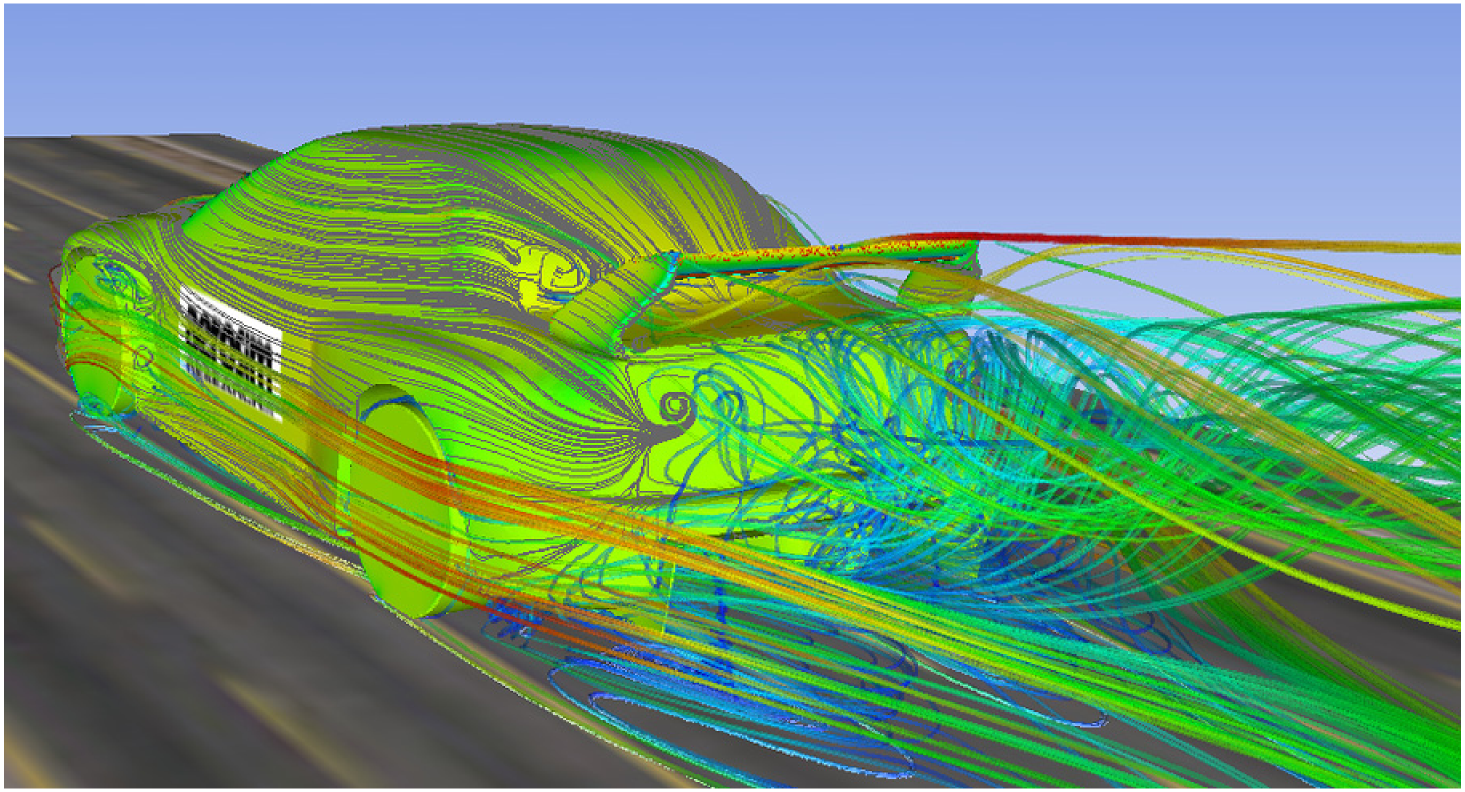
Florida International University, Department of Civil and Environmental Engineering

CWR 3201 Fluid Mechanics, Fall 2019

Fluids in Motion



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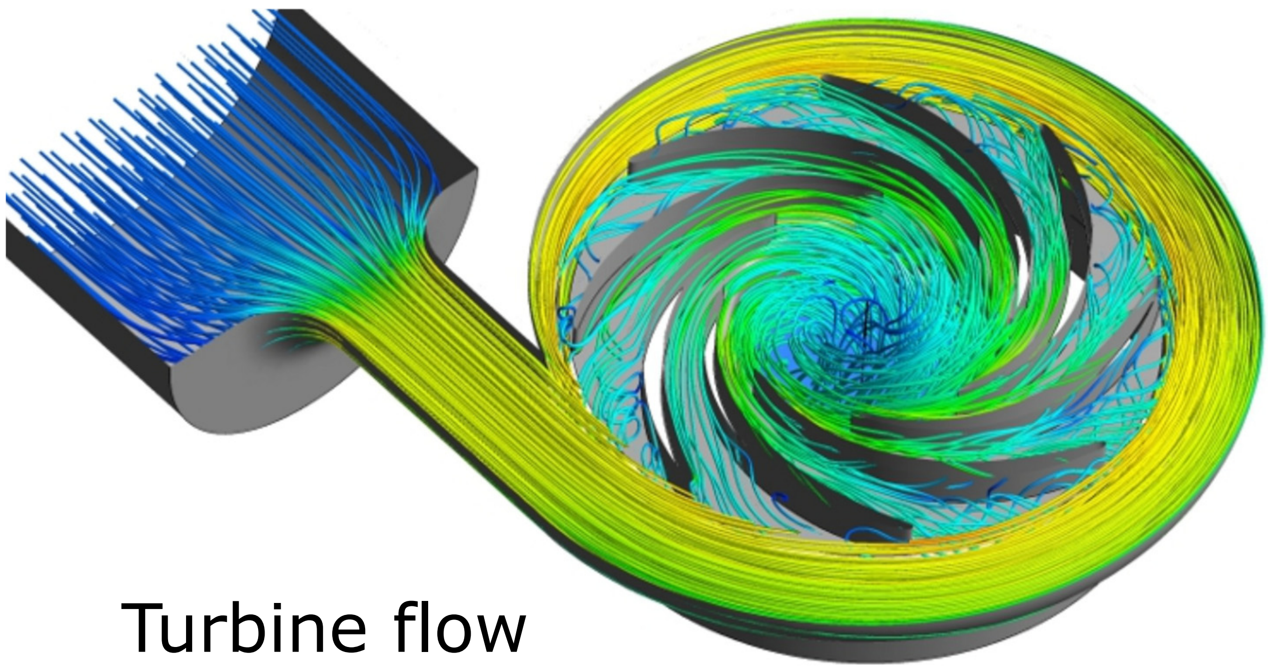
3.1 Introduction

- General equations of motion in fluid flow are very difficult to solve.
  - Need simplifying assumptions.
  - In some cases viscosity can be neglected.

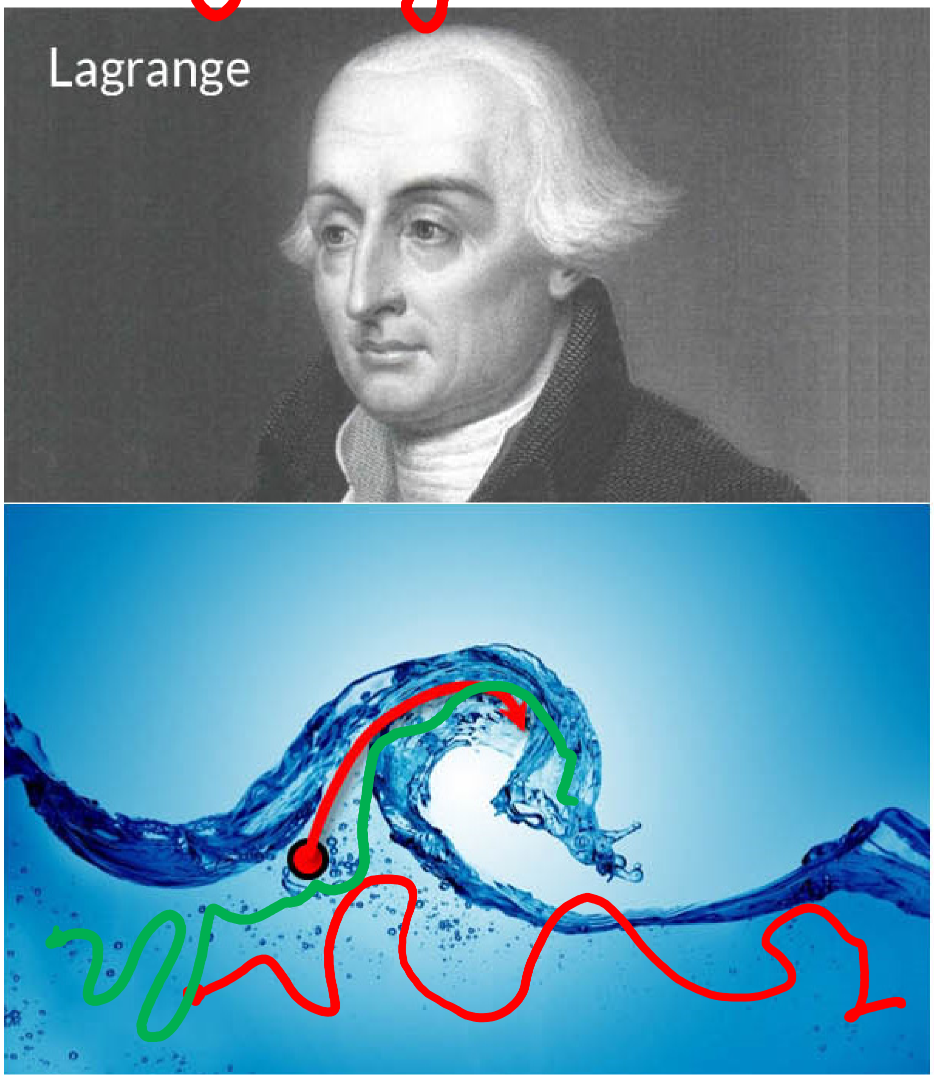


Turbine flow

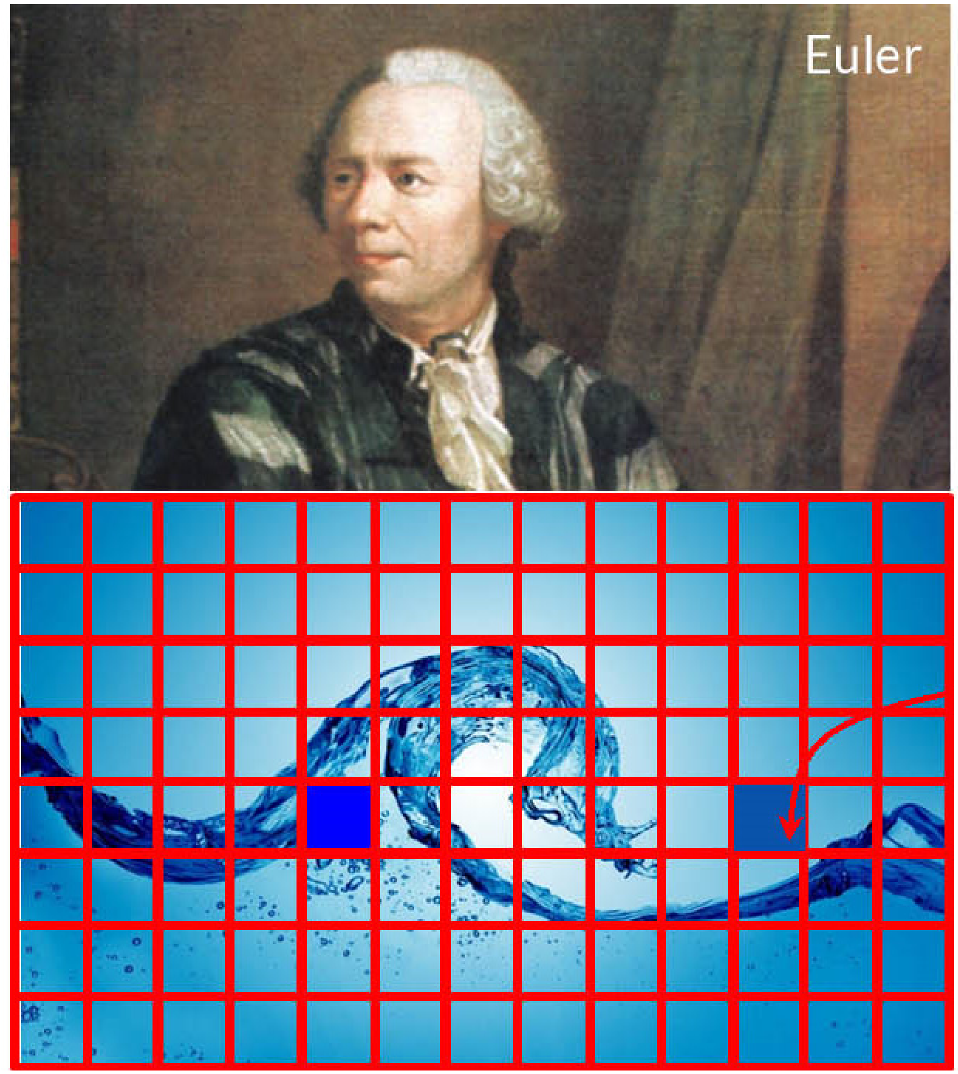
3.2 Description of Fluid Motion

3.2.1 Lagrangian and Eulerian Descriptions of Motion (Cont.)

Lagrangian



Eulerian



3.2 Description of Fluid Motion

3.2.2 Pathlines, Streaklines and Streamlines

$t+10\Delta t$

- **Pathline** is the locus of points traversed by a given particle as it travels in a field of flow. The pathline provides us with a "history" of the particle's locations.

t $t+\Delta t$

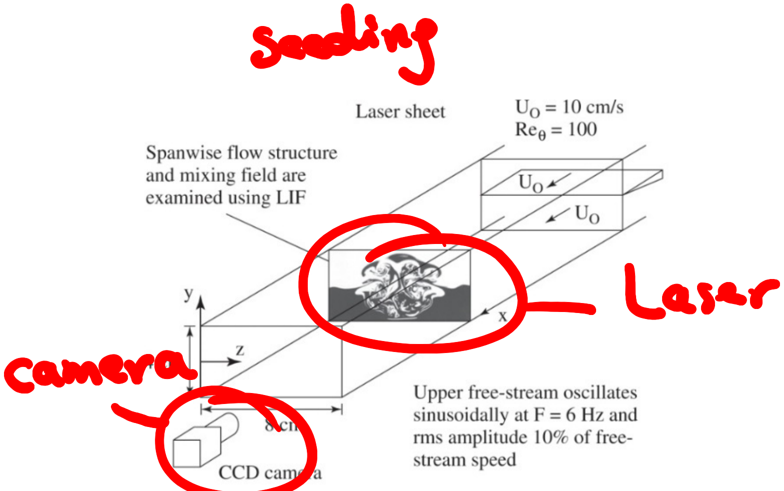
- **Streakline** is defined as an instantaneous line whose points are occupied by all particles originating from some specified point in the flow field. Streaklines tell us where the particles are "right now."

- **Streamline** is a line in the flow possessing the following property: the velocity vector of each particle occupying a point on the streamline is tangent to the streamline

In a steady flow, pathlines and streamlines are all coincident.

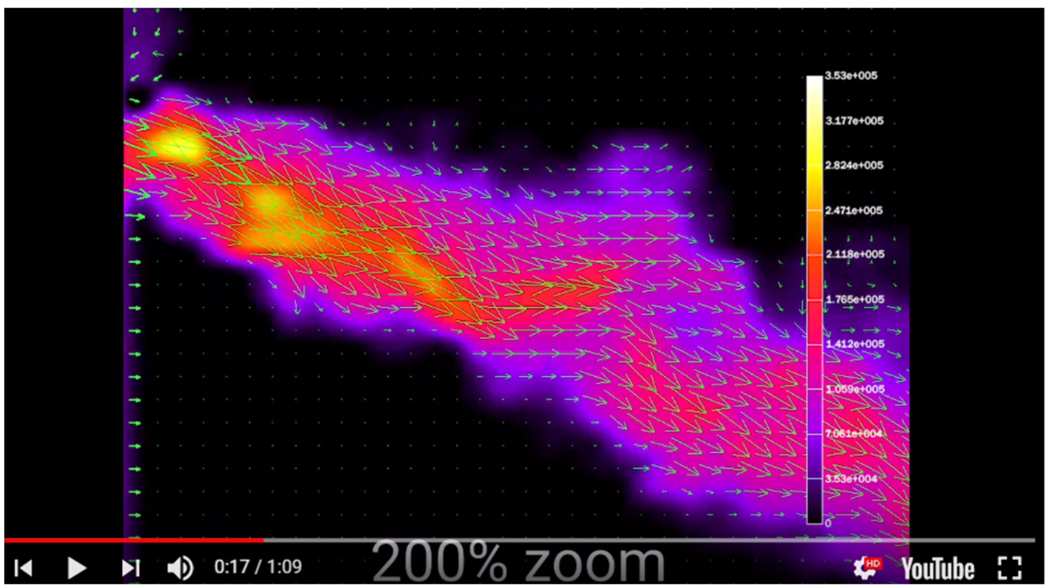
https://www.youtube.com/watch?v=Dqa1IdG_6cs

Flow Visualization: **Photography and Lighting**



(a) **Fig. 13.23** Laser Induced Fluorescence (LIF): (a) experimental layout

(a)



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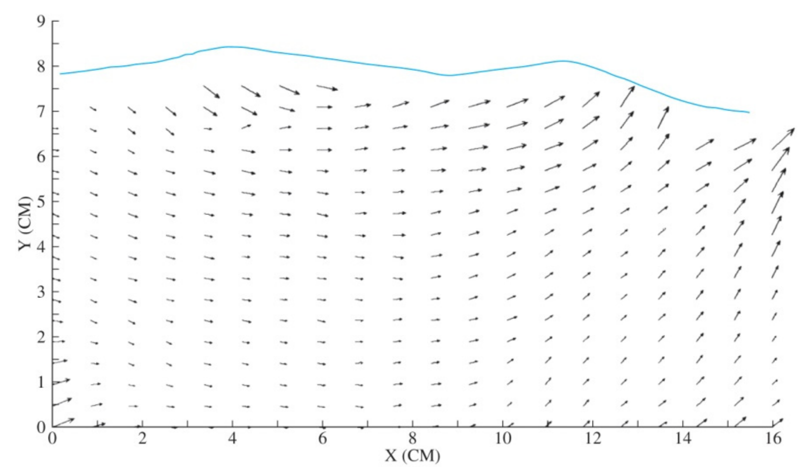


Fig. 13.21 Particle Image Velocimetry (PIV): (a) photograph of particle pathlines; (b) scaled velocity vectors. (Courtesy of R. Bouwmeester.)

3.2 Description of Fluid Motion

3.2.3 Acceleration

$\hat{i}, \hat{j}, \hat{k}$ (unitary vectors)

- Acceleration is the derivative of velocity (with respect to time).

