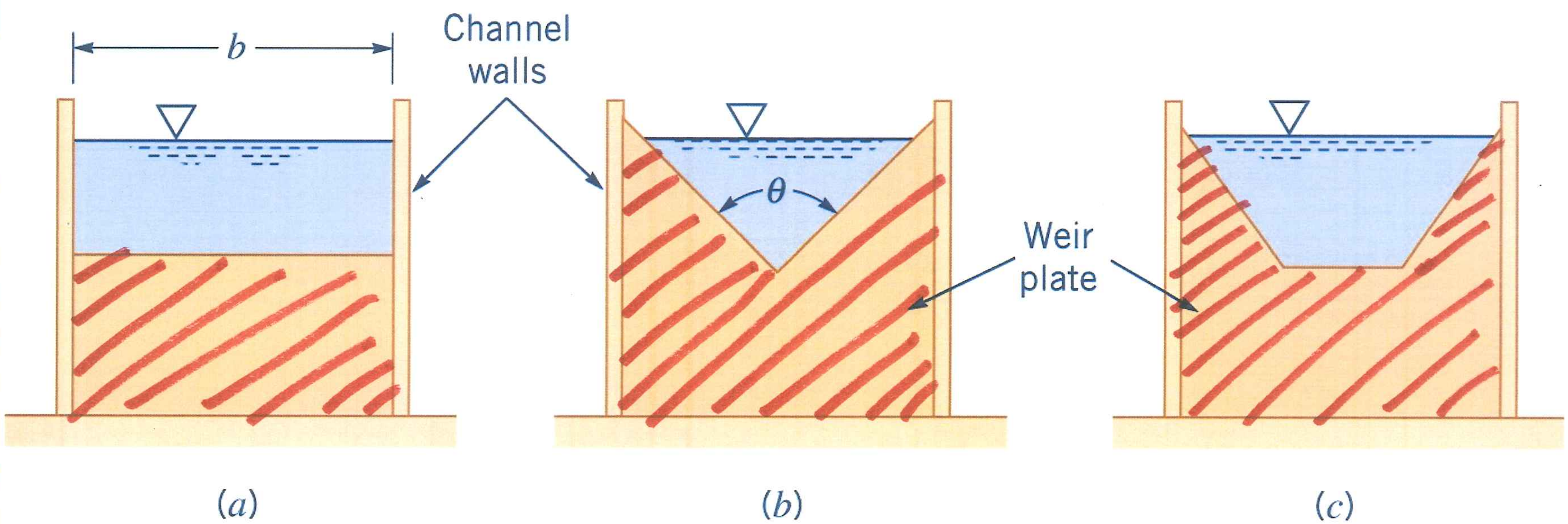
# Sharp-Crested Weir

A sharp-crested weir is essentially a vertical-edged flat plate placed across the channel.



# Sharp-Crested Weir - Geometry

(a) rectangular, (b) triangular, (c) trapezoidal.



