Florida International University Department of Civil and Environmental Engineering
CWR 3201 Fluid Mechanics,
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Homework Assignment 8 Solutions

Mechanics of Fluids (Fifth edition), by M.C. Potter, D.C. Wiggert and B.H. Ramadan.

1. $A = \frac{Q}{V} = \frac{75}{1.75} = 42.86\, m^2$

Efficient Trapezoid designs are usually when half of the top width is equal to one of the sloping sides:

$$\frac{B + 2lh}{2} = h\sqrt{l^2 + 1} = \frac{B + (2 * 2) * h}{2} = h\sqrt{2^2 + 1}$$

$$B = 0.472h$$

$$where\ l\ is\ the\ horizontal\ of\ the\ side\ slope\ and\ h\ is\ the\ height\ of\ the\ channel$$

$$Area\ of\ a\ trapizoid = \frac{(B + 2lh)h}{2}$$

$$42.86 = \frac{(0.472h + 2 * 2 * h)h}{2}$$

$$h = 4.38\ m$$

$$B = 2.07\ m$$

If flow is uniform, $C = \frac{1}{n} m^{1/6}\ where\ m = \frac{h}{2}$

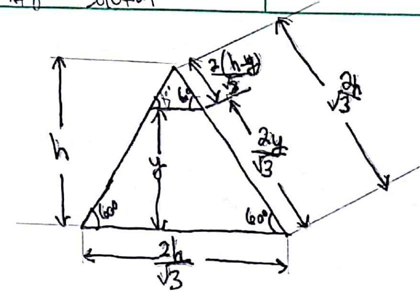
$$C = \frac{1}{0.03}(\frac{4.38}{2})^{1/6} = 38$$

$$Q = AC\sqrt{m * i}$$

$$75 = 42.86 * 38 * \sqrt{2.19 * i}$$

$$i = 0.0016$$

HW #8 Solution THAO DO



MANNING'S EQN. FOR VELOCITY : $V = \frac{k}{n}(R_h)^{2/3}(S_0)^{1/2}$

FLOW RATE: $Q = VA = \frac{k}{n}(R_h)^{2/3}(S_0)^{1/2} * A$

REARRANGE : $Q = \frac{k}{n}(S_0)^{1/2} * ((R_h)^{2/3} * A)$

$Q = C * A * (R_h)^{2/3}$

WHEN PIPE IS HALF-FULL: R_h = HYDRAULIC RADIUS

CROSS-SECTIONAL AREA : $A = \frac{1}{2}bh = \frac{1}{2}\left(\frac{2h}{\sqrt{3}} + \frac{2(h-y)}{\sqrt{3}}\right)y = \left(\frac{2h-y}{\sqrt{3}}\right)y$

WETTED PERIMETER : $P = 2 * \frac{2y}{\sqrt{3}} + \frac{2h}{\sqrt{3}} = \frac{2h+4y}{\sqrt{3}}$

$$R_h = \frac{\left(\frac{2h-y}{\sqrt{3}}\right)y}{\left(\frac{2h+4y}{\sqrt{3}}\right)} = \frac{2hy - y^2}{2h+4y}$$

PLUGGING BACK : $Q = C * \left[\left(\frac{2h-y}{\sqrt{3}}\right)y\right] * \left[\frac{2hy-y^2}{2h+4y}\right]^{2/3}$

AT MAX FLOW RATE, $\frac{dQ}{dy} = 0 \Rightarrow \frac{d}{dy}\left[C\left(\frac{2h-y}{\sqrt{3}}\right)y * \left(\frac{2hy-y^2}{2h+4y}\right)^{2/3}\right] = 0$

$$\Rightarrow \left(\frac{2hy-y^2}{\sqrt{3}}\right) * \frac{2}{3}\left(\frac{2hy-y^2}{2h+4y}\right)^{-1/3} \frac{(2h+4y)(2h-2y)-(2hy-y^2)*4}{(2h+4y)^2} + \left(\frac{2hy-y^2}{2h+4y}\right)^{2/3}\left(\frac{2h-2y}{\sqrt{3}}\right) = 0$$

SIMPLIFY $\rightarrow$ $4h^2 - 4hy - 6y^2 = 3(2h+4y)(y-h)$

$4h^2 - 4hy - 4y^2 = -6h^2 - 6hy + 12y^2$

$10h^2 + 2hy - 16y^2 = 0$

$y = 0.856h$

$\therefore \frac{y}{h} = 0.856$

2.

3. 10.8

$$Area\ of\ gutter = by + 0.5y^2(m_1 + m_2)$$
$$b\ and\ m_2 = 0, m_1 = 8$$
$$Area\ of\ gutter = 0.5 * (8)y^2 = 4y^2$$
$$Wetted\ Perimeter = b + y\left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2}\right) = y\left(\sqrt{64^2 + 1} + 1\right)$$
$$= 9.06y$$
$$Q = AR^{\frac{2}{3}}\frac{\sqrt{S_0}}{n} = 4y^2\left(\frac{4y^2}{9.06y}\right)^{\frac{2}{3}}\frac{\sqrt{0.0005}}{0.015} = 3.456y^{\frac{8}{3}}$$
$$When\ y = 0.12\ m, Q = 0.0121\ m^3/s$$
$$When\ Q = 0.08\frac{m^3}{s}, y = 0.244m$$

4. 10.15

$$q = \frac{Q}{b_1} = \frac{4.8}{2} = 2.4\ m^2/s$$
$$y_{c1} = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{2.4^2}{9.81}} = 0.84\ m$$
$$y_2 = y_1 + h = 1.22 + 0.1 = 1.32\ m$$

Energy Eqn from upstream to transition:

$$y_1 + \frac{q_1^2}{2gy_1^2} + h = y_2 + \frac{q_2^2}{2gy_2^2} = 1.22 + \frac{2.4^2}{2 * 9.81 * 1.22^2} + 0.1$$
$$= 1.32 + \frac{q_2^2}{2 * 9.81 * 1.32^2}$$
$$q_2 = 2.62\ m^2/s$$
$$b_2 = \frac{Q}{q_2} = