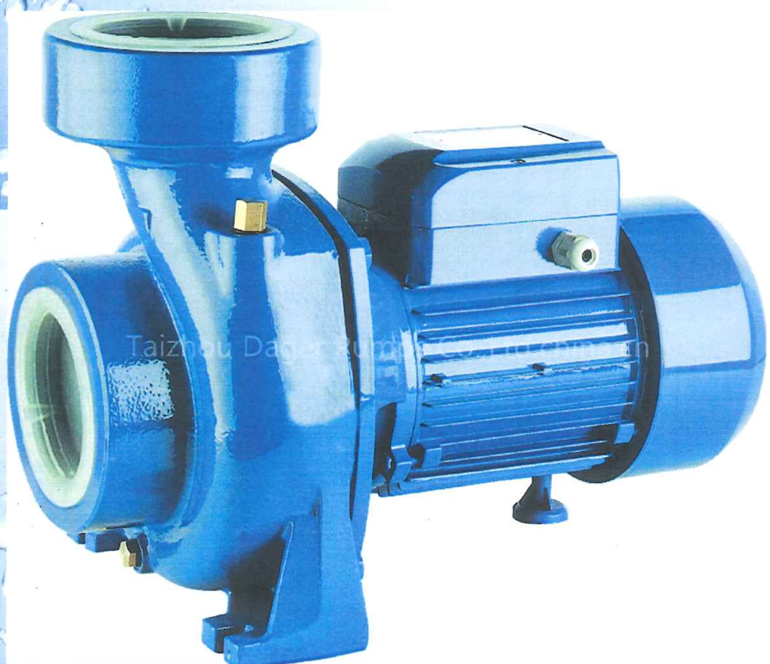
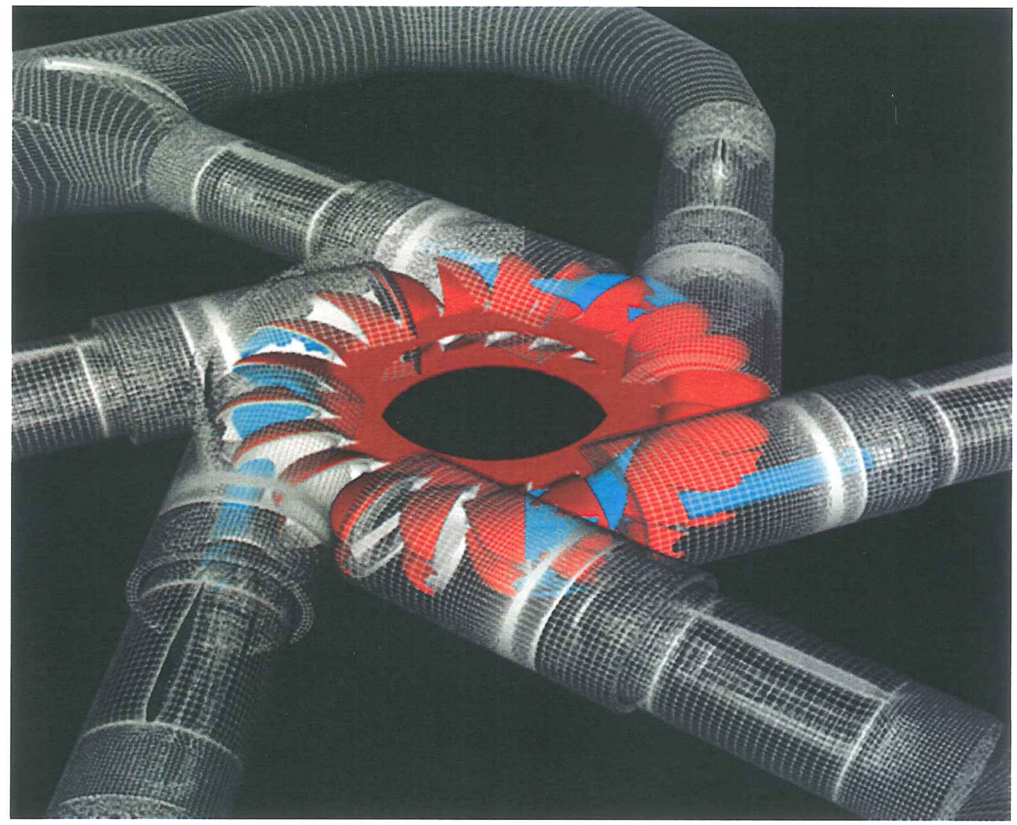
Lecture 16, 02/24/2014
Pumps and Turbines





Arturo Leon, Oregon State University

Pumps and Turbines

- Pumps: Add energy to the fluid - they do work on the fluid.
- Turbines: Extract energy from the fluid - the fluid does work on them.

Pumps
Introduction and Pump Performance Characteristics

Archimedes Pump

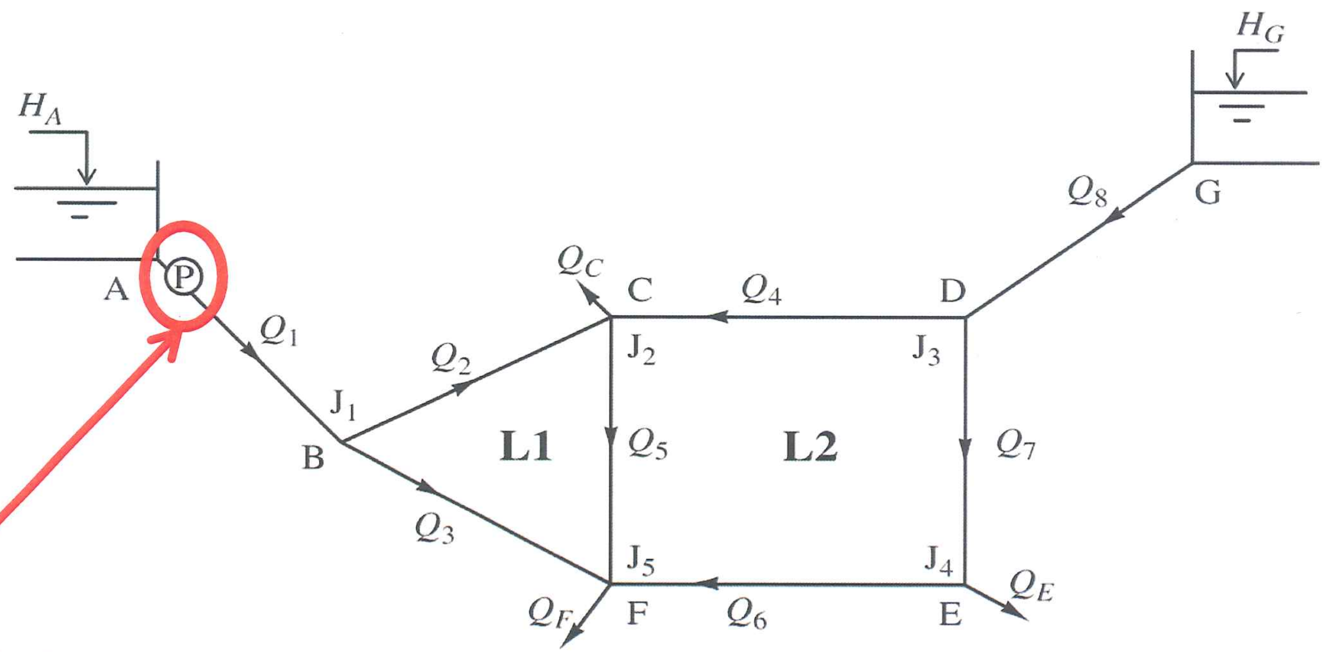
Hydraulic Pumps Videos:

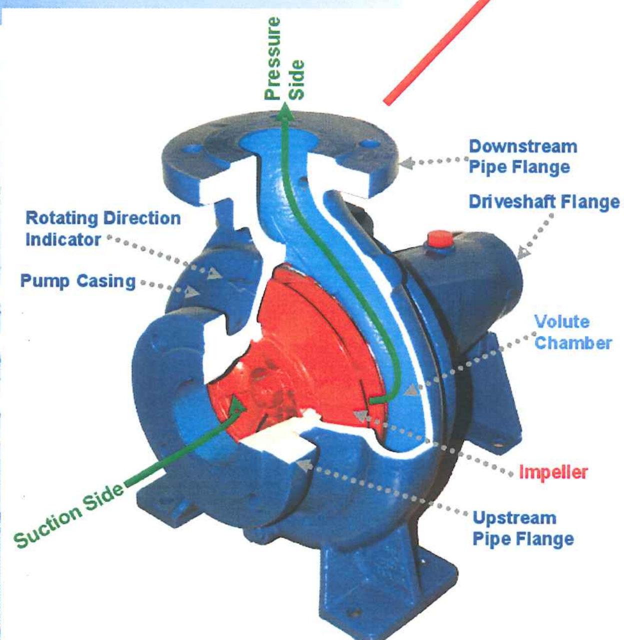
Centrifugal pump
https://www.youtube.com/watch?v=BaEHVpKc-1Q

Screw pump and Chain Pump
https://www.youtube.com/watch?v=gNAiWCOHh8k)

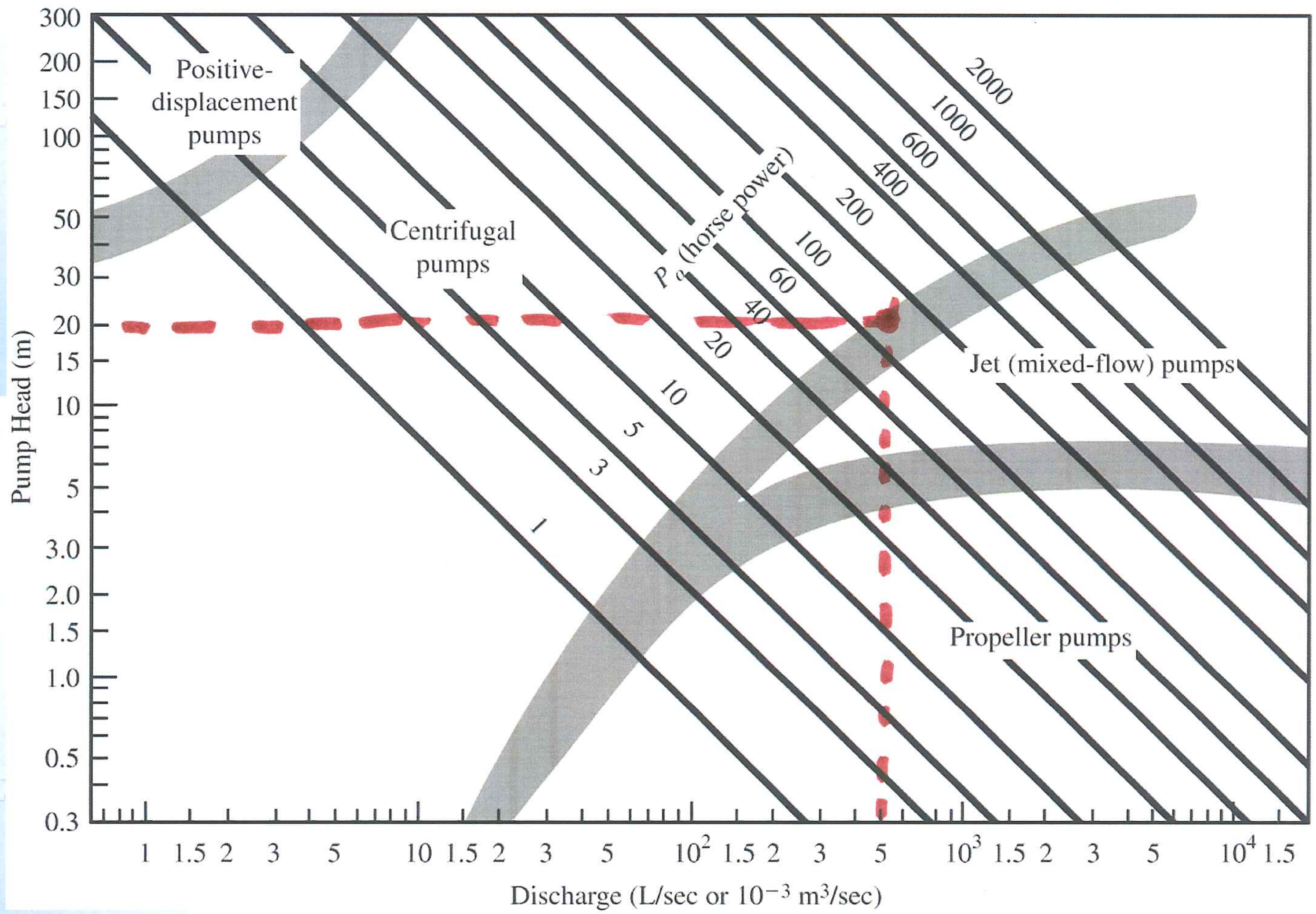
RAM PUMP (No energy is required)
https://www.youtube.com/watch?v=plSqyQGuwWc

Pipe Network with Hydraulic pumps





Typical discharge, head, and power requirements for different types of pumps



Reference: Houghtalen et al.