# Rectangular Weir – Flowrate

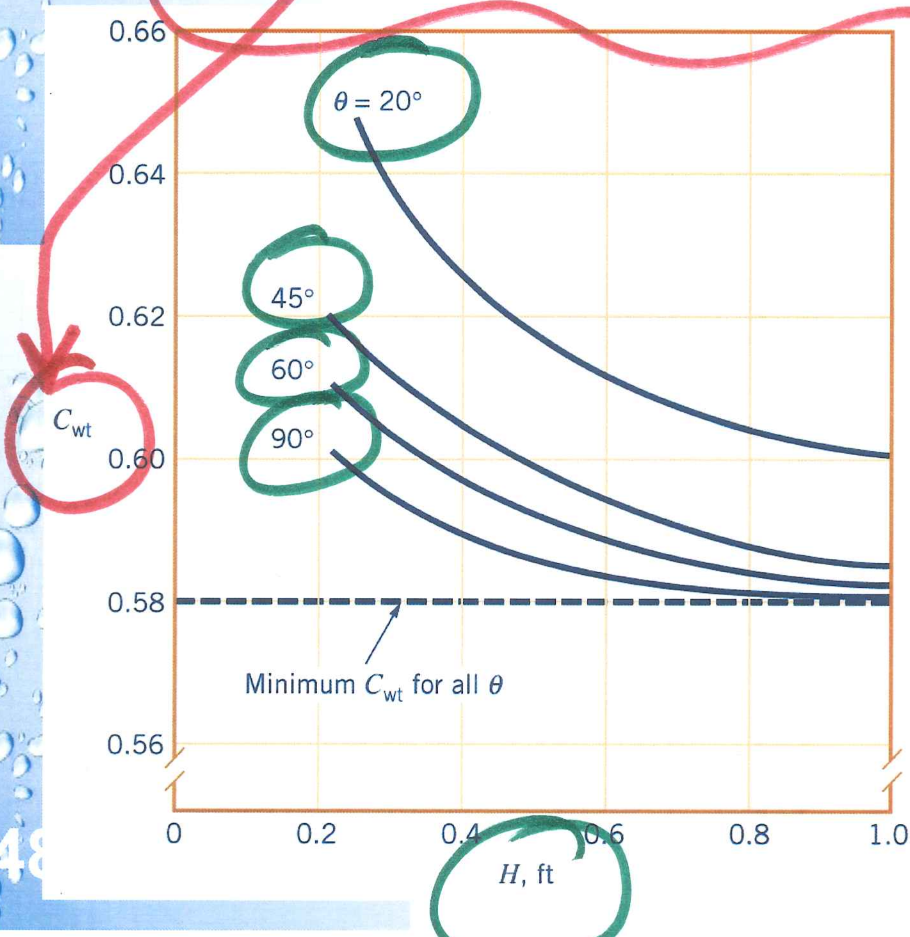
$$Q = C_{wr} \frac{2}{3} \sqrt{2g}\, b\, H^{3/2}$$

Where: $C_{wr}$ is the rectangular weir coefficient given by

$$C_{wr} = 0.611 + 0.075 \left( \frac{H}{P_w} \right)$$

# Triangular Weir – Flowrate

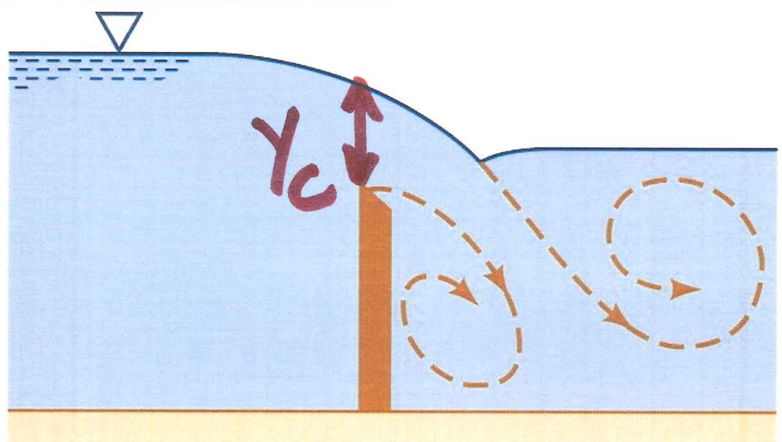
$$Q = C_{wt} \frac{8}{15} \tan\left(\frac{\theta}{2}\right) \sqrt{2g}\, H^{5/2}$$



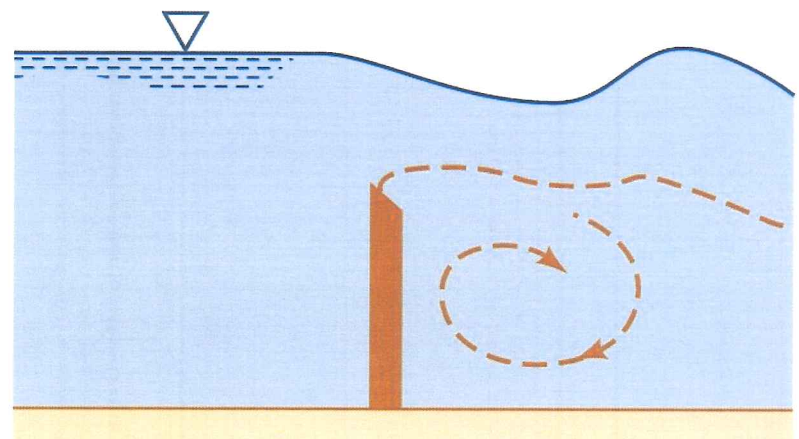

Weir coefficient for triangular sharp-crested weirs

# Free and Submerged Nappe

Flowrate over a weir depends on whether the **nappe is free or submerged**.




