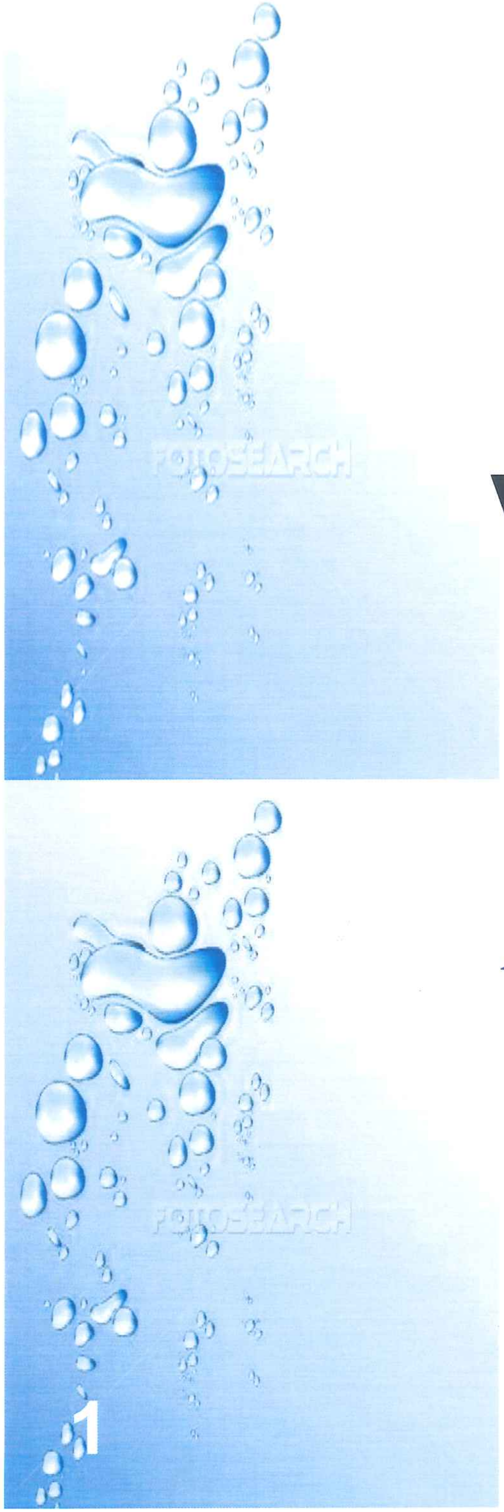
①



Viscous Flow in Pipes
Lecture 1, 01/08/2014

Arturo Leon, Oregon State University

2

Learning Objectives of the section of Pipe flows

(1) Identify and understand various characteristics of flows in pipes

(2) Discuss the main properties of laminar and turbulent flows and appreciate their differences

(3) Calculate losses in straight portions of pipes as well as those in various pipe system components

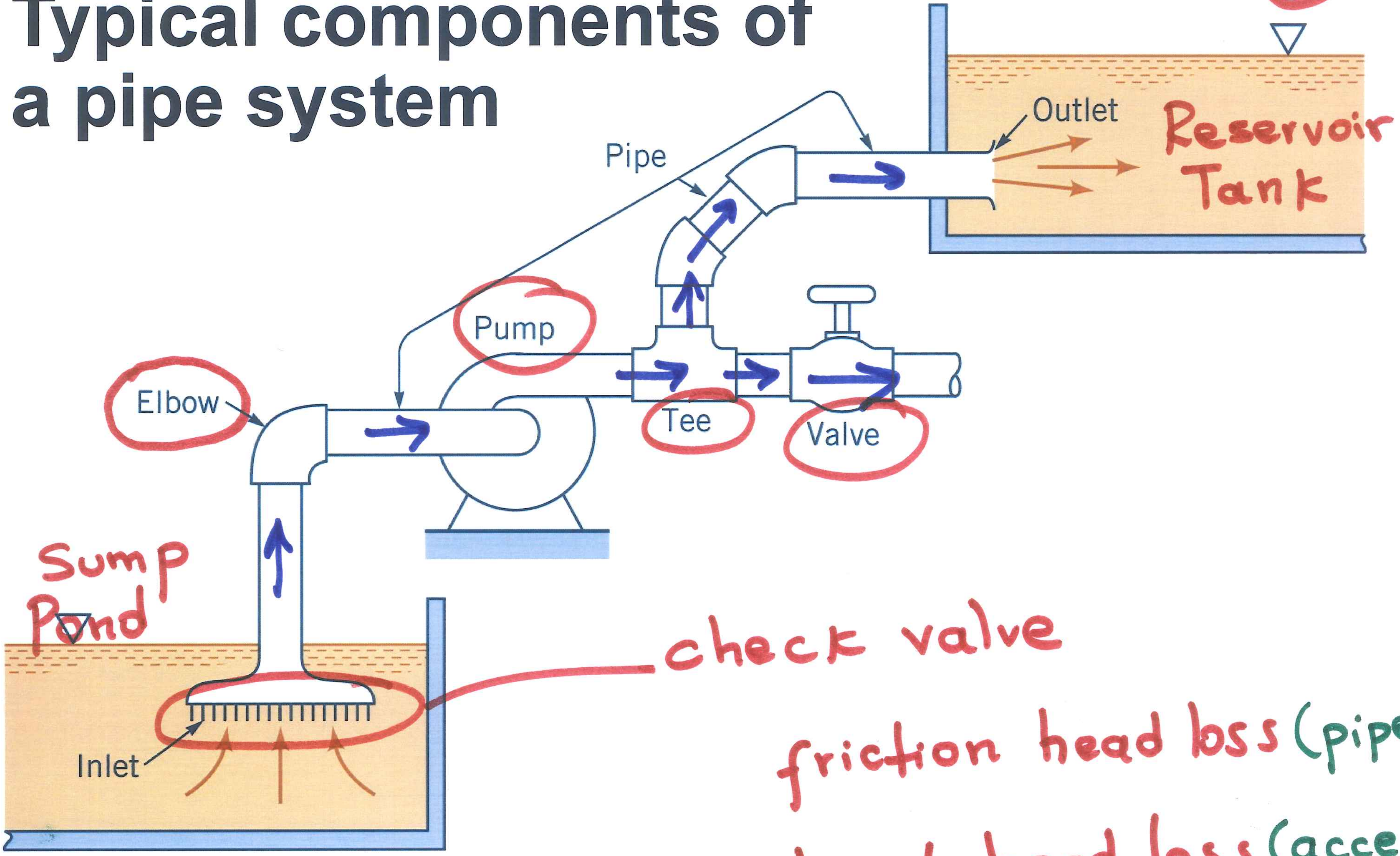
(4) Apply appropriate equations and principles to analyze a variety of pipe flow situations

(5) Predict the flow rate in a pipe by use of common flow meters

turbulent flow

Video 8.1
© A. M. Fincham

Typical components of a pipe system



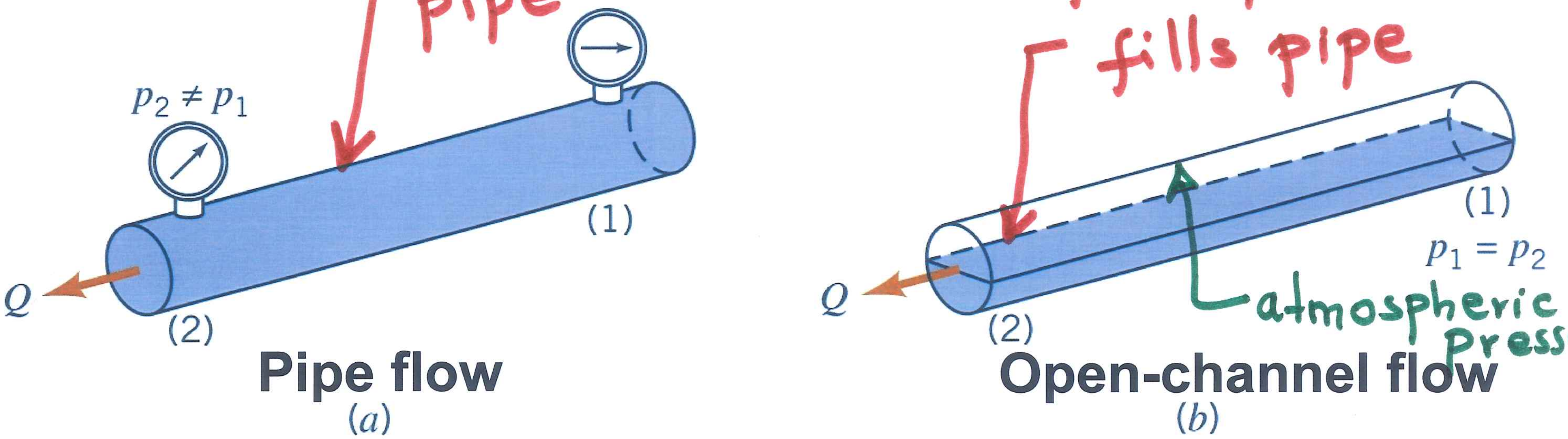
check valve

friction head loss (pipe)

Local head loss (accessories)

Figure 8.1

General Characteristics of pipe flow



fluid fills the pipe

fluid partially fills pipe

atmospheric pressure

Pipe flow
(a)

Open-channel flow
(b)

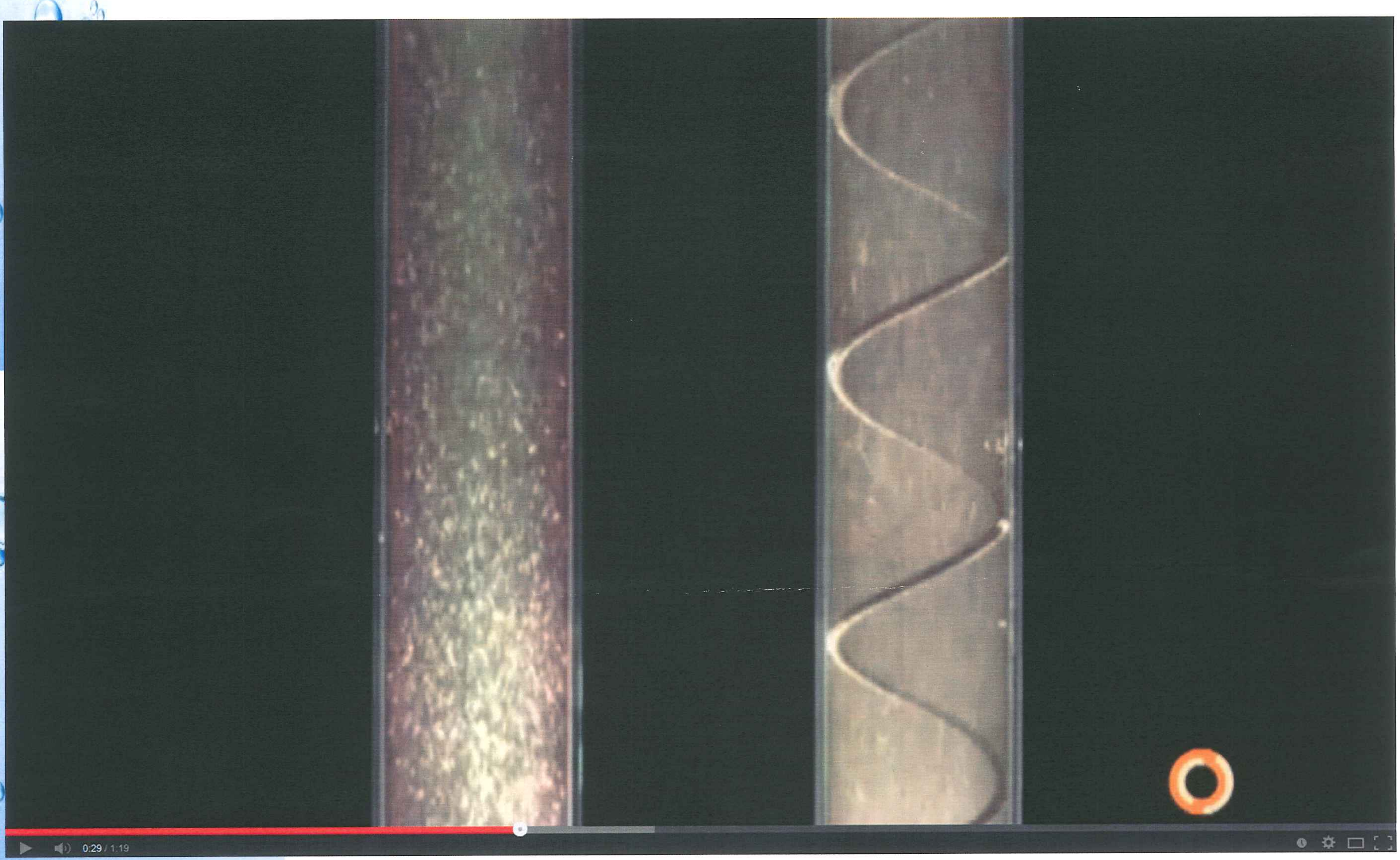
Figure 8.2

→ main driving force is pressure

→ driving force is gravity

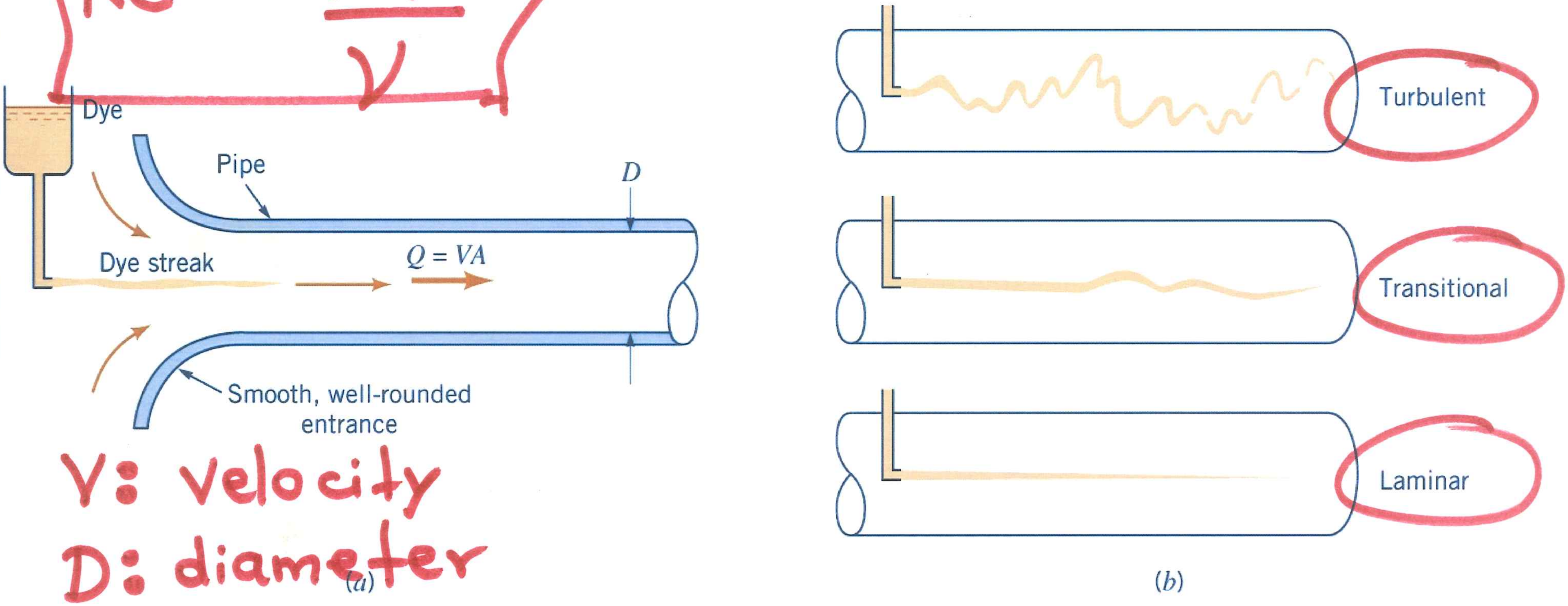
Laminar or Turbulent Flow?

(http://www.youtube.com/watch?v=WG-YCpAGgQQ)



Laminar or Turbulent Flow?

$$\boxed{Re = \frac{V \cdot D}{\nu}}$$



(a)

(b)

Figure 8.3

V: velocity

D: diameter

ν: kinematic viscosity

Typical dye streaks

Pipe flows

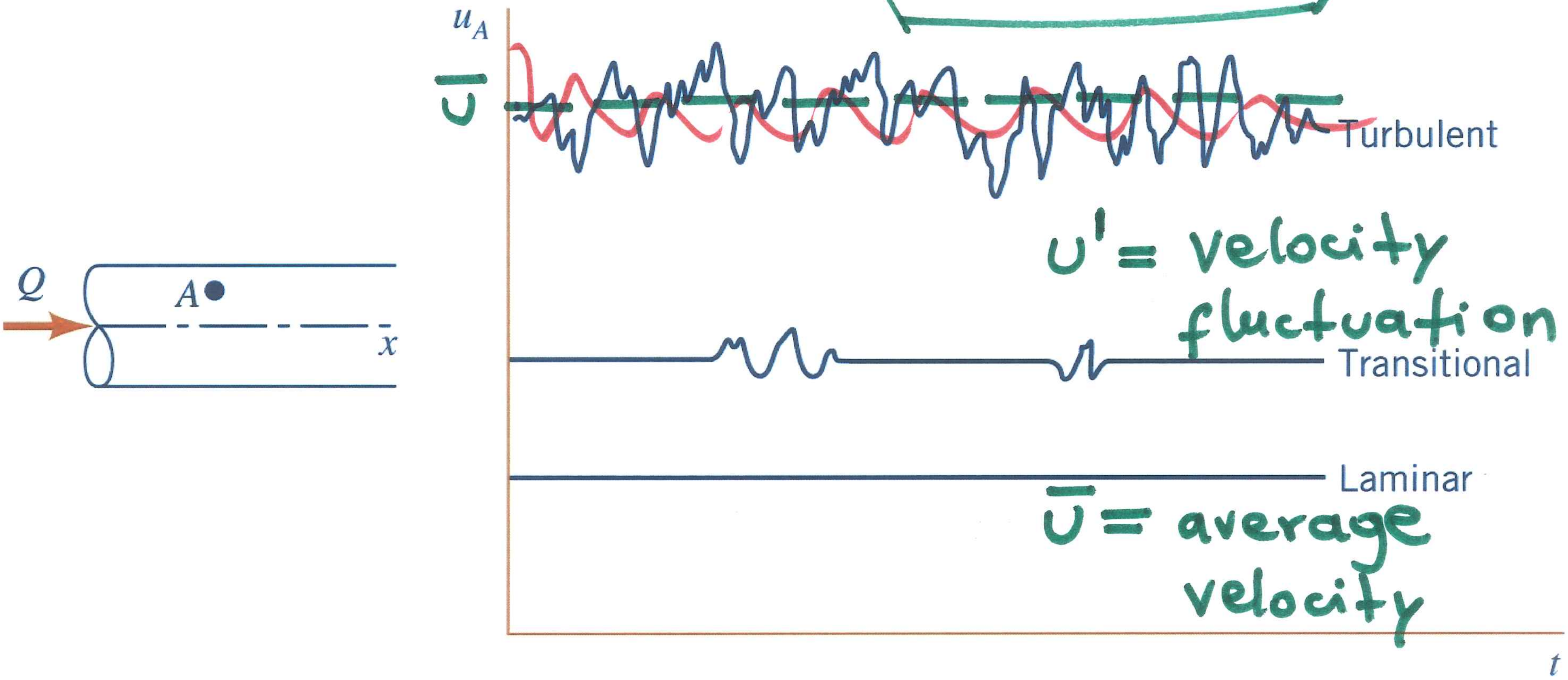
$Re \leq 2100$ (Laminar)

$2100 < Re \leq 4000$ (Transit.)

$Re > 4000$ (Turbulent)

Time dependence of fluid velocity at a point

$$\boxed{u = \bar{u} + u'}$$



$u' =$ velocity fluctuation

$\bar{u} =$ average velocity

Figure 8.4

Example (Proposed problems):

8.9 WILEY (See Fluids in the News article titled "Nanoscale Flows," Section 8.1.1.) ⓐ Water flows in a tube that has a diameter of $D = 0.1$ m. Determine the Reynolds number if the average velocity is 10 diameters per second. ⓑ Repeat the calculations if the tube is a nanoscale tube with a diameter of $D = 100$ nm.

$$Re = \frac{V \cdot D}{\nu} = \frac{10 \times 0.1 \cancel{m}/\cancel{s} \times 0.1 \cancel{m}}{1.12 \times 10^{-6} \cancel{m^2/s}}$$

$$\boxed{Re = 89{,}286}$$

Turbulent flow

$T = 15.6° \rightarrow \nu = 1.12 \times 10^{-6}\ m^2/s$

$D = 100\ nm = 100 \times 10^{-9}\ m$

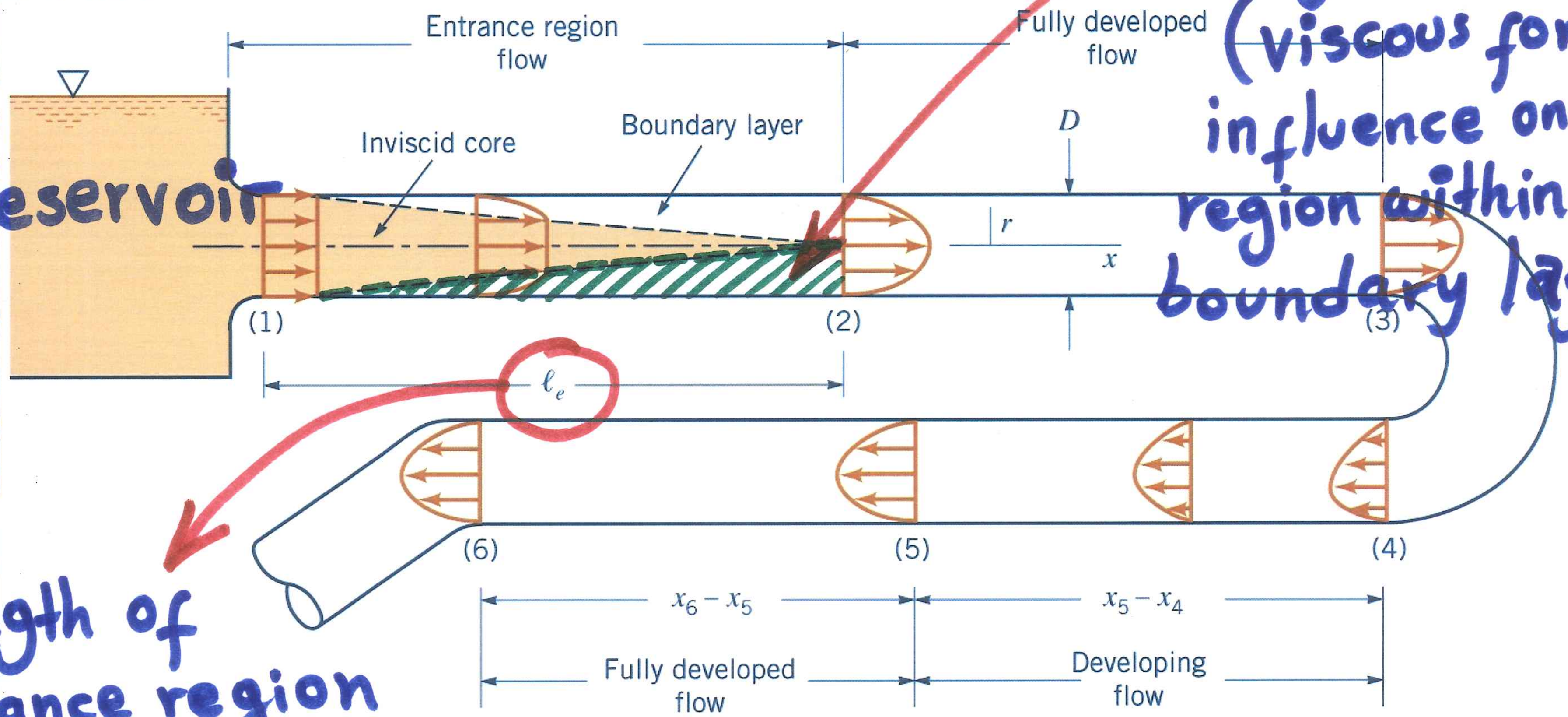
$D = 10^{-7}\ m$

$$Re = \frac{10 \times 10^{-7} \times 10^{-7}}{1.12 \times 10^{-6}}$$

$$\boxed{Re = 8.9 \times 10^{-8}}$$

Laminar flow

Fully Developed Flow



Reservoir

boundary layer (viscous forces influence only region within boundary layer)

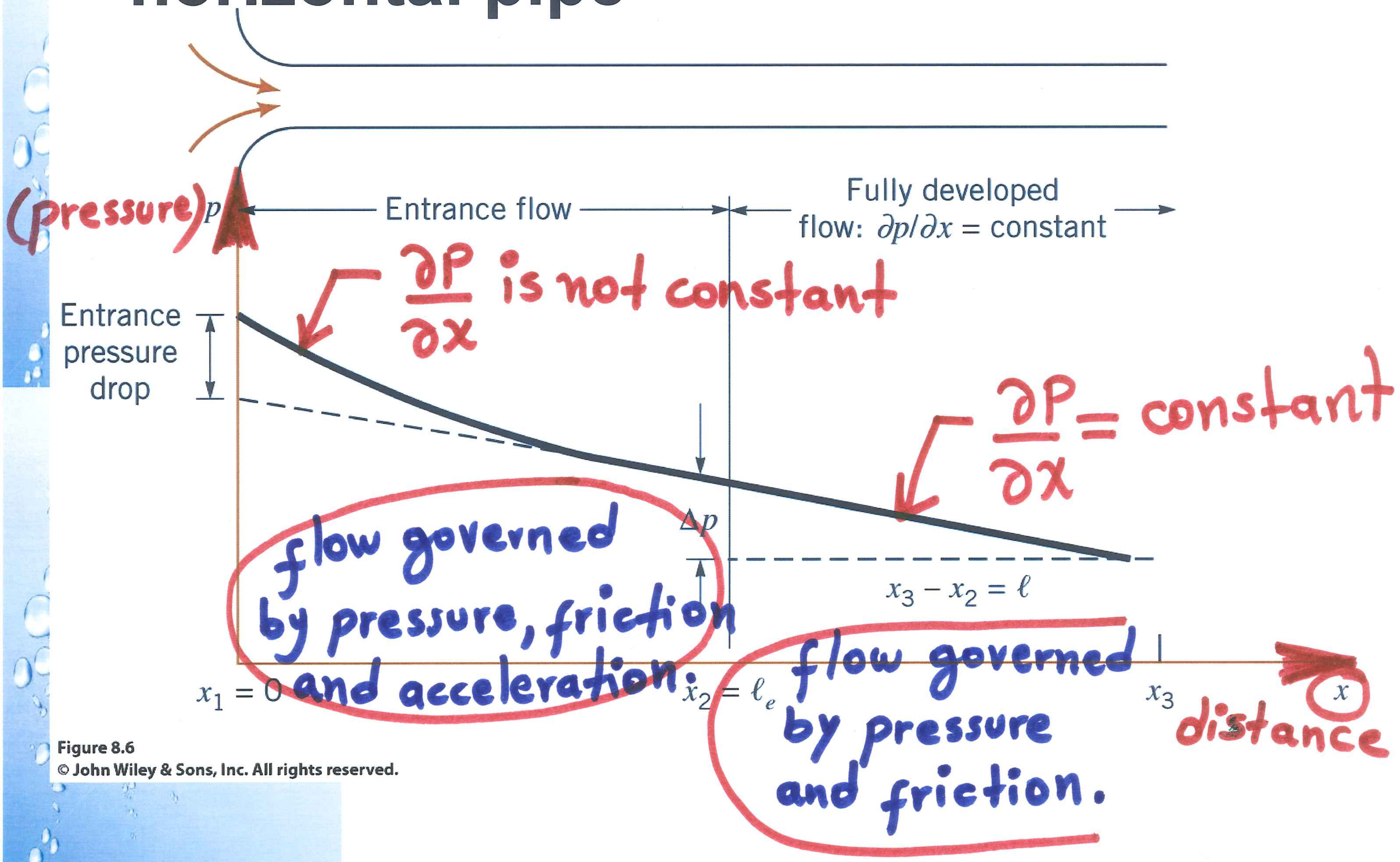
Length of entrance region

Figure 8.5

$$\boxed{\frac{\ell_e}{D} = 0.06\ \mathrm{Re}}\ \text{for laminar flow}$$

$$\boxed{\frac{\ell_e}{D} = 4.4\ (\mathrm{Re})^{1/6}}\ \text{for turbulent flow}$$

Pressure distribution along a horizontal pipe



$\frac{\partial P}{\partial x}$ is not constant

$\frac{\partial P}{\partial x} =$ constant

flow governed by pressure, friction and acceleration.

flow governed by pressure and friction.