Pump Performance Characteristics 1/8

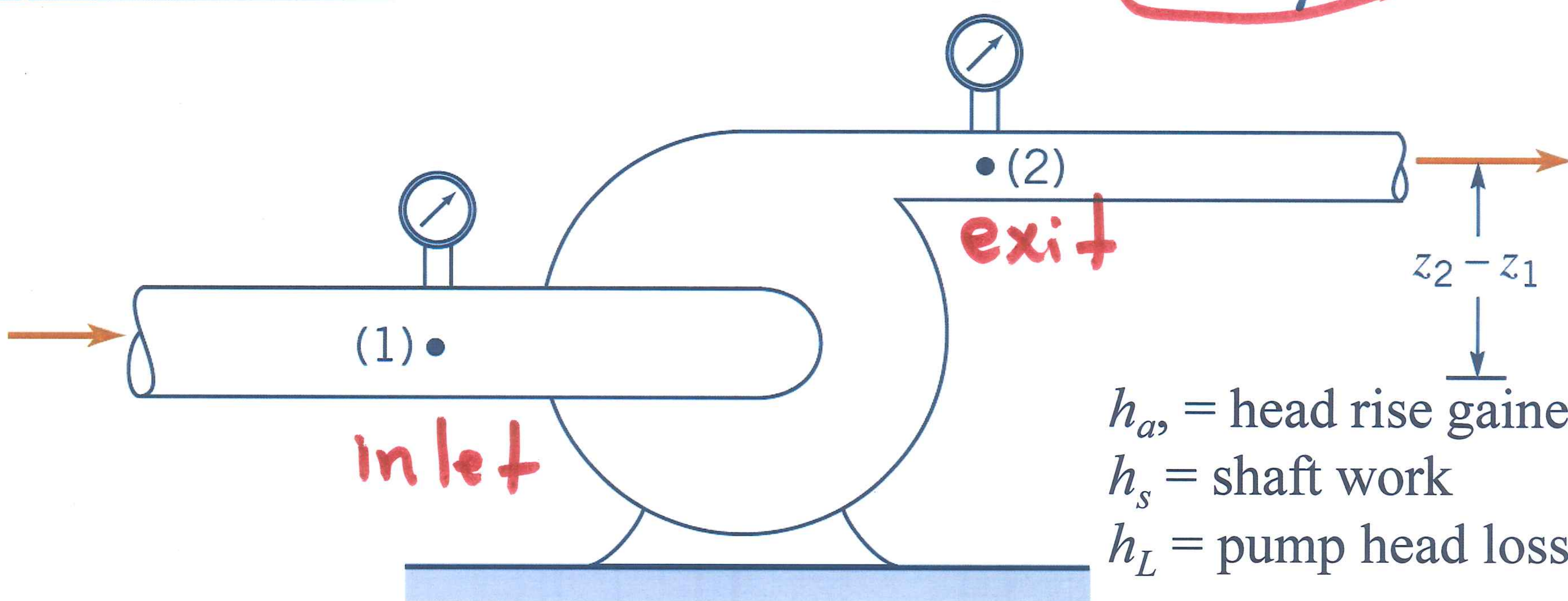
Using the energy equation with $h_a = h_s - h_L$

$E_1 + h_a = E_2$

$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g}$$

The differences in elevations and velocities are typically small

$$h_a \approx \frac{p_2 - p_1}{\gamma}$$



h_a, = head rise gained
h_s = shaft work
h_L = pump head loss

Pump Performance Characteristics 2/8

The power gained by the fluid $\boxed{\mathscr{P}_f = \gamma Q h_a}$

$$\mathscr{P}_f = \underset{\text{(HP)}}{\text{water horsepower}} = \frac{\gamma Q h_a}{550}$$

Where γ in lb/ft^3, Q in ft^3/s and h_a in ft

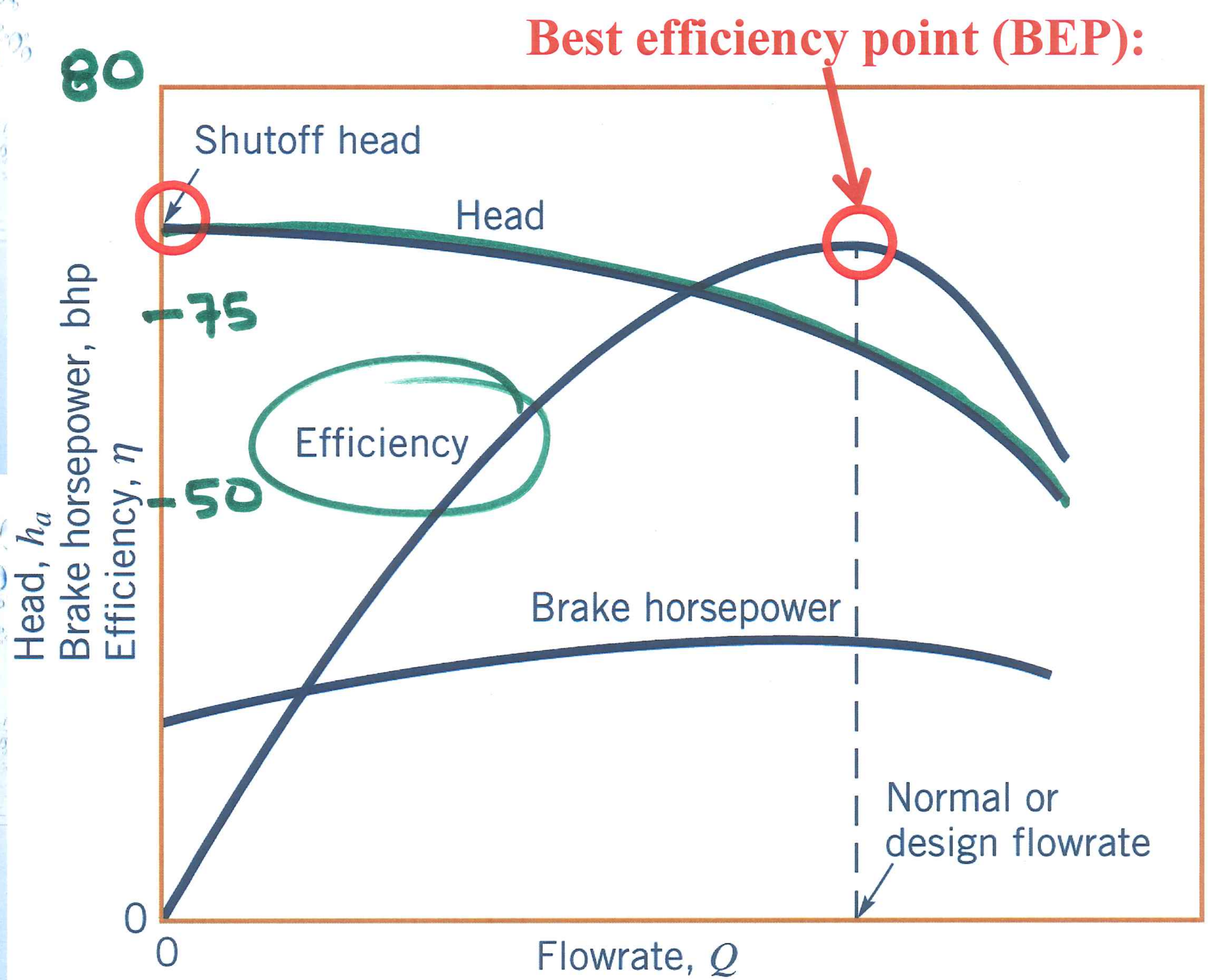
Overall efficiency $\eta = \dfrac{\gamma Q h_a/550}{\text{bhp}}$

Pump Performance Characteristics

The overall efficiency arises from three sources, the **hydraulic efficiency** (hydraulic losses), η_h, the **mechanical efficiency** (bearings and seals), η_m, and the **volumetric efficiency** (leakage losses), η_v

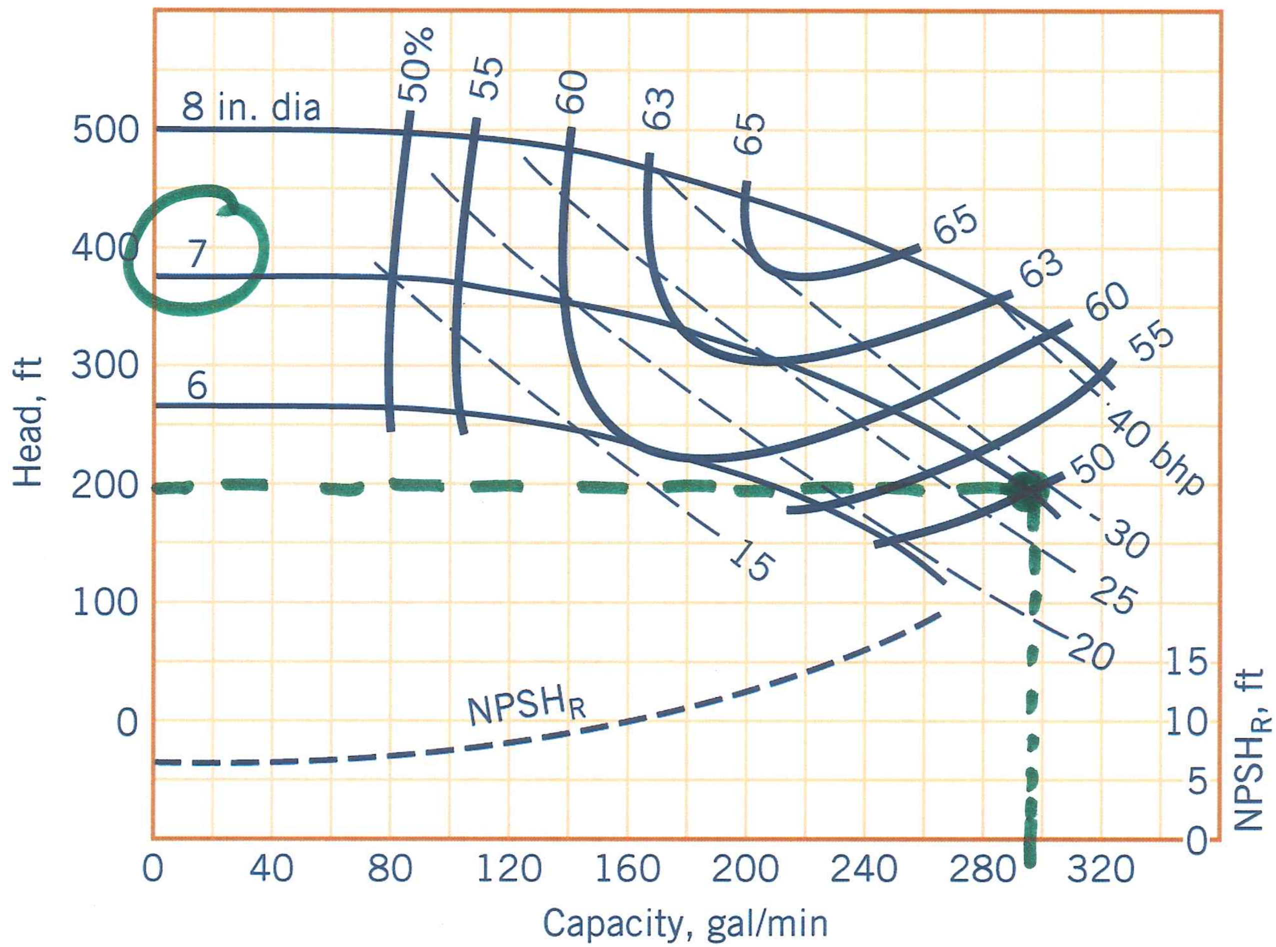
$$\boxed{\eta = \eta_h \eta_m \eta_v}$$

Pump Performance Characteristics



Pump Performance Characteristics

$NPSH_R$ = Required net positive suction head



Performance curves for a two-stage centrifugal pump operating at 3500 rpm. Data given for three different impeller diameters.

Example of application (P12.17):

The performance characteristics of a centrifugal pump are determined from an experimental setup similar to that shown below. When the flow rate of a liquid ($SG = 0.9$) through the pump is 120 GPM, the pressure gage at (1) indicates a vacuum of 95 mm of mercury and the pressure gage at (2) indicates a pressure of 80 kPa. The diameter of the pipe at the inlet is 110 mm and at the exit it is 55 mm. if z_2-z_1 = 0.5 m, what is the actual head rise across the pump? Explain how you would estimate the pump motor power requirement.

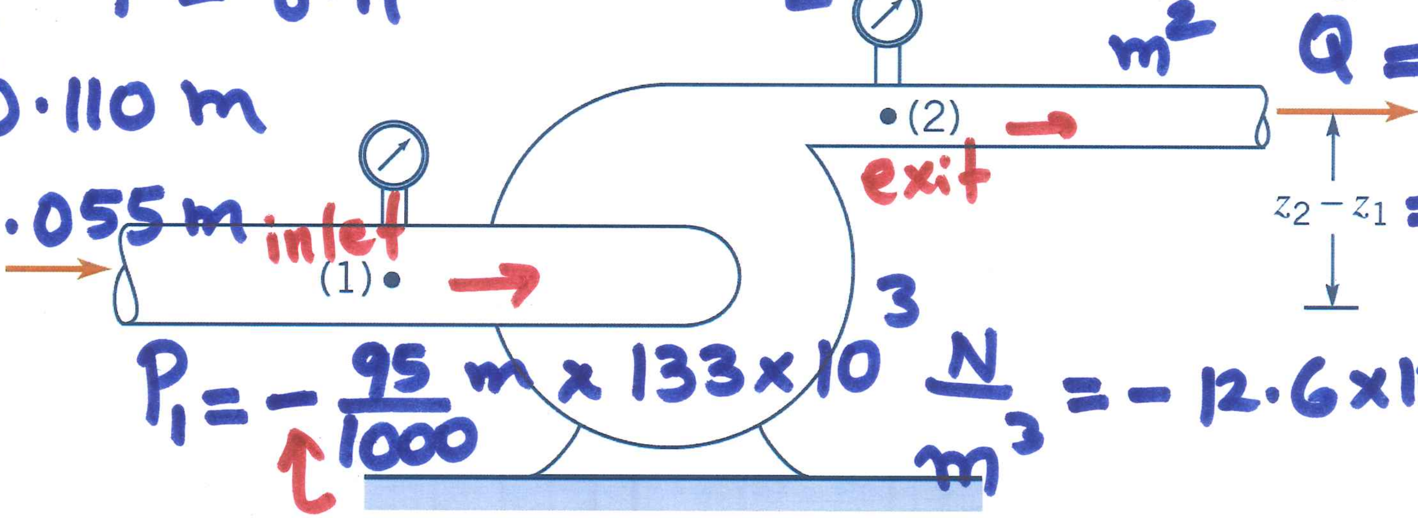
$P = \gamma \cdot h$

$P_2 = 80\times10^3 \frac{N}{m^2}$ $\quad SG = 0.9$

$Q = 120\,gpm$

$D_1 = 0.110\,m$

$D_2 = 0.055\,m$



$P_1 = -\frac{95}{1000}\,m \times 133\times10^3 \frac{N}{m^3} = -12.6\times10^3\,N/m^2$

Vacuum

metric units

$$V_1 = \frac{Q}{A_1} = \frac{120\ \text{gpm}\left(6.309\times10^{-5}\ \frac{m^3/s}{\text{gpm}}\right)}{\pi\times(0.110)^2/4} = 0.797\ m/s$$

$$V_1A_1 = V_2A_2 \rightarrow V_2 = 3.19\ m/s$$

head rise

$$h_a = \frac{P_2-P_1}{\gamma} + Z_2 - Z_1 + \frac{V_2^2 - V_1^2}{2g}$$

$$h_a = \frac{80\times10^3\ \frac{N}{m^2} - (-)\,12.6\times10^3\ \frac{N}{m^2}}{0.9\times 9.8\times10^3\ \frac{N}{m^3}} + 0.5 + \frac{3.19^2 - 0.797^2}{2\times9.8}$$

SG → 0.9

$$\boxed{h_a = 11.5\,m}$$

$$\eta = \frac{\gamma Q h_a/550}{bhp} \rightarrow \boxed{bhp = \frac{\gamma Q h_a/550}{\eta}}$$

Lecture 17, 02/26/2014
Net Positive Suction Head (NPSH)

Arturo Leon, Oregon State University

Net Positive Suction Head (NPSH)

- On the suction side of a pump, low pressures are very common. Check for cavitation.
- Cavitation occurs when the liquid pressure at a given location is reduced to the vapor pressure of the liquid.
- How to characterize the potential for cavitation…

NPSH

Net Positive Suction Head

To characterize the potential for cavitation, define the net positive suction head (NPSH) as

$$\mathrm{NPSH} = \underbrace{\frac{p_s}{\gamma} + \frac{V_s^2}{2g}} - \frac{p_v}{\gamma}$$