(*a*)



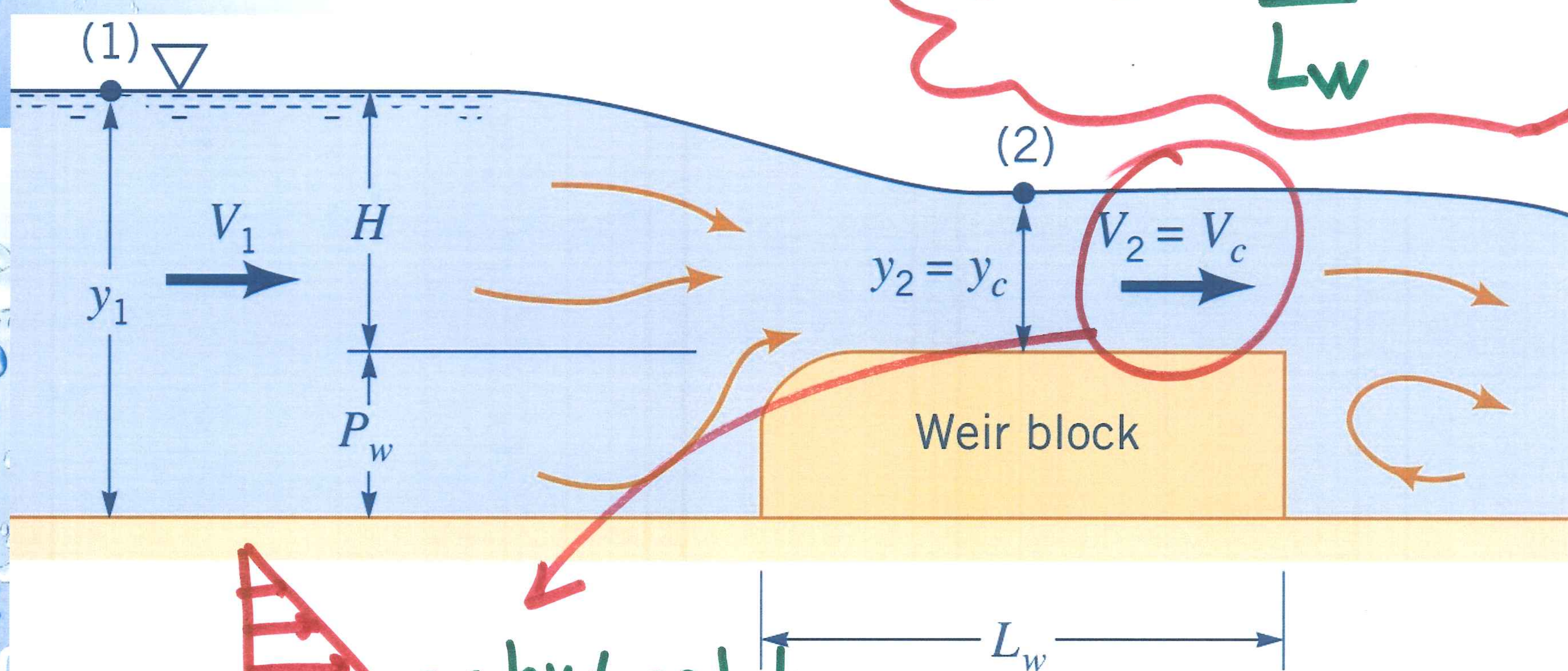
(*b*)

Flow conditions over a weir without a free nappe: (*a*) plunging nappe, (*b*) submerged nappe.

# Broad-Crested Weir in rectangular channels

A broad-crested weir is a structure that has a horizontal crest above which the fluid pressure may be considered hydrostatic.