Figure 8.6

Viscous Flow in Pipes
Lecture 2, 01/10/2014

Arturo Leon, Oregon State University

Energy considerations

elevation head

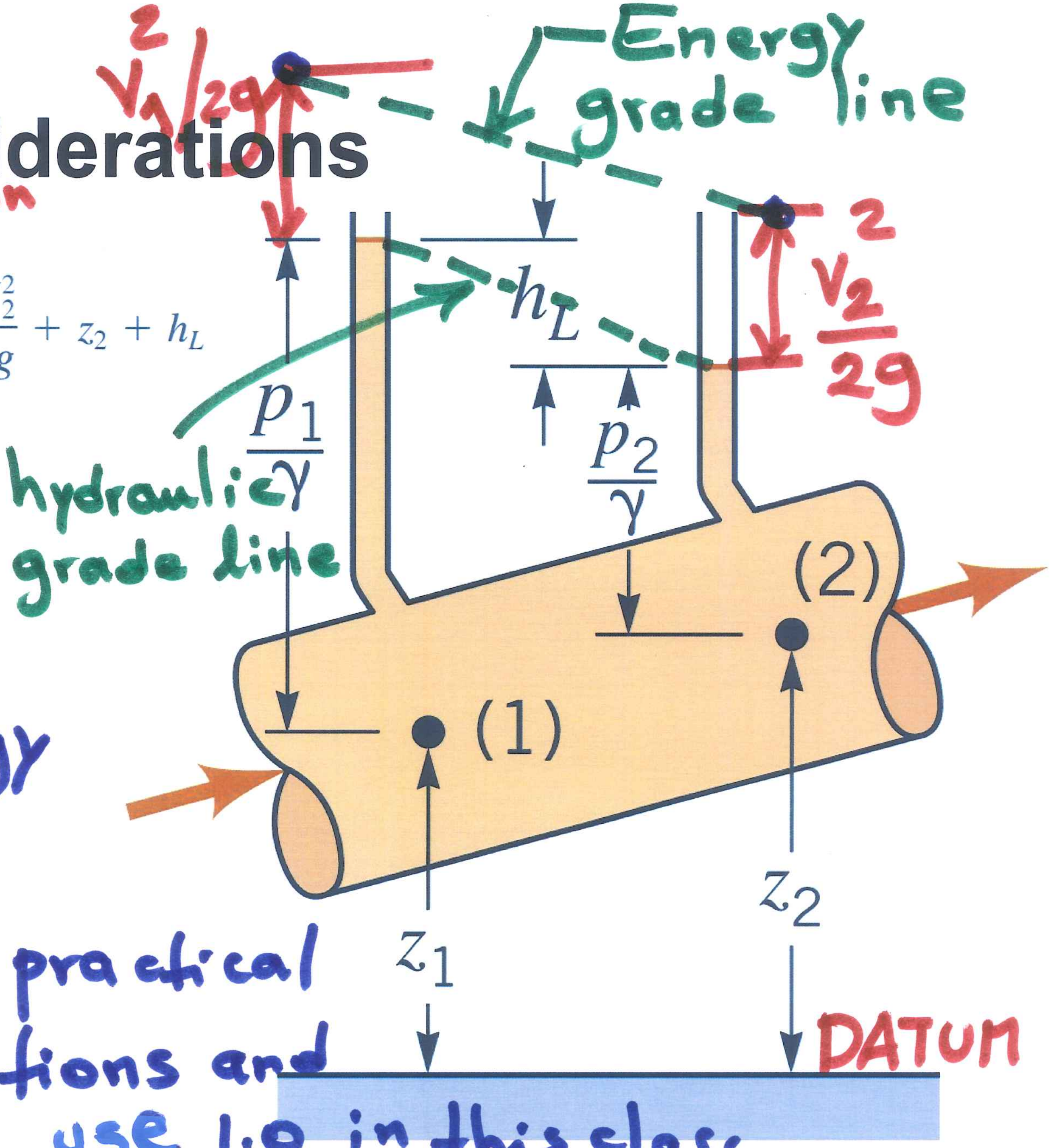
$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

Pressure head

velocity head

α = Kinetic energy coefficient

α = 1.0 (most practical applications and we will use 1.0 in this class



bered 8 p414b

Major losses

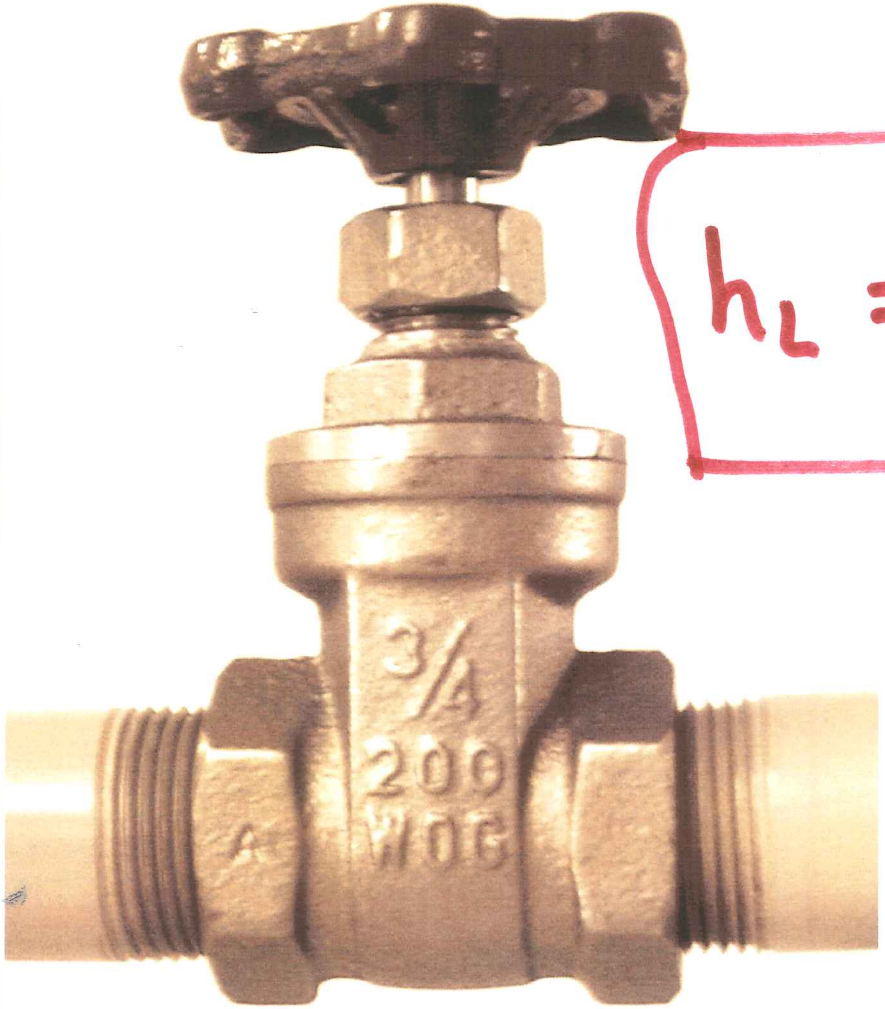
$$\boxed{(h_L)_{total} = h_{L_{major}} + h_{L_{minor}}}$$

Unnumbered 8 p429a
Photograph courtesy of CorrView

In design account for corrosion build-up and sediments

Minor losses (due to pipe accessories)

$$\boxed{h_L = k\frac{V^2}{2g}}$$



(*a*)

Figure 8.21a
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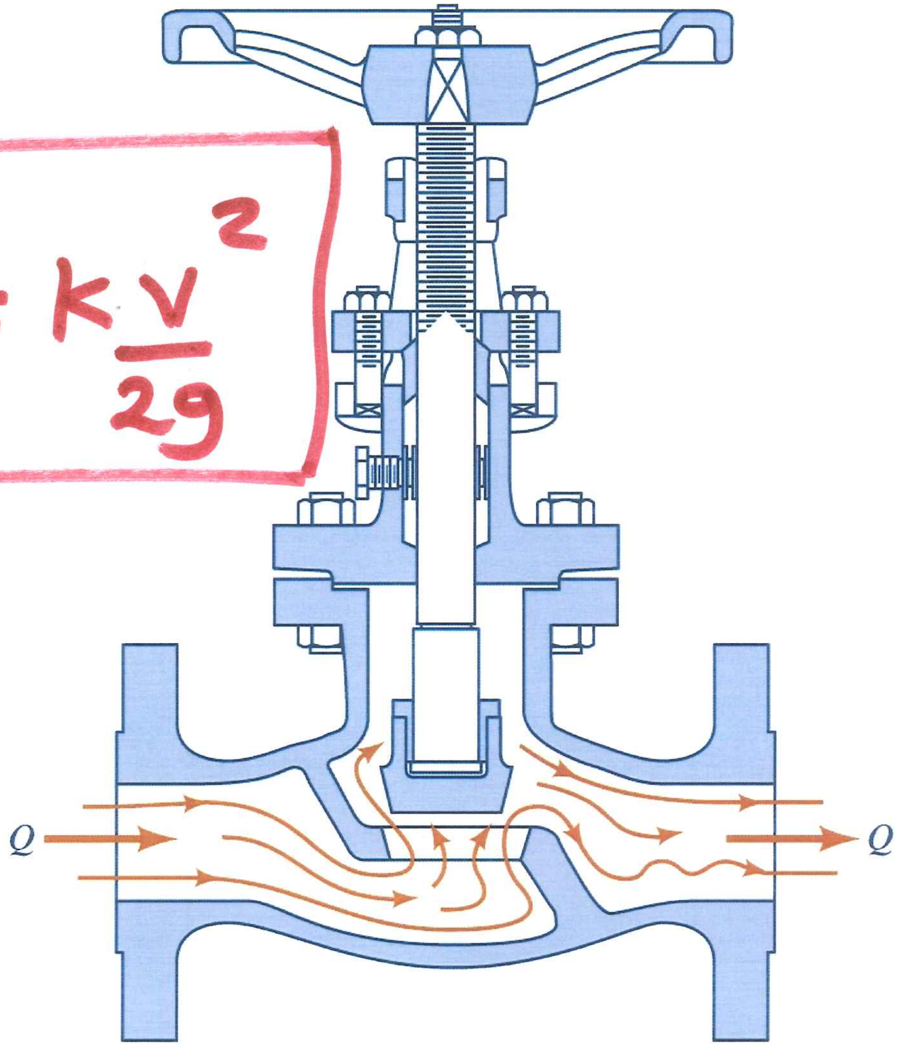
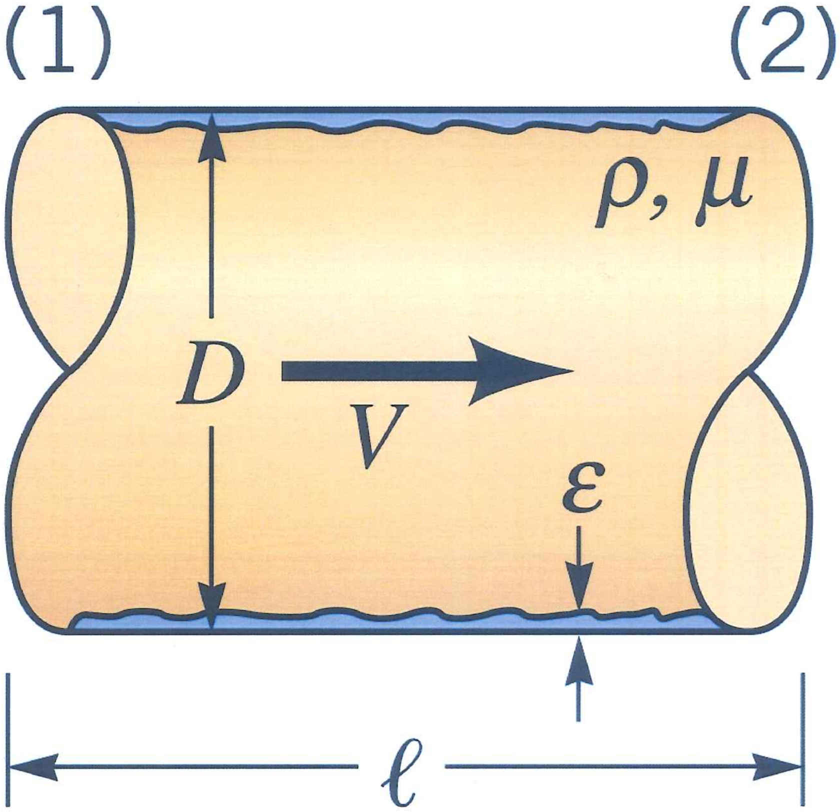


(*b*)

Figure 8.21b

Major losses

$$\Delta p = p_1 - p_2$$



Unnumbered 8 p427

Dimensional Analysis

$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$$

Equation 8.32

$$\boxed{\frac{\Delta p}{\rho g} = f\frac{\ell}{D}\frac{\cancel{\rho}V^2}{2\cancel{\rho}g}}$$

Equation 8.33

$$\boxed{h_{L\,\text{major}} = f\frac{\ell}{D}\frac{V^2}{2g}}$$

f: friction factor

ε: Pipe equivalent roughness

Table 8.1

Equivalent Roughness for New Pipes [Adapted from Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

rough

smooth

Table 8.1

The Moody chart

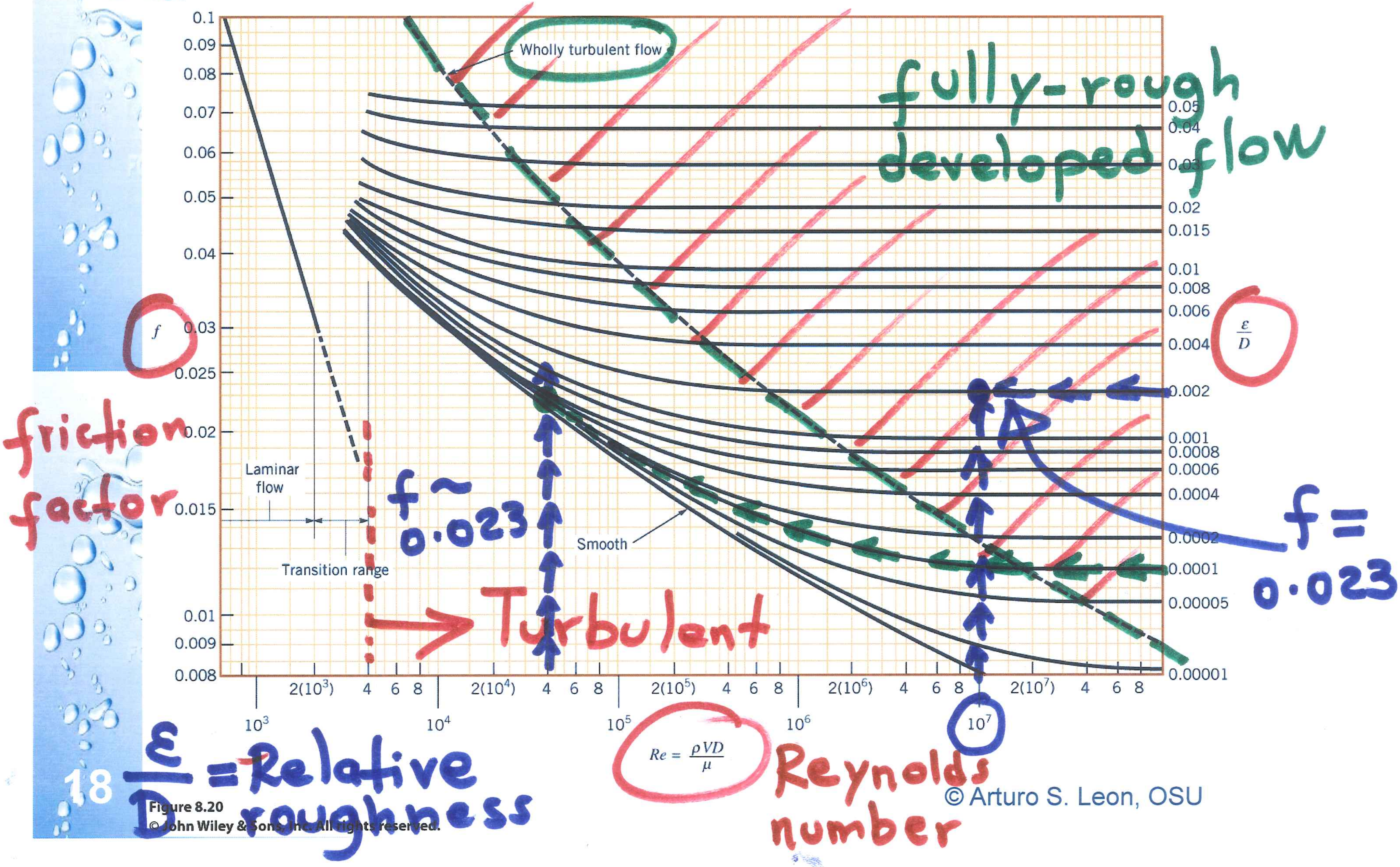


Figure 8.20
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Friction factor for the entire nonlaminar range

Colebrook formula

Implicit equation (requires iteration)

$$\boxed{\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)}$$

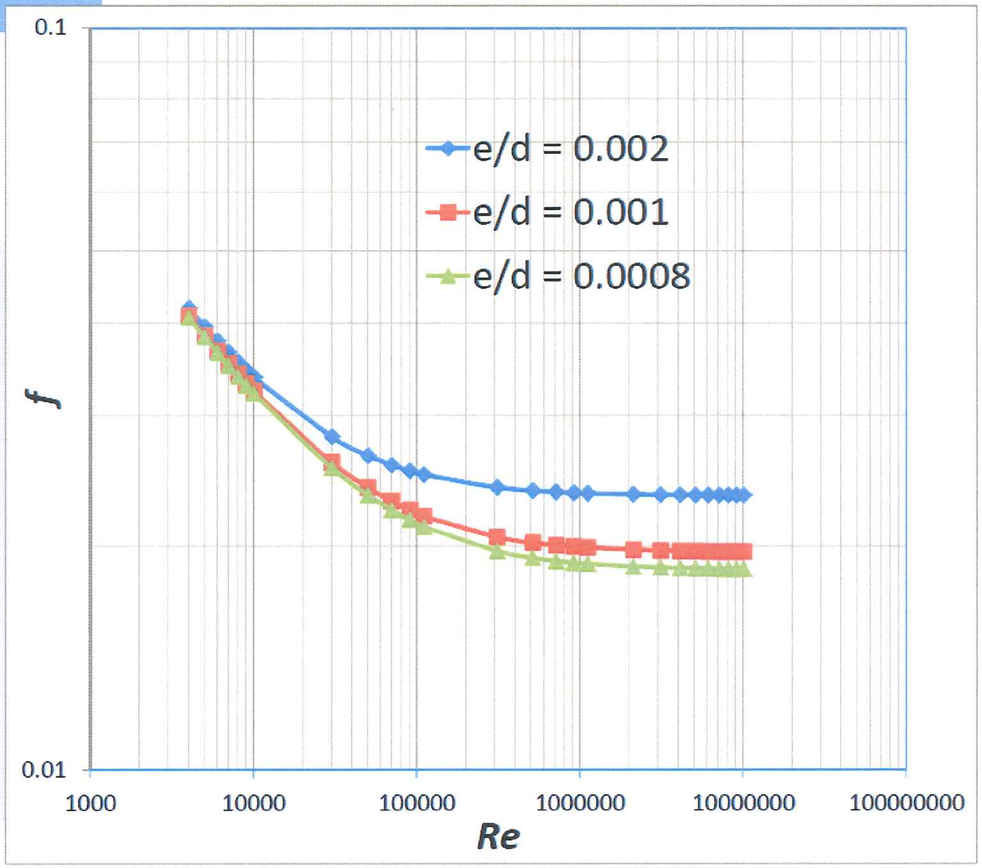
Haaland formula

$$\boxed{\frac{1}{\sqrt{f}} = -1.8\log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]}$$

Explicit (No iteration)

Plotting the Moody chart

Re	friction factors			Residuals		
		e/d			e/d	
	0.002	0.001	0.0008	0.002	0.001	0.008
4000	0.041890915	0.040912	0.040712	-1.7E-07	-0.00013	0
5000	0.039566021	0.038495	0.038277	0	0	0
6000	0.0378475	0.036696	0.036461	0	0	0
7000	0.036511522	0.035287	0.035036	0	0	0
8000	0.035435345	0.034144	0.033878	0	0	0
9000	0.034545211	0.033191	0.032912	0	0	0
10000	0.033793731	0.032382	0.032089	0	0	0
30000	0.02809364	0.02597	0.025507	0	0	0
50000	0.026505592	0.024021	0.02346	0	0	0
70000	0.025733725	0.023022	0.022394	0	0	0
90000	0.025273725	0.022404	0.021727	0	0	0
110000	0.024967401	0.021981	0.021266	0	0	0
310000	0.02400585	0.020574	0.019699	0	0	0
510000	0.023781793	0.020224	0.019299	0	0	0
710000	0.023681811	0.020065	0.019114	0	0	0
910000	0.023625177	0.019973	0.019007	0	-3.1E-05	0
1110000	0.023588721	0.019914	0.018938	0	-5.7E-08	0
2110000	0.023509481	0.019784	0.018786	0	-7.4E-08	0
3110000	0.023480988	0.019736	0.01873	0	0	0
4110000	0.023466316	0.019712	0.018701	0	0	0
5110000	0.023457372	0.019697	0.018684	0	0	0
6110000	0.02345135	0.019687	0.018672	0	0	0
7110000	0.023447018	0.01968	0.018663	0	0	0
8110000	0.023443753	0.019674	0.018657	0	0	0
9110000	0.023441204	0.01967	0.018652	0	0	2.13E-14
10110000	0.023439158	0.019667	0.018648	0	0	1.92E-07
				-1.7E-07	-1.3E-07	1.92E-07
				-1.1E-07		



Example (Proposed problems):

8.38 WILEY During a heavy rainstorm, water from a parking lot completely fills an 18-in.-diameter, smooth, concrete storm sewer. If the flowrate is 10 ft^3/s, a) determine the pressure drop in a 100-ft horizontal section of the pipe. b) Repeat the problem if there is a 2-ft change in elevation of the pipe per 100 ft of its length.