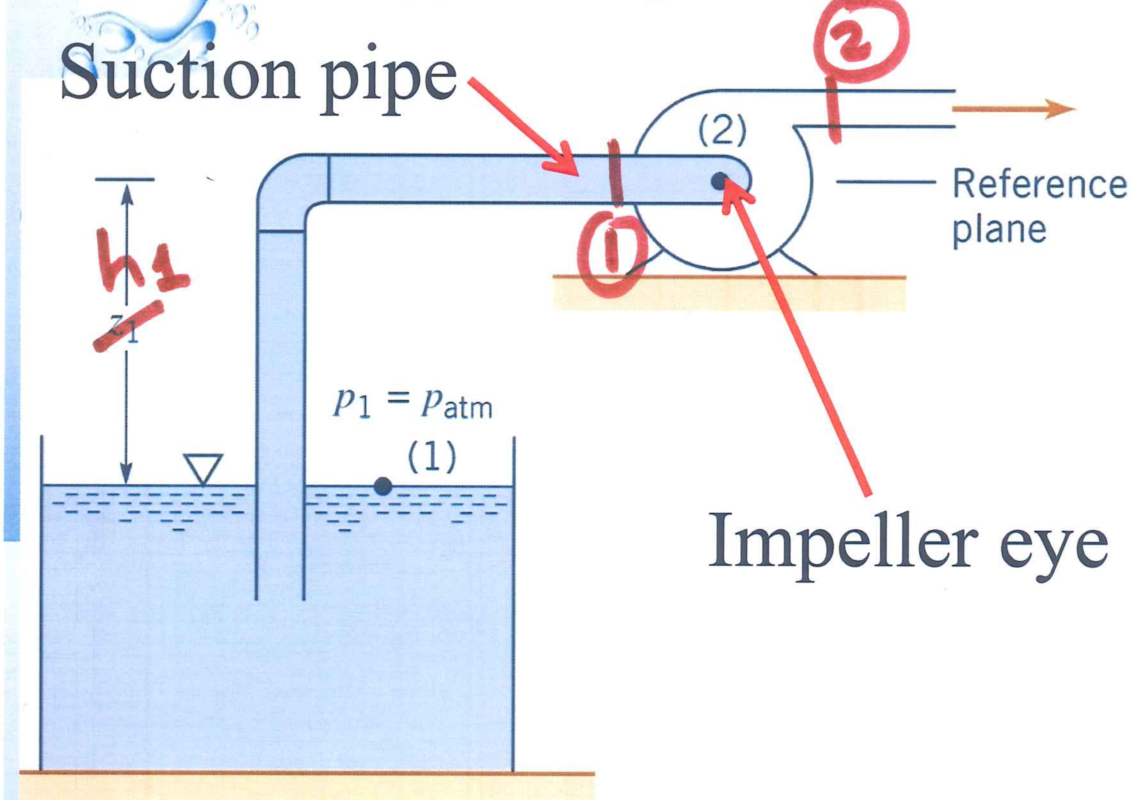
The total head on the suction side near the pump impeller inlet

The liquid vapor pressure head

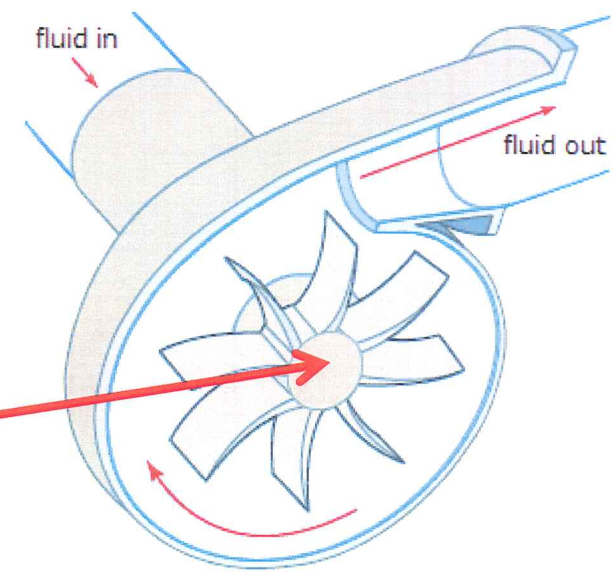
There are actually two values of NPSH of interest.

$\mathbf{NPSH_R}$ (Required NPSH to avoid cavitation)

Suction pipe



Impeller eye

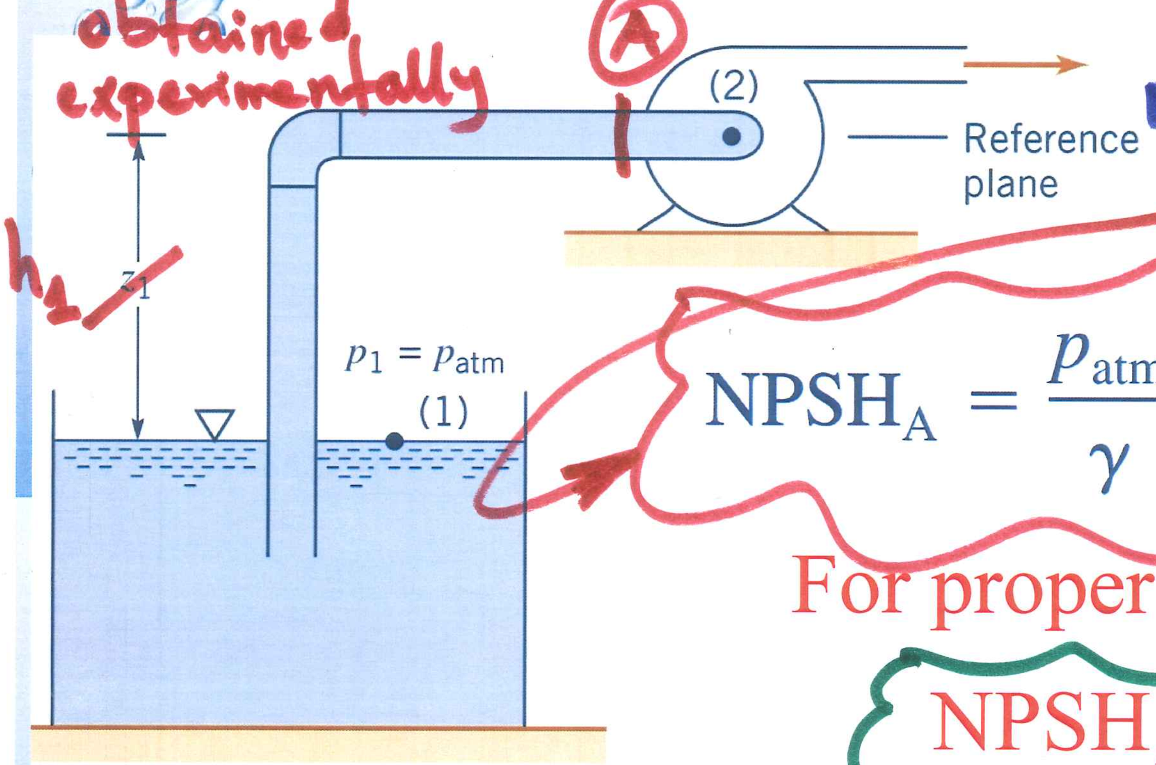


Because pressures lower than those in the suction pipe will develop in the impeller eye, $\mathbf{NPSH_R}$ is determined experimentally

$NPSH_R$ and $NPSH_A$

available

obtained experimentally



$$E_1 = E_A$$

$$\frac{P_{atm}}{\gamma} + \cancel{\frac{V_1^2}{2g}} + elev_1 - \frac{P_v}{\gamma} = \frac{P_A}{\gamma} + \frac{V_A^2}{2g} + elev_A + \Sigma h_L - \frac{P_v}{\gamma}$$

NPSH

$$NPSH_A = \frac{p_{atm}}{\gamma} - \cancel{z_1} - \sum h_L - \frac{p_v}{\gamma}$$

h_1

For proper pump operation

$$NPSH_A \geq NPSH_R$$

for avoiding cavitation

$\sum h_L$ =Head losses between free surface and pump impeller inlet

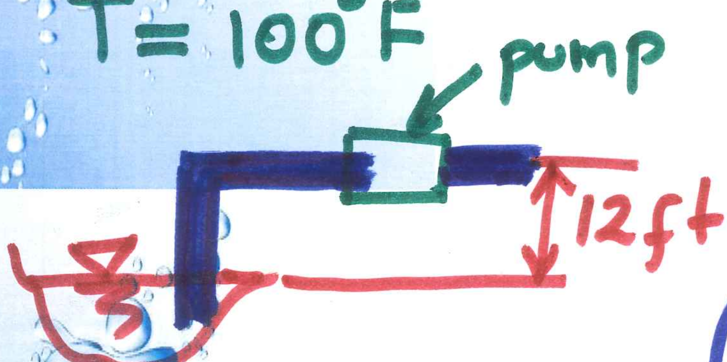
$NPSH_A$ = Available NPSH (head that actually occurs)

$NPSH_R$ = Required NPSH to avoid cavitation

Example of application (P12.21):

A centrifugal pump with a 7-in diameter impeller has the performance characteristics shown below. The pump is used to pump water at 100° F, and the pump inlet is located 12ft above the open water surface. When the flow rate is 200 gpm, the head loss between the water surface and the pump inlet is 6 ft of water. Would you expect cavitation in the pump to be a problem? Assume standard atmospheric pressure. Explain how you arrived at your answer.

$T = 100°F$

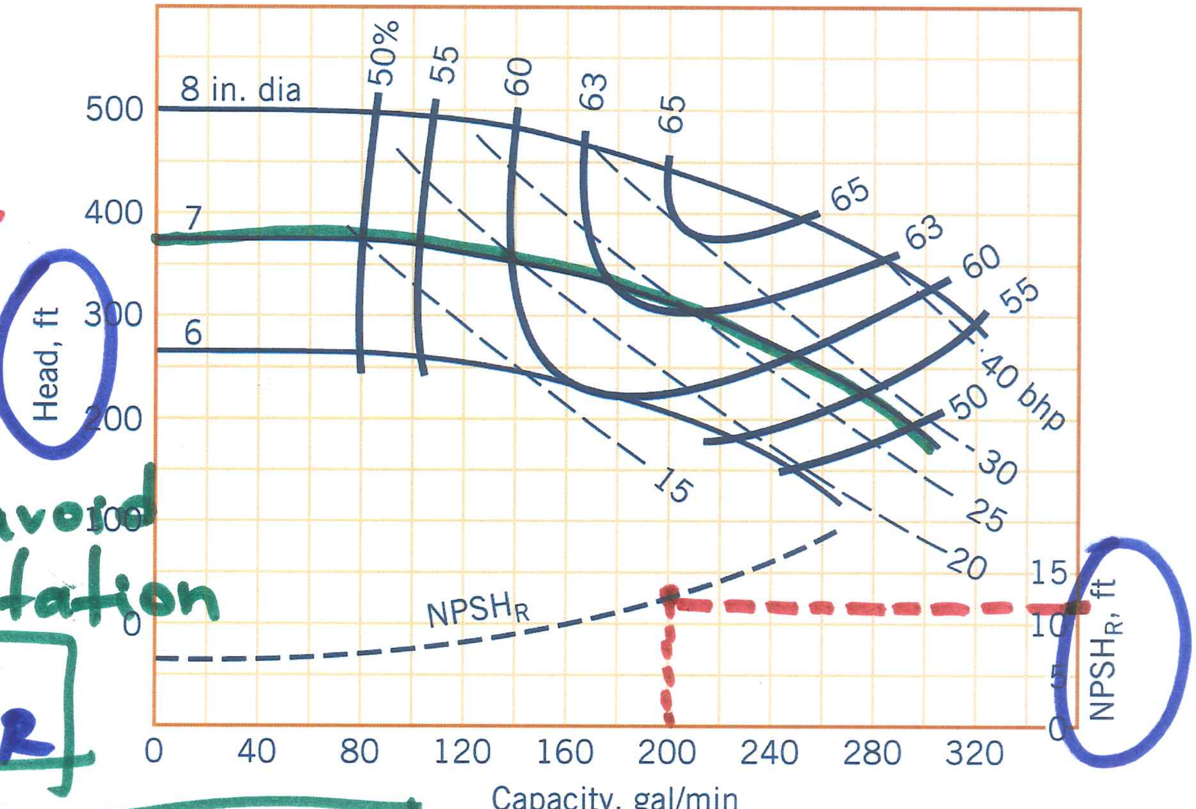


$Q = 200$ gpm

$\Sigma h_L = 6$ ft

To avoid cavitation

$\boxed{NPSH_A > NPSH_R}$



$\boxed{NPSH_R = 12 \text{ ft}}$

$$NPSH_A = \frac{P_{atm}}{\gamma} - h_1 - \Sigma h_L - \frac{P_v}{\gamma}$$

At 100°F (Table B-1) $\left\{ \begin{array}{l} P_v = 0.9493 \text{ Psia} \\ \gamma = 62 \text{ lb/ft}^3 \end{array} \right.$

$P_{atm} = 14.7$ Psia (standard atmospheric pressure)

$$NPSH_A = \frac{14.7 \frac{lb}{in^2} \times \frac{144\ in^2}{ft^2}}{62 \frac{lb}{ft^3}} - 12\ ft - 6\ ft - \frac{0.9493 \times 144}{62}$$

$$\boxed{NPSH_A = 13.9\ ft}$$

because

$NPSH_A > NPSH_R$ (No cavitation)

Lecture 18, 02/28/2014
Pumps
System Characteristics and Pump Selection

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System Characteristics and Pump Selection

