Open-Channel Flows

Arturo S. Leon, Ph.D., P.E., D.WRE
Learning Objectives

1. Describe various types of open-channel flows
2. Use energy and momentum principles for rapidly varied flow configurations
3. Sketch water surface profiles
Animations of Unsteady Open Channel Flows

- Emergency water releases at 25 dams
  https://www.youtube.com/watch?v=o3E4s59OSLQ

- Road Collapse- Maine 2008
  https://www.youtube.com/watch?v=NTbhyHNA1Vc
What is Open-channel Flow?
Types of Open-channel

**Canal**: A canal is usually a long and mild-sloped channel built in the ground.

\[ S_0 = \frac{10^{-3}}{10} \]
Types of Open-channel (Cont.)

**Chute**: A chute is a channel with a steep slope.
Types of Open-channel (Cont.)

**Drop**: A drop is a channel with a **sudden change in elevation**
Types of Open-channel (Cont.)

**Culvert:** A culvert is a *covered channel* flowing usually partly full.
Types of Open-channel (Cont.)

**Natural channel:** A natural channel has irregular geometry. Examples include, rivers and creeks.
Classification of open-channel flows

- **FLOW IN OPEN CHANNEL**
  - **TEMPORAL (Time)**
    - **STEADY FLOW**
    - **UNSTEADY FLOW**
  - **SPATIAL (Space)**
    - **UNIFORM FLOW**
    - **NON-UNIFORM FLOW**
      - **RAPIDLY VARIED FLOW**
      - **GRADUALLY VARIED FLOW**
Classification of open-channel flows

Fig. 10.2  Steady nonuniform flow in a channel.

Rapidly varied flow: \( \frac{dy}{dx} \sim 1 \)

gradually varied flow: \( \frac{dy}{dx} \ll 1 \)
Wave speed in open channel flows

For wide channels or rectangular canals

\[ C = \sqrt{g y} \]

For any cross-section

\[ C = \sqrt{\frac{gA}{T}} \]
Propagation of a disturbance in subcritical, critical and supercritical flows

\[ u < c \]
Subcritical flow
\[ u < \sqrt{\frac{gA}{T}} \]

\[ u = c \]
Critical flow

\[ u > c \]
Supercritical flow
\[ u > \sqrt{\frac{gA}{T}} \]
Froude Number:

When $Fr > 1$, the flow possesses a relatively high velocity and shallow depth; on the other hand, when $Fr < 1$, the velocity is relatively low and the depth is relatively deep.

$$Fr = \frac{U}{\sqrt{g A/T}}$$

$Fr < 1$ (subcritical)

$Fr = 1$ (critical)

$Fr > 1$ (supercritical)
Uniform Flow \(
\frac{dy}{dx} = 0
\)
Cross-section Representation

A composite section

Fig. 10.5  Generalized section representation: (a) actual cross section; (b) composite cross section.
Regular cross sections

Fig. 10.4  Representative regular cross sections: (a) rectangular; (b) trapezoidal; (c) circular.
The Chezy-Manning Equation

Uniform flow occurs in a channel when the depth and velocity do not vary along its length.

\[ Q = \frac{C_1}{n} A R^{2/3} S_0^{1/2} \]

Where:
- \( c_1 = 1 \) for SI units and \( c_1 = 1.49 \) for English units.
- \( n \) = Manning roughness coefficient
- \( A \) = Hydraulic area
- \( R \) = Hydraulic radius
- \( S_0 \) = slope of the channel bottom

The depth associated with uniform flow is designated \( y_0 \); it is called either \textit{uniform depth} or \textit{normal depth}. 
Average values of the Manning Coefficient, $n$