$D = 18'' = 1.5\,ft$

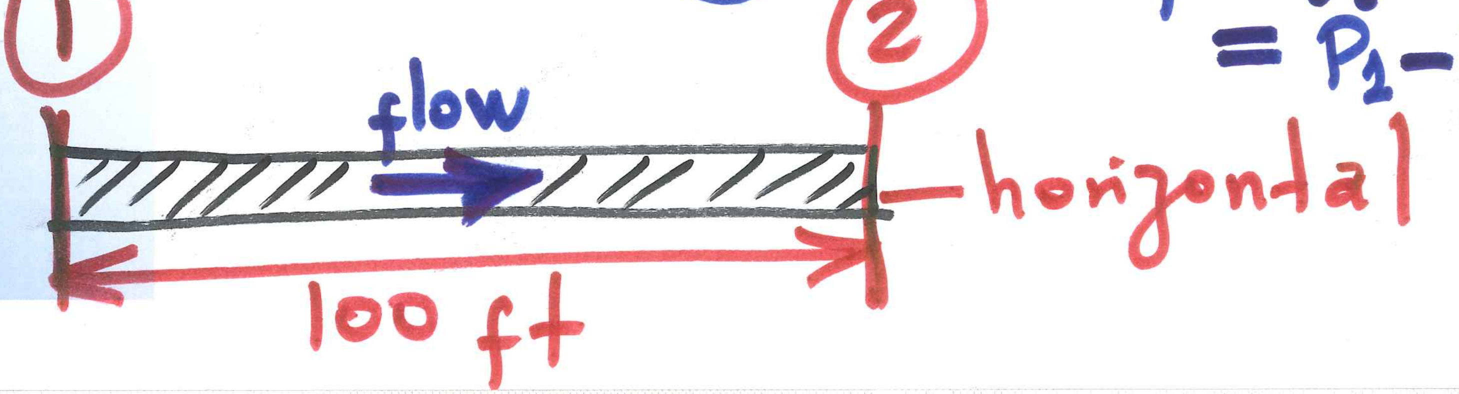
$Q = 10\,ft^3/s$

Table 8.1

$\varepsilon = 0.001\,ft$

$\Delta P = ??$

$= P_1 - P_2$



Energy equation:

$$\underline{\underline{\frac{P_1}{\gamma}}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \underline{\underline{\frac{P_2}{\gamma}}} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f$$

$V_1 = V_2$ (steady flow)

$$\Delta P \text{---} \frac{P_1 - P_2}{\gamma} = h_f + z_2 - z_1$$

$$\boxed{\Delta p = \gamma (h_f + z_2 - z_1)}$$

$z_2 - z_1 = 0$ (horizontal)

$$\Delta p = \gamma h_f = \gamma f \cdot \frac{L}{D} \frac{V^2}{2g}$$

V : velocity. $= \frac{Q}{A} = 5.66 \text{ ft/s}$

$$\overset{\text{friction factor}}{\boxed{f}} = \text{function}\left(\frac{\varepsilon}{D}, Re\right)$$

$$\frac{\varepsilon}{D} = \frac{0.001\ \cancel{ft}}{1.5\ \cancel{ft}} = \boxed{0.00066}$$

$$Re = \frac{V \cdot D}{\nu} = \frac{\overset{ft/s}{5.66} \times \overset{ft}{1.5}}{\boxed{\nu_{T=15.6^\circ}}} \rightarrow ft^2/s$$

$$\nu_{T=15.6^\circ} \rightarrow 1.21 \times 10^{-5}\ \frac{ft^2}{s}$$

$$Re = \overset{701,652}{\cancel{704,238}}$$

$$Re \sim \boxed{7.0\overset{1}{\cancel{4}} \times 10^{5}}$$

$$f\ (\text{Moody Chart}) = \underline{0.0185}\ ✓$$

$$\Delta p = 62.4 \, \frac{\overrightarrow{lb}}{ft^3} \times (0.61 \, ft) \checkmark$$

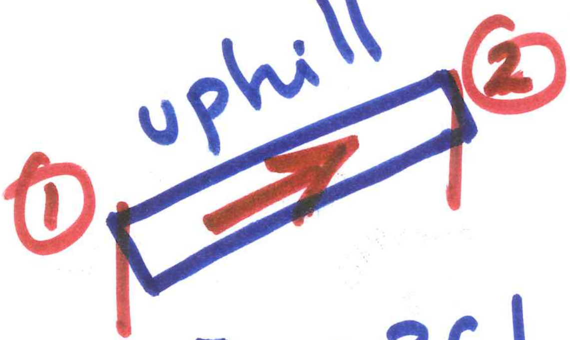
$$\boxed{\Delta p = 38.06 \, \frac{\overrightarrow{lb}}{ft^2}}$$

Conversion

$$1 \, psi = 144 \, \frac{\overrightarrow{lb}}{ft^2}$$

$$\boxed{\Delta p = 0.26 \, psi}$$ for horizontal pipe $(Z_2 - Z_1 = 0)$

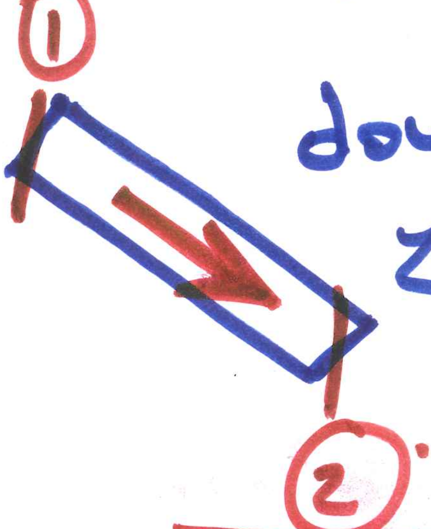
b)



$Z_2 - Z_1 = 2 ft$

$h_f = 2.61 \, ft$ ✓

$$\boxed{\Delta p = 1.13 \, psi}$$



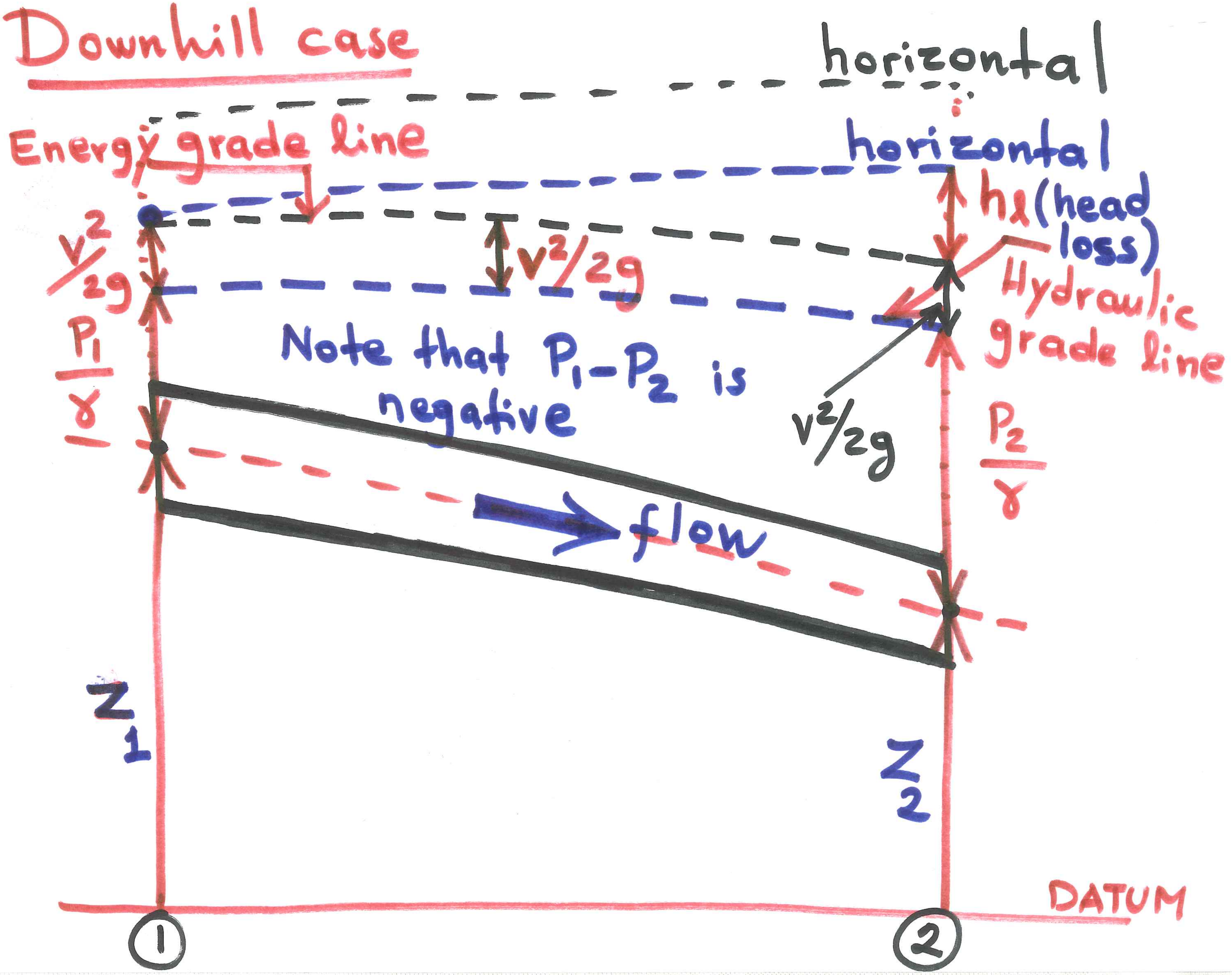
downhill

$h_f = -1.39 \, ft$

$Z_2 - Z_1 = -2 ft$

$$\boxed{\Delta p = -0.601 \, psi}$$

Downhill case

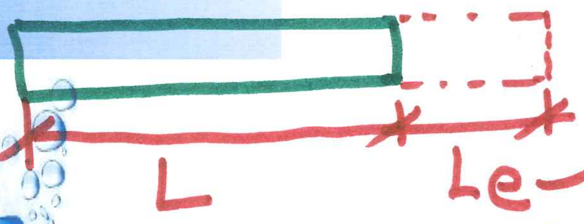


Viscous Flow in Pipes
Lecture 3, 01/13/2014

Arturo Leon, Oregon State University

Minor losses

$$h_{L\,\text{minor}} = K_L \frac{V^2}{2g}$$

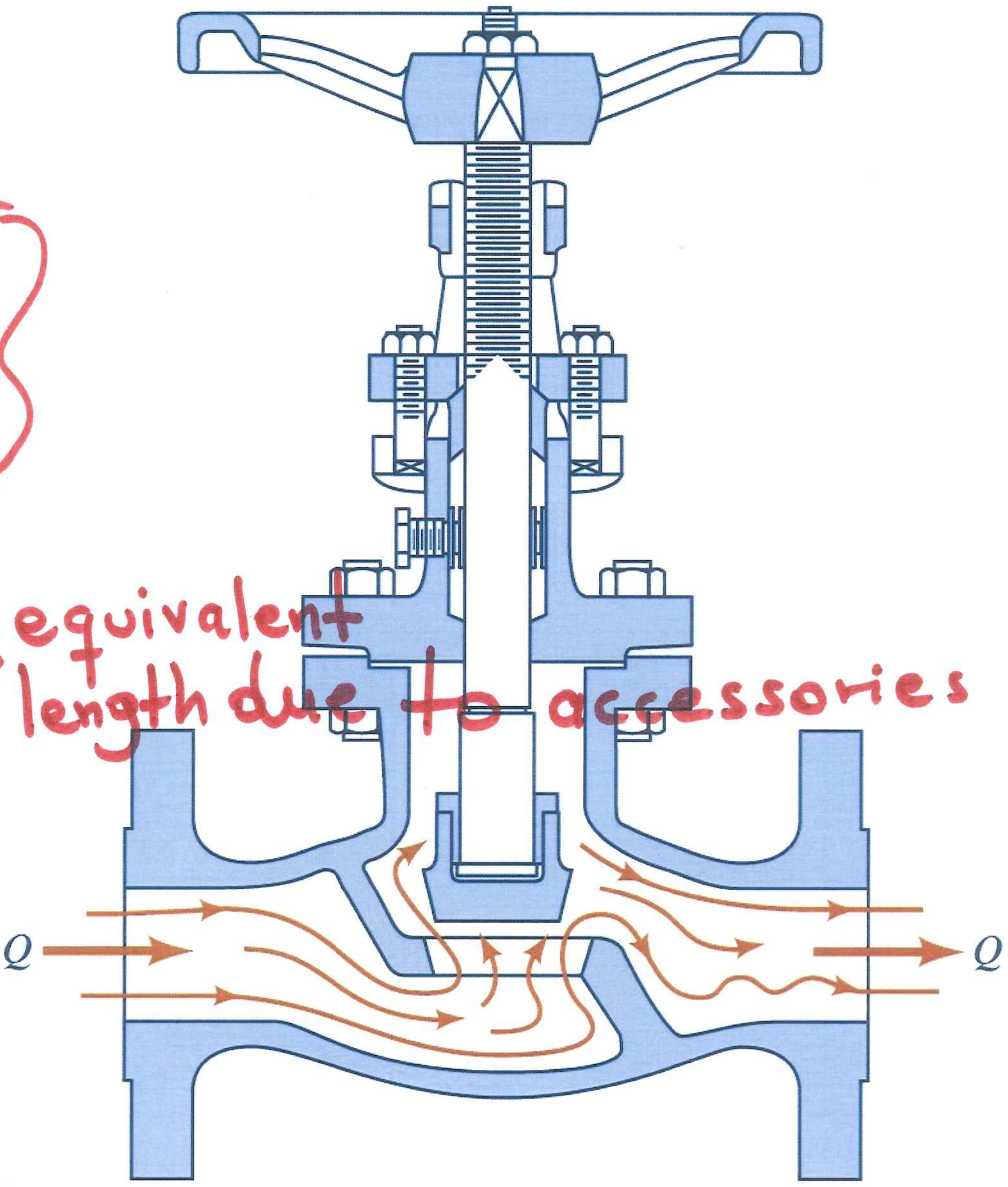


equivalent length due to accessories

$$h_{L_{major}} = f * \frac{L}{D} \frac{V^2}{2g}$$

$$f\frac{Le}{D} = K_L$$

$$\boxed{Le = \frac{K_L \cdot D}{f}}$$

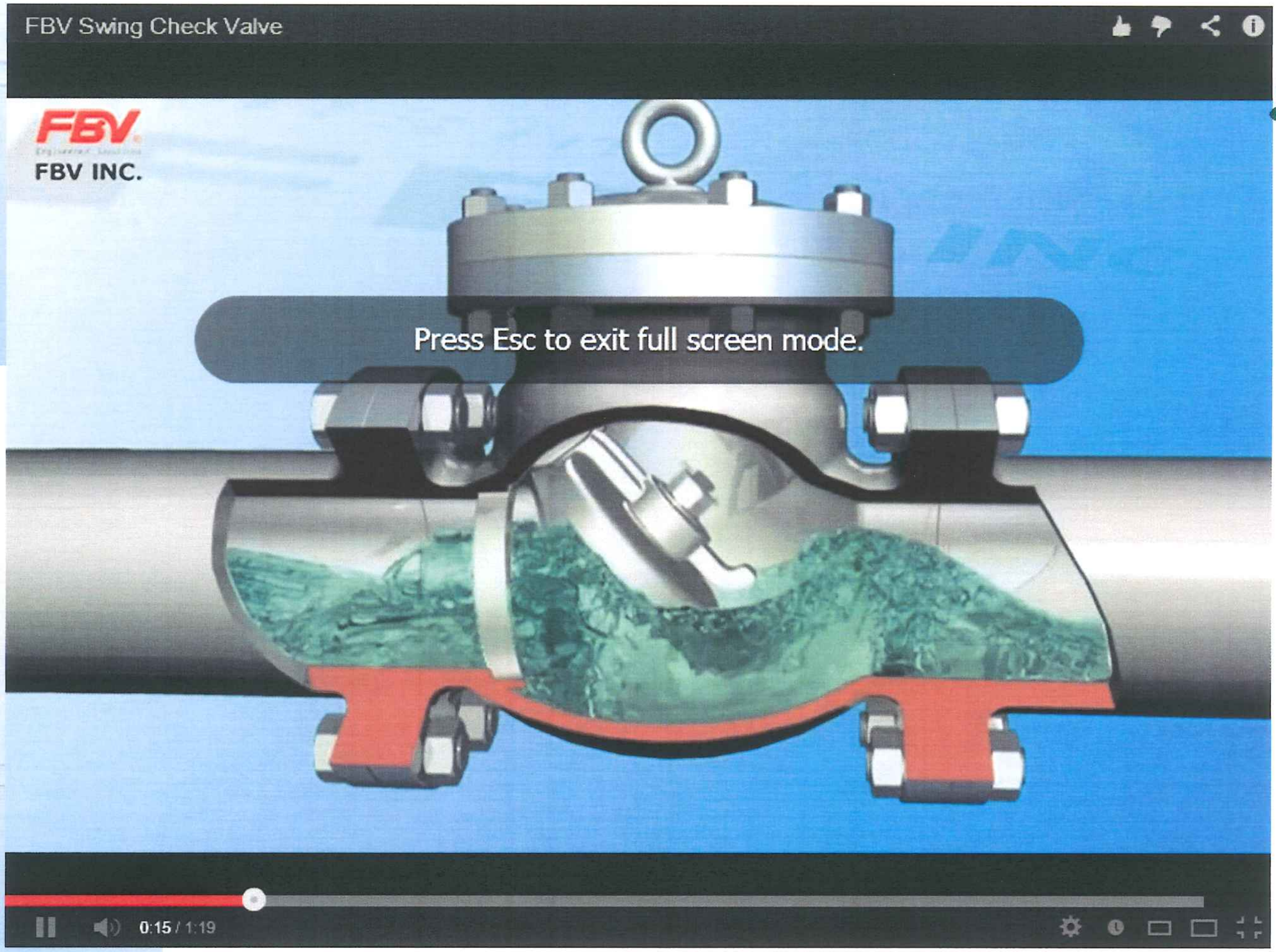


(b)

Figure 8.21b

Swing check valve video

http://www.youtube.com/watch?v=Krp6pOnaNsk



SCADA
system
for
real-time
control.

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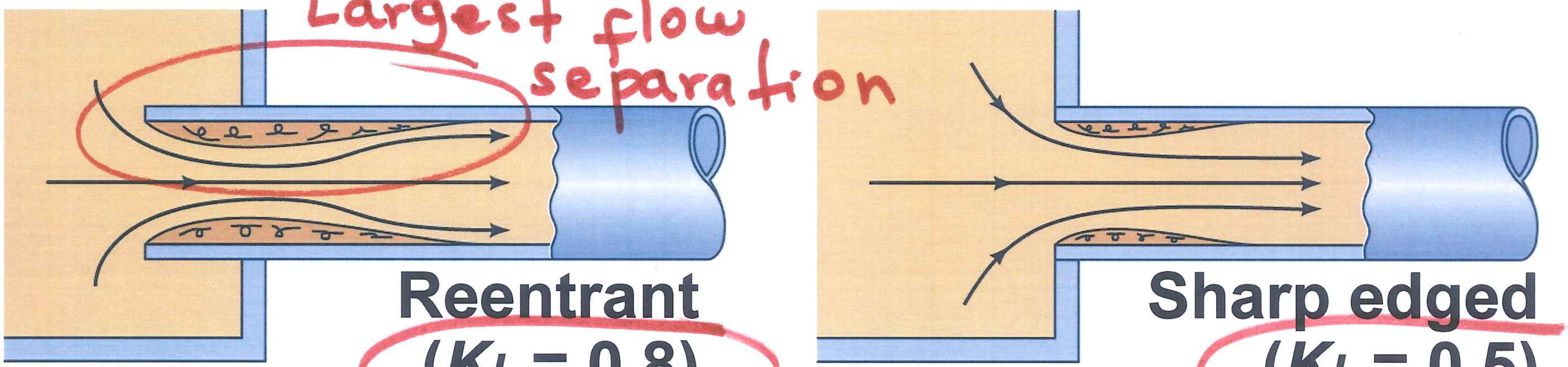
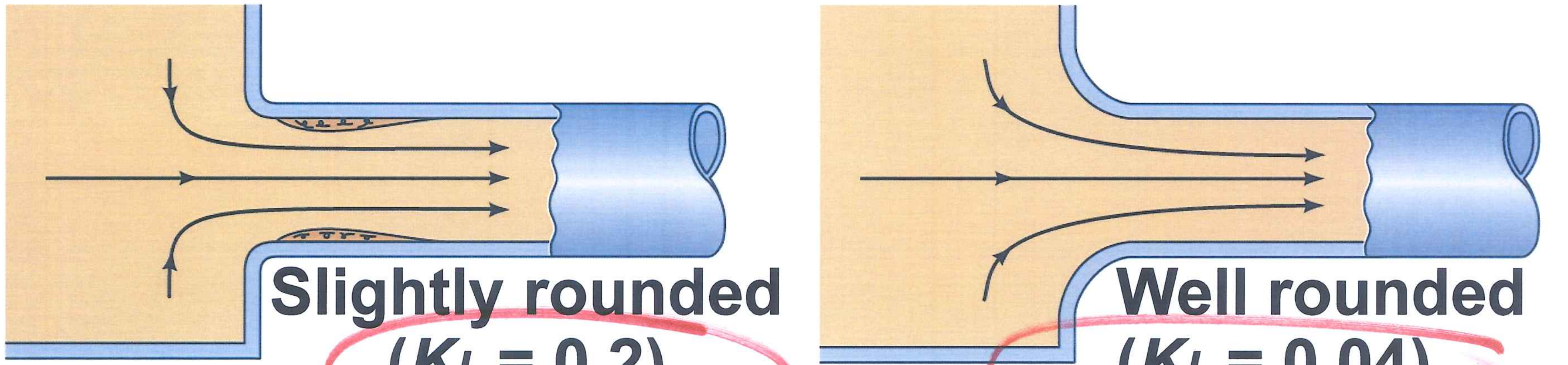
Entrance flow conditions and loss coefficient

Largest flow separation



Reentrant
$(K_L = 0.8)$
(a)

Sharp edged
$(K_L = 0.5)$
(b)



Slightly rounded
$(K_L = 0.2)$
(c)

Well rounded
$(K_L = 0.04)$
(d)