The energy equation between points (1) and (2) gives

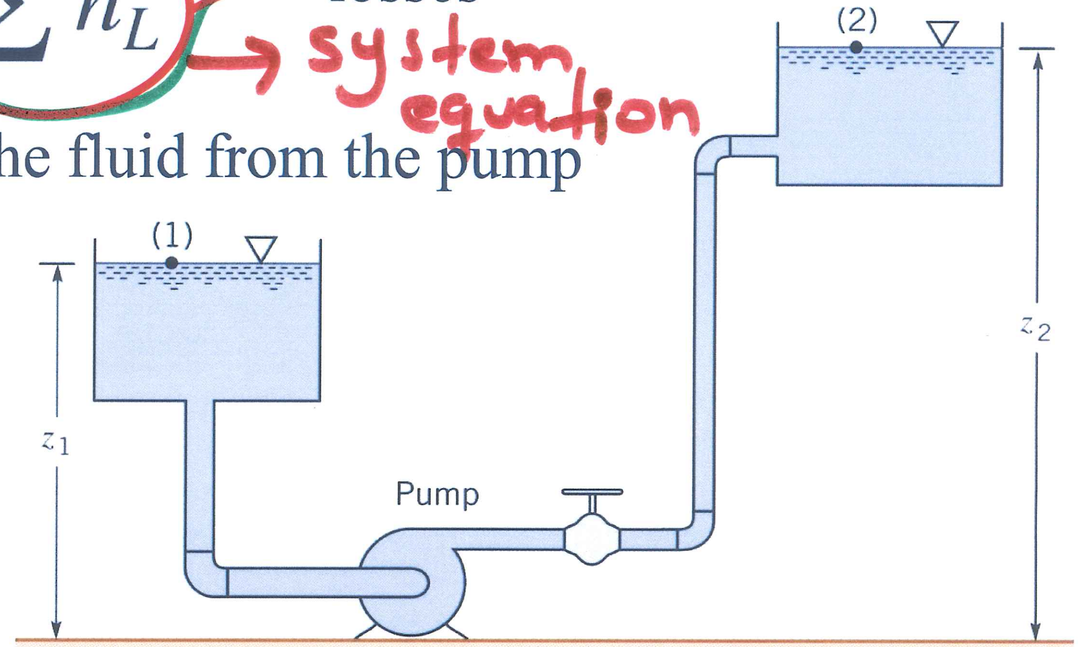
$$\cancelto{0}{\frac{p_1}{\gamma}} + \cancelto{0}{\frac{V_1^2}{2g}} + z_1 + h_a = \cancelto{0}{\frac{p_2}{\gamma}} + \cancelto{0}{\frac{V_2^2}{2g}} + z_2 + \Sigma h_L$$

$$h_a = z_2 - z_1 + \sum h_L$$

All friction losses and minor losses

system equation

h_a = actual head gained by the fluid from the pump



System Characteristics and Pump Selection

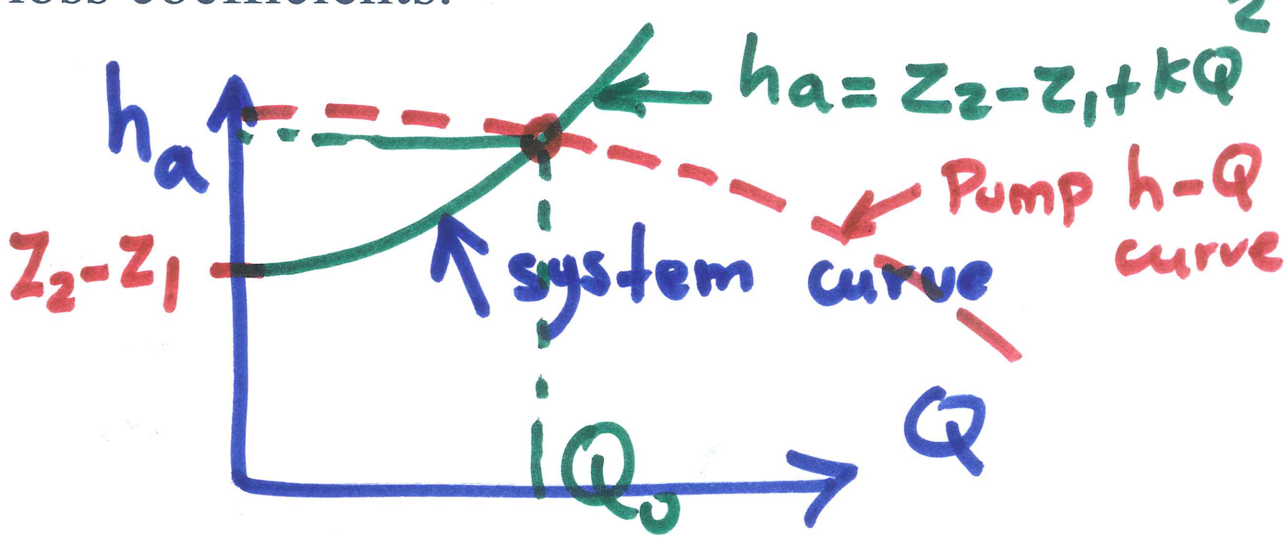
$$h_a = z_2 - z_1 + \left(f\frac{L}{D} + \Sigma K\right)\frac{V^2}{2g} \qquad V = \frac{Q}{A}$$

$$h_a = z_2 - z_1 + \underbrace{\frac{\left(f\frac{L}{D} + \Sigma K\right)}{2gA^2}}_{K} Q^2$$

$$h_a = z_2 - z_1 + KQ^2$$ **system equation**

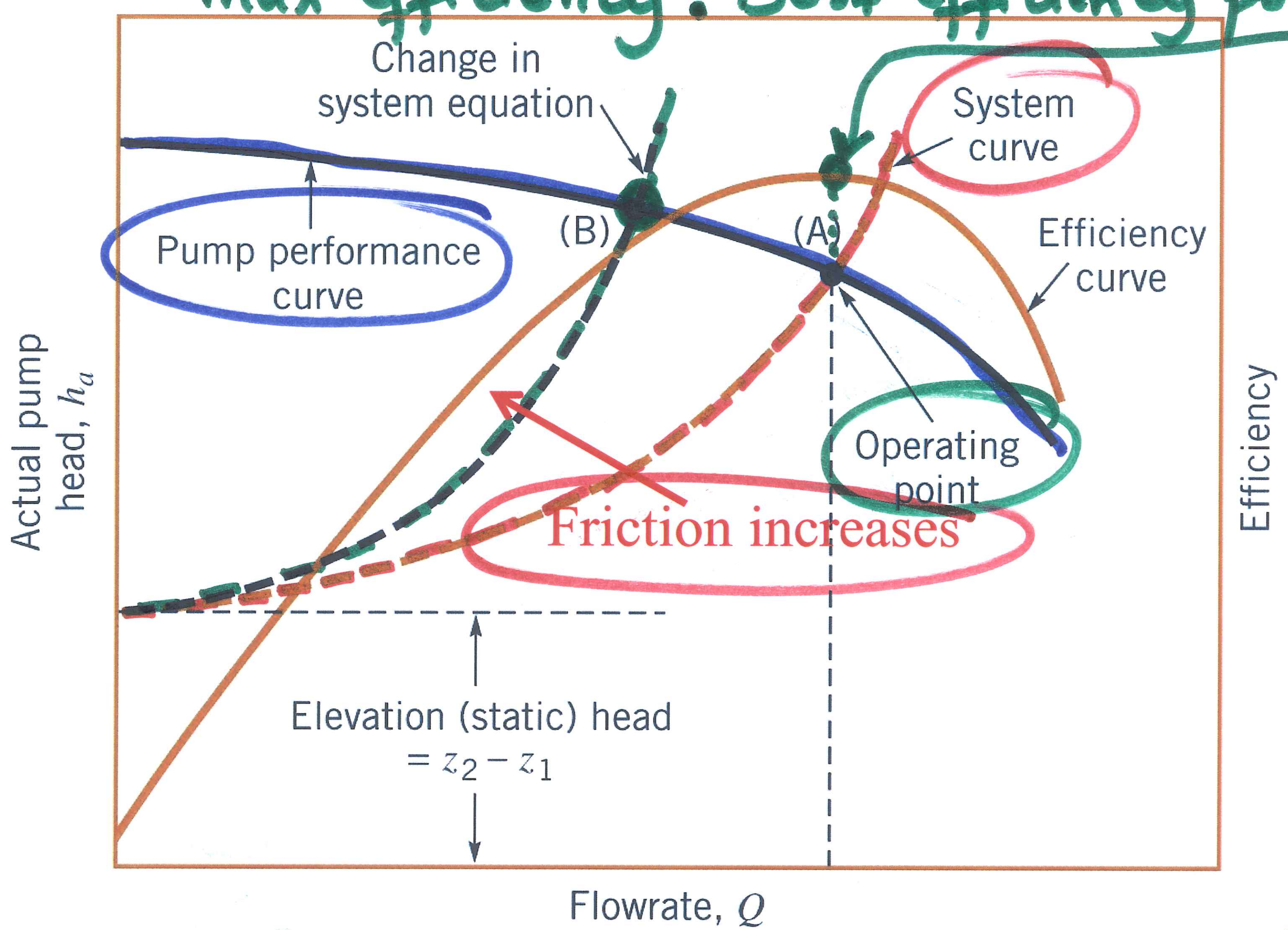
K depends on the pipe size and length, friction factor, and minor loss coefficients.

Plot of h_a vs Q



System Characteristics and Pump Selection

max efficiency : Best efficiency point



Utilization of the system curve and the pump performance curve to obtain the operating point for the system

System Characteristics and Pump Selection

system hydraulics (YOU will get it)

- To select a pump, it is necessary to utilize both the **system curve** (determined by the system equation), and the **pump performance curve**.

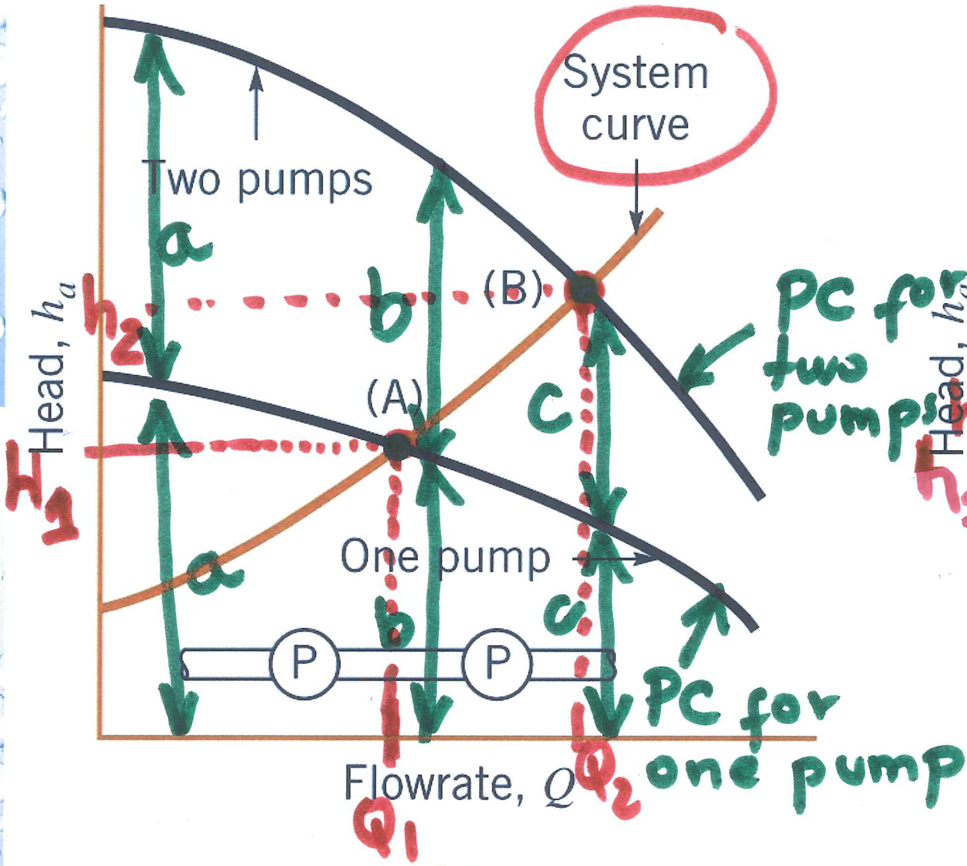
pump manufacturer provides these curves.

- The intersection of both curves represents the **operating point for the system**.

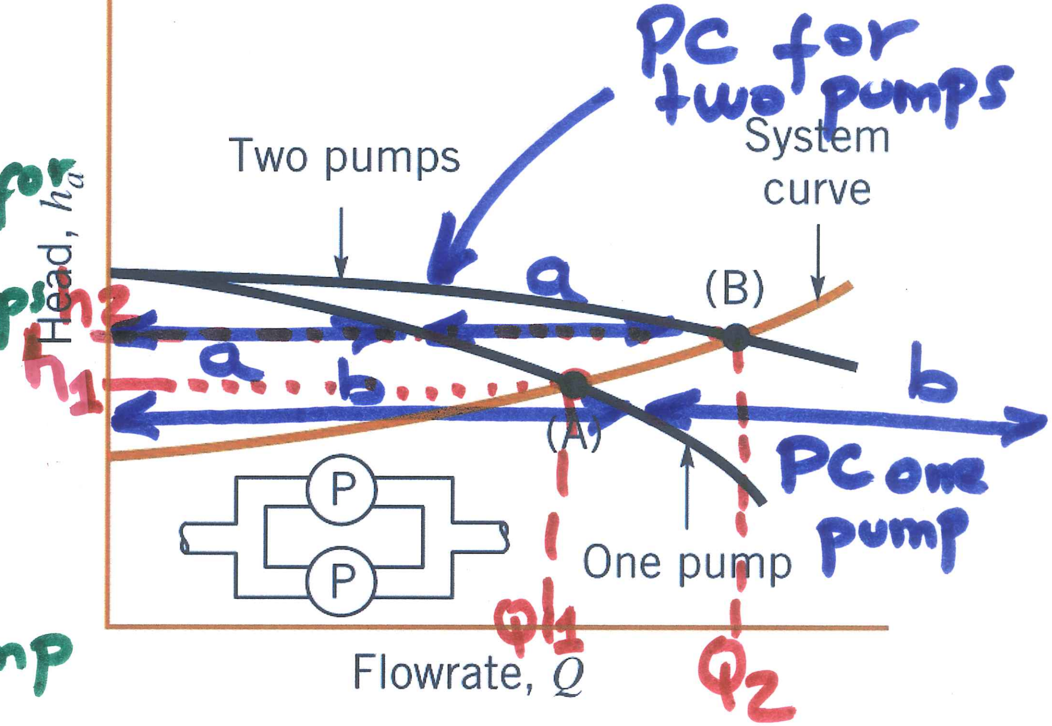
The operating point wanted to be near the best efficiency point (BEP).

Pumps in Series or Parallel

PC : Performance curve



(a)

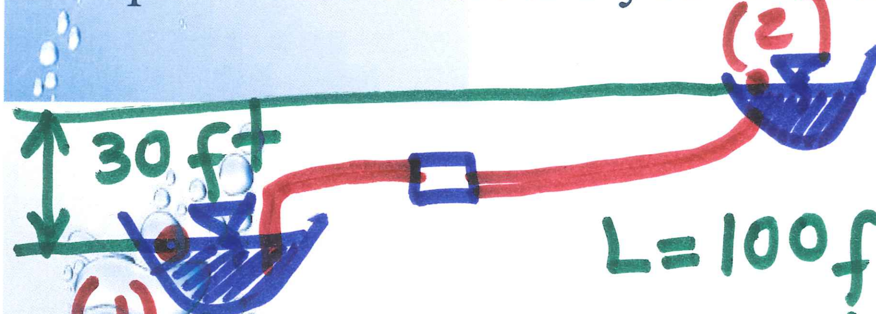


(b)

 Effect of operating pumps in (*a*) series and (*b*) in parallel.

Example of application (P12.30):

A centrifugal pump having the characteristics shown below is used to pump water between two large open tanks through 100 ft of 8-in diameter pipe. The pipeline contains four regular flanged 90° elbows, a check valve, and a fully open globe valve. Other minor losses are negligible. Assume the friction factor $f = 0.02$ for the 100-ft section of pipe. If the static head (difference in height of fluid surfaces in the two tanks) is 30 ft, what is the expected flowrate? Do you think this pump is a good choice? Explain



Q=?