<table>
<thead>
<tr>
<th>Wall material</th>
<th>Manning n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planed wood</td>
<td>0.012</td>
</tr>
<tr>
<td>Unplaned wood</td>
<td>0.013</td>
</tr>
<tr>
<td>Finished concrete</td>
<td>0.012</td>
</tr>
<tr>
<td>Unfinished concrete</td>
<td>0.014</td>
</tr>
<tr>
<td>Sewer pipe</td>
<td>0.013</td>
</tr>
<tr>
<td>Brick</td>
<td>0.016</td>
</tr>
<tr>
<td>Cast iron, wrought iron</td>
<td>0.015</td>
</tr>
<tr>
<td>Concrete pipe</td>
<td>0.015</td>
</tr>
<tr>
<td>Riveted steel</td>
<td>0.017</td>
</tr>
<tr>
<td>Earth, straight</td>
<td>0.022</td>
</tr>
<tr>
<td>Corrugated metal flumes</td>
<td>0.025</td>
</tr>
<tr>
<td>Rubble</td>
<td>0.03</td>
</tr>
<tr>
<td>Earth with stones and weeds</td>
<td>0.035</td>
</tr>
<tr>
<td>Mountain streams</td>
<td>0.05</td>
</tr>
</tbody>
</table>

*The values in this table result in flow rates too large for hydraulic radii greater than about 3 m (10 ft). The Manning $n$ should be increased by 10 to 15% for such large conduits.*
The Most Efficient Section (or best hydraulic cross section)

The Most Efficient cross-section is defined as the section of maximum flow rate ($Q$) for a constant hydraulic area ($A$), slope ($S_o$), and roughness coefficient ($n$). Alternatively, the Most Efficient cross-section can be defined as the section of minimum hydraulic area ($A$) for a constant flow rate ($Q$).

For a rectangular channel:

$$Q = \frac{C_l}{n} AR^{2/3} S_o^{1/2}$$

$$0 = \frac{C_l}{n} S_o^{1/2} d\left( \frac{A^{5/3}}{p^{2/3}} \right) \rightarrow A \left( -\frac{2}{3} \right) p^{-5/3} + P \left( \frac{5}{3} A \right) \frac{dA}{dy} = 0$$

$$\frac{dp}{dy} = 0$$

$P$ is minimum
The best hydraulic cross-section for various shapes

\[ A = by, \]
\[ \frac{dA}{dy} = b \cdot \frac{dy}{dy} + y \cdot \frac{db}{dy} = 0 \ldots (1) \]
\[ p = b + 2y \]
\[ \frac{dp}{dy} = 0 \quad \frac{db + 2}{dy} = 0 \ldots (2) \]

In (1)
\[ b + y(-2) = 0 \]
\[ b = 2y \]
Example:
The following data are obtained for a particular reach of the Provo River in Utah: $A = 183 \, ft^2$, free-surface width = 55 ft, average depth = 3.3 ft, $R_h = 3.32 \, ft$, $V = 6.56 \, ft/s$, length of reach = 116 ft, and elevation drop of reach = 1.04 ft. Determine (a) the Manning coefficient, $n$, and (b) the Froude number of the flow.

\[
\begin{align*}
A &= 183 \, ft^2 \\
R &= 3.32 \, ft \\
V &= 6.56 \, ft/s \\
L &= 116 \, ft \\
\Delta z &= 1.04 \, ft
\end{align*}
\]

(a) "n" = ??

\[
\begin{align*}
Q &= \frac{C_a}{n} AR S_0^{2/3} \\
V &= \frac{Q}{A}
\end{align*}
\]

\[
6.56 = \frac{1.49}{n} (3.32)^{2/3} \left( \frac{1.04}{116} \right)^{1/2}
\]

\[
\eta = 0.0469
\]
b) \[ Fr = \frac{V}{\sqrt{9A^2}} = \frac{6.56}{\sqrt{32.2 \times \frac{183}{55}}} \]

\[ Fr = 0.63 \quad \text{[Subcritical flow, } Fr < 1 \text{]} \]
Example:
At a given location, under normal conditions a river flows with a Manning coefficient of 0.030, and a cross section as indicated in part (a) of the figure below. During flood conditions at this location, the river has a Manning coefficient of 0.040 (because of trees and brush in the floodplain) and a cross section as shown in part (b) of the figure below. Determine the ratio of the flowrate during flood conditions to that during normal conditions.

