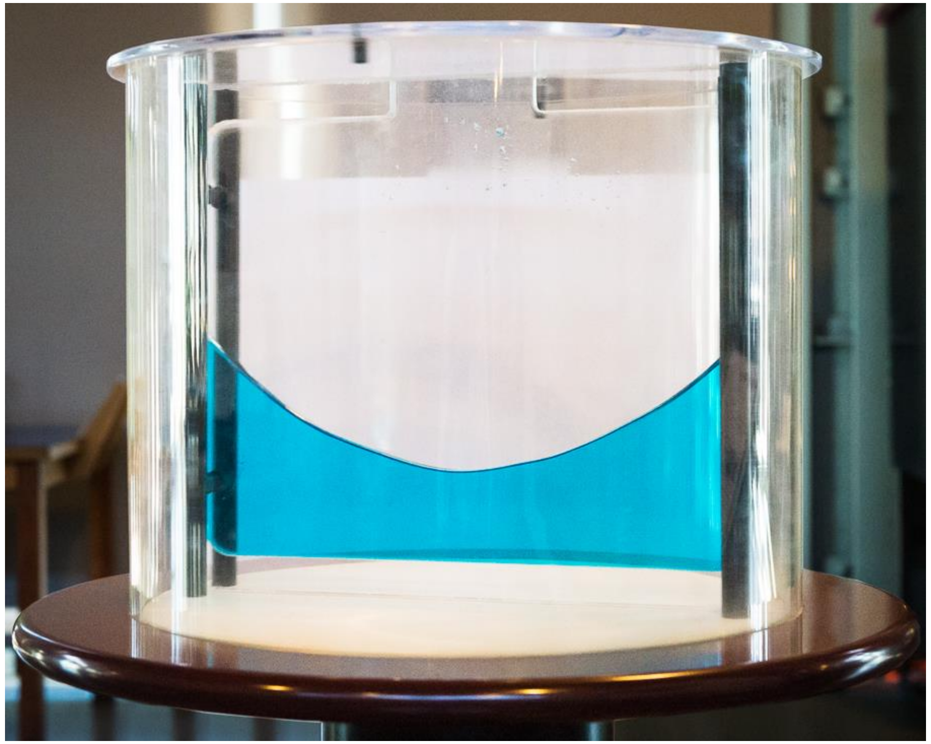
Florida International University, Department of Civil and Environmental Engineering

CWR 3201 Fluid Mechanics, Fall 2018

Fluid Statics



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2.1 INTRODUCTION

Fluid Statics: Study of fluids with no relative motion between fluid particles.

- No shearing stress (no velocity gradients)
- Only normal stress exists (pressure)

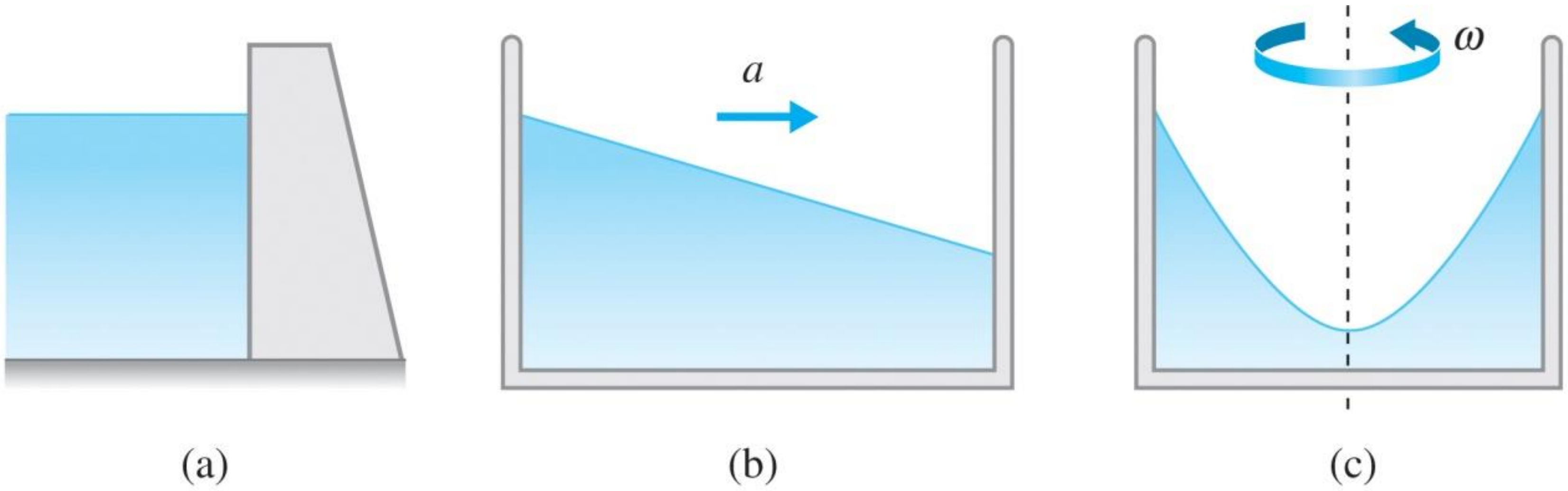
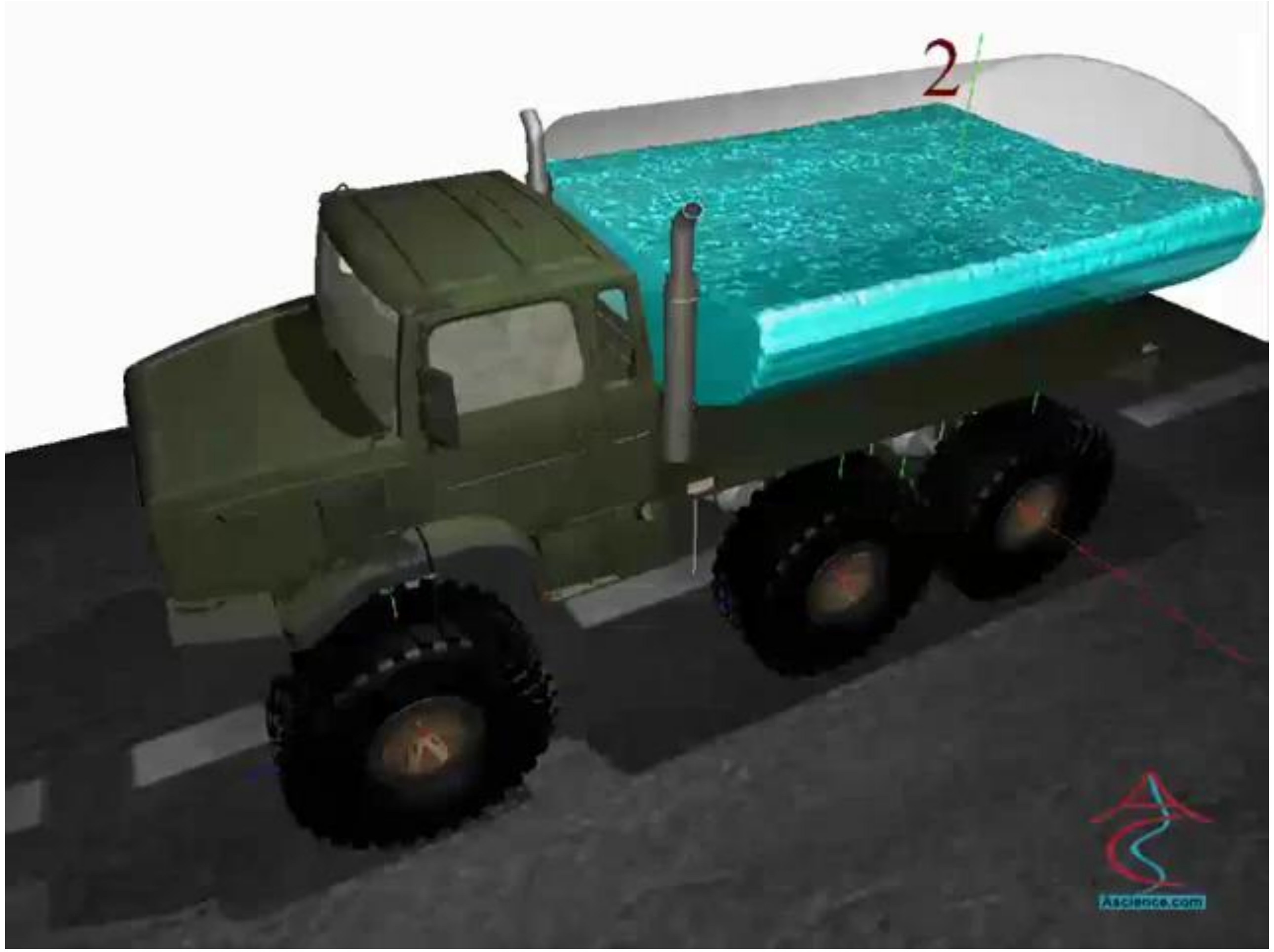


Fig. 2.1 Examples included in fluid statics: (a) liquids at rest; (b) linear acceleration; (c) angular rotation.

MOTIVATION



Source: asciencecom, Youtube
(https://www.youtube.com/watch?v=jqpl4ME6rRY)

MOTIVATION (CONT.)

Youtube (https://www.youtube.com/watch?v=Zip9ft1PgV0)

MOTIVATION (CONT.)

Youtube (https://www.youtube.com/watch?v=9jLQx3kD7p8)

2.2 PRESSURE AT A POINT

- Pressure is an infinitesimal normal compressive force divided by the infinitesimal area over which it acts.

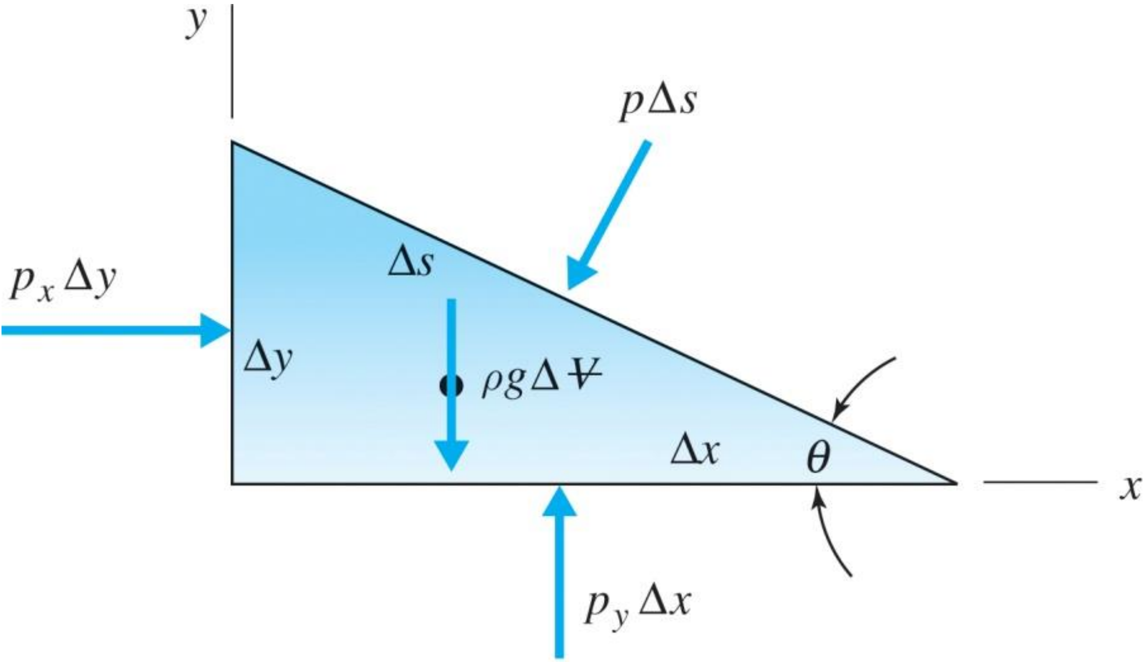


Fig. 2.2 Pressure at a point in a fluid.

- From Newton's Second Law (for *x*- and *y*-directions):

2.2 PRESSURE AT A POINT

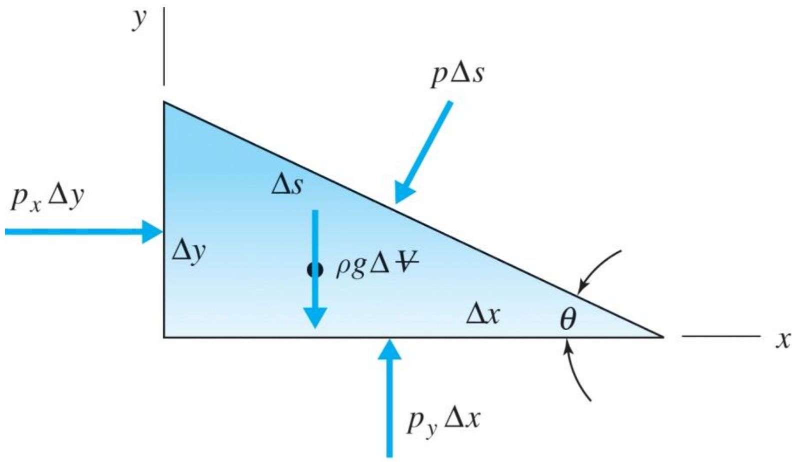


Fig. 2.2 Pressure at a point in a fluid.

- As the element goes to a point (Δx, $\Delta y \rightarrow 0$)

- Pressure in a fluid is **constant** at a point.
- Pressure is a **scalar** function.
- It acts **equally in all directions** at a point for both static and dynamic fluids.

2.3 DERIVATION OF GENERAL FORM OF PRESSURE VARIATION

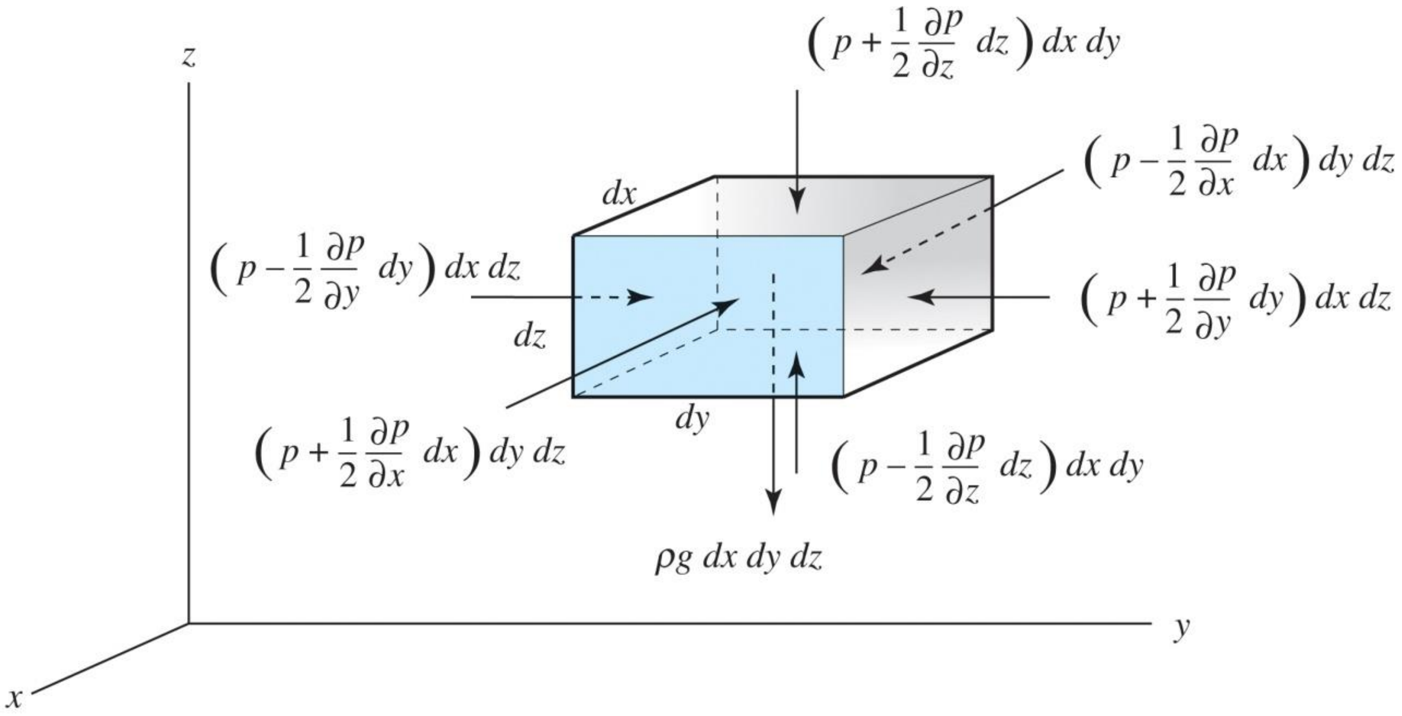


Fig. 2.3 Forces acting on an infinitesimal element that is at rest in the *xyz*-reference frame. The reference frame may be accelerating or rotating.

- Newton's Second Law in "*x*", "*y*" and "z"-directions:

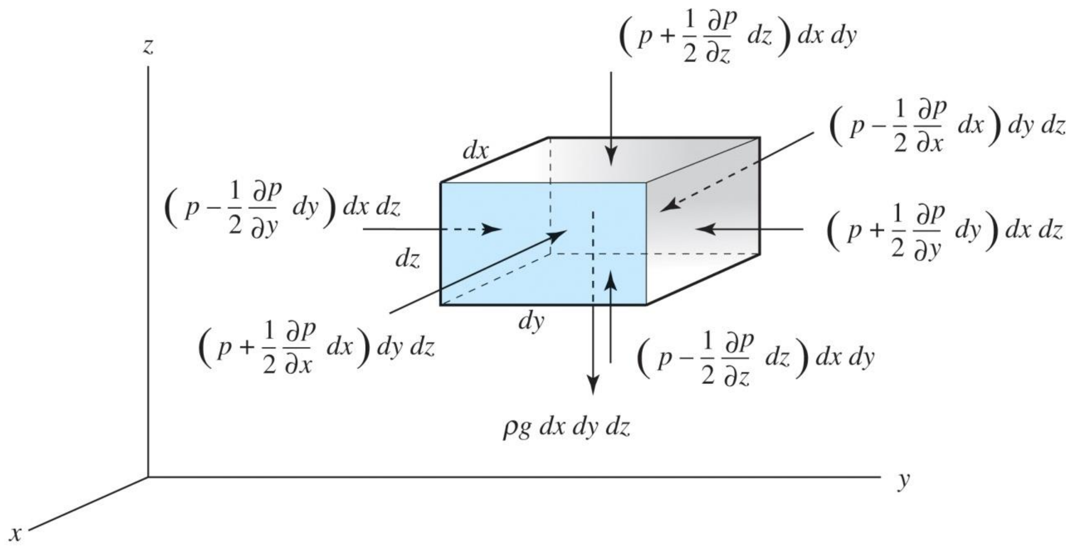


Fig. 2.3 Forces acting on an infinitesimal element that is at rest in the *xyz*-reference frame. The reference frame may be accelerating or rotating.