$$0.08 < \frac{H}{L_w} < 0.50$$



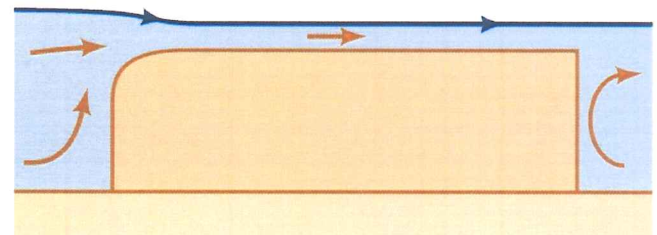

~ hydrostatic pressure distribution

# Rectangular broad-crested weir – Flowrate

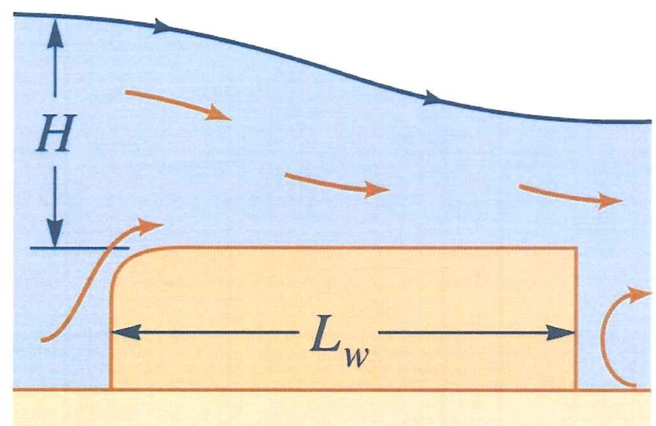
$$Q = C_{wb}\, b \sqrt{g} \left(\frac{2}{3}\right)^{3/2} H^{3/2}$$

