Solution of problem 4.3.3 (textbook) using Matlab script.

01/29/2012 6:20 pm

Data:

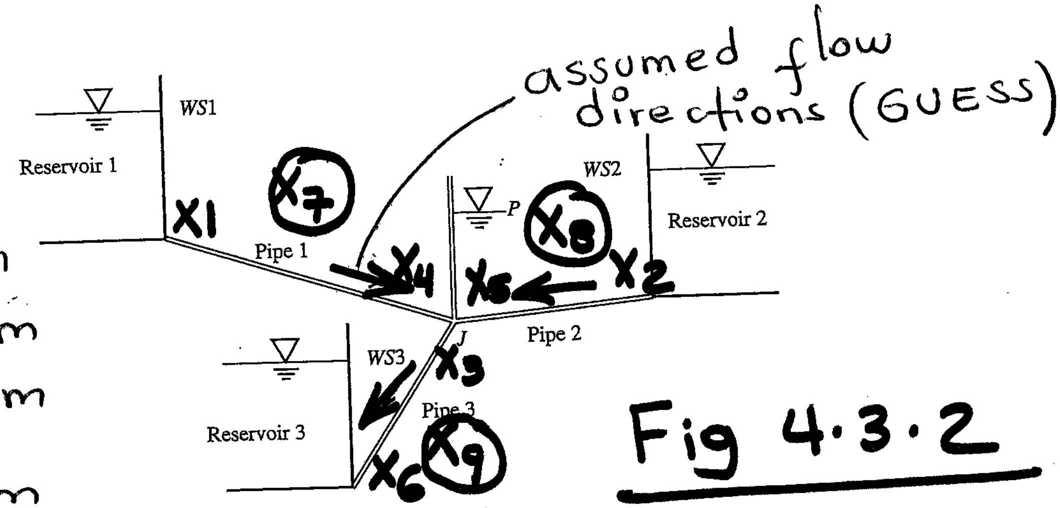


Fig 4.3.2

$WS1 = 2100\ m$

$WS2 = 2080\ m$

$WS3 = 2060\ m$

$L_1 = 5000\ m$

$L_2 = 4000\ m$

$L_3 = 5000\ m$

$D_1 = 1.0\ m$

$D_2 = 0.3\ m$

$D_3 = 1.0\ m$

$\nu = 10^{-6}\ m^2/s$ ($T = 20^\circ C$)

$e = 0.045$ mm (all pipes)

Flow rates ??

Unknowns : 9 (3x3)

Solution 1 (neglecting local head losses at the entrance/exit of reservoirs).

Equations:

* Head loss in each pipe (See Fig. 4.3.2)

In flow direction of pipe 1 $X(1) = X(4) + \underbrace{\left(f_1 \frac{L_1}{D_1} \frac{Q_1|Q_1|}{(2gA_1^2)}\right)}_{HLcof\,1}$

or $x(1) - x(4) - HLcof_1\, X(7)\,|X(7)| = 0$ ← absolute value

For pipe 2 : $X(2) - X(5) - HLcof\,2\ x(8)\,|x(8)| = 0$

For pipe 3 : $X(3) - X(6) - HLcof\,3\ X(9)\,|X(9)| = 0$

②

* Boundary conditions at reservoirs

Neglecting local head losses

For pipe 1 $\quad 2100 = X(1)$ [In flow direction]

or $\quad 2100 - X(1) = 0$

For pipe 2 : $2080 - X(2) = 0$

For pipe 3 : $X(3) - 2060 = 0$

* Continuity equation (according to assumed flow direction)

$$X_7 + X_8 = X_9$$

or $X_7 + X_8 - X_9 = 0$

* Compatibility conditions of Total Heads

(See fig 4.3.2 in this solution).