\begin{align*}
\eta_a &= 0.030 \\
\eta_b &= 0.040 \\
\frac{Q_b}{Q_a} &= ? \\
A_a &= 12 \times 800 = 9600 \text{ ft}^2 \\
P_a &= 800 + 2 \times 12 = 824 \text{ ft} \\
Ra &= \frac{A_a}{P_a} = 11.65 \text{ ft} \\
A_b &= 24,000 \text{ ft}^2 \\
P_b &= 20 + 800 + 12 + 1000 + 8 = 1840 \text{ ft} \\
R_b &= \frac{A_b}{P_b} = 13.04 \text{ ft}
\end{align*}
\[ \frac{Q_b}{Q_a} = \frac{\frac{C_I}{n_b} A_b R_b^{2/3} S_o^{1/2}}{\frac{C_I}{n_a} A_a R_a^{2/3} S_o^{1/2}} = \frac{n_a A_b R_b^{2/3}}{n_b A_a R_a^{2/3}} \]

\[ \frac{Q_b}{Q_a} = \frac{0.030}{0.040} \times \frac{24000}{9600} \times \left( \frac{13.04}{11.65} \right)^{2/3} \]

\[ \frac{Q_b}{Q_a} = 2.02 \]
10.4 Energy Concepts

**Total energy**: The sum of the vertical distance to the channel bottom measured from a horizontal datum, the depth of flow, and the kinetic energy head.

\[ H = z + y + \frac{v^2}{2g} \]

Energy is actually an energy head.

\[ H_1 = H_2 + h_L \]

\( h_L \) is the head loss.
10.4 Energy Concepts

**Specific energy**: Measurement of energy relative to the bottom of the channel.

\[ E = y + \frac{v^2}{2g} \]

**Specific discharge**: The total discharge divided by the channel width (valid only for a rectangular channel).

\[ q = \frac{Q}{b} \]

\[ E = q + \frac{q^2}{2gy^2} \]
Critical depth

For a rectangular channel \((q = Q/b)\):

\[ \frac{dE}{dy} = 0 \]

For any cross-section:

\[ 1 - \frac{Q^2 T}{gA^3} = 0 \]

\[ \frac{V_c^2}{2g} = \frac{1}{2} \frac{A_c}{T_c} \]

\[ y_c = \sqrt[3]{\frac{Q^2}{g}} \]

\[ \frac{V_c^2}{2g} = \frac{1}{2} y_c \]
The condition of choked flow or a choking condition implies that minimum specific energy exists within the transition.
Flow Choking

$h_{max} = E_1 - E_c$

For a rectangular channel:

$h_{max} = E_1 - \frac{3}{2} y_c$

For a non-rectangular channel:

$h_{max} = E_1 - \left[ y_c + \frac{1}{2} \frac{A_c}{T_c} \right]$
Example:
Consider a channel where the upstream velocity is 5.0 m/s and the upstream flow depth is 0.6 m. The flow then passes over a bump 15 cm in height.
(a) Compute the flow depth and velocity on the crest of the bump.
(b) Compute the maximum allowable bump height that keeps water from backing up upstream.

\[
\begin{align*}
V &= 5 \text{ m/s} \\
0.6 \text{ m} \\
0.66 \text{ m} \\
0.15 \text{ m}
\end{align*}
\]
a) \( y_2, V_2 \)?

\[
0.6 + \frac{5^2}{2 \times 9.8} = y_2 + \frac{V_2^2}{2g} + 0.15 \quad \cdots (1)
\]

*Continuity*

\[
0.6(5)y = V_2 y_2 / y
\]

\[
V_2 y_2 = 3 \rightarrow V_2 = \frac{3}{y_2}
\]

