FIU, Department of Civil and Environmental Engineering

CWR 3201 Fluid Mechanics, Fall 2018

Open-Channel Flows

Arturo S. Leon, Ph.D., P.E., D.WRE

Learning Objectives

1. Describe various types of open-channel flows
2. Use energy and momentum principles for rapidly varied flow configurations
3. Sketch water surface profiles

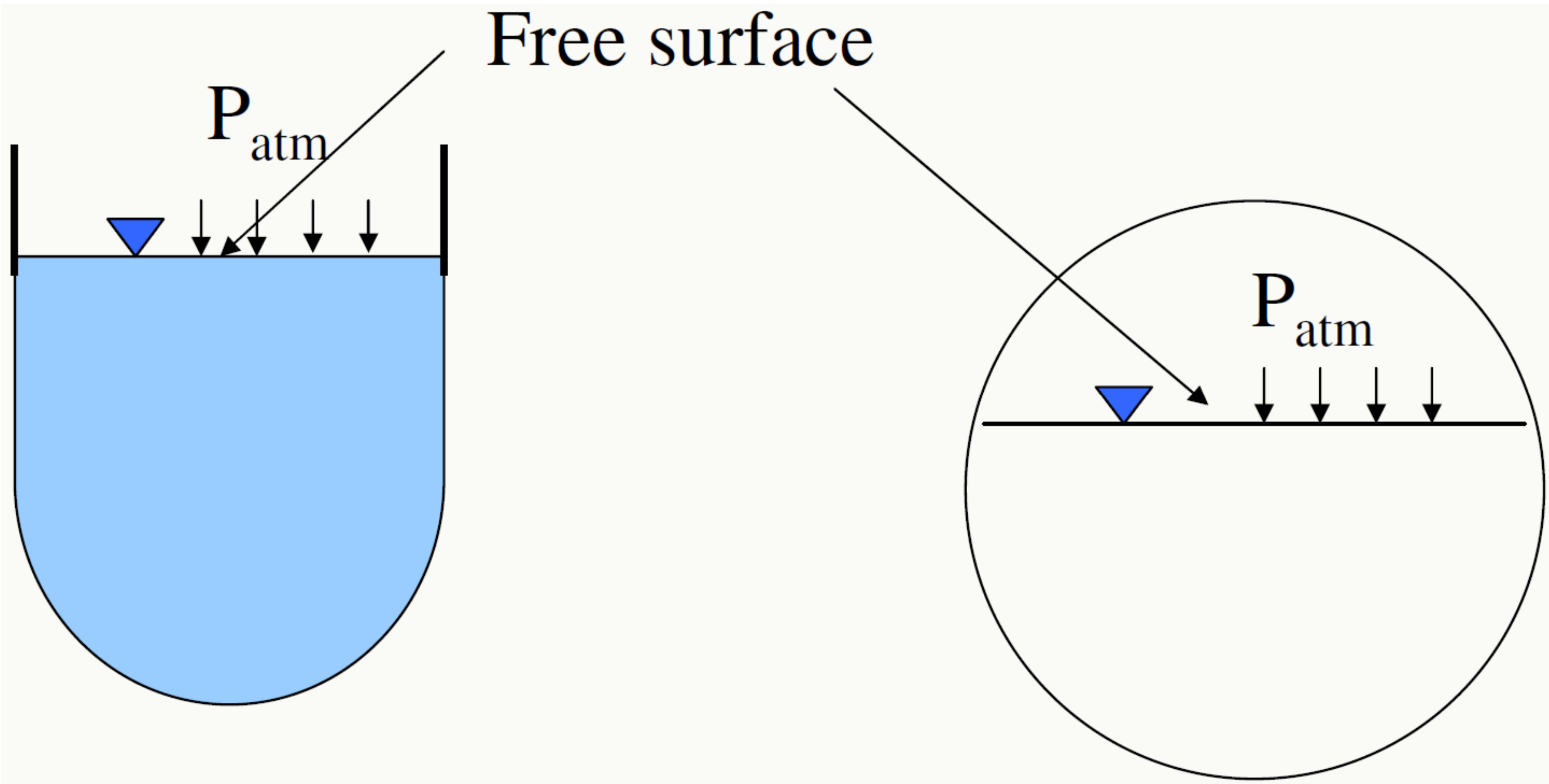
Animations of Unsteady Open Channel Flows

- **Emergency water releases at 25 dams**
  https://www.youtube.com/watch?v=o3E4s59OSLQ

- **Road Collapse- Maine 2008**

https://www.youtube.com/watch?v=NTbhyHNA1Vc

What is Open-channel Flow?



Types of Open-channel

Canal: A canal is usually a long and **mild-sloped** channel built in the ground

Types of Open-channel (Cont.)

<u>Chute</u>: A chute is a channel with a **steep slope**

Types of Open-channel (Cont.)

<u>Drop</u>: A drop is a channel with a **sudden change in elevation**

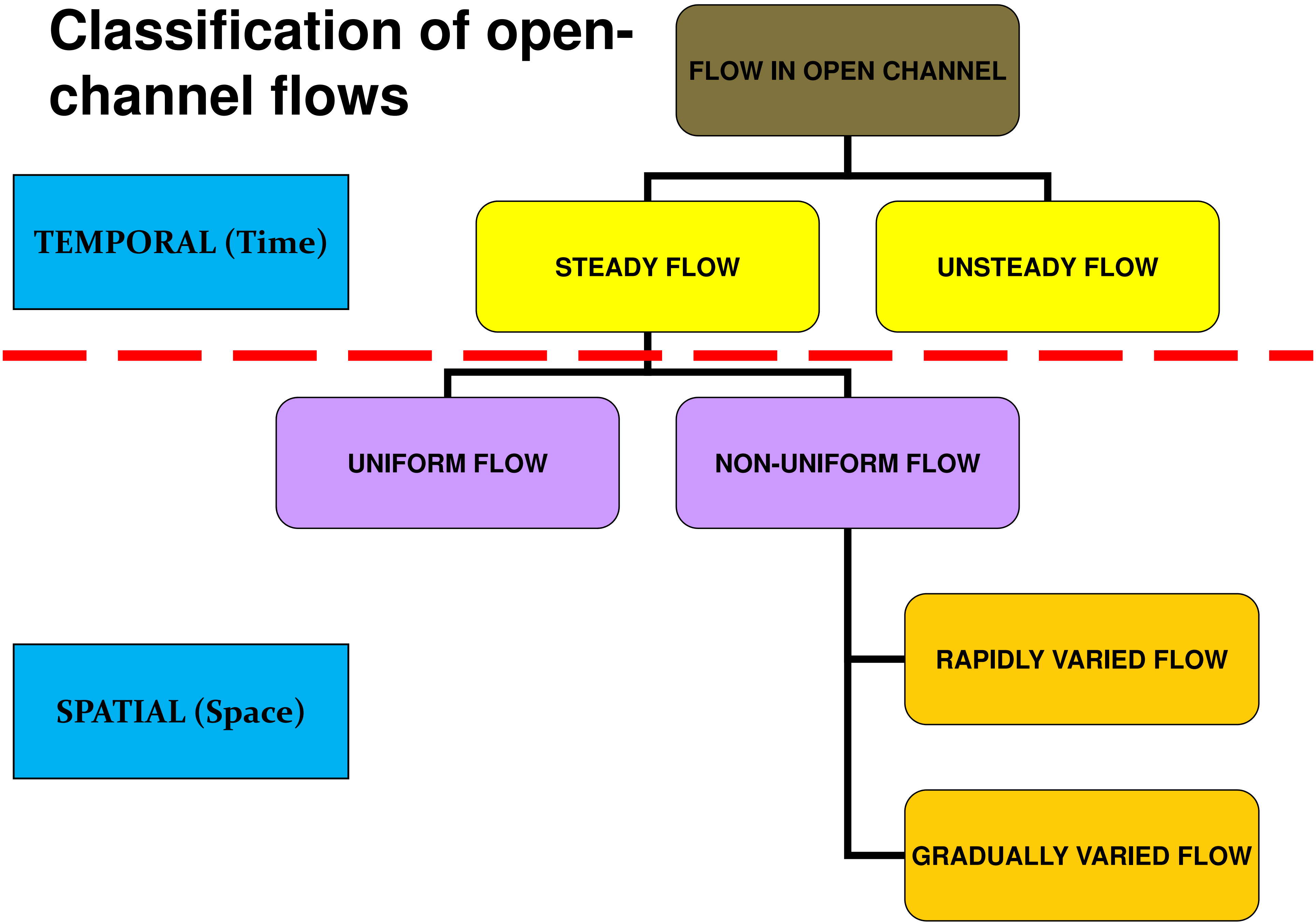
Types of Open-channel (Cont.)

<u>Culvert</u>: A culvert is a **covered channel** flowing usually partly full.

Types of Open-channel (Cont.)

<u>Natural channel</u>: A natural channel has **irregular geometry**. Examples include, rivers and creeks.

Classification of open-channel flows



Classification of open-channel flows

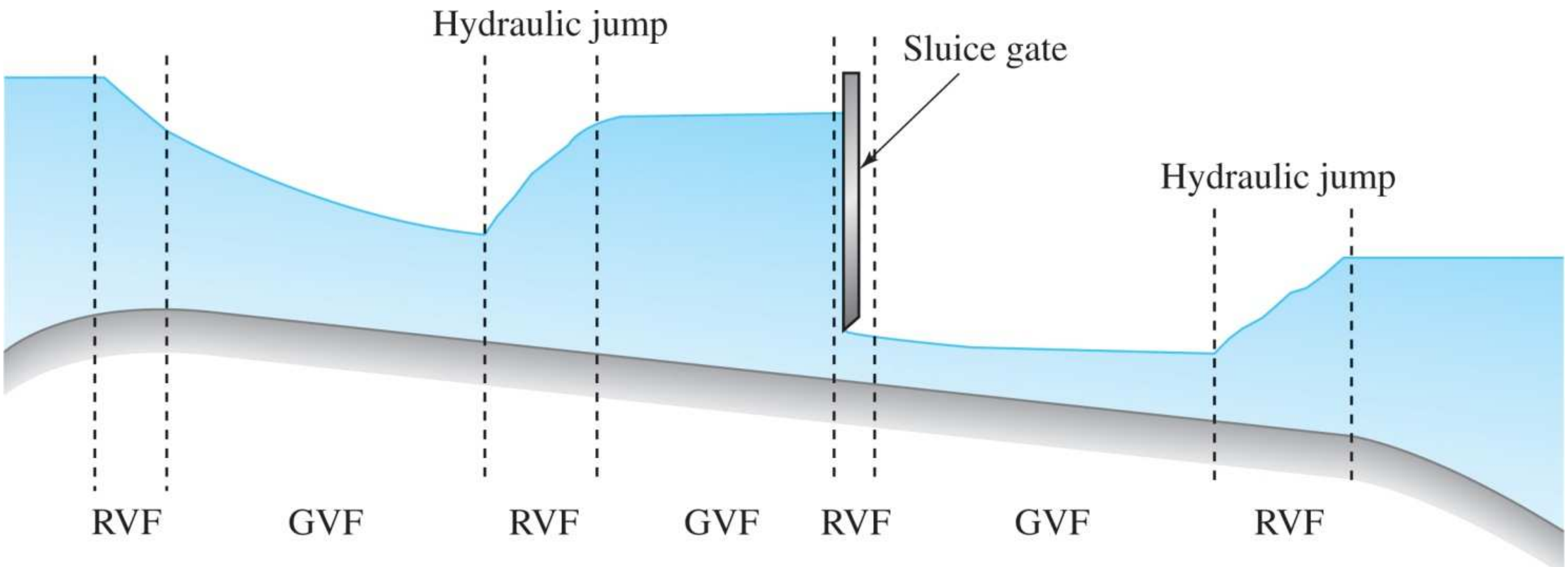
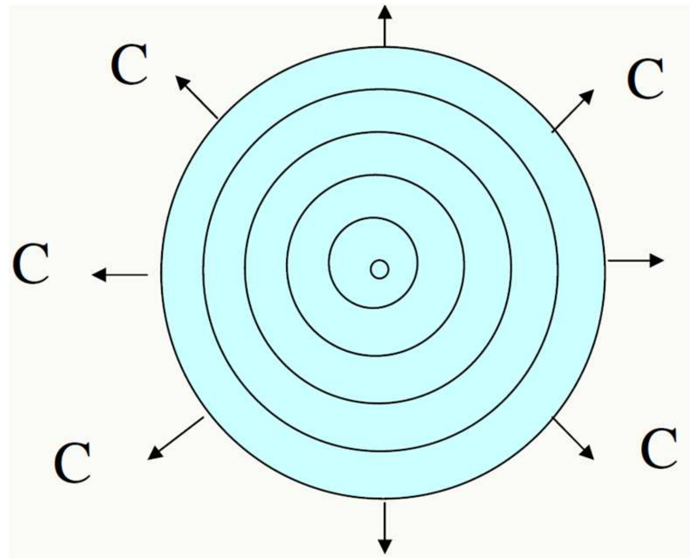
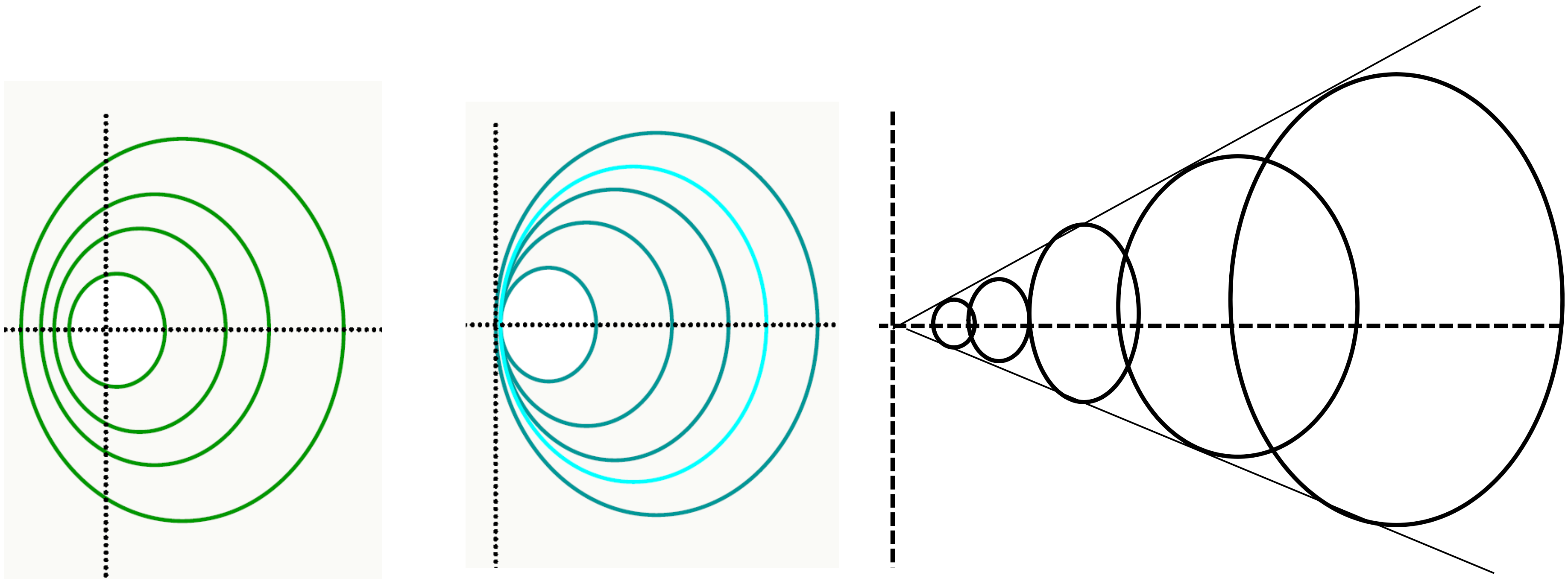


Fig. 10.2 Steady nonuniform flow in a channel.