$$C_{wb} = 1.125 \left(\frac{1 + H/P_w}{2 + H/P_w}\right)^{1/2}$$

$$0.08 < \frac{H}{L_w} < 0.50$$

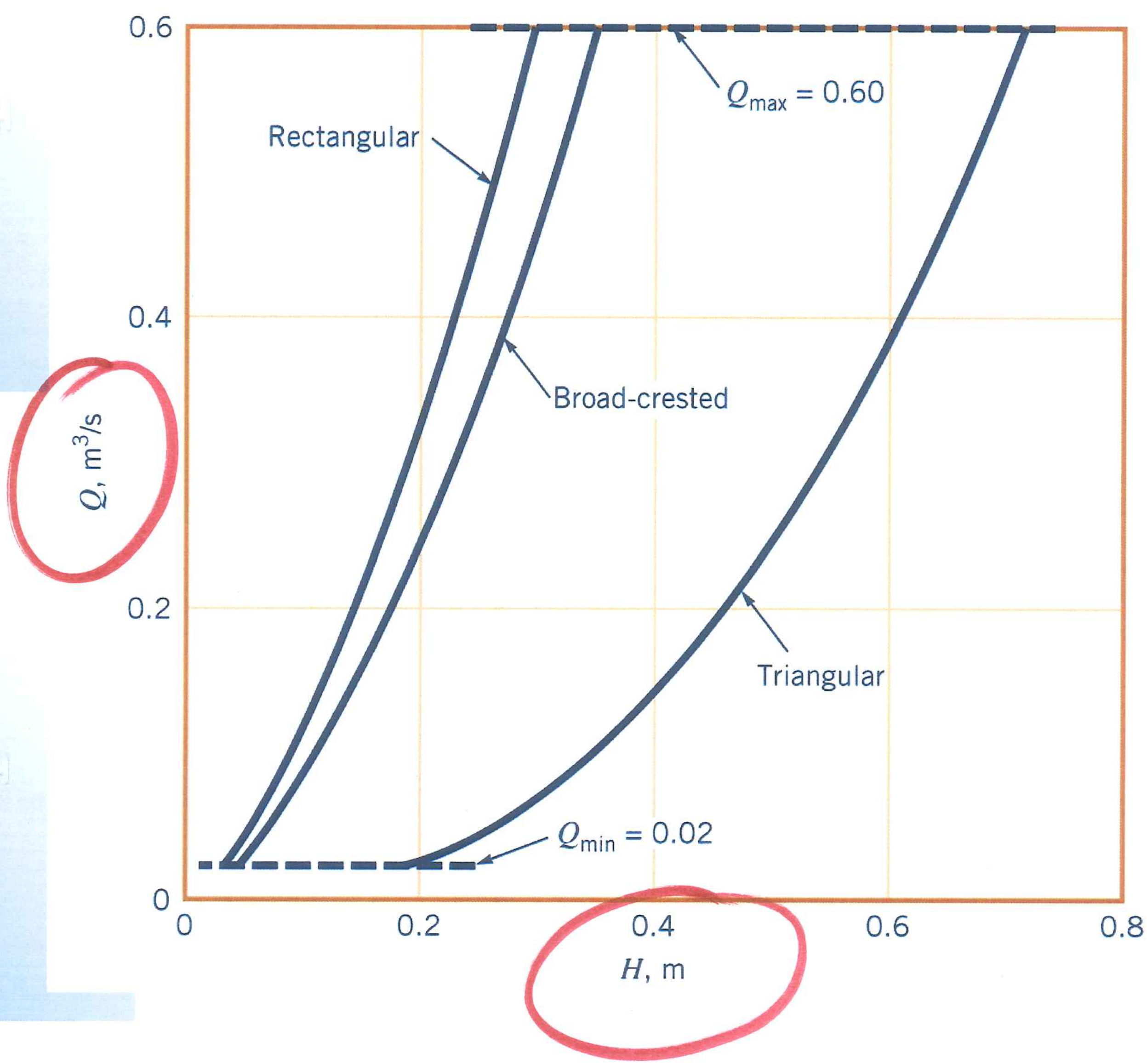


$H/L_w = 0.08$




$H/L_w = 0.50$

# Example of Comparison of Flowrates for Various Weirs



# Example of application (P10.113):

Water flows over the rectangular sharp-crested weir in a wide channel as shown in Fig. P10.113. If the channel is lined with unfinished concrete with a bottom slope of 2m/300m, will it be possible to produce a hydraulic jump in the channel downstream of the weir? Explain.

$n = 0.014$ (Table 10.1)