At node J : $X_4 = X_5$ and $X_4 = X_3$

$X_4 - X_5 = 0$ and $X_4 - X_3 = 0$

or

$$\begin{bmatrix} f(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Matlab: (don't forget to input the initial gues value for X in Main Script.m)

$X(1) = 2100$ m [HL(1)] $\quad X(6) = 2060$ m

$X(2) = 2080$ m [HL(2)] $\quad X(7) = 2.052$ m³/s [Q(1)]

$X(3) = 2079.999$ m $\quad X(8) = \sim 0$ m³/s [Q(2)]

$X(4) = 2079.999$ m

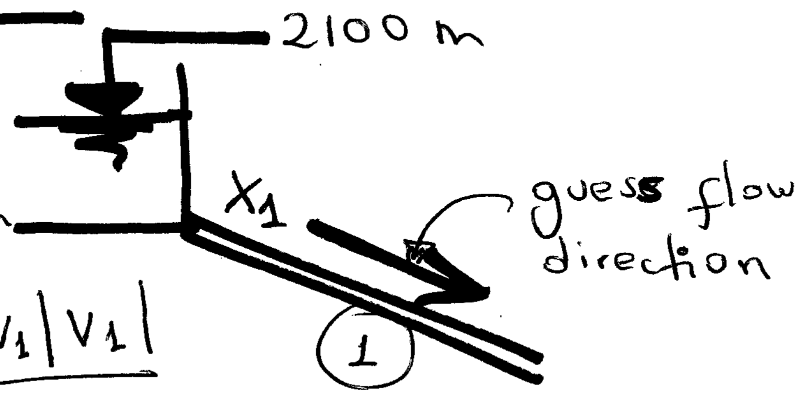
$X(5) = 2079.999$ m $\quad X(9) = 2.052$ m³/s [Q(3)]

Solution 2 (including local head losses at the entrance/exit of reservoirs).

For this case, all equations are the same except the boundary conditions at the reservoirs.

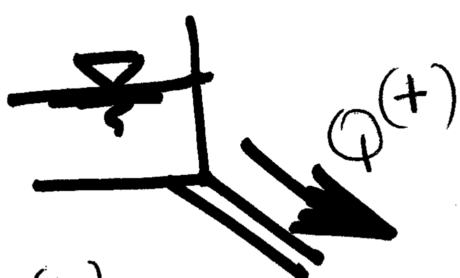
For reservoir 1

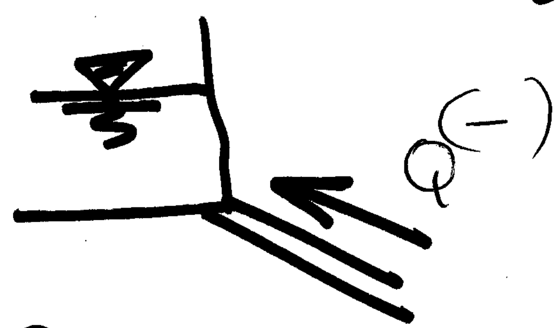
Energy equation along flow direction



$$E_{res\,1} = E_{x_1} + K \frac{V_1 |V_1|}{2g}$$

which "K" should be used?





if $Q^{(+)}$, $K = 0.5$

if $Q^{(-)}$, $K = 1.0$

for pipe 1

$$Q(1) = X(7)$$

How to express this in matlab?

(4)

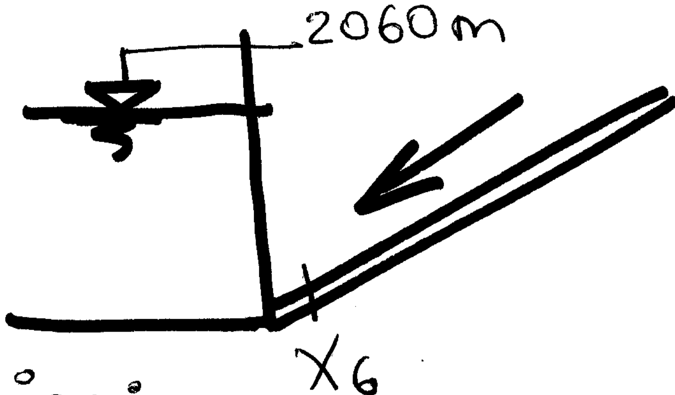
```
if x(7) >= 0
        K = 0.5
else
        K = 1.0
endif
```