- Using the Chain rule, the pressure change in any direction can be calculated as:

- Then the pressure differential becomes:

2.4 FLUIDS AT REST

- The pressure differential (from the previous slide) is:

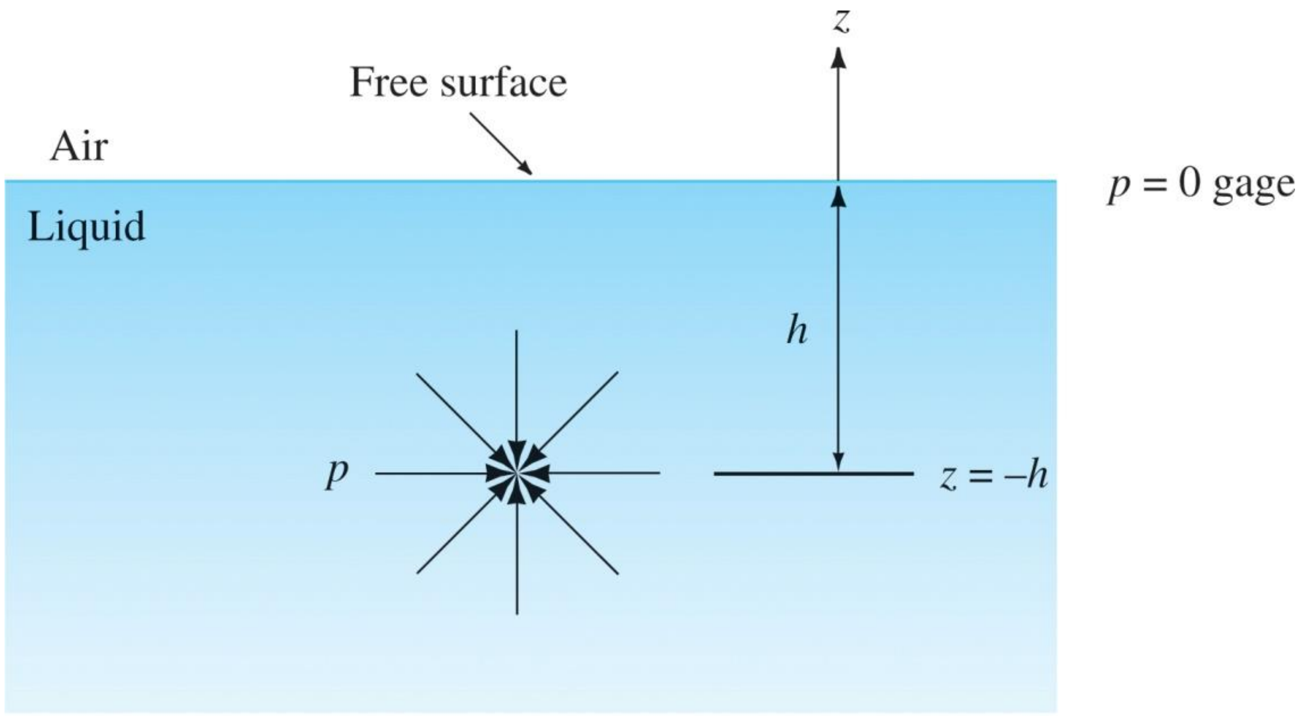


Fig. 2.4 Pressure below a free surface.

- At rest, there is no acceleration (a = 0):

No pressure variation in the x- and y-directions (horizontal plane). Pressure varies in the z-direction only (dp is negative if dz is positive).

Pressure decreases as we move up and increases as we move down.

2.4.1 Pressure in Liquids at Rest

- At a distance *h* below a free surface, the pressure is:

$p = 0$ at $h = 0$.

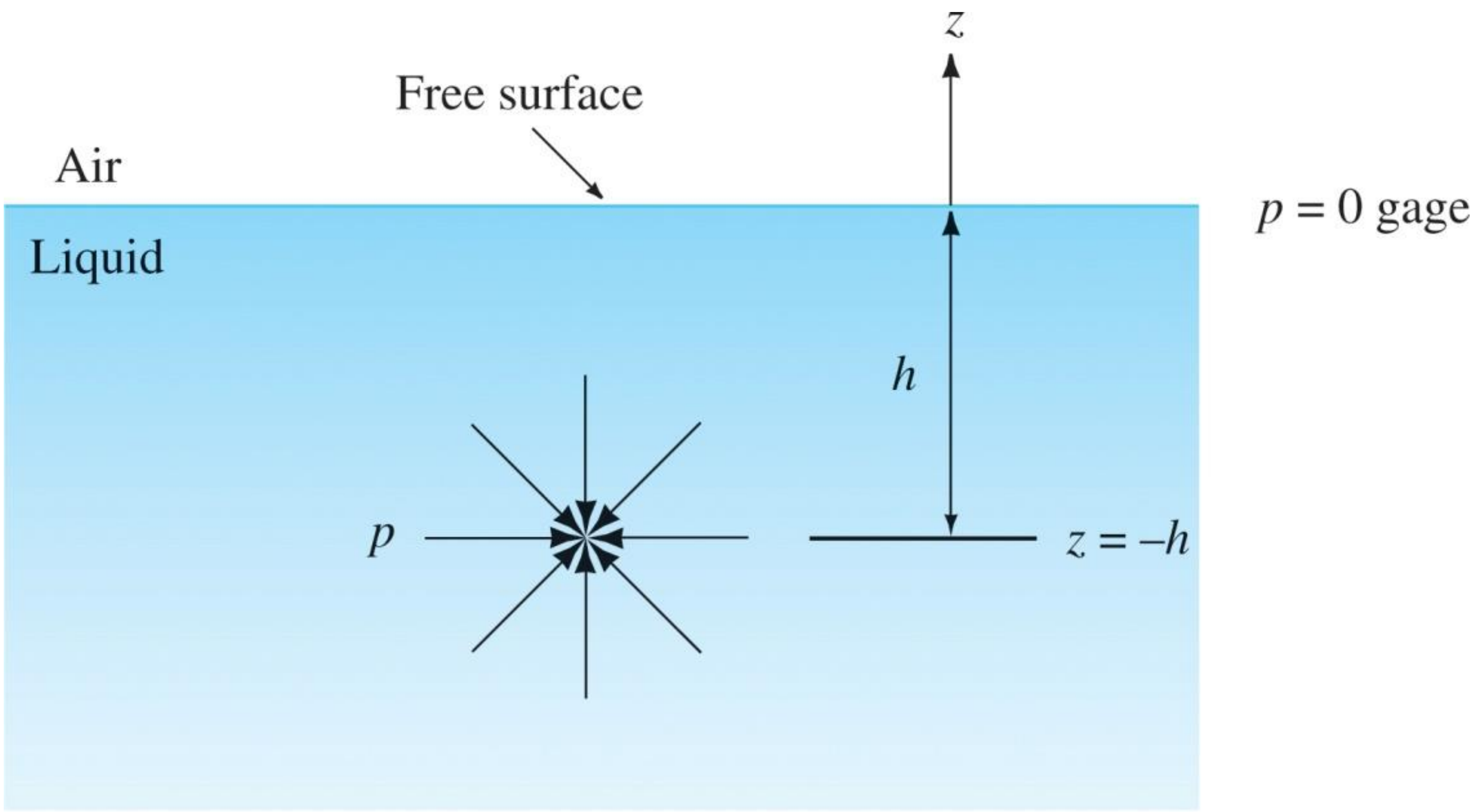


Fig. 2.4 Pressure below a free surface.

2.4 FLUIDS AT REST

2.4.3 Manometers

Manometers are instruments that use columns of liquid to measure pressures.

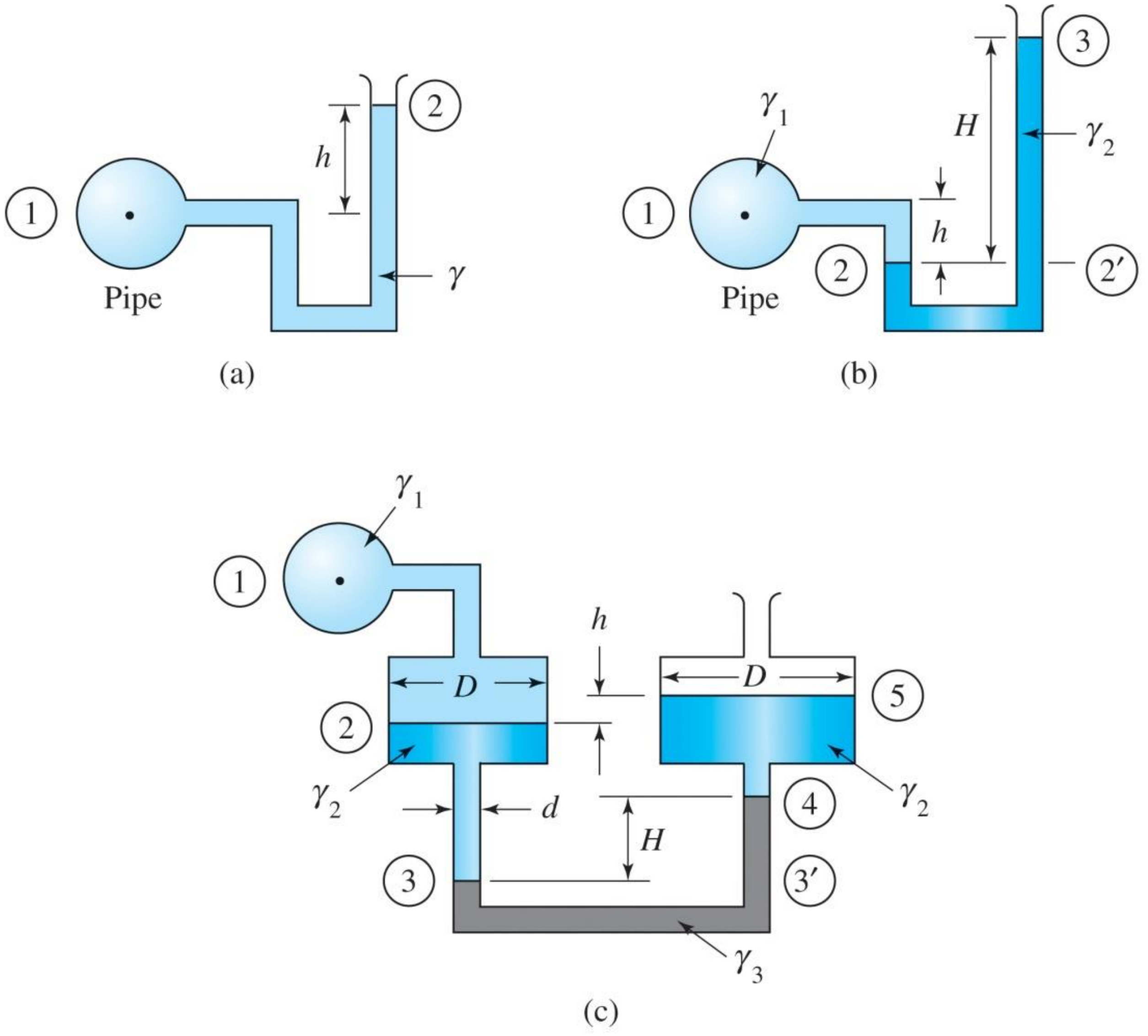


Fig. 2.7 Manometers: (a) U-tube manometer (small pressures); (b) U-tube manometer (large pressures); (c) micromanometer (very small pressure changes).

- (a) displays a U-tube manometer used to measure **relatively small pressures**
- (b): **Large pressures** can be measured using a liquid with large γ_2.
- (c): **Very small pressures** can be measured as small pressure changes in p_1, leading to a relatively large deflection H.

Example: **P.2.40**. Find the gage pressure in the water pipe shown in Fig. P2.40

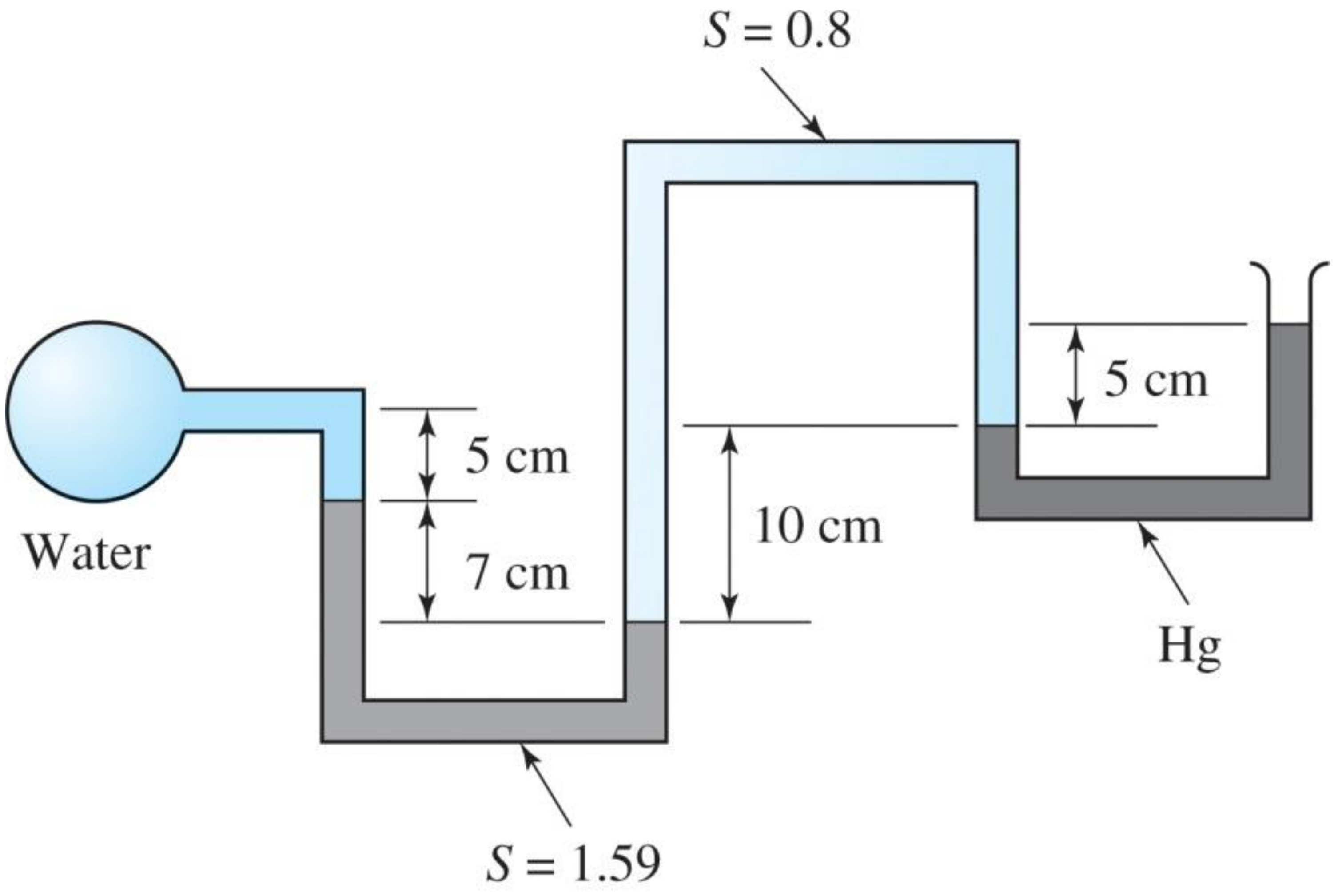


Fig. P2.40

Example: **P.2.41**. For the inclined manometer containing mercury, shown in Fig. P2.41, determine the pressure in pipe *B* if the pressure in pipe *A* is 10 kPa. Pipe *A* has water flowing through it, and oil is flowing in pipe *B*.

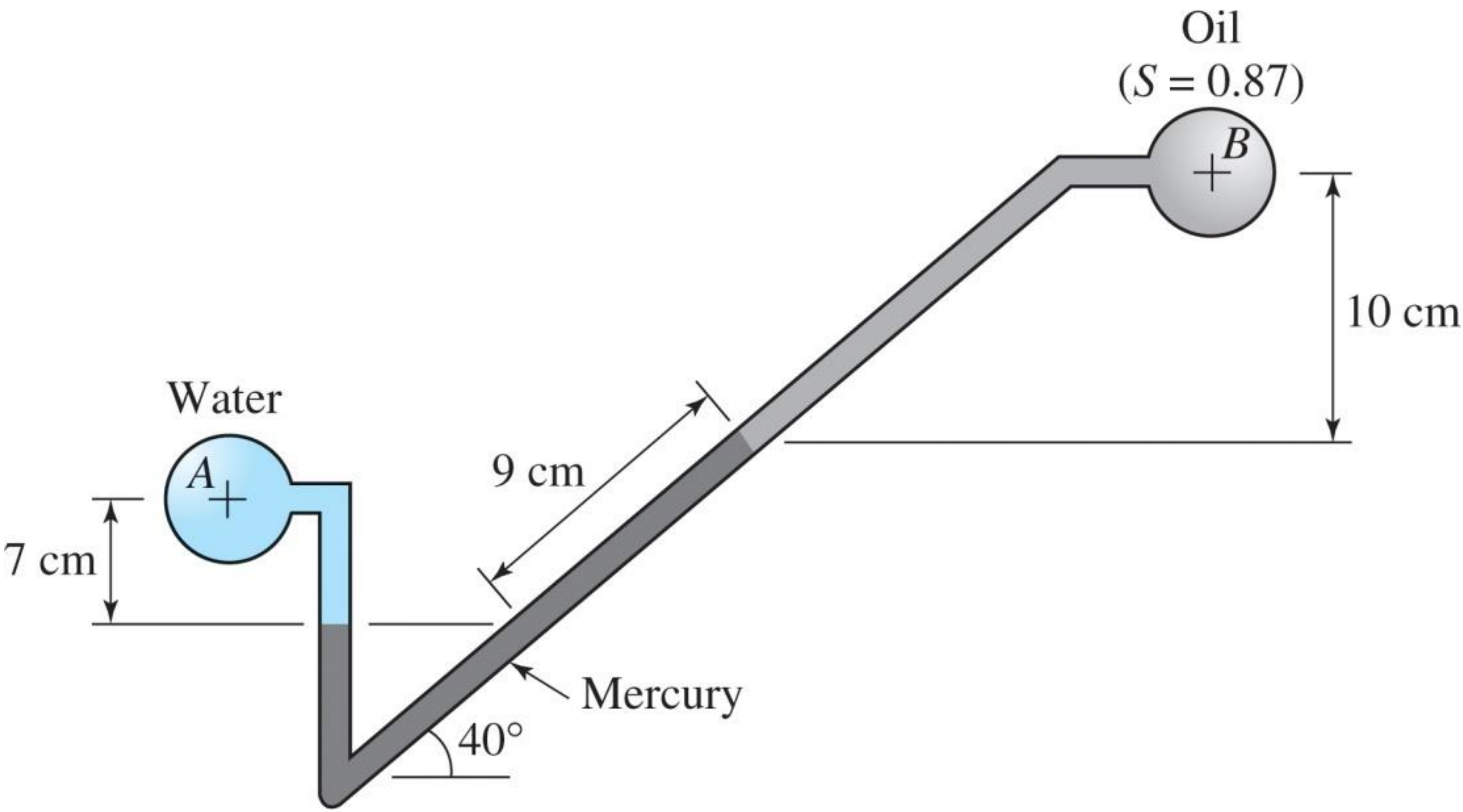


Fig. P2.41

Example: **P.2.42**. The pressure in pipe B in Problem P2.41 is reduced slightly. Determine the new pressure in pipe B if the pressure in pipe A remains the same and the reading along the inclined leg of the manometer is 11 cm (**Tip: See problems 2.41 and 2.42)**