Wave speed in open channel flows



Propagation of a disturbance in subcritical, critical and supercritical flows



Froude Number:

When $Fr > 1$, the flow possesses a relatively high velocity and shallow depth; on the other hand, when $Fr < 1$, the velocity is relatively low and the depth is relatively deep.

Uniform Flow

Arturo S. Leon, Ph.D., P.E., D.WRE

Cross-section Representation

A composite section

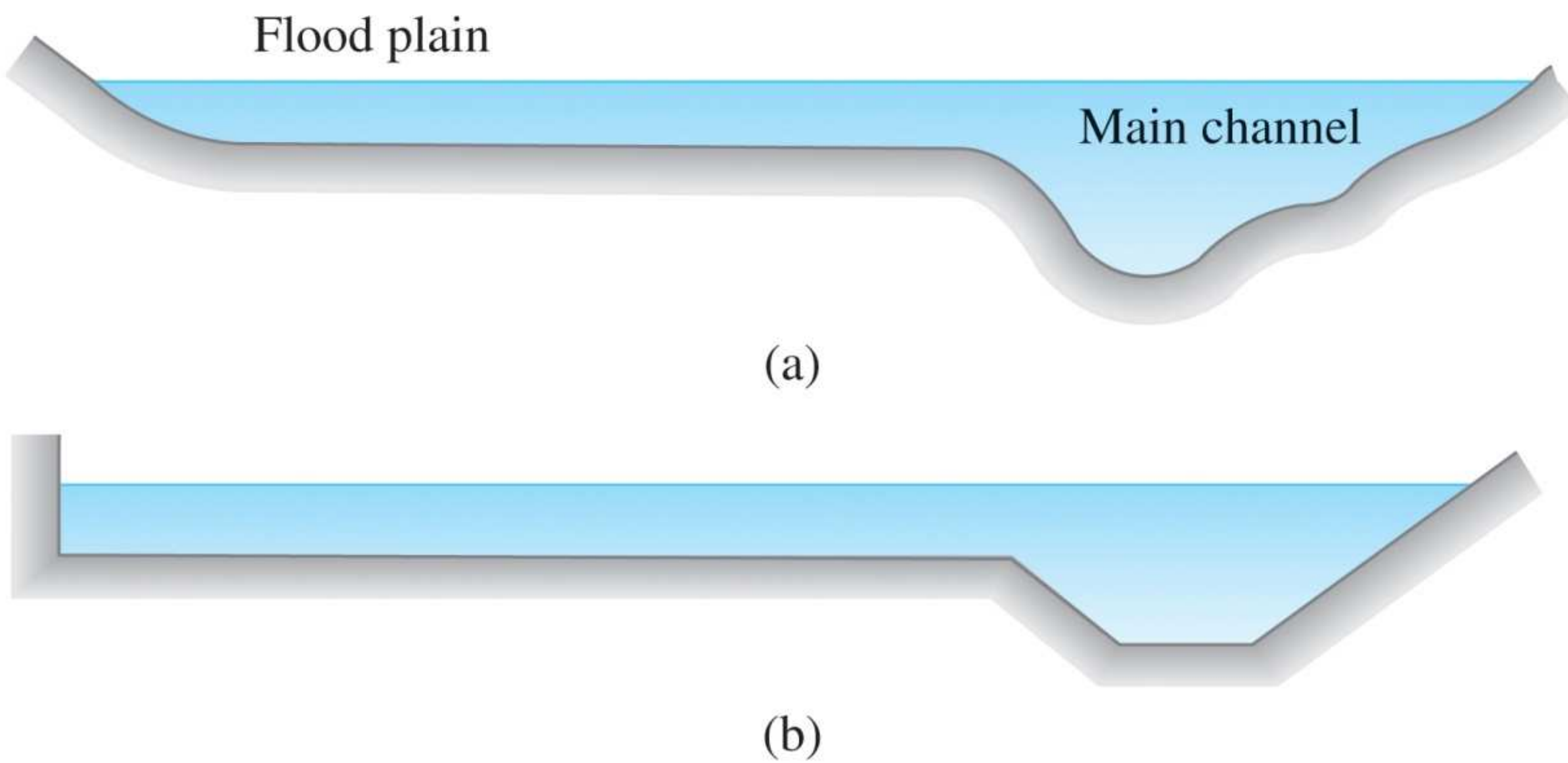


Fig. 10.5 Generalized section representation: (a) actual cross section; (b) composite cross section.

Regular cross sections

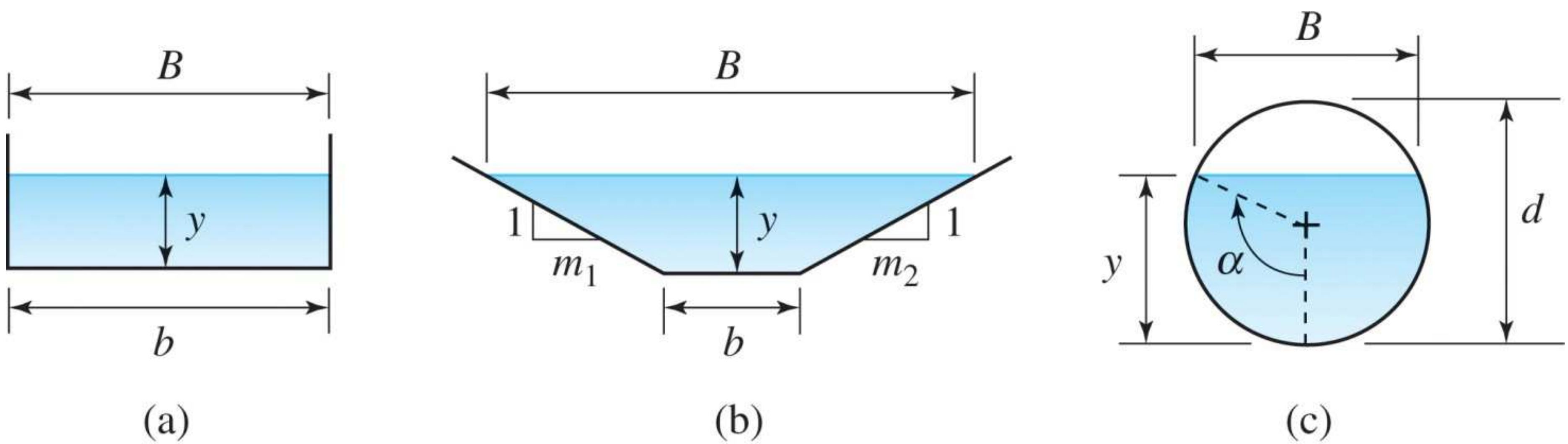


Fig. 10.4 Representative regular cross sections: (a) rectangular; (b) trapezoidal; (c) circular.

Equation for Uniform Flow

Uniform flow occurs in a channel when the depth and velocity do not vary along its length

The Chezy-Manning Equation

Where:

c_1 = 1 for SI units and c_1 = 1.49 for English units.

n = Manning roughness coefficient

A = Hydraulic area

R = Hydraulic radius

S_0 = slope of the channel bottom

The depth associated with uniform flow is designated y_0; it is called either ***uniform depth*** or ***normal depth***.

Average values of the Manning Coefficient, *n*

TABLE 7.3 Average Values[a] of the Manning *n*

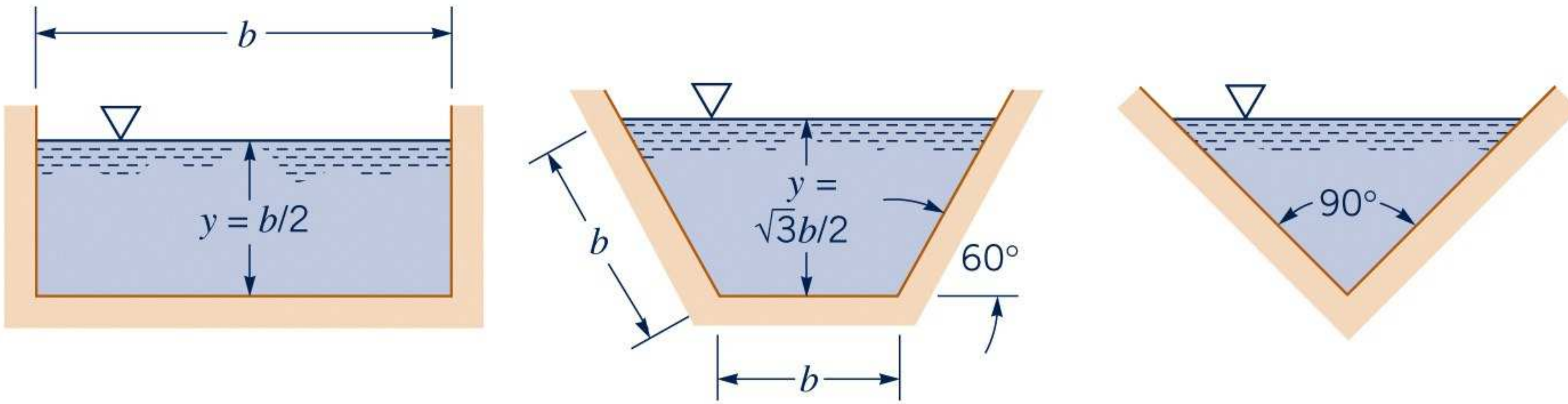
Wall material	*Manning n*
Planed wood	0.012
Unplaned wood	0.013
Finished concrete	0.012
Unfinished concrete	0.014
Sewer pipe	0.013
Brick	0.016
Cast iron, wrought iron	0.015
Concrete pipe	0.015
Riveted steel	0.017
Earth, straight	0.022
Corrugated metal flumes	0.025
Rubble	0.03
Earth with stones and weeds	0.035
Mountain streams	0.05

[a]The values in this table result in flow rates too large for hydraulic radii greater than about 3 m (10 ft). The Manning n should be increased by 10 to 15% for such large conduits.

The Most Efficient Section (or best hydraulic cross section)

The ***Most Efficient*** cross-section is defined as the section of maximum flow rate (*Q*) for a constant hydraulic area (*A*), slope (S_o), and roughness coefficient (*n*). Alternatively, the ***Most Efficient*** cross-section can be defined as the section of minimum hydraulic area (*A*) for a constant flow rate (*Q*).

