Final Exam of CE 313 (Hydraulic Engineering), Winter 2013

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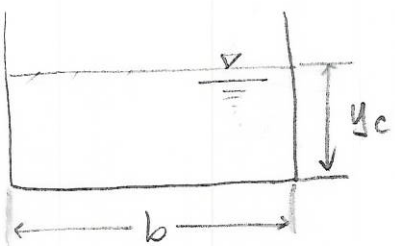
Name: Solution **Date:** 03/21/2013

- ✓ **You will have 1h 50 minutes to complete the exam. The exam is closed book, closed notes and open mind.**
- ✓ **The procedure will be graded. Please justify your answers**
- ✓ **The exam consists of 4 questions. Each question is worth 25 points.**
- ✓ **Only specified calculators in the syllabus will be permitted**

1. You are asked to design a rectangular channel that has the minimum wetted perimeter and that conveys flow in critical conditions. Find the relationship between the critical depth and the channel width if the flow discharge is constant. Your answer should look something like "$Y_c = cb$", where "c" is a constant and "b" is the channel width.



$P = b + 2y_c$

$P_{min} \rightarrow dP/dy_c = 0 = db/dy_c + 2$ 5pt

$\Rightarrow db/dy_c = -2$

$Fr = 1 \rightsquigarrow V = \sqrt{gy_c}$ 5pt or $Q = by_c\sqrt{gy_c} = bg^{1/2}y_c^{3/2}$

Since Q is constant, $dQ/dy_c = 0 = bg^{1/2}(3/2)y_c^{1/2} + \frac{db}{dy_c}g^{1/2}y_c^{3/2}$ 5pt

$$0 = \frac{bg^{1/2}(3/2)y_c^{1/2}}{g^{1/2}y_c^{1/2}} + \frac{db/dy_c\, g^{1/2}y_c^{3/2}}{g^{1/2}y_c^{1/2}}$$

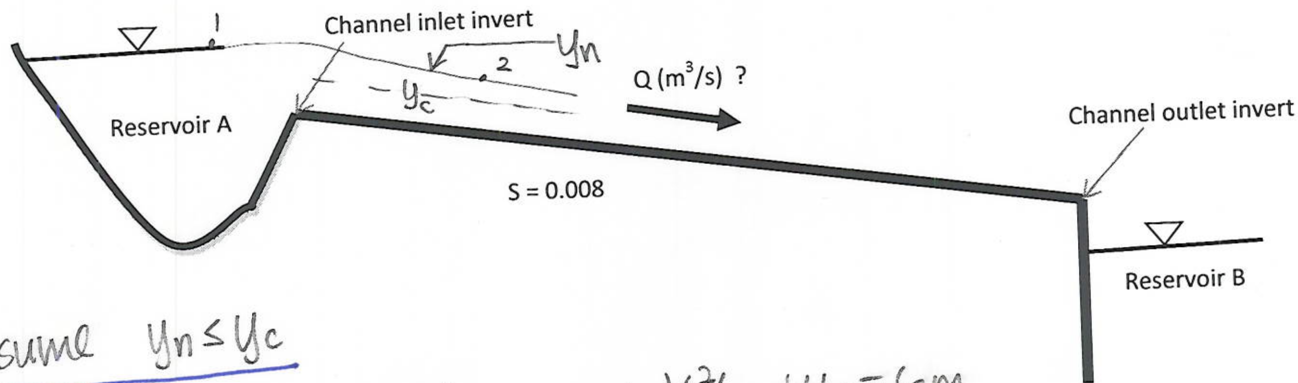
10pt for overall process.

$$0 = b(3/2) + db/dy_c\, y_c$$

$$0 = b(3/2) - 2y_c$$

$$\therefore y_c = 3b/4$$

2. A long rectangular channel connects reservoirs "A" and "B" as sketched below. The channel has a width of 8 m, a longitudinal slope of 0.008 and a Manning's roughness "n" of 0.030. If the upstream reservoir water surface is 6.00m above the channel inlet invert and the downstream reservoir water surface is below the channel outlet invert, determine the flow discharge in the channel in m³/s. Neglect local head losses.