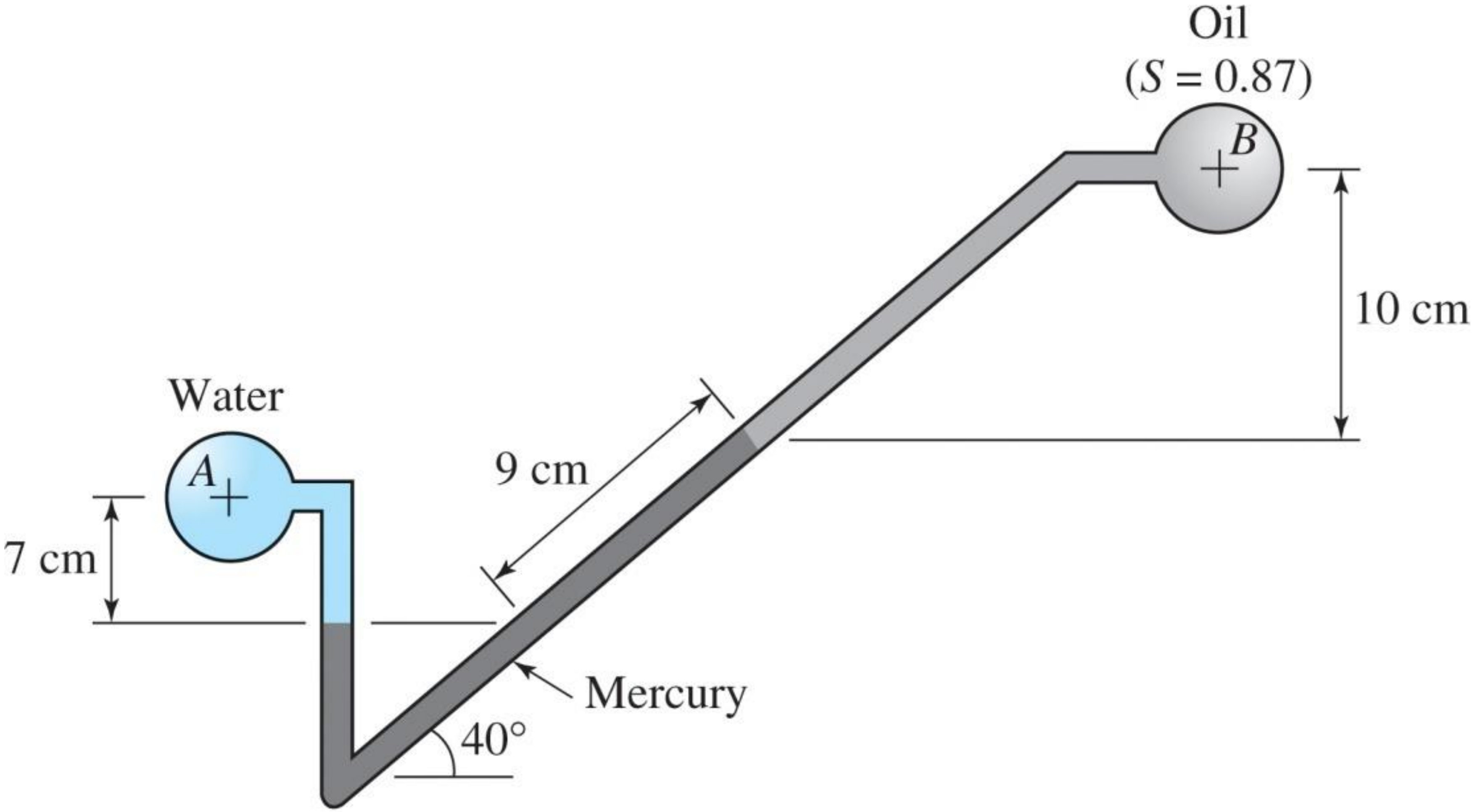


Fig. P2.41

2.4.4 Forces on Plane Areas

- The total force of a liquid on a plane surface is:

- After knowing the equation for pressure ($P = \gamma h$):

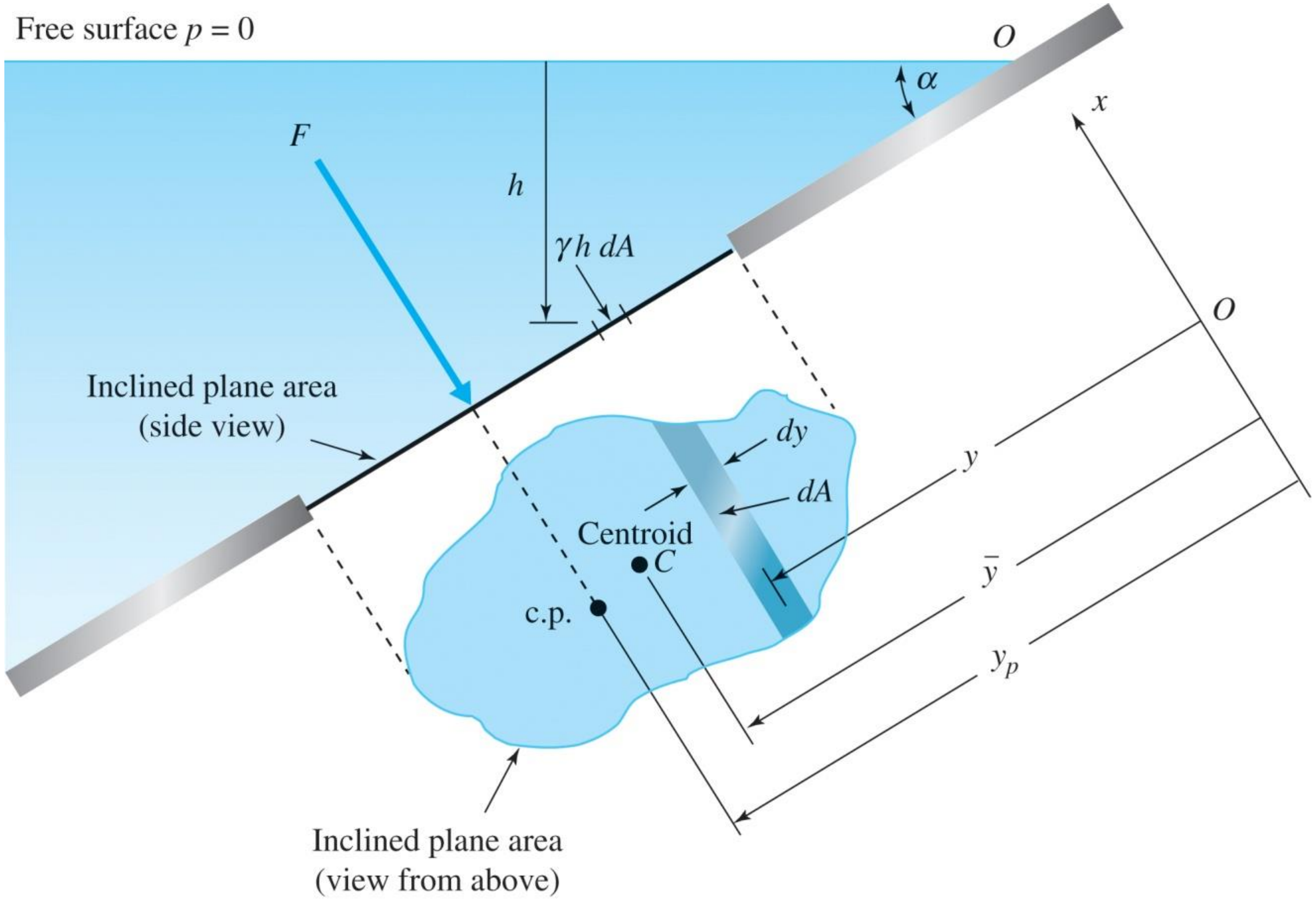


Fig. 2.8 Force on an inclined plane area.

2.4.4 Forces on Plane Areas

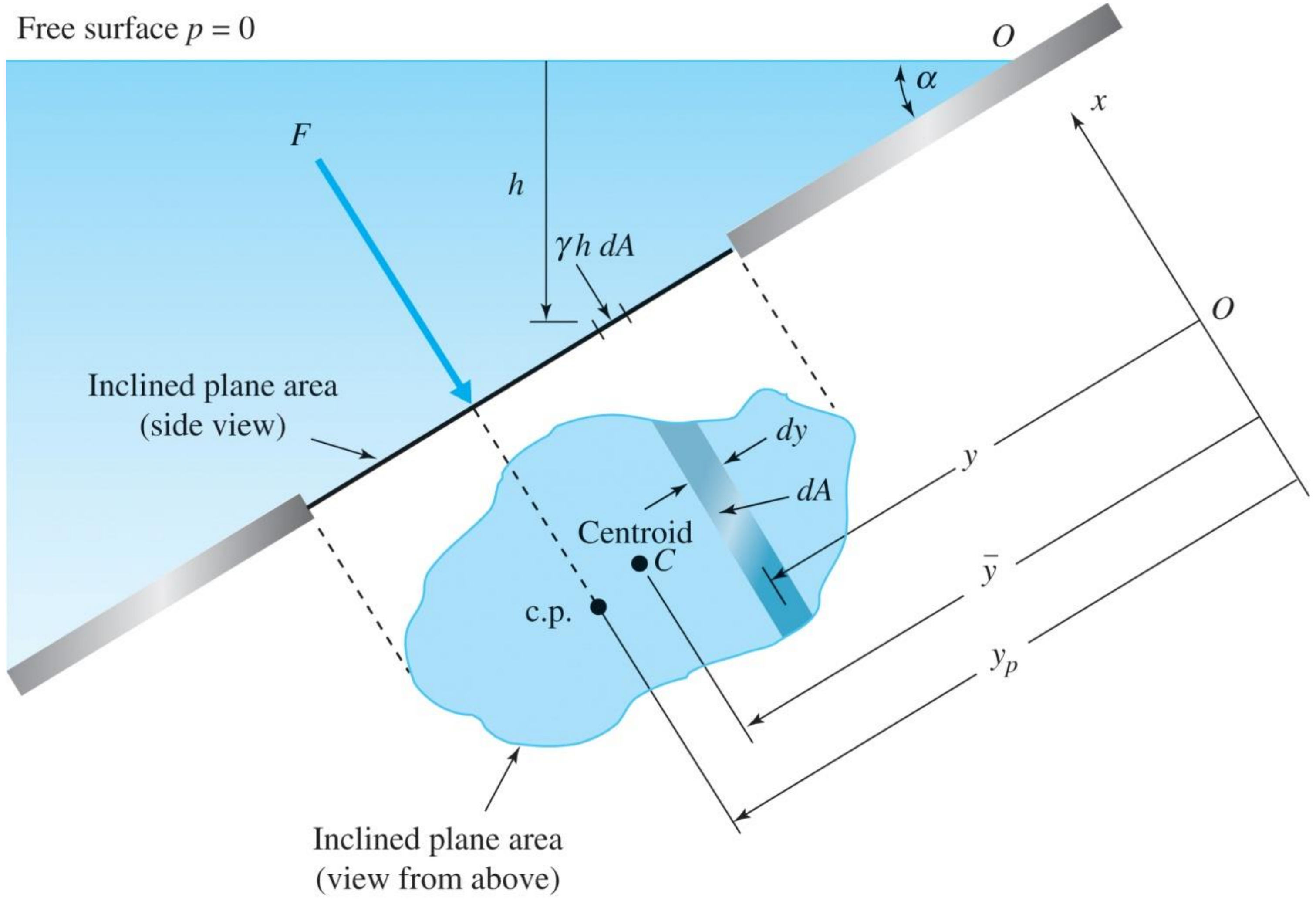


Fig. 2.8 Force on an inclined plane area.

$\bar{h}$: Vertical distance from the free surface to the centroid of the area

p_C: Pressure at the centroid

The **centroid** or geometric center of a plane figure is the arithmetic mean ("average") position of all the points in the shape.

2.4 FLUIDS AT REST

- The center of pressure is the point where the resultant force acts:
    - Sum of moments of all infinitesimal pressure forces on an area, A, equals the moment of the resultant force.

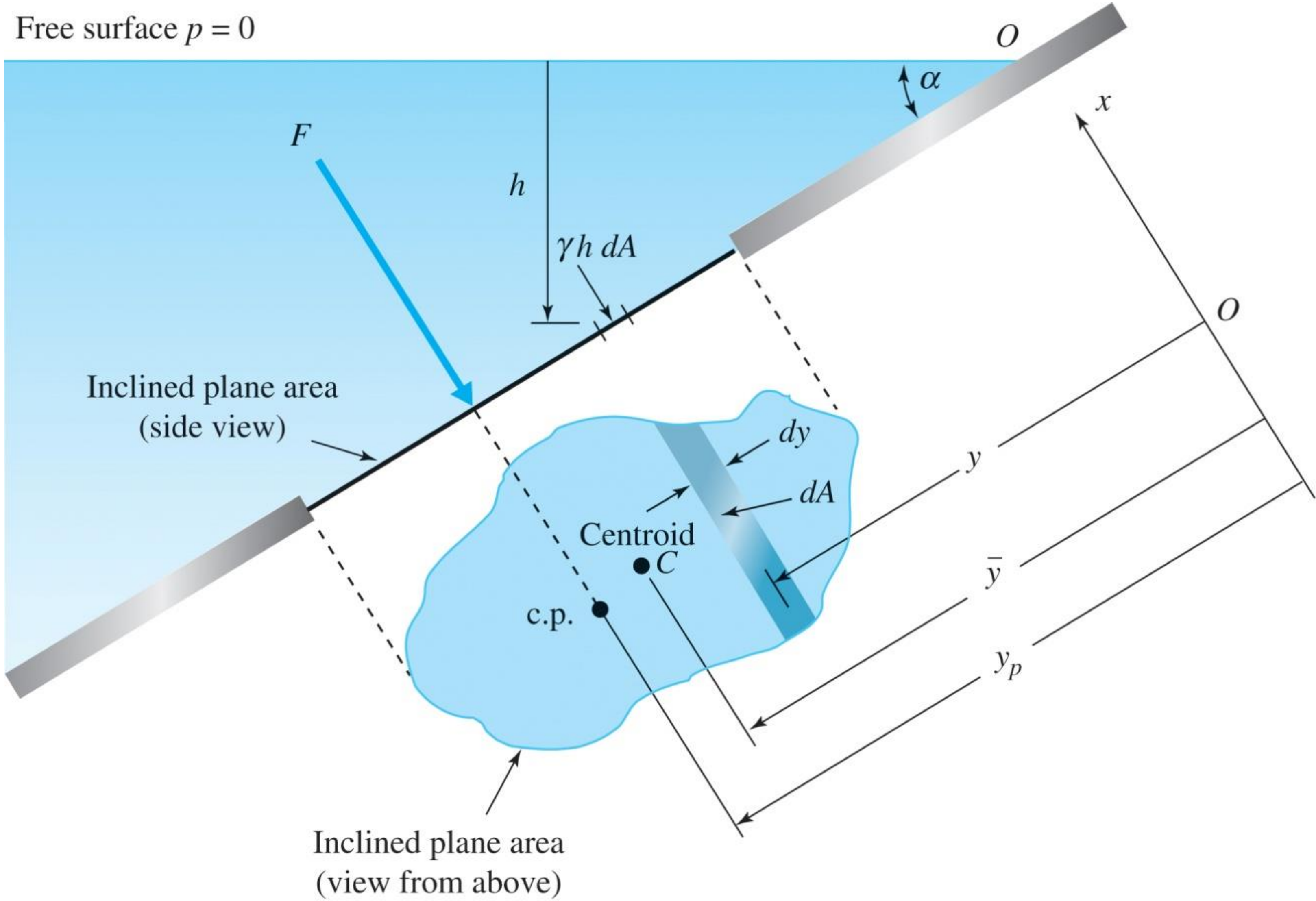


Fig. 2.8 Force on an inclined plane area.

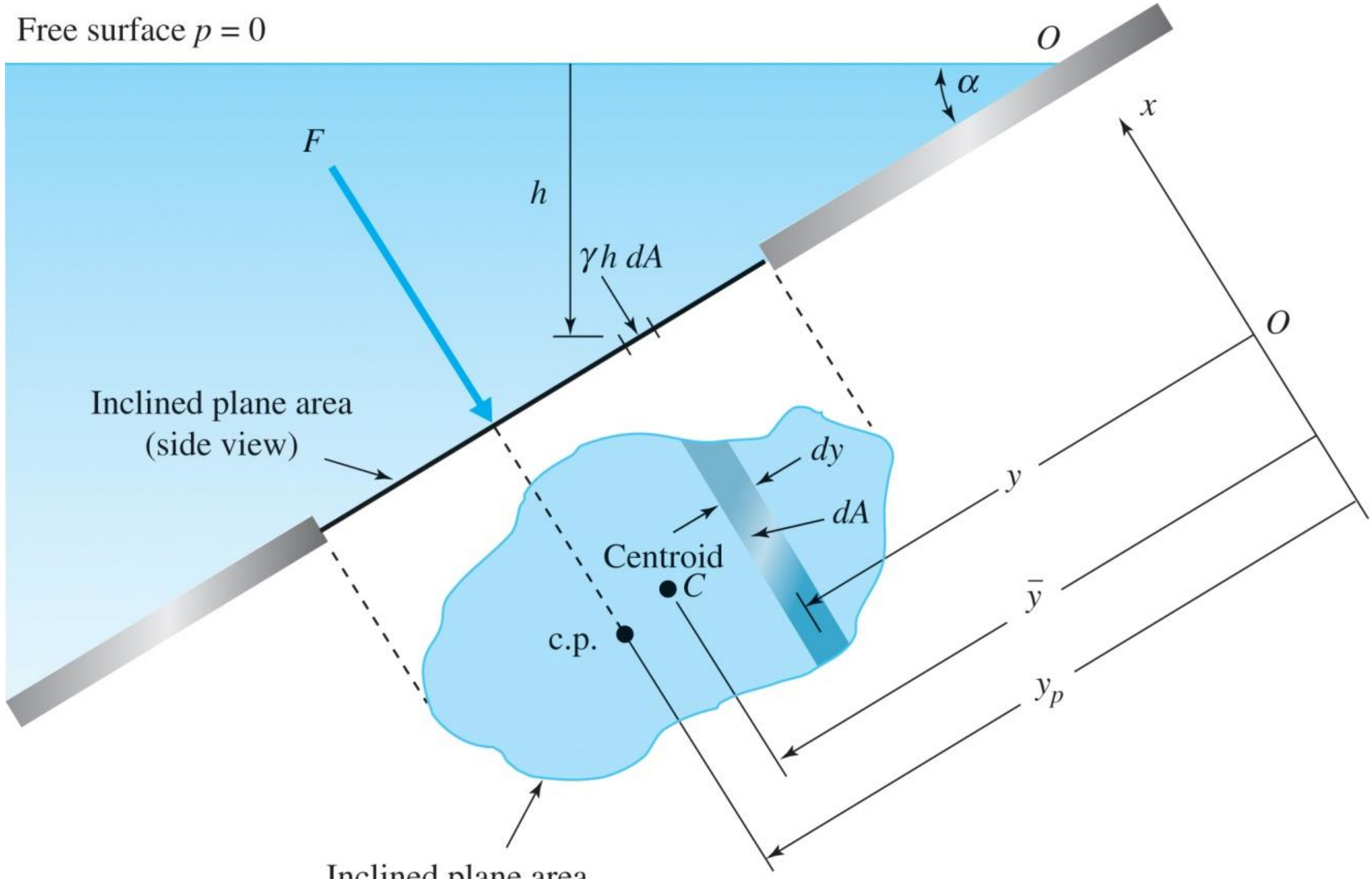


Fig. 2.8 Force on an inclined plane area.