The best hydraulic cross-section for various shapes

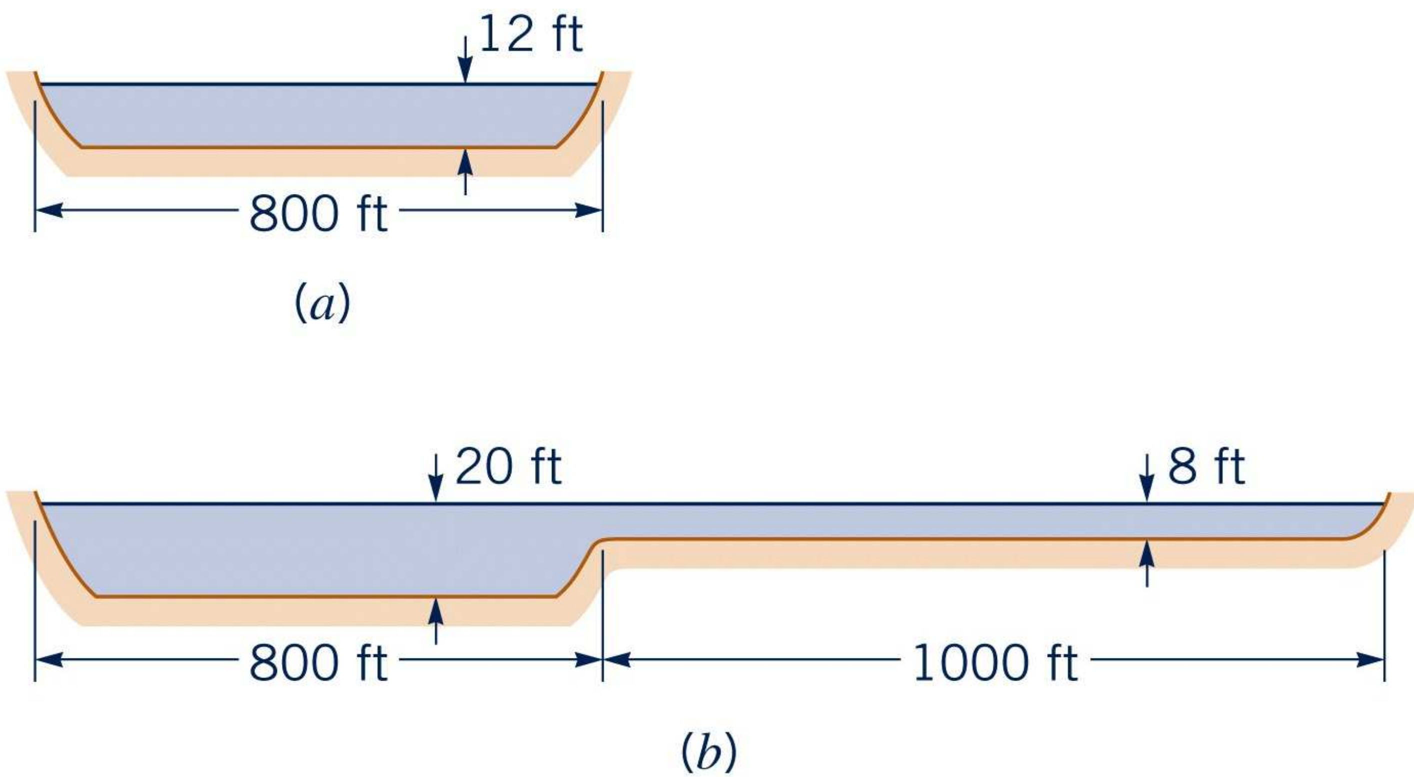


Example:

The following data are obtained for a particular reach of the Provo River in Utah: $A = 183\ ft^2$, free-surface width = 55 ft, average depth = 3.3 ft, $R_h = 3.32$ ft, $V = 6.56$ ft/s, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine (a) the Manning coefficient, *n*, and (b) the Froude number of the flow.

Example:

At a given location, under normal conditions a river flows with a Manning coefficient of 0.030, and a cross section as indicated in part (a) of the figure below. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of tress and brush in the floodplain) and a cross section as shown in part (b) of the figure below. Determine the ratio of the flowrate during flood conditions to that during normal conditions.

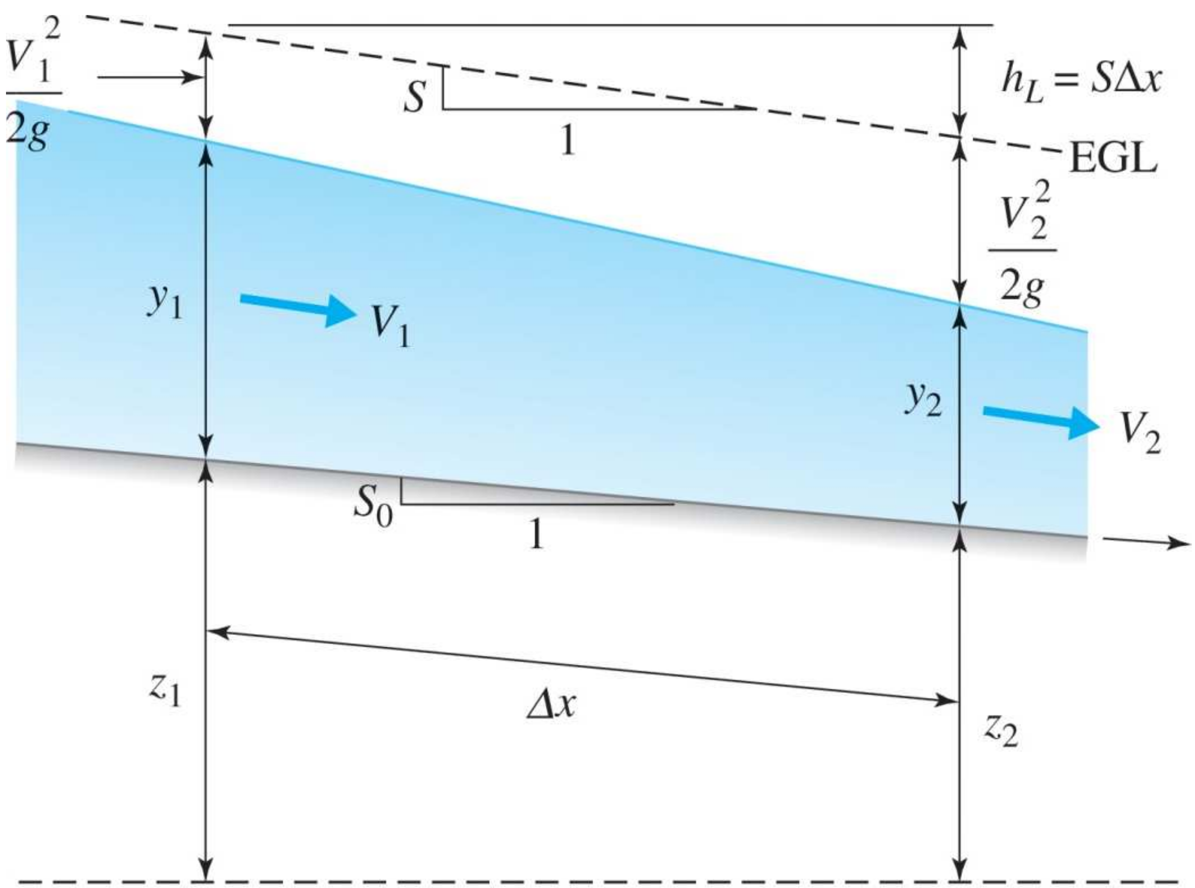


Energy concepts

Arturo S. Leon, Ph.D., P.E., D.WRE

10.4 Energy Concepts

Total energy: The sum of the vertical distance to the channel bottom measured from a horizontal datum, the depth of flow, and the kinetic energy head.



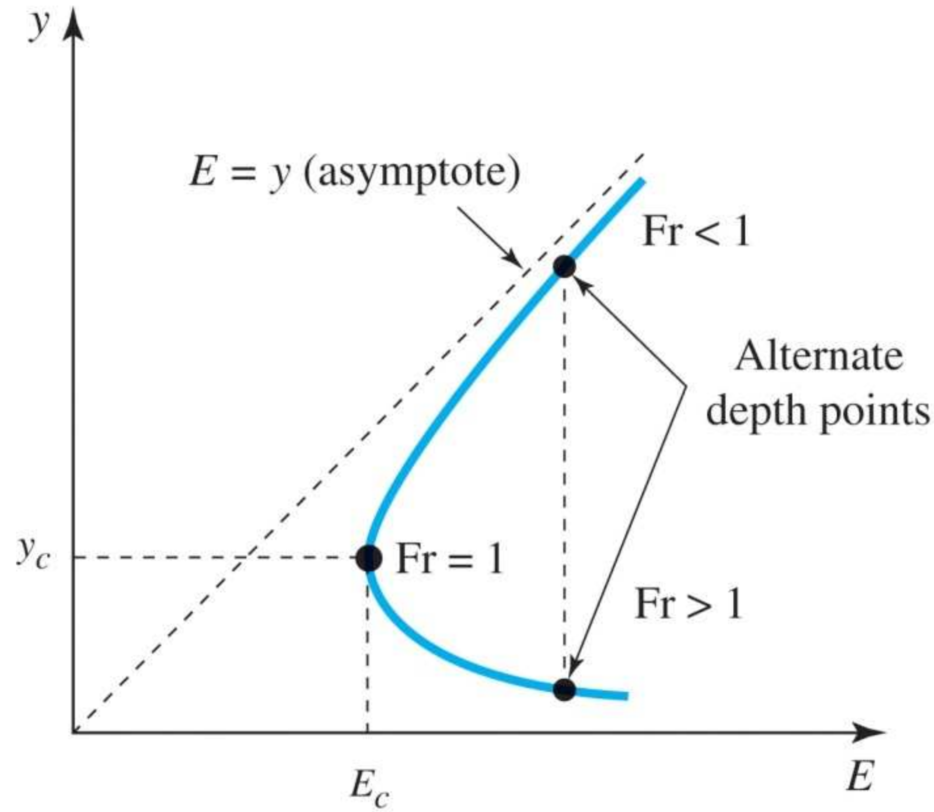
Energy is actually an energy head.

h_L is the head loss.

10.4 Energy Concepts

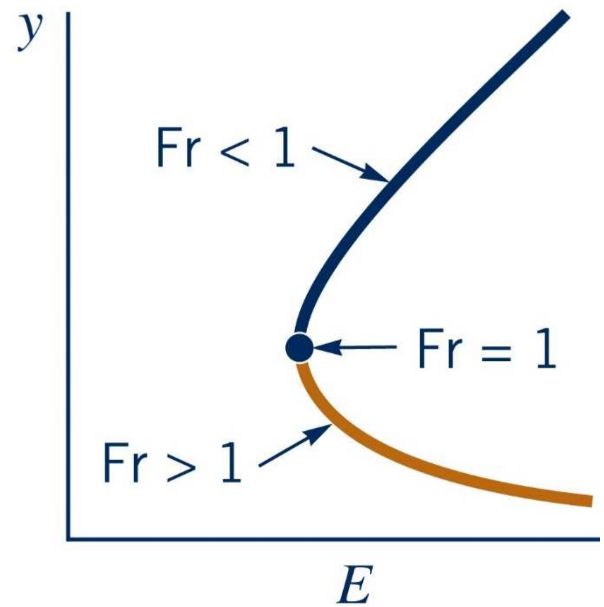
Specific energy: Measurement of energy **relative to the bottom of the channel**.

Specific discharge: The total discharge divided by the channel width (**valid only for a rectangular channel**).



Specific energy diagram

Critical depth



For any cross-section:

For a rectangular channel ($q = Q/b$)

Energy Equation in Transitions

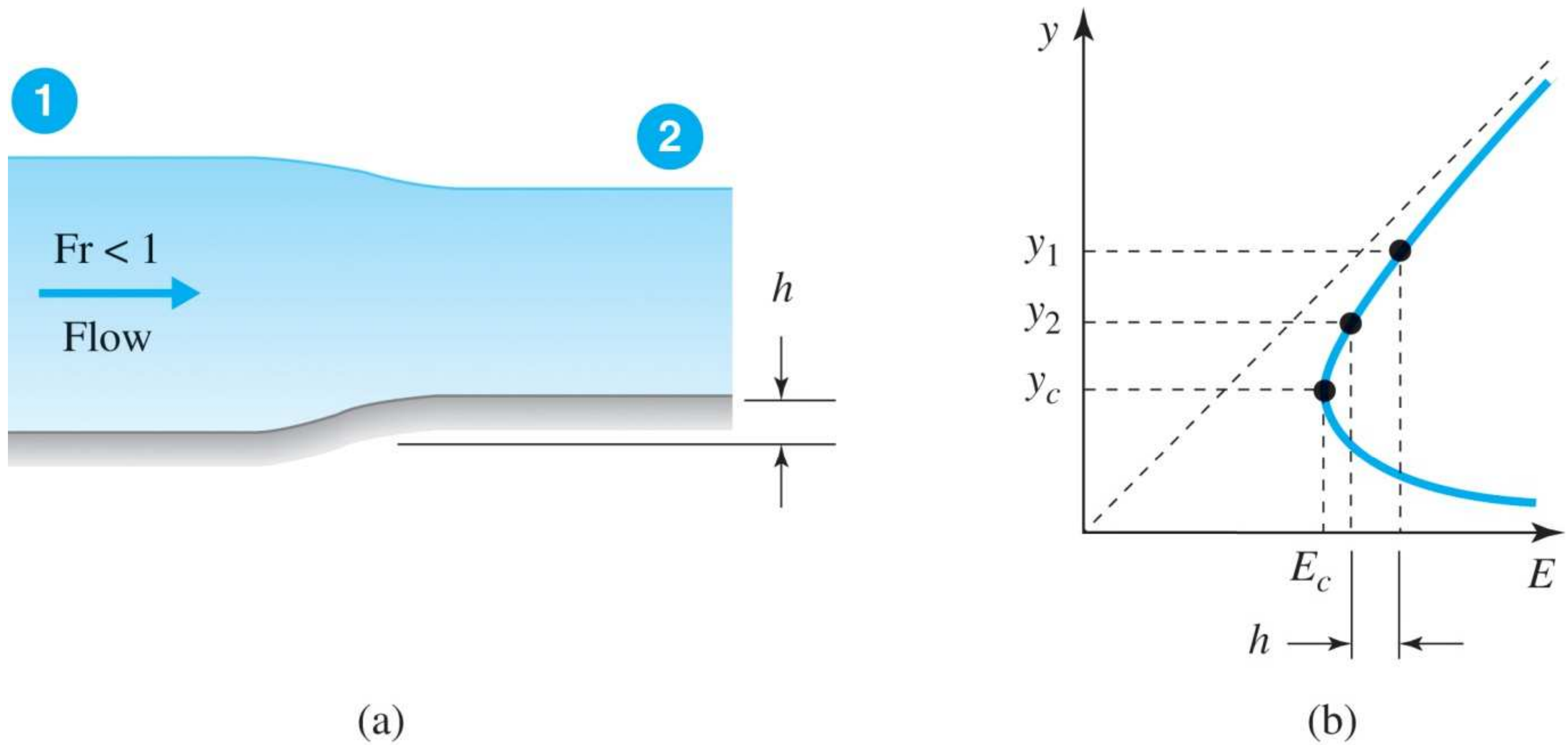
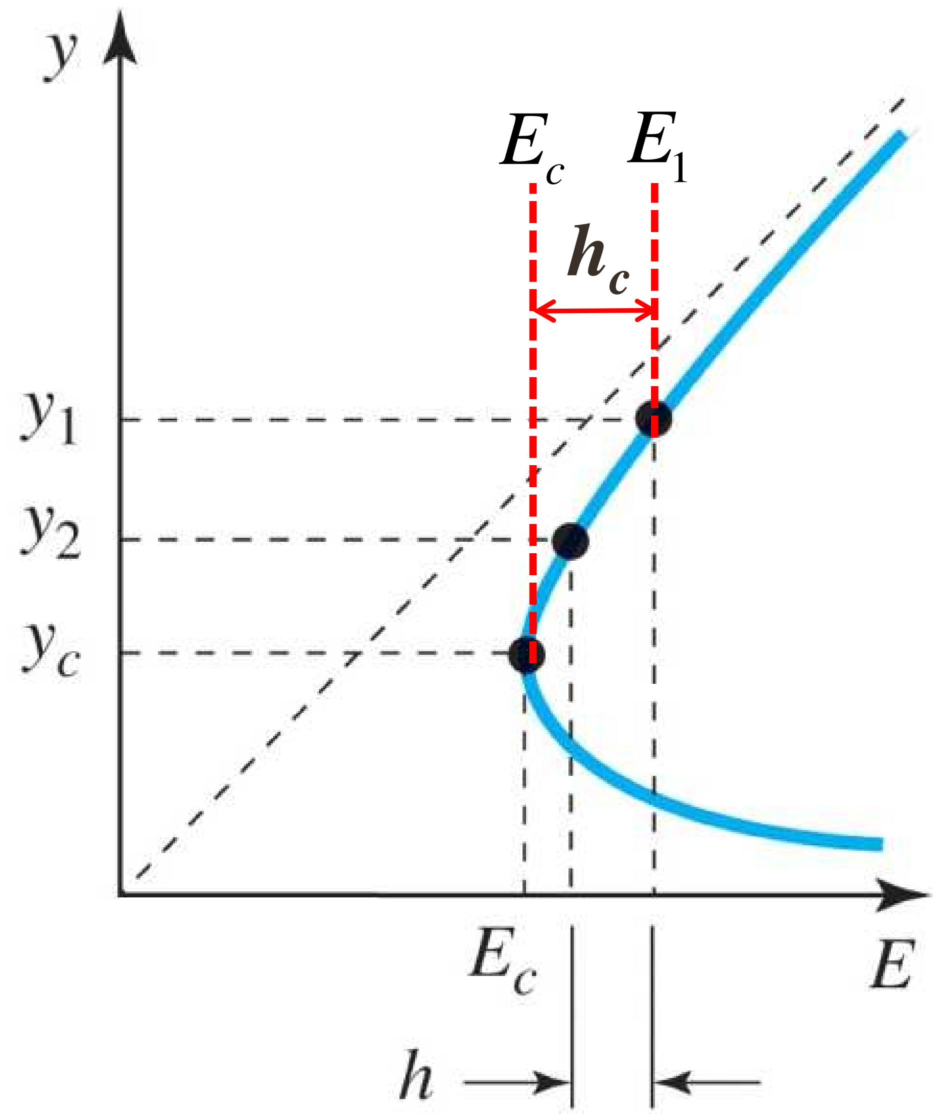