L = 100 ft

D = 8-in

$K_{\text{regular flanged } 90^\circ} = 0.3$ (Table 8.2)

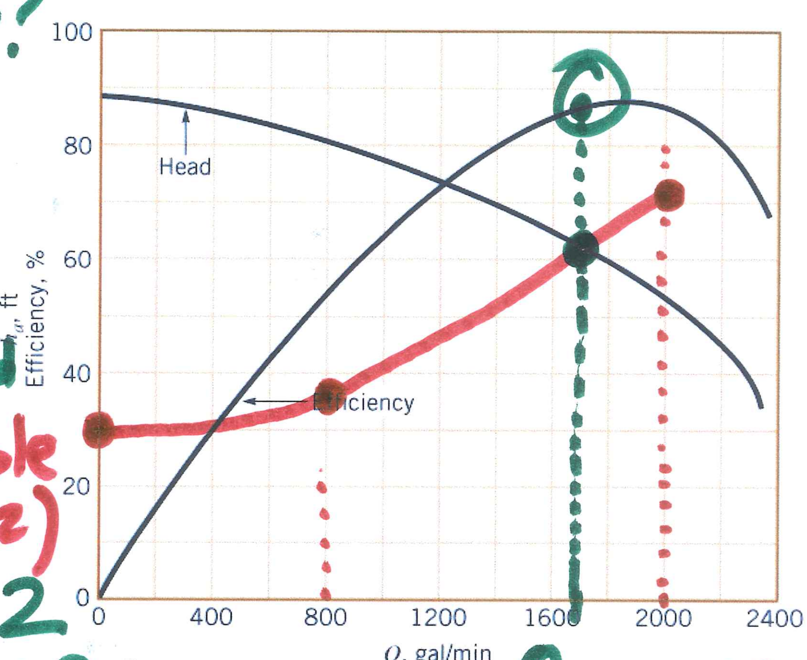
$K_{\text{check valve}} = 2$

$K_{\text{globe valve}} = 10$

$f = 0.02$

$E_1 = E_2$

$$h_p + \cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + z_2 + \left(f\frac{L}{D} + \Sigma K\right)\frac{V^2}{2g}$$



Q = 1740 gal/min

$$h_p = 30 + \left(f\frac{L}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

$$h_p = 30 + \left(0.02 \times \frac{100}{0.667} + 2 + 10 + 4 \times 0.3\right)\frac{Q^2}{2 \times 32.2 \times \left(\frac{\pi}{4} \times 0.667^2\right)^2}$$

$$h_p = 30 + 2.06\left(Q_{(ft^3/s)}\right)^2$$

Convert Q to gal/min to use performance curve

$$h_p = 30 + 1.02 \times 10^{-5}\left(Q_{(gal/min)}\right)^2$$

$$\boxed{Q = 1740 \text{ gal/min}}$$

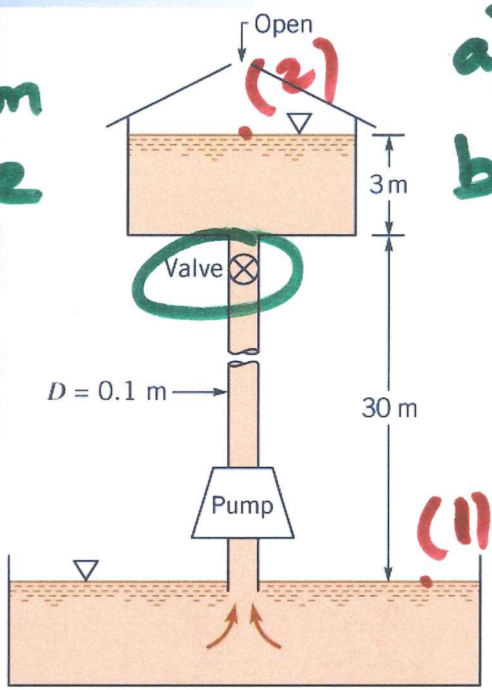
because Q is near peak efficiency, yes, this pump is a good choice.

Example of application (12.31):

In a chemical processing plant a liquid is pumped from an open tank, through a 0.1-m-diameter vertical pipe, and into another open tank as shown in Fig. P12.31 (a). A valve is located in the pipe, and the minor loss coefficient for the valve as a function of the valve setting is shown in Fig. P12.31(b). The pump head-capacity relationship is given by the equation $h_a = 52.0 - 1.01 \times 10^3 Q^2$ with h_a in meters when Q is in m^3/s. Assume the friction factor $f = 0.02$ for the pipe, and all minor losses, except for the valve, are negligible. The fluid levels in the two tanks can be assumed to remain constant. (a) Determine the flow rate with the valve wide open. (b) Determine the required valve setting (percent open) to reduce the flowrate by 50%.

$D = 0.1\,m$

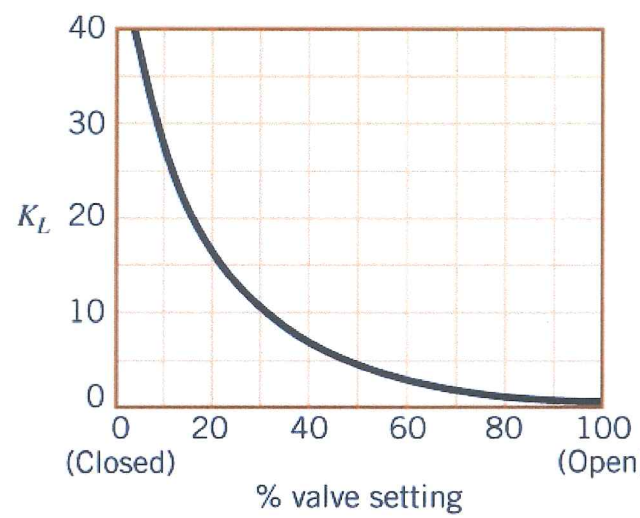
$f = 0.02$



(a)

a) $Q_0 = ?$ valve open $K = 1$

b) $Q = \frac{Q_0}{2}$, % of valve opening



(b)

System curve

$$hp = 33 + \left(\frac{0.02\times 30}{0.1} + k\right)\frac{Q^2}{2\times 9.8\times\left(\frac{\pi\times 0.1^2}{4}\right)^2}$$

a) $k = 1.0$

$$\boxed{hp = 33 + 5.78\times 10^{3}\left[Q(m^3/s)\right]^2}$$

Pump head-capacity curve

$$\boxed{ha = 52 - 1.01\times 10^{3}\, Q^2_{(m^3/s)}}$$

$$ha = hp \rightarrow \boxed{Q = 0.0529\ m^3/s}$$

b) $Q = \frac{0.0529}{2} = 0.026\ m^3/s \rightarrow ha = 50.6\,m$

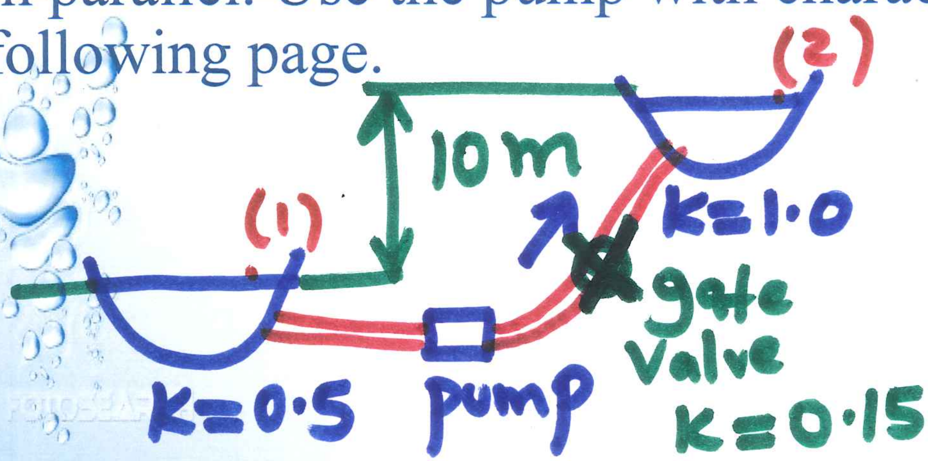
System curve $\quad 50.6 = 33 + \left(\frac{0.02\times 30}{0.1} + k\right)\frac{0.026^2}{2g(\pi D^2/4)^2}$

$$\boxed{k = 24.3}$$

$$\boxed{\text{Valve opening } 13\%}$$

Example of application

Two reservoirs are connected by a 300-m-long, asphalt-lined, cast-iron pipeline ($\varepsilon = 0.12$ mm), 40 cm in diameter. The minor losses include a sharp-edged entrance ($k = 0.5$), the exit ($k = 1.0$), and a fully-open gate valve ($k = 0.15$). The elevation difference between the reservoirs is 10 m, and the water temperature is 10°C ($v = 1.31 \times 10^{-6}$ m²/s). Determine the discharge, head, and the efficiency using (a) one pump, (b) two pumps in series, (c) two pumps in parallel, (d) three pumps in series, and (e) three pumps in parallel. Use the pump with characteristics depicted in the following page.



$L = 300$ m

$\varepsilon = 0.12$ mm

$D = 0.4$ m

$T = 10°C$

$\nu = 1.31 \times 10^{-6}$ m²/s

System curve: $h_p = 10 + \left(\frac{f \times 300}{0.4} + 1.65\right)\frac{V^2}{2 \times 9.8}$

Q (L/s)	V (m/s)	$Re = V \cdot D / \nu$	f	h_p
0	0	—	—	10
100	0.80	2.44×10^5	0.0175	10.5
300	2.39	7.3×10^5	0.016	14.0
500	3.98	1.22×10^6	0.0155	20.7
700	5.57	1.70×10^6	0.0155	31.0

a) $Q = 420$ L/s, $h = 18$ m, $\eta = 40\%$

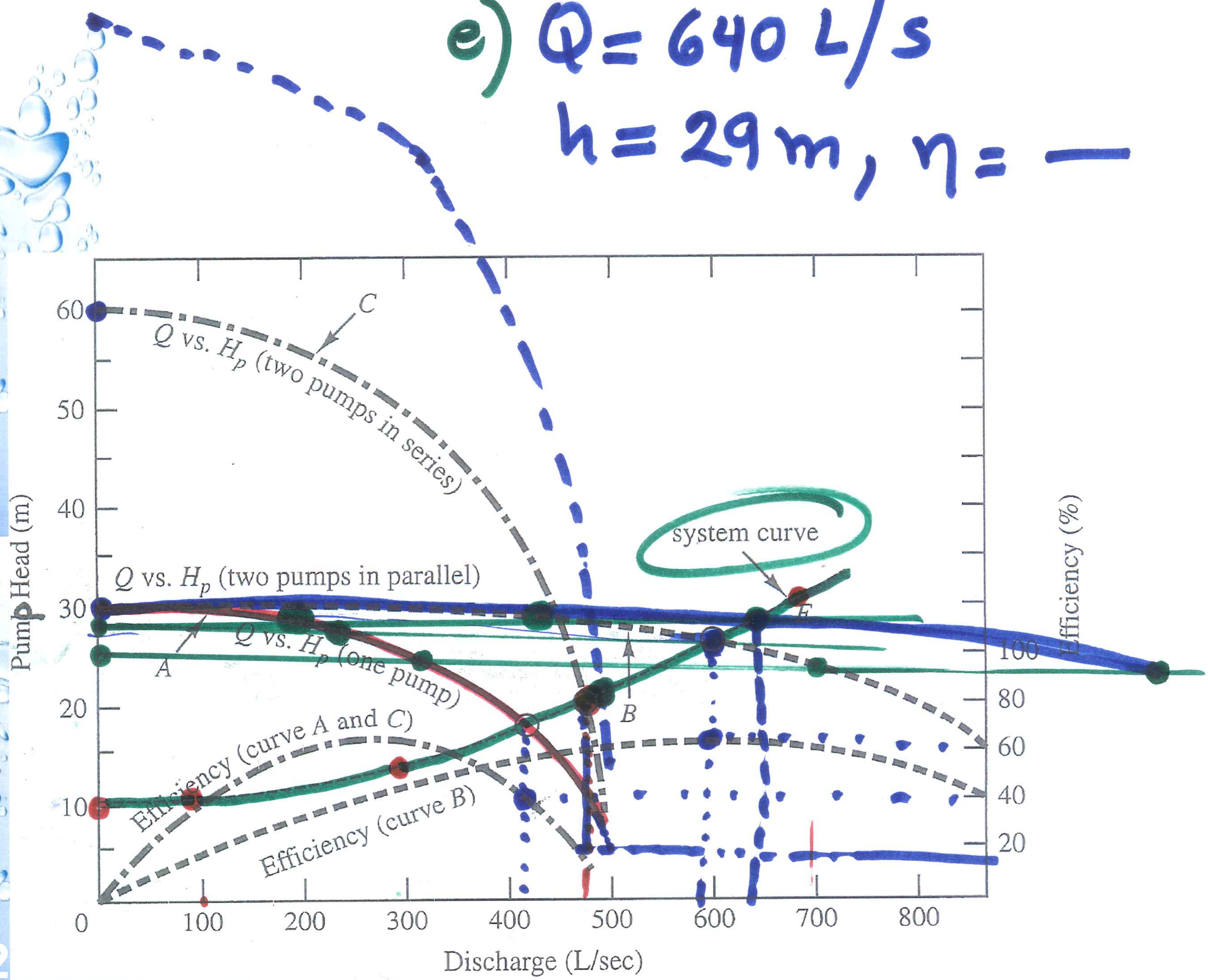
b) $Q = 470$ L/s, $h = 20$ m, $\eta = 15\%$