Figure 8.22

Flow pattern and pressure distribution for a sharp-edged entrance

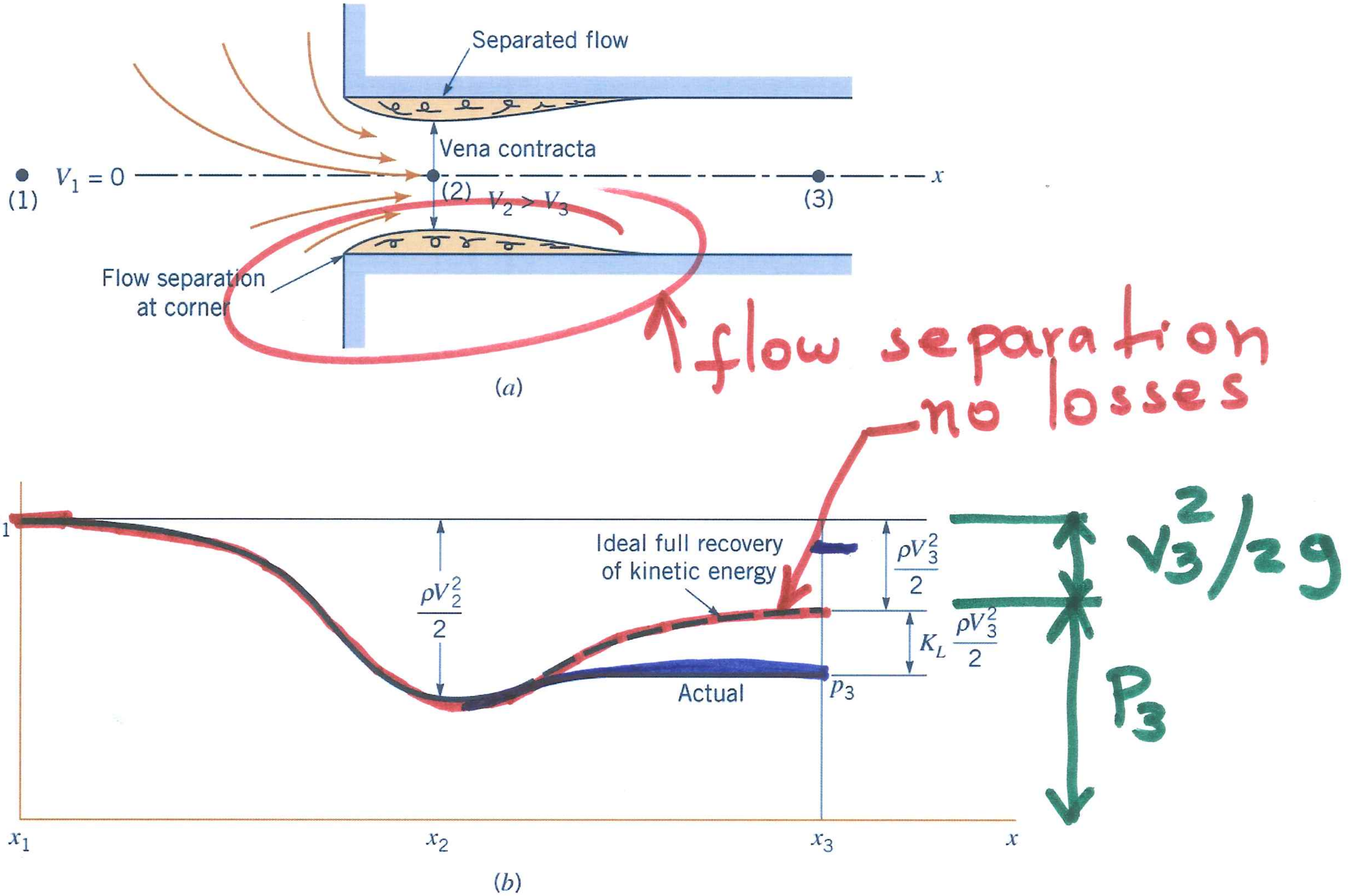


Figure 8.23

Entrance loss coefficient as a function of rounding of the inlet edge

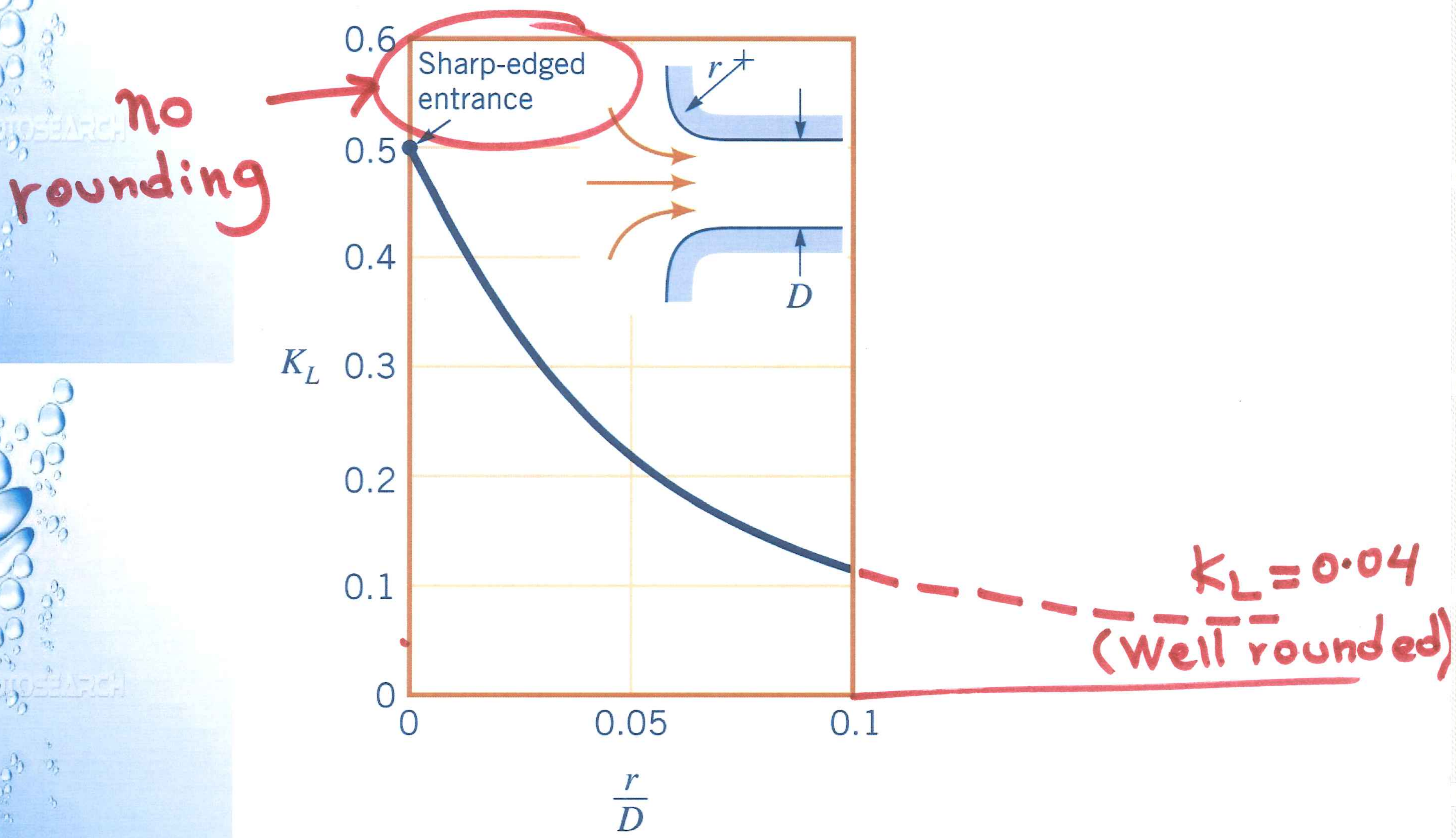


Figure 8.24
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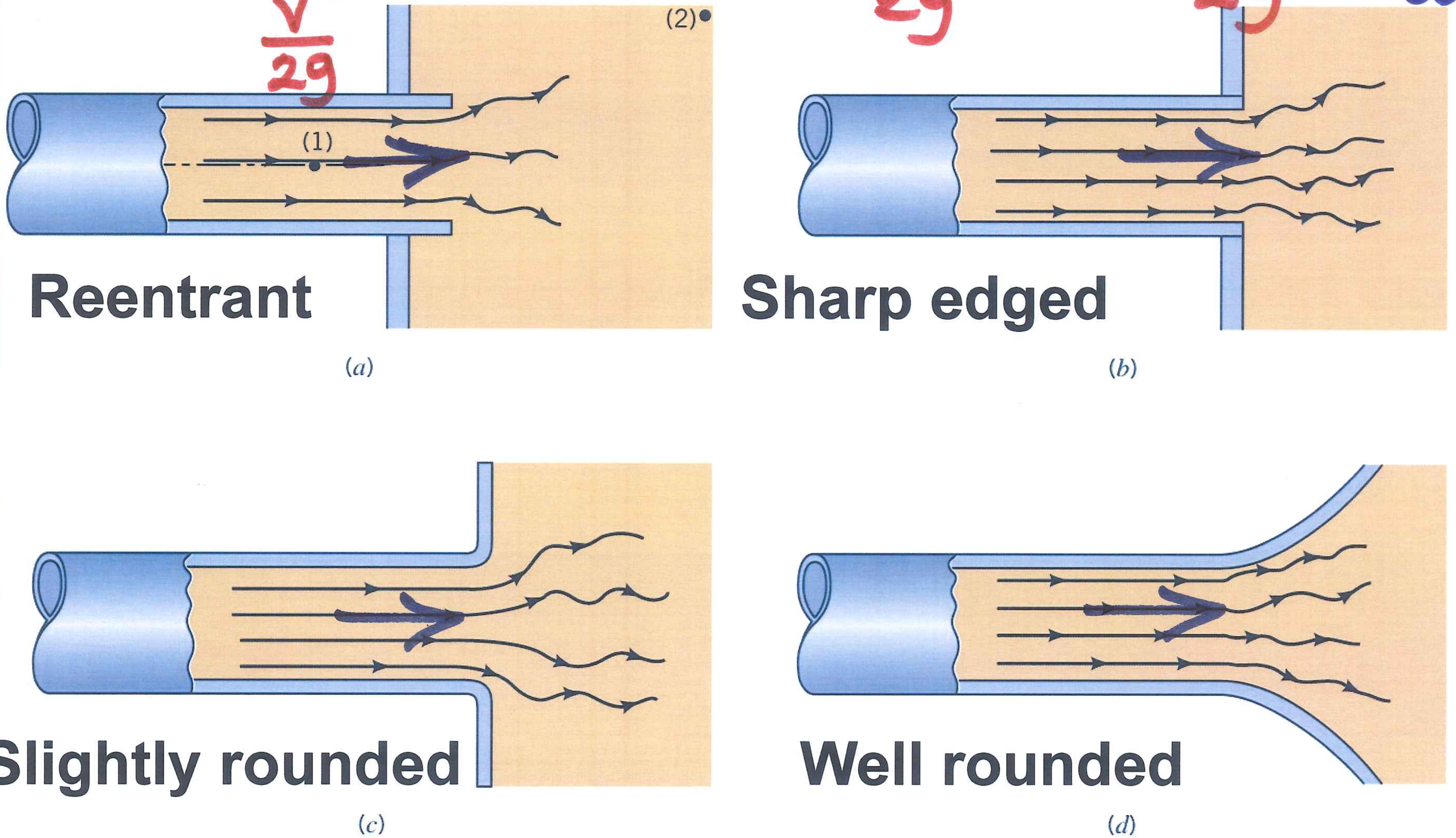
Exit flow conditions and loss coefficient

$$h_L = \frac{V^2}{2g} - 0 = \frac{V^2}{2g}$$

$$\boxed{K_L = 1.0}$$ for all.



Reentrant

(a)

Sharp edged

(b)

Slightly rounded

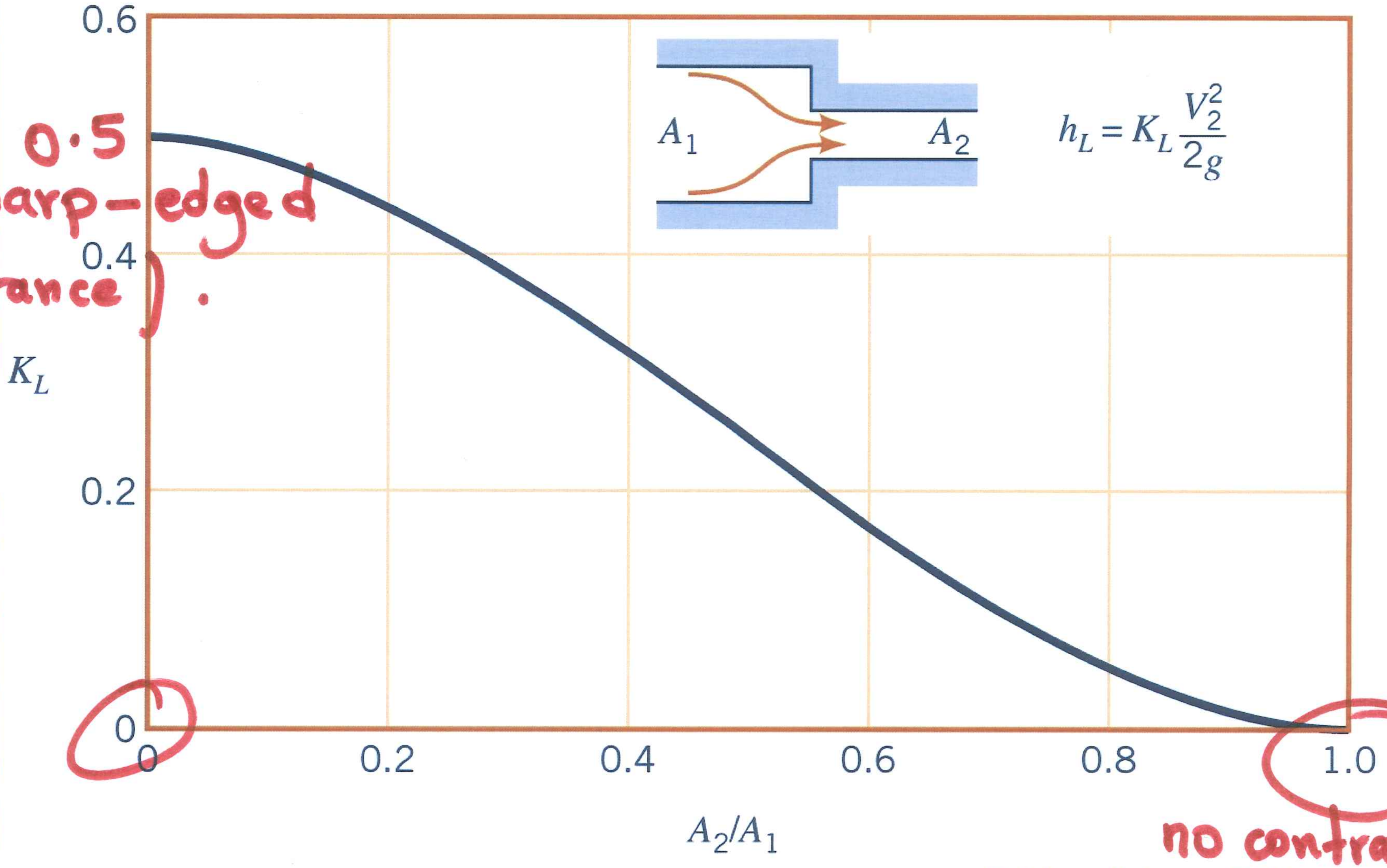
(c)

Well rounded

(d)

Figure 8.25

Loss coefficient for a sudden contraction



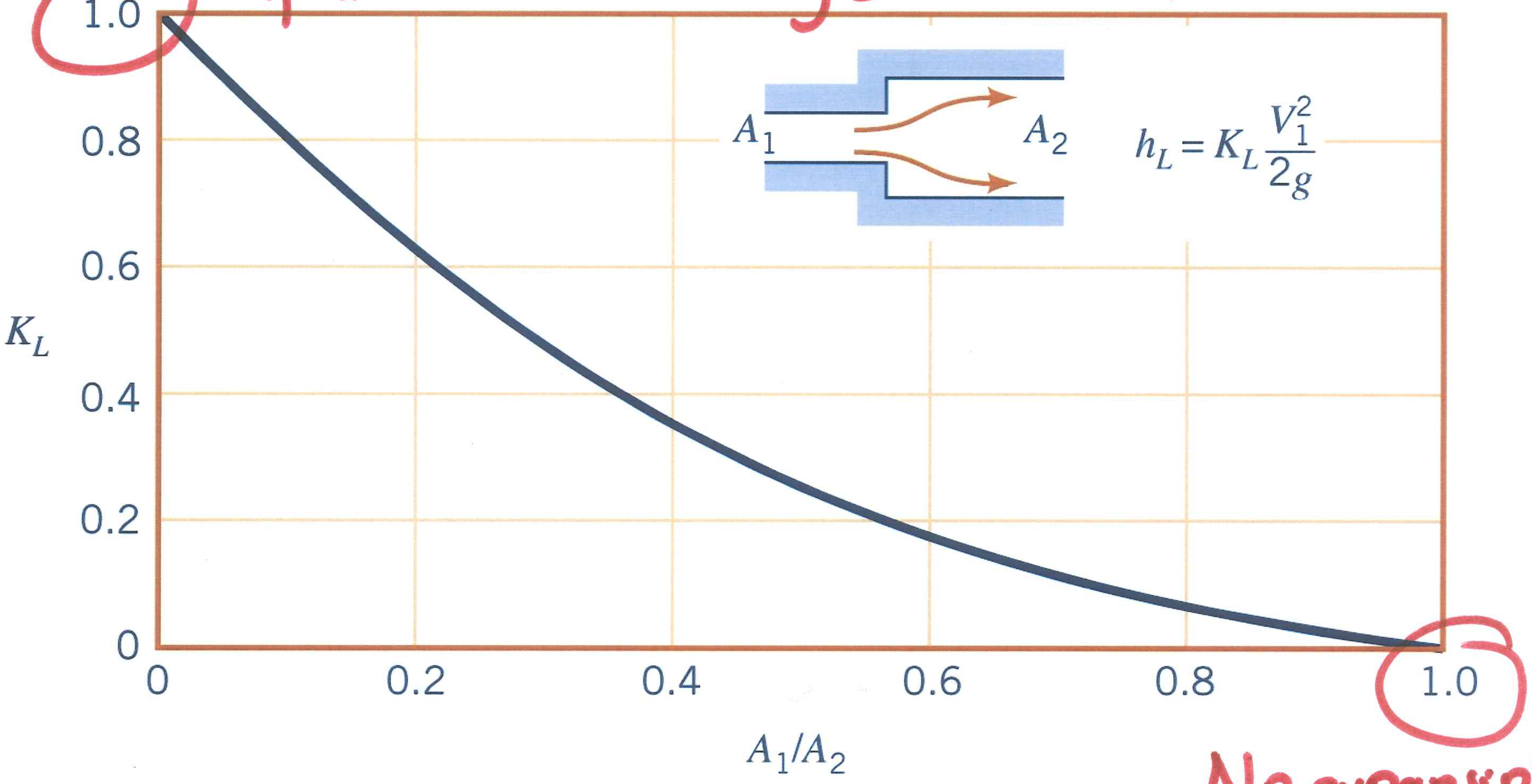
0.5 (sharp-edged entrance).

no contraction

Figure 8.26

Loss coefficient for a sudden expansion

expansion to a large reservoir



No expansion

Figure 8.27
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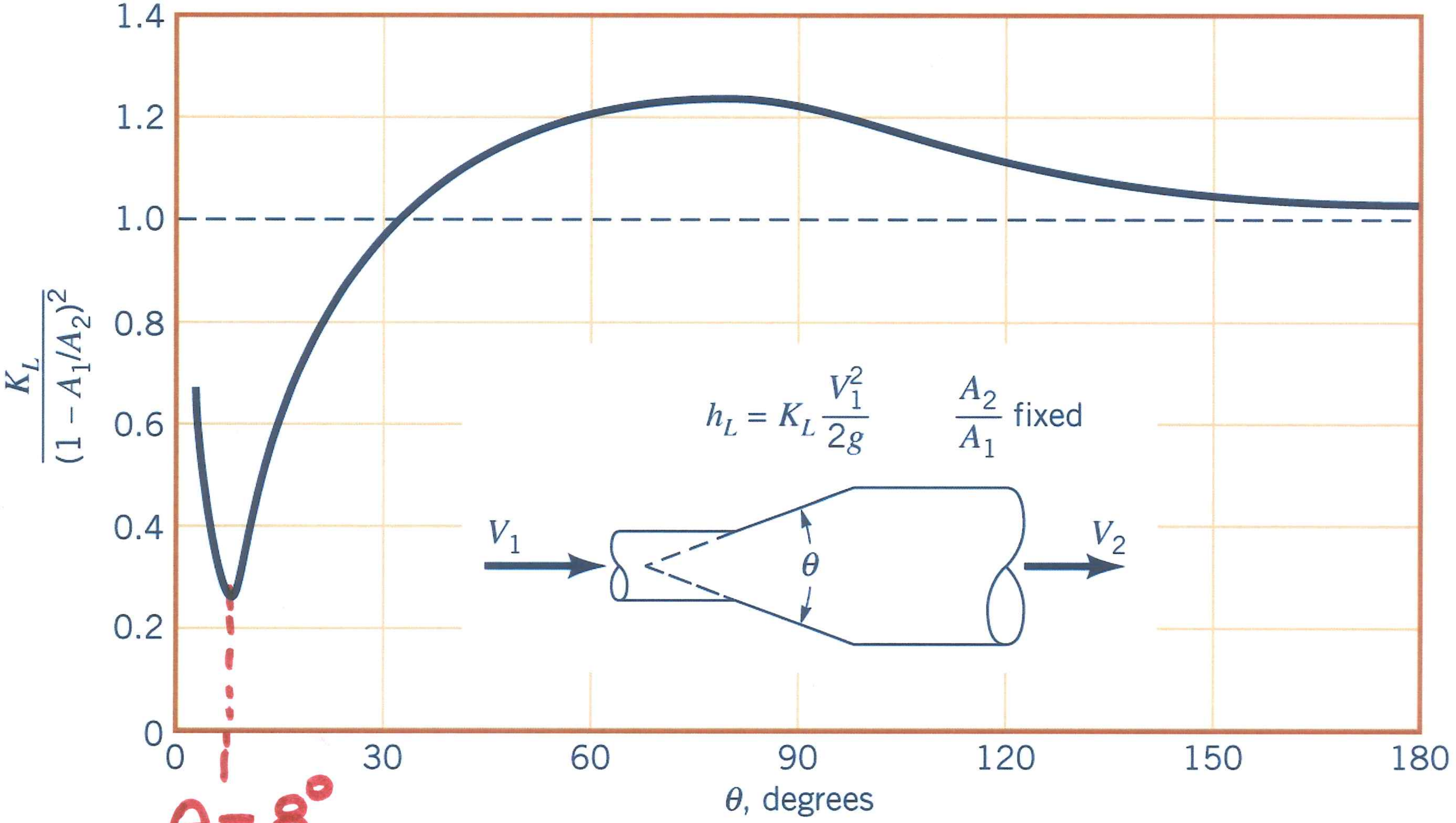
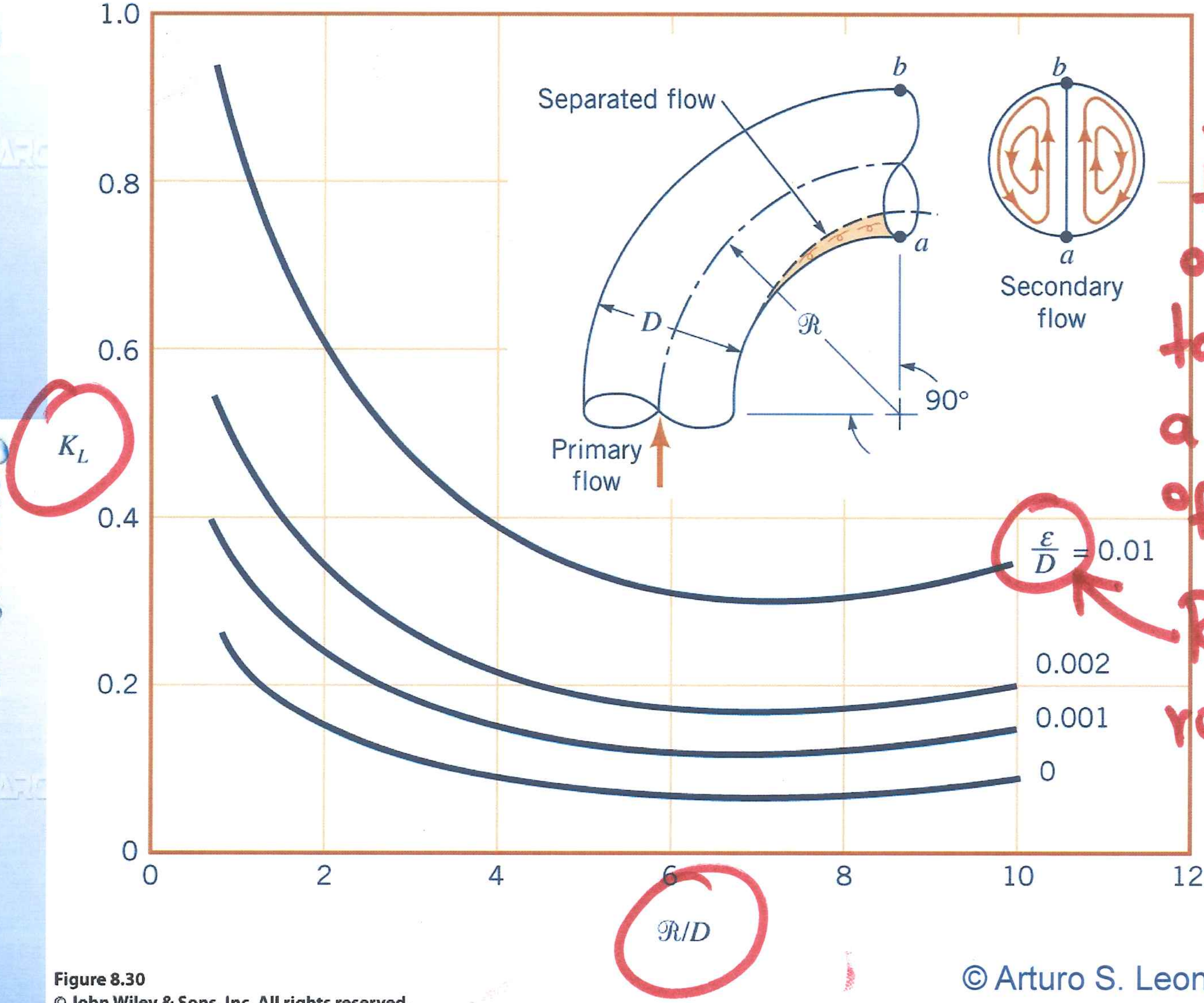
Loss coefficient for a typical conical diffuser



$\theta = 8°$

Figure 8.29

Loss coefficient in a 90° bend

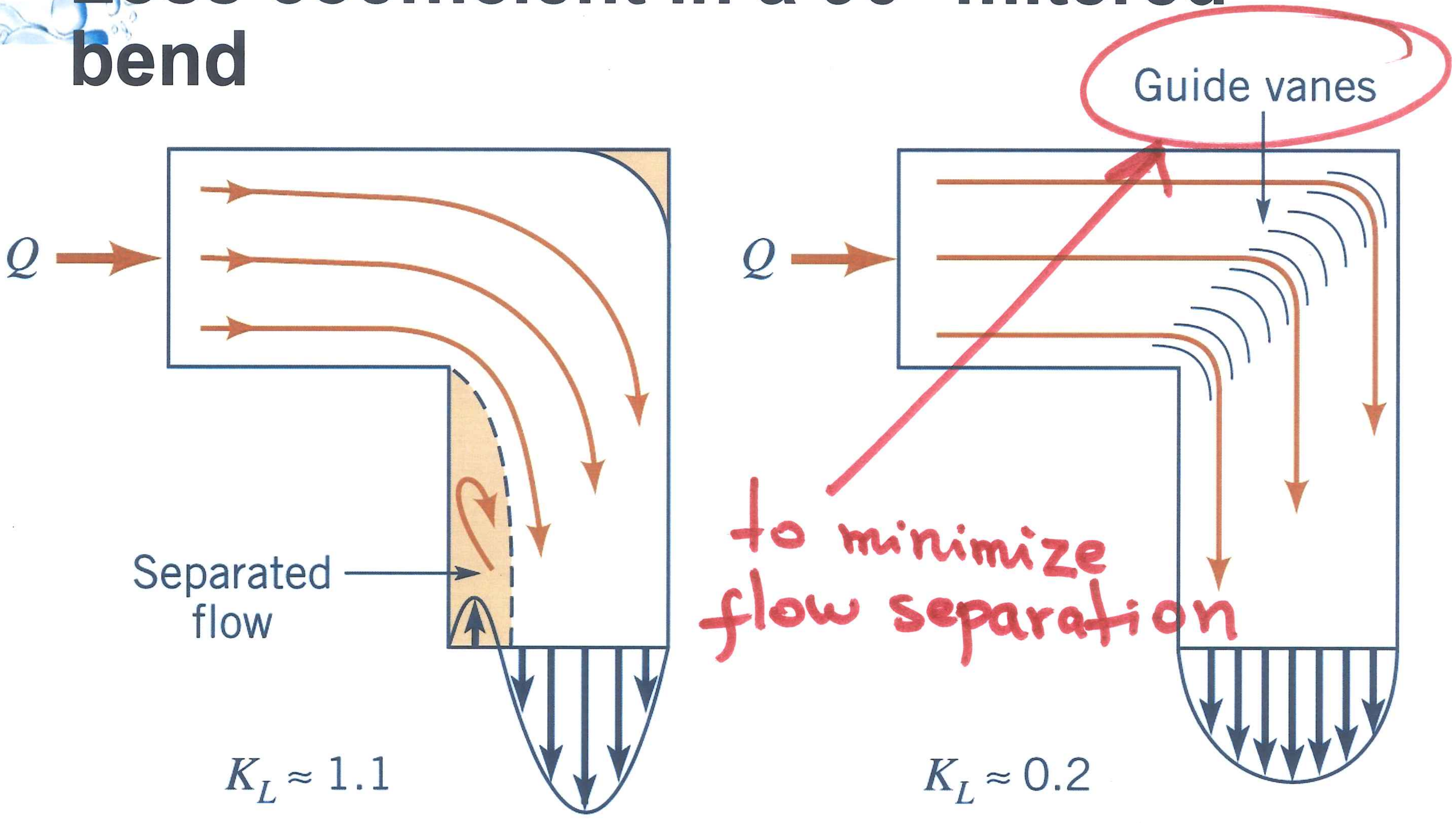


swirling flow due to imbalance of centripetal forces as a result of curvature

Relative roughness

Figure 8.30

Loss coefficient in a 90° mitered bend



$K_L \approx 1.1$

$K_L \approx 0.2$

to minimize flow separation

(*a*)
Without guide vanes

(*b*)
With guide vanes

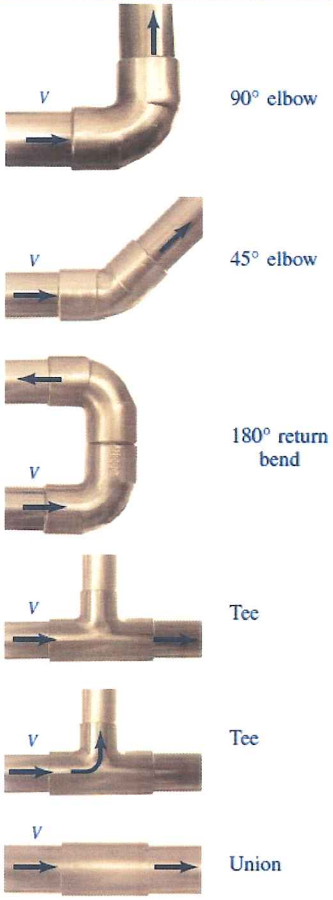
Figure 8.31

Loss coefficients for Pipe Components

Table 8.2

Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ **(Data from Refs. 5, 10, 27)**

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



*See Fig. 8.32 for typical valve geometry.