$\bar{y}$: Measured parallel to the plane area to the free surface

- The moments of area can be found using:

See Appendix C for centroids and moments

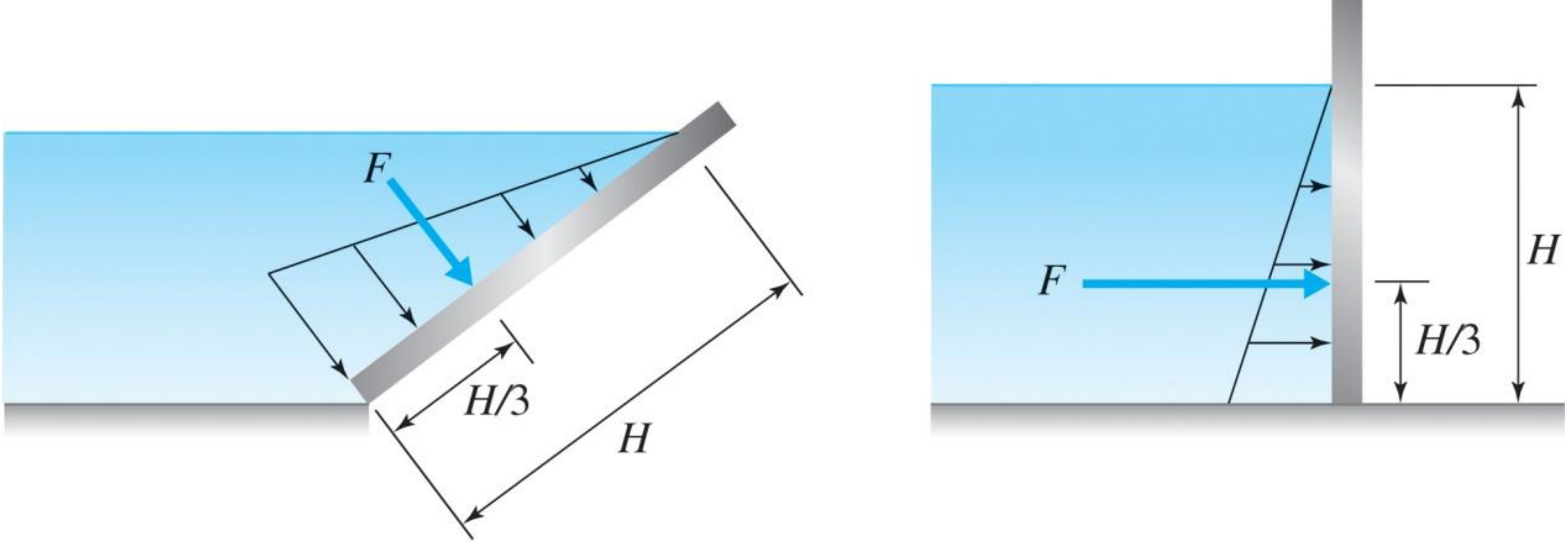


Fig. 2.9 Force on a plane area with top edge in a free surface.

Example: **P.2.56**. Determine the force *P* needed to hold the 4-m wide gate in the position shown in Fig. P2.56.

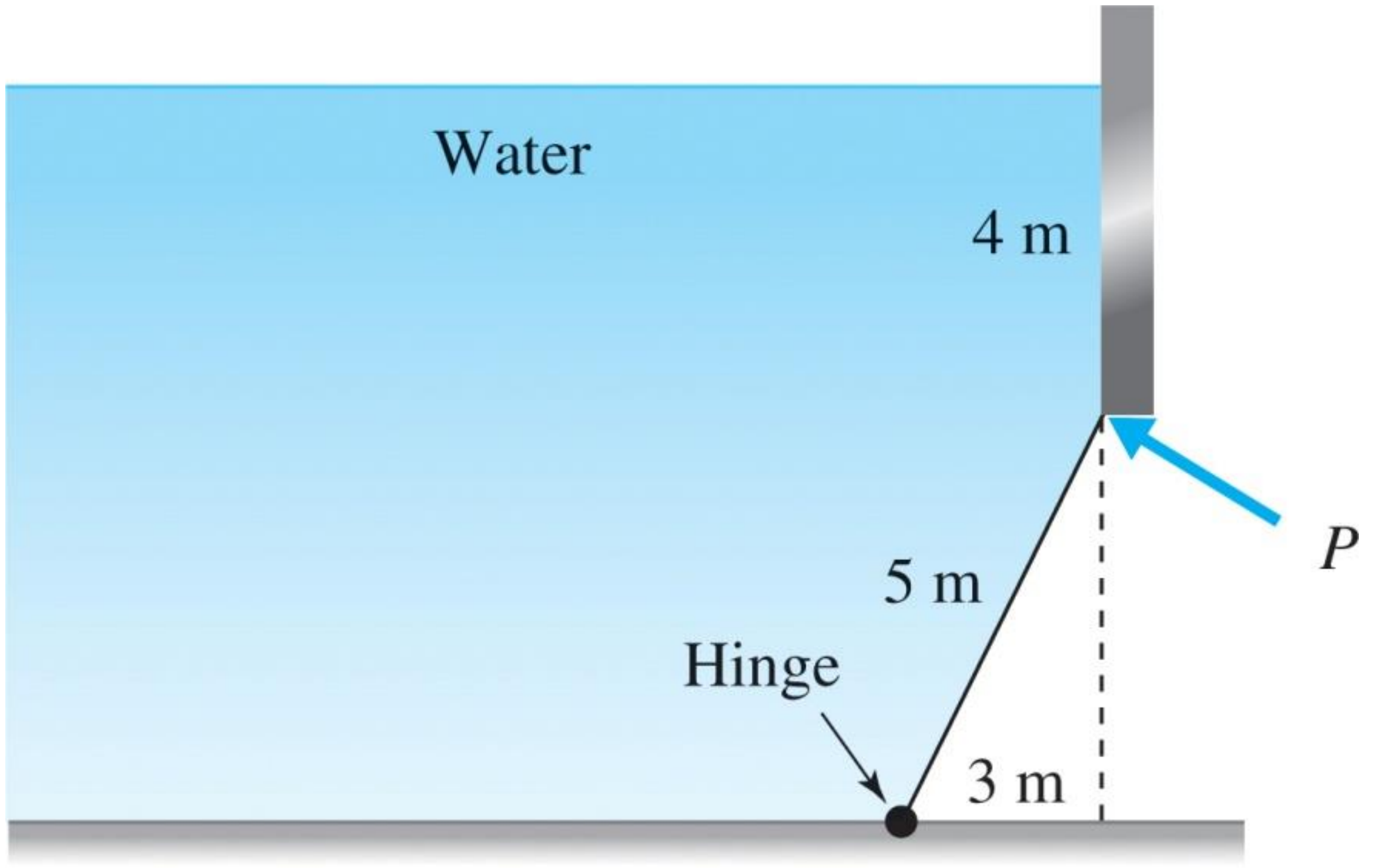


Fig. P2.56

Example: **P.2.62**. At what height H will the rigid gate, hinged at a central point as shown in Fig. P2.62, open if h is:

a) 0.6 m? b) 0.8 m? c) 1.0 m?

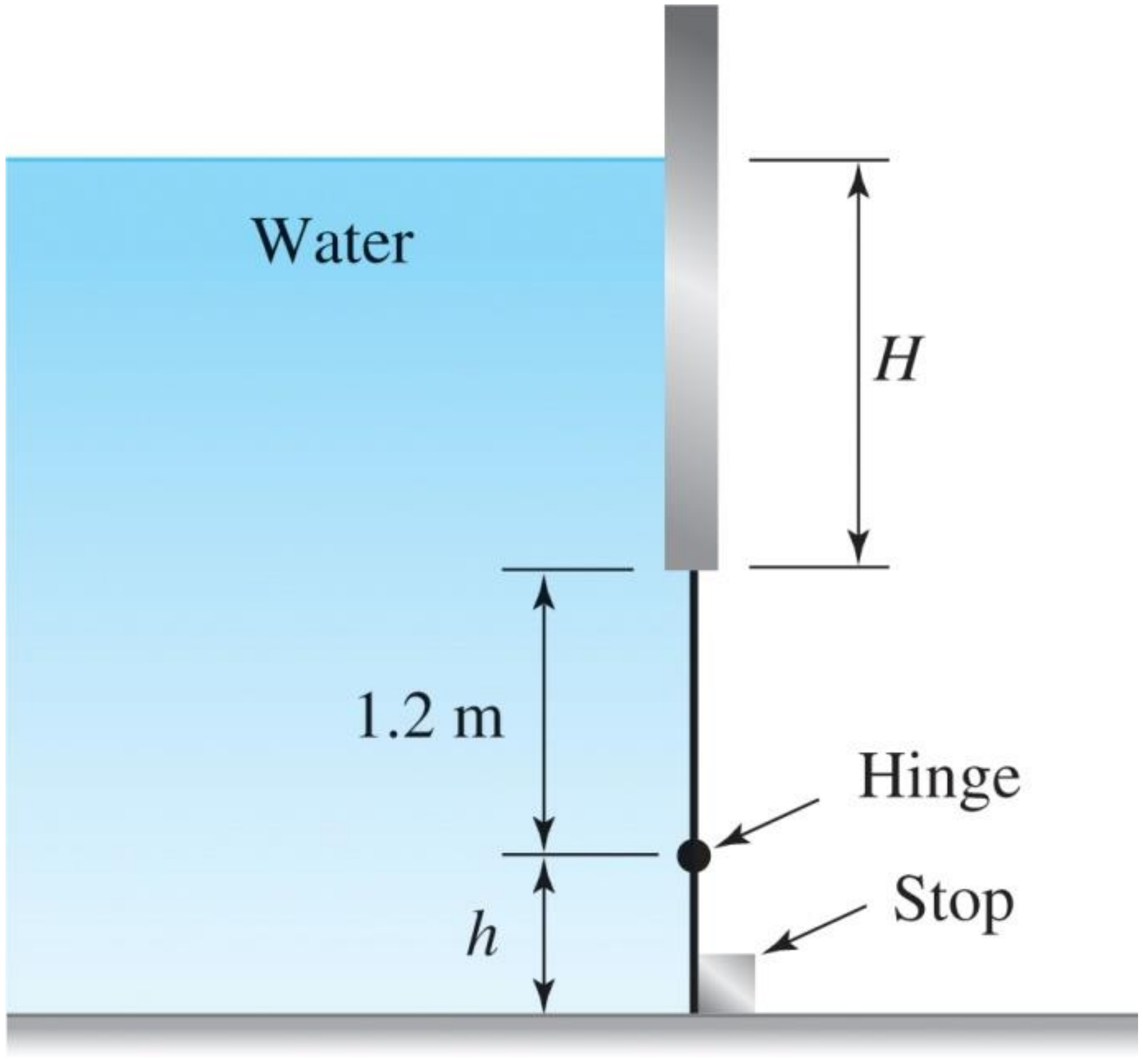


Fig. P2.62

2.4.5 Forces on Curved Surfaces

https://www.youtube.com/watch?v=zV-JO-l7Mx4

- Direct integration cannot find the force due to the hydrostatic pressure on a curved surface.
- A free-body diagram containing the curved surface and surrounding liquid needs to be identified.

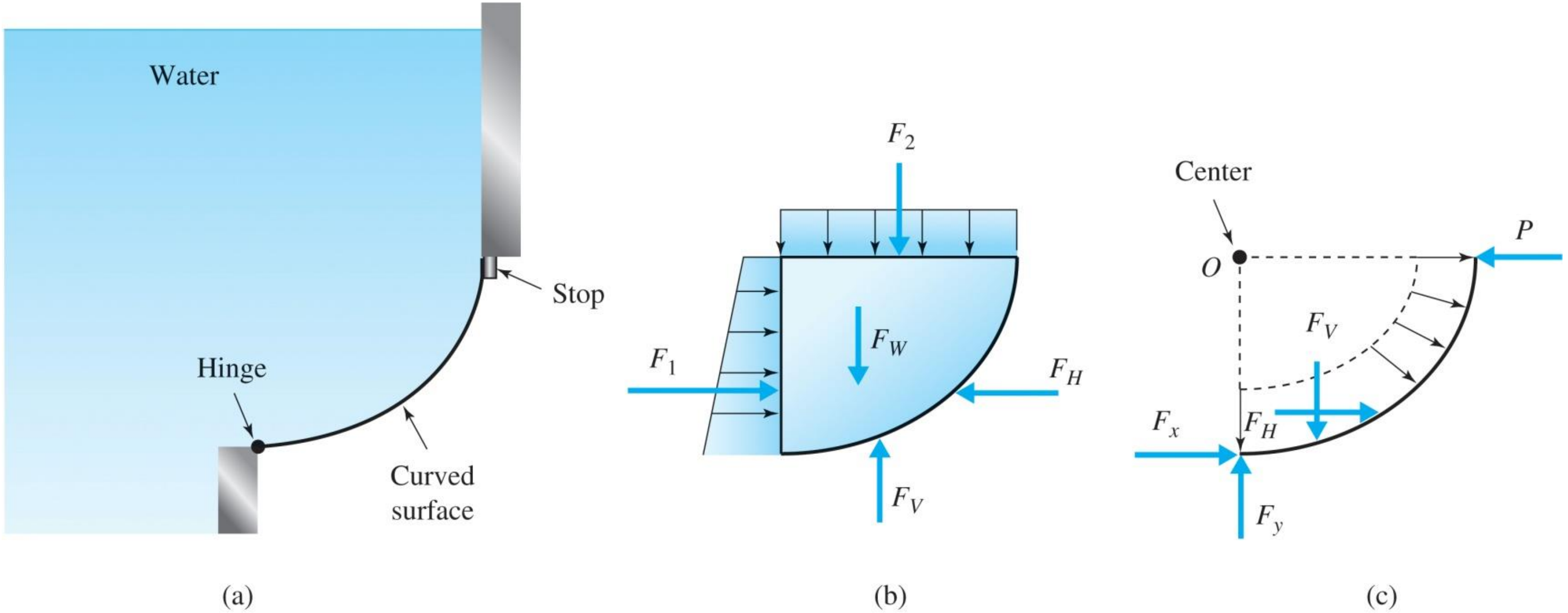


Fig. 2.11 Forces acting on a curved surface: (a) curved surface; (b) free-body diagram of water and gate; (c) free-body diagram of gate only.

Example: P.2.72. Find the force *P* required to hold the gate in the position shown in Fig. P.2.72 . The gate is 5-m wide.

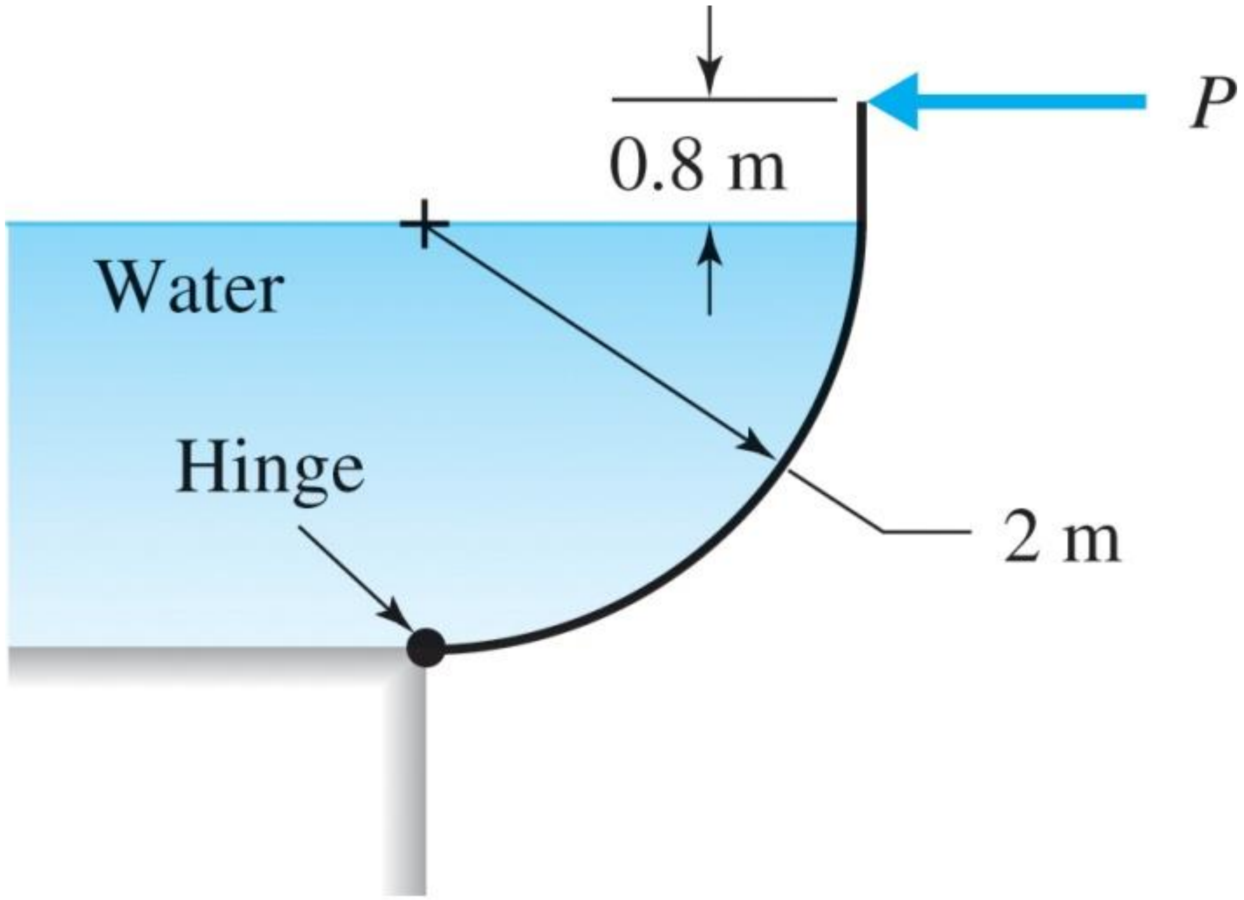


Fig. P2.72

Example: **P.2.77**. Find the force P if the parabolic gate shown in Fig. P.2.77 is

a) 2-m wide and $H = 2$ m
b) 4-ft wide and $H = 8$ ft.