c) $Q = 590$ L/s, $h = 26$ m, $\eta = 60\%$

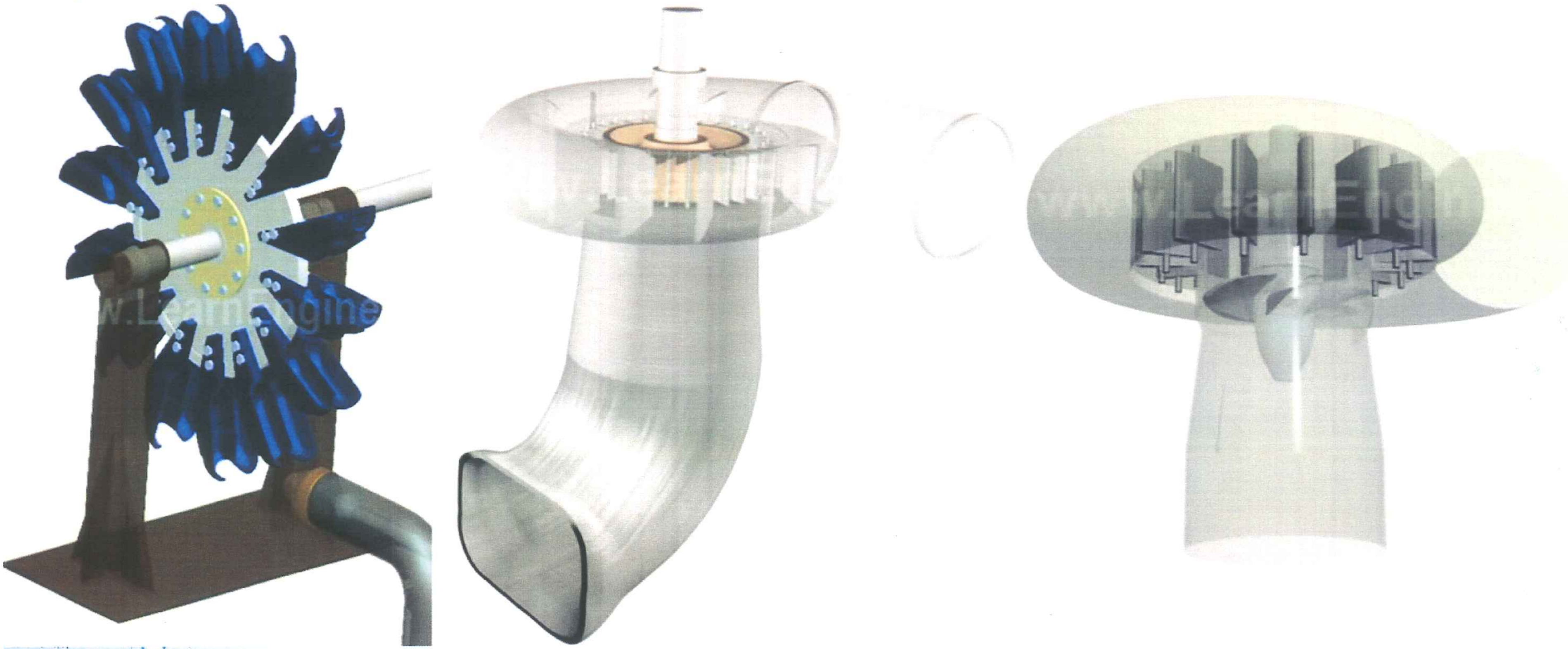
d) $Q = 500$ L/s, $h = 21$ m, $\eta =$ —

e) $Q = 640\ L/s$

$h = 29\ m$, $\eta = $ —



Lecture 19, 03/03/2014
Turbines

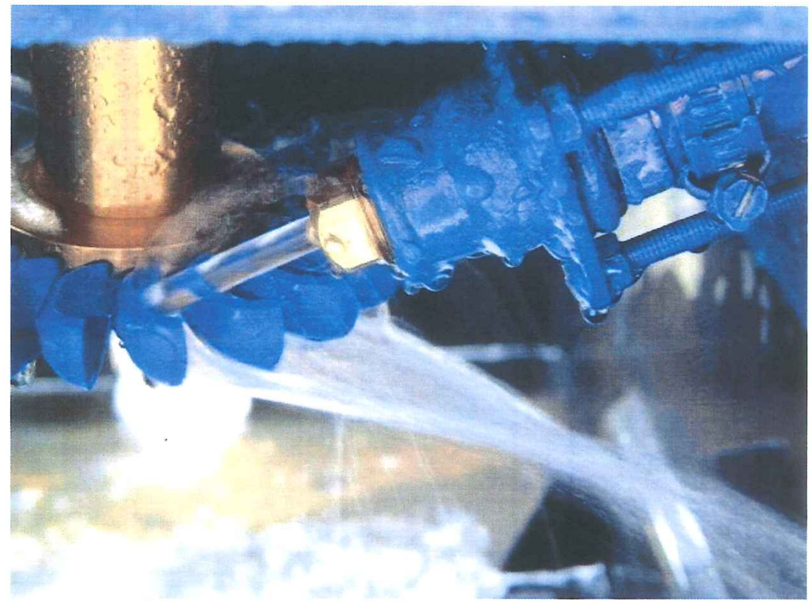


Comparison of Pelton, Francis & Kaplan Turbine

https://www.youtube.com/watch?v=k0BLOKEZ3KU

Arturo Leon, Oregon State University

Impulse Turbines



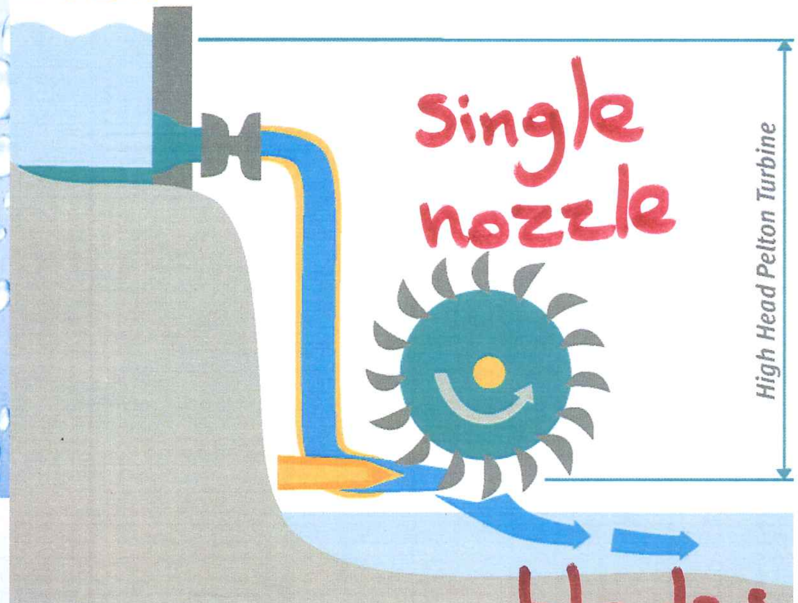
Turbines

- Turbines are devices that extract energy from a flowing fluid.
- The two basic types of hydraulic turbines are **impulse and reaction turbines.**
  - **Impulse turbines**: High-head, low flowrate devices.

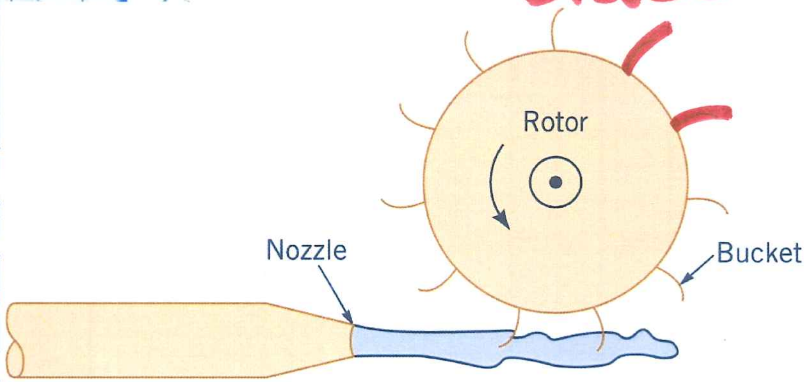
    >~ 100 - 200 ft
  - **Reaction turbines**: Low-head, high flowrate devices.

    <~ 100 ft

Impulse Turbines



blades



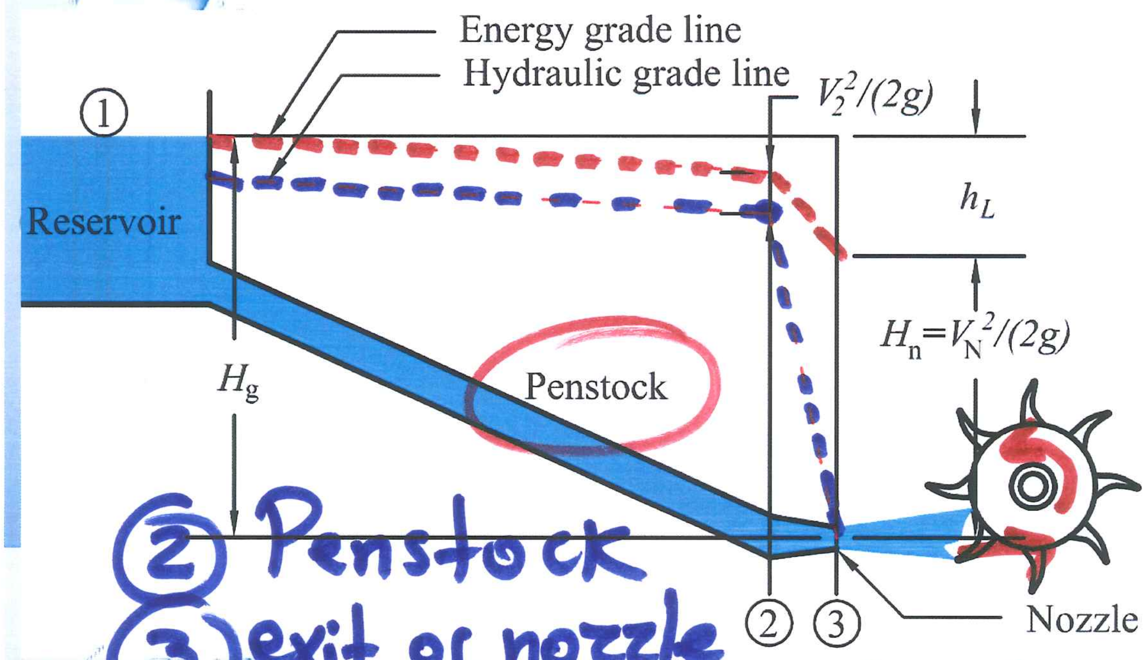
multiple nozzles

100

37 The Pelton wheel is a classical example of an impulse turbine

Impulse Turbines (Definition of variables)

$E_1 = E_3$



② Penstock
③ exit of nozzle

Gross head (H_g)

Penstock length (L)

Nozzle velocity coefficient (C_V)

Turbine efficiency (η_t)

Generator efficiency (η_g)

overall hydroelectric unit efficiency (η)

net head

$$P = \eta\gamma Q(H_g - h_L)$$

$$h_L = \frac{Q^2}{2gA_2^2}\underbrace{\left[f\frac{L}{D_2} + \sum k_{1-2} + k_N\left(\frac{A_2}{A_N}\right)^2\right]}_{C_L}$$

$$\eta\ (= \eta_t \times \eta_g)$$

$$k_N = \frac{1}{C_V^2} - 1$$

Impulse Turbines (Cont.)

$$C_L = f\frac{L}{D_2} + \sum k_{1-2} + k_N\left(\frac{A_2}{A_N}\right)^2$$ for an impulse turbine

If P is specified

$$Q_{\text{opt}} = \frac{45}{38}\left(\frac{P}{\eta\gamma H_g}\right) \qquad \frac{(C_L)_{\text{opt}}}{A_2^2} \underset{=}{\leq} \frac{14}{45}\frac{gH_g}{Q^2}$$

If Q is specified

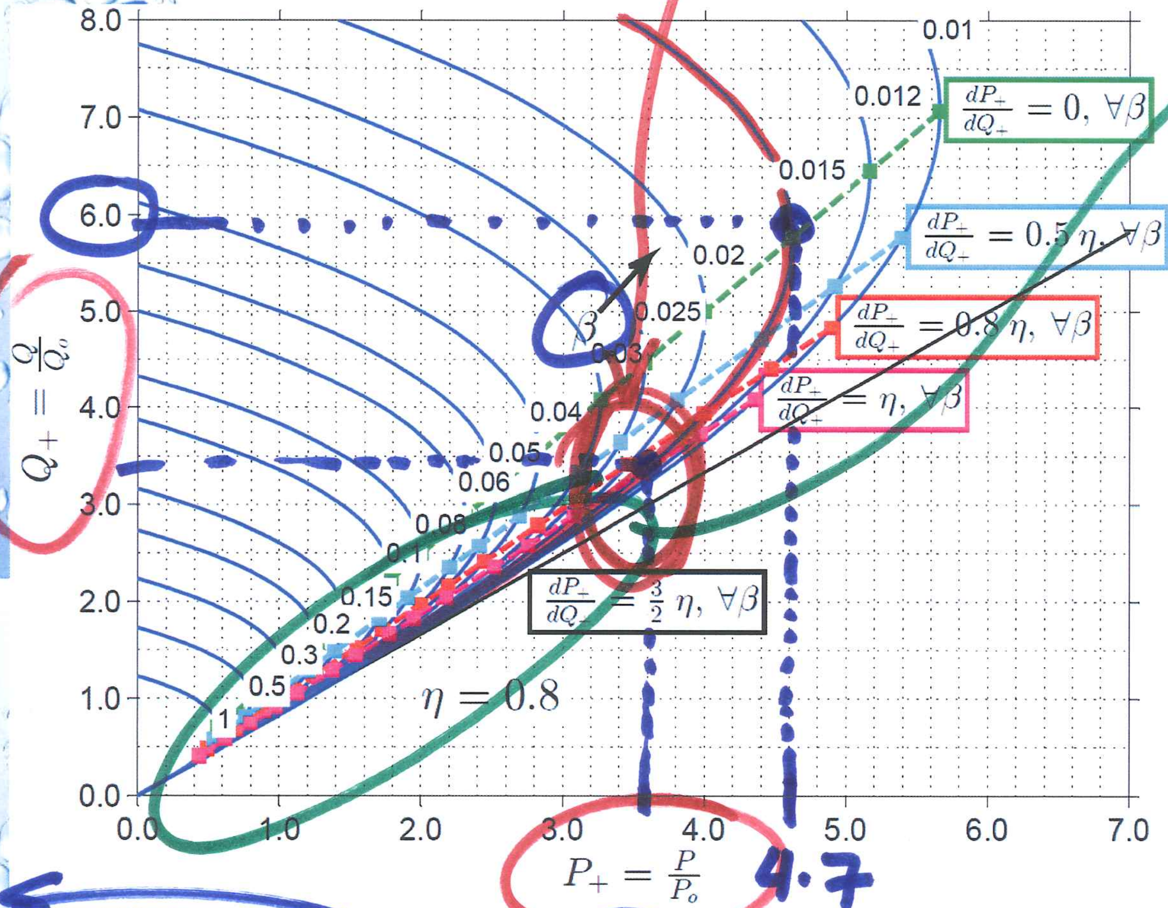
$$\frac{(C_L)_{\text{opt}}}{A_2^2} \underset{=}{\leq} \frac{14}{45}\frac{gH_g}{Q^2} \qquad P_{\text{opt}} = \frac{38}{45}\eta\gamma H_g Q$$

Source: Leon A. S. and Zhu, L (2014). A dimensional analysis for determining optimal discharge and penstock diameter in impulse and reaction water turbines.