Table 8.2

Threaded elbow

Flanged elbow

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Head loss in a valve

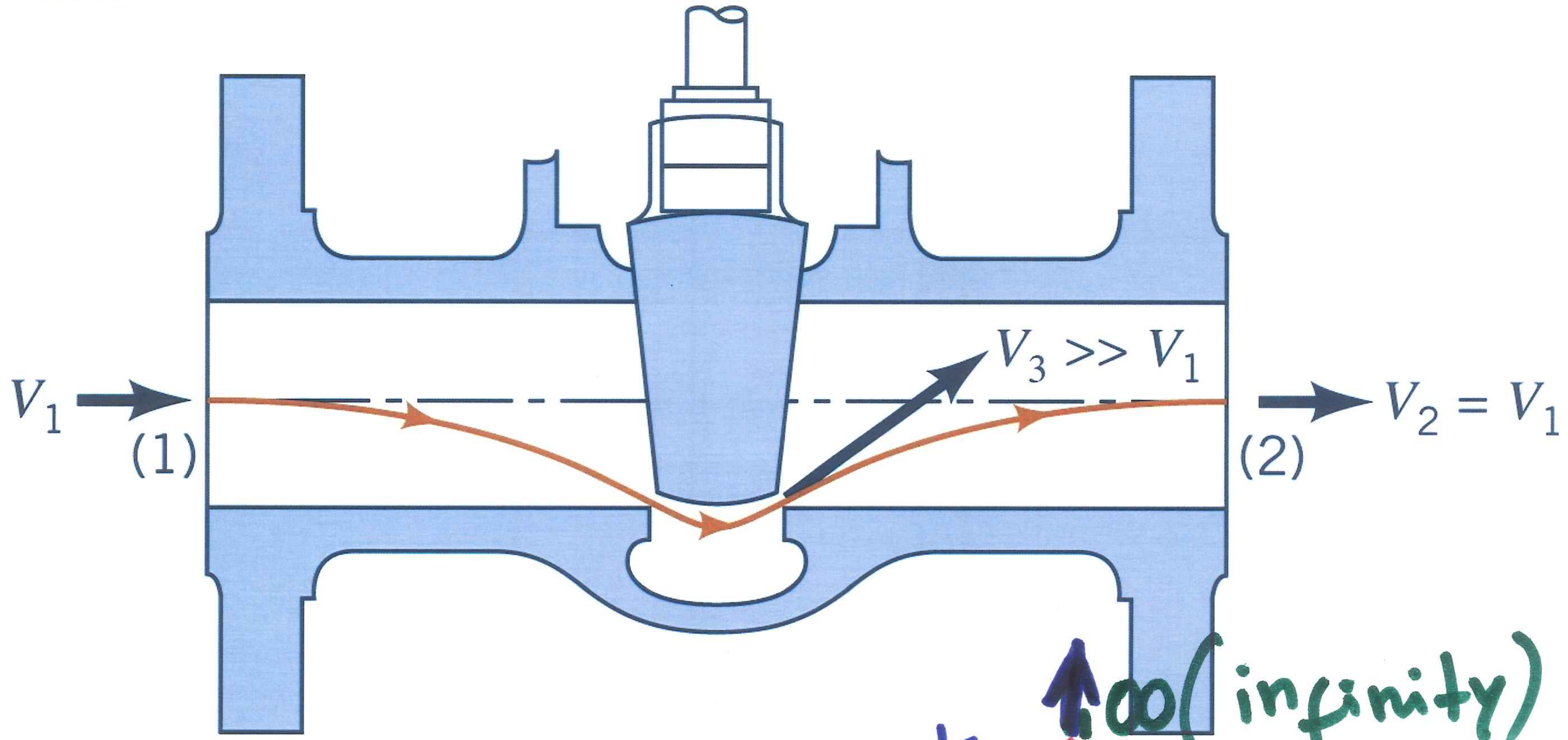
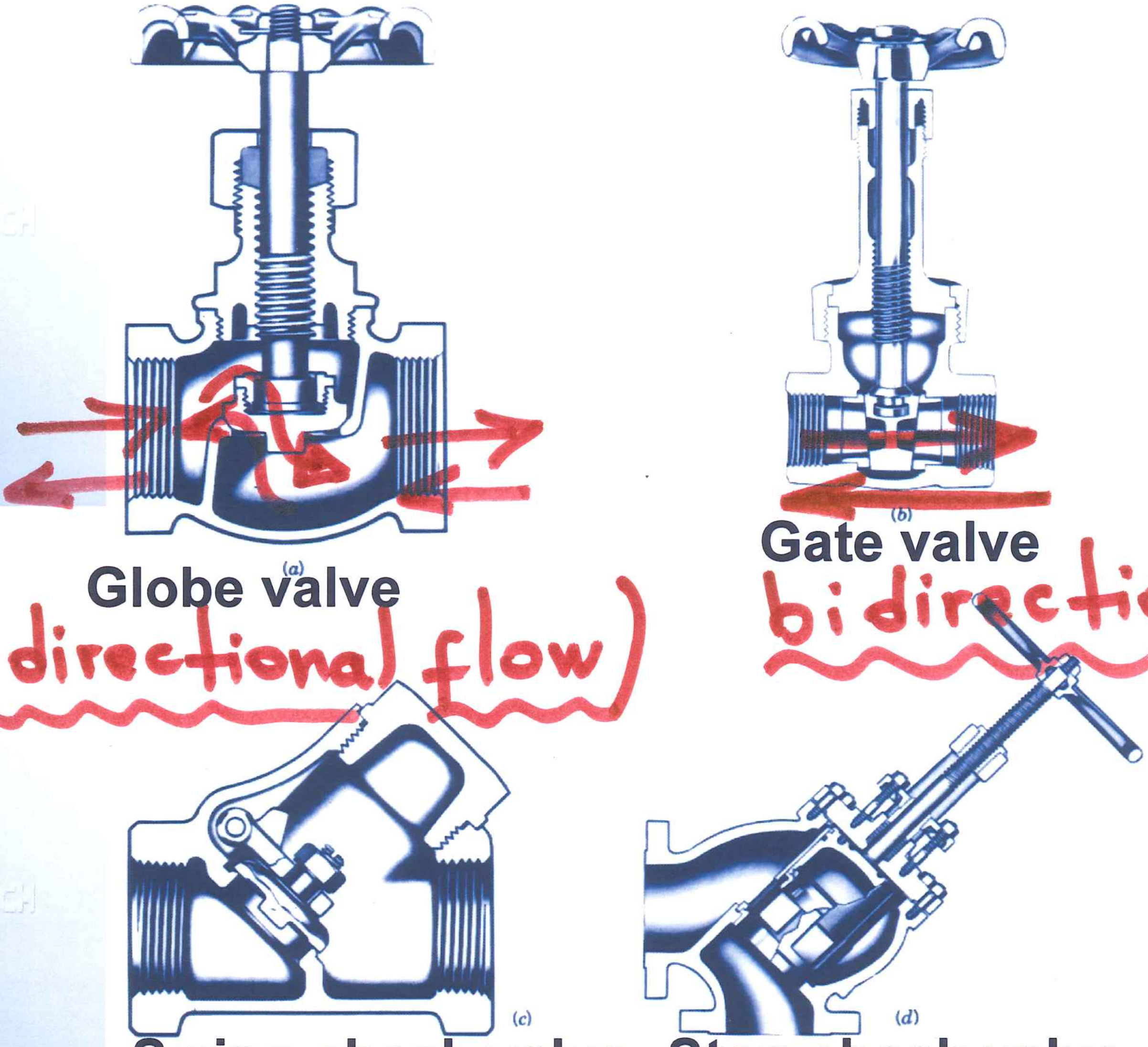


Figure 8.33

⑩

Common valves



Globe valve

(bi directional flow)

Gate valve

bi directional flow

Swing check valve

Stop check valve

(one direc-tional flow)

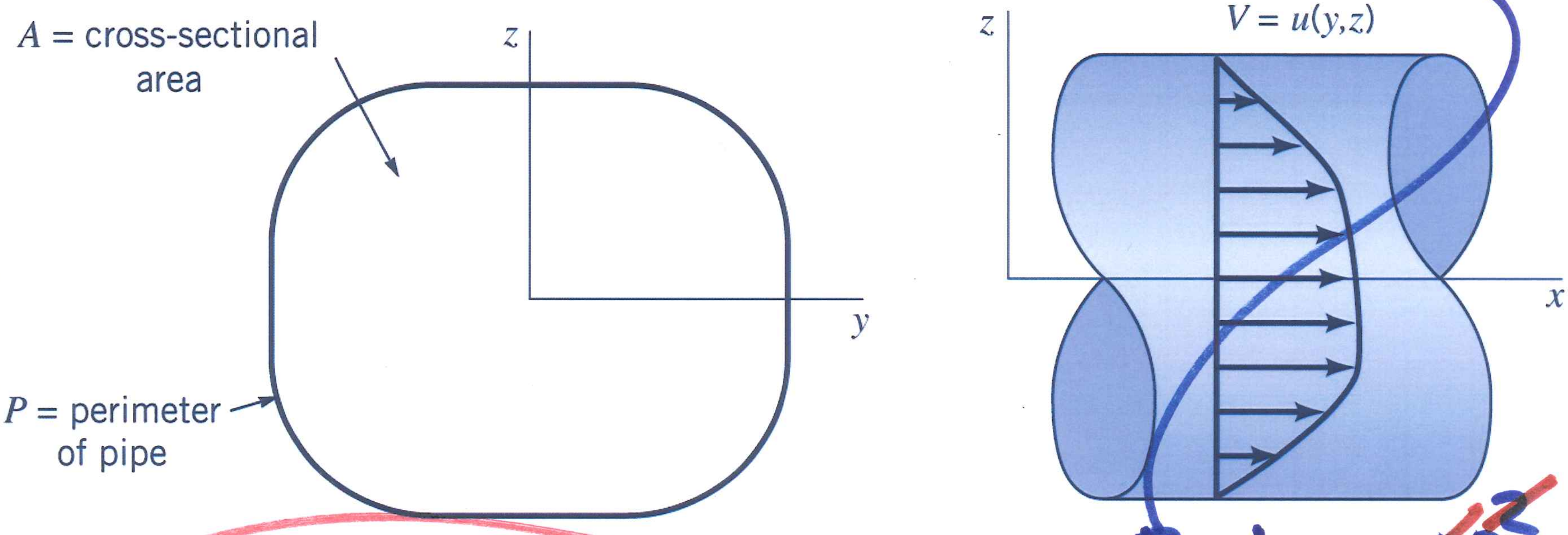
Figure 8.32
Courtesy of Crane Co., Fluid Handling Division

Noncircular Conduits

Circular pipe

A,

$P = \pi D$



$D_h = 4A/P$ = hydraulic diameter

(a)

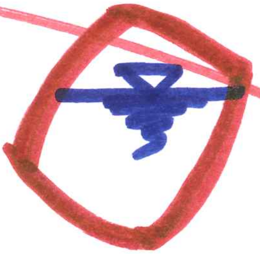
(b)

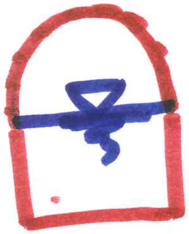
$$\frac{A}{P} = \frac{\cancel{\pi} D^{\cancel{2}}}{4\cancel{\pi}\cancel{D}} = \frac{D}{4}$$

$$\boxed{\frac{4A}{P} = D_h}$$

Dh = Hydraulic diameter

For a circular pipe, Dh = D





tunnel

Figure 8.34
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$$h_L = f\frac{L}{D_h}\frac{V^2}{2g}$$

$$\frac{\varepsilon}{D} = \frac{\varepsilon}{D_h}$$

⑬

Example (Proposed problems):

Water flows from the container shown in Fig. 8.59. Determine the loss coefficient needed in the valve if the water is to "bubble up" 3 in above the outlet pipe. The entrance is slightly rounded.

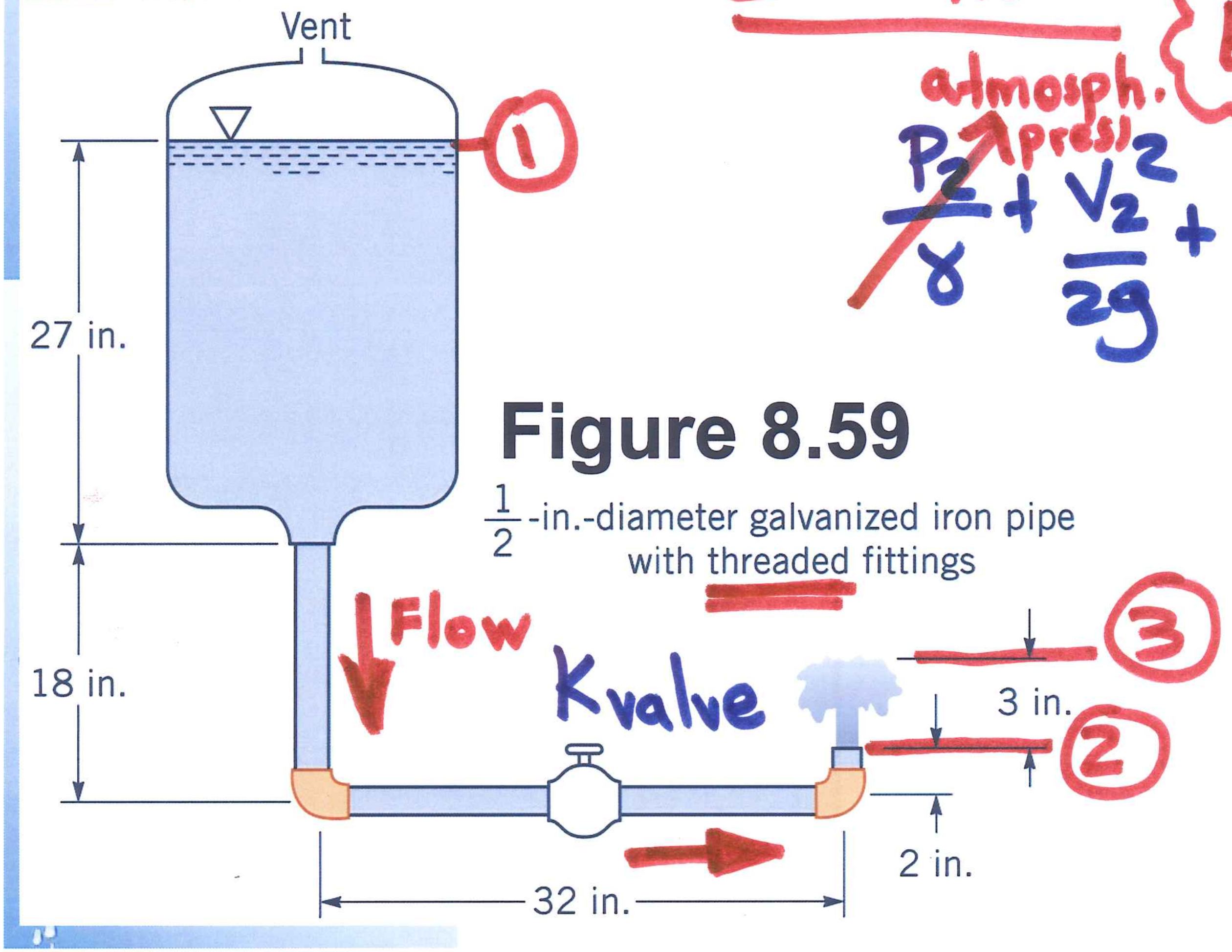

Figure 8.59

Solution

$$\boxed{E_2 = E_3}$$

$$\cancelto{\text{atmosph. press.}}{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 = \cancelto{}{\frac{P_3}{\gamma}} + \cancelto{0}{\frac{V_3^2}{2g}} + z_3 + h_L$$

atmospher. pressure

$$\frac{V_2^2}{2g} = \frac{3}{12} + \overset{\approx}{0}$$

$$\boxed{V_2 = 4.01\,ft/s}$$

⑭

$E_1 = E_2$

Vented

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + Z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + Z_2 + f\frac{L}{D}\frac{V_2^2}{2g} + K_a\frac{V_2^2}{2g}$$

$$\left(\frac{27+18-2}{12}\right) = \frac{V_2^2}{2g}\left(1 + f * \frac{52}{0.5} + \sum K_a\right) \cdots (*)$$

$$\Sigma K_a = \underbrace{K_v}_{\text{Kvalve}} + 2 \times \underbrace{1.5}_{\text{threaded bends}} + \underbrace{0.2}_{\text{Kentrance flow}}$$

$$f = fn\left(Re, \frac{\varepsilon}{D}\right) \rightarrow Re = \frac{V \cdot D}{\nu} = \frac{4.01 \times \frac{0.5}{12}}{1.21 \times 10^{-5}}$$

$$\boxed{Re = 13,808}$$

$\varepsilon = 0.0005\,ft$ ⑮

$$\frac{\varepsilon}{D} = \frac{0.0005}{0.5} \times 12 = 0.012$$

$f\,(\text{Moody chart}) = 0.043$

In $(*) \rightarrow \boxed{K_v = 5.68}$

Viscous Flow in Pipes
Lecture 4, 01/15/2014

Arturo Leon, Oregon State University

②

Pipe flow examples

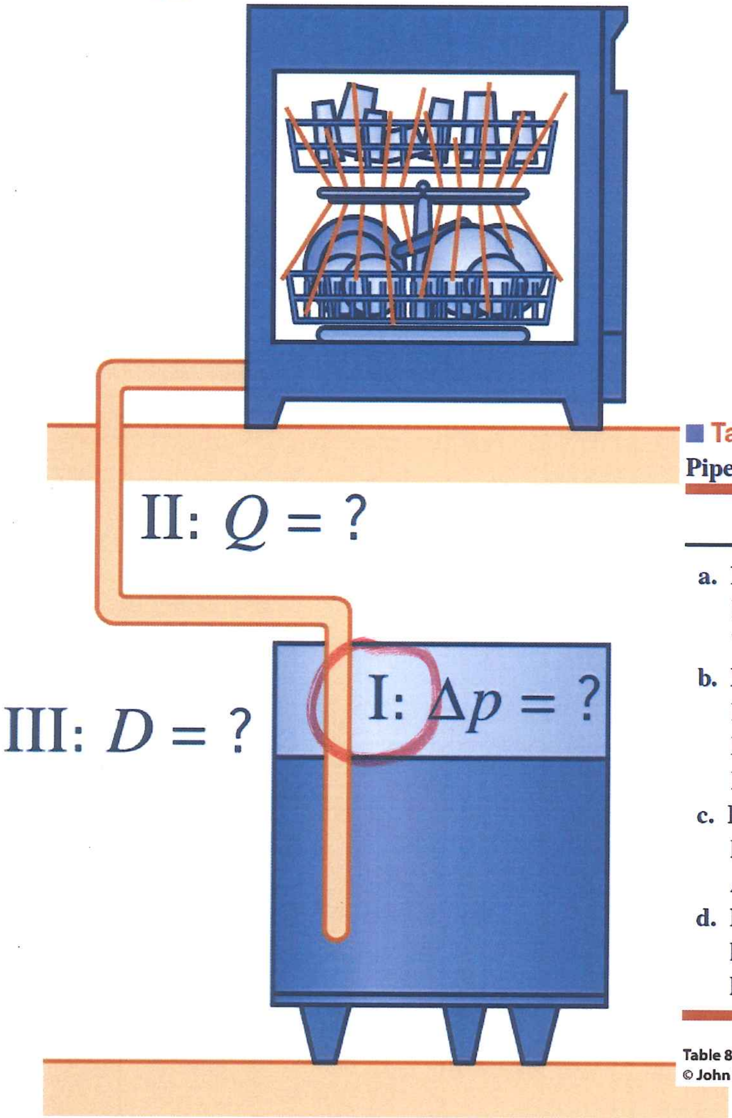

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Table 8.4
Pipe Flow Types

Variable	Type I	Type II	Type III
a. Fluid			
Density	Given	Given	Given
Viscosity	Given	Given	Given
b. Pipe			
Diameter	Given	Given	Determine
Length	Given	Given	Given
Roughness	Given	Given	Given
c. Flow			
Flowrate or Average Velocity	Given	Determine	Given
d. Pressure			
Pressure Drop or Head Loss	Determine	Given	Given

Table 8.4

🟥 Determine
🟦 Given

③

Example (Proposed problems):

A 40-m long, 12-mm diameter pipe with a friction factor of 0.020 is used to siphon 30°C water from a tank as shown in Fig. 8.50. Determine the maximum value of *h* allowed if there is to be no cavitation within the hose. Neglect minor losses.

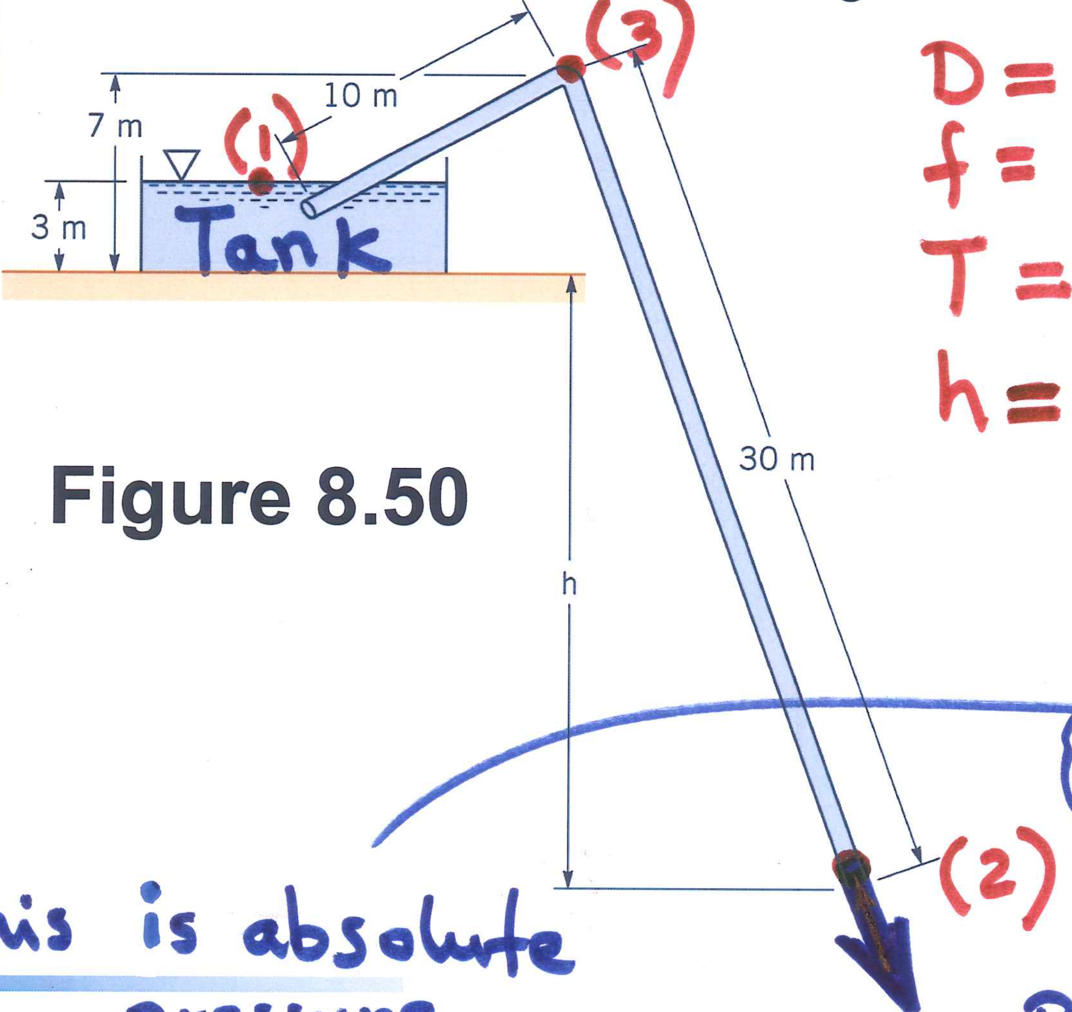


Figure 8.50

$D = 12\,mm$

$f = 0.020$ $\quad \Sigma k = 0$

$T = 30°C$

$h = ??$ (No cavitation)

$P_3 = P_v$ (Vapor pressure)

$\boxed{P_3 = 4.243 \times 10^3 \frac{N}{m^2}}$ (Table B.2)

$1\,Pa = 1 \frac{N}{m^2}$

$P_3 = 4.243\ kN/m^2\ (kPa)$

This is absolute pressure

④

$$\boxed{E_1 = E_3}$$

$$\frac{P_1}{\gamma} + \overset{0}{\cancel{\frac{V_1^2}{2g}}} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 + f\frac{L}{D}\frac{V_3^2}{2g}$$

$$P_{abs} = P_{man} + P_{atm}$$

P_1 is $\boxed{1.013\times10^5 \frac{N}{m^2}}$ (Table C.2) $= 101.3 \frac{kN}{m^2}$ (absolute pressure)

$\boxed{\gamma(30°C) = 9.765 \frac{kN}{m^3}}$ (Table B.2)

$$\frac{101.3}{9.765} \frac{\frac{kN}{m^2}}{\frac{kN}{m^3}} + 0 = \frac{4.243}{9.765} + \frac{V_3^2}{2g} + \overset{z_3 - z_1}{\overset{\downarrow}{4}} + h_L$$

⑤

$$\underset{\text{known}}{Ⓒ} = \frac{V_3^2}{2g}\left(1 + 0.020 \times \frac{10}{12} \times 1000\right)$$

$$\boxed{V_3 = 2.56\ m/s}$$

$\rightarrow \boxed{E_1 = E_2}$

$$\cancel{\frac{P_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}}^{\,0} + Z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + Z_2 + f\frac{L}{D}\frac{V_2^2}{2g}$$

$\boxed{P_1 = P_2}$

$$\underset{Z_1 - Z_2}{\boxed{h+3}} = \frac{V_2^2}{2g}\left(1 + 0.020 \times \frac{40}{12} \times 1000\right)$$

$\boxed{V_2 = V_3 \text{ (same pipe)}}$

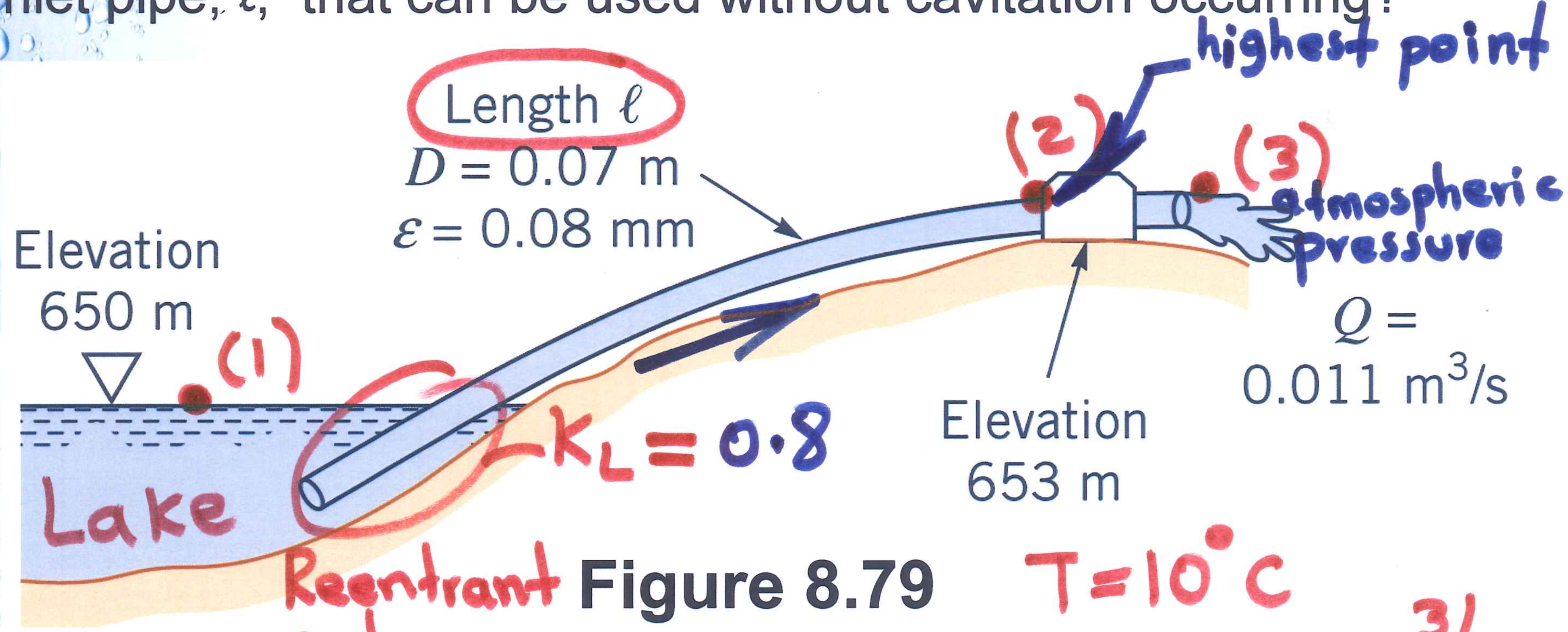
$$h+3 = \frac{2.56^2}{2 \times 9.8}\left(1 + 0.02 \times \frac{40}{12} \times 1000\right)$$

$$\boxed{h = 19.62\ m} \checkmark$$

⑥

Example (Proposed problems):

Water at 10°C is pumped from a lake as shown in Fig. 8.79. If the flow rate is 0.011 m3/s, what is the maximum length of the inlet pipe,, ℓ, that can be used without cavitation occurring?