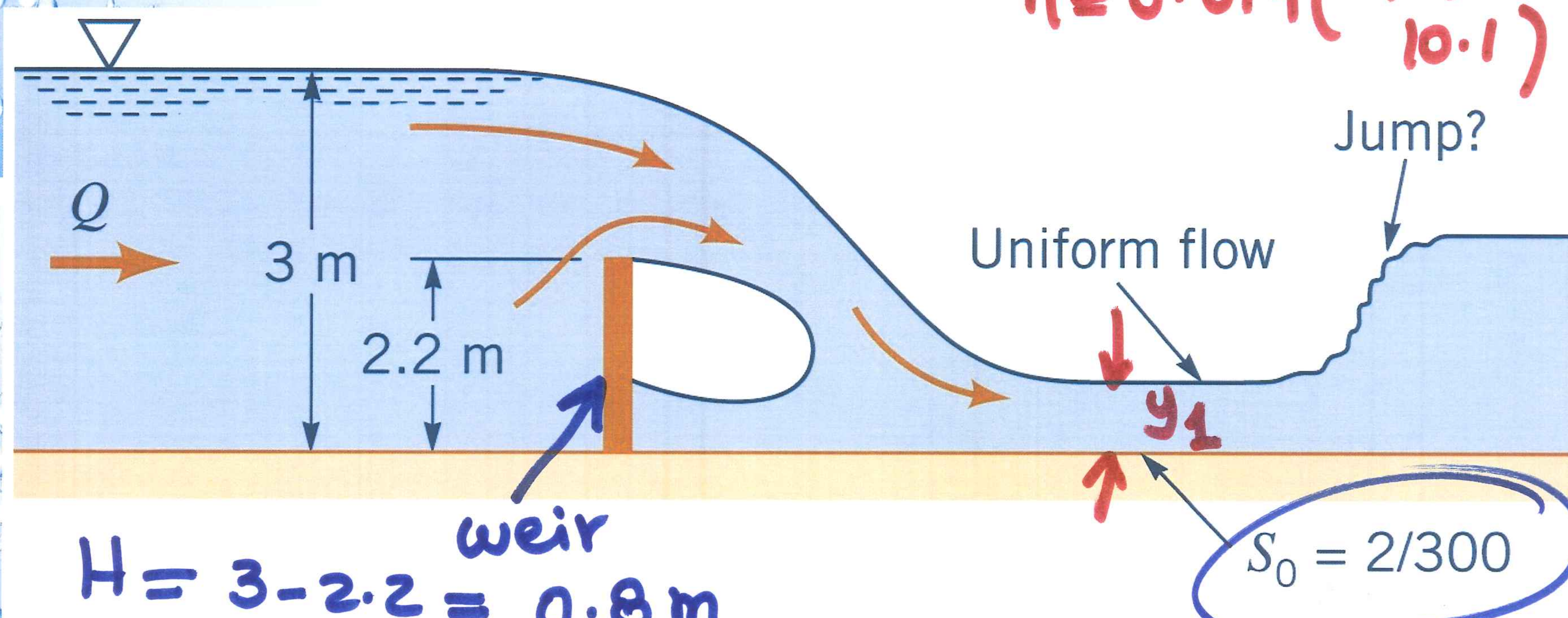



Figure P10.113

$H = 3 - 2.2 = 0.8\,m$

$P_w = 2.2\,m$

$$Q = C_{wr}\,\frac{2}{3}\sqrt{2g}\,b\,H^{3/2}$$

$$C_{wr} = 0.611 + 0.075\left(\frac{H}{P_w}\right)$$

$$Q = 1.349\, b \ \frac{m^3}{s} \qquad q = \frac{Q}{b} = 1.349 \ \frac{m^2}{s}$$

**Uniform flow**

$$Q = \frac{k}{n} A R^{2/3} S_o^{1/2} \qquad k = 1 \ (SI)$$

$$1.349\cancel{b} = \frac{1}{0.014} \cancel{b} y_1 \, y_1^{2/3} \left(\frac{2}{300}\right)^{1/2}$$

$$R = \frac{b y_1}{b + 2y_1} = \frac{\frac{\cancel{b} y_1}{\cancel{b}}}{\frac{\cancel{b}}{\cancel{b}} + \cancel{\frac{2y_1}{b}}}$$

$$\frac{2y_1}{b} \approx 0$$

$$y_1 \ll b$$

$$R = y_1$$

$$\boxed{y_1 = 0.415 \, m}$$

$$V = \frac{Q}{A}$$

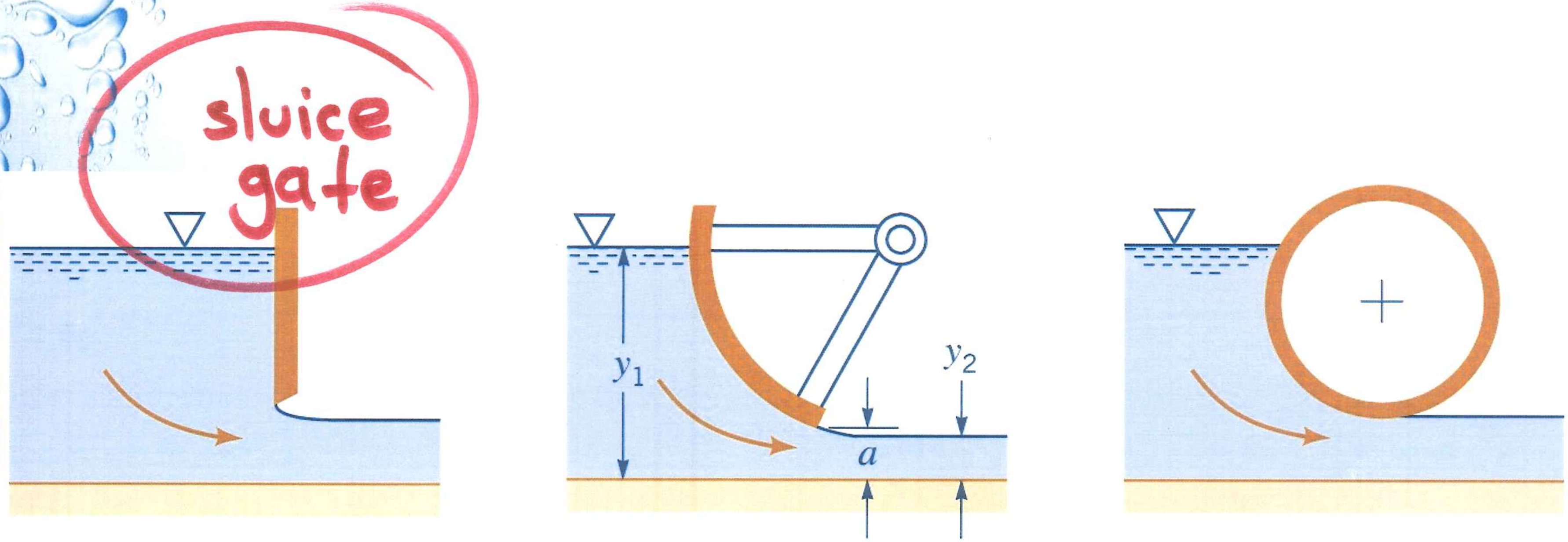
$$V_1 = 3.25 \, m/s$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{3.25}{\sqrt{9.8 \times 0.415}} = 1.61$$