In \((1)\)

\[
0.6 + \frac{5^2}{2 \times 9.8} = y_2 + \frac{9}{2 \times 9.8 y_2} + 0.15
\]

3 roots \(\begin{cases} \text{(1)} & \quad y_2 = 0.66m \\ \text{(2)} & \quad y_1 = 1.52m \\ \end{cases}\)

\[*Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{5}{\sqrt{9.8 \times 0.6}} = 2.06 (supercritical)\]

\(\gamma_2 = 0.66m\)
\[
V_2 = \frac{3}{y_2} = 4.54 \text{ m/s}
\]

b) Maximum bump height

\[
h_{\text{max}} = E_1 - E_c
\]

\[
y_c = \sqrt{\frac{q^2}{g}}
\]

\[
h_{\text{max}} = 1.87 - \frac{3}{2} y_c
\]

\[
h_{\text{max}} = 0.41 \text{ m}
\]
Example:

Compute the critical depth in a trapezoidal channel for a flow of 30 m$^3$/s. The channel bottom width is 10 m, side slopes are 2H:1V.

\[ Q = 30 \text{ m}^3/\text{s} \]
\[ T = 10 + 4Y_c \]
\[ A = \left(\frac{10 + 10 + 4Y_c}{2}\right) Y_c \]
\[ A = (10 + 2Y_c)Y_c = 10Y_c + 2Y_c^2 \]

*critical depth*

\[ 1 - \frac{Q^2T}{gA^3} = 0 \quad \ldots \text{Eq} \]
In (1)

\[ 1 - \frac{30^2(10+4y_c)}{9.8(10y_c+2y_c^2)^3} = 0 \]

\[ y_c = 0.9 \text{ m} \]
Momentum Concepts

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Hydraulic Jump

Hydraulic jump:
https://www.youtube.com/watch?v=cRnIsqSTX7Q
Low head dams:

https://www.youtube.com/watch?v=XsYgODmniAM
10.5 Momentum Concepts

Fig. 10.13 Channel flow over an obstacle: (a) idealized flow; (b) control volume

Linear momentum equation is:

$$\gamma A_1 \bar{y}_1 - \gamma A_2 \bar{y}_2 = \int Q (V_2 - V_1)$$

Let’s define $M$ (momentum function) as:

$$M = A \bar{y} + \frac{Q^2}{gA}$$

For a rectangular section:

$$M = b \left[ \frac{y^2}{2} + \frac{q^2}{gy} \right]$$
Momentum function $M$ for various channels

- **Rectangular**
  $$M = Ay + \frac{Q^2}{gb}$$
  $$by^2/2 + \frac{Q^2}{(gb)y}$$

- **Trapezoidal**
  $$by^2/2 + my^3/3 + \frac{Q^2}{[gy(b + my)]}$$

- **Triangular**
  $$my^3/3 + \frac{Q^2}{(gmy^2)}$$

- **Circular**
  $$[3 \sin(\theta/2) - \sin^3(\theta/2) - 3(\theta/2) \cos(\theta/2)] \frac{d^3}{24} + \frac{Q^2}{[gd^2(\theta - \sin\theta)/8]}$$
  $$\theta = 2 \cos^{-1}[1 - 2(y/d)]$$
10.5 Momentum Concepts

Fig. 10.14  Variation of the momentum function with depth.
Hydraulic Jump in a rectangular channel (Cont.)