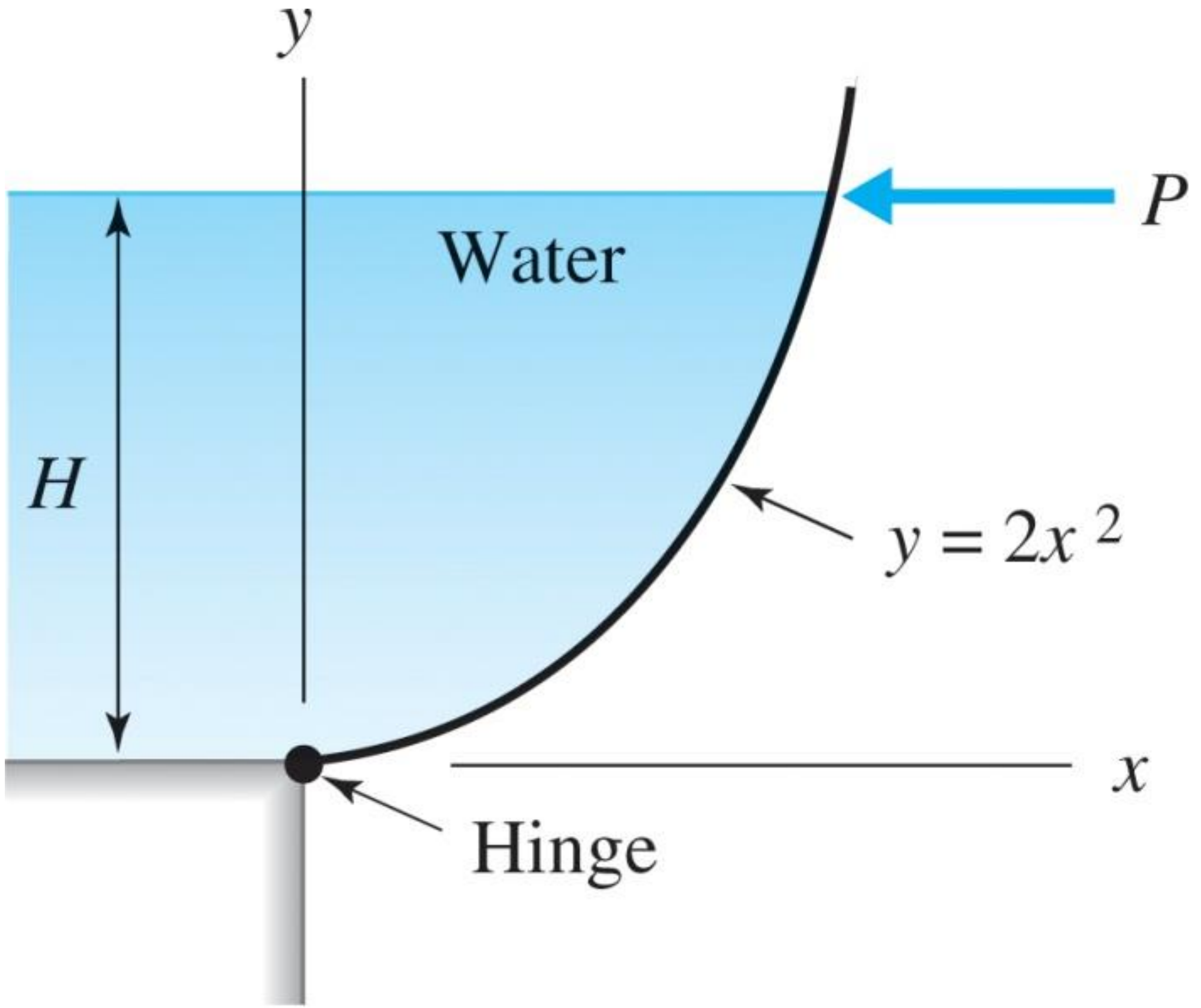


Fig. P2.77

2.4.6 Buoyancy (Archimedes' principle)

https://www.youtube.com/watch?v=2RefIvqaYg8

- **Buoyancy force on an object equals the weight of displaced liquid.**

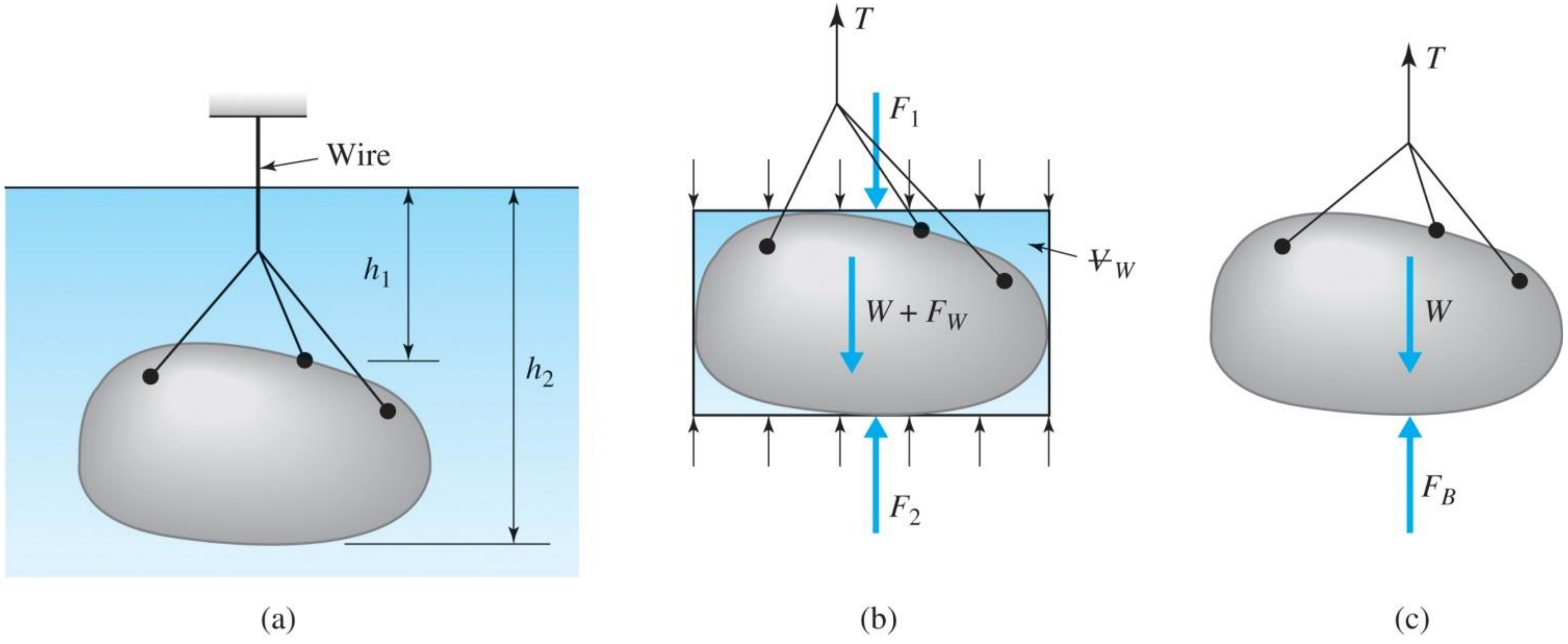


Fig. 2.12 Forces on a submerged body: (a) submerged body; (b) free-body diagram; (c) free body showing the buoyant force F_B.

V is the volume of displaced fluid and *W* is the weight of the floating object.

2.4.6 Buoyancy (Archimedes' principle)

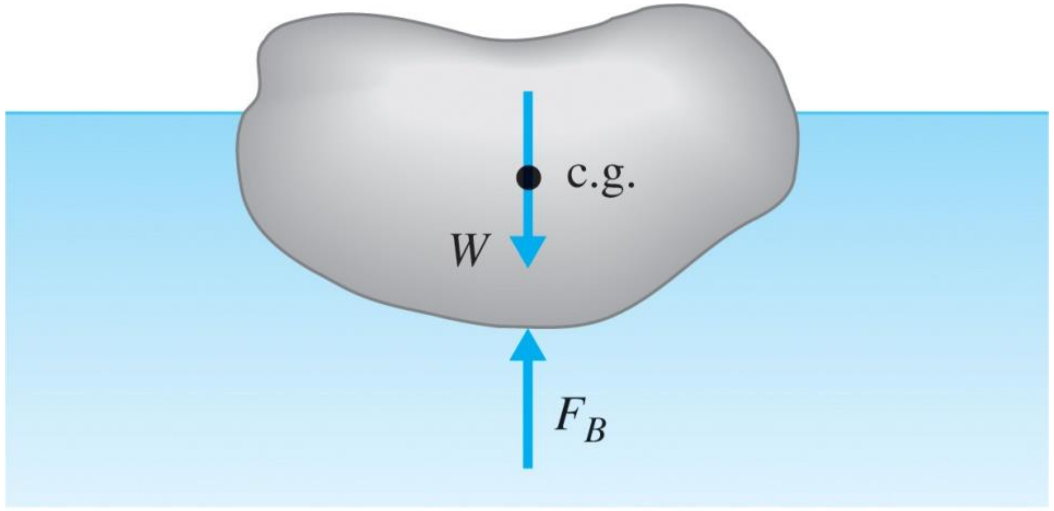


Fig. 2.13 Forces on a floating object.

- The buoyant force acts through the centroid of the displaced liquid volume.
- An application of this would be a hydrometer that is used to measure the specific gravity of liquids.
  - For pure water, this is 1.0

2.4 FLUIDS AT REST

2.4.6 Buoyancy (Hydrometers)

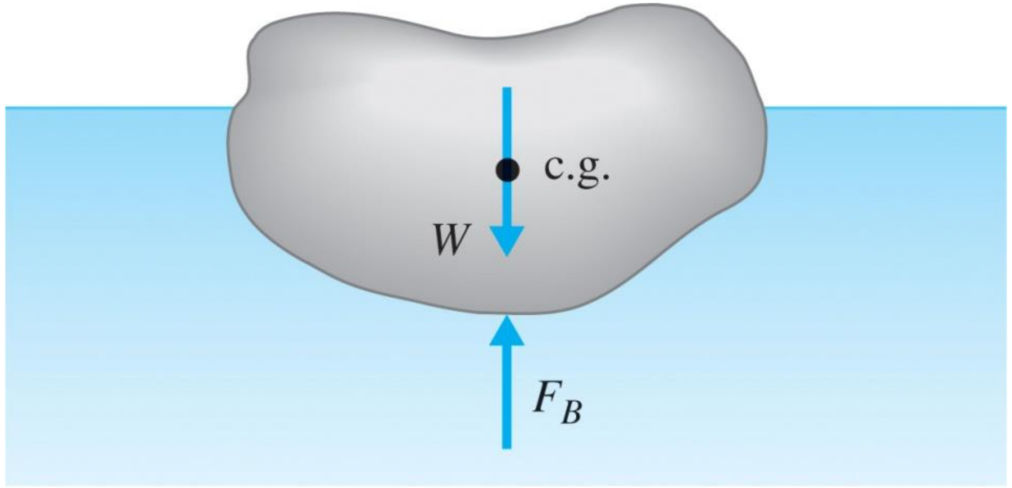


Fig. 2.13 Forces on a floating object.

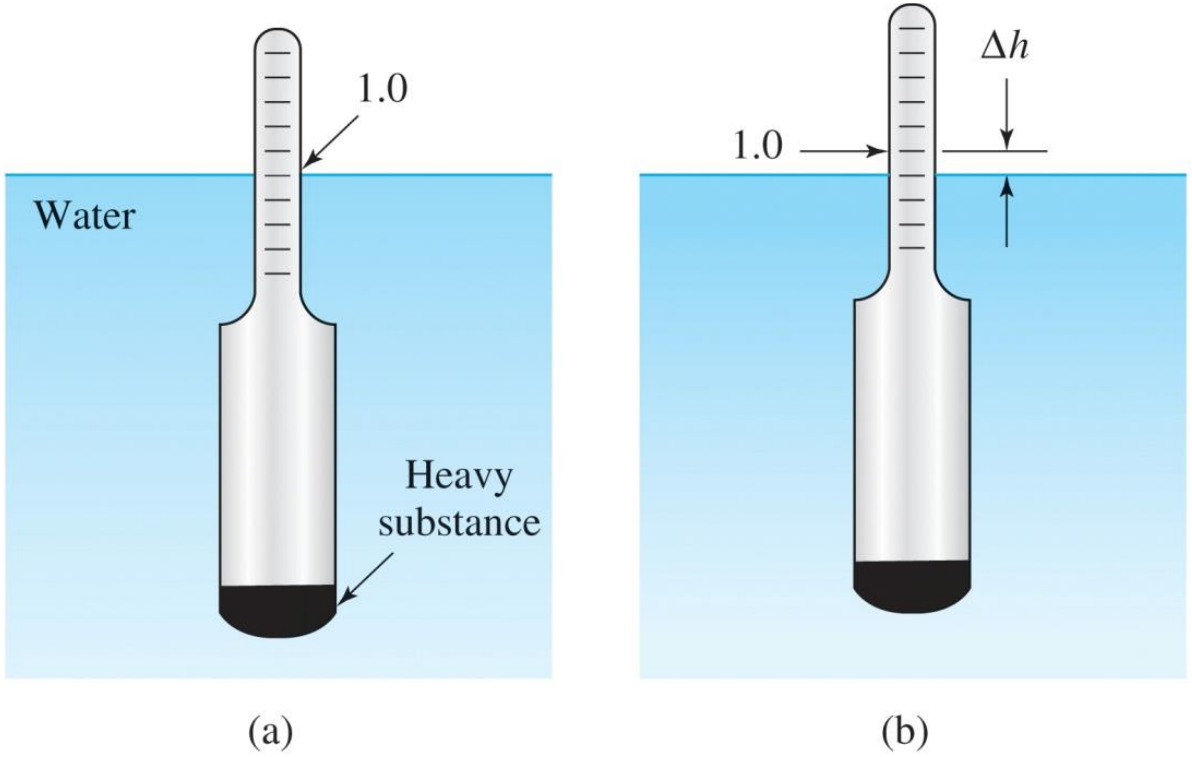


Fig. 2.14 Hydrometer: (a) in water; (b) in an unknown liquid.

- Where Δh is the displaced height
- A: Cross-sectional area of the stem
- $S_x = \frac{\gamma_x}{\gamma_{water}}$
- For a given hydrometer, $\forall$ and A are fixed.

Example: **P.2.78**. The 3-m wide barge shown in Fig. P.2.78 weighs 20 kN empty. It is proposed that it carry a 250-kN load. Predict the draft in:

a) Fresh water
b) Salt water (S = 1.03)

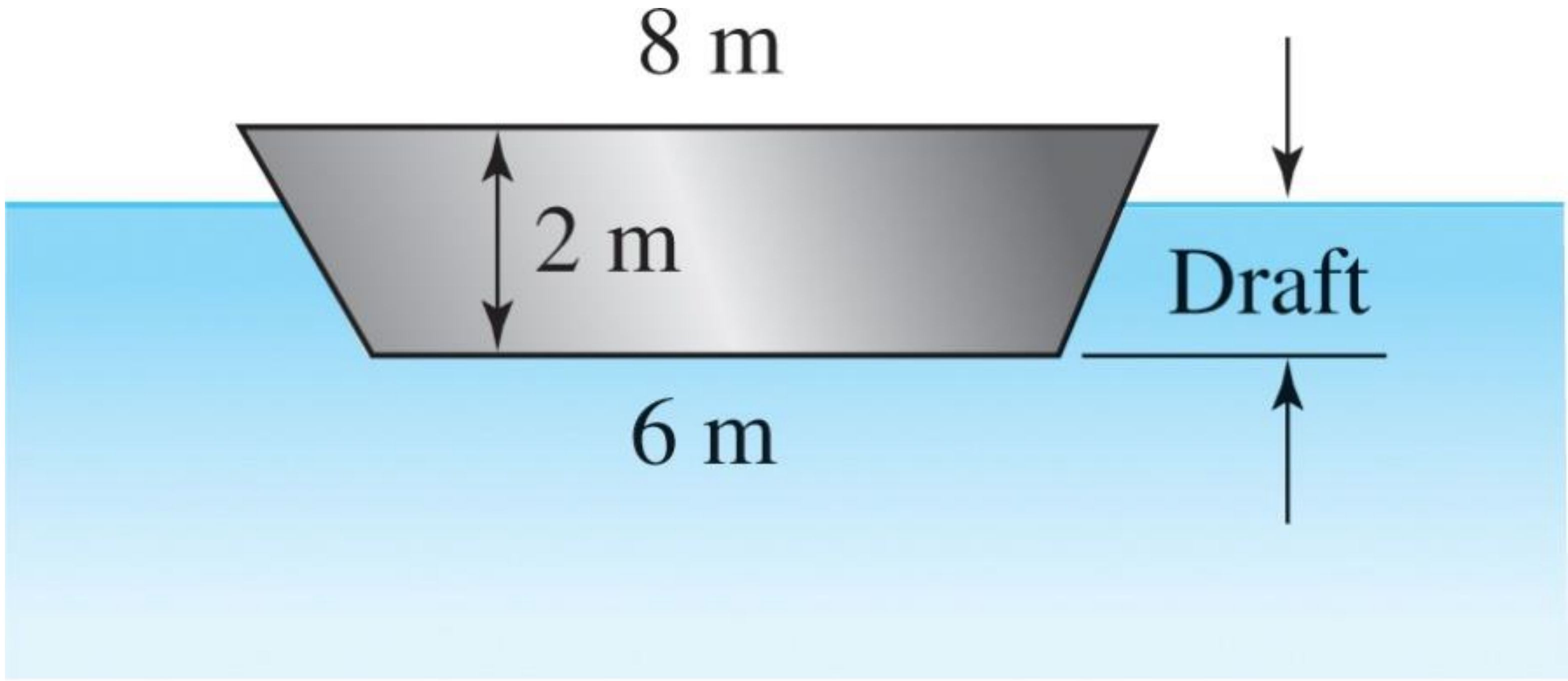


Fig. P2.78

2.4.7 Stability

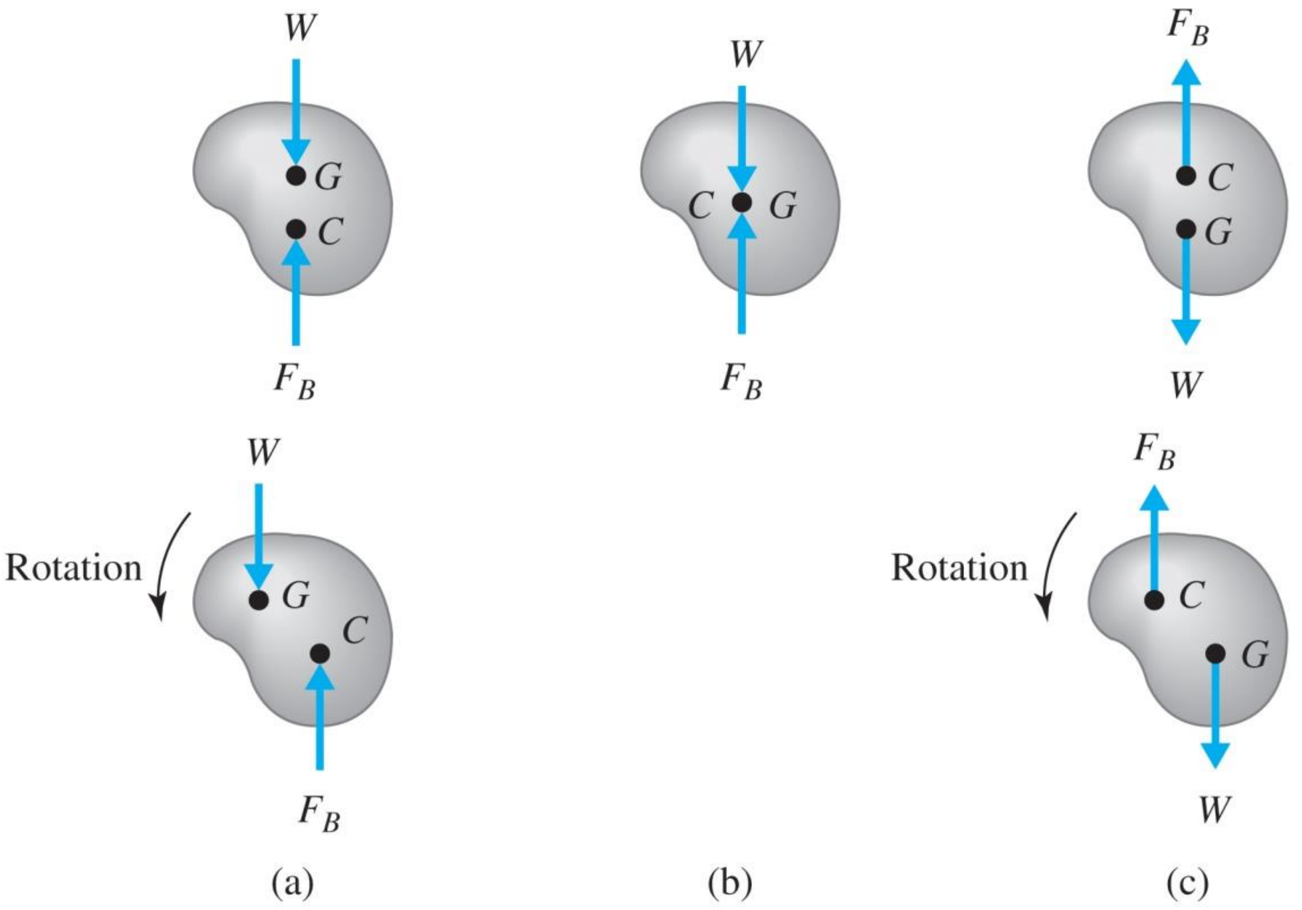


Fig. 2.15 Stability of a submerged body: (a) unstable; (b) neutral; (c) stable.