Reentrant entrance condition

Figure 8.79

Figure P8.79

$T = 10^\circ C$

$Q = 0.011\ m^3/s$

$\ell = ?$ (Without cavitation)

$$\left.\begin{aligned}\gamma &= 9.804\ \frac{kN}{m^3}\\ P_v &= 1.228\ \frac{kN}{m^2}\end{aligned}\right\}$$ Table B.2

⑦

$$\boxed{E_1 = E_2}$$

$$\frac{P_1}{\gamma} + \overset{0}{\cancel{\frac{V_1^2}{2g}}} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \frac{V_2^2}{2g}\left(\Sigma k + f\frac{L}{D}\right)$$

$P_2 = P_v$

$$\frac{101.3}{9.804}\ \frac{\frac{kN}{m^2}}{\frac{kN}{m^3}} + 0 = \frac{1.228}{9.804} + \frac{V_2^2}{2g}\left(1 + \overset{0.8}{\Sigma k} + f\frac{L}{\underset{0.07}{D}}\right) + \overset{z_2 - z_1}{3}\ (*)$$

$$V_2 = \frac{Q}{A} = \frac{0.011}{\frac{\pi \times 0.07^2}{4}} = 2.86\ m/s$$

$$f = fn\left(Re, \frac{\varepsilon}{D}\right) \rightarrow Re = \frac{2.86 \times 0.07}{1.307 \times 10^{-6}}\ \frac{\cancel{\frac{m}{s}} \cdot \cancel{m}}{\cancel{\frac{m^2}{s}}}$$

Table B.2

$$Re = 1.53 \times 10^5$$

Used in practice

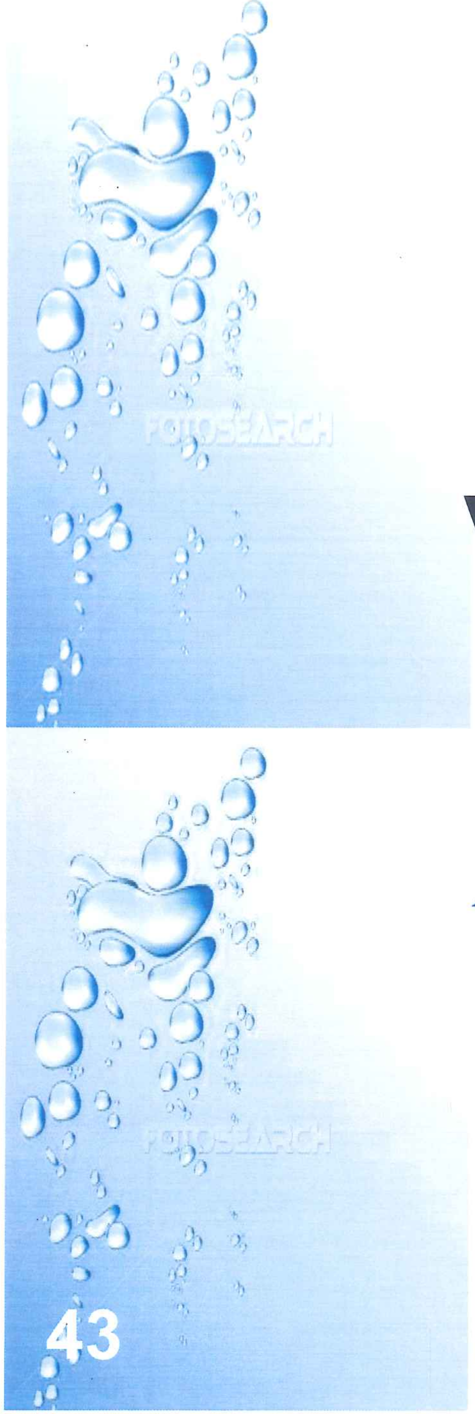
$$\nu = 10^{-6}\ \frac{m^2}{s}$$

⑧

$$\frac{\varepsilon}{D} = \frac{0.08}{1000 \times 0.07} = \boxed{0.00114}$$

f (Moody chart) $= 0.0216$

In (*) $\underline{\ell = 49.9\,m}$

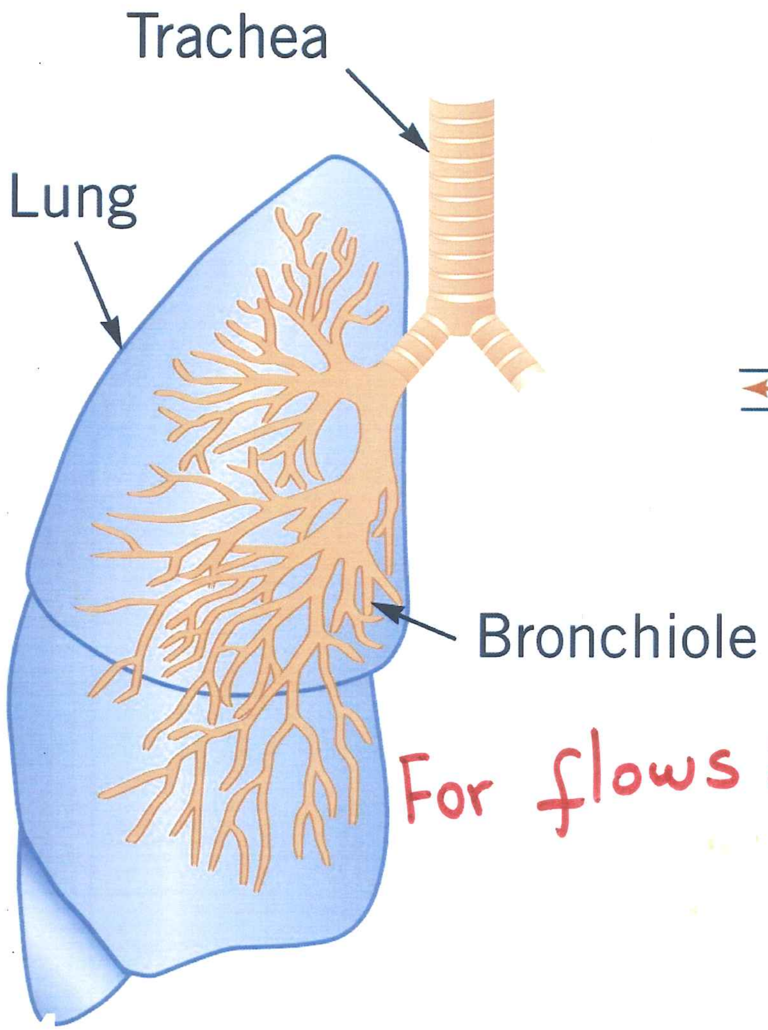


Viscous Flow in Pipes
Lecture 5, 01/17/2014

Arturo Leon, Oregon State University

Multiple pipe systems and complex pipe networks (steady flows)

Hardy Cross (1950's)



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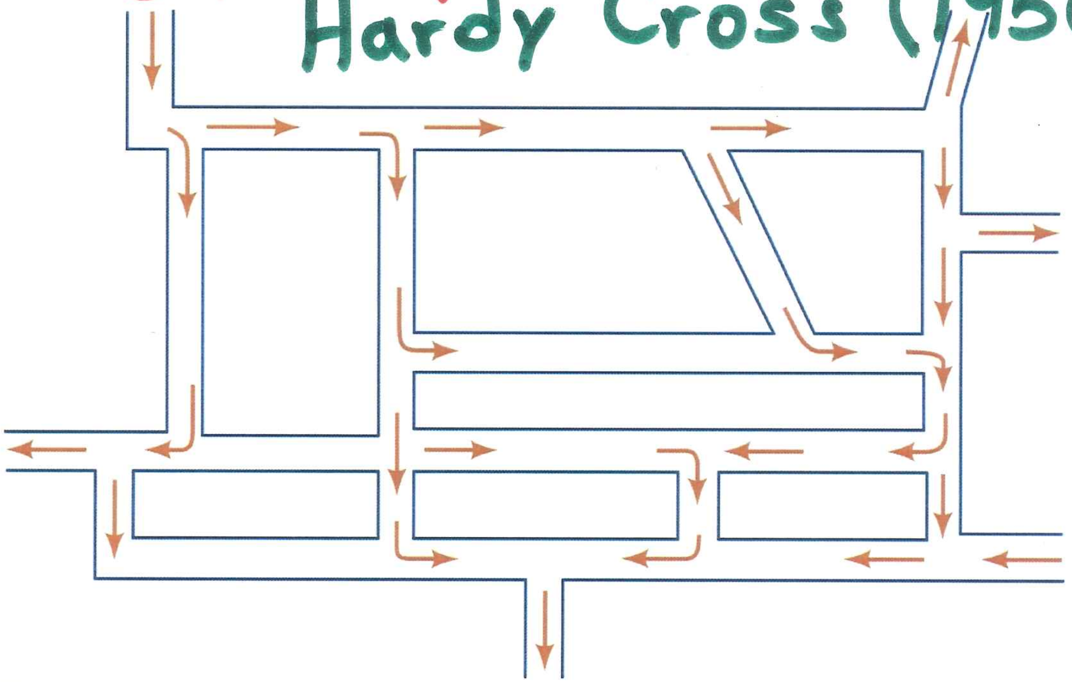
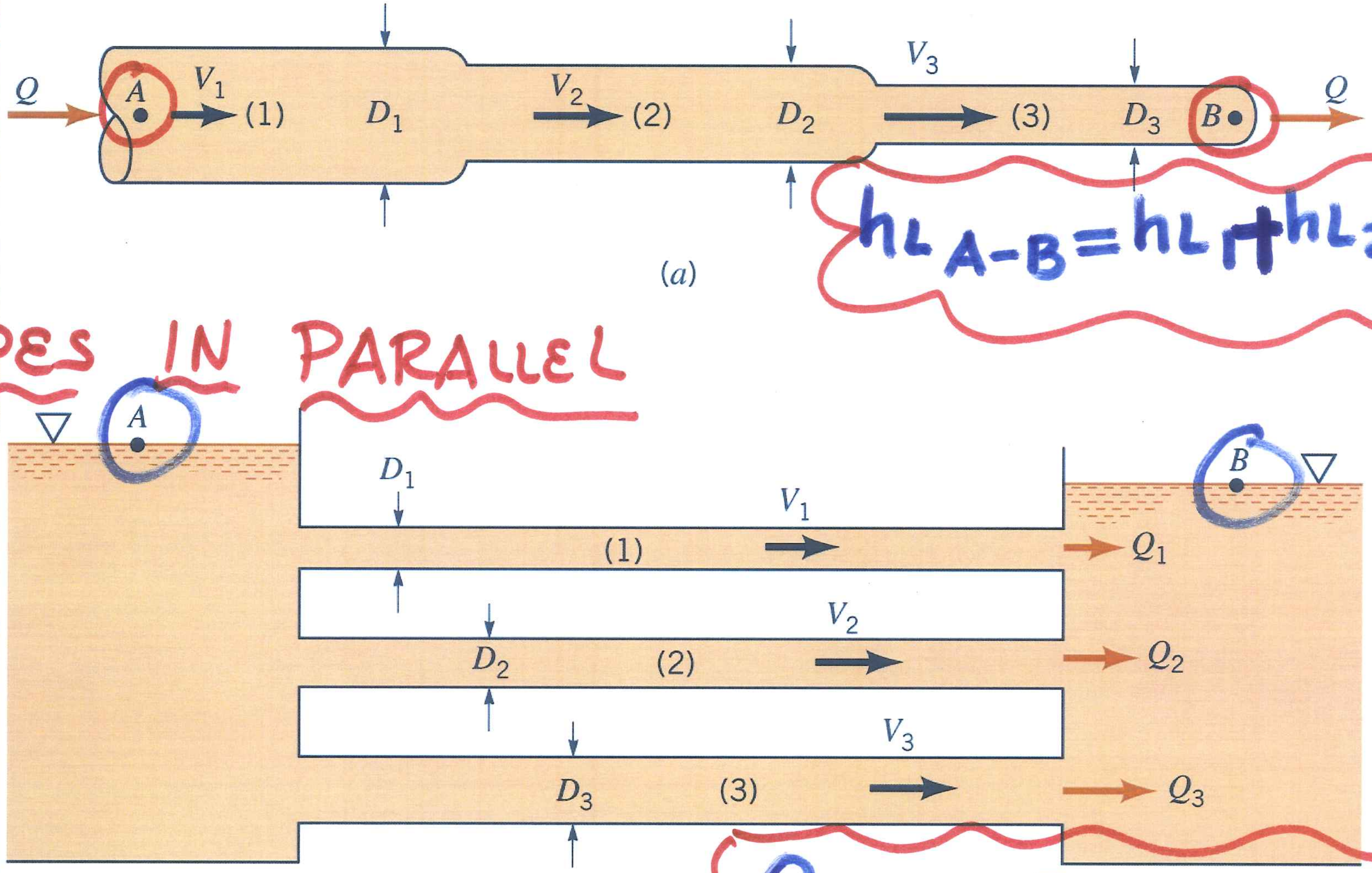


For flows: one-dimensional
bi-directional

uniform flow

PIPES IN SERIES

$Q_1 = Q_2 = Q_3$



(*a*)

$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$

PIPES IN PARALLEL

(*b*)

$Q_{A-B} = Q_1 + Q_2 + Q_3$

$h_{L_{A-B}} = h_{L_1} = h_{L_2} = h_{L_3} = \cdots h_{L_n}$

Figure 8.35

PIPE LOOP

$$\boxed{Q_1 = Q_2 + Q_3}$$ Continuity

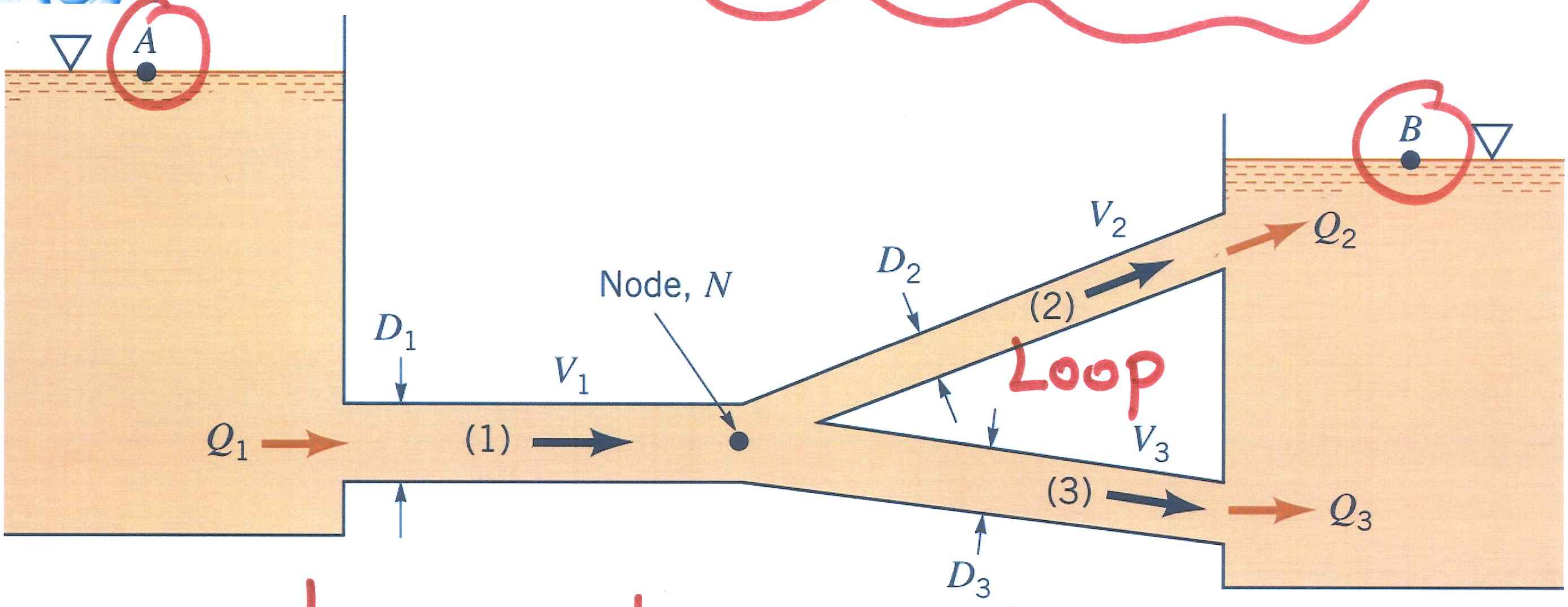


Figure 8.36

$$h_{L\,1-2} = h_{L\,1-3}$$

$$\cancel{h_{L1}} + h_{L2} = \cancel{h_{L1}} + h_{L3}$$

$$\boxed{h_{L2} = h_{L3}}$$

Arturo Leon Method for pipes:

① assume flow directions

② Name pipes

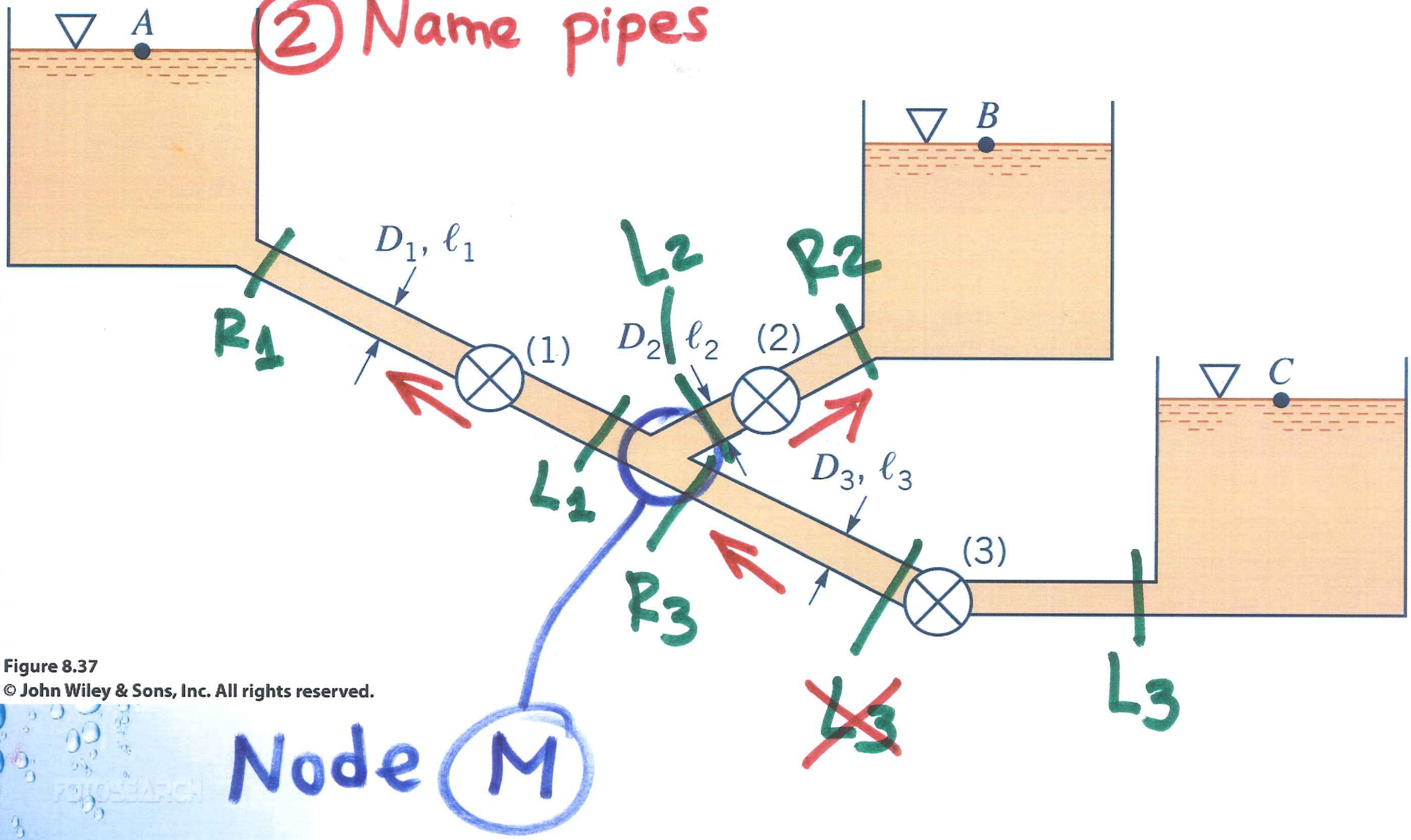
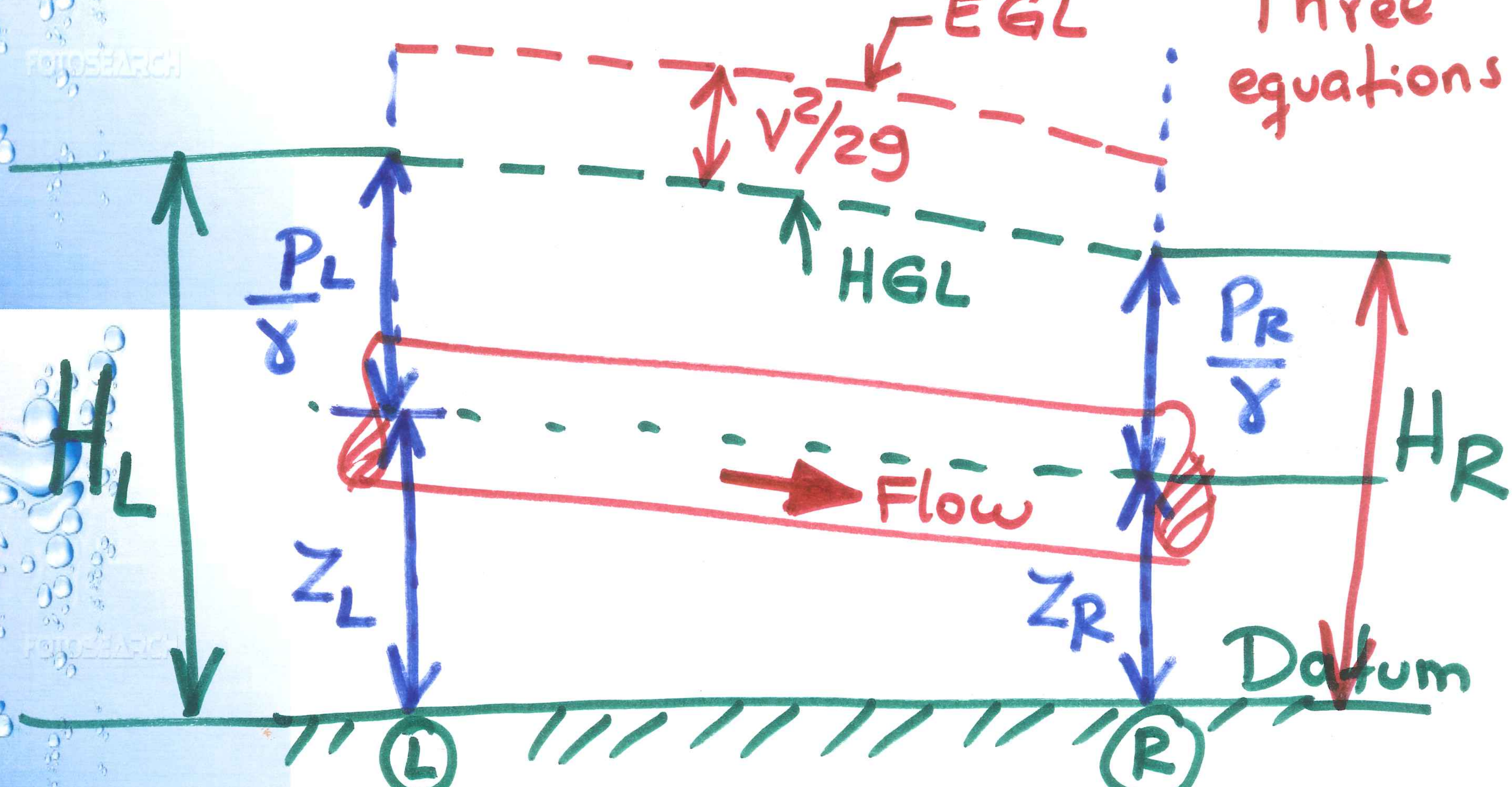


Figure 8.37

$$E_L = E_R$$

$$H_L + \cancel{\frac{V^2}{2g}} = H_R + \cancel{\frac{V^2}{2g}} + \Sigma h_L \rightarrow \boxed{H_{L_i} - H_{R_i} - h_{L_i} = 0}$$

Three equations



Unknowns:

3N where N is number of pipes

Continuity equation

$$\sum Q_{in} - \sum Q_{out} = 0$$

$$\boxed{Q_3 = Q_1 + Q_2}$$

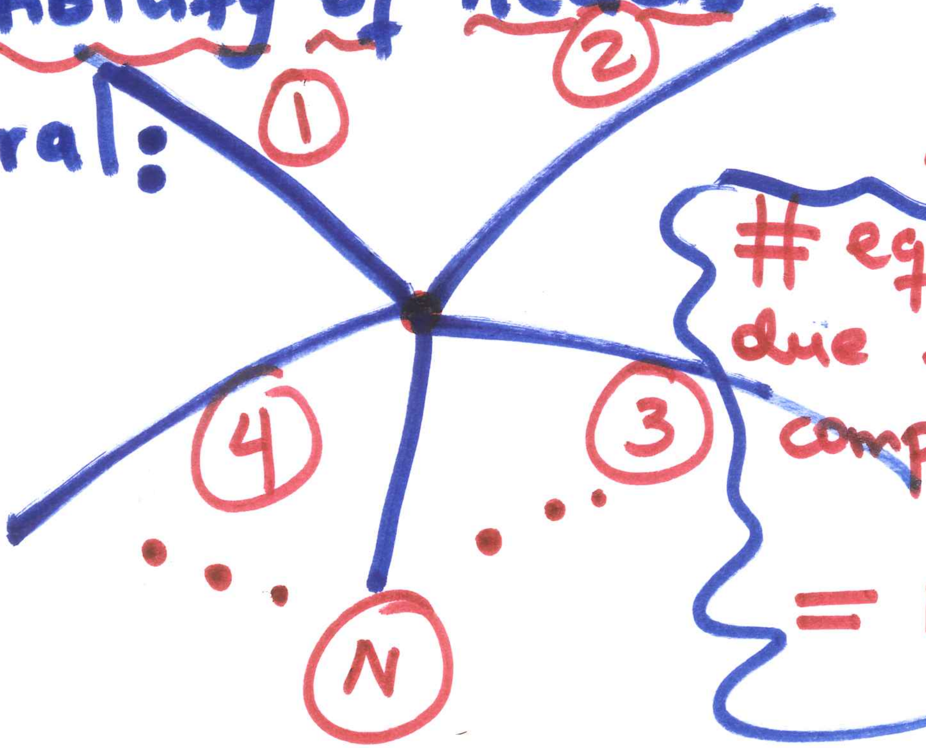
So far
4 equations

Compatibility of Heads

$$\left.\begin{array}{l}\boxed{H_{L_1} + \frac{V_1^2}{2g} = H_{L_2} + \frac{V_2^2}{2g}} \\ \boxed{H_{L_1} + \frac{V_1^2}{2g} = H_{R_3} + \frac{V_3^2}{2g}}\end{array}\right\} \text{two equations}$$

Compatibility of heads

In general:



N pipes

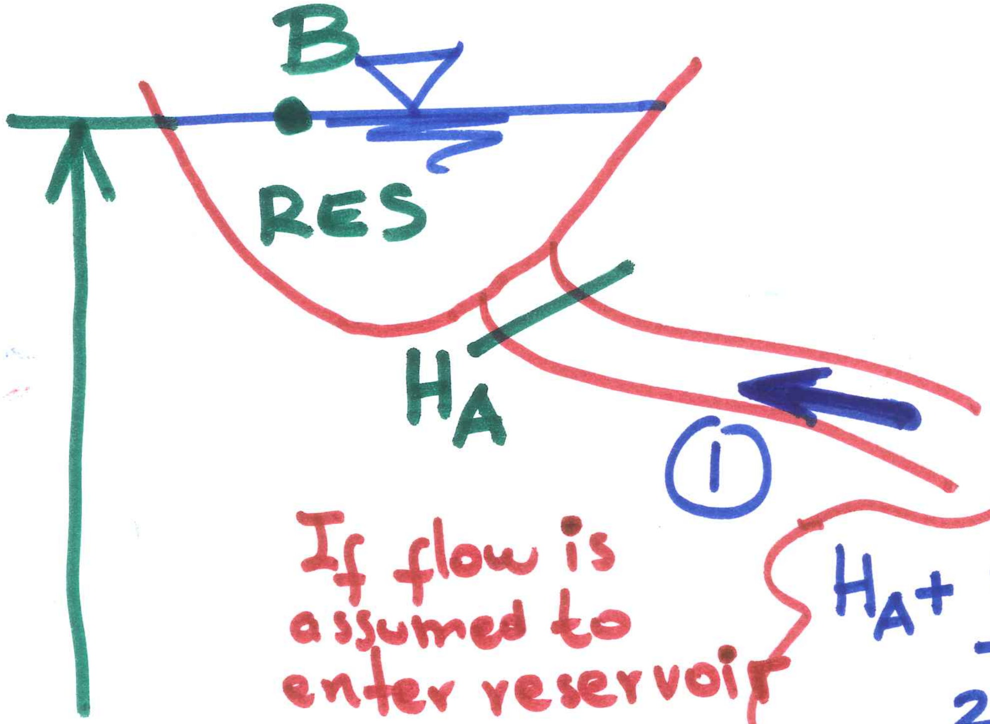
\# equations due to compatibility

$= N - 1$

So far 6 equations.

Lecture 06, 2014

Boundary Conditions (Reservoirs)



$$\boxed{E_{H_A} = E_B}$$

$$H_A + \frac{V_{HA}^2}{2g} = H_{res} + \frac{KV^2}{2g}$$

$H_{res} = Z_B$

If flow is assumed to enter reservoir

$$H_A + \frac{Q_1^2}{2gA_1^2} - H_{res} \enclose{circle}{-} k\frac{Q_1^2}{2gA_1^2} = 0$$

If flow is assumed to leave reservoir

$$H_{res} = H_A + \frac{V_{HA}^2}{2g} + \frac{KV^2}{2g}$$

$$H_A + \frac{Q_1^2}{2gA_1^2} - H_{res} \enclose{circle}{+} k\frac{Q_1^2}{2gA_1^2} = 0$$

In general:

If flow is assumed to enter reservoir:

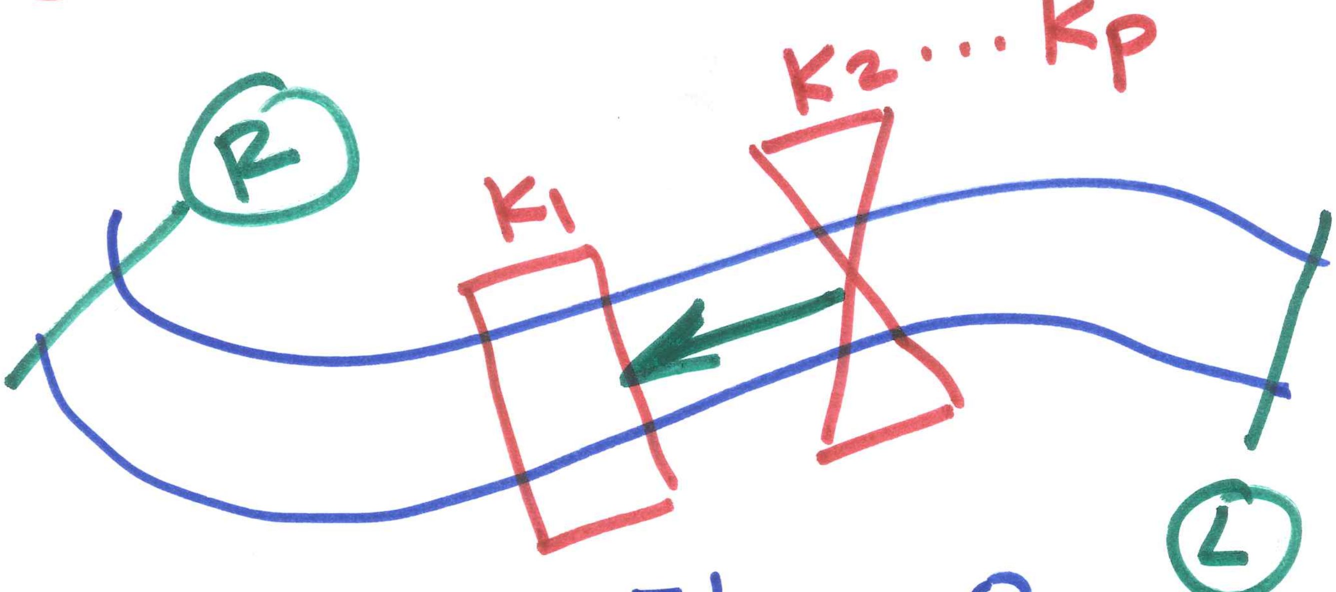
$$\boxed{H_A + \frac{Q_1^2}{2gA_1^2} - H_{res} - \frac{k\,Q_1|Q_1|}{2gA_1^2} = 0}$$

If flow is assumed to leave reservoir:

$$\boxed{H_A + \frac{Q_1^2}{2gA_1^2} - H_{res} + \frac{k\,Q_1|Q_1|}{2gA_1^2} = 0}$$

$$\left.\begin{array}{l}\text{9 equations}\\ \text{9 unknowns}\end{array}\right\} \text{Problem is solved.}$$

Revisiting head losses in a pipe:



$$H_{L_i} - H_{R_i} - \Sigma h_{L_i} = 0$$

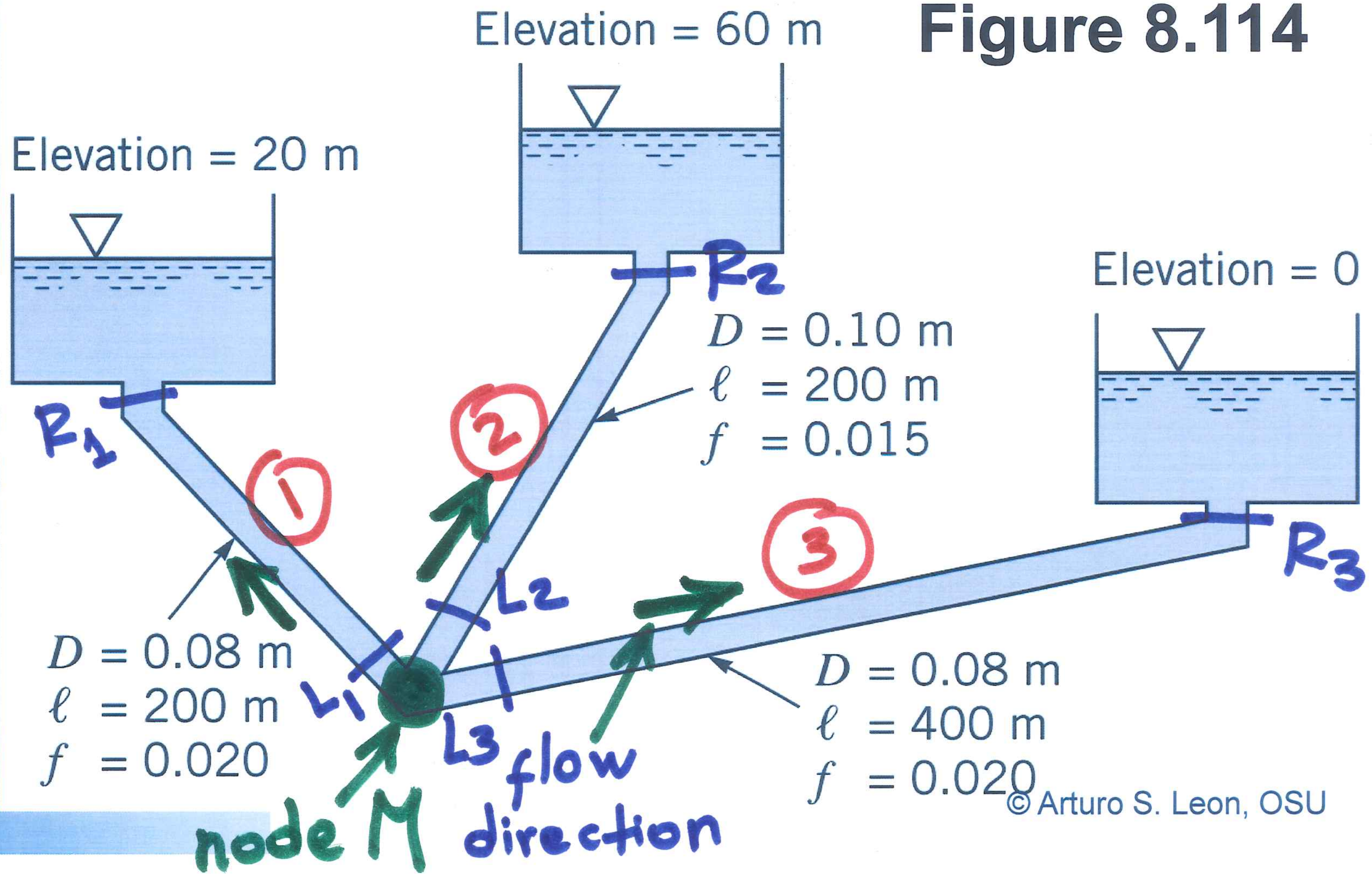
$$H_{L_i} - H_{R_i} - f\frac{L}{D}\frac{Q|Q|}{2gA^2} - (k_1 + k_2 + \dots k_p)\frac{Q|Q|}{2gA^2} = 0$$

$$\boxed{H_{L_i} - H_{R_i} - \left(f\frac{L}{D} + \Sigma K\right)\frac{Q|Q|}{2gA^2} = 0}$$

Example (Proposed problems):

8.114. The three water filled-tanks shown in Fig P.8.114 are connected by pipes as indicated. If minor losses are neglected, determine the flow rate in each pipe.

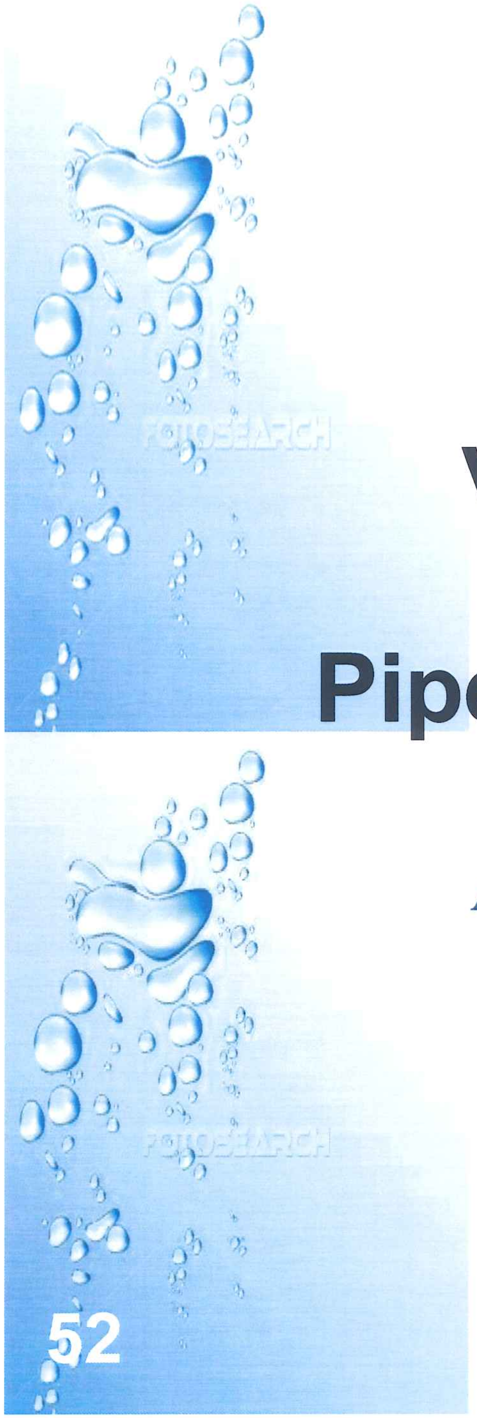
Figure 8.114



For "N" pipes, 3N equations

$$\rightarrow\begin{array}{l}HL_1\\HL_2\\\vdots\\HL_N\\HR_1\\HR_2\\\vdots\\HR_N\\Q_1\\Q_2\\\vdots\\Q_N\end{array}$$

$$3N\left\{\begin{array}{l}
2N\left\{\begin{array}{l}
N\left\{\begin{array}{l}x(1)\\x(2)\\\vdots\\x(N)\end{array}\right\}H_L\\
N\left\{\begin{array}{l}x(N+1)\\x(N+2)\\\vdots\\x(2N)\end{array}\right\}H_R
\end{array}\right.\\
N\left\{\begin{array}{l}x(2N+1)\\x(2N+2)\\\vdots\\x(3N)\end{array}\right\}Q
\end{array}\right.$$

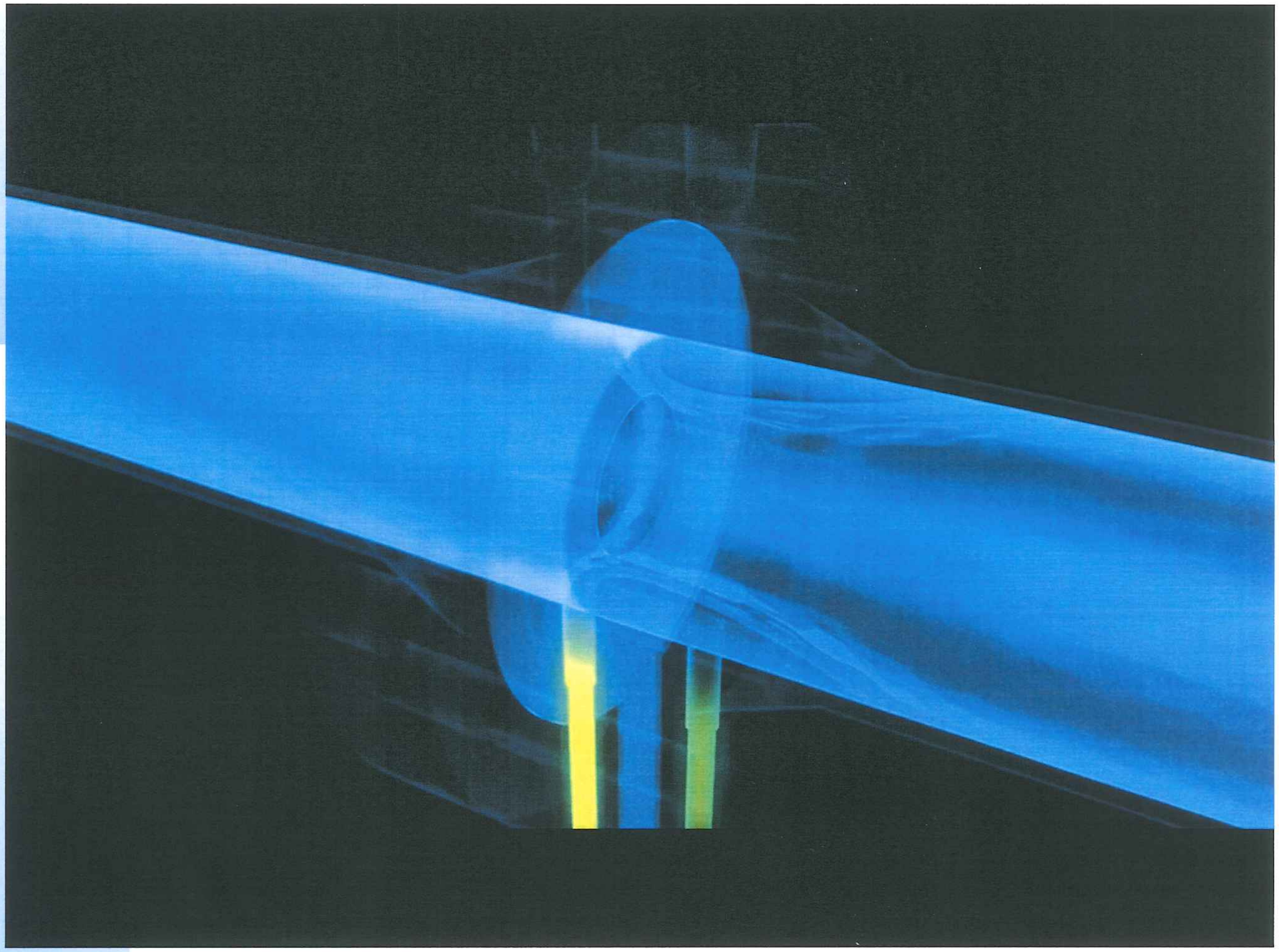


Viscous Flow in Pipes
Lecture 7, 01/24/2014
Pipe flow Rate Measurement

Arturo Leon, Oregon State University

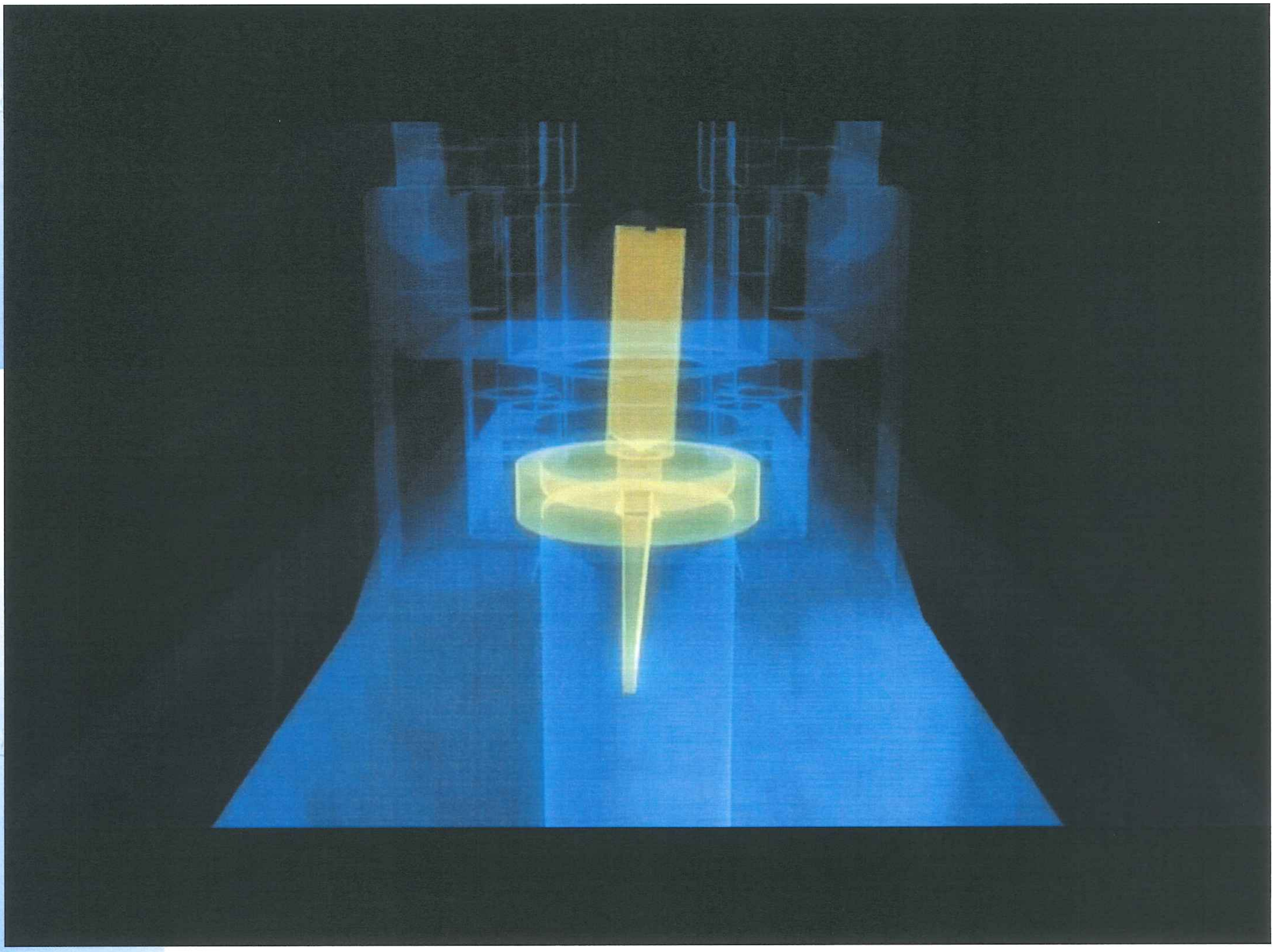
The Differential Pressure Flow Measuring Principle (Orifice-Nozzle-Venturi)

http://www.youtube.com/watch?v=oUd4WxjoHKY



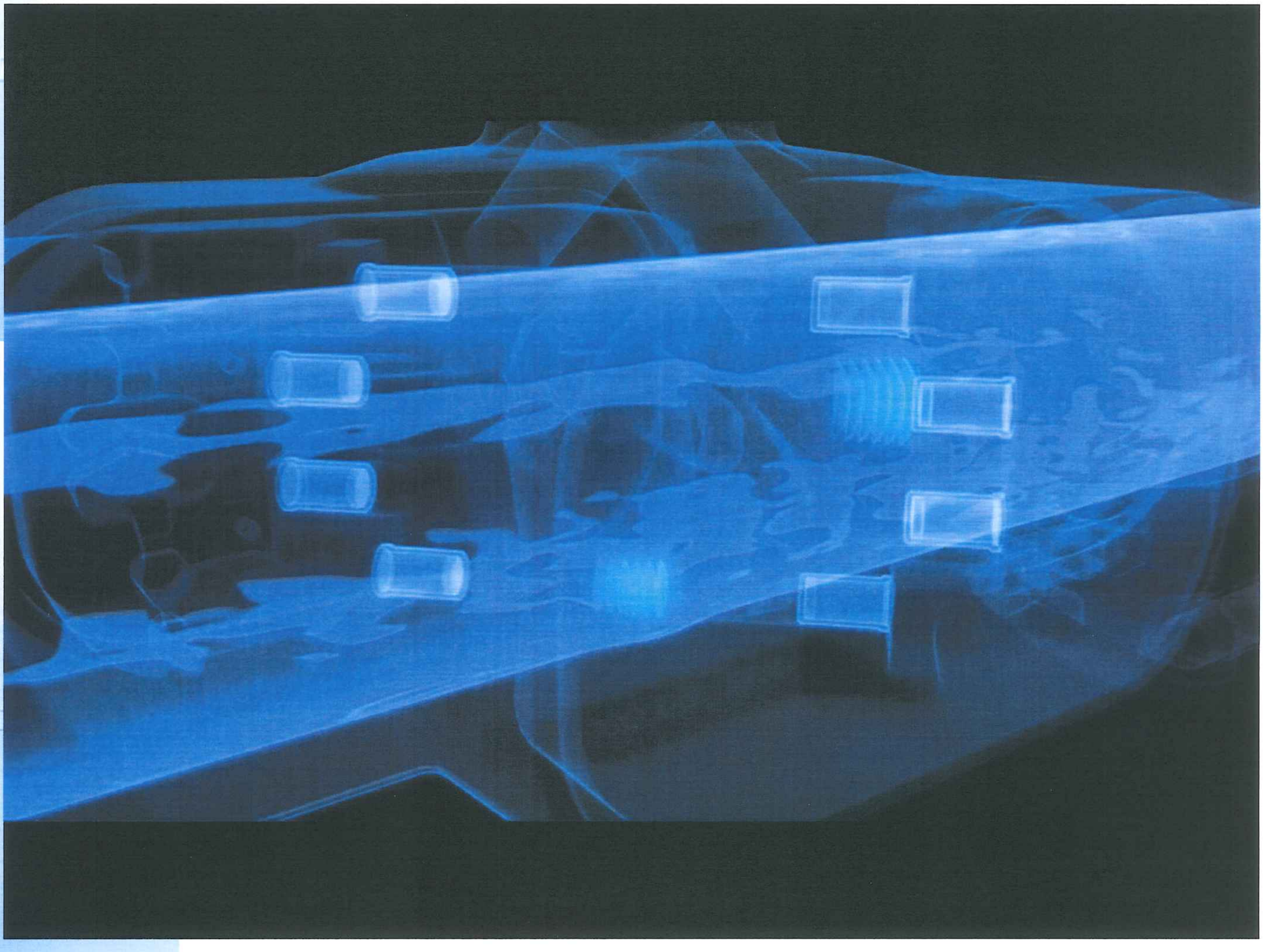
The Vortex Flow Measuring Principle

http://www.youtube.com/watch?v=GmTmDM7jHzA



The Ultrasonic Flow Measuring Principle

http://www.youtube.com/watch?v=Bx2RnrfLkQg



Pipe flowrate Measurement

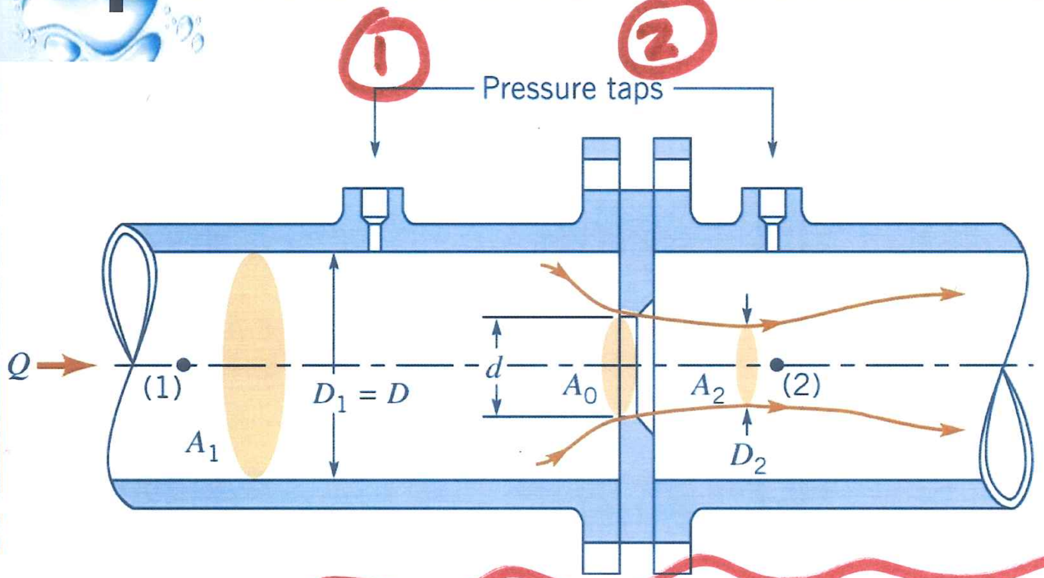


Figure 8.40

Typical orifice meter

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_1$$

$$A_1V_1=A_2V_2$$

combining

$$Q_{\text{ideal}} = A_2V_2 = A_2\sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}$$