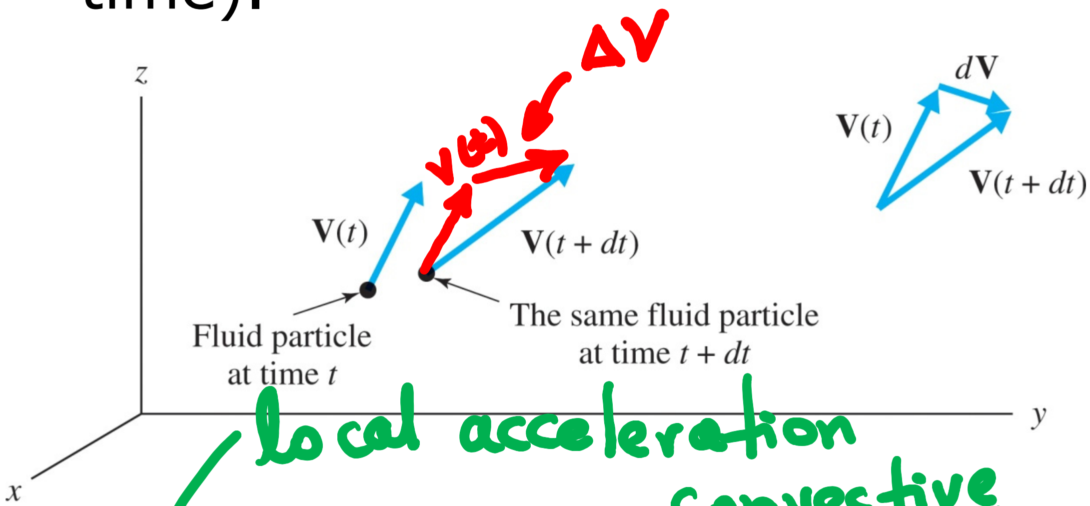


Fig. 3.4 Velocity of a fluid particle.

$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$

$u = \frac{dx}{dt}, \quad v = \frac{dy}{\Delta t}$

$w = \frac{dz}{dt}$

local acceleration

convective acceleration

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt} + \frac{\partial \vec{V}}{\partial z}\frac{dz}{dt}$$

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

3.2.3 Acceleration

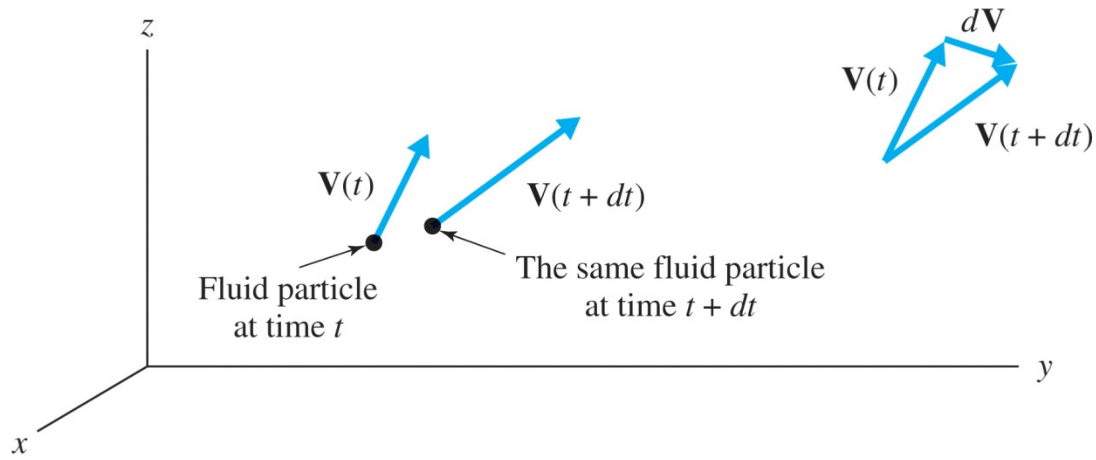


Fig. 3.4 Velocity of a fluid particle.

- The acceleration is:

$$\vec{a} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$

- The scalar components of the above equation in rectangular coordinates are:

$$a_x = \frac{\partial V_x}{\partial t} + u\frac{\partial V_x}{\partial x} + v\frac{\partial V_x}{\partial y} + w\frac{\partial V_x}{\partial z}$$

$$\vdots$$

Earth

another Planet

3.2.3 Acceleration

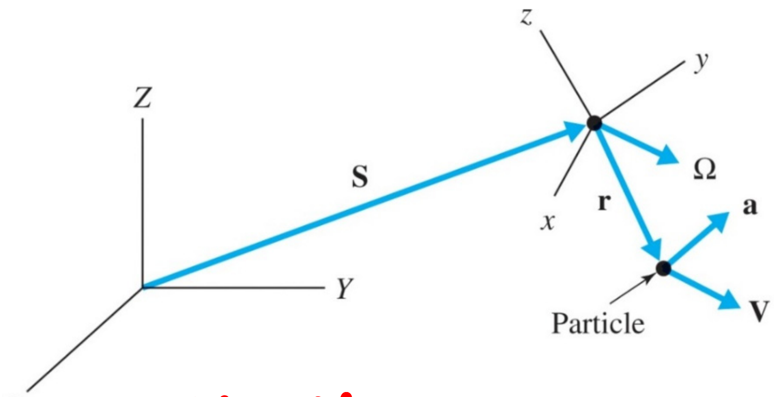
- If the observer's reference frame is accelerating:
  - Acceleration of a particle relative to a fixed reference frame is needed.

$$A = a + \underbrace{\frac{d^2 s}{dt^2}}_{\text{acceleration of reference frame}} + \underbrace{2\Omega \times V}_{\text{Coriolis acceleration}} + \underbrace{\Omega \times (\Omega \times \vec{r})}_{\text{Normal acceleration}} + \underbrace{\frac{d\Omega}{dt} \times \vec{r}}_{\text{Angular acceleration}}$$

a: Acceleration given by the equation in previous slide
V: Velocity vector of the particle
r: Position vector of the particle
Ω: Angular velocity of the observer's reference frame

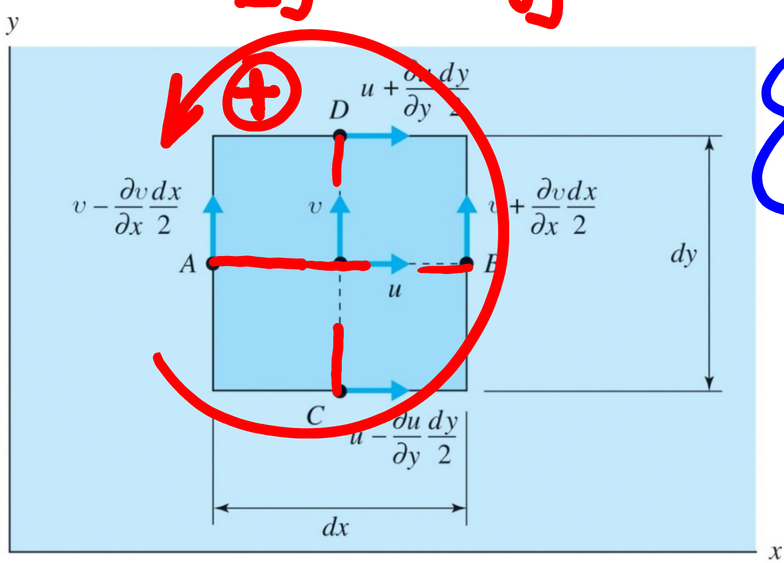
- If **A = a**, the reference frame is **inertial**: a reference frame that moves with constant velocity without rotating.
- If **A ≠ a**, the reference frame is **noninertial.**

3.2.4 Angular Velocity and Vorticity



$$\Omega_{AB} = \frac{V_B - V_A}{\Delta x} = \frac{\partial v}{\partial x}$$

$$\Omega_{CD} = \frac{V_C - V_D}{\Delta y} = -\frac{\partial u}{\partial y}$$



As a fluid particle moves it may rotate or deform. In certain flows or regions, fluid particles do not rotate. These are called **irrotational flows**

- **Angular Velocity (Ω)**: The average velocity of two perpendicular line segments of a fluid particle.

$$\Omega_z = \frac{\Omega_{AB} + \Omega_{CD}}{2}$$

$$\Omega_z = \frac{\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}}{2}$$

- **Vorticity (ω)**: Twice the angular velocity.

$$\omega = 2\Omega$$

  - **An irrotational flow has no vorticity**

Table 3.1 The Substantial Derivative, Acceleration, and Vorticity in Cartesian, Cylindrical, and Spherical Coordinates

Substantial Derivative

Cartesian

$$\frac{D}{Dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \frac{\partial}{\partial t}$$

Cylindrical

$$\frac{D}{Dt} = v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z} + \frac{\partial}{\partial t}$$

Vorticity

Cartesian

$$\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \qquad \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \qquad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Cylindrical

$$\omega_r = \frac{1}{r}\left(\frac{\partial v_z}{\partial \theta}\right) - \frac{\partial v_\theta}{\partial z} \qquad \omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \qquad \omega_z = \frac{1}{r}\left(\frac{\partial (r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial \theta}\right)$$

Acceleration

Cartesian

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Cylindrical

$$a_r = \frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}$$

$$a_\theta = \frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}$$

$$a_z = \frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

$$\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k} = \underline{u}\,\hat{i} + \underline{v}\,\hat{j} + \underline{w}\,\hat{k}$$

Example: The velocity field in a flow is given by V = 2x$\hat{\imath}$ + 2y$\hat{\jmath}$ m/s. Find the acceleration, the angular velocity and the vorticity vector at the point (2,-1,3) at *t* = 2 s.

$$\vec{V} = 2x\,\hat{i} + 2y\,\hat{j}$$

$$\vec{a} = \frac{d\vec{V}}{dt} = \cancel{\frac{\partial \vec{V}}{\partial t}}^{\,0} + \underline{u}\,\frac{\partial \vec{V}}{\partial x} + \underline{v}\,\frac{\partial \vec{V}}{\partial y} + w\,\cancel{\frac{\partial \vec{V}}{\partial z}}^{\,0}$$

$u = 2x$

$v = 2y$

$$\frac{\partial \vec{V}}{\partial x} = 2\hat{i}$$

$$\frac{\partial \vec{V}}{\partial y} = 2\hat{j}$$

$$\vec{a} = 2x(2\hat{i}) + 2y(2\hat{j})$$

$$\vec{a} = 4x\,\hat{i} + 4y\,\hat{j}$$

at point (2,-1,3)

$$\vec{a} = 4(2)\hat{i} + 4(-1)\hat{j}$$

$$\boxed{\vec{a} = 8\,\hat{i} - 4\,\hat{j}}$$

angular velocity

vorticity

$$* \; \Omega_z = \frac{1}{2}\left(\cancel{\frac{\partial v}{\partial x}}_{0} - \cancel{\frac{\partial u}{\partial y}}_{0}\right) = 0 \qquad \Omega_x = 0, \quad \Omega_y = 0$$

$$\omega = 2\Omega = 0$$

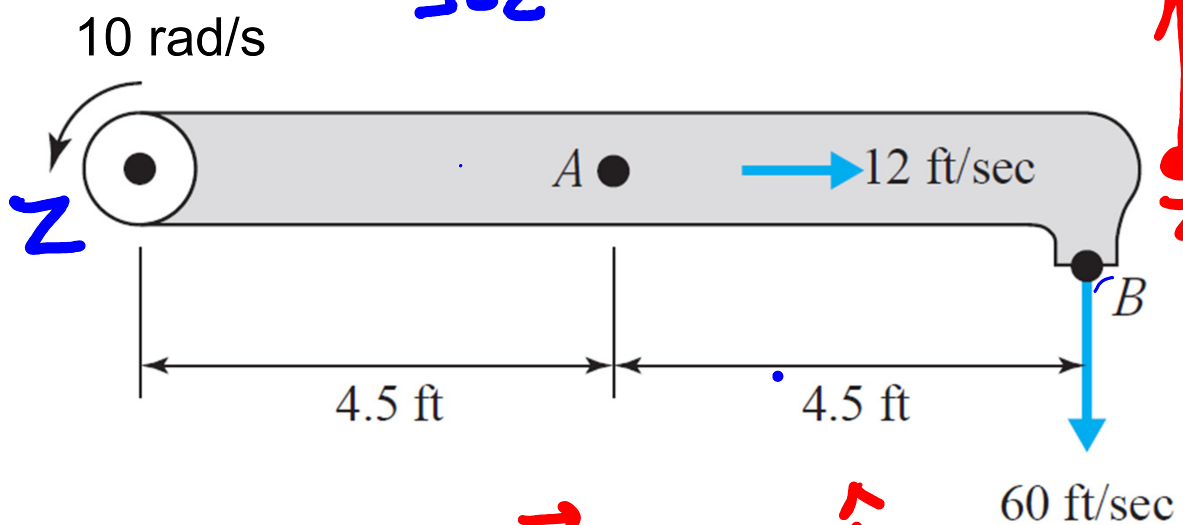
Example: For the flow shown in the figure below, relative to a fixed reference frame, find the acceleration of a fluid particle at:

(a) Point *A*

(b) Point *B*

The water at *B* makes an angle of 45° with respect to the ground and the sprinkler arm is horizontal.

$\Omega_z = 10\hat{k}$



$-$ $\hat{i}\ \hat{j}\ \hat{k}\ \hat{i}\ \hat{j}$ $+$

O (reference frame is not accelerating)

$\vec{r}_A = 4.5\hat{i}$

reference

$$A = a + \cancel{\frac{d^2 S}{dt^2}} + 2\Omega \times \vec{V} + \vec{\Omega} \times (\Omega \times \vec{r}) + \cancel{\frac{d\Omega}{dt}} \times \vec{r}$$

0

$V_A = 12\hat{i}$

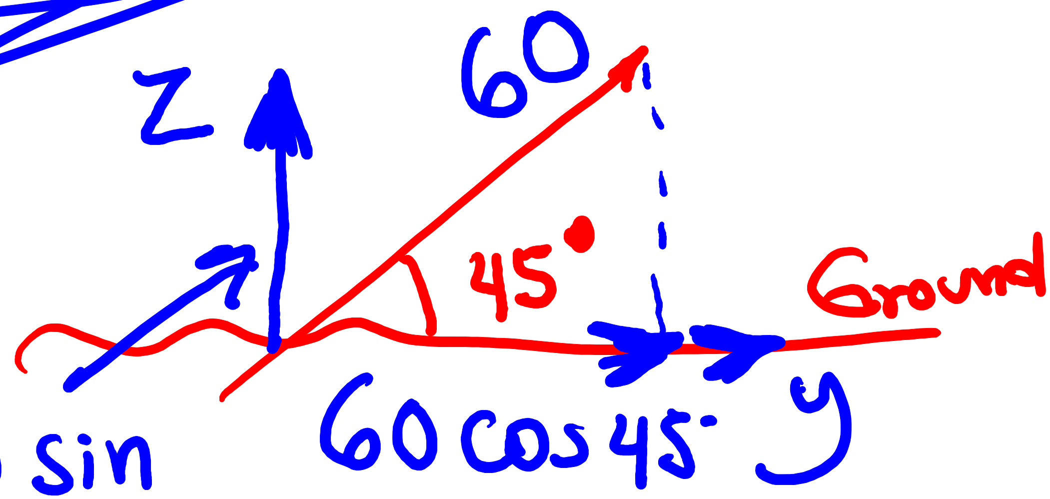
$a_A = 0$ $= 0$

$$A = 2\Omega \times \vec{V} + \Omega \times (\Omega \times \vec{r})$$

$$A = 2(10\hat{k} \times 12\hat{i}) + 10\hat{k} \times (10\hat{k} \times 4.5\hat{i})$$

$$A_A = 240\hat{j} - 450\hat{i} \text{ ft/s}^2$$

$*A_B = ??$ $\quad \vec{r}_B = 9\hat{i}$



$$\vec{V}_B = -60\cos 45^\circ \hat{j} + 60 \sin 45^\circ \hat{k}$$

$$\vec{a}_B = 0$$

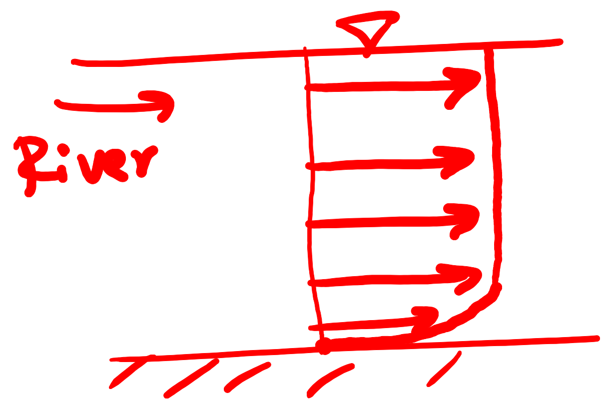
$$A_B = 2\left(10\hat{k} \times \left[-60\cos 45^\circ \hat{j} + 60 \sin 45^\circ \hat{k}\right]\right) + \Omega \times (\Omega \times \vec{r}_B)$$

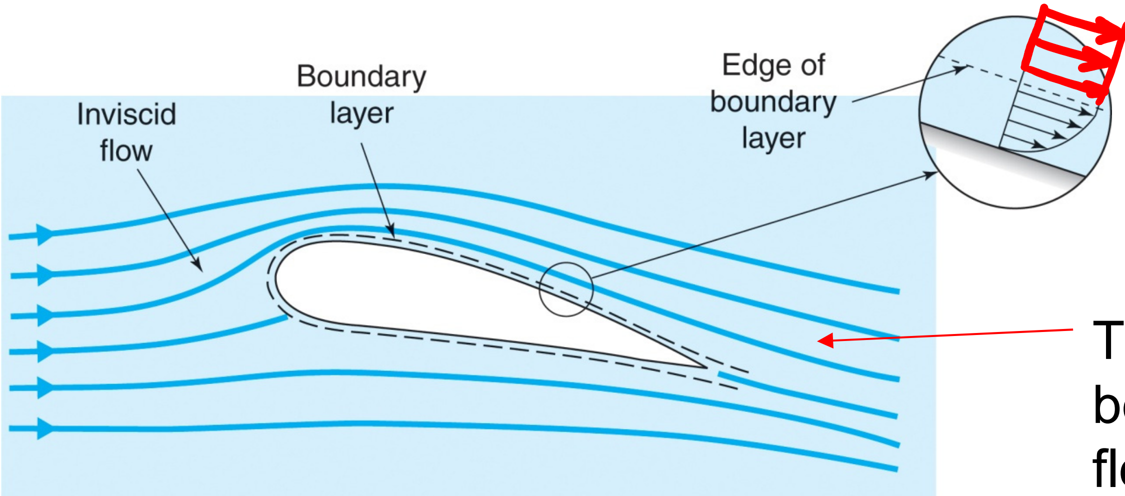
$$= 848.5\hat{i} - 900\hat{i} = -51.5\hat{i}$$

3.3.2 Viscous and Inviscid Flows

- A fluid flow can either be a viscous flow or an inviscid flow.
  - **Inviscid flow**: Viscous effects do not significantly influence the flow.
  - **Viscous flow**: Effects of viscosity are important.

- Any viscous effects that (may) exist are confined to a thin **boundary layer**.
  - The velocity in this layer is always zero at a fixed wall (due to viscosity).



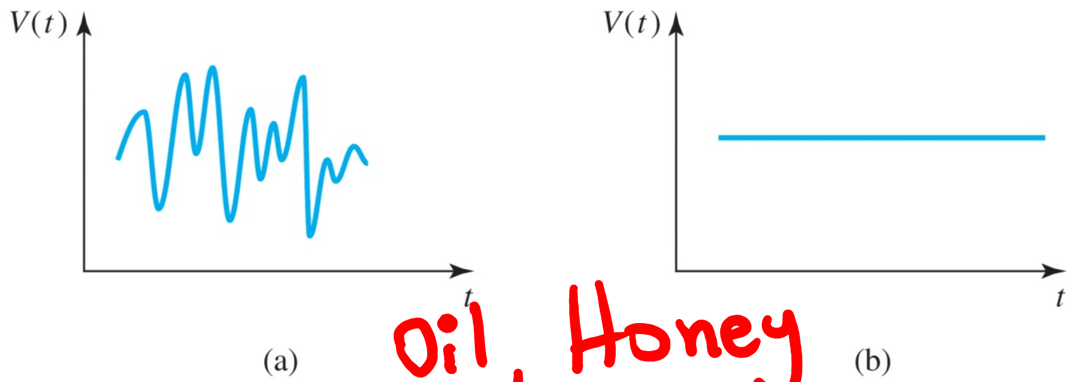


The inviscid flow outside the boundary layer in an external flow is called the **free stream**.