- In (a) the center of gravity of the body is above the centroid C (center of buoyancy), so a small angular rotation leads to a moment that increases rotation: unstable.
- (b) shows neutral stability as the center of gravity and the centroid coincide.
- In (c), as the center of gravity is below the centroid, a small angular rotation provides a restoring moment and the body is stable.

Metacentric height

https://www.youtube.com/watch?v=QUgXf2Rj2YQ

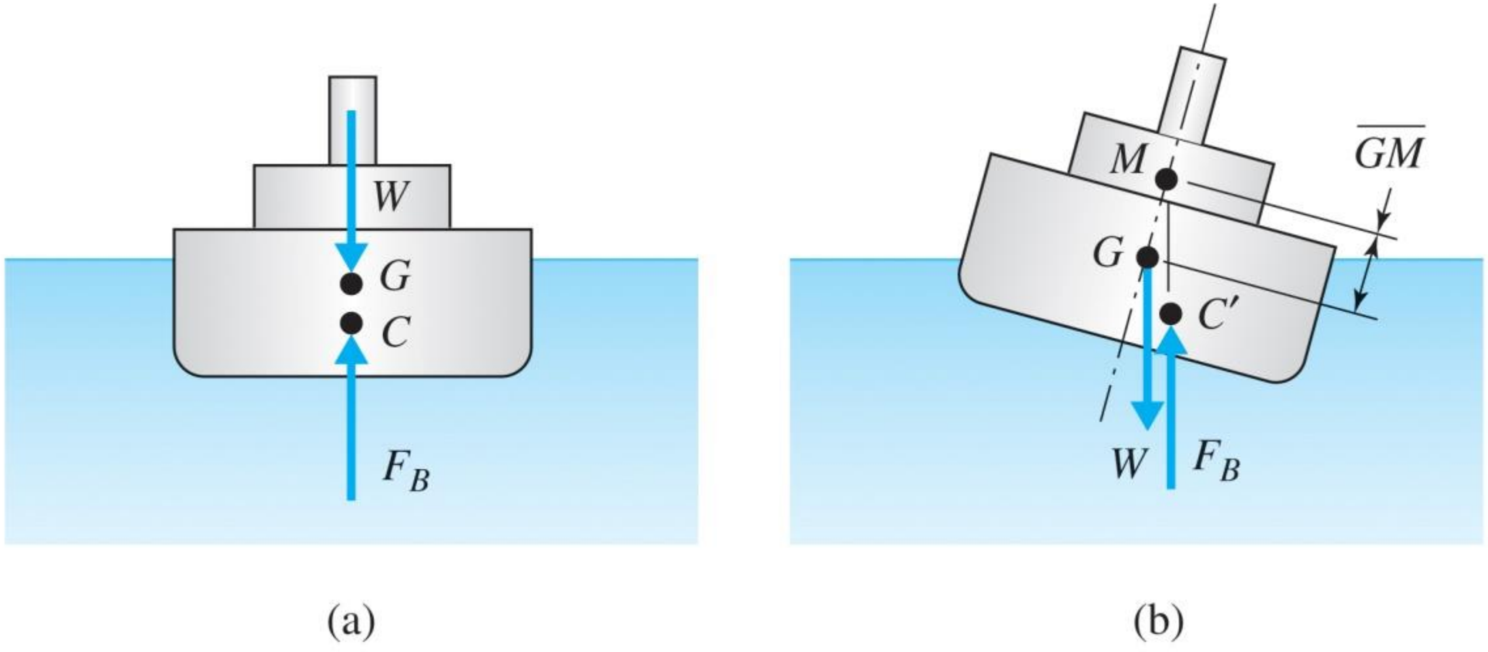


Fig. 2.16 Stability of a floating body: (a) equilibrium position; (b) rotated position.

- The **metacentric height $\overline{GM}$** is the distance from G to the point of intersection of the buoyant force before rotation with the buoyant force after rotation.
- If $\overline{GM}$ is positive: Stable
- If $\overline{GM}$ is negative: Unstable

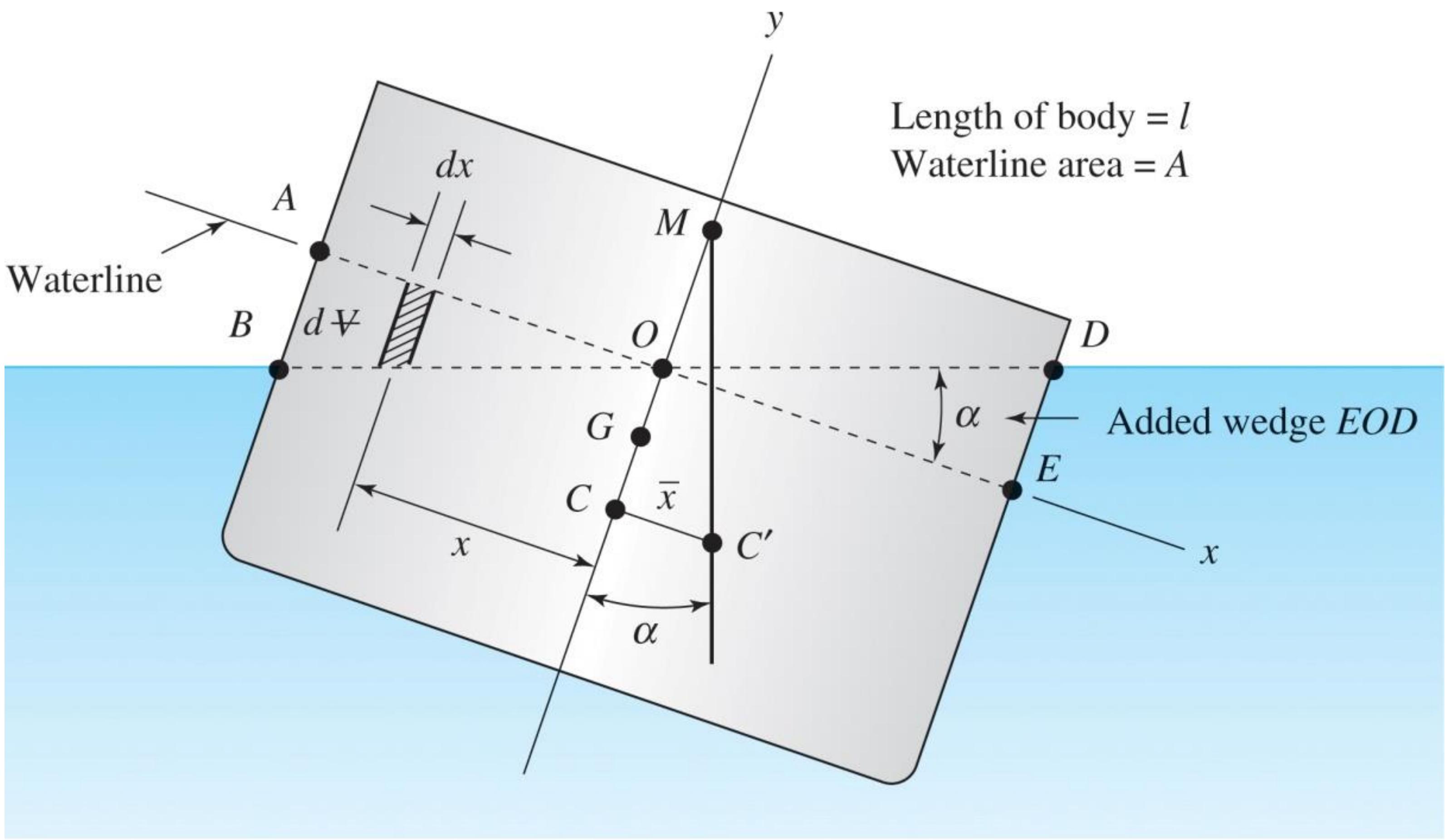


Fig. 2.17 Uniform cross section of a floating body.

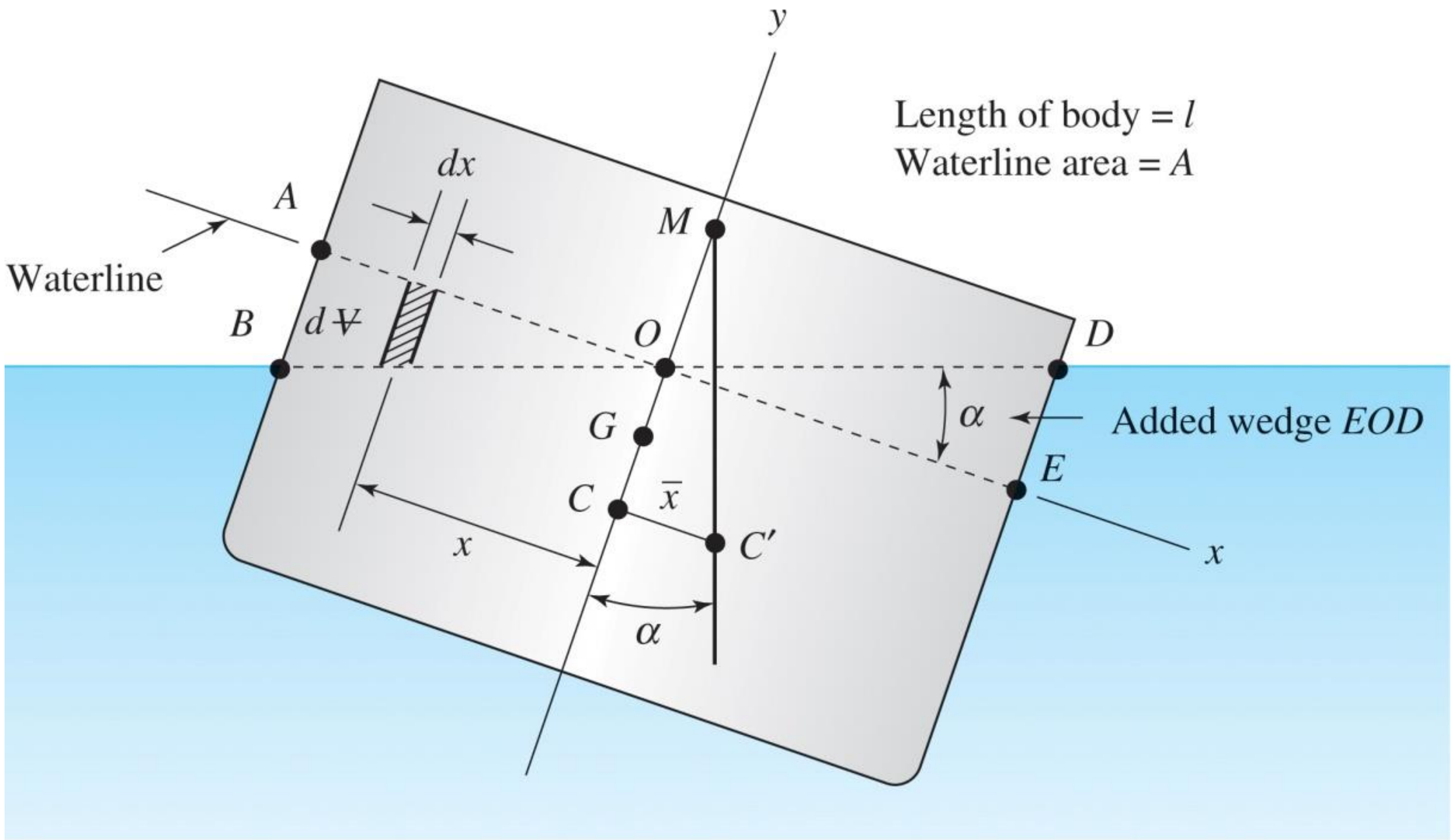


Fig. 2.17 Uniform cross section of a floating body.

Example: **P.2.94**. The barge shown in Fig. P2.94 is loaded such that the center of gravity of the barge and the load is at the waterline. Is the barge stable?

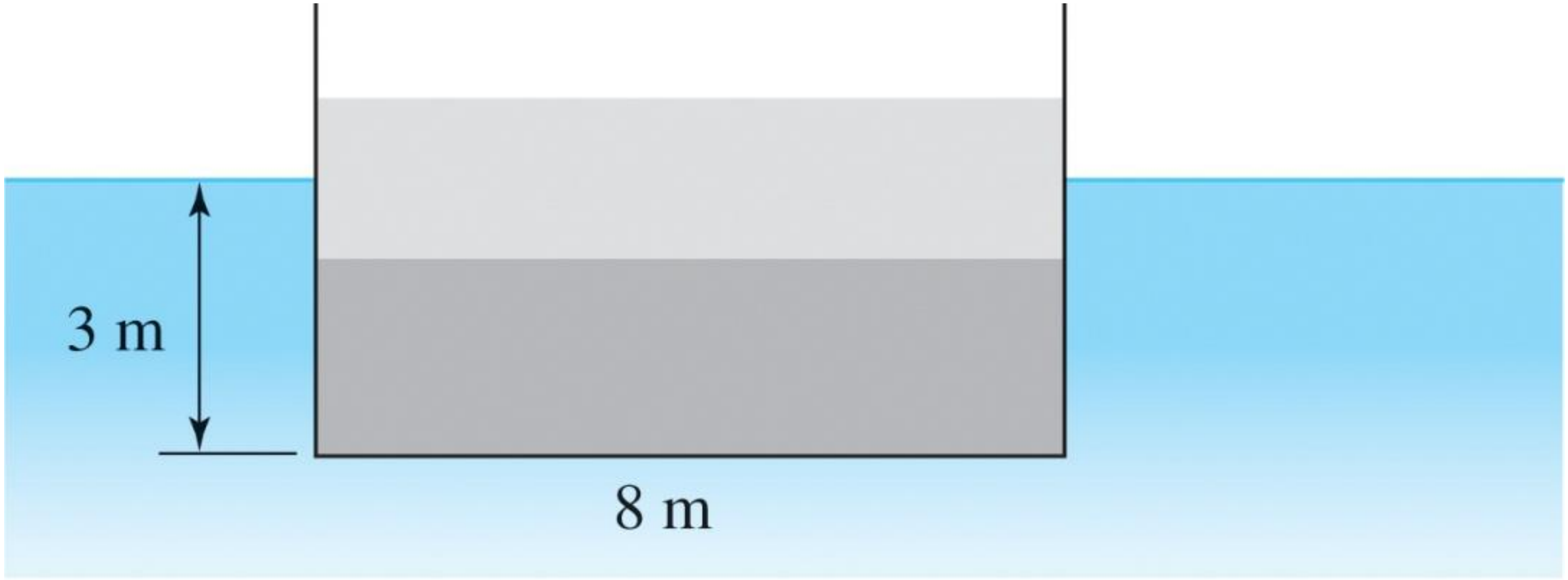


Fig. P2.94

Example: P.2.92. For the object shown in Fig. P2.92, calculate S_A for neutral stability when submerged.