Fig. 10.7 Channel constriction: (a) raised channel bottom; (b) specific energy diagram.

The condition of choked flow or a choking condition implies that minimum specific energy exists within the transition.

Flow Choking



For a rectangular channel:

For a non-rectangular channel:

Example:

Consider a channel where the upstream velocity is 5.0 m/s and the upstream flow depth is 0.6 m. The flow then passes over a bump 15 cm in height.

(a) Compute the flow depth and velocity on the crest of the bump.

(b) Compute the maximum allowable bump height that keeps water from backing up upstream.

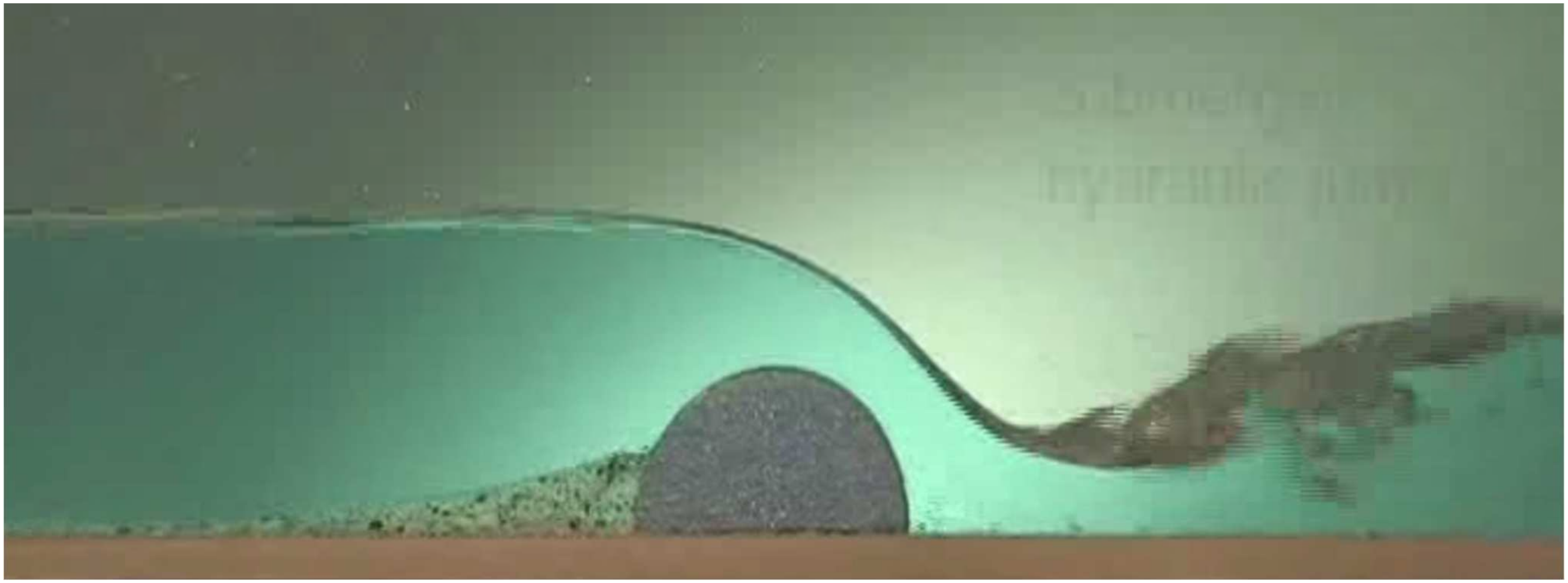
Example:

Compute the critical depth in a trapezoidal channel for a flow of 30 m^3/s. The channel bottom width is 10 m, side slopes are 2H:1V.

Momentum Concepts

Arturo S. Leon, Ph.D., P.E., D.WRE

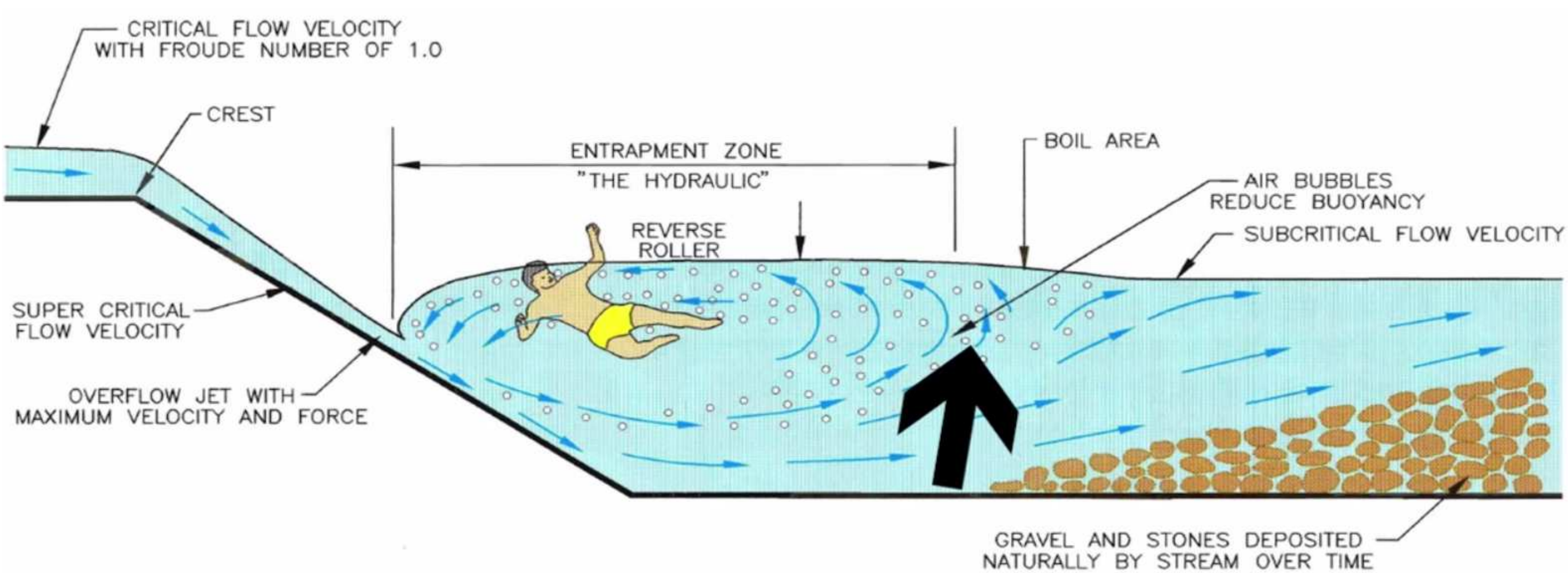
Hydraulic Jump



Hydraulic jump:
https://www.youtube.com/watch?v=cRnIsqSTX7Q

Low head dams:

https://www.youtube.com/watch?v=XsYgODmmiAM



10.5 Momentum Concepts

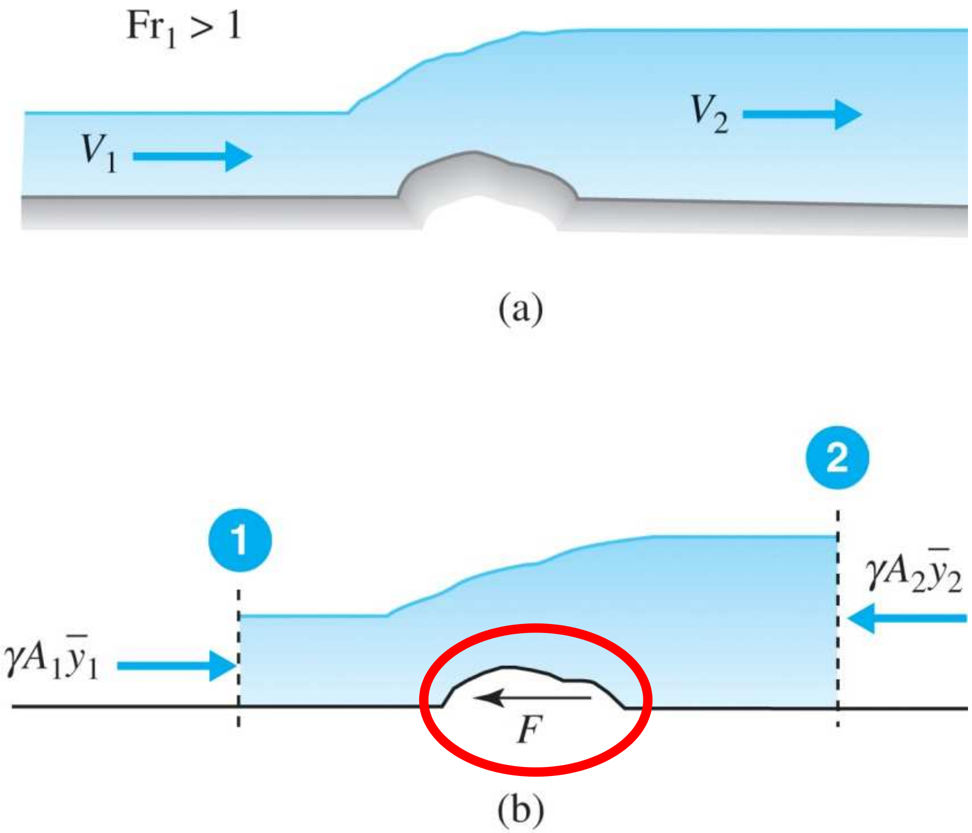


Fig. 10.13 Channel flow over an obstacle: (a) idealized flow; (b) control volume

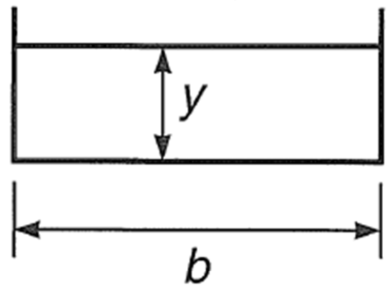
Linear momentum equation is:

Let's define ***M*** (momentum function) as:

For a rectangular section:

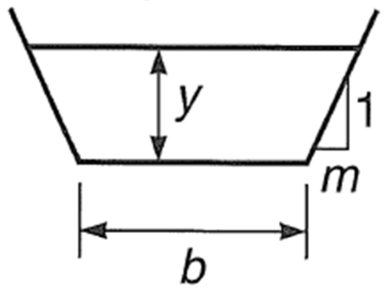
Momentum function *M* for various channels

Rectangular



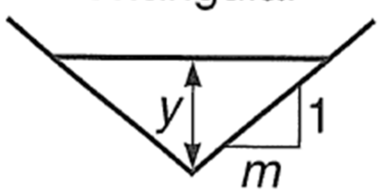
$$by^2/2 + Q^2/(gby)$$

Trapezoidal



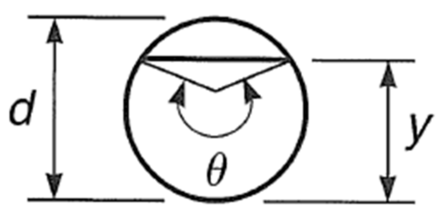
$$by^2/2 + my^3/3 + Q^2/[gy(b + my)]$$

Triangular



$$my^3/3 + Q^2/(gmy^2)$$

Circular†



$$[3\sin(\theta/2) - \sin^3(\theta/2) - 3(\theta/2)\cos(\theta/2)]\, d^3/24 + Q^2/[gd^2(\theta - \sin\theta)/8]$$

† $\theta = 2\cos^{-1}[1 - 2(y/d)]$

10.5 Momentum Concepts

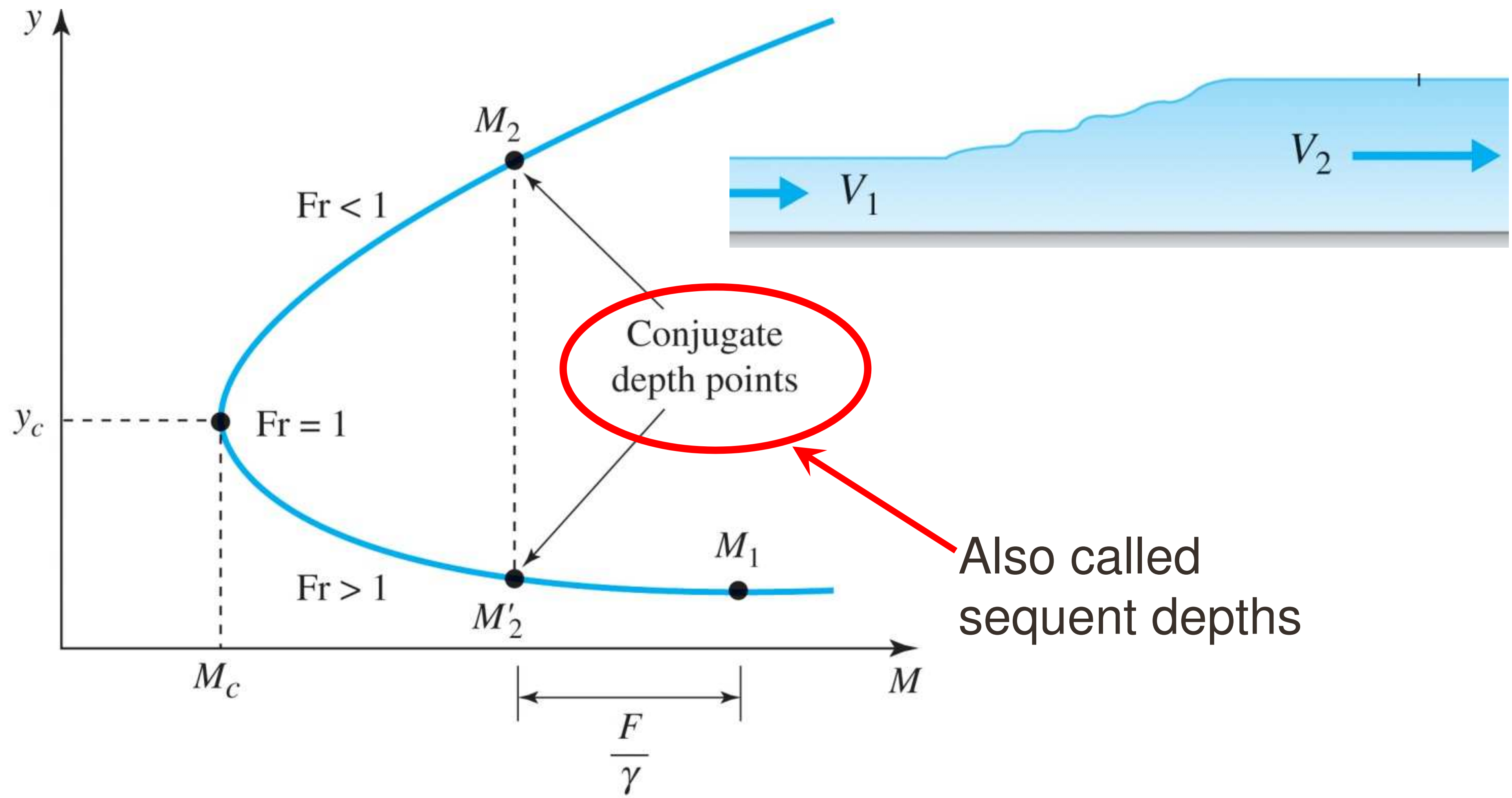


Fig. 10.14 Variation of the momentum function with depth.

Hydraulic Jump in a rectangular channel (Cont.)

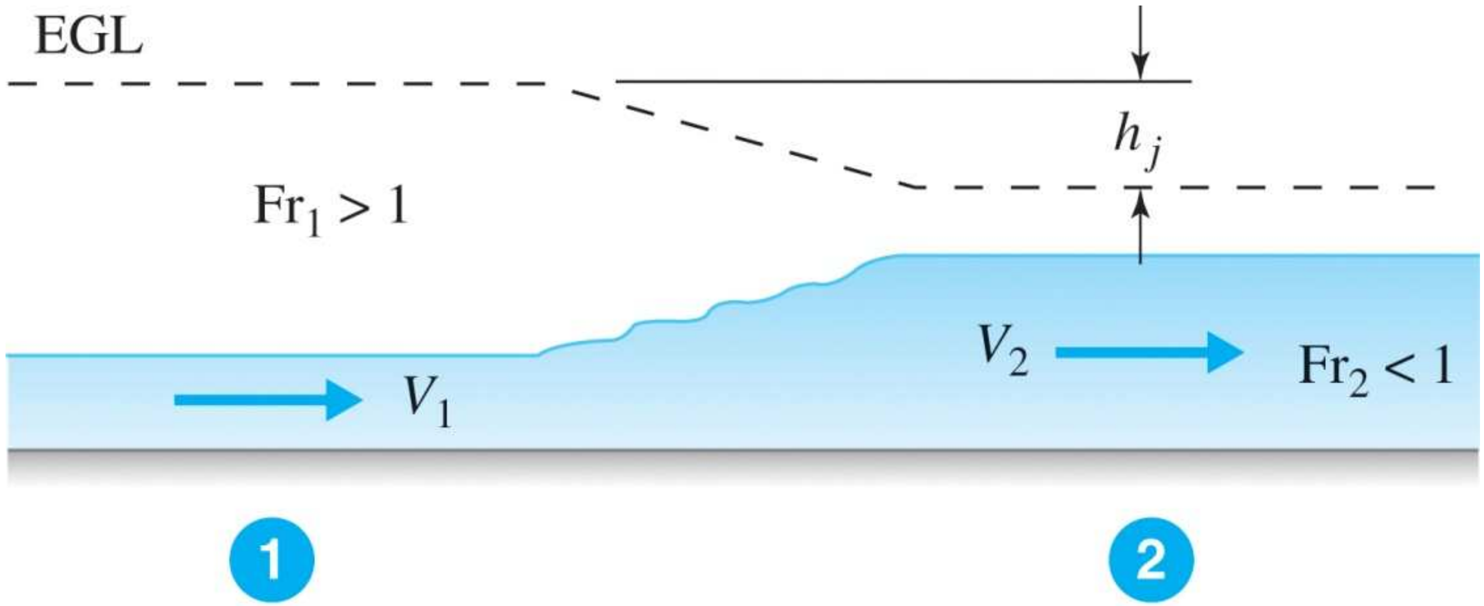
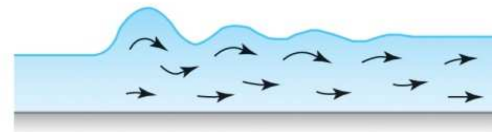
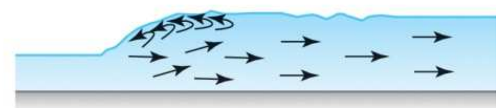
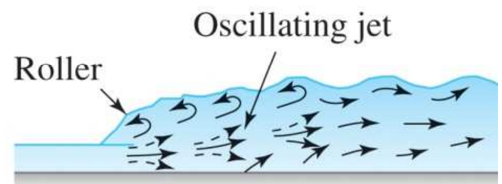


Fig. 10.15 Idealized hydraulic jump.

Classification of Hydraulic Jumps

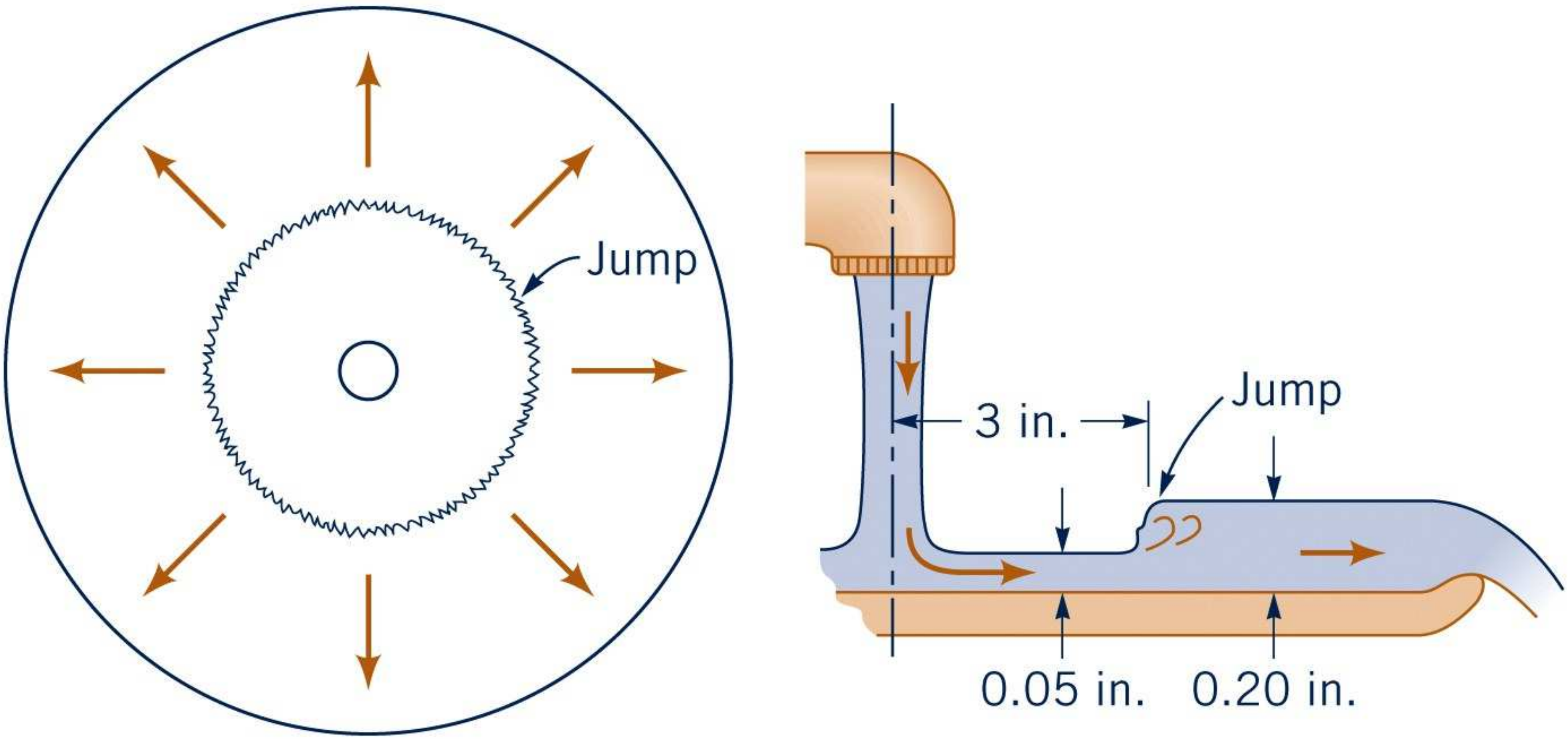
Table 10.2 Hydraulic Jumps in Horizontal Rectangular Channels

Upstream Fr	*Type*	*Description*	
1.0–1.7	Undular	Ruffled or undular water surface; surface rollers form near Fr = 1.7	
1.7–2.5	Weak	Prevailing smooth flow; low energy loss	
2.5–4.5	Oscillating	Intermittent jets from bottom to surface, causing persistent downstream waves	
4.5–9.0	Steady	Stable and well-balanced; energy dissipation contained in main body of jump	
>9.0	Strong	Effective, but with rough, wavy surface downstream	

Source: Adapted with permission from Chow, 1959. (Adapted from Chow, 1959)

Example:

Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in the figure below. Consider a situation where a jump forms 3.0 in from the center of the plate with depths upstream and downstream of the jump of 0.05 in and 0.20 in, respectively. Determine the flow rate from the faucet.