Then for pipe 1 :

$$2100 - X(1) - K(1) * \frac{V(1)\,|V(1)|}{2g} = 0$$

for pipe 2 : $2080 - X(2) - K(2)\,\frac{V(2)\,|V(2)|}{2g} = 0$

for pipe 3 :



In flow direction;

$$X_6 = 2060 + K_3\,\frac{V_3|V_3|}{2g}$$

or

$$X(6) - 2060 - K(3)\,\frac{V_3\,|V_3|}{2g} = 0$$

Using Matlab

$X(1) = 2099.828$ m

$X(2) = 2080.0001$ m

$X(3) = 2080.0511$ m

$X(4) = 2080.0511$ m

$X(5) = 2080.0511$ m

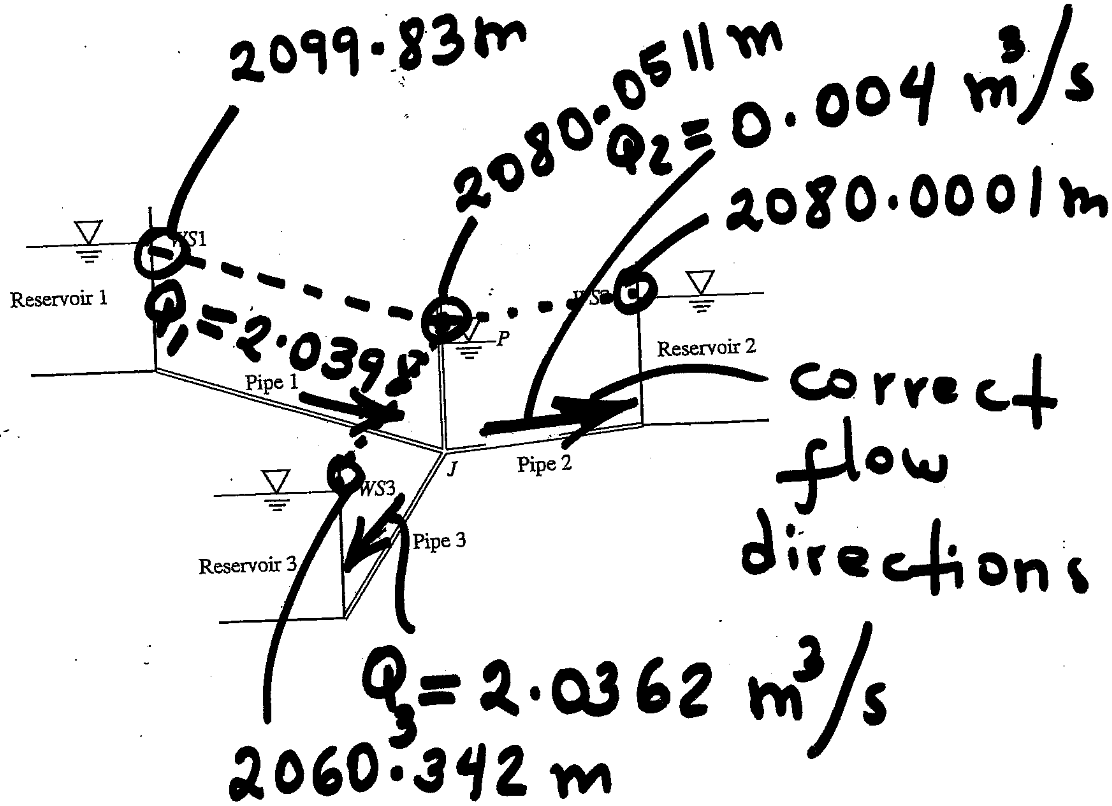
$X(6) = 2060.342$ m

$X(7) = 2.0398\ m^3/s$ **Q(1)**

$X(8) = -0.004\ m^3/s$ **Q(2)**

$X(9) = 2.0362\ m^3/s$ **Q(3)**

Results considering local head losses



Note that the results including and not considering local head losses are very similar. In long pipelines, it is okay to neglect local head losses.

Tips in Matlab

% (means the line is commented)

If a line in the code will not be used, it should be commented.