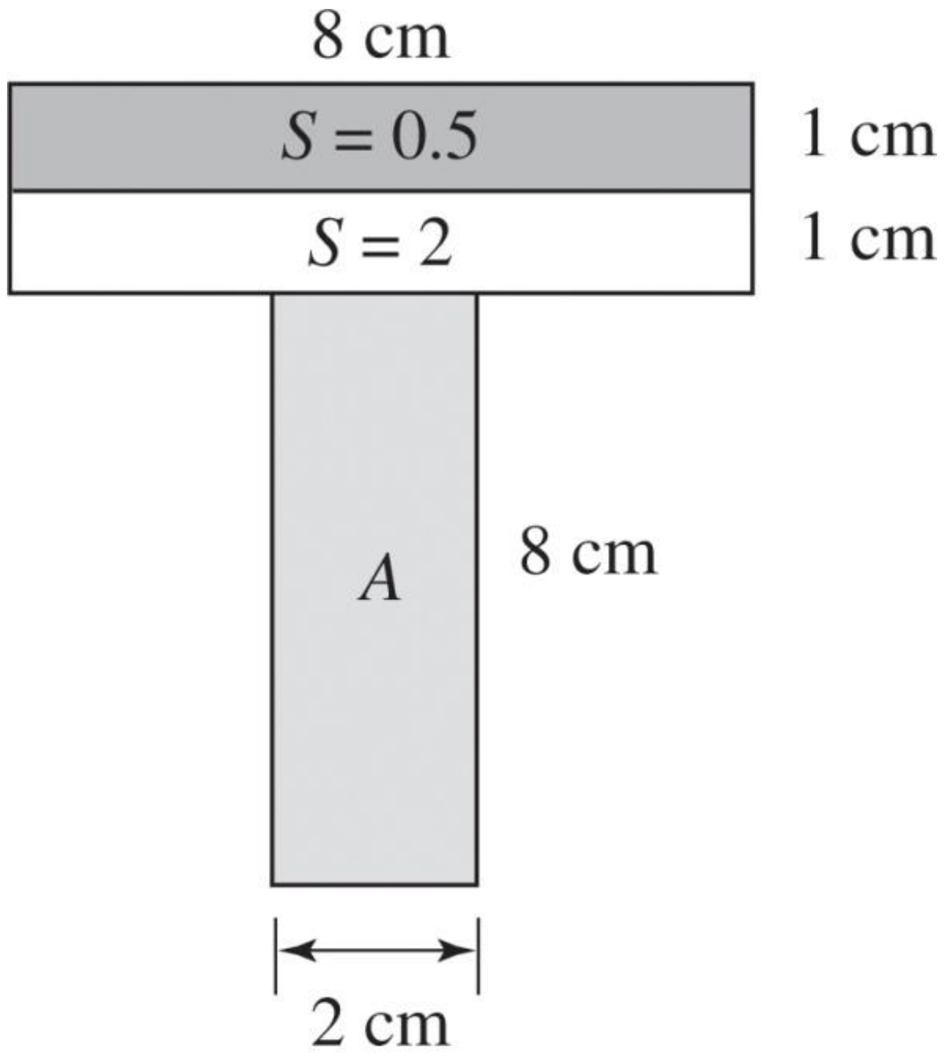
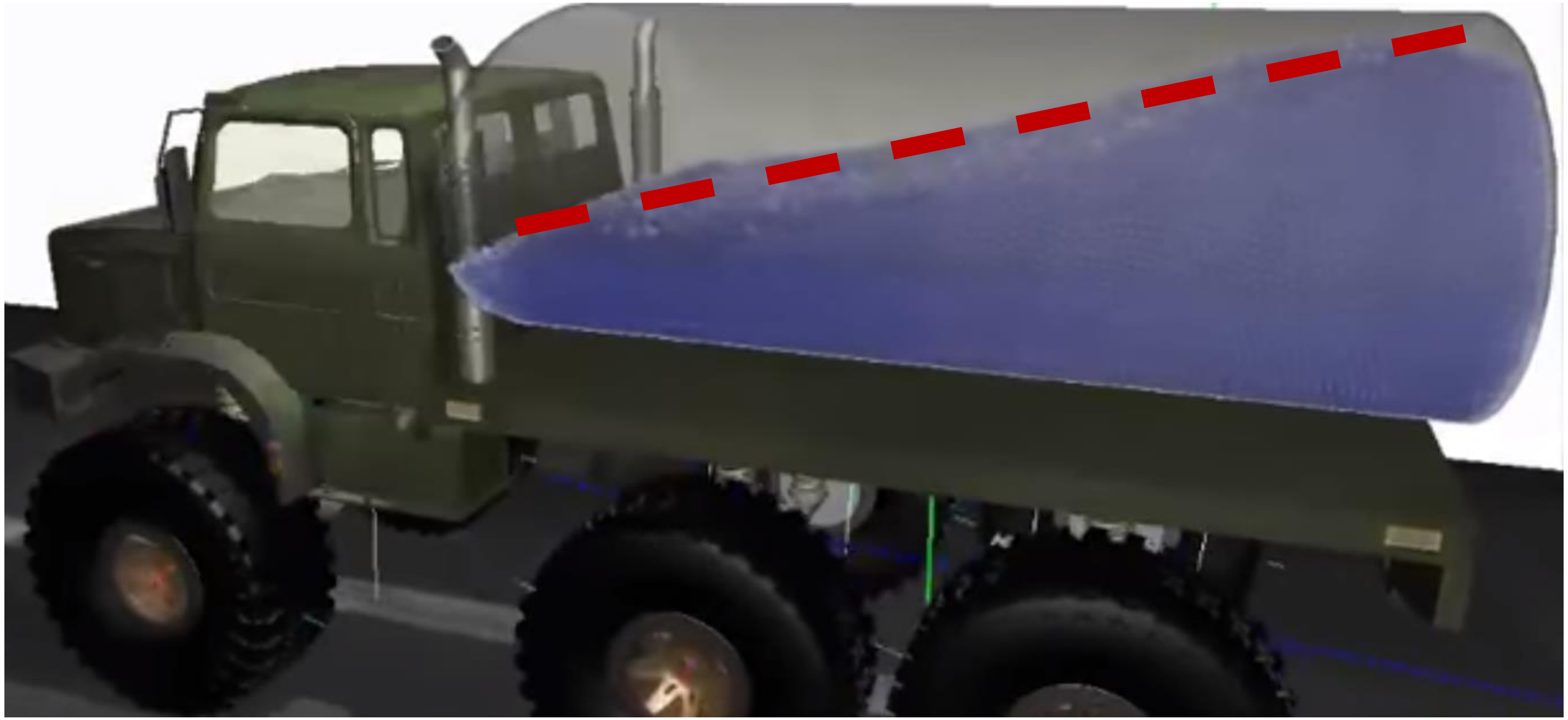


Fig. P2.92

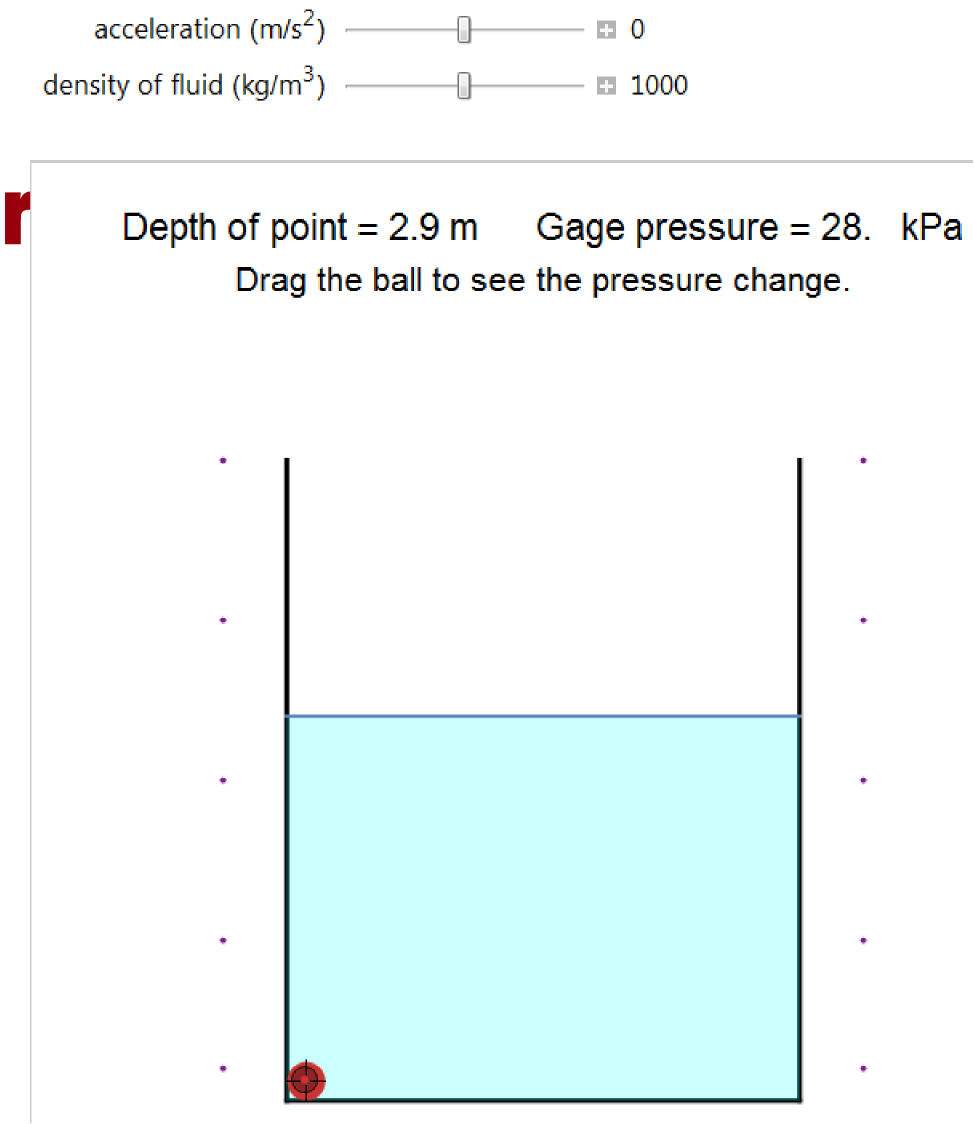
Linearly Accelerating Containers



Source: asciencecom, Youtube (https://www.youtube.com/watch?v=jqpl4ME6rRY)

DEMONSTRATION

Pressure within an Accelerating Container



Source: Jon Barbieri and Peter Hassinger, "Pressure within an Accelerating Container"
http://demonstrations.wolfram.com/PressureWithinAnAcceleratingContainer/

2.5 LINEARLY ACCELERATING CONTAINERS

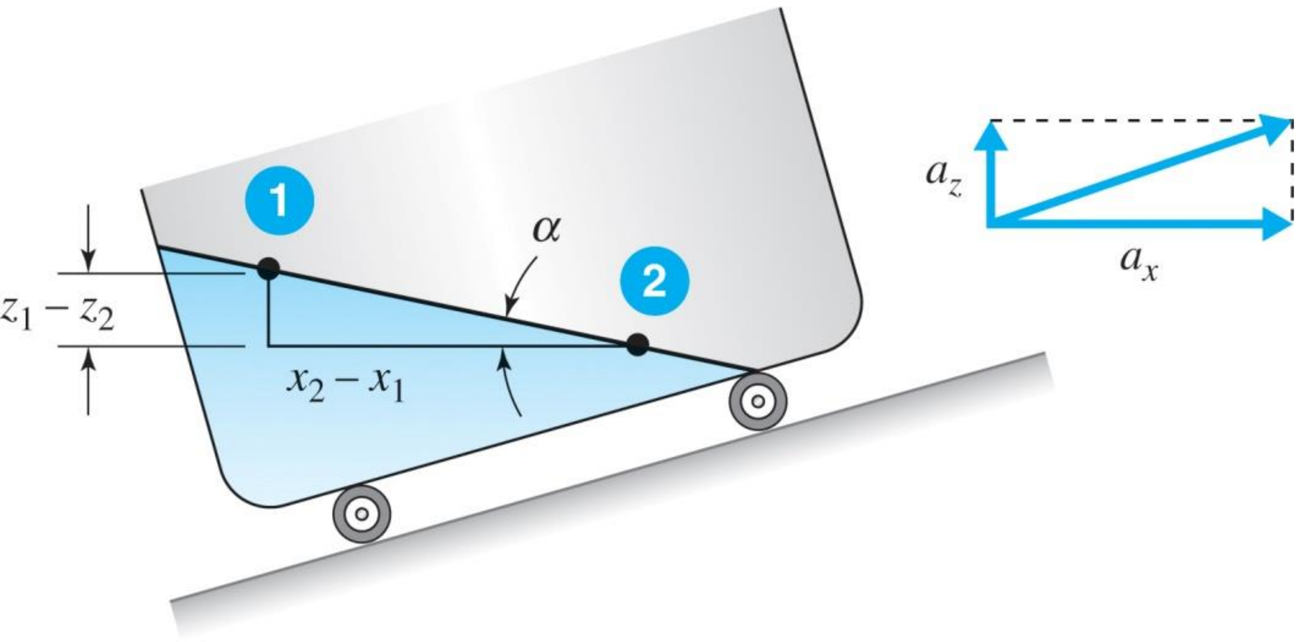


Fig. 2.18 Linearly accelerating tank.

- The derived pressure differential equation is:

- When the fluid is linearly accelerating with horizontal (a_x) and vertical (a_z) components:

2.5 LINEARLY ACCELERATING CONTAINERS

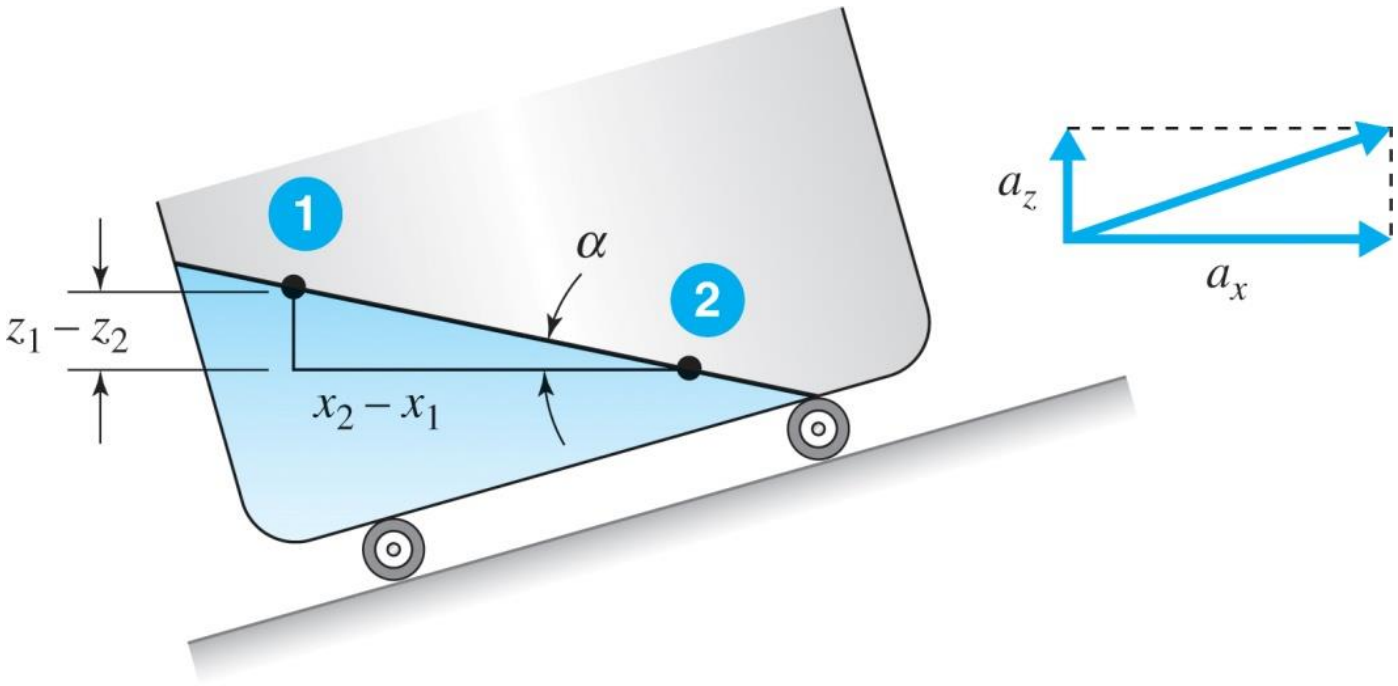


Fig. 2.18 Linearly accelerating tank.

α = angle that the constant-pressure line makes with the horizontal.

- As points 1 and 2 lie on a constant-pressure line:

Example: **P.2.97**. The tank shown in Fig. P2.97 is accelerated to the right at 10 m/s^2. Find:

a)P_A, b)P_B, c)P_C

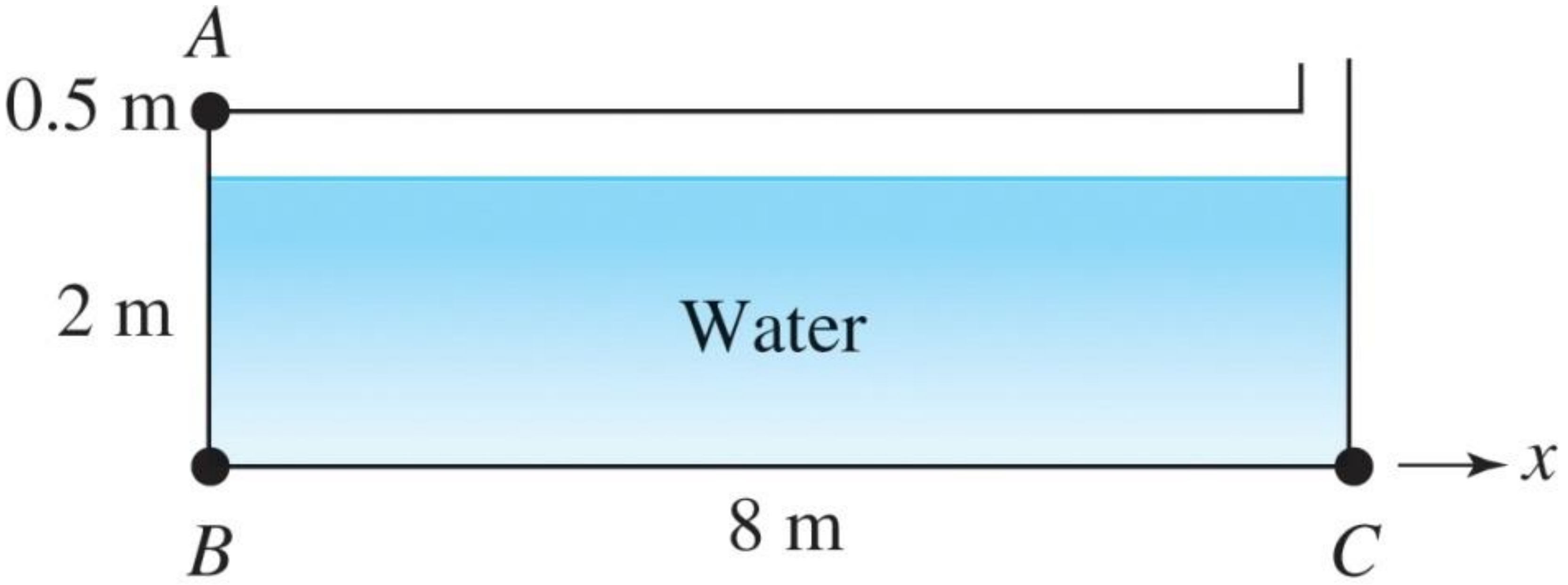


Fig. P2.97

Example: **P.2.99**. The tank shown in Fig. P2.99 is filled with water and accelerated. Find the pressure at point *A* if $a = 20\ m/s^2$ and $L = 1$ m.