Gradually varied flow

Arturo S. Leon, Ph.D., P.E., D.WRE

Gradually varied Flows

Differential Equation for Gradually Varied Flow

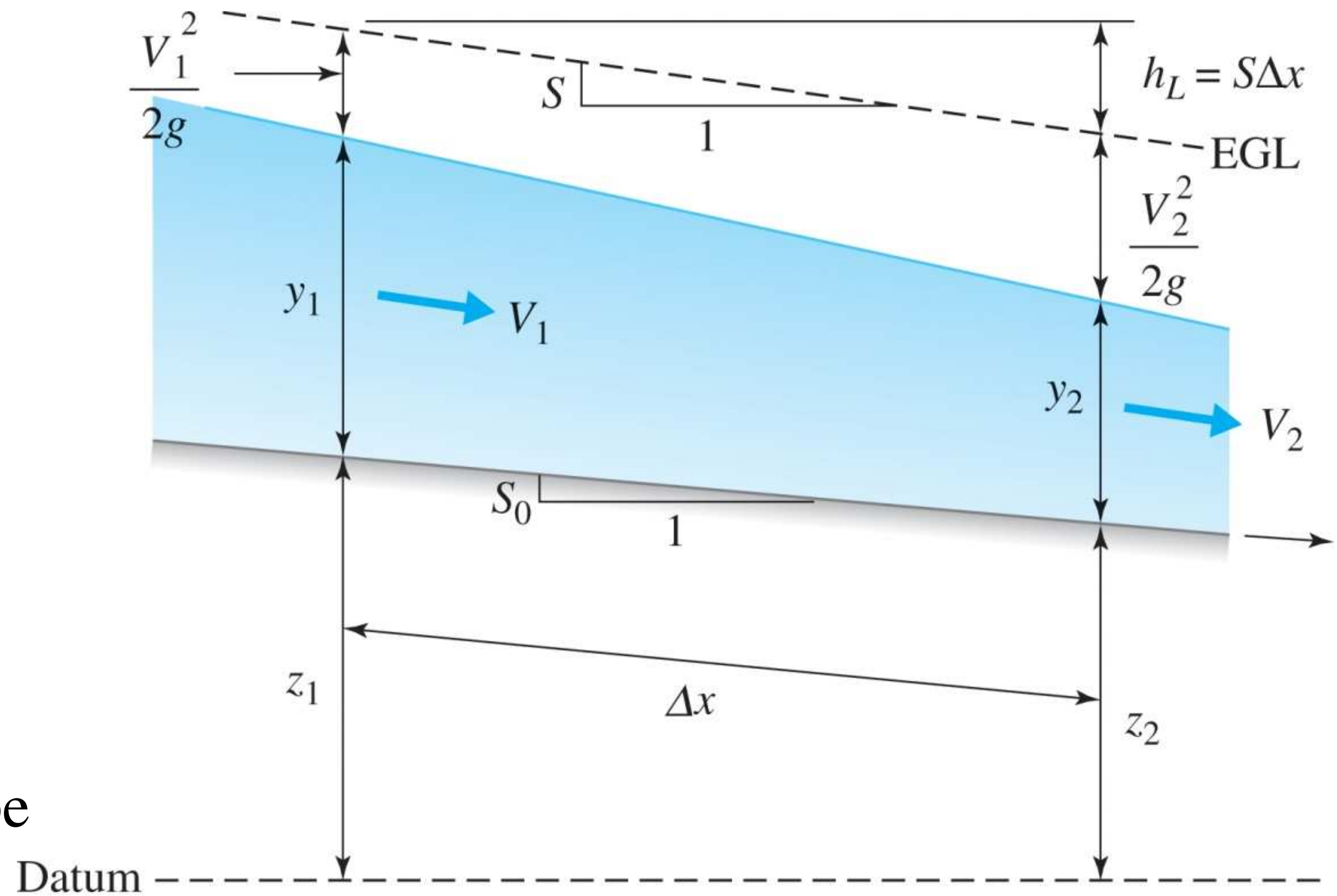


Fig. 10.18 Nonuniform gradually varied flow.

Where:

S = total energy slope

S_o = bed slope

Fr = Froude Number

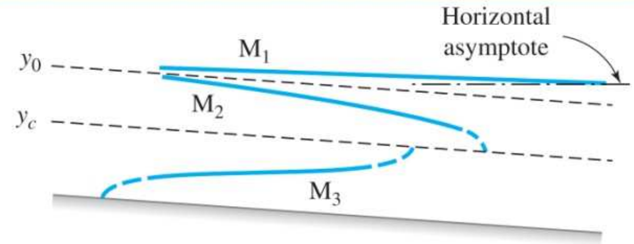
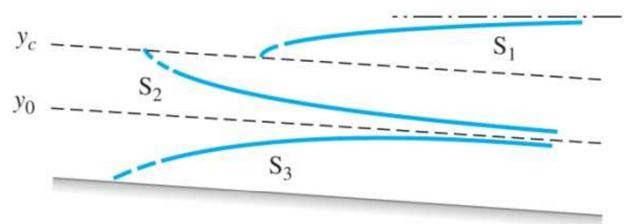
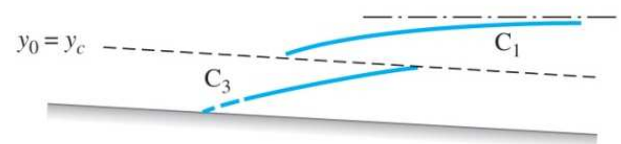
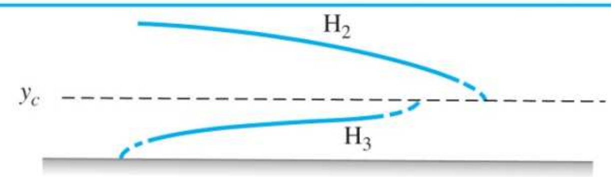
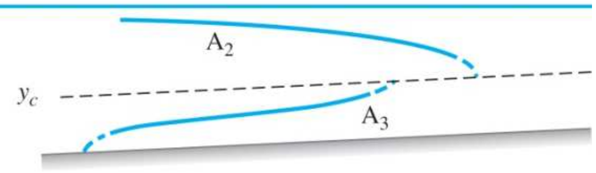
Does water depth increase or decrease in *x* direction?

Is $\frac{dy}{dx}$ positive or negative?

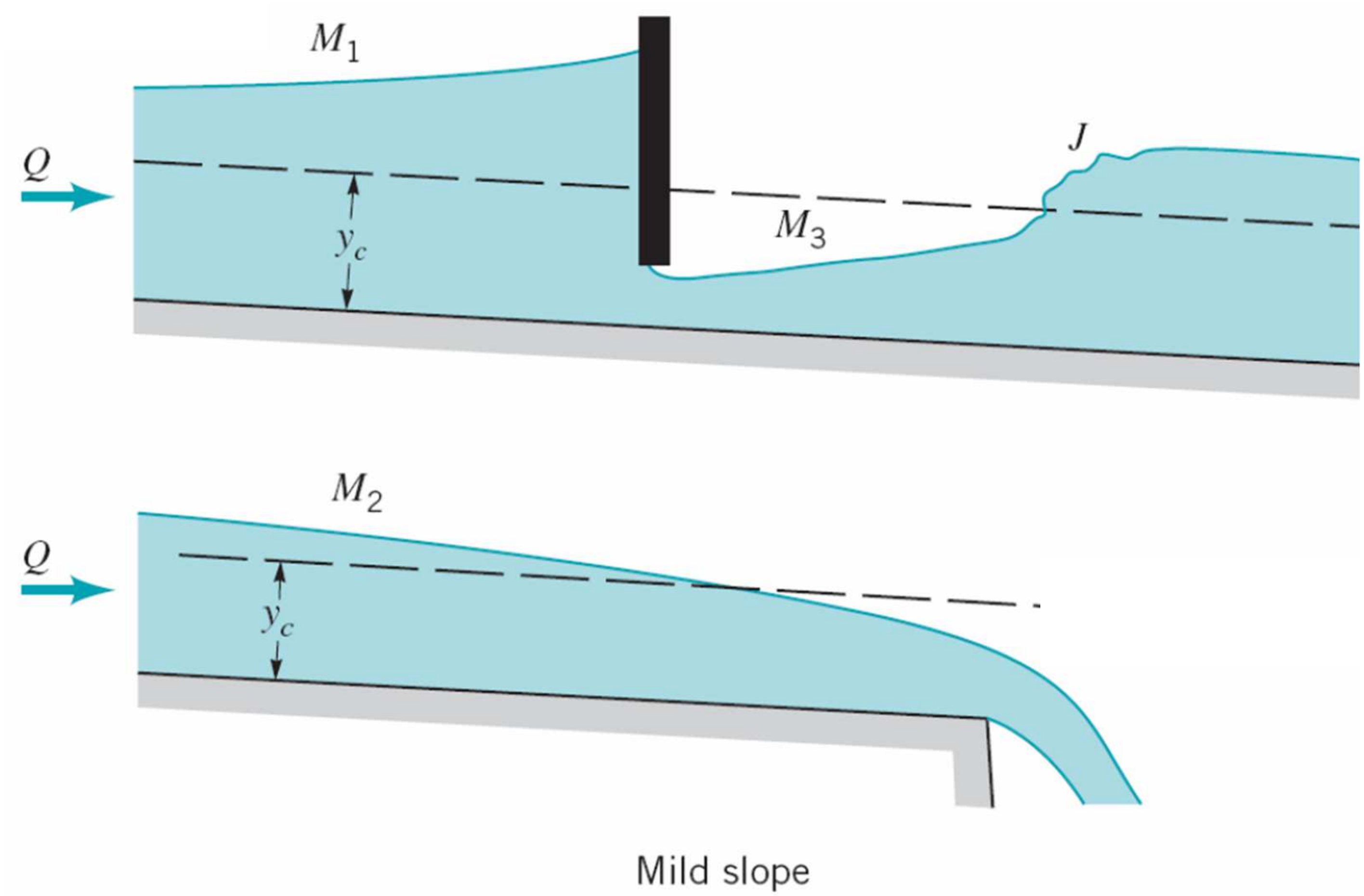
Assuming a wide rectangular channel:

Classification of Surface Profiles

Table 10.3 Classification of Surface Profiles

Channel slope	*Profile type*	*Depth range*	Fr	$\frac{dy}{dx}$	$\frac{dE}{dx}$	
Mild $S_0 < S_c$ $y_0 > y_c$	M_1	$y > y_0 > y_c$	< 1	> 0	> 0	
	M_2	$y_0 > y > y_c$	< 1	< 0	< 0	
	M_3	$y_0 > y_c > y$	> 1	> 0	< 0	
Steep $S_0 > S_c$ $y_0 < y_c$	S_1	$y > y_c > y_0$	< 1	> 0	> 0	
	S_2	$y_c > y > y_0$	> 1	< 0	> 0	
	S_3	$y_c > y_0 > y$	> 1	> 0	< 0	
Critical $S_0 = S_c$ $y_0 = y_c$	C_1	$y > y_c$ or y_0	< 1	> 0	> 0	
	C_3	y_c or $y_0 > y$	> 1	> 0	< 0	
Horizontal $S_0 = 0$ $y_0 \to \infty$	H_2	$y > y_c$	< 1	< 0	< 0	
	H_3	$y_c > y$	> 1	> 0	< 0	
Adverse $S_0 < 0$ y_0 undefined	A_2	$y > y_c$	< 1	< 0	< 0	
	A_3	$y_c > y$	> 1	> 0	< 0	