Impulse Turbines (Cont.)

optimal flow discharge and optimal penstock diameter

optimal flow discharge



Dimensionless discharge (Q_+) versus dimensionless power (P_+) for $\eta = 0.8$ and a typical range of β for impulse turbines

$P_+ = 4.7$
$Q_+ = 6.0$

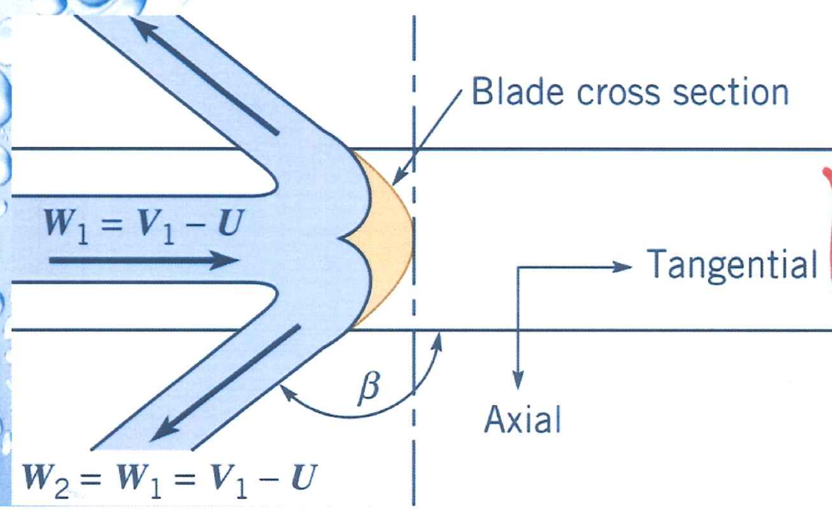
$P_+ = 3.6$
$Q_+ = 3.3$

f(penstock, nozzle)

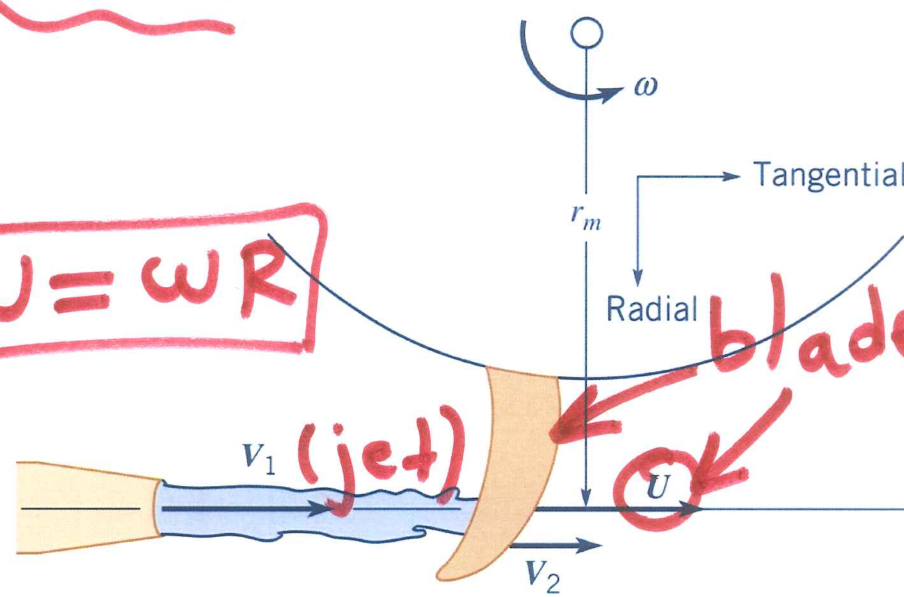
$$\beta = \left(\frac{A_N}{A_2}\right)^2 \left(f\frac{L}{D_2} + \sum k_{1-2} + k_N\left(\frac{A_2}{A_N}\right)^2\right)$$

Source: Leon A. S. and Zhu, L (2014). A dimensional analysis for determining optimal discharge and penstock diameter in impulse and reaction water turbines.

Pelton Wheel Turbine



$U = \omega R$



$$\dot{W}_{shaft} = T_{shaft}\omega = \dot{m}U(U - V_1)(1 - \cos\beta)$$

$\dot{m} = \rho Q$

$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$

$\cos\beta = 180$

For maximum power:

$$\frac{d\dot{W}_{shaft}}{dU} = 0 \qquad 2U - V_1 = 0$$

$\boxed{U = V_1/2}$ $\rightarrow$ $\dot{W}_{shaft} = \rho Q \frac{V_1}{2}\left(-\frac{V_1}{\cancel{2}}\right)(\cancel{2})$

$$\boxed{\dot{W}_{shaft\,max} = -\rho Q \frac{V_1^{\,2}}{2}}$$

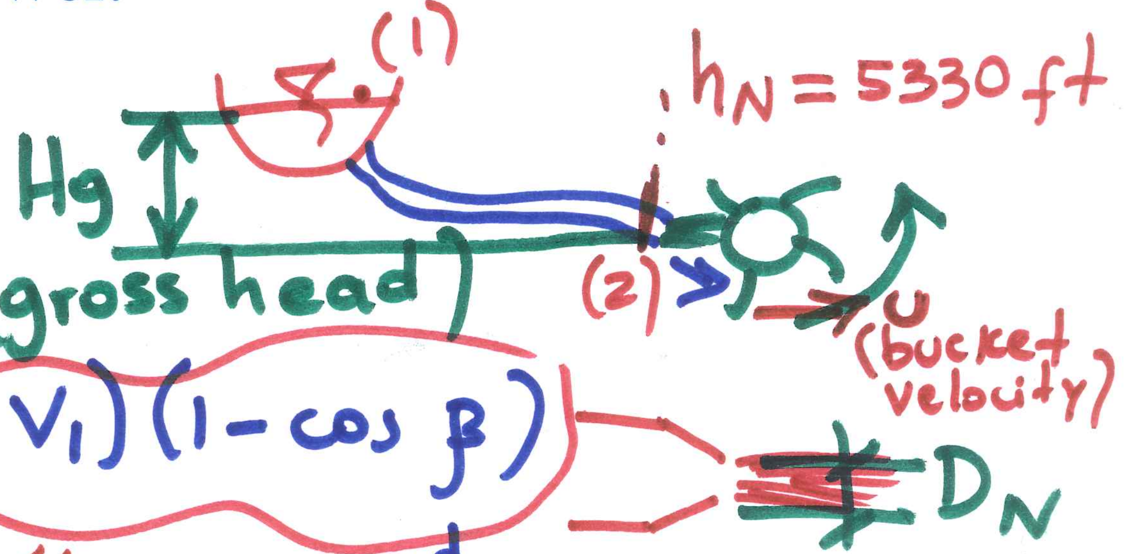
Example of application (P 12.59):

A 10.9-ft diameter Pelton wheel operates at 500 rpm with a total head just upstream of the nozzle of 5330 ft. Estimate the diameter of the nozzle of the single-nozzle wheel if it develops 25,000 horsepower.

$D = 10.9\ ft$

$\omega = 500\ RPM$

$P = 25{,}000\ HP$



$\dot{W}_{shaft} = \rho Q U (U - V_1)(1 - \cos\beta)$

$U = \omega \cdot R = 500 \frac{\cancel{rev}}{min} \left(\frac{1\ min}{60\ s}\right) \frac{2\pi\ rad}{1\ \cancel{rev}} \times \frac{10.9}{2} ft$

$\cos\beta \approx -1$

$U = 286\ ft/s$

$E_1 = E_2$

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$Hg = \frac{V_2^2}{2g} + h_L \rightarrow Hg - h_L = \frac{V_2^2}{2g}$$

Net head (h_N)

$$\frac{V_2^2}{2g} = 5330\text{ ft} \rightarrow \boxed{V_2 = 586\text{ ft/s}}$$

Velocity of jet

In our problem $V_1 = 586$ ft/s

$$-25000\text{ Hp}\left(\frac{550\ \frac{\text{ft-lb}}{\text{s}}}{1\text{ Hp}}\right) = 1.94\ \frac{\text{slugs}}{\text{s}} \times Q \times 286\ \frac{\text{ft}}{\text{s}} \times (286-586)(1-(-1))$$

42.2

$$\boxed{Q = 41.3\ ft^3/s}$$

$$Q = A \cdot V \rightarrow A = \frac{41.3}{586} \checkmark$$

$$\frac{\pi d_N^2}{4} = A \rightarrow \boxed{d_N = 0.3\ ft}$$

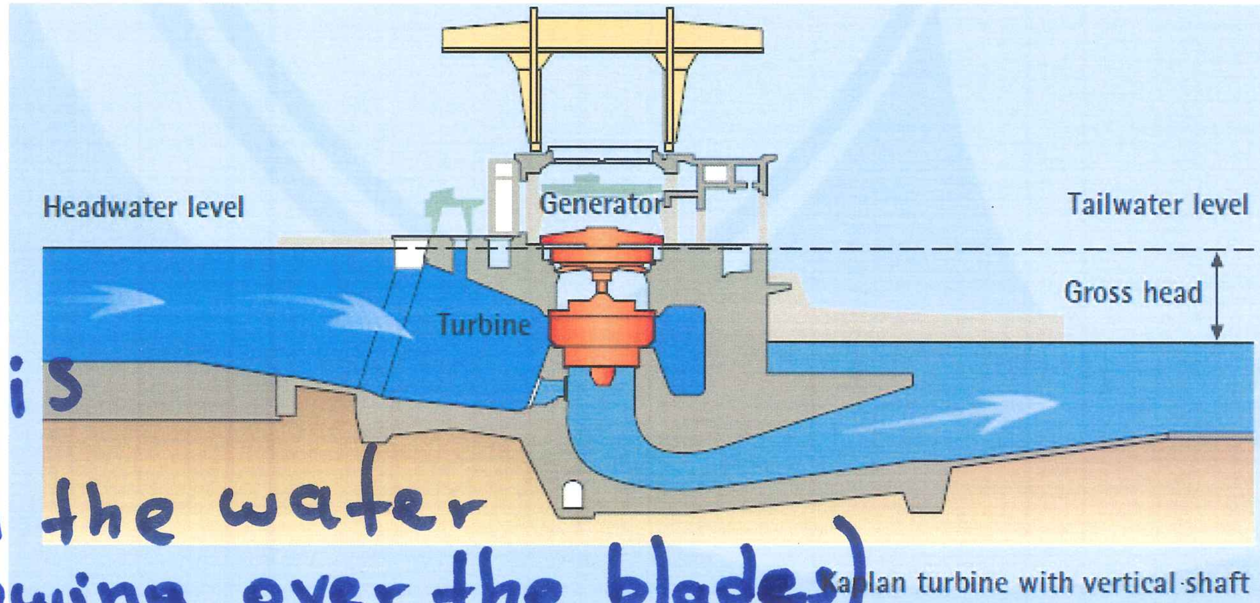
42.3

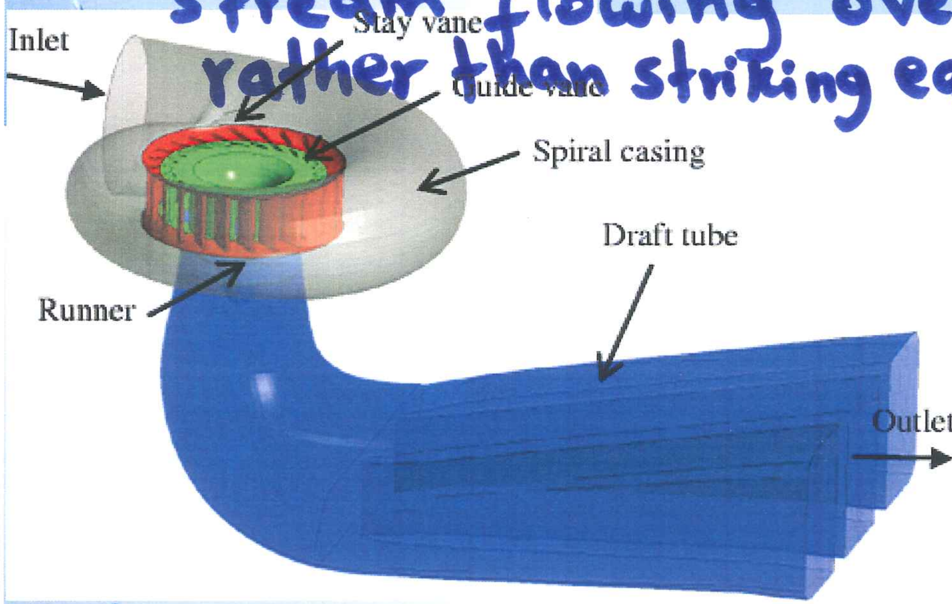
Lecture 20, 03/05/2014
Turbines – Reaction Turbines

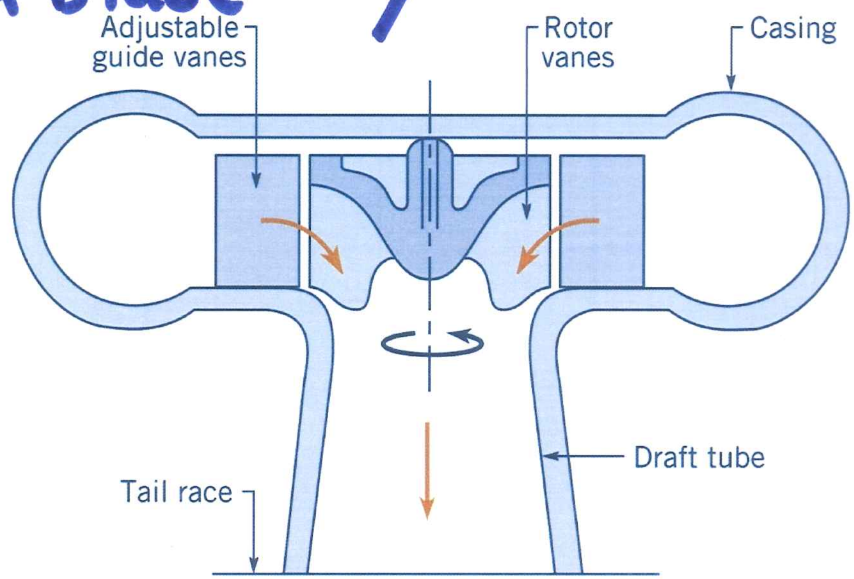
Arturo Leon, Oregon State University

Reaction Turbines

(turbine is located in the water stream flowing over the blades rather than striking each blade)