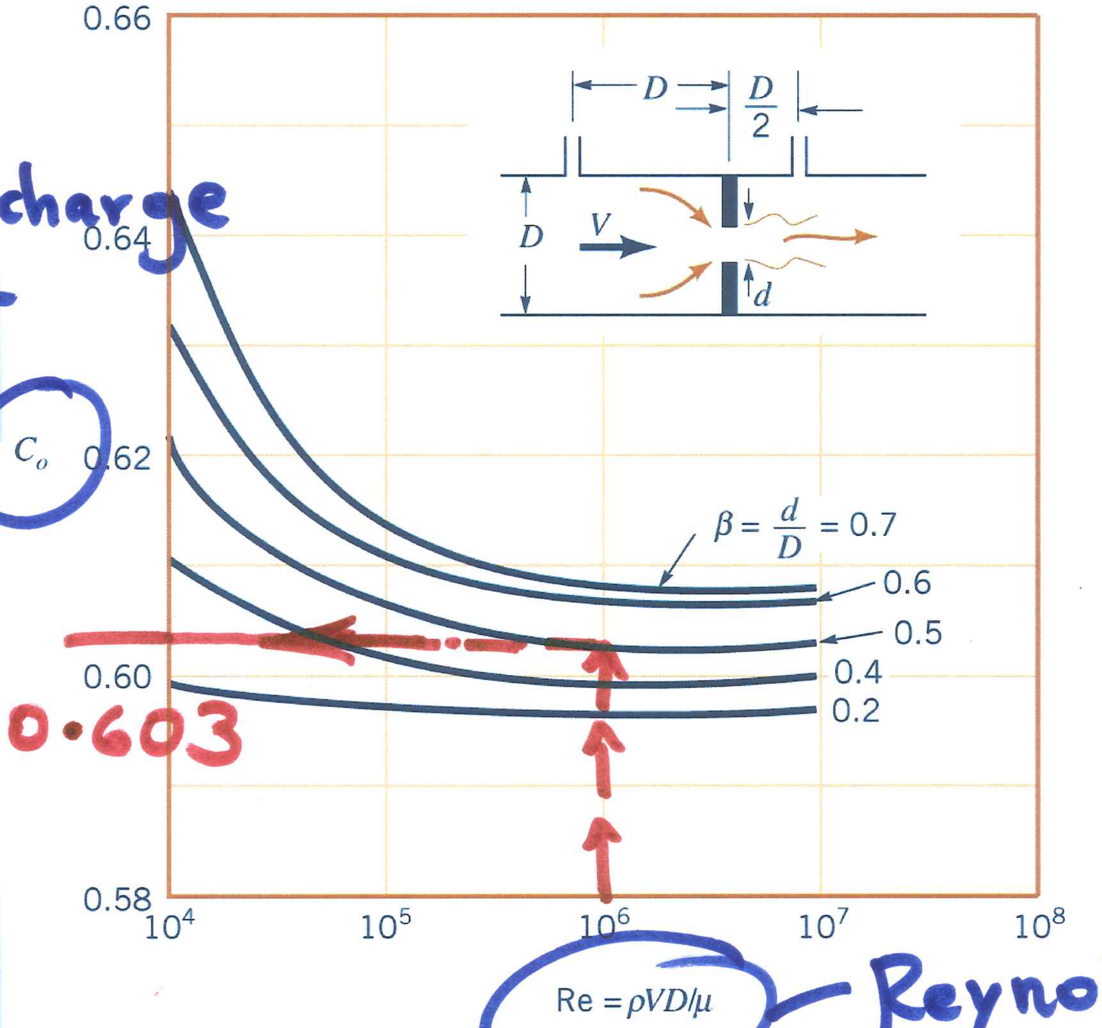
No head losses

$$\beta=\frac{d}{D}$$

$$\underset{\text{actual}}{Q} = C_o Q_{\text{ideal}} = C_o A_o \sqrt{\frac{2(p_1-p_2)}{\rho(1-\beta^4)}}$$

C_o = orifice coefficient

Orifice meter discharge coefficient



orifice discharge coefficient

Co ~ 0.603

Reynolds number

Figure 8.41

Nozzle meter

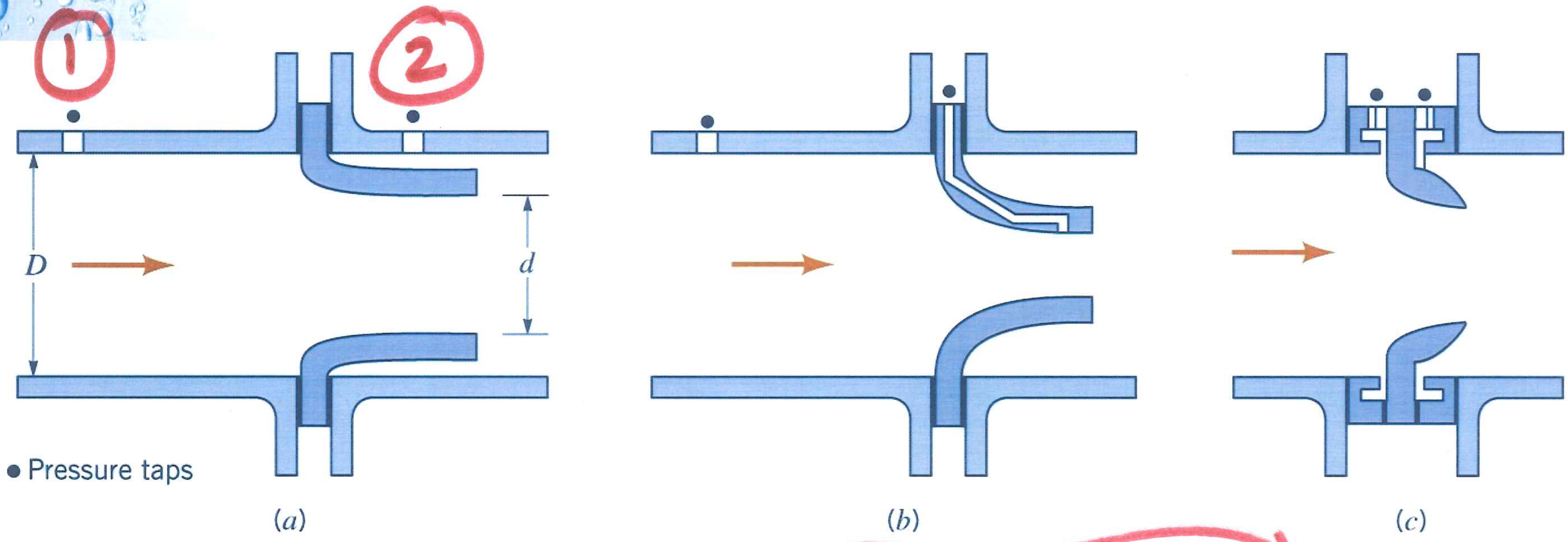


Figure 8.42
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$$Q = C_n Q_{\text{ideal}} = C_n \boxed{A_n} \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$

Equation 8.39
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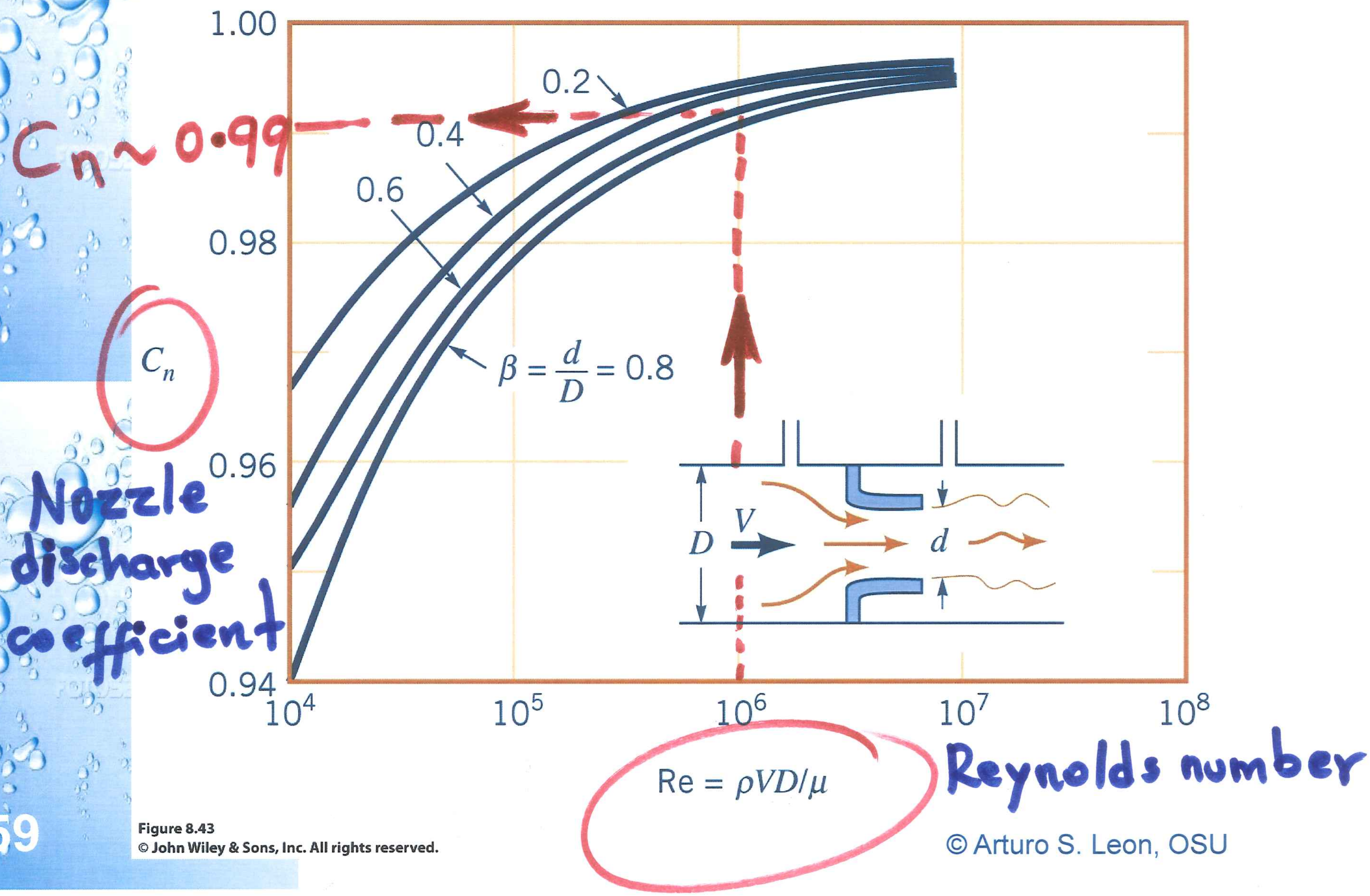
Cn : Nozzle discharge coefficient

nozzle area

$A_n = \pi d^2/4$

© Arturo S. Leon, OSU

Nozzle meter discharge coefficient



$C_n \sim 0.99$

Nozzle discharge coefficient

Reynolds number

Figure 8.43
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Venturi meter

(Most efficient of the three differential pressure meters)

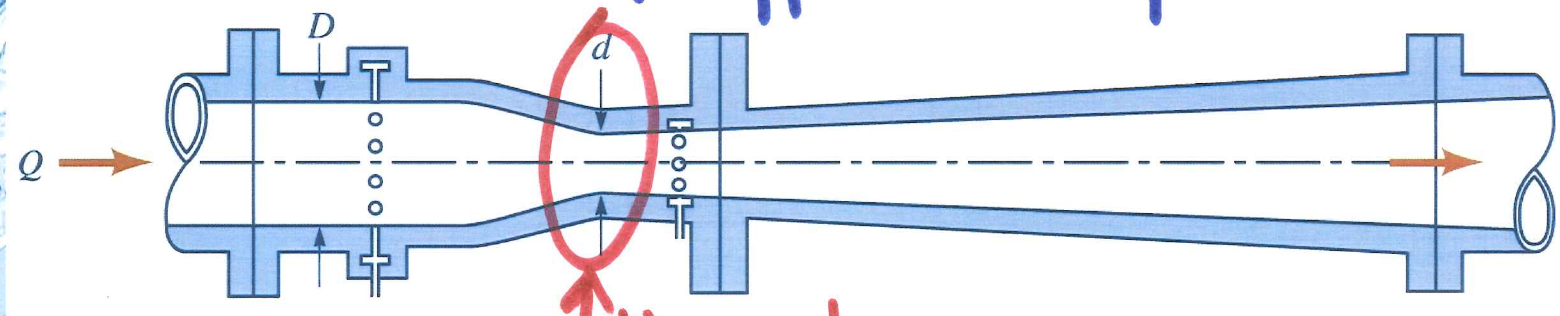

Figure 8.44
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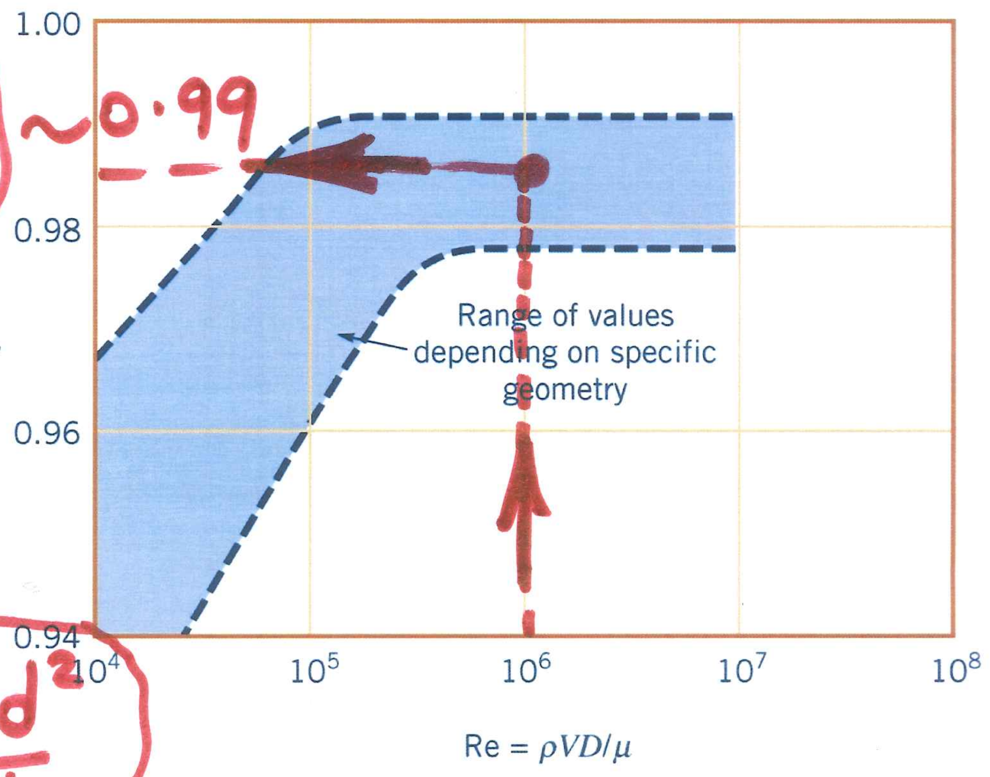
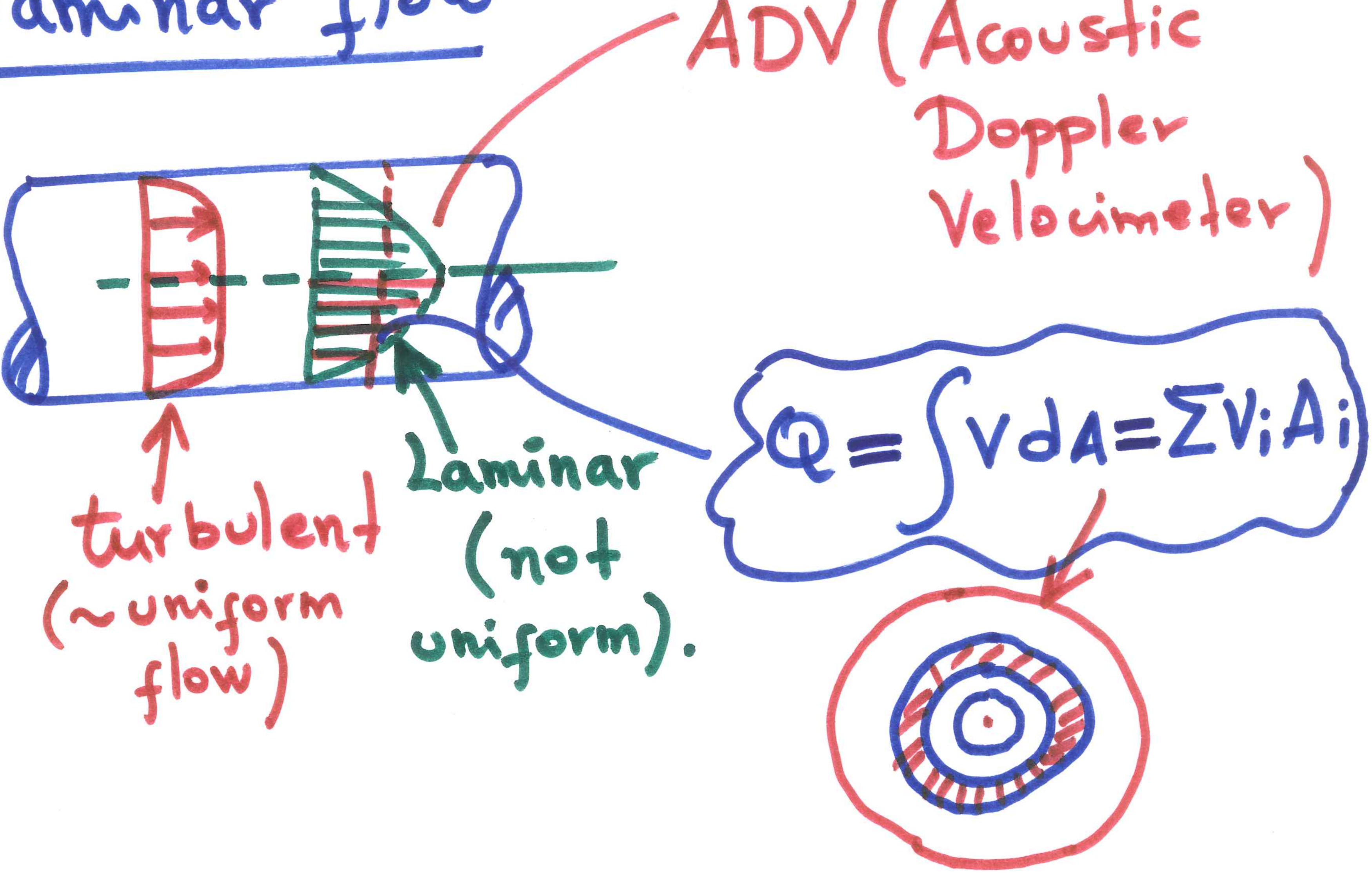
$$Q = C_v Q_{\text{ideal}} = C A_T \sqrt{\frac{2(p_1 - p_2)}{\rho(1 - \beta^4)}}$$



C_v = **Venturi meter discharge coefficient**

A_T = throat area = $\frac{\pi d^2}{4}$

Laminar flow



Example of application: Water flows through the Venturi meter shown in Fig. 8.125. The specific gravity of the manometer fluid is 1.52. Determine the flowrate.

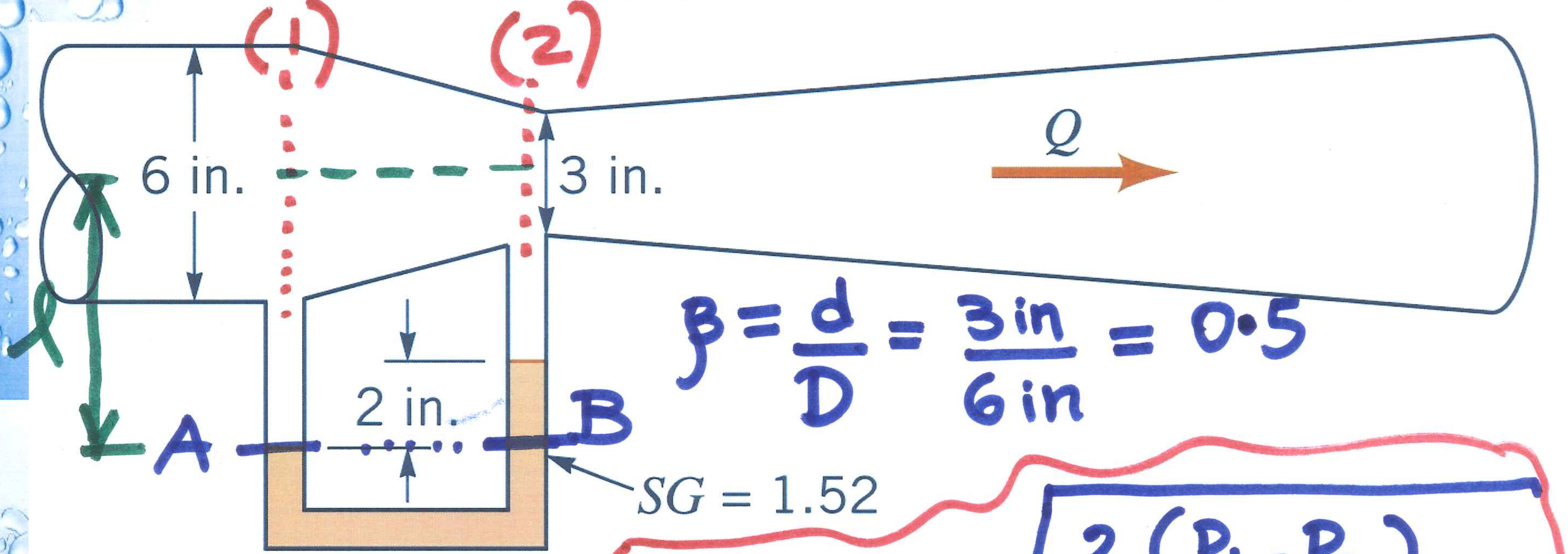


Figure P8.125

$$\beta = \frac{d}{D} = \frac{3in}{6in} = 0.5$$

$$\boxed{Q = C_v A_T \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}}$$

(*)

$$A_T = \frac{\pi}{4}\left(\frac{3}{12}\right)^2 = \frac{\pi}{64}$$

$$\boxed{P_A = P_B}$$

$$P_1 + \cancel{\gamma_w l} = P_2 + \gamma_w(\cancel{l} - \tfrac{2}{12}) + \gamma_f(\tfrac{2}{12})$$

$$P_1 - P_2 = \frac{2}{12}\left(\frac{\gamma_f - \gamma_w}{\gamma_w}\right)\gamma_w \xleftarrow{} 0.52$$

$$P_1 - P_2 = 0.0867\,\rho_w \cdot g \qquad g = 32.2\ ft/s^2$$

In (*) $Q = 0.1198\,C_v$

assume $C_v = 0.98$ (see graph)

$$Q = 0.1174\ ft^3/s$$

$$V = 0.596\ ft/s$$

$$Re = 2.46 \times 10^4$$

from graph $\boxed{C_v = 0.96}$

$V = 0.586$ ft/s

$Re = 2.42 \times 10^4$

From graph:

$\boxed{C_v = 0.96}$ ✓ ✓ Same as previous.

$$\boxed{Q = 0.115 \text{ ft}^3/\text{s}}$$