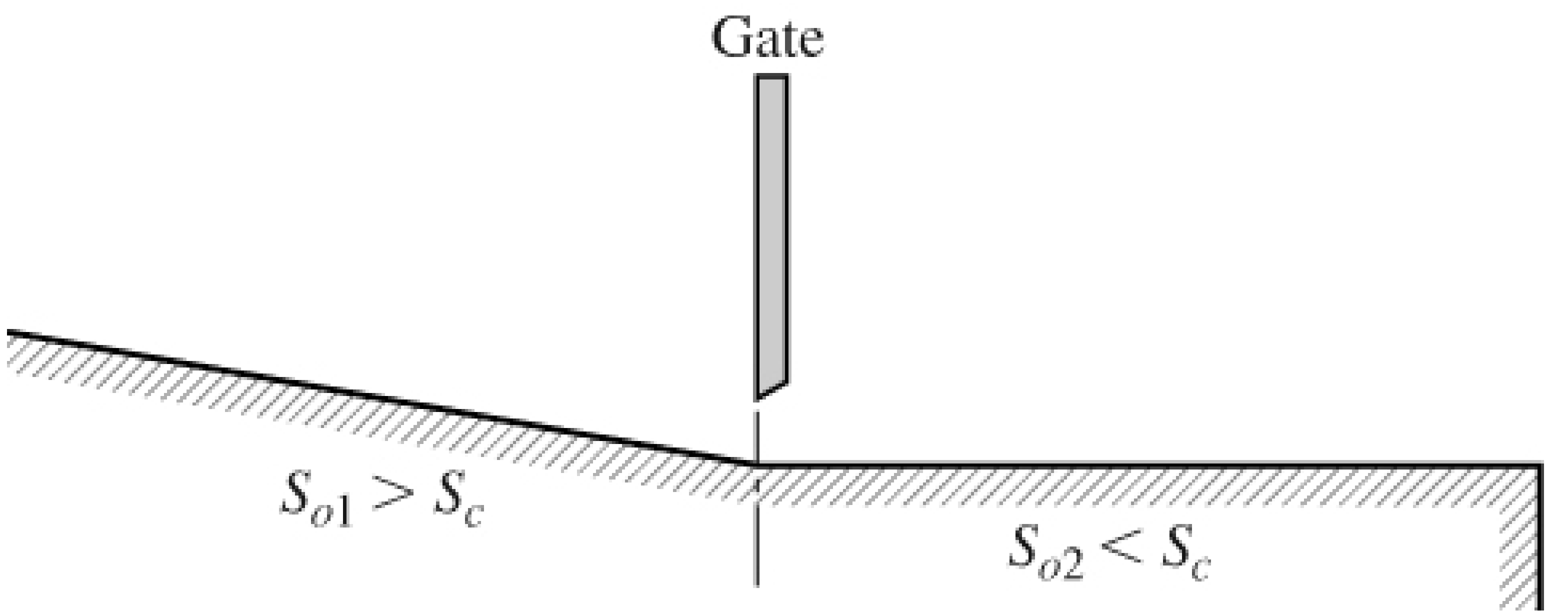
Examples of Gradually Varied Flows



Typical surface configurations for nonuniform depth flow with a **mild slope**

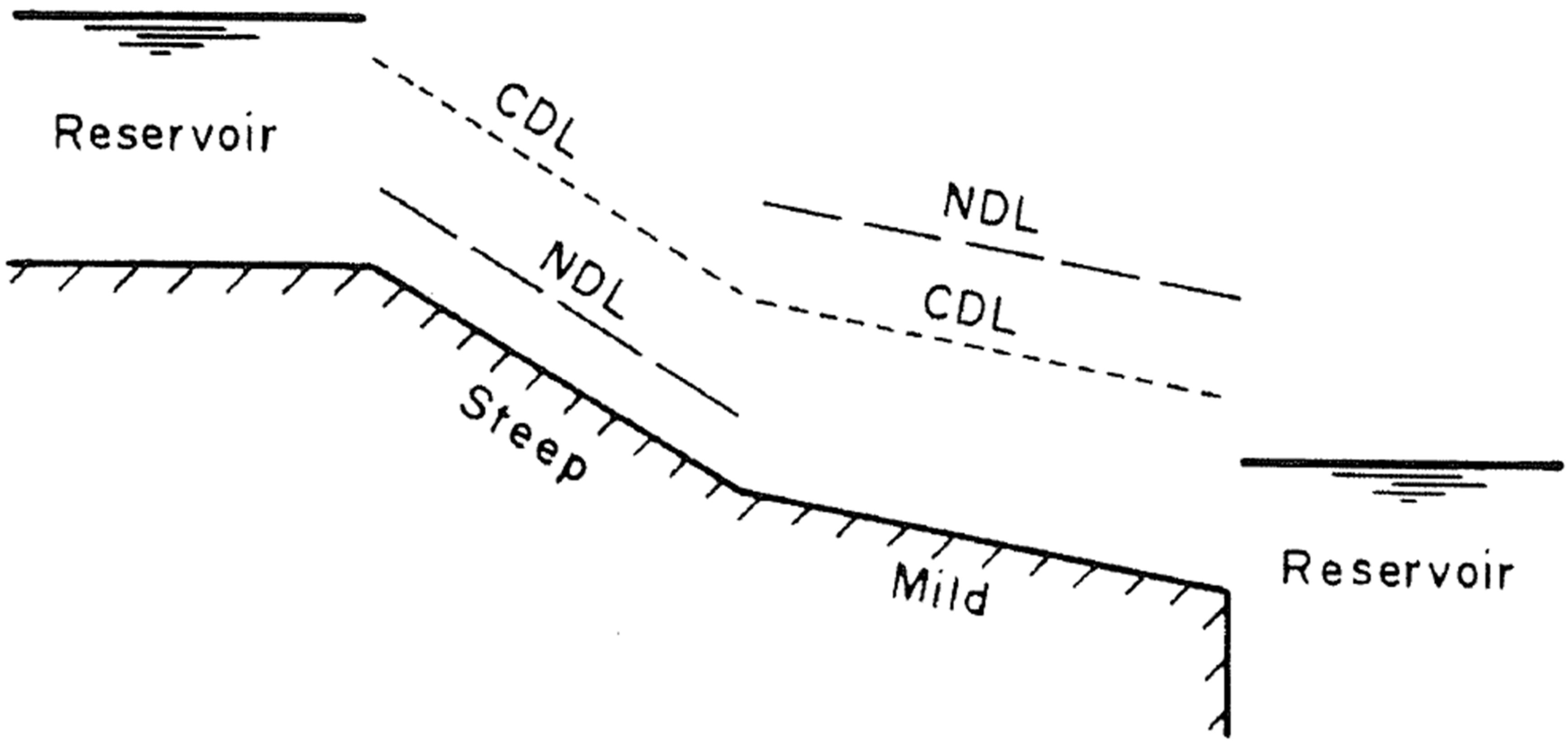
Example:

Sketch the water surface profile for the two-reach open-channel system below. A gate is located between the two reaches and the second reach ends with a sudden fall.



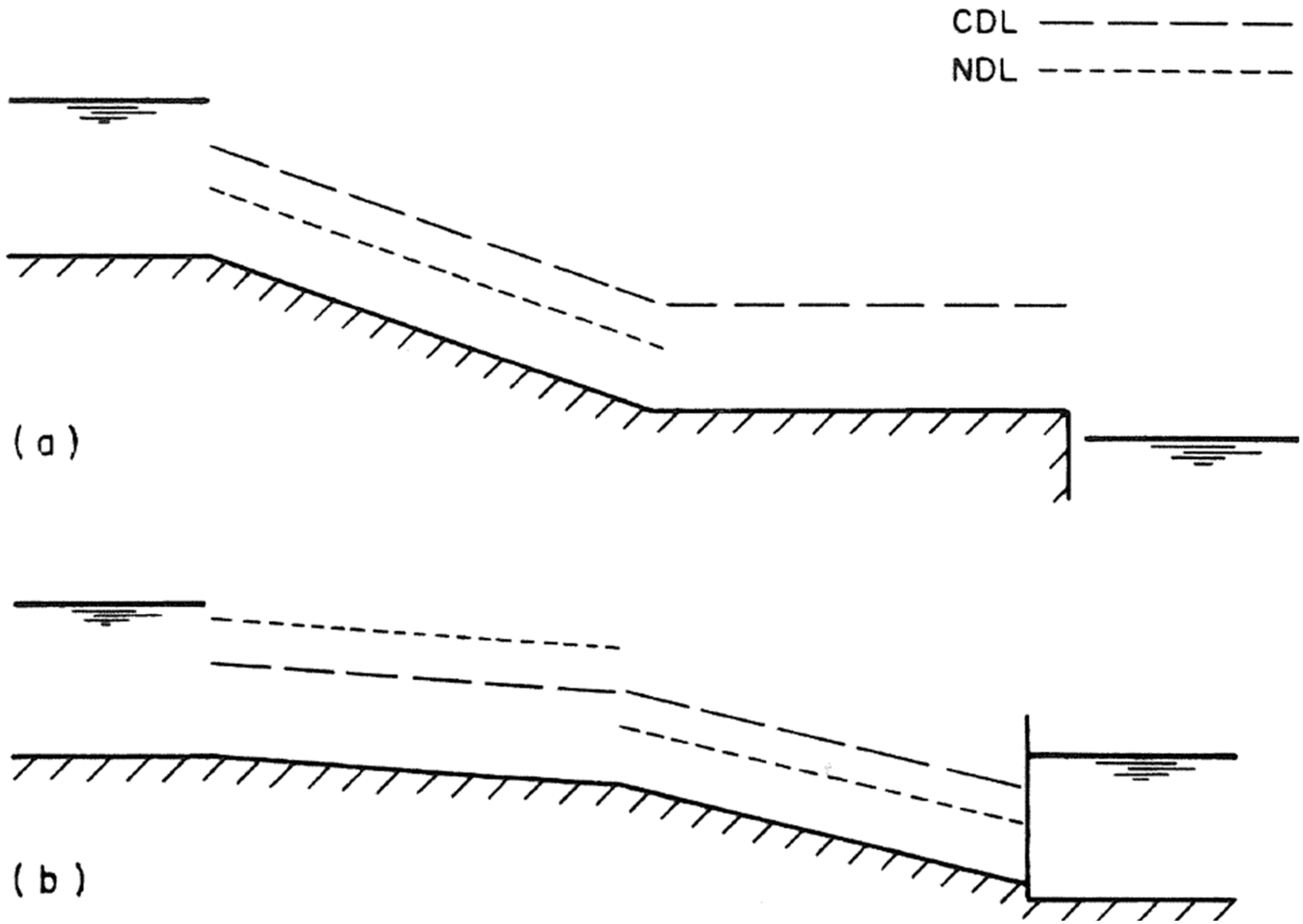
Example:

Sketch the water surface profile for the open-channel system below.



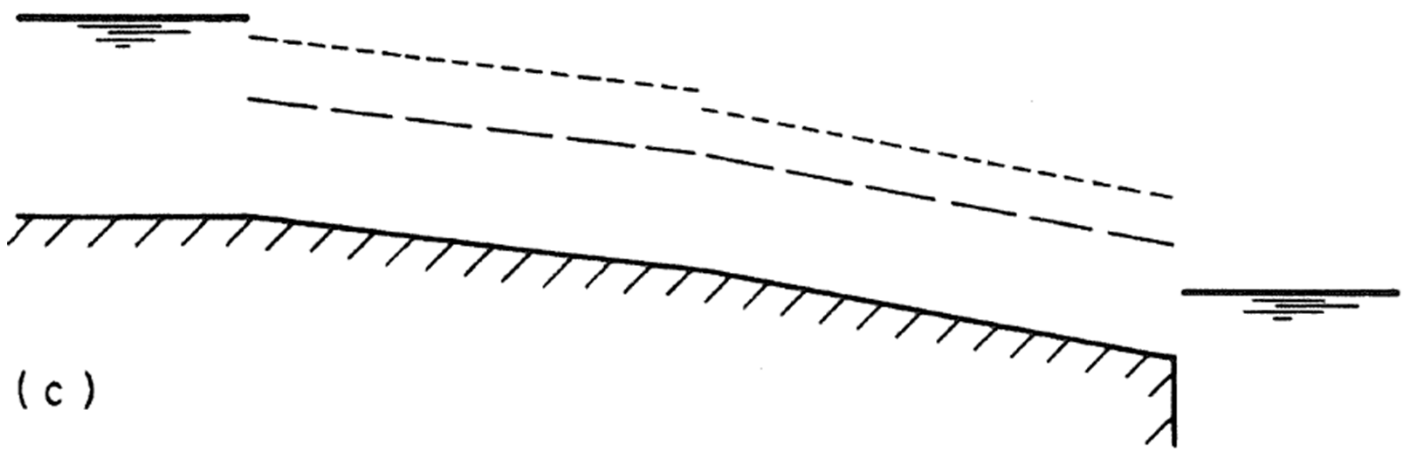
Example:

Sketch the water surface profile for the open-channel system below.

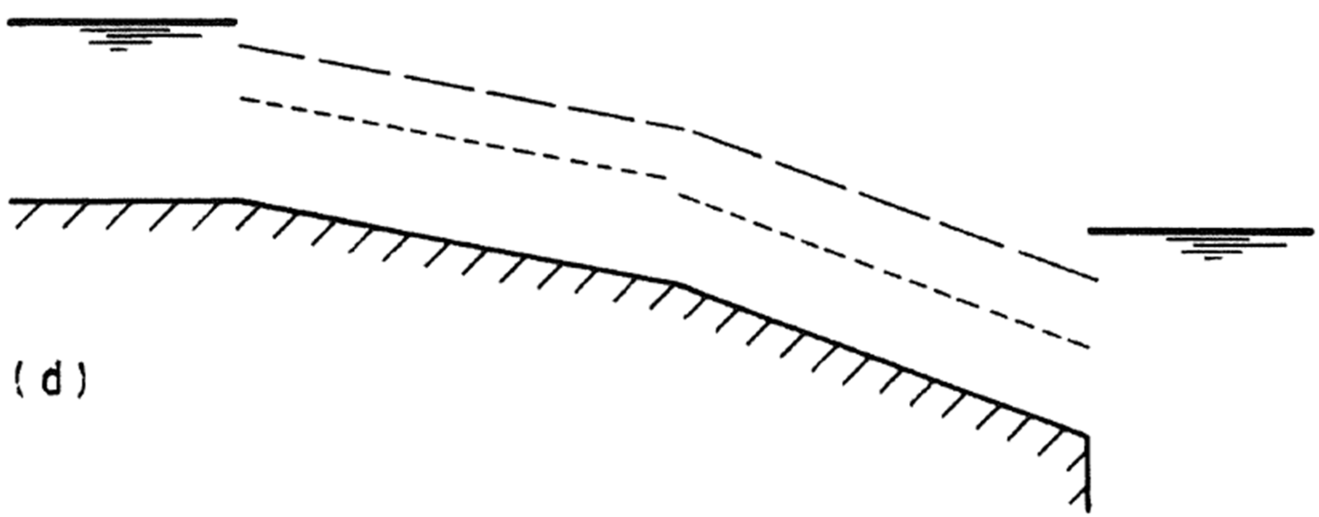


Example:

Sketch the water surface profile for the open-channel system below.



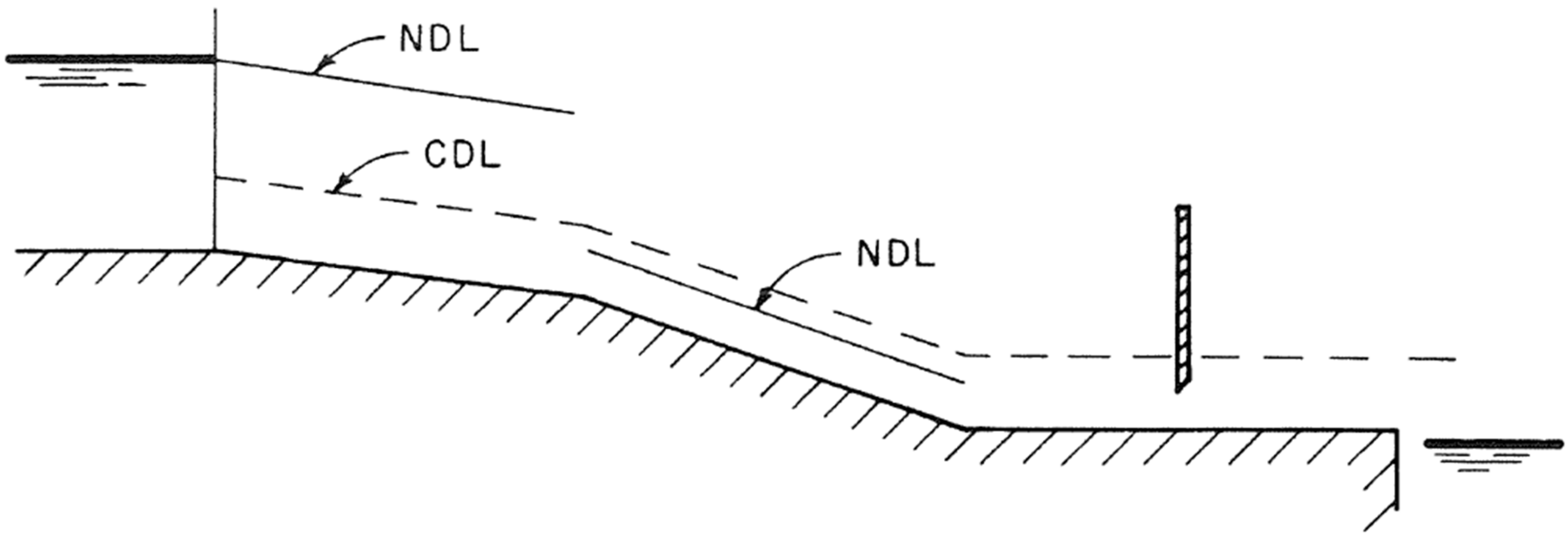
(c)



(d)

Example:

Sketch the water surface profile for the open-channel system below.