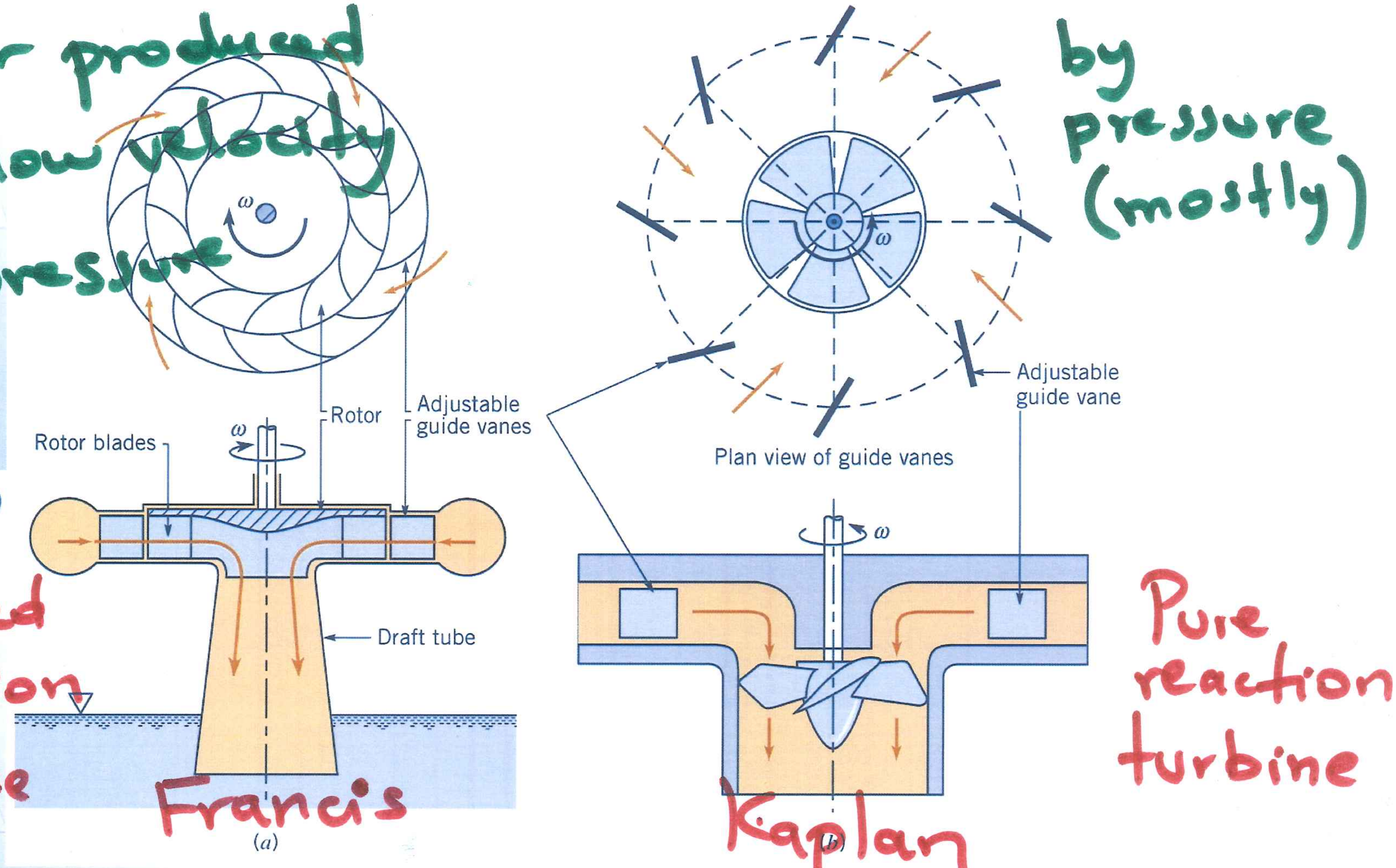


Schematic of a reaction turbine

Reaction Turbines

Power produced by flow velocity and pressure

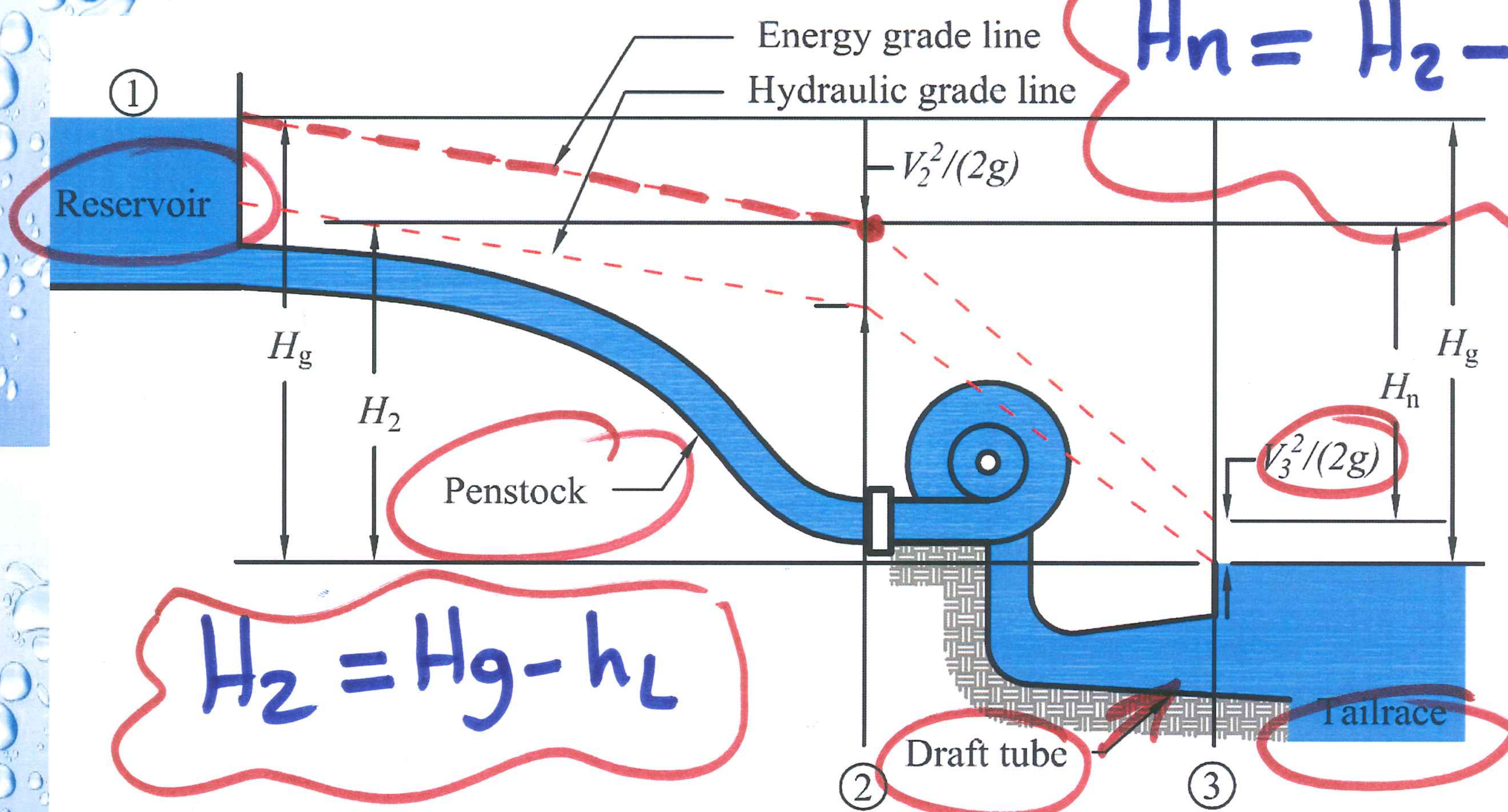
by pressure (mostly)



(*a*) Typical radial-flow Francis turbine. (*b*) typical axial-flow Kaplan turbine.

Reaction Turbines (Definition of variables)

$$H_n = H_2 - \frac{V_3^2}{2g}$$

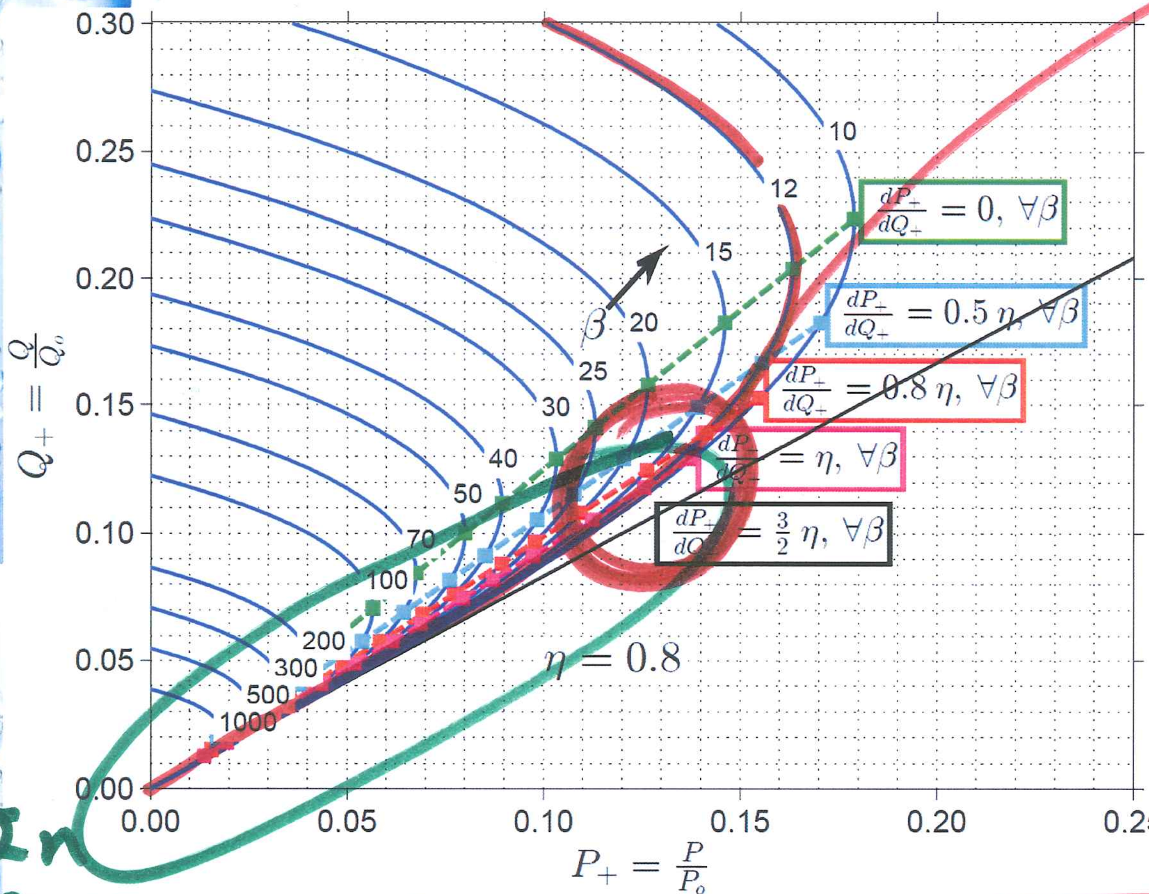


$$H_2 = H_g - h_L$$

A_2 = penstock cross-sectional area

A_d = draft tube cross-sectional area $= A_3$

Reaction Turbines (Cont.)



right before high gradient

we optimize both flow discharge and penstock diameter

Dimensionless discharge (Q_+) versus dimensionless power (P_+) for $\eta = 0.8$ and a typical range of β for reaction turbines

$\beta = f$ (penstock, draft tube)

In linear region, we optimize flow discharge

$$\beta = \left(\frac{A_d}{A_2}\right)^2 \left(f\frac{L}{D_2} + \sum k_{1-2} + \left(\frac{A_2}{A_d}\right)^2 \right)$$

Source: Leon A. S. and Zhu, L (2014). A dimensional analysis for determining optimal discharge and penstock diameter in impulse and reaction water turbines.

Reaction Turbines (Cont.)

$$C_L = f\frac{L}{D_2} + \sum k_{1-2} + \left(\frac{A_2}{A_d}\right)^2$$ for a reaction turbine

If P is specified

P: Power

$$Q_{opt} = \frac{45}{38}\left(\frac{P}{\eta\gamma H_g}\right)$$

$$\frac{(C_L)_{opt}}{A_2^2} \leq \frac{14}{45}\frac{gH_g}{Q^2}$$

=

If Q is specified

Q: flow discharge

$$\frac{(C_L)_{opt}}{A_2^2} \leq \frac{14}{45}\frac{gH_g}{Q^2}$$

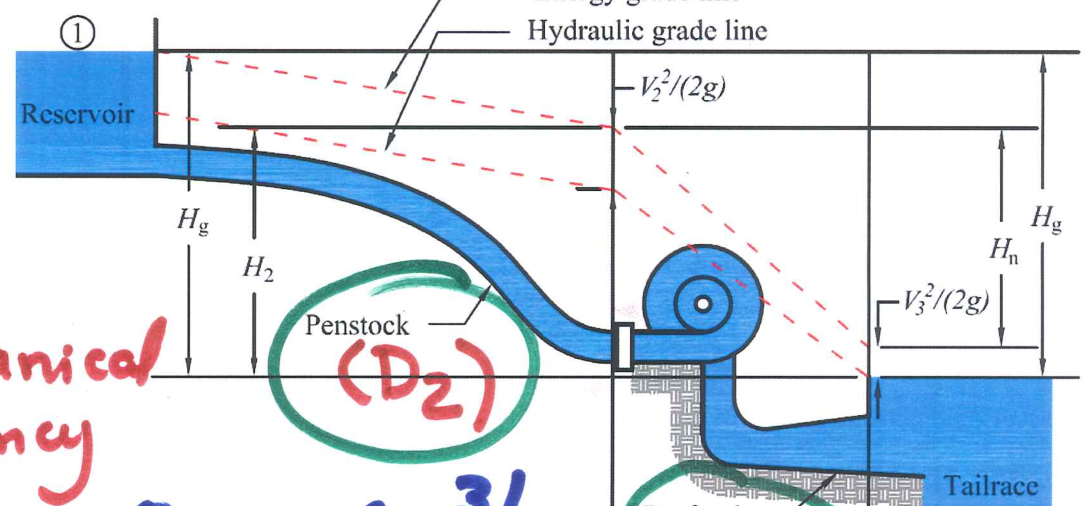
=

$$P_{opt} = \frac{38}{45}\eta\gamma H_g Q$$

Source: Leon A. S. and Zhu, L (2014). A dimensional analysis for determining optimal discharge and penstock diameter in impulse and reaction water turbines.

Example of application:

The site, penstock and nozzle characteristics for a **reaction turbine** are as follows:
Gross head (H_g) = 200 m, Penstock length (L) = 500 m, $A_2/A_d = 1/3$, Sum of coefficients of local losses in penstock = 1.5, roughness height of penstock material = 0.045 mm (commercial steel), kinematic viscosity = 10^{-6} m^2/s, turbine efficiency = 82%, generator efficiency = 90%. If the design flow discharge Q is 0.6 m^3/s, determine the optimal penstock diameter and the electric power that can be extracted using this flow.



$\frac{A_2}{A_d} = 1/3$

turbine efficiency = mechanical efficiency

generator efficiency = 90%

$Q = 0.6\ m^3/s$

Doptimal penstock, Power ??

$$\frac{C_L}{A_2^2} = \frac{14}{45} g \frac{H_g}{Q^2}$$

$$M = \frac{f \times \frac{500}{D_2} + 1.5 + \left(\frac{1}{3}\right)^2}{\left(\frac{\pi D_2^2}{4}\right)^2} - \frac{14}{45} \times 9.8 \times \frac{200}{0.6^2} = 0$$

D_2	A_2	$V_2 = \frac{Q}{A_2}$	Re	f	M
0.5	✓	✓	✓	✓	✓
0.3696 m	49.2				0

guess

iterate until M = 0

$$\boxed{P = \eta \gamma Q (H_g - h_L)}$$

Eq. (6) in paper (Blackboard)

$$P = \eta \gamma Q \left(H_g - C_L \frac{Q^2}{2gA_2^2} \right)$$

$$C_L = 17.60$$

$$\boxed{P = 787.01 \text{ kW}}$$

$$\eta = \eta_t \times \eta_g = 0.82 \times 0.90$$
$$= 0.738$$

49.3