Assume $y_n \le y_c$

Then $Fr = 1$, $V_c = \sqrt{g y_c}$ and $V_c^2/2g + y_c = 6m$

Solving for $y_c = 2/3 H = 4m$

On the other hand, $q = Q^*/b = \sqrt{g y_c^3} = 25.06\ m^2/s$

$Q^* = q \times b = 25.06 m^2/s \times 8m = 200.45\ m^3/s$

$= k/n\, A R_h^{2/3} S^{1/2}$

$$= \frac{1}{0.030}(y_n \cdot b)\frac{(y_n \cdot b)^{2/3}}{(2y_n + b)^{2/3}}(0.008)^{1/2} = 200.45\ m^3/s$$

4pt

$y_n = 4.95\,m \ge y_c = 4m$ → Assumption is wrong

4pt

Assume $y_n \ge y_c$

$$z_1 - z_2 = V_2^2/2g = Q^2/(8^2 y_2^2 \cdot 2 \cdot g)$$

$$(6 - y_2)(8^2 y_2^2\, 2g) = \left[\frac{1}{0.030}(y_2 \cdot 8)\left[\frac{y_2 \cdot 8}{2y_2 + 8}\right]^{2/3}(0.008)^{1/2}\right]^2$$

$y_2 = 4.73\,m$ (Normal) 4pt ⇝ $Q = \frac{1}{0.030}(y_2 8)\left[\frac{y_2 8}{2y_2\ 8}\right]^{2/3}(0.008)^{1/2} = 188.9\ m^3/s$

4pt

Find Critical depth

$V_c = \sqrt{g y_c}$ ⇝ $Q = b y_c \sqrt{g y_c} = b\, y_c^{3/2} g^{1/2}$

$$y_c = \left(\frac{Q}{b g^{1/2}}\right)^{2/3} = 3.84\,m$$

4pt

$y_n = 4.73\,m \geq y_c = 3.84\,M$

∴ Assumption is correct, $Q = 188.9\,m^3/s$

5pt for overall process

3. A trapezoidal channel is to be designed to carry a discharge of 75 m^3/s at **maximum hydraulic efficiency**. The side slopes of the channel are 2H:1V (2 Horizontal and 1 Vertical) and the Manning's roughness n is 0.030.
   (a) If the maximum allowable velocity in the channel is 1.75m/s, what should be the dimensions of the channel (bottom width and height)?
   (b) What should be the longitudinal slope of the channel if the flow is uniform?

a) 15pt

$A = Q/V = 75/1.75 = 42.857 m^2$

$b = 2y(\sqrt{1+z^2} - z) = 0.472y \quad (z=2)$ — 10pt

$A = by + zy^2 = 0.472y^2 + 2y^2 = 2.472y^2$

5pt for process and answer

$A = 42.86 = 2.472y^2$

$y^2 = 17.336 \rightarrow y = 4.16 m \Rightarrow b = 0.472y = 1.96m$

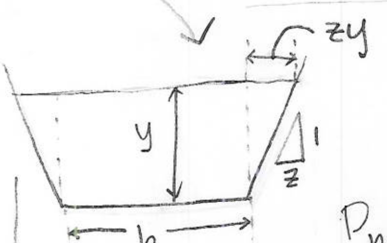
b) 10pt

$P = b + 2y\sqrt{1+z^2} = 1.96 + 2(4.16)\sqrt{1+2^2} = 20.56m$

$R = A/P = 42.86/20.56 = 2.08 m$

$V = 1.75 = 1/0.030 (2.08)^{2/3} S_o^{1/2}$

$S_o = 0.00104 m/m$



$A = by + zy^2$ (A and z are constants)

$P = b + 2y\sqrt{1+z^2}$

$P_{min} \Rightarrow$ maximum hydraulic efficiency, $\frac{dP}{dy} = 0$

$dP/dy = db/dy + 2\sqrt{1+z^2} = 0 \Rightarrow db/dy = -2\sqrt{1+z^2}$

$dA/dy = b + y\,db/dy + 2zy = 0$

$0 = b - 2y\sqrt{1+z^2} + 2zy \Rightarrow b = 2y(\sqrt{1+z^2} - z)$

4. [ε = 0.0009 m for steel pipe, kinematic viscosity of water at 16°C = 1.15 x 10^{-6} m^2/s]

A pump station is used to fill a tank on a hill above from a lake below as shown in Figure below. The flow rate is 10.5 l/s at a temperature of 16°C. The pump is 4 m above the lake and the tank water level is 115 m above the pump. The suction and discharge lines are 10.2-cm-diameter steel pipe. The equivalent length of the inlet line between the lake and the pump is 100 m while the total equivalent length between the lake and the tank 2300 m. The overall efficiency of the pump and motor is 70%.