Numerical Analysis of Water Surface Profiles

Regardless of the type of method follows these steps:

1. The channel geometry, channel slope S_0, roughness coefficient n, and discharge Q are given or assumed.
2. Determine normal depth y_0 and critical depth y_c.
3. Establish the controls (i.e., the depth of flow) at the upstream and downstream ends of the channel reach.

To find y_0

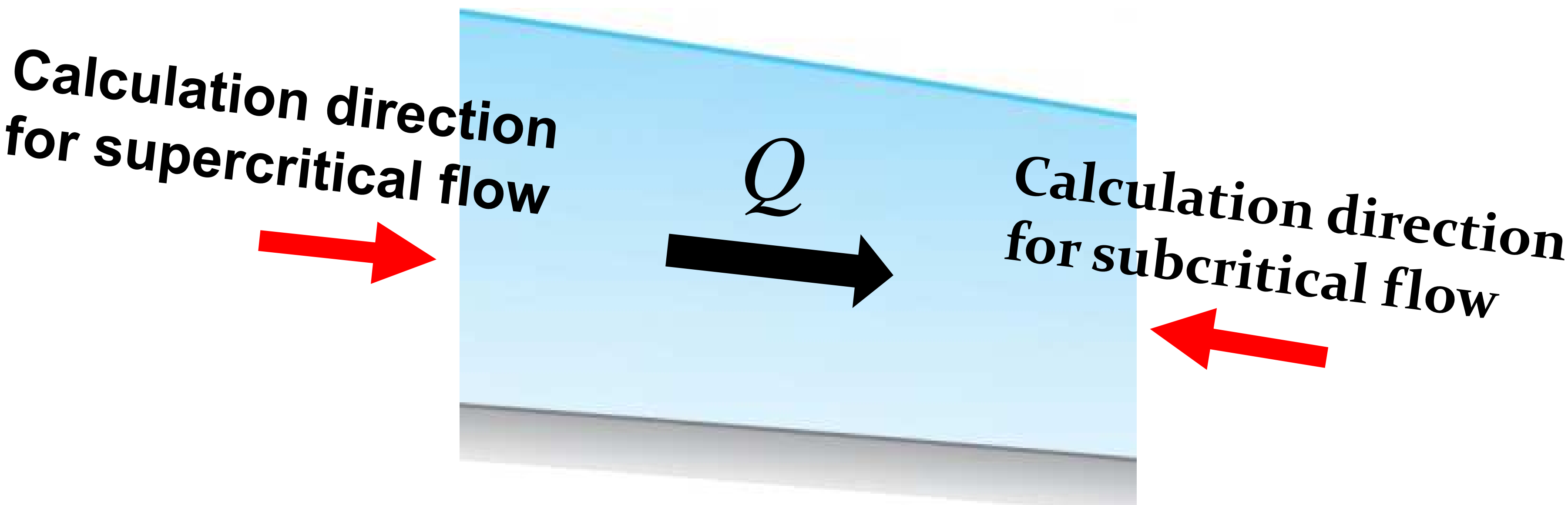
To find y_c

Energy slope s

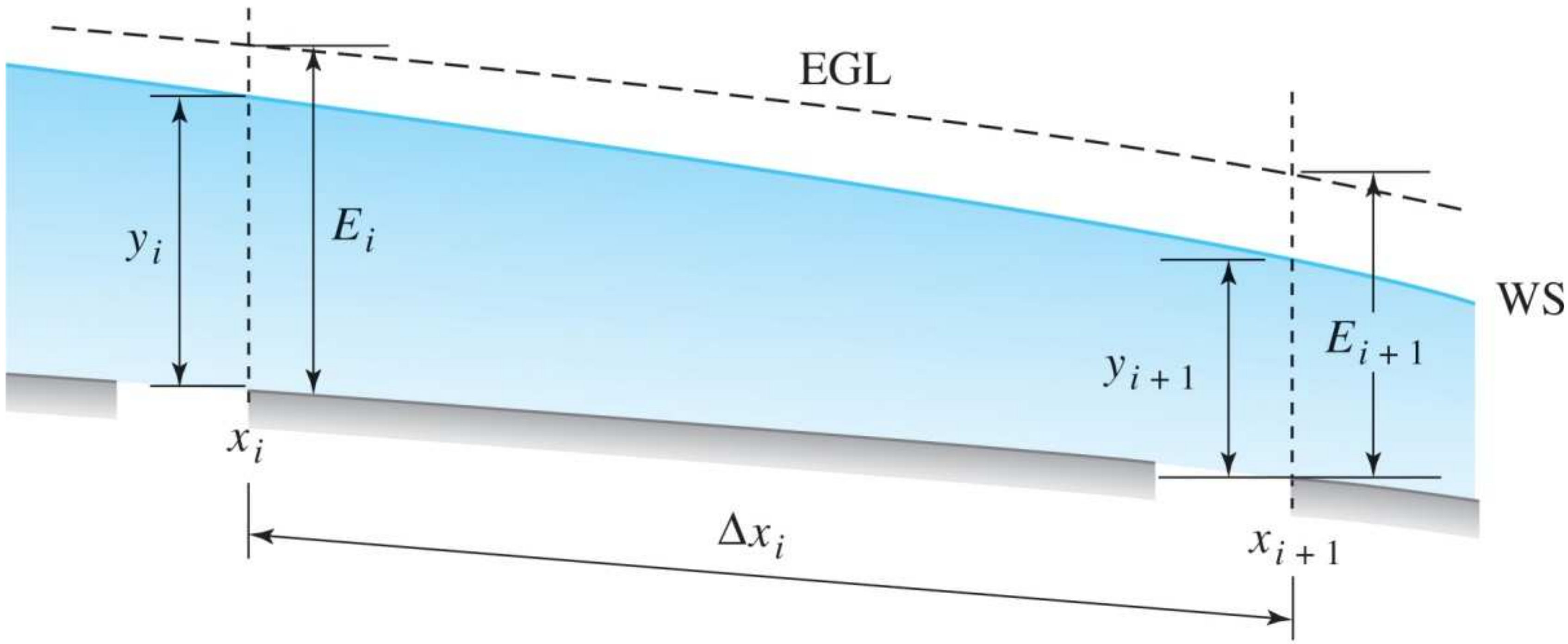
"Standard Step" Method

This method solves sequentially for y_1, y_2, y_3, … starting at the control section (upstream or downstream end) with known water depth. The computation procedure is to determine the depth at a section a distance Δx away from a section with a known depth.

Step size (Δx) must be small enough so that changes in water depth aren't very large. Otherwise estimates of the friction slope and the velocity head are inaccurate



"Standard Step" Method (cont.)



"Standard Step" Method (cont.)

In general:

For Subcritical flow:

For Supercritical flow:

Solve sequentially for unknown water depth (y) starting at the control section. The computation procedure is to determine the depth at a section a distance Δx away from a section with a known depth.

Example:

A rectangular concrete-lined channel (*n* = 0.015) has a constant bed slope of 0.0001 and a bottom width of 40 m. A control gate at the dam increased the depth at the dam to **12 m** when the discharge is 300 m^3/s. Compute the water surface profile from the dam up to 200 km upstream of the dam. (See Excel spreadsheet for rectangular channels).

Solution

The first step is to calculate the <u>critical</u> and <u>normal</u> depths.
y_o is computed using the Chezy-Manning formula

$$y_o = 4.65 \text{ m}$$

y_c is computed using the critical flow condition:

$$y_c = 1.79 \text{ m}$$

Because $\boldsymbol{y} > \boldsymbol{y_o} > \boldsymbol{y_c}$, the profile is **M1**

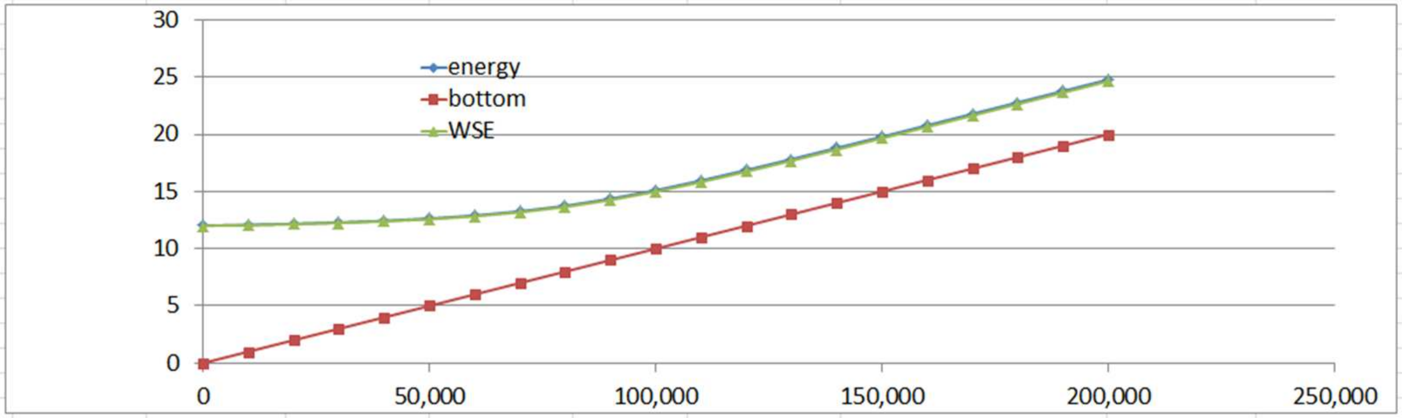
Show exercises using Excel for rectangular channels

CE 544 Gradually Varied Flow for rectangular channels, Arturo Leon

Q (m3/s) =	300
So (Slope)	0.0001
n Manning =	0.015
b (m) =	40
Initial depth (m)	12
delta X (m) =	10000
tolerance =	0.0000001

write energy equation in flow direction

x	depth y	Z (m)	A (m^2)	P (m)	R (m)	V (m/s)	WSE (z + y)	H = z + y + v^2/(2g)	Sf	average Sf	F(y) = 0
0	12.00000	0.00000	480.00000	64.00000	7.50000	0.62500	12.00000	12.01993	5.98679E-06		0.00000
10000	11.06414	1.00000	442.56542	62.12827	7.12341	0.67787	12.06414	12.08758	7.54314E-06	6.76497E-06	-2.27596E-15
20000	10.14579	2.00000	405.83144	60.29157	6.73115	0.73922	12.14579	12.17367	9.6742E-06	8.60867E-06	-7.91311E-14
30000	9.25171	3.00000	370.06821	58.50341	6.32558	0.81066	12.25171	12.28523	1.26394E-05	1.11568E-05	-1.17609E-12
40000	8.39175	4.00000	335.67000	56.78350	5.91140	0.89373	12.39175	12.43250	1.68143E-05	1.47269E-05	-1.19906E-11
50000	7.58016	5.00000	303.20623	55.16031	5.49682	0.98943	12.58016	12.63010	2.27056E-05	1.976E-05	-9.61358E-11
60000	6.83666	6.00000	273.46631	53.67332	5.09501	1.09703	12.83666	12.89806	3.08857E-05	2.67956E-05	-6.307E-10
70000	6.18607	7.00000	247.44299	52.37215	4.72471	1.21240	13.18607	13.26107	4.17166E-05	3.63011E-05	-3.23127E-09
80000	5.65380	8.00000	226.15198	51.30760	4.40777	1.32654	13.65380	13.74358	5.47855E-05	4.8251E-05	-1.17373E-08
90000	5.25580	9.00000	210.23180	50.51159	4.16205	1.42700	14.25580	14.35969	6.84361E-05	6.16108E-05	-2.761E-08
100000	4.98810	10.00000	199.52386	49.97619	3.99238	1.50358	14.98810	15.10344	8.03144E-05	7.43752E-05	-4.02669E-08
110000	4.82619	11.00000	193.04767	49.65238	3.88798	1.55402	15.82619	15.94940	8.88785E-05	8.45964E-05	-3.639E-08
120000	4.73663	12.00000	189.46524	49.47326	3.82965	1.58340	16.73663	16.86455	9.41501E-05	9.15143E-05	-2.11176E-08
130000	4.69011	13.00000	187.60431	49.38022	3.79918	1.59911	17.69011	17.82057	9.70554E-05	9.56027E-05	-8.53727E-09
140000	4.66685	14.00000	186.67390	49.33370	3.78390	1.60708	18.66685	18.79862	9.85533E-05	9.78043E-05	-2.68647E-09
150000	4.65546	15.00000	186.21836	49.31092	3.77641	1.61101	19.65546	19.78788	9.92981E-05	9.89257E-05	-7.28113E-10
160000	4.64994	16.00000	185.99771	49.29989	3.77278	1.61292	20.64994	20.78267	9.96615E-05	9.94798E-05	-1.82296E-10
170000	4.64729	17.00000	185.89140	49.29457	3.77103	1.61385	21.64729	21.78017	9.98373E-05	9.97494E-05	-4.39921E-11
180000	4.64601	18.00000	185.84032	49.29202	3.77019	1.61429	22.64601	22.77896	9.99219E-05	9.98796E-05	-1.04935E-11
190000	4.64540	19.00000	185.81580	49.29079	3.76979	1.61450	23.64540	23.77839	9.99625E-05	9.99422E-05	-2.52731E-12
200000	4.64510	20.00000	185.80404	49.29020	3.76959	1.61460	24.64510	24.77811	9.9982E-05	9.99723E-05	-6.28386E-13
										SUM	0.00000



Show examples using Annel2