Fig. 10.15  Idealized hydraulic jump.

\[
\frac{y_2}{y_1} = \frac{1}{2} \left[ \sqrt{1 + 8 \text{Fr}_n^2} - 1 \right]
\]

\[
h_L = \frac{(y_2 - y_1)^3}{4y_1y_2}
\]

\[\text{Fr}_1 > 1\]

\[\frac{V_1^2}{2g} \]

\[h_j\]

\[\frac{V_2^2}{2g}\]
# Classification of Hydraulic Jumps

## Table 10.2  Hydraulic Jumps in Horizontal Rectangular Channels

<table>
<thead>
<tr>
<th>$Fr_{Upstream}$</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0–1.7</td>
<td>Undular</td>
<td>Ruffled or undular water surface; surface rollers form near $Fr = 1.7$</td>
</tr>
<tr>
<td>1.7–2.5</td>
<td>Weak</td>
<td>Prevailing smooth flow; low energy loss</td>
</tr>
<tr>
<td>2.5–4.5</td>
<td>Oscillating</td>
<td>Intermittent jets from bottom to surface, causing persistent downstream waves</td>
</tr>
<tr>
<td>4.5–9.0</td>
<td>Steady</td>
<td>Stable and well-balanced; energy dissipation contained in main body of jump</td>
</tr>
<tr>
<td>$&gt;9.0$</td>
<td>Strong</td>
<td>Effective, but with rough, wavy surface downstream</td>
</tr>
</tbody>
</table>

*Source: Adapted with permission from Chow, 1959. (Adapted from Chow, 1959)*
Example:
Under appropriate conditions, water flowing from a faucet, onto a flat plate, and over the edge of the plate can produce a circular hydraulic jump as shown in the figure below. Consider a situation where a jump forms 3.0 in from the center of the plate with depths upstream and downstream of the jump of 0.05 in and 0.20 in, respectively. Determine the flow rate from the faucet.

\[ y_1 = 0.05 \text{ in} \]
\[ y_2 = 0.20 \text{ in} \]
\[ Q = ?? \]
\[
\frac{y_2}{y_1} = \frac{1}{2} \left[ \sqrt{1 + 8F_{R_1}} - 1 \right]
\]
\[
\frac{0.20}{0.05} = \frac{1}{2} \left[ \sqrt{1 + 8F_{R_1}} - 1 \right]
\]

\[
F_{R_1} = 3.16
\]

\[
\frac{V_1}{\sqrt{9y_1}} = \frac{V_1}{\sqrt{32.2 \times 0.05}} = 3.16
\]

\[
V_1 = 1.16 \text{ ft/s}
\]

\[
Q = V_1 \cdot A_1
\]

\[
A_1 = P_1 \cdot y_1 = 2\pi \left( \frac{3}{12} \right) \left( \frac{0.05}{12} \right)
\]
\[ Q = 0.00759 \text{ ft}^3/\text{s} \]

HW 8 [1-6]
Gradually varied flow

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Gradually varied Flows

Differential Equation for Gradually Varied Flow

From Energy Eq.
\[ \frac{dy}{dx} = \frac{S_0 - S}{1 - Fr^2} \]

Fr = Froude number

For solving numerically gradually varied flows.

Where:
\( S \) = total energy slope
\( S_o \) = bed slope
\( Fr \) = Froude Number

\[ h_L = S\Delta x \]

Fig. 10.18 Nonuniform gradually varied flow.
Does water depth increase or decrease in $x$ direction?

Is $\frac{dy}{dx}$ positive or negative?

Assuming a wide rectangular channel:

$$\frac{dy}{dx} = S_0 \frac{1 - (y_0/y)^{10/3}}{1 - (y_c/y)^3}$$

$y_c$: critical depth

$y_0$: normal depth

$y$: flow depth

Use this equation to evaluate if water depth increases or decreases with $x$. 
## Classification of Surface Profiles