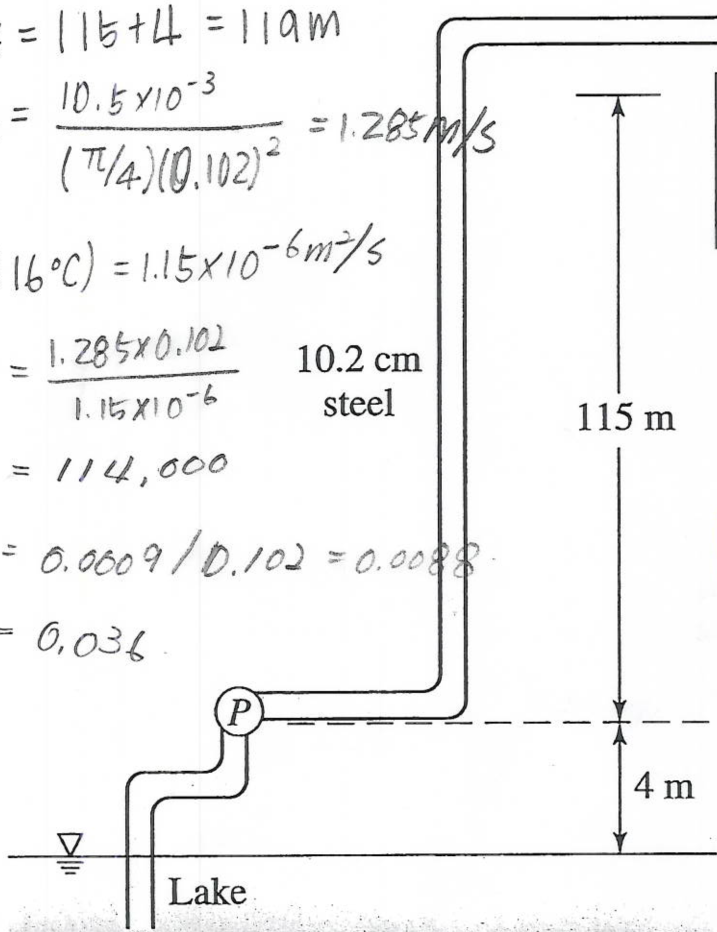
$\gamma_{16°c} = 9.79 \times 10^3 \frac{N}{m^3}$

$P_{atm} = 101 \times 10^3 N/m^2$

$P_{vap\ 16°c} = 1.783 \times 10^3 \frac{N}{m^2}$

a. What is the required power of the motor in kW? 10 pt

b. What is the NPSH for this application? 15 pt



a. $\Delta z = 115 + 4 = 119m$

$V_1 = V_2 = \frac{10.5 \times 10^{-3}}{(\pi/4)(0.102)^2} = 1.285 m/s$

$\nu (@16°C) = 1.15 \times 10^{-6} m^2/s$

$R_n = \frac{Vd}{\nu} = \frac{1.285 \times 0.102}{1.15 \times 10^{-6}}$

$= 114,000$

$\epsilon/d = 0.0009/0.102 = 0.0088$

$\rightarrow f = 0.036$

$h_f = f \frac{L}{d} \frac{V^2}{2g} = 0.036 \frac{2300}{0.102} \frac{1.285^2}{2g}$

$= 68.31 m$

3 pt

$h_{pump} = 119 + 68.31 = 187.31 m$

4 pt

$P = \gamma Q h$

$= 9.79 \times 10^3 \frac{N}{m^3} \times 0.0105 \frac{m^3}{s} \times 187.31 m$

$= 19254 W = 19.3 kW$

3 pt

$P_{70\%\ efficiency} \rightarrow P/0.7 = 28 kW$

b) $NPSH = \frac{P_s}{\gamma} + \frac{V_s^2}{2g} - \frac{P_v}{\gamma}$ ———————— eq ①

$E_{lake} = E_{suction\ side\ of\ pump}$

$$P_1/\gamma + V_1^2/2g + Z_1 = \underbrace{P_s/\gamma + V_s^2/2g}_{M} + Z_s + f\frac{L}{D}V_s^2/2g$$

5pt

$$\frac{P_{atm}}{\gamma} = M + 4 + 0.036\frac{L}{D}V_s^2/2g \quad (L = 100m)$$

$$= M + 4 + 0.036\frac{100}{0.102}\, 1.285^2/19.6$$

$$= M + 6.97 \longrightarrow M = \frac{P_{atm}}{\gamma} - 6.97$$

5pt

$$NPSH = \frac{P_{atm}}{\gamma} - 6.97 - \frac{P_v}{\gamma} \quad \text{(based on eq ①)}$$

$$= \frac{101 \times 10^3}{9.79 \times 10^3} - 6.97 - \frac{1.783 \times 10^3}{9.79 \times 10^3}$$

$$= 10.3 - 6.97 - 0.18$$

$$= 3.15\,m$$

∴ No cavitation 2pt for answer

3pt for overall process