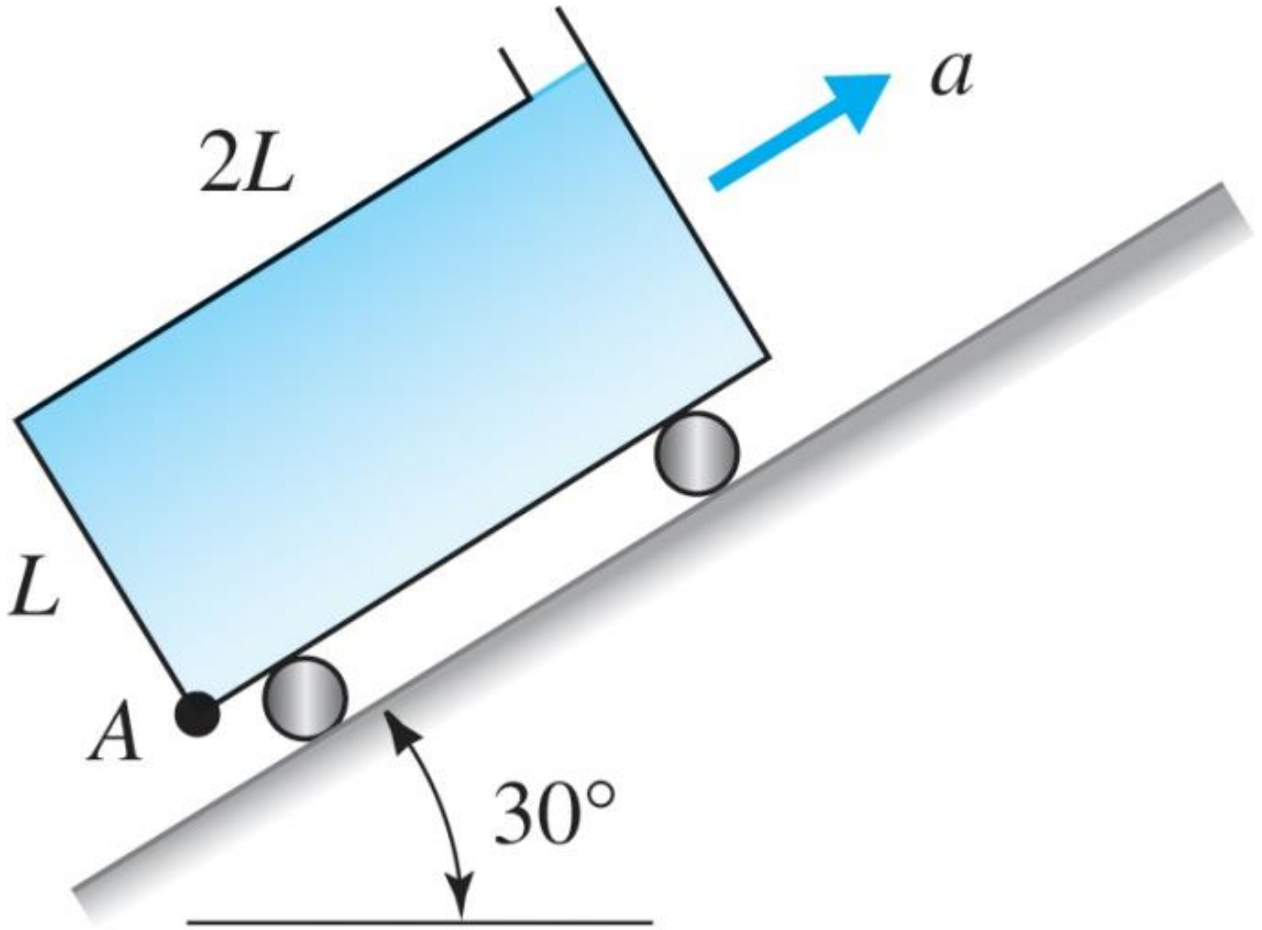


Fig. P2.99

2.6 ROTATING CONTAINERS

https://www.youtube.com/watch?v=RdRnB3jz1Yw

- For a liquid in a rotating container (cross-section shown):

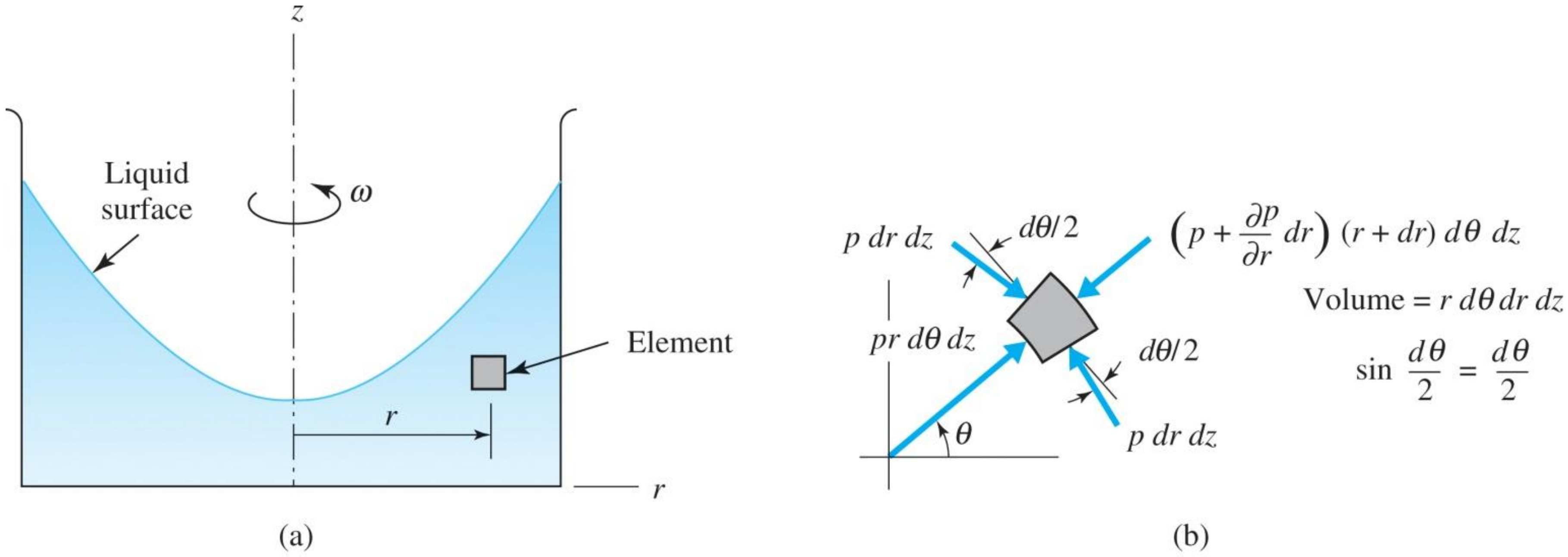


Fig. 2.19 Rotating container: (a) liquid cross section; (b) top view of element.

- In a short time, the liquid reaches static equilibrium with respect to the container and the rotating *rz*-reference frame.
- Horizontal rotation will not affect the pressure distribution in the vertical direction.
- No variation in pressure with respect to the θ-coordinate.

2.6 ROTATING CONTAINERS

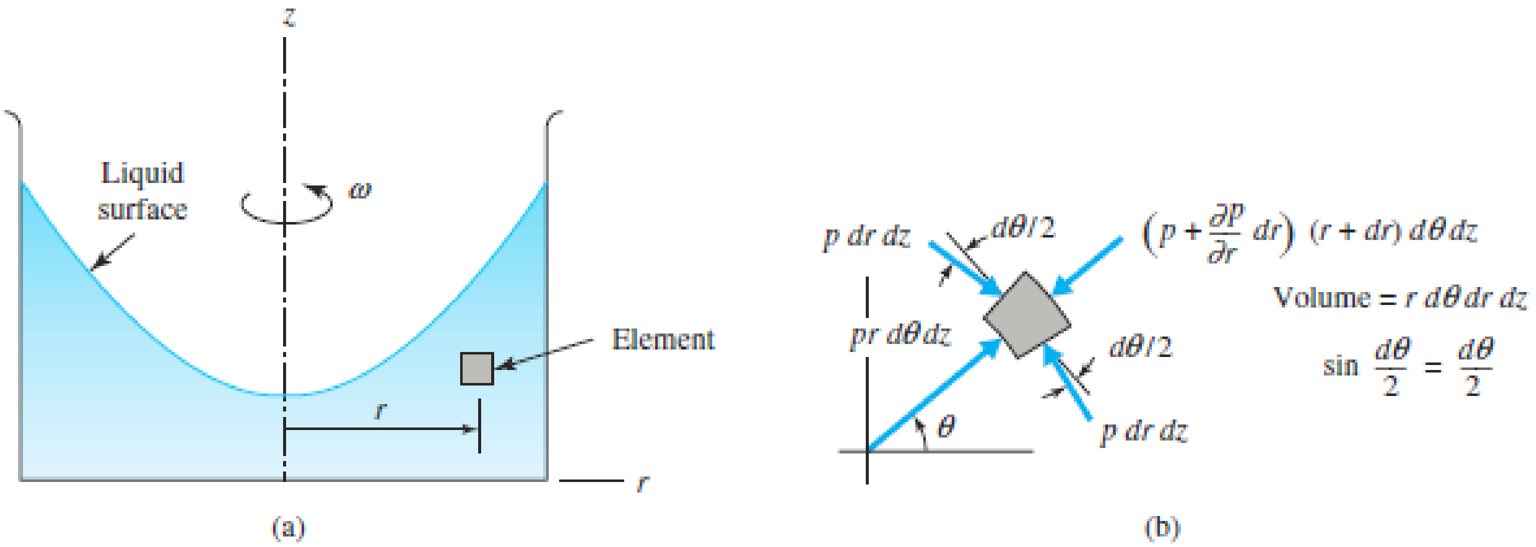


Figure 2.19 Rotating container: (a) liquid cross section; (b) top view of element.

- Between two points (r_1,z_1) and (r_2,z_2) on a rotating container, the static pressure variation is:

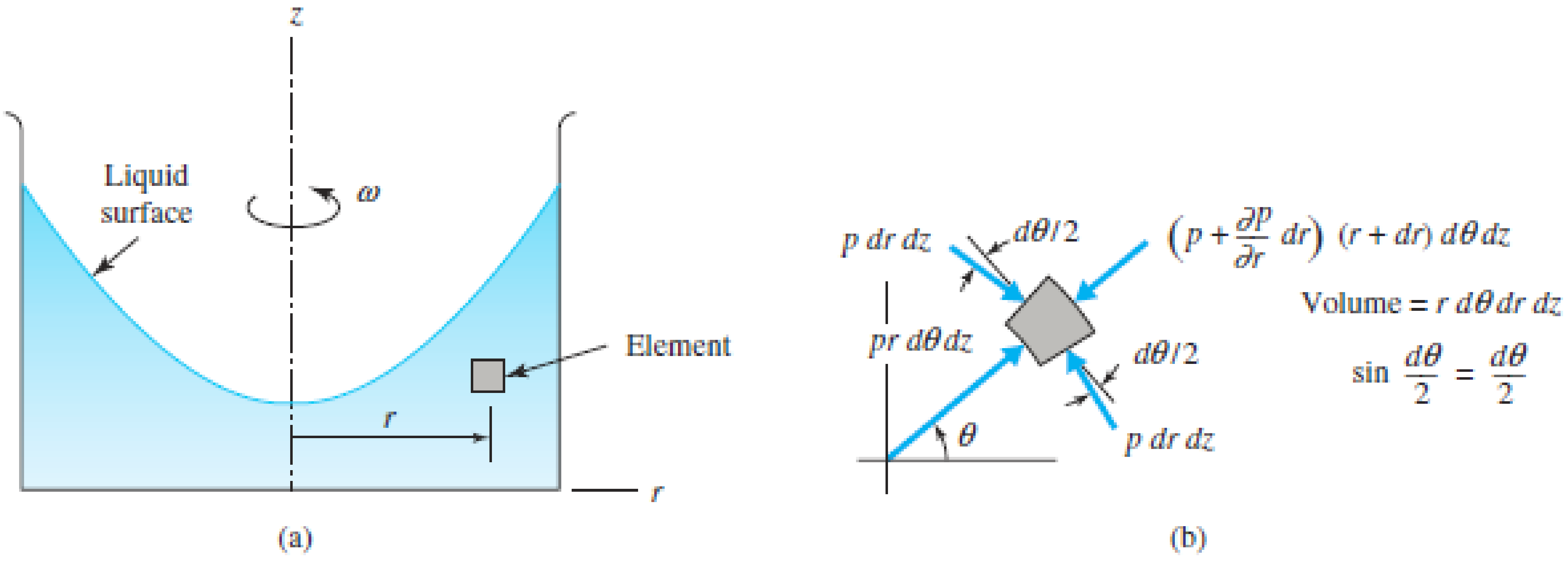


Figure 2.19 Rotating container: (a) liquid cross section; (b) top view of element.

- If two points are on a constant-pressure surface (e.g., free surface) with point 1 on the *z*-axis [r_1=0]:

- The free surface is a **paraboloid of revolution**.

Example: P.2.106. For the cylinder shown in Fig. P2.106, determine the pressure at point *A* for a rotational speed of 5 rad/s.

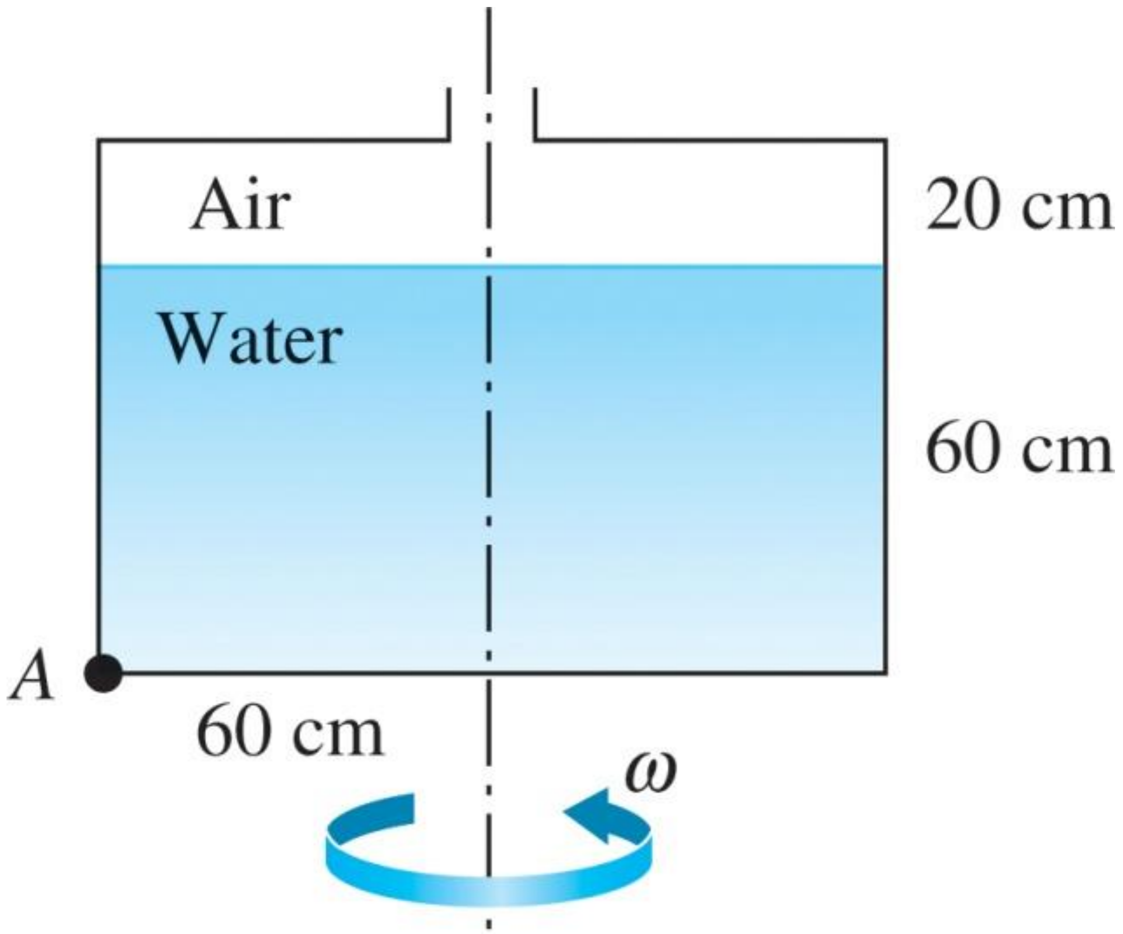


Fig. P2.106

Example: **P.2.107**. The hole in the cylinder of Problem P2.106 is closed and the air pressurized to 25kPa. Find the pressure at point *A* if the rotational speed is 5 rad/s.

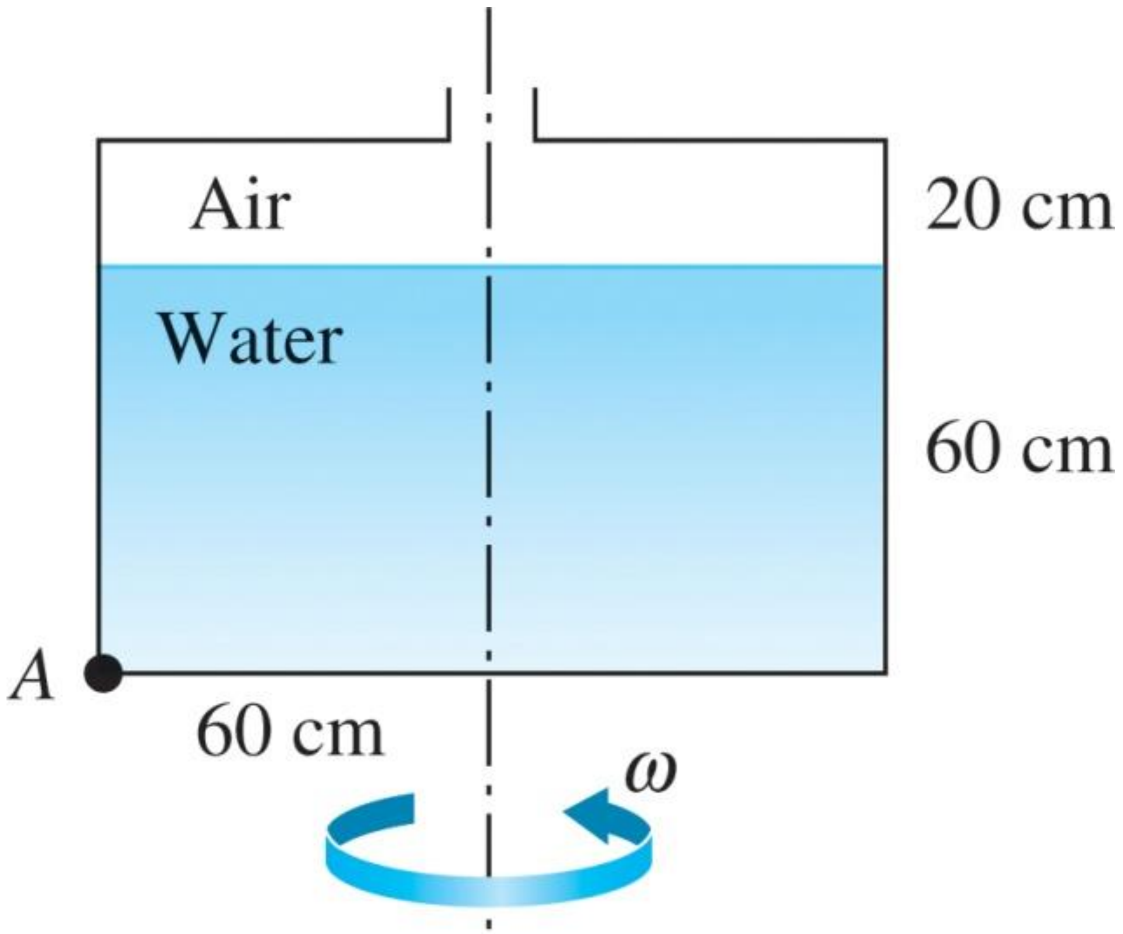


Fig. P2.106