It is possible to produce a hydraulic jump because $Fr_1 > 1$

# Underflow gates

# Underflow Gates

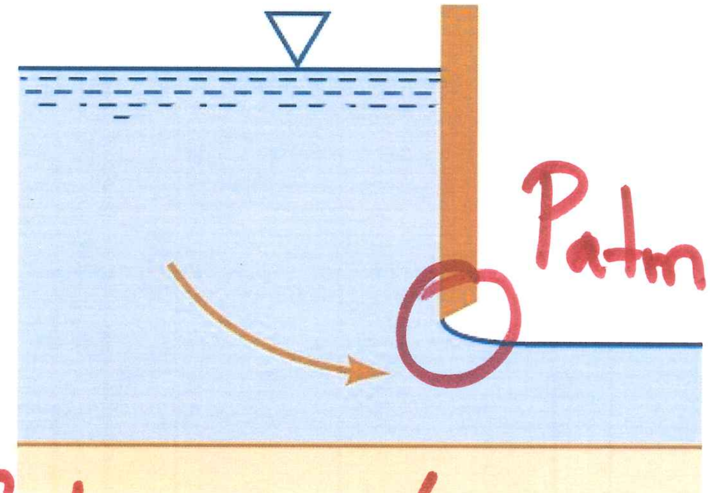



(*a*)　　(*b*)　　(*c*)

(*a*) vertical gate, (*b*) radial gate, (*c*) drum gate

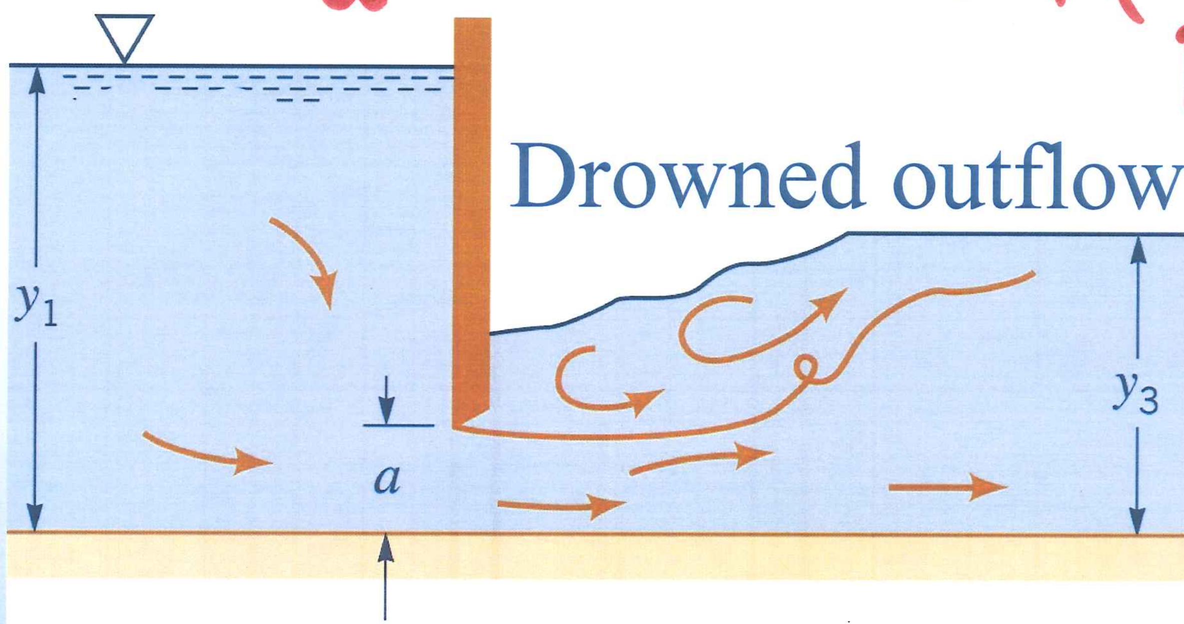
# Underflow Gates

Free outflow




$\frac{y_1}{a} = 6$, $\frac{y_3}{a} = 4$ $C_d = 0.4$ (see next page)