Fig. 3.10 Flow around an airfoil.

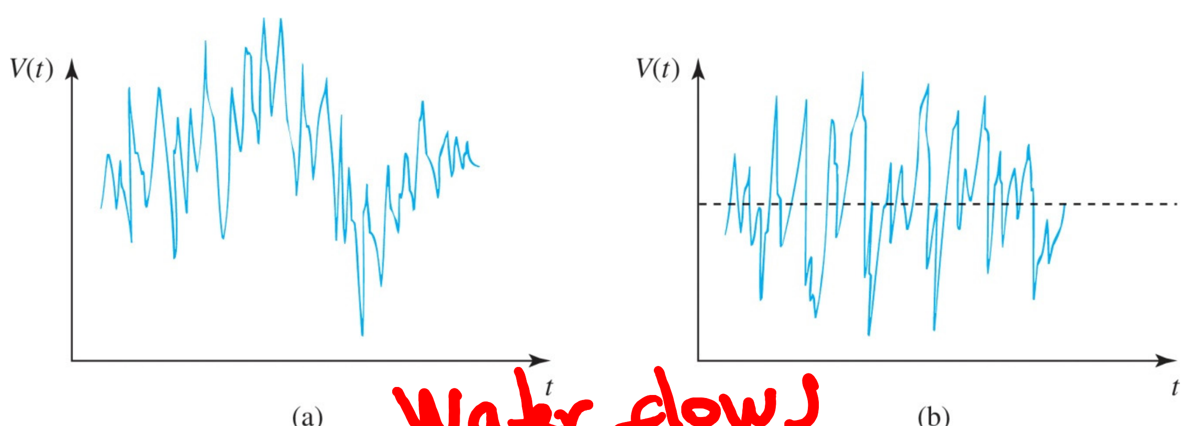
3.3.3 Laminar and Turbulent Flows

Viscous flow is either laminar or turbulent.



Oil, Honey

Fig. 3.11 Velocity as a function of time in a laminar flow: (a) unsteady flow; (b) steady flow.



Water flows

Fig. 3.12 Velocity as a function of time in a turbulent flow: (a) unsteady flow; (b) “steady” flow.

- **Laminar flow**: Flow with no significant mixing of particles but with significant viscous shear stresses.

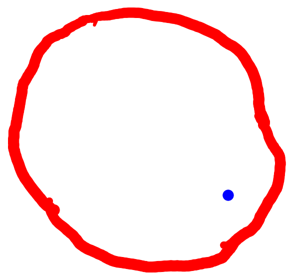
  Turbulent flow: Flow varies irregularly so that flow quantities (velocity/pressure) show random variation.
  - A “steady” turbulent flow is one in which the time-average physical quantities do not change in time.

3.3 Classification of Fluid Flows

3.3.3 Laminar and Turbulent Flows

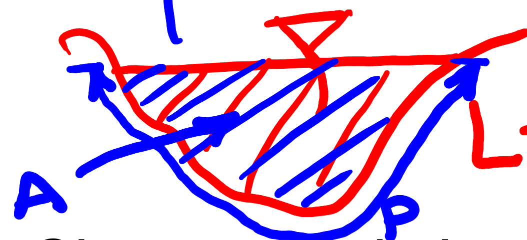
- Whether a flow is laminar or turbulent depends on The **Reynolds Number:**

$$Re = \frac{V \cdot L}{\nu}$$



$L = D$

$L = R$



$L = R = \frac{A}{P}$

L: Characteristic Length

V: Characteristic Velocity

ν: Kinematic Viscosity

$Re < Re_{crit}$ (Laminar flow)

$Re > Re_{crit}$ (Turbulent flow)

$$R = \frac{\cancel{\pi} D^{\cancel{2}}}{4 \times \cancel{\pi} \cancel{D}} = \frac{D}{4}$$

- If the Reynolds number is greater than the critical Reynolds number ($Re > Re_{crit}$) then the flow is turbulent:
    - **Pipe flow**: $Re_{crit} \approx 2000$
    - **Rivers and canals**: $Re_{crit} \approx 500$

3.4 The Bernoulli Equation

Along a streamline

The Bernoulli equation states that for an inviscid fluid flow, an increase in fluid velocity causes a decrease in pressure

$$\frac{\partial}{\partial s}\left(\frac{V^2}{2}+\frac{P}{\rho}+gh\right)=0$$

$$\frac{V^2}{2}+\frac{P}{\rho}+gh=\text{constant}$$

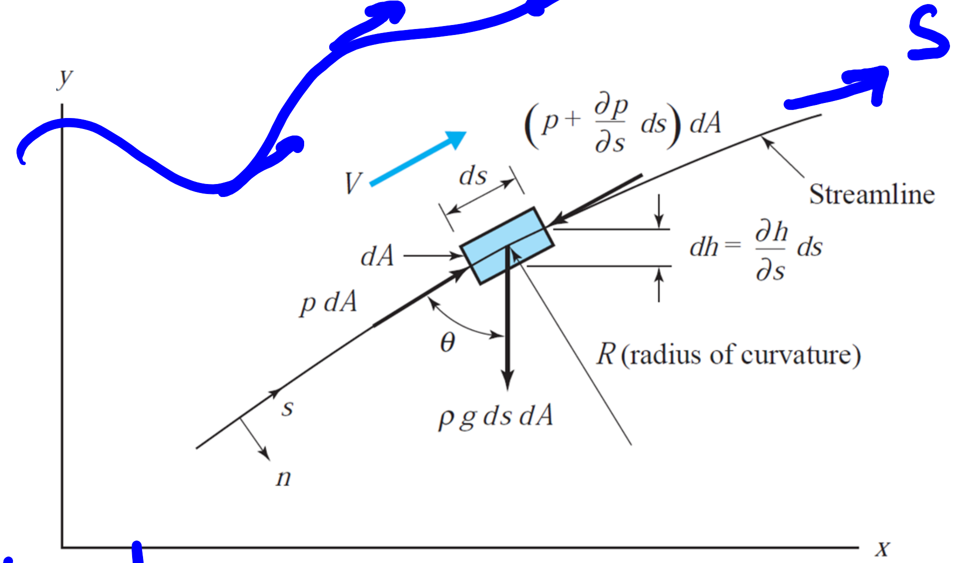


Fig. 3.17 Particle moving along a streamline.

Between two points on the same streamline:

1

2

$$\frac{V_1^2}{2}+\frac{P_1}{\rho_1}+gh_1=\frac{V_2^2}{2}+\frac{P_2}{\rho_2}+gh_2$$

Assumptions

- Inviscid flow (no shear stress)
- Steady flow $\frac{\partial V}{\partial t}=0$
- Along a streamline
- Constant density
- Inertial reference frame

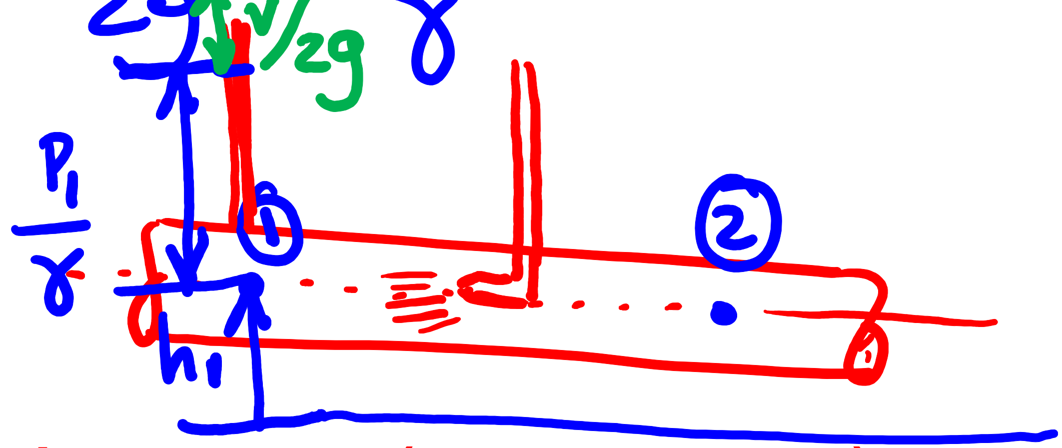
3.4 The Bernoulli Equation

- Another form of the equation (by dividing by g) is:

dynamic pressure

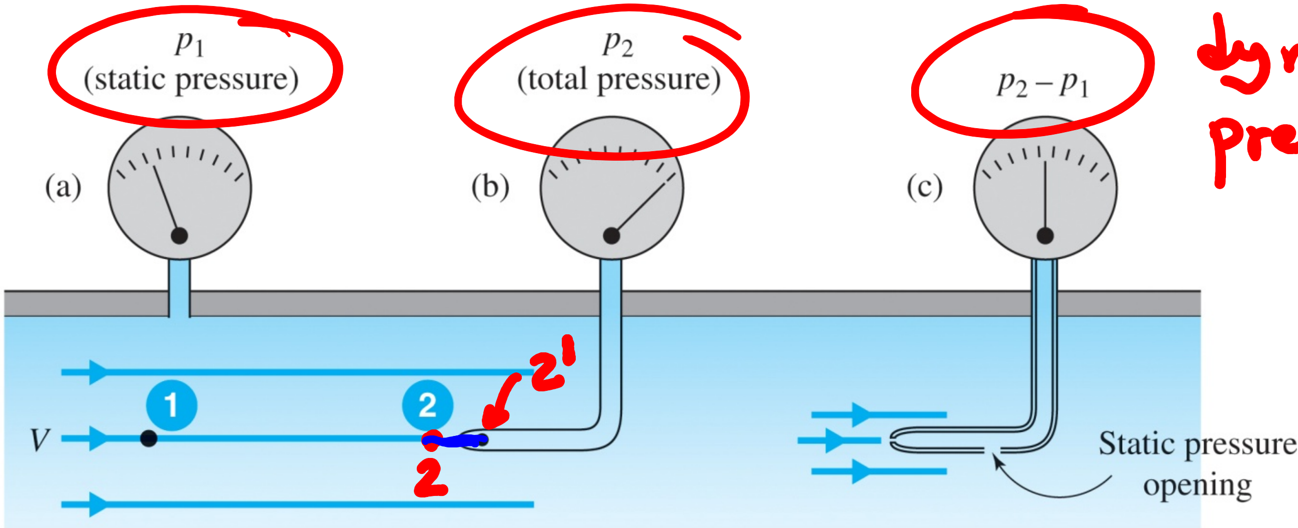
static pressure

$$\underbrace{\frac{V_1^2}{2g}}_{\text{velocity head}} + \underbrace{\underbrace{\frac{P_1}{\gamma}}_{\text{pressure head}} + h_1}_{\text{piezometric head}} = \frac{V_2^2}{2g} + \frac{P_2}{\gamma} + h_2$$



1. Pressure *p*, is called the **static pressure (gage pressure)**.
2. Piezometric head is $\frac{p}{\gamma} + h$ and the total head is $\frac{p}{\gamma} + h + \frac{V^2}{2g}$
3. The total pressure at a stagnation point (local fluid velocity is zero) is the **stagnation pressure**. $p + \rho\frac{V^2}{2} = p_T$

3.4 The Bernoulli Equation



dynamic pressure

Fig. 3.18 Pressure probes: (a) piezometer; (b) pitot probe; (c) pitot-static probe.

1. A piezometer (left) is used to measure static pressure.
2. A pitot probe (center) is used to measure total pressure.
   a) Point 2 is a stagnation point.
3. A pitot-static probe (right) is used to measure the difference between total and static pressure.

Bernoulli 2 - 2'

$$\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} = \frac{p_2'}{\gamma} + \cancel{\frac{V_2'^2}{2g}}^{\,0} + \cancel{z_2'}$$

$$\boxed{p_2' = p_2 + \frac{\rho}{2} V_2^2}$$

3.4 The Bernoulli Equation

Newton's second law in normal direction to the streamline

$$\boxed{\Delta p = -\rho \frac{V^2}{R} \Delta n}$$

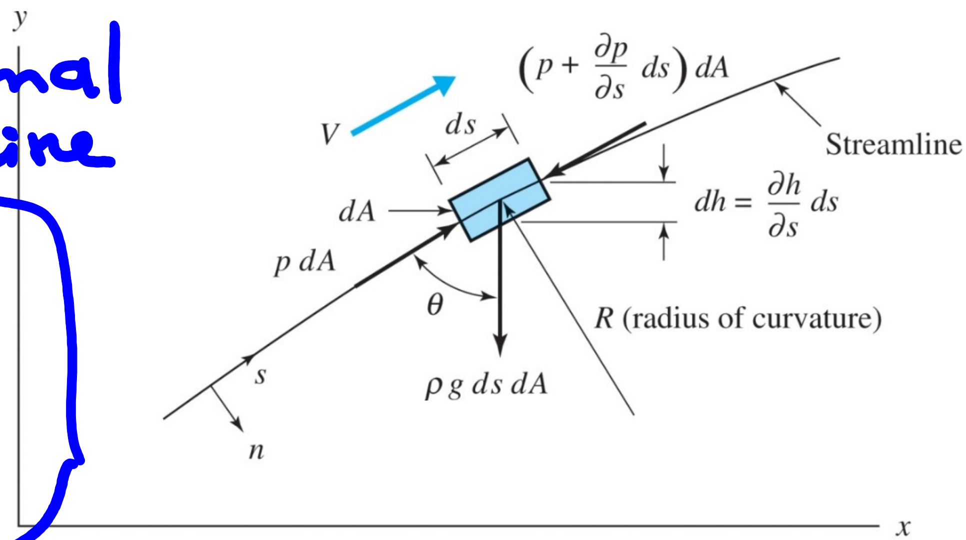


Fig. 3.17 Particle moving along a streamline.

- The equation above shows how the pressure changes normal to the streamline.
  - Δp: Incremental pressure change
  - Δn: Short distance
  - R: Radius of curvature

- Pressure decreases in the *n*-direction.
- Decrease is directly proportional to ρ and V^2
- Decrease is inversely proportional to R

Example: P.3.70. In the pipe contraction shown in Fig. P3.70, water flows steadily with a velocity of V_1 = 0.5 m/s and V_2 = 1.125 m/s. Two piezometer tubes are attached to the pipe at sections 1 and 2. Determine the height *H.* Neglect any losses through the contraction.

Bernoulli

$z_1 = z_2$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2}$$

$\frac{P_1}{\gamma}$ streamline

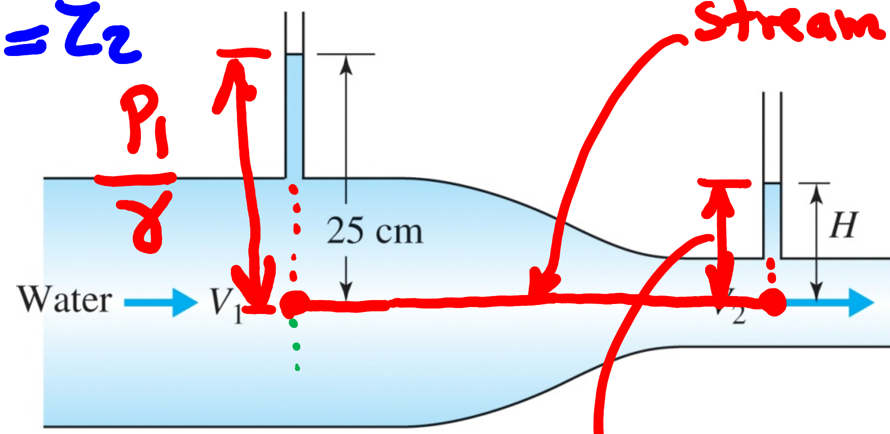


Fig. P3.70

P_2/γ

$$\frac{P_1}{\gamma} = 0.25\,m$$

$$0.25 + \frac{0.5^2}{19.6} = H + \frac{1.125^2}{19.6}$$

$$\boxed{H = 0.20\ m}$$

Flow measurement

13.2 Measurement of Local Flow Parameters

Pressure

- Manometer

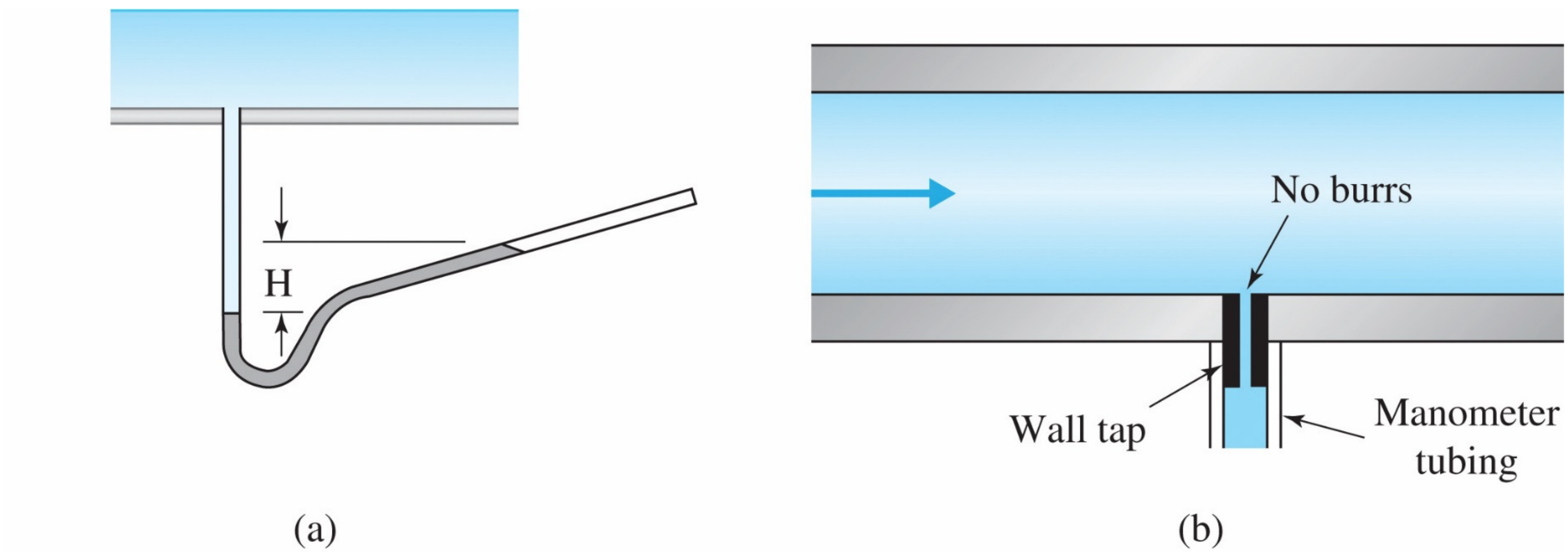


Fig. 13.1 Manometer used to measure pressure: (a) inclined tube manometer; (b) piezometer opening.