<table>
<thead>
<tr>
<th>Channel slope</th>
<th>Profile type</th>
<th>Depth range</th>
<th>Fr</th>
<th>( \frac{dy}{dx} )</th>
<th>( \frac{dE}{dx} )</th>
<th>Normal depth</th>
<th>Critical depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mild ( S_0 &lt; S_c ) ( y_0 &gt; y_c )</td>
<td>M_1</td>
<td>( y &gt; y_0 &gt; y_c )</td>
<td>&lt; 1</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td><img src="image" alt="M_1" /></td>
<td><img src="image" alt="M_1" /></td>
</tr>
<tr>
<td>M_2</td>
<td>( y_0 &gt; y &gt; y_c )</td>
<td>&lt; 1</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="M_2" /></td>
<td><img src="image" alt="M_2" /></td>
<td></td>
</tr>
<tr>
<td>M_3</td>
<td>( y_0 &gt; y_c &gt; y )</td>
<td>&gt; 1</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="M_3" /></td>
<td><img src="image" alt="M_3" /></td>
<td></td>
</tr>
<tr>
<td>Steep ( S_0 &gt; S_c ) ( y_0 &lt; y_c )</td>
<td>S_1</td>
<td>( y &gt; y_c &gt; y_0 )</td>
<td>&lt; 1</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td><img src="image" alt="S_1" /></td>
<td><img src="image" alt="S_1" /></td>
</tr>
<tr>
<td>S_2</td>
<td>( y_c &gt; y &gt; y_0 )</td>
<td>&gt; 1</td>
<td>&lt; 0</td>
<td>&gt; 0</td>
<td><img src="image" alt="S_2" /></td>
<td><img src="image" alt="S_2" /></td>
<td></td>
</tr>
<tr>
<td>S_3</td>
<td>( y_c &gt; y_0 &gt; y )</td>
<td>&gt; 1</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="S_3" /></td>
<td><img src="image" alt="S_3" /></td>
<td></td>
</tr>
<tr>
<td>Critical ( S_0 = S_c ) ( y_0 = y_c )</td>
<td>C_1</td>
<td>( y &gt; y_c ) or ( y_0 )</td>
<td>&lt; 1</td>
<td>&gt; 0</td>
<td>&gt; 0</td>
<td><img src="image" alt="C_1" /></td>
<td><img src="image" alt="C_1" /></td>
</tr>
<tr>
<td>C_3</td>
<td>( y_c ) or ( y_0 &gt; y )</td>
<td>&gt; 1</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="C_3" /></td>
<td><img src="image" alt="C_3" /></td>
<td></td>
</tr>
<tr>
<td>Horizontal ( S_0 = 0 ) ( y_0 \to \infty )</td>
<td>H_2</td>
<td>( y &gt; y_c )</td>
<td>&lt; 1</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="H_2" /></td>
<td><img src="image" alt="H_2" /></td>
</tr>
<tr>
<td>H_3</td>
<td>( y_c &gt; y )</td>
<td>&gt; 1</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="H_3" /></td>
<td><img src="image" alt="H_3" /></td>
<td></td>
</tr>
<tr>
<td>Adverse ( S_0 &lt; 0 ) ( y_0 ) undefined</td>
<td>A_2</td>
<td>( y &gt; y_c )</td>
<td>&lt; 1</td>
<td>&lt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="A_2" /></td>
<td><img src="image" alt="A_2" /></td>
</tr>
<tr>
<td>A_3</td>
<td>( y_c &gt; y )</td>
<td>&gt; 1</td>
<td>&gt; 0</td>
<td>&lt; 0</td>
<td><img src="image" alt="A_3" /></td>
<td><img src="image" alt="A_3" /></td>
<td></td>
</tr>
</tbody>
</table>
Examples of Gradually Varied Flows

Typical surface configurations for nonuniform depth flow with a mild slope \((F_r < 1)\)
Example:

Sketch the water surface profile for the two-reach open-channel system below. A gate is located between the two reaches and the second reach ends with a sudden fall.

\[ S_{o1} > S_c \]
\[ S_{o2} < S_c \]

Sc : critical slope
So > Sc (steep slope)
So < Sc (mild slope)
Example:

Sketch the water surface profile for the open-channel system below.

CDL: Critical depth line
NDL: Normal depth line
Example:

Sketch the water surface profile for the open-channel system below.