Drowned outflow



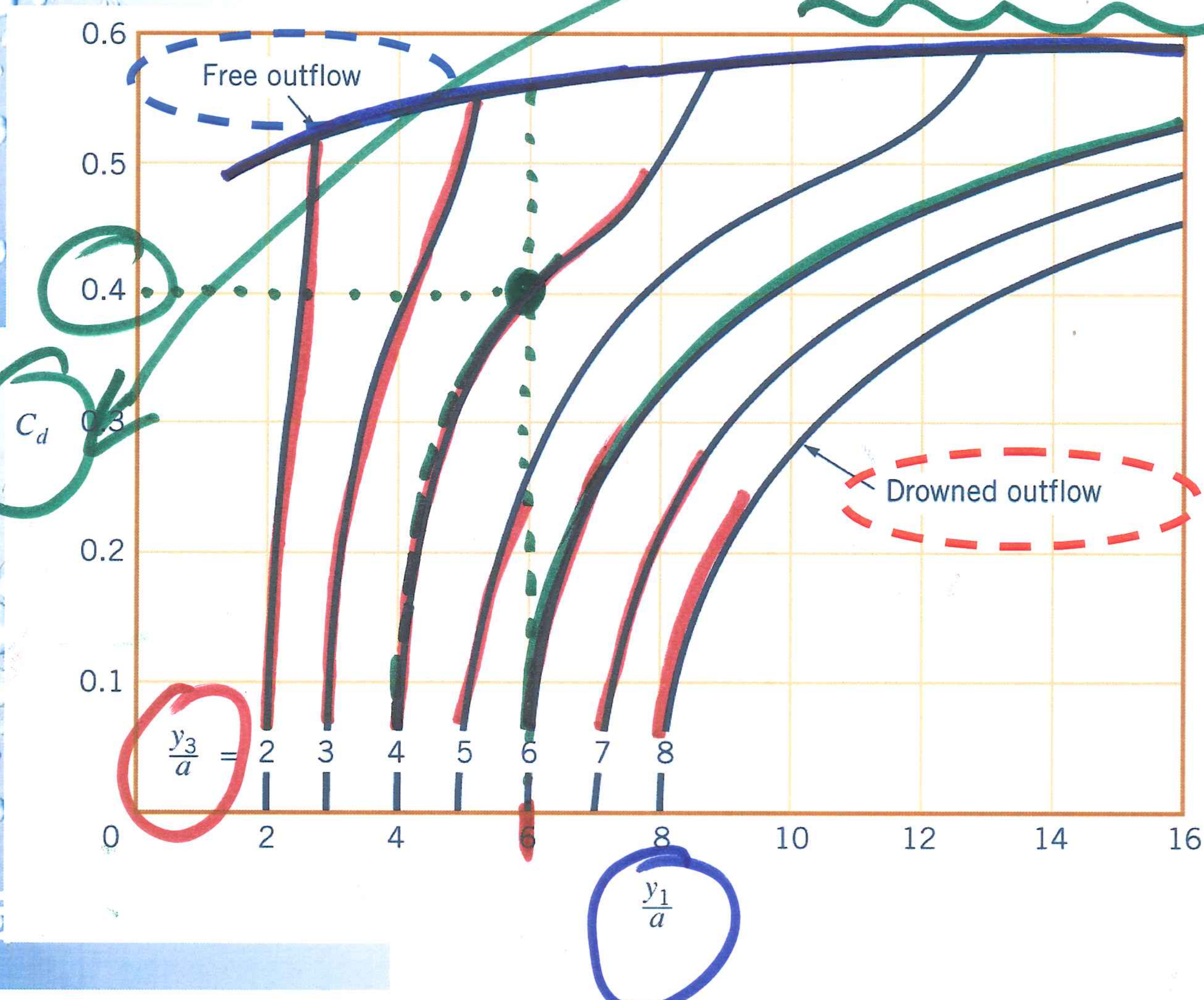

high tailwater that submerges the exit.

# Underflow Gates

$$q = C_d a \sqrt{2gy_1}$$ m²/s, ft²/s

Where $q$ is the flowrate per unit width

$C_d$ : discharge coefficient




Typical discharge coefficients for underflow gates