13.2 Measurement of Local Flow Parameters

Velocity

- Pitot-Static Probe

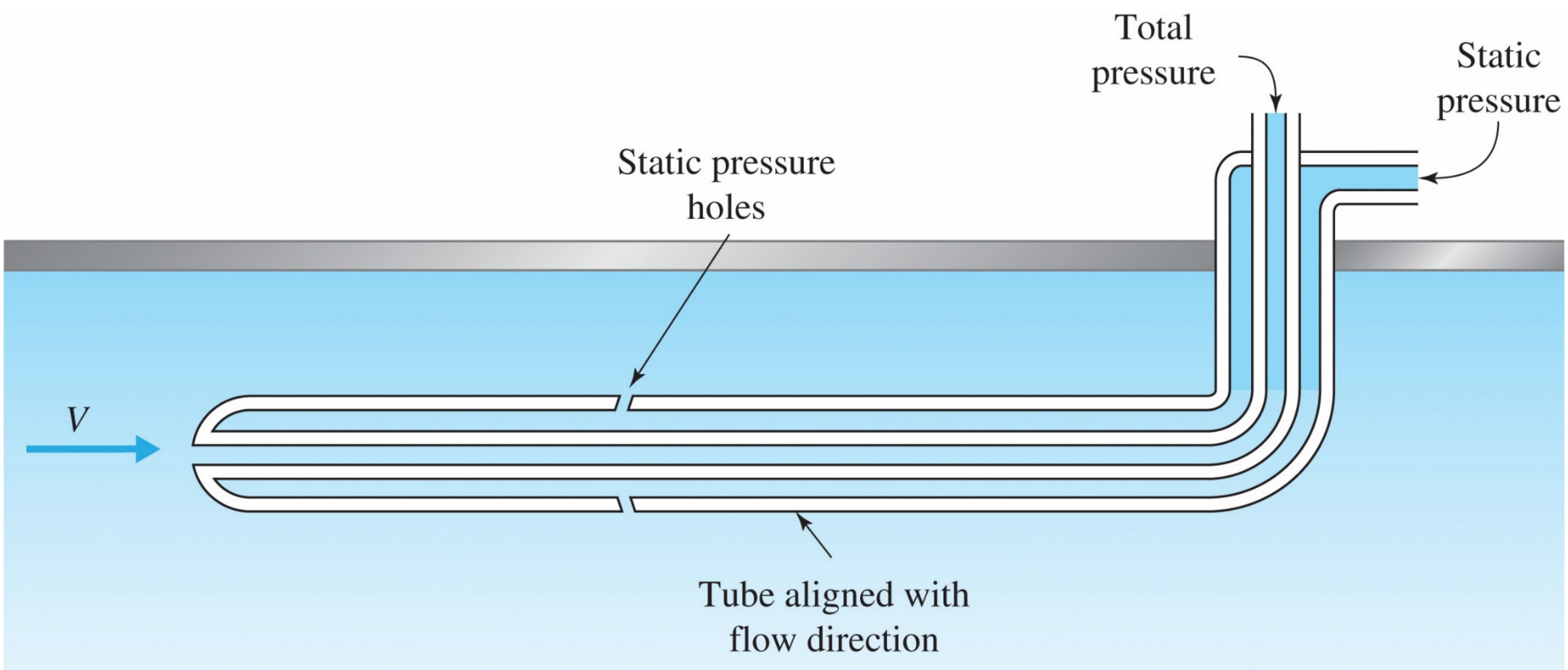
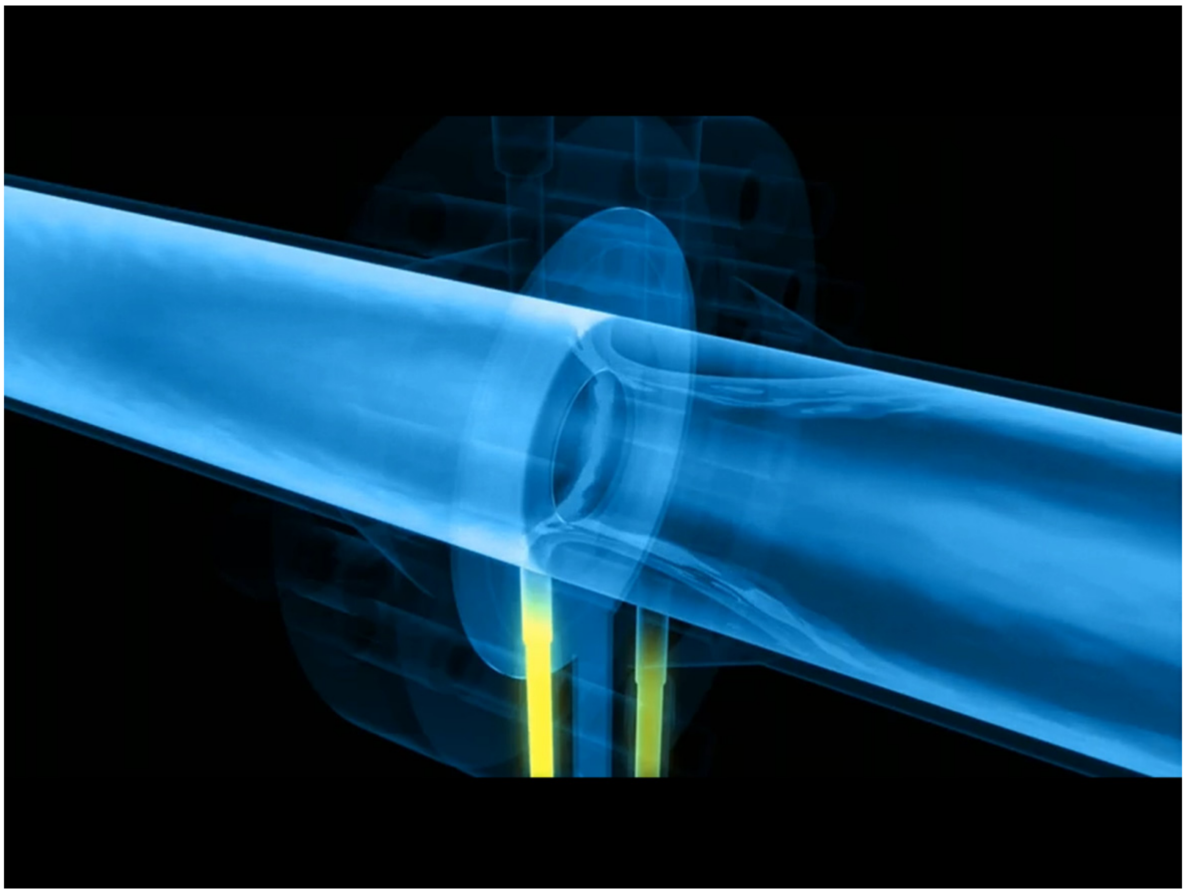


Fig. 13.4 Pitot-static probe.

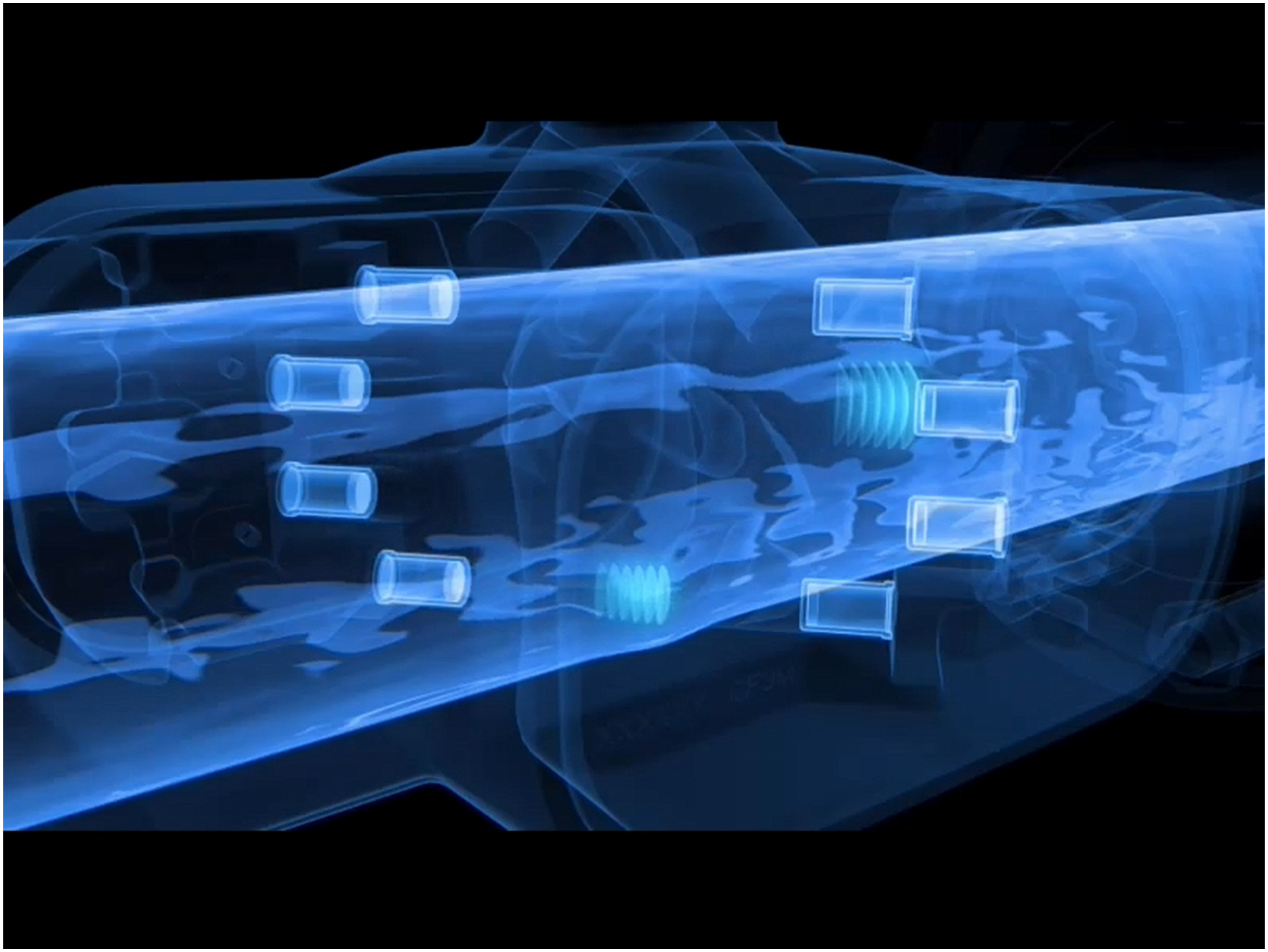
Flow Rate Measurement

The Differential Pressure Flow Measuring Principle (Orifice-Nozzle-Venturi)



http://www.youtube.com/watch?v=oUd4WxjoHKY

The Ultrasonic Flow Measuring Principle



http://www.youtube.com/watch?v=Bx2RnrfLkQg

13.3 Flow Rate Measurement

Differential Pressure Meters

Downstream of the restriction, the streamlines converge to form a minimal flow area A_c, termed the *vena contracta*.

$$V_1 A_1 = V_c A_c \quad \text{(Continuity)}$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_c}{\gamma} + \frac{V_c^2}{2g} + z_c$$

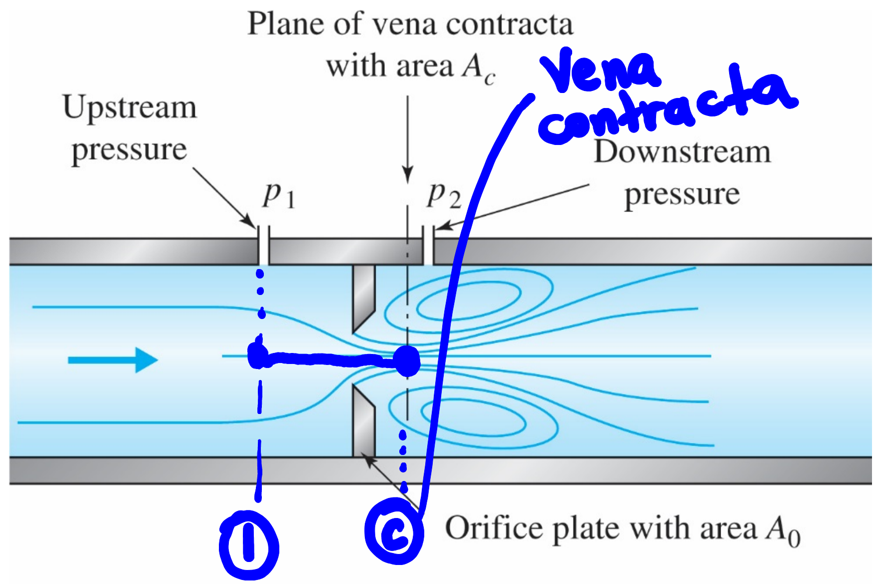


Fig. 13.8 Flow through an orifice meter.

$$h_1 = \frac{P_1}{\gamma} + z_1$$

$$h_c = \frac{P_c}{\gamma} + z_2$$

Combining these two Equations and solving for V_c yields

$$\boxed{V_c = \sqrt{\frac{2g(h_1 - h_c)}{1 - \left(\frac{A_c}{A_1}\right)^2}}}$$

ideal velocity at the vena contracta

13.3 Flow Rate Measurement (Cont.)

Ideal flow rate (Q_i)

$$Q_i = V_c A_c$$

$$Q_i = A_c \sqrt{\frac{2g(h_1 - h_c)}{1 - \left(\frac{A_c}{A_1}\right)^2}}$$

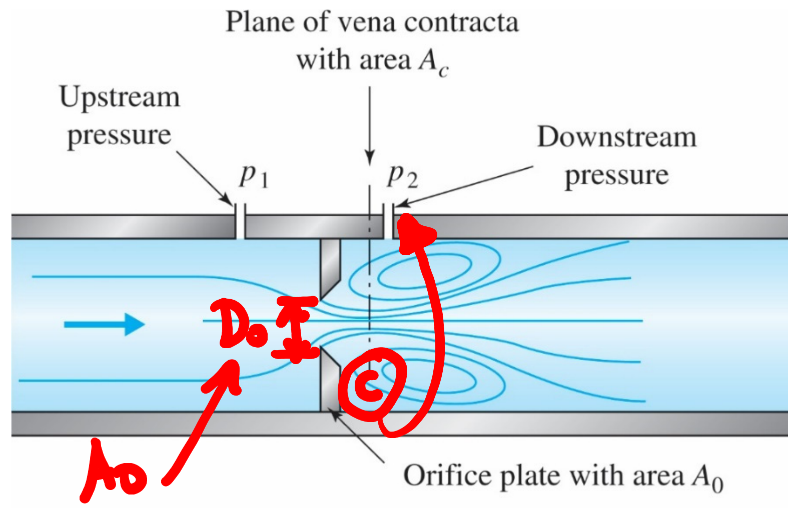


Fig. 13.8 Flow through an orifice meter.

Actual flow rate (Q_{actual})

$$\boxed{Q_{actual} = \frac{C_d A_o \sqrt{2g(h_1 - h_2)}}{\sqrt{1 - \left(\frac{C_c A_o}{A_1}\right)^2}}}$$

$C_c = \frac{A_c}{A_o}$, $C_d = C_c C_v$

C_c: contraction coeff.

C_v: velocity coeff.

C_d: discharge coefficient

13.3 Flow Rate Measurement (Cont.)

Orifice Meter

$$\boxed{Q_{actual} = K A_o \sqrt{2g(h_1 - h_2)}}$$

$$K = \frac{C_d}{\sqrt{1 - C_c^2 \beta^4}}$$

$$\beta = \frac{D_o}{D}$$

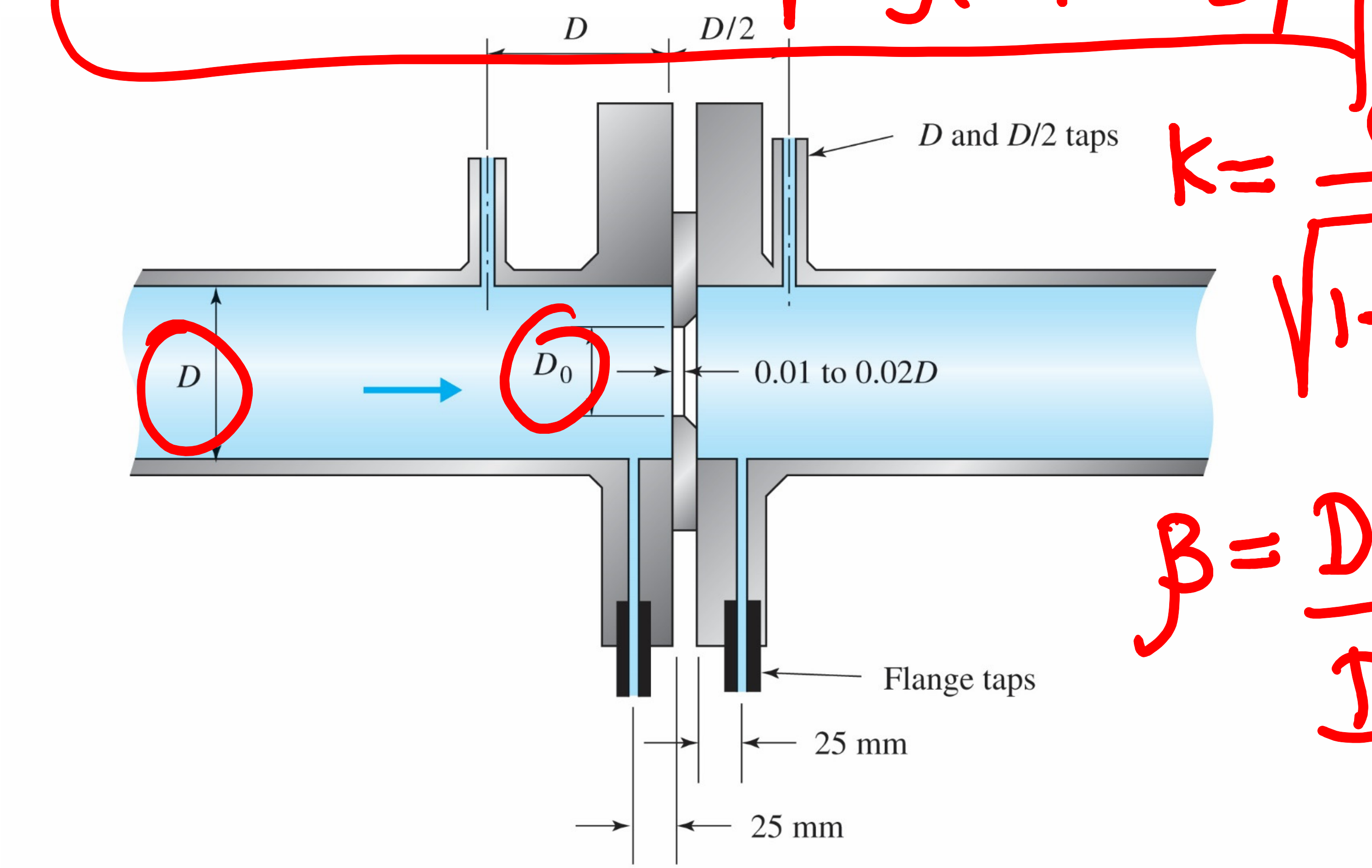


Fig. 13.9 Details of a thin-plate orifice meter. (FLUID MECHANICS MEASUREMENTS by G. E. Mattingly. Copyright 1996 by Taylor & Francis Group LLC-Books. Reproduced with permission of Taylor & Francis Group LLC-Books in the format Textbook via Copyright Clearance Center.)

13.3 Flow Rate Measurement (Cont.)

Venturi Meter

The venturi meter has a shape that attempts to mimic the flow patterns through a streamlined obstruction in a pipe.