(a) Hydraulic jump

(b) Hydraulic jump
Example:

Sketch the water surface profile for the open-channel system below.
Example:

Sketch the water surface profile for the open-channel system below.
Numerical Analysis of Water Surface Profiles

Regardless of the type of method follows these steps:
1. The channel geometry, channel slope $S_0$, roughness coefficient $n$, and discharge $Q$ are given or assumed.
2. Determine normal depth $y_0$ and critical depth $y_c$.
3. Establish the controls (i.e., the depth of flow) at the upstream and downstream ends of the channel reach.

To find $y_0$

$$\frac{Qn}{C_1 AR^{2/3} \sqrt{S_0}} - 1 = 0$$

To find $y_c$

$$\frac{Q^2 T}{9A^3} - 1 = 0$$

Energy slope $s$

$$S(y) = \frac{Q^2 n^2}{C_1^2 \left[ A(y) \right]^2 \left[ R(y) \right]^{4/3}}$$

$$R = \frac{A}{P}$$
“Standard Step” Method

This method solves sequentially for $y_1$, $y_2$, $y_3$, … starting at the control section (upstream or downstream end) with known water depth. The computation procedure is to determine the depth at a section a distance $\Delta x$ away from a section with a known depth.

Step size ($\Delta x$) must be small enough so that changes in water depth aren’t very large. Otherwise estimates of the friction slope and the velocity head are inaccurate.

Calculation direction for supercritical flow

$\Delta x \sim 100 \text{ m (supercritical flow)}$

Calculation direction for subcritical flow

$\Delta x \sim 5 \text{ m (subcritical flow)}$
“Standard Step” Method (cont.)

\[ \frac{V_i^2}{2g} \]

\[ y_i \]

\[ E_i \]

\[ s \Delta x \]

\[ WS \]

\[ y_{i+1} \]

\[ E_{i+1} \]

\[ x_i \]

\[ x_{i+1} \]

\[ \Delta x_i \]

\[ H_i = H_{i+1} + h_L \]

\[ Z_{i+1} \]

\[ h = Z + y + \frac{V_i^2}{2g} \]

\[ S = s(y_m)^2 \]

\[ y_m = \frac{y_i + y_{i+1}}{2} \]
"Standard Step" Method (cont.)

In general:

For Subcritical flow:

$$H_i - H_{i+1} - S\Delta x = 0$$

$H_{i+1}$ is known

For Supercritical flow:

$$H_i - H_{i+1} - S\Delta x = 0$$

$H_i$ is known

Solve sequentially for unknown water depth ($y$) starting at the control section. The computation procedure is to determine the depth at a section a distance $\Delta x$ away from a section with a known depth.

$y_m = y_i + y_{i+1}$
Example:
A rectangular concrete-lined channel ($n = 0.015$) has a constant bed slope of 0.0001 and a bottom width of 40 m. A control gate at the dam increased the depth at the dam to **12 m** when the discharge is 300 m$^3$/s. Compute the water surface profile from the dam up to 200 km upstream of the dam. (See Excel spreadsheet for rectangular channels).
Solution

The first step is to calculate the critical and normal depths. $y_o$ is computed using the Chezy-Manning formula:

\[
\frac{Qn}{C_1 AR^{2/3} \sqrt{S_0}} - 1 = 0
\]

$y_o = 4.65$ m

$y_c$ is computed using the critical flow condition:

\[
\frac{Q^2 T}{g A^3} - 1 = 0
\]

$y_c = 1.79$ m

Because $y > y_o > y_c$, the profile is M1.
Show exercises using Excel for rectangular channels

<table>
<thead>
<tr>
<th>x</th>
<th>depth y</th>
<th>Z (m)</th>
<th>A (m^2)</th>
<th>P (m)</th>
<th>R (m)</th>
<th>V (m/s)</th>
<th>WSE (z + y)</th>
<th>H = z + y + v^2/(2g)</th>
<th>Sf</th>
<th>average Sf</th>
<th>F(y) = 0</th>
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</tbody>
</table>

- **SUM**: 0.00000

- write energy equation in flow direction
Show examples using Annel2