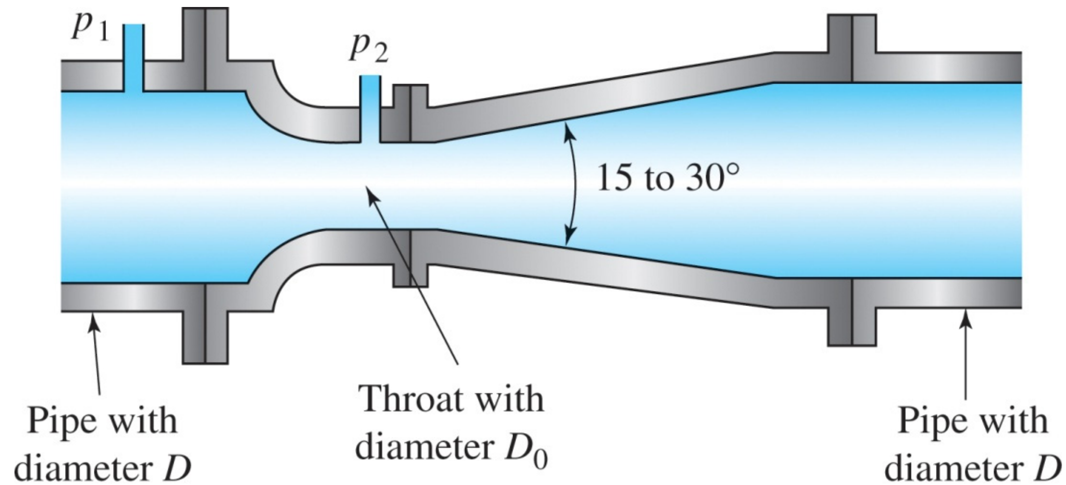


Fig. 13.11 Venturi meter.

Flow Nozzle

The flow nozzle consists of a standardized shape with pressure taps typically located one diameter upstream of the inlet and one-half diameter downstream.

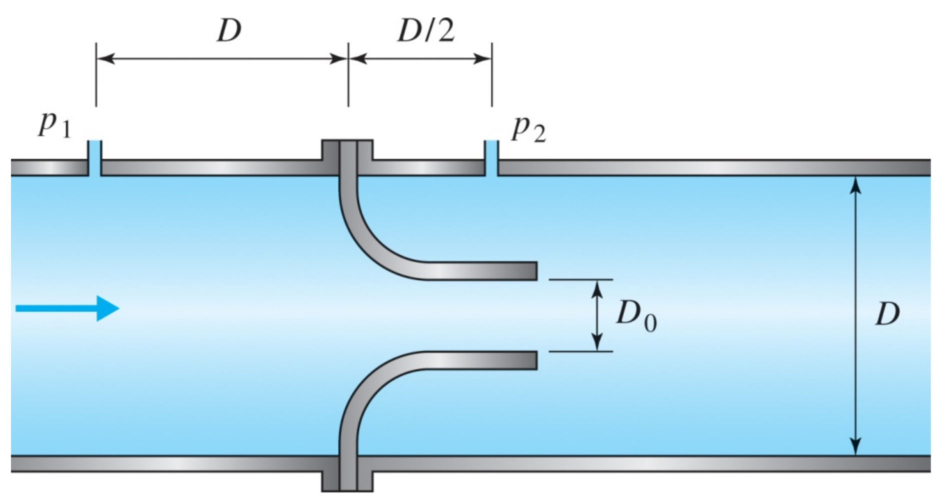


Fig. 13.12 Flow nozzle.

13.3 Flow Rate Measurement (Cont.)

Flow coefficient *K*

$D_o = 0.5\ m$

$D = 1\ m$

$\beta = 0.5$

$k = 1.0$

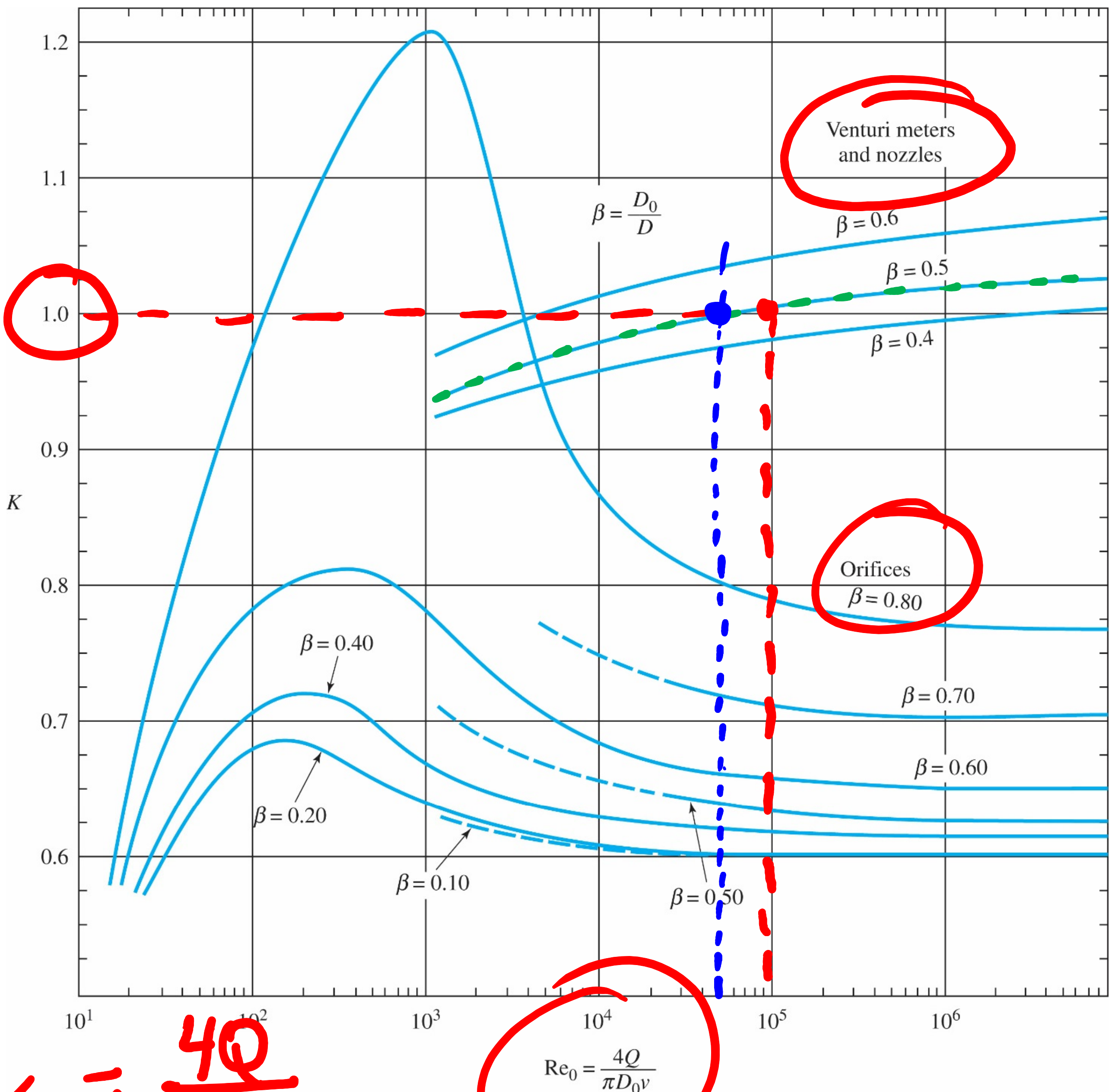
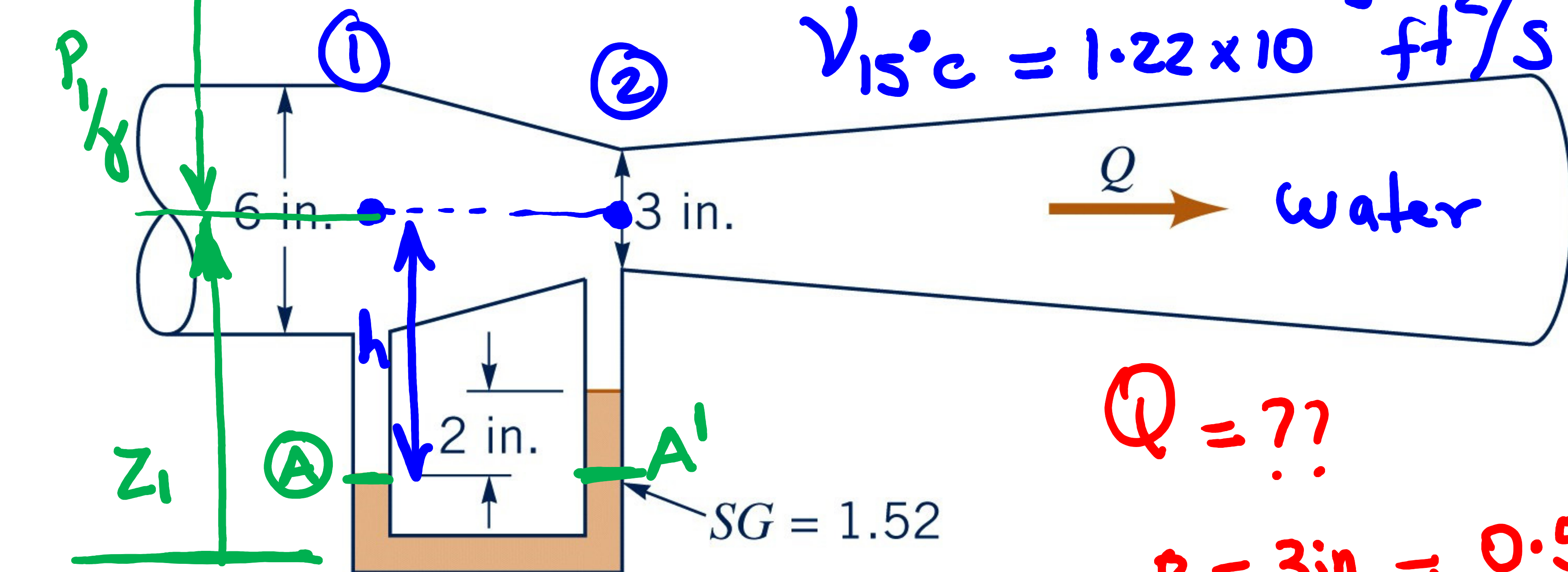


Fig. 13.10 Flow coefficient *K* versus the Reynolds number for orifices, nozzles, and venturi meters. (Adapted from Engineering Fluid Mechanics, Robertson and Crowe, © 1990 John Wiley & Sons, Inc., New York. Reproduced with permission of John Wiley & Sons, Inc.)

$$Re_o = \frac{V_o D_o}{\nu} = \frac{Q}{\pi D_o^{\cancel{2}}/4}\frac{\cancel{D_o}}{\nu} = \frac{4Q}{\pi D_o \nu}$$

Example of application: Water flows through the Venturi meter shown in the figure below. The specific gravity of the manometer fluid is 1.52. Determine the flowrate.

$\nu_{15^\circ C} = 1.22 \times 10^{-5}\ ft^2/s$



$Q = ??$

$\beta = \frac{3in}{6in} = 0.5$

$A_o = \frac{\pi}{4}\left(\frac{3}{12}\right)^2 = \frac{\pi}{64}$

$Q_{actual} = K A_o \sqrt{2g(h_1 - h_2)}$

here: $(z_1 = z_2)$

$h_1 = \frac{P_1}{\gamma} + z_1$

$h_2 = \frac{P_2}{\gamma} + z_2$

$\therefore h_1 - h_2 = \frac{P_1 - P_2}{\gamma_w}$

* from manometer $P_A = P_{A'}$

$$P_1 + \cancel{\gamma_w h} = P_2 + \gamma_w\left(\cancel{h} - \frac{2}{12}\right) + 1.52\,\gamma_w\left(\frac{2}{12}\right)$$

$$\frac{P_1 - P_2}{\gamma_w} = -2\frac{\cancel{\gamma_w}}{12\,\cancel{\gamma_w}} + 2\times 1.52\frac{\cancel{\gamma_w}}{12\,\cancel{\gamma_w}}$$

$$h_1 - h_2 = \frac{P_1 - P_2}{\gamma_w} = 0.08667$$

* K requires Re, $Re = f(Q)$

Guess K to find Q, Repeat process until convergence.

$$K = 1.0, \quad Q_{calculated} = 1.0 \times \frac{\pi}{64}\sqrt{2\times 32.2\,(0.08667)}$$

$$Q_{calc} = 0.116\ ft^3/s$$

$$Re_o = \frac{4Q}{\pi D_o \nu} = \frac{4 \times 0.116}{3.1416 \times \frac{3}{12} \times 1.22 \times 10^{-5}} = 48425$$

$K_{new} = 1.0$ Because $K_{guess} = K_{new}$,

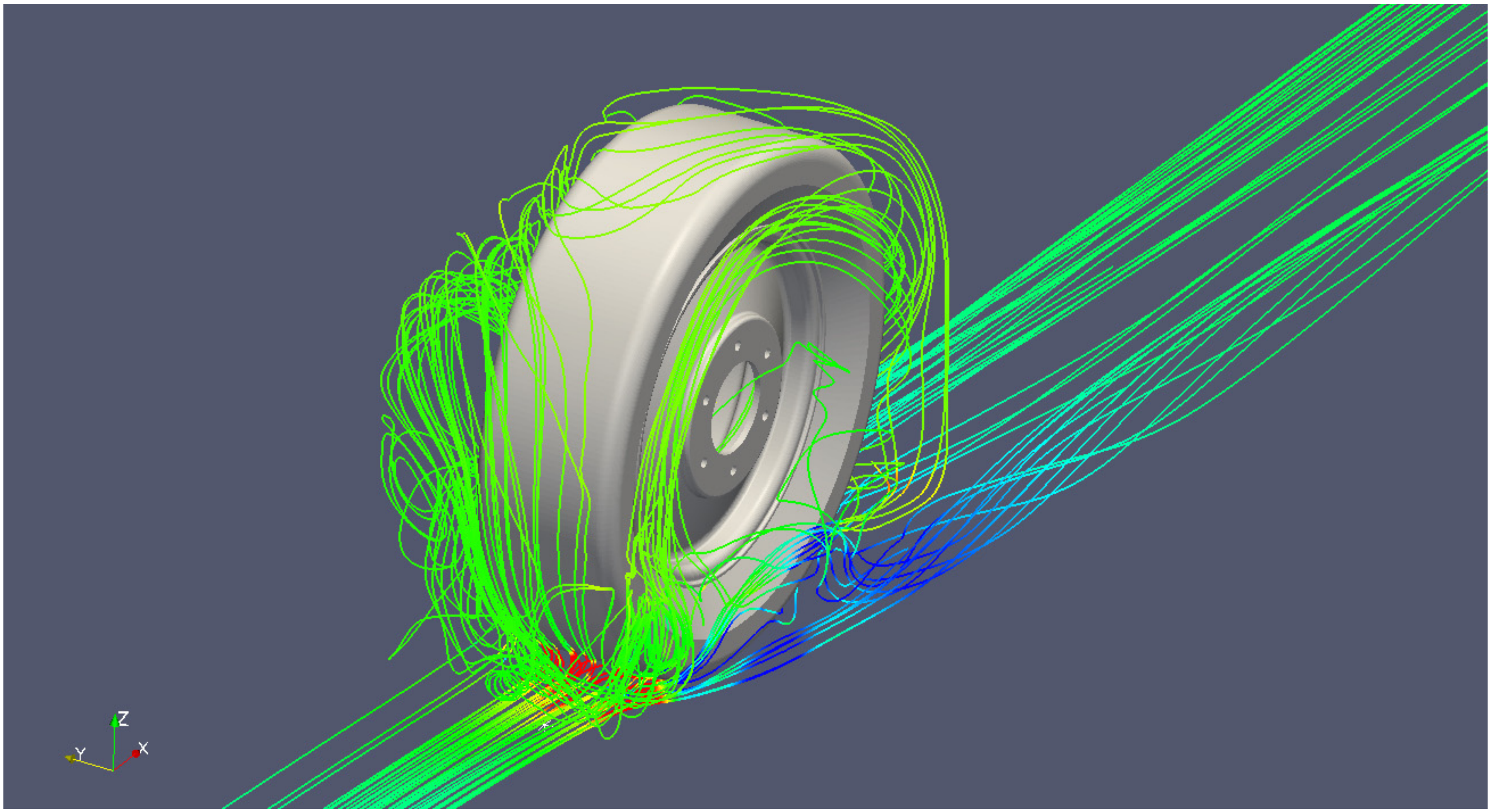
Then $Q_{actual} = 0.116 \ ft^3/s$

Let's say $K_{new} = 0.97 \rightarrow$ New Q using $K = 0.97$.

K
1.00
0.97
0.96
0.96

$Re \rightarrow K$

The Integral Forms of the Fundamental Laws



Arturo S. Leon, Ph.D., P.E., D.WRE

4.2 The Three Basic Laws

- The integral quantities in fluid mechanics are contained in the three laws:
  - Conservation of Mass
  - First Law of Thermodynamics
  - Newton's Second Law
- They are expressed using a Lagrangian description in terms of a system (fixed collection of material particles).

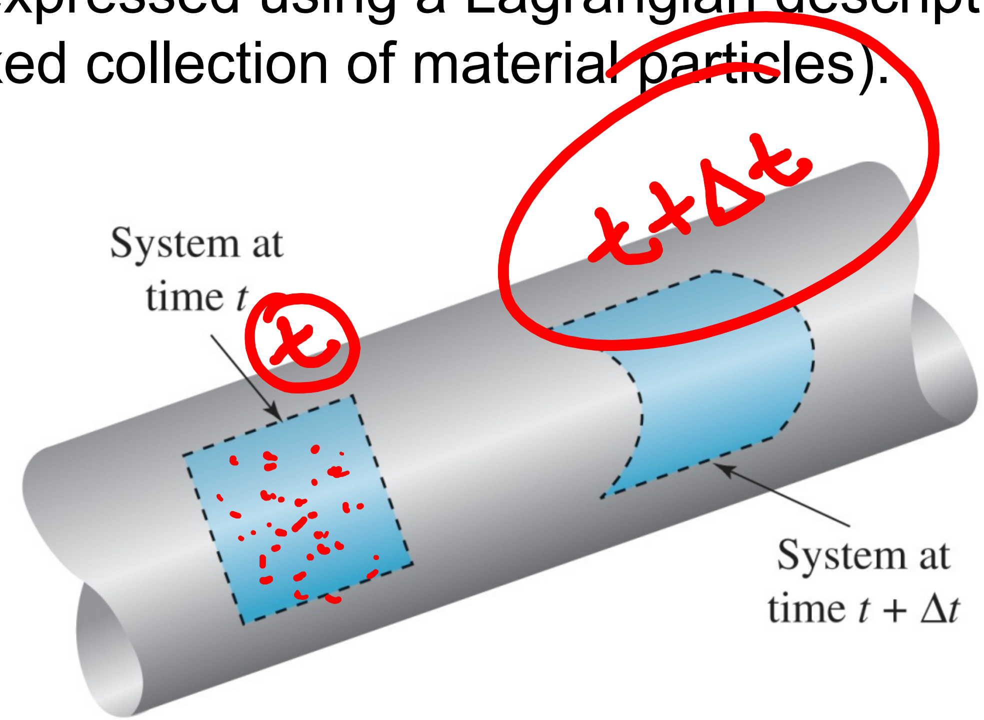


Fig. 4.1 Example of a system in fluid mechanics.

4.2 The Three Basic Laws

$\rho = \frac{m}{\forall}$

$m = \rho \forall$

- **CONSERVATION OF MASS: Mass of a system remains constant.**

$\int dm = \int \rho \, d\forall$

$\frac{DM}{Dt} = 0, \quad \frac{D}{Dt} \underbrace{\int \rho \, d\forall}_{1} = 0$

Integral form of the mass-conservation equation. ρ = Density; $d\forall$ = Volume occupied by the particle

- **FIRST LAW OF THERMODYNAMICS: Rate of heat transfer to a system minus the rate at which the system does work equals the rate at which the energy of the system is changing.**

$\dot{Q} - \dot{W} = \frac{D}{Dt} \underbrace{\int e \rho \, d\forall}_{e}$

Specific energy (e): Accounts for kinetic energy per unit mass ($0.5V^2$), potential energy per unit mass (gz), and internal energy per unit mass ($\tilde{u}$).

$\dot{Q}$ = Rate of heat transfer to the system

$\dot{W}$ = The rate at which the system does work

4.2 The Three Basic Laws

- **NEWTON'S SECOND LAW: Resultant force acting on a system equals the rate at which the momentum of the system is changing.**

  In an inertial frame of reference

$$\sum F = \frac{D}{Dt}\int \underbrace{V \rho \, d\forall}$$

momentum of a fluid particle is given by $V\,dm$

4.2 The Three Basic Laws

- **Control Volume**: A region of space into which fluid enters and/or from which fluid leaves.

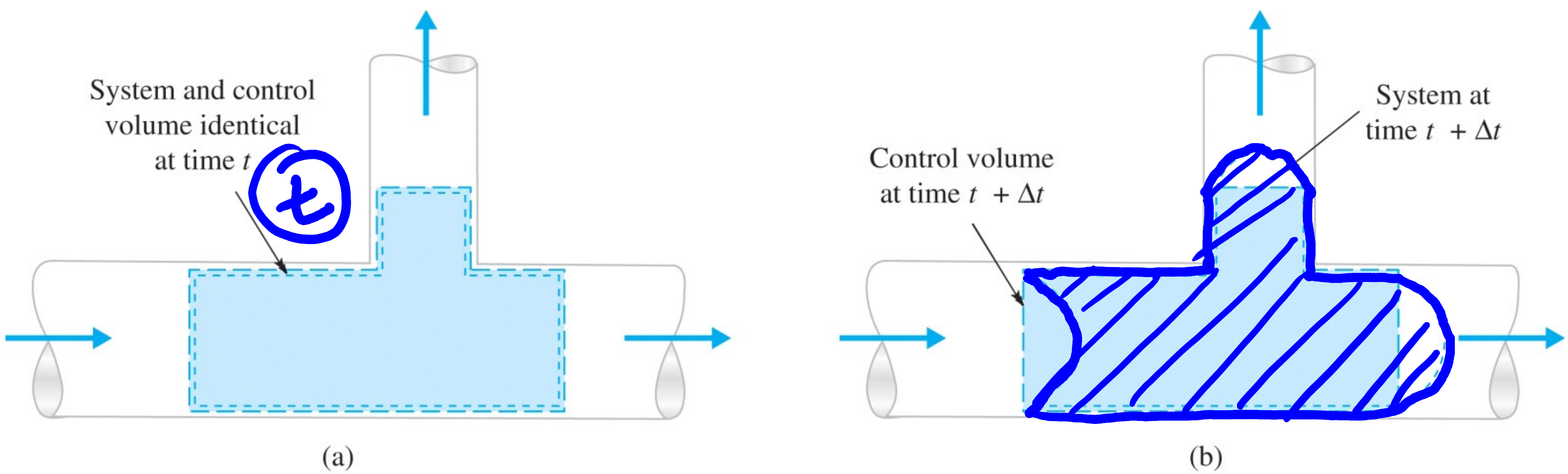


Fig. 4.2 Example of a fixed control volume and a system: (a) time t; (b) time $t + \Delta t$.

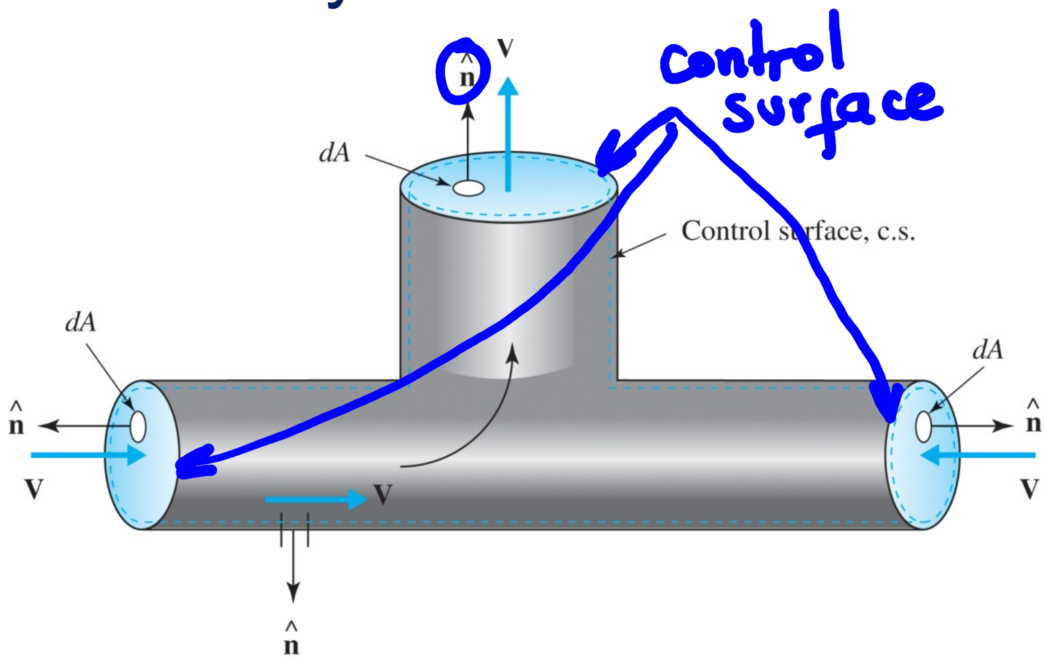
4.3 System-to-Control-Volume Transformation

- Interested in the time rate of change of an **extensive property** to be expressed in terms of quantities related to a control volume.
  - Involves fluxes of an extensive property in and out of a control volume.
  - **Flux** is the measure of the rate at which an extensive property crosses an area.

Extensive property N_{system}	Intensive property η
mass	1
energy	e
momentum	V

$$N_{system} = \int \eta \rho \, d\forall$$

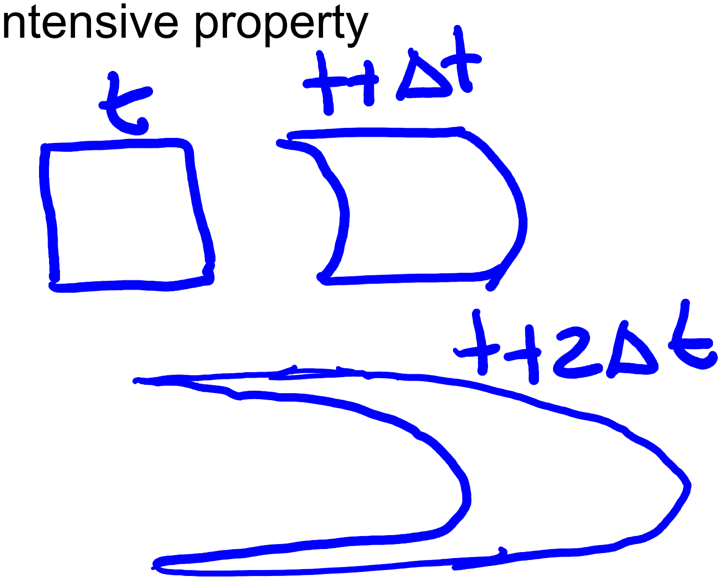
4.3 System-to-Control-Volume Transformation



Control surface: The surface area that completely **encloses** the control volume.

$\hat{n}$: Unit vector normal to *dA* (always points out of the control volume)

η: Intensive property



- The flux across an element *dA* is:

$$\text{flux} = \int \eta \rho \hat{n} \cdot \vec{V} \, dA$$

$\hat{n}$ = unitary vector

- Only the normal component of $\hat{n}$·V contributes to this flux.

example:

$$\text{flux of momentum} = \int V \rho \hat{n} \cdot \vec{V} \, dA$$

4.3 System-to-Control-Volume Transformation

Reynolds Transport Theorem

- The Reynolds transport theorem is a system-to-control-volume transformation

$$\boxed{\frac{DN_{syst}}{Dt} = \frac{d}{dt}\int_{c.v} \eta \rho \, d\forall + \int_{c.s} \eta \rho \, \hat{n} \cdot \vec{V} \, dA}$$

c.v. = control volume, c.s. = control surface

- This is a Lagrangian-to-Eulerian transformation of the rate of change of an extensive quantity.
  - First part of integral: Rate of change of an extensive property in the control volume.
  - Second part of integral: Flux of the extensive property across the control surface (nonzero where fluid crosses the control surface).

4.3 System-to-Control-Volume Transformation

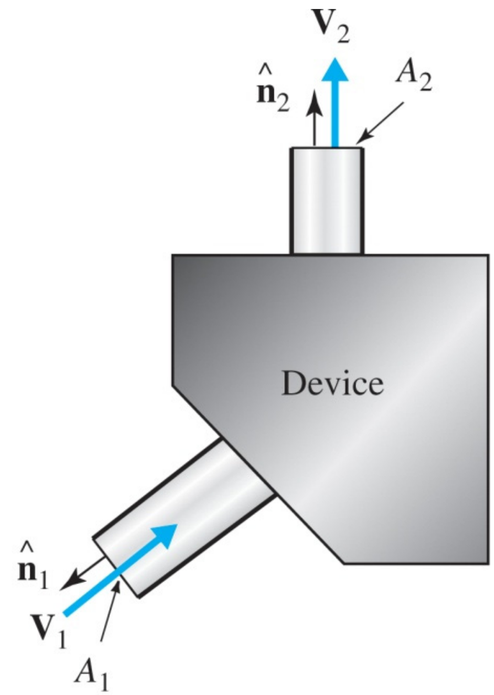
Reynolds Transport Theorem

- An equivalent form of the control volume is: Applying the Leibnitz rule:

$$\frac{DN_{syst}}{Dt} = \int_{C.V} \frac{\partial}{\partial t}(\rho \eta)\, d\forall + \int_{C.S} \eta \rho \hat{n} \cdot \vec{V}\, dA$$

- The time derivative of the control volume is moved inside the integral:
  - For a fixed control volume, the limits on the volume integral are independent of time.

4.3.1 Simplifications of the Reynolds Transport Theorem



- **Steady-state (time derivative is zero):**

$$\frac{DN_{syst}}{Dt} = \int_{c.s.} \eta \rho \hat{n} \cdot \vec{V} \, dA$$

(steady state)

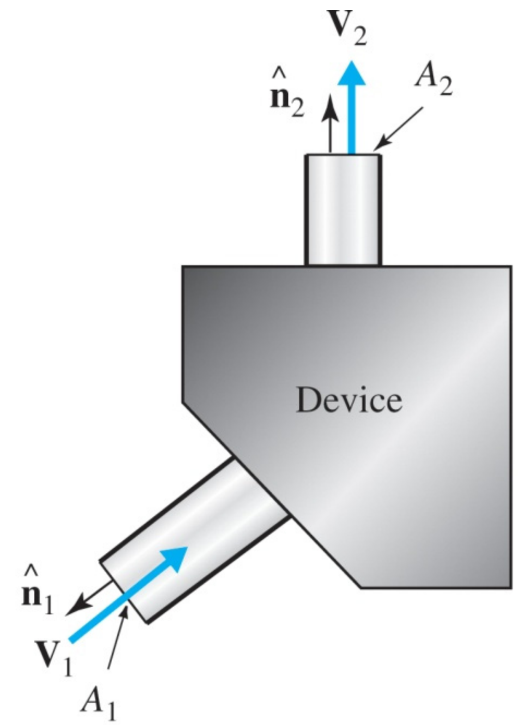
- **Often one inlet (A_1), and one outlet (A_2):**

$$\frac{DN_{syst}}{Dt} = \int_{A_2} \eta_2 \rho_2 V_2 \, dA - \int_{A_1} \eta_1 \rho_1 V_1 \, dA$$

- **For uniform properties over a plane area:**

$$\frac{DN_{syst}}{Dt} = \eta_2 \rho_2 V_2 A_2 - \eta_1 \rho_1 V_1 A_1$$

4.3.1 Simplifications of the Reynolds Transport Theorem (cont.)



- **Unsteady flow with uniform flow properties**:

$$\frac{DN_{syst}}{Dt} = \forall_{c.v.} \frac{d}{dt}(\eta \rho) + \eta_2 \rho_2 V_2 A_2 - \eta_1 \rho_1 V_1 A_1$$

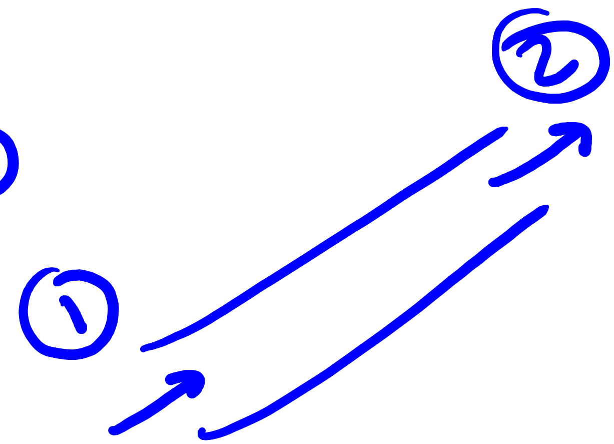
4.4 Conservation of Mass

$$\frac{D\,Mass}{Dt} = \frac{d}{dt}\int_{c.v.} \rho\, d\forall + \int_{c.s.} \rho\, \hat{n}\cdot\vec{V}\, dA$$

Mass of a system is fixed.

- For a steady flow, this simplifies to:

$$\frac{D\,Mass}{Dt} = \int_{c.s} \rho\, \hat{n}\cdot\vec{V}\, dA = 0$$



- Uniform flow with one entrance and one exit:

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For constant density, the continuity equation is only dependent on *A* and *V*

$$\underbrace{V_1 A_1}_{\text{flow rate}} = V_2 A_2 \quad (\text{only when } \rho \text{ is constant})$$

4.4 Conservation of Mass (Cont.)

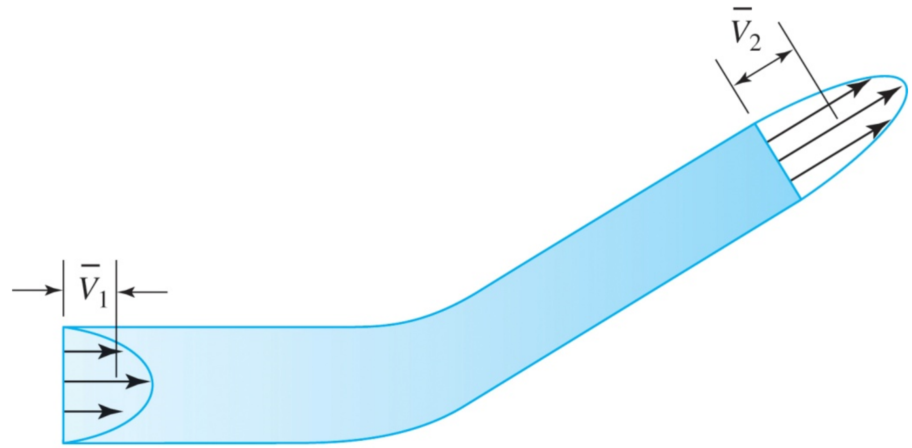


Fig. 4.7 Nonuniform velocity profiles.

- If the density is uniform over each area, with nonuniform velocity profiles:

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2$$

(averages can also be used)

- The *mass flux* $\dot{m}$ (kg/s or slug/s) is the mass rate of flow:

$$\dot{m} = \int \rho V_n dA \quad \sim \quad \dot{m} = \rho A V$$

  - Where V_n is the normal component of velocity.

4.4 Conservation of Mass (Cont.)

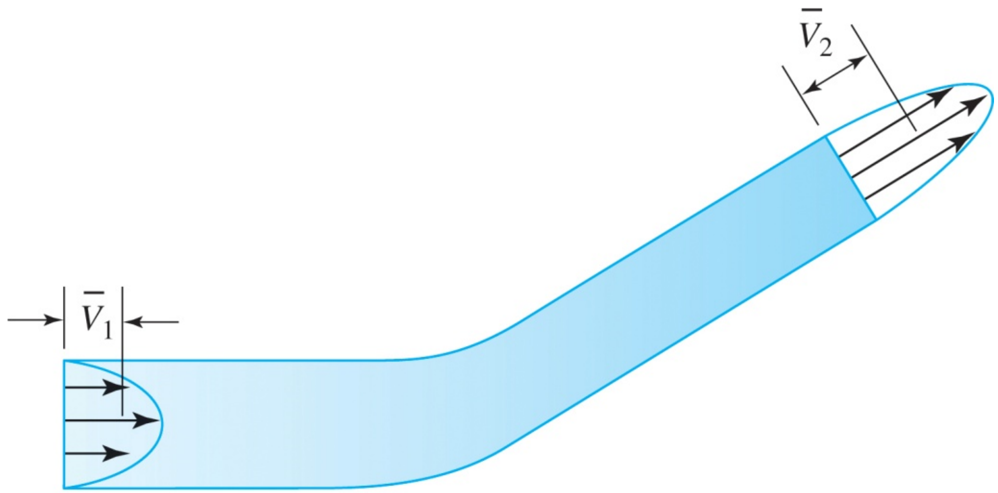


Fig. 4.7 Nonuniform velocity profiles.

- The *flow rate (or discharge) Q* (m^3/s or ft^3/s) is the volume rate of flow:

$$Q = \bar{V} \cdot A = \int_A V \, dA$$

  - Mass flow rate is often used in compressible flow. The flow rate is often used to specify incompressible flow.

Example: P.4.63. A 1-m diameter cylindrical tank initially contains liquid fuel and has a 2-cm diameter rubber plug at the bottom as shown in the figure below. If the plug is removed, how long will it take to empty the tank.

$$\frac{DM}{Dt} = \frac{d}{dt}\int_{c.v.} \rho \, d\forall \; +$$

$$\int V dA = Q$$

$$\int_{c.s.} \rho \, \hat{n}\cdot\vec{V} \, dA = 0$$

$$\cancel{\rho}\frac{d\forall}{dt} + \cancel{\rho}\int_{c.s.} \hat{n}\cdot\vec{V} \, dA = 0$$

$$\boxed{\frac{d\forall}{dt} + V_e A_e = 0} \quad \ldots\ldots (1)$$

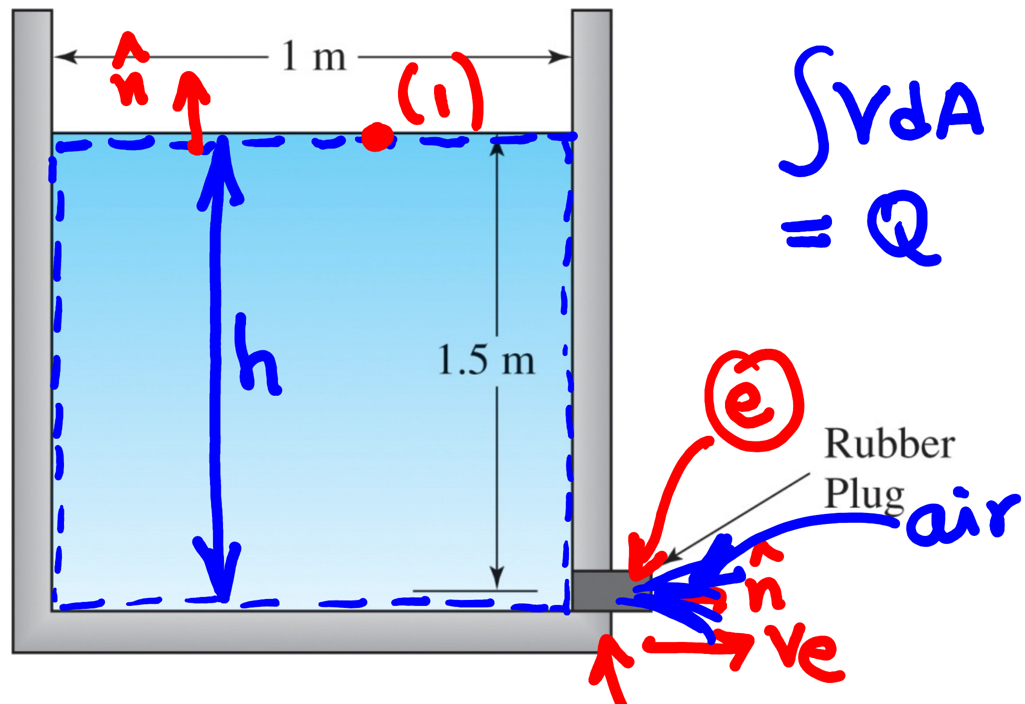


Fig. P4.63

$$\forall = \pi \times 0.5^2 h, \quad A_e = \frac{\pi \times 0.02^2}{4}$$

$V_e = ??$ Bernoulli Eq.

$$\cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0\,(\text{negligible})} + \underline{\underline{Z_1}} = \cancel{\frac{P_e}{\gamma}}^{0\,(\text{atmospheric})} + \frac{V_e^2}{2g} + \underline{\underline{Z_e}}$$

$$h = \frac{V_e^2}{2g} \rightarrow V_e = \sqrt{2gh}$$

In (1)

$$\cancel{\pi} \times 0.5^2 \frac{dh}{dt} + \sqrt{2g}\, h^{1/2} \times \cancel{\pi} \times 0.01^2 = 0$$

$$\int_{1.5}^{0} \underbrace{-\frac{0.5^2}{0.01^2\sqrt{2g}}}_{M} \underline{h^{-1/2}\, dh} = \int_0^t dt$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$-2Mh^{1/2}\Big|_{1.5}^{0} = \text{time}$$

$M = 564.7$

$$\boxed{\text{time} = 1383 \text{ seconds}}$$

4.5 Energy Equation

$\eta = e$

- This equation is required if heat is transferred (boiler/compressor) or work is done (pump/turbine).
  - Can relate pressures/velocities when Bernoulli's equation cannot be used.

$$\dot{Q} - \dot{W} = \frac{D}{Dt} \int_{system} e \rho \, d\forall$$

Where *e* is the specific energy and consists of the specific kinetic energy, specific potential energy, and specific internal energy.

$$e = \frac{V^2}{2} + gz + \tilde{u}$$

$\tilde{u}$: internal energy

- In terms of a control volume:

$$\dot{Q} - \dot{W} = \frac{d}{dt} \int_{c.v.} e \rho \, d\forall + \int_{c.s.} e \rho \, \hat{n} \cdot \vec{V} \, dA$$

- $\dot{Q}$: Rate-of-energy transfer across the control surface due to a temperature difference.
- $\dot{W}$: Work-rate term due to work being done by the system.

4.5 Energy Equation

4.5.1 Work-Rate Term

- The work-rate term is from the work being done by the system.
- Rate of work (Power) is the dot product of force with its velocity.

$$\dot{W} = P = -\vec{F} \cdot \vec{V}$$

The velocity is measured with respect to a fixed inertial reference frame. Negative sign is because work done on the control volume is negative.

- If the force is from variable shear stress over a control surface:

$$\dot{W} = -\int_{c.s.} \vec{\tau} \cdot \vec{V} \, dA$$

- $\vec{\tau}$ is a stress vector acting on an elemental area *dA*

4.5 Energy Equation

4.5.1 Work-Rate Term

$$\dot{W} = \int_{c.s.} p\,\hat{n}\cdot\vec{V}\,dA + \dot{W}_{shear} + \dot{W}_S + \dot{W}_I$$

pressure

The terms are summarized as follows:

$\int p\,\hat{\mathbf{n}}\cdot\mathbf{V}\,dA$	Work rate resulting from the force due to pressure moving at the control surface. It is often referred to as **flow work**.
$\dot{W}_S$	Work rate resulting from rotating shafts such as that of a pump or turbine, or the equivalent electric power.
$\dot{W}_{\text{shear}}$	Work rate due to the shear acting on a moving boundary, such as a moving belt.
$\dot{W}_I$	Work rate that occurs when the control volume moves relative to a fixed reference frame.

4.5 Energy Equation

4.5.2 General Energy Equation

- From the previous equation, the energy equation can be rewritten as:

$$\dot{Q}-\dot{W}_S-\dot{W}_{shear}-\dot{W}_I=\frac{d}{dt}\int_{C.V}e\rho\, d\forall+\int_{C.S.}\left(e+\frac{p}{\rho}\right)\rho\,\hat{n}\cdot\vec{V}\,dA$$

- Losses are the sum of all terms for unusable forms of energy

$$\text{Losses}=\frac{d}{dt}\int_{C.V.}\tilde{u}\rho\, d\forall+\int_{C.S}\tilde{u}\rho\,\hat{n}\cdot\vec{V}\,dA-\dot{Q}$$

  - Can be due to viscosity (causes friction resulting in increased internal energy).
  - Or due to changes in geometry resulting in separated flows.

4.5 Energy Equation

4.5.3 Steady Uniform Flow

- For steady-flow with one inlet and one outlet (with uniform profile) and no shear work, the following energy equation is used:

$$\dot{Q} - \dot{W}_s = \int \left(\frac{V^2}{2} + gz + \frac{p}{\rho} + \tilde{u} \right) \rho \hat{n} \cdot \vec{V} \, dA$$

$$\dot{m} = \rho A V$$

$$\frac{\dot{Q} - \dot{W}_s}{\dot{m} g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + \frac{p_2}{\gamma_2} - \frac{p_1}{\gamma_1} + z_2 - z_1 + h_L$$

head losses

- Where h_L is the head loss (dimensions of length).

$$h_L = \frac{\tilde{u}_2 - \tilde{u}_1}{g} - \frac{\dot{Q}}{\dot{m} g}$$

$$h_L = K \frac{V^2}{2g}$$

Where **K** is the loss coefficient

- $\frac{V^2}{2g}$ is the velocity head, and $\frac{p}{\gamma}$ is the pressure head.

4.5 Energy Equation

4.5.3 Steady Uniform Flow

- For steady-flow with one inlet and one outlet (with uniform profiles) and no shear work, negligible losses, and no shaft work:

$$\boxed{\frac{p_1}{\gamma_1} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma_2} + \frac{V_2^2}{2g} + z_2}$$

Identical to Bernoulli's equation for a constant density flow.

4.5 Energy Equation

4.5.3 Steady Uniform Flow

- If a turbine/pump is used, the efficiency of a device is needed, $\boldsymbol{\eta_T}$
  - The power generated by the turbine is:

$$\boxed{\dot{W}_T = \gamma Q H_T \eta_T}$$

η_T : efficiency of Turbine

  - The power required by a pump is:

$$\boxed{\dot{W}_P = \frac{\gamma Q H_P}{\eta_P}}$$

The power is calculated in Watts, ft-lb/s, or horsepower (1 Hp = 746 W = 550 ft-lb/s)

- The ***pump head***, H_P is the energy term associated for a pump $[\frac{\dot{W}_S}{\dot{m}g}]$. If a turbine is involved, the energy term is called the ***turbine head*** **(**H_T**)**.

4.5 Energy Equation

4.5.4 Steady Nonuniform Flow

- If a uniform velocity profile assumption cannot be used, the velocity distribution should be corrected:
  - Using a **kinetic-energy correction factor** α

Use α to correct non-uniform velocity profile

Fig. 4.7 Nonuniform velocity profiles.

$$\alpha = \frac{\int V^3 \, dA}{\bar{V}^3 \cdot A}$$

$\bar{V}$: average velocity

- The final equation that account for this nonuniform velocity distribution is:

$$\boxed{\frac{P_1}{\gamma} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2 + h_L}$$

$\alpha \approx 1.0$

Example: P.4.74. Find the velocity V_1 of the water in the vertical pipe shown in Figure P4.74. Assume no losses.

Bernoulli: ① – ②

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2 \quad \cdots (1)$$

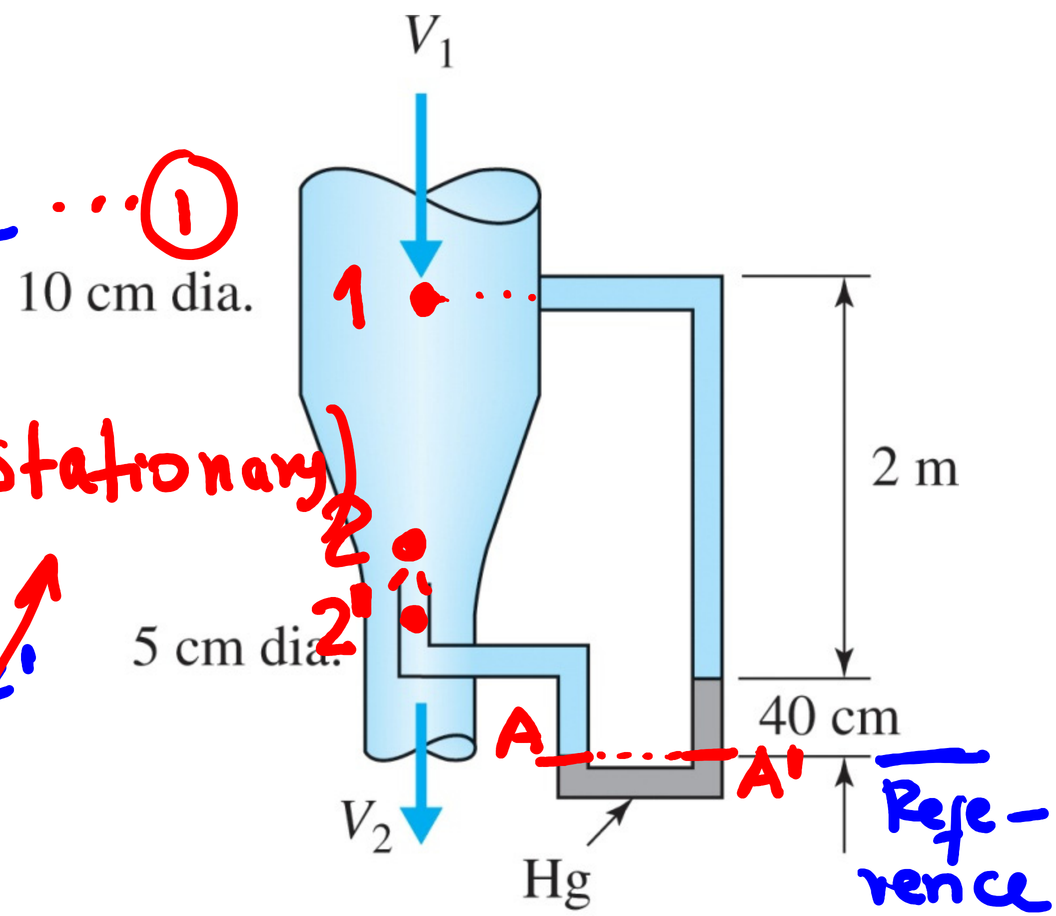


Fig. P4.74

Bernoulli: ② – ②'

$$\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+\cancel{z_2}=\frac{P_2'}{\gamma}+\cancel{\frac{V_2'^2}{2g}}+\cancel{z_2'}$$

$$P_2' = P_2+\rho\frac{V_2^2}{2}$$

In (1)

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2'}{\gamma}+z_2 \qquad (2)$$

* Manometers $P_A = P_{A'}$

$$\frac{P_{2'}}{\gamma_w} + \frac{\cancel{\gamma_w} Z_{2'}}{\cancel{\gamma_w}} = \frac{P_1}{\gamma_w} + \frac{\cancel{\gamma_w}(2)}{\cancel{\gamma_w}} + \frac{13.6\cancel{\gamma_w}(0.4)}{\cancel{\gamma_w}} \quad \ldots 3$$

$2 = 3$

$$\cancel{\frac{P_1}{\gamma_w}} + \frac{V_1^2}{2g} + \underset{2.4}{Z_1} = \cancel{\frac{P_1}{\gamma_w}} + 2 + 13.6(0.4)$$

$$\frac{V_1^2}{2\times 9.8} = 2 + 13.6(0.4) - 2.4$$

$$\boxed{V_1 = 9.94\ m/s}$$

4.6 Momentum Equation

4.6.1 General Momentum Equation

- Newton's second law (momentum equation): The resultant force acting on a system equals the rate of change of momentum of the system in an inertial reference frame.

$$\Sigma F = \frac{D}{Dt}\int_{syst} \rho V \, d\forall$$

- For a control volume:

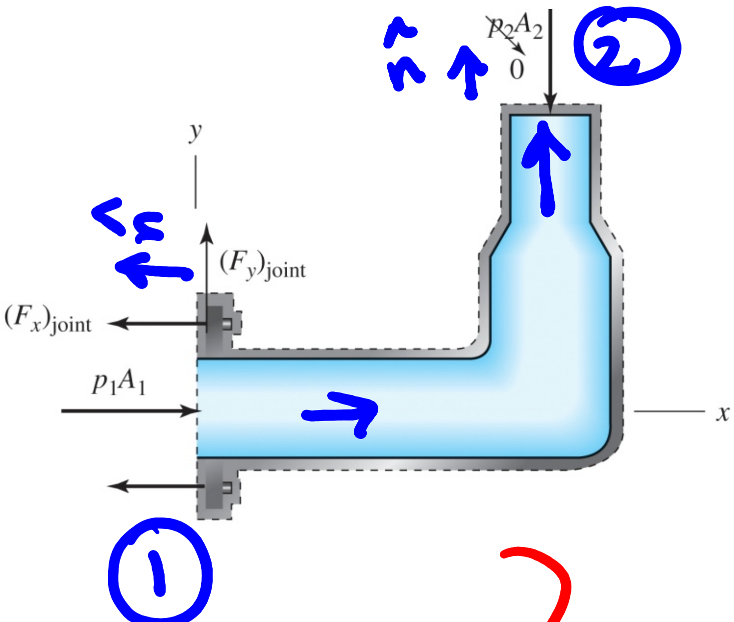
$$\Sigma F = \frac{d}{dt}\int_{c.v.} \rho V \, d\forall + \int_{c.s.} \rho V (\vec{V}\cdot\hat{n})\, dA$$

4.6 Momentum Equation

4.6.2 Steady Uniform Flow

- If flow is uniform and steady, for ***N* number of entrances and exits**, the previous equation can be simplified to:

$$\Sigma F = \sum_{i=1}^{N} \rho_i A_i V_i (V_i \cdot \hat{n})$$



Horizontal nozzle with one entrance and one exit

The momentum equation simplifies to:

$$\Sigma F = \underbrace{\rho_2 A_2 V_2}(V_2) - \underbrace{\rho_1 A_1 V_1}(V_1)$$

With continuity: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2 = \dot{m}$

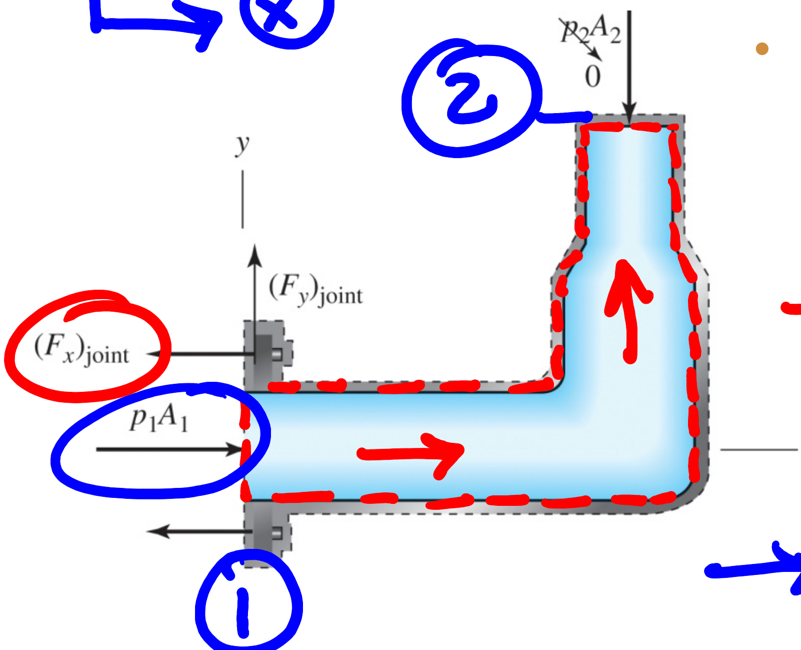
$$\Sigma F = \dot{m}(V_2) - \dot{m} V_1$$

$$\boxed{\Sigma F = \dot{m}(V_2 - V_1)}$$

$$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$$

$$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$$

4.6.2 Steady Uniform Flow



Horizontal nozzle with one entrance and one exit

- To determine the *x*-component of the force of the joint on the nozzle:

$$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$$

$$-F_{x\,joint} + P_1A_1 = \dot{m}(0 - V_1)$$

$$\boxed{F_{x\,joint} = P_1A_1 + \dot{m}V_1}$$

As $(V_1)_x = V_1$ and $(V_2)_x = 0$

- To determine the *y*-component of the force of the joint on the nozzle:

$$\Sigma F_y = \dot{m}(V_{2y} - V_{1y})$$

$$F_{y\,joint} - P_2A_2 = \dot{m}(V_2 - 0)$$

$$\boxed{F_{y\,joint} = \dot{m}V_2 + P_2A_2}$$

4.6 Momentum Equation

4.6.2 Steady Uniform Flow

- To find the force of the gate on the flow:

$$F_1 = \gamma \bar{h} A = \gamma \frac{h_1}{2}(h_1 b) = \gamma \frac{h_1^2}{2} b$$

$$F_2 = \gamma \bar{h} A = \gamma \frac{h_2}{2}(h_2 b) = \gamma \frac{h_2^2}{2} b$$

$$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$$

$$F_1 - F_2 - F_{gate} = \dot{m}(V_2 - V_1)$$

$$\boxed{F_{gate} = F_1 - F_2 - \dot{m}(V_2 - V_1)}$$

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

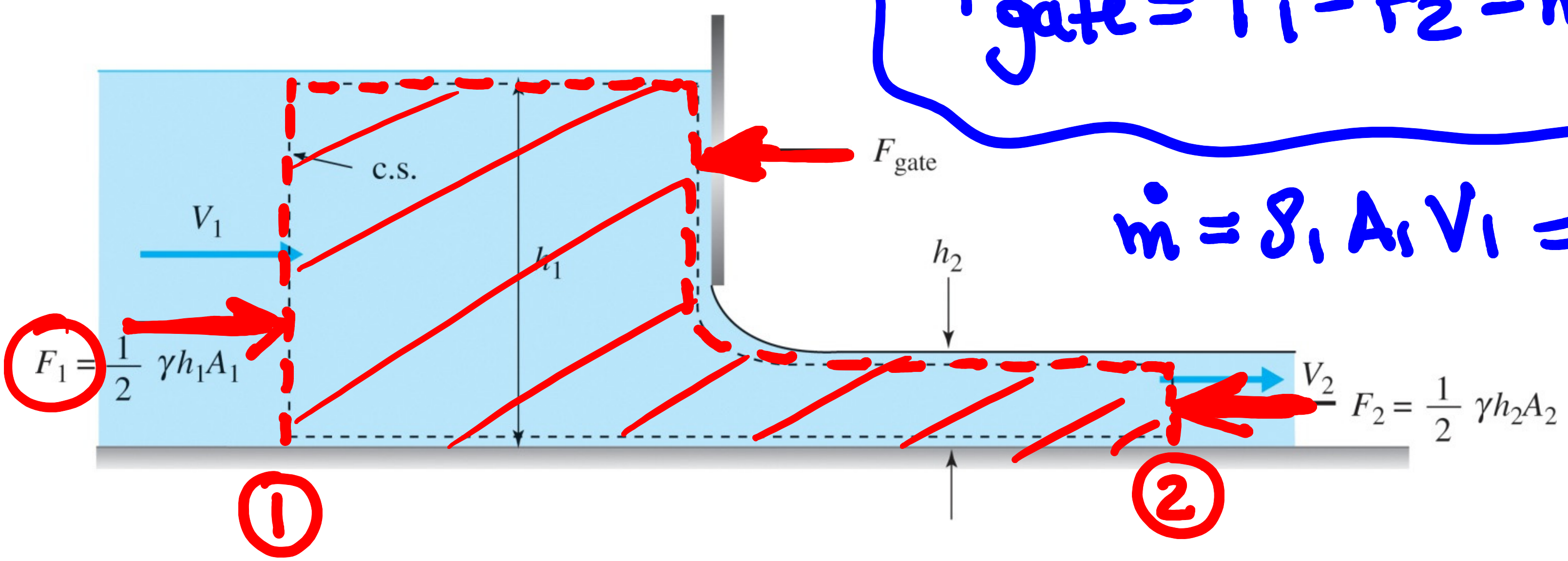


Fig. 4.12 Force of the flow on a gate in a free-surface flow.

Example: P4.124. Assuming hydrostatic pressure distributions, uniform velocity profiles, and negligible viscous effects, find the horizontal force needed to hold the sluice gate in the position shown in Fig. P4.124.

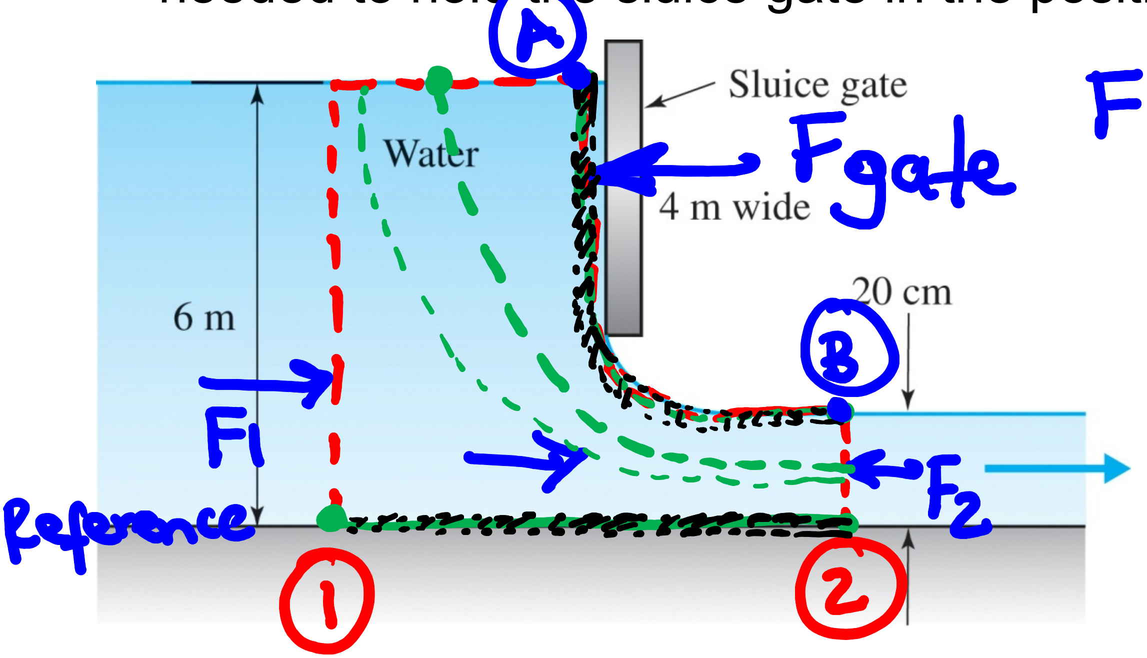


Fig. P4.124

$$F_1 = \frac{1}{2}\gamma h_1^2 b = \frac{1}{2}(1000)(9.81)6^2(4) = 706{,}320\ N$$

$$F_2 = \frac{1}{2}\gamma h_2^2 b = 784.8\ N$$

$$\Sigma F_x = \dot{m}(V_{2x} - V_{1x})$$

$$F_1 - F_2 - F_{gate} = \dot{m}(V_2 - V_1) \quad \cdots\cdots (1)$$

$$\dot{m} = \cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 V_2$$

* Bernoulli Eq. (top streamline)

$$\cancelto{0}{\frac{P_A}{\gamma}} + \frac{V_A^2}{2g} + Z_A = \cancelto{0}{\frac{P_B}{\gamma}} + \frac{V_B^2}{2g} + Z_B$$

$$\boxed{\frac{V_1^2}{2g} + 6 = \frac{V_2^2}{2g} + 0.2} \quad \ldots (2)$$

* $A_1 V_1 = A_2 V_2 \; (\rho_1 = \rho_2)$

$$6\cancel{6} V_1 = 0.2\cancel{6} V_2 \rightsquigarrow \boxed{V_2 = 30 V_1} \quad \ldots (3)$$

(3) in (2)

$V_1 = 0.356 \ m/s$

$V_2 = 10.67 \ m/s$

$$\dot{m} = \rho_1 A_1 V_1 = 8544$$

In (i)

$$F_{gate} = F_1 - F_2 - \dot{m}(V_2 - V_1)$$

$$F_{gate} = 706{,}320 - 784.8 - 8544(10.67 - 0.356)$$

$$F_{gate} = 617{,}412 \text{ N}$$

$$= 617 \text{ kN}$$

4.7 Moment-of-Momentum Equation

- Needed to find the line of action of a given force component.
- Needed to analyze flow situations in devices with rotating components (to relate rotational speed to other flow parameters)

4.7 Moment-of-Momentum Equation

- The general equation with attached inertial forces is:

$$\sum M - M_I = \frac{D}{Dt}\int_{sys} \vec{r} \times \vec{V} \rho \, d\forall$$

$$M_I = \int \vec{r} \times \left[\frac{d^2 S}{dt^2} + 2\Omega \times \vec{V} + \Omega \times (\Omega \times \vec{r}) + \frac{d\Omega}{dt} \times \vec{r} \right] \rho \, d\forall$$

M_I is the inertial moment that accounts for the noninertial reference frame.

4.7 Moment-of-Momentum Equation

- When a system-to-control volume transformation is applied, the moment-of-momentum equation becomes:

$$\sum M - M_I = \frac{d}{dt}\int_{C.V.} \vec{r}\times\vec{V}\,\rho\, d\forall + \int_{C.S.} \vec{r}\times\vec{V}(\vec{V}\cdot\hat{n})\,\rho\, dA$$

Example: Water flows out the 6-mm slots as shown in Fig. P4.166. Calculate Ω if 20 kg/s is delivered by the two arms.

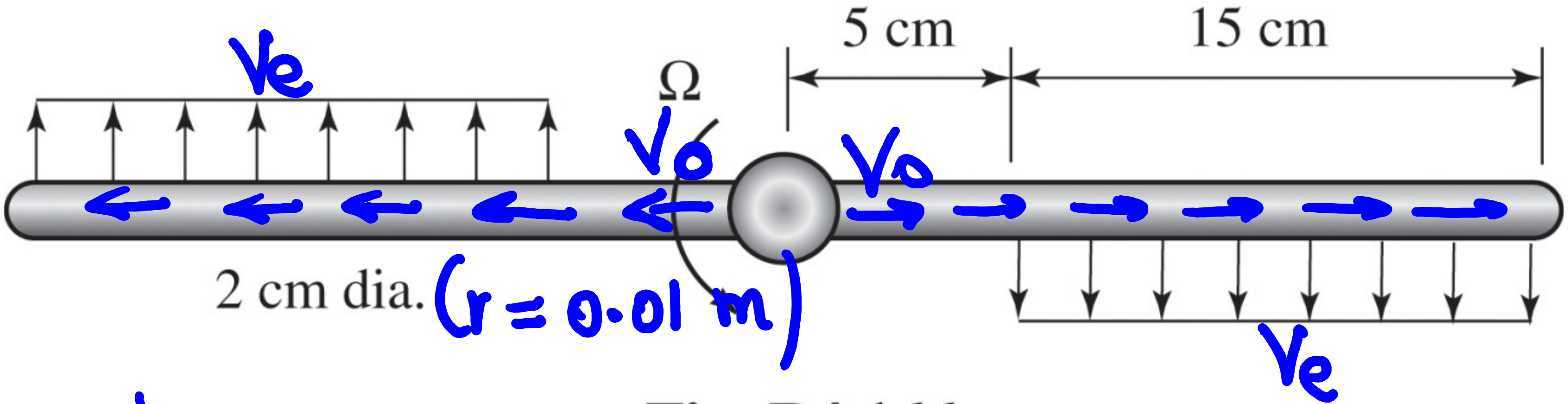


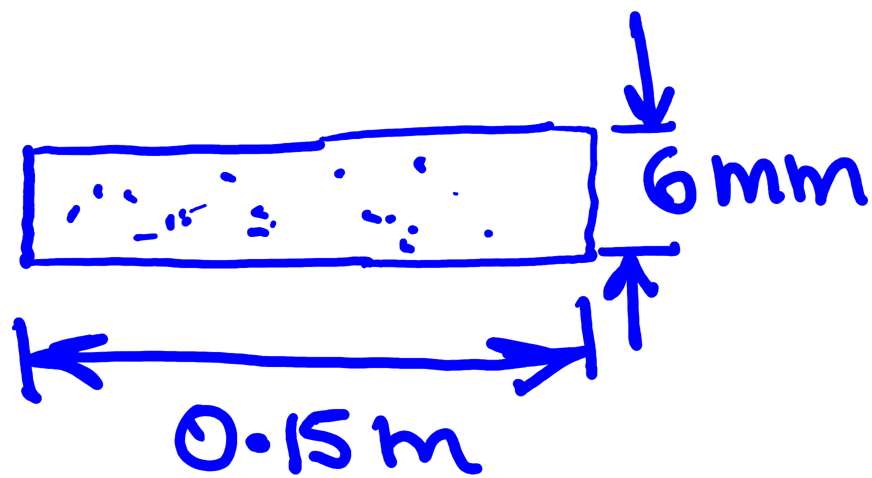
Fig. P4.166

For each arm

$$\dot{m} = \frac{20}{2} = 10 \text{ kg/s}$$

$$10 = \rho A V = 1000\, \pi \times 0.01^2 \times V_o$$

$$V_o = 31.8 \text{ m/s}$$



* Continuity

Flow in = Flow out

$$V_o A_o = V_e A_e$$

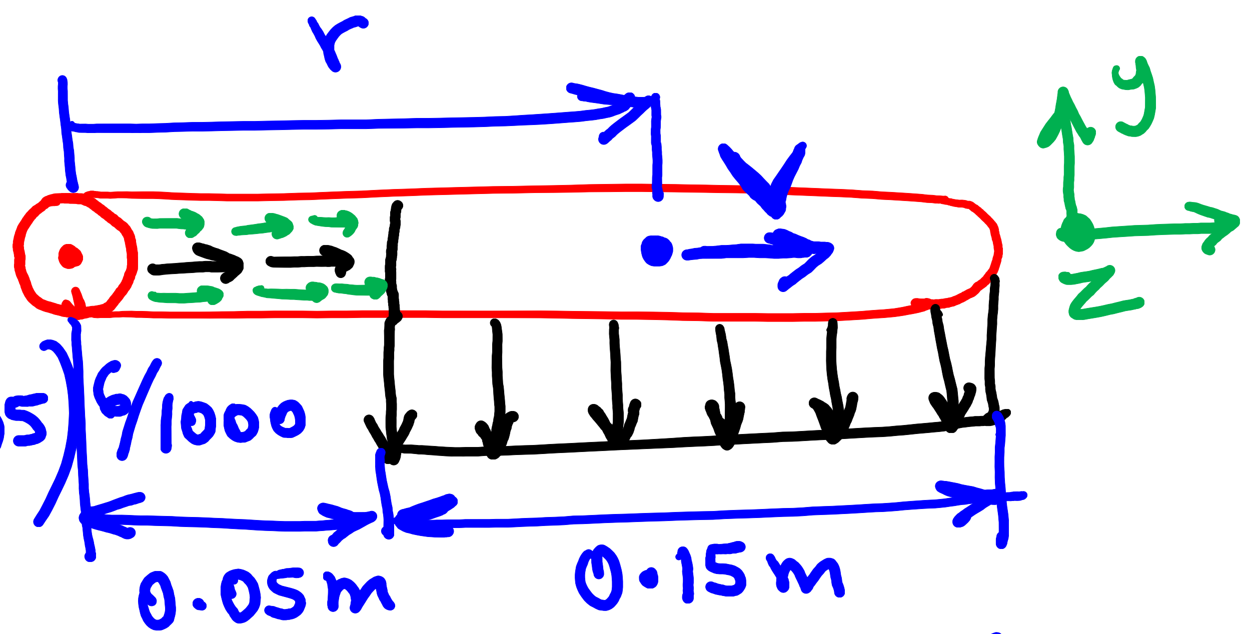
$$31.8 \times \pi \times 0.01^2 = V_e \left(\frac{6}{1000}\right)(0.15)$$

$$\boxed{V_e = 11.1 \text{ m/s}}$$

$$d\forall = A\,dr$$

* Continuity:

$$V_o A_o = V A_o + V_e (r - 0.05)\, 6/1000$$



$$31.8\, \pi \times 0.01^2 = V\, \pi \times 0.01^2 + 11.1 \left(\frac{6}{1000}\right)(r - 0.05)$$

$$V = (42.4 - 212 r)\,\hat{i} \quad (r \geq 0.05\,m)$$

$$V = 31.8\ m/s \quad (r < 0.05\,m)$$

$$\frac{d}{dt}(\ ) = 0$$

AA

$$* \ \cancel{\sum M} - M_I = \cancel{\frac{d}{dt}\int_{c.v.} \vec{r} \times \vec{V}\,\rho\, d\forall} + \int_{c.s.} \vec{r} \times \vec{V}\,(\vec{V}\cdot\hat{n})\,\rho\, dA \quad \ldots (1)$$

0 0 0

$$M_I = \int \vec{r} \times \left[\cancel{\frac{d^2 S}{dt^2}} + 2\Omega \times \vec{V} + \cancel{\Omega \times (\Omega \times \vec{r})} + \cancel{\frac{d\Omega}{dt} \times \vec{r}} \right] \rho\, d\forall$$

0 0 0

(stationary reference)

(Ω = constant)

i j k i j k

$$\vec{r} = r\hat{i} \qquad r\hat{i} \times \Omega\hat{k} \times (\Omega_k \times r\hat{i}) = 0$$

$$\Omega = \Omega\hat{k}$$

j

$-\hat{i}$

0

$$M_I = \int \vec{r} \times (2\Omega \times \vec{V})\rho\, d\forall$$

ijkij

$$M_I = \int_0^{0.05} r\hat{i} \times (2\Omega\hat{k} \times V_0\hat{i})\rho A_0\, dr +$$

$$\int_{0.05}^{0.20} r\hat{i} \times (2\Omega\hat{k} \times (42.4 - 212r)\hat{i})\rho A_0\, dr$$

$$M_I = 2\Omega V_0 \rho A_0 \int_0^{0.05} r\,dr + 2\Omega \rho A_0 \int_{0.05}^{0.2} r(42.4 - 212r)\,dr$$

$V_0 = 31.8$ m/s

$\rho = 1000$ kg/m^3

$A_0 = \pi \times 0.01^2$

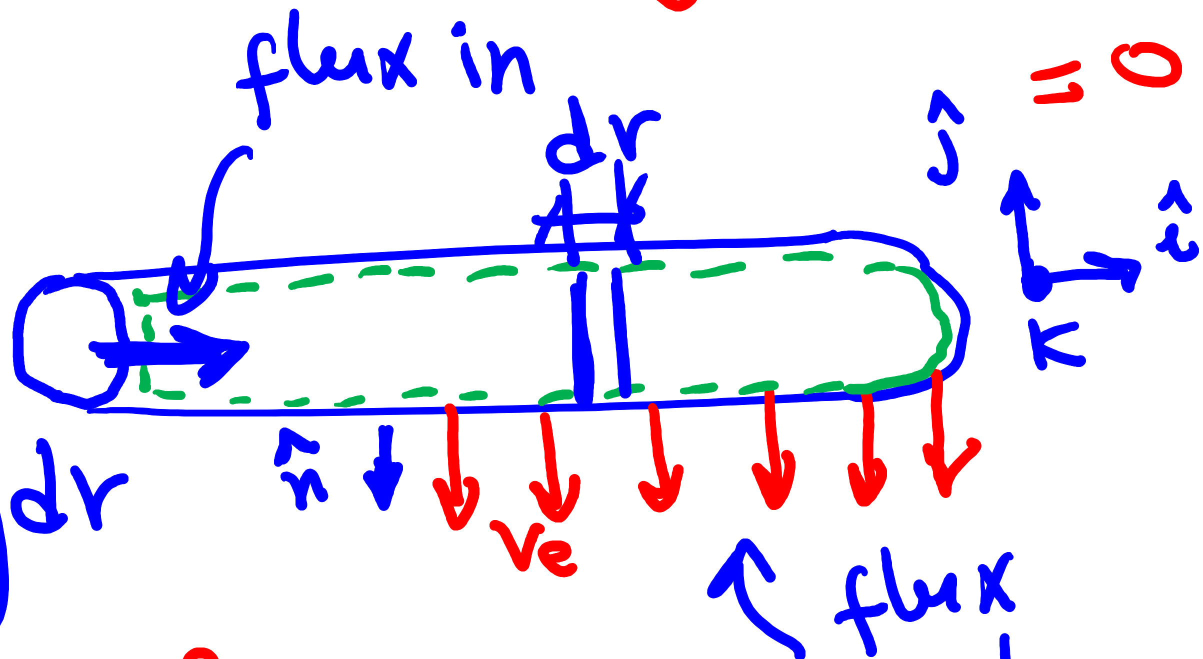
$M_I = 0.175\,\Omega\,\hat{k}$ [for one arm]

For two arms: $M_I = 0.35\,\Omega\,\hat{k}$

$$AA = \int_{c.s.} \vec{r} \times \vec{V} (\vec{V} \cdot \hat{n}) \rho \, dA$$

flux in

$$\int r\hat{i} \times V_0 \hat{i} (\ ) = 0$$



flux out:

$$\int_{0.05}^{0.2} r\hat{i} (-V_e \hat{j}) V_e \rho \left(\frac{6}{1000}\right) dr$$

$$= -\frac{27.72}{2} \hat{k} \text{ (for one arm)}$$

$$dA = \left(\frac{6}{1000}\right) dr$$

For two arms: $-27.72 \hat{k}$

$$0 - 0.35 \Omega \hat{k} = -27.72 \hat{k}$$

$$\Omega = 27.72 / 0.35 = 79.2 \text{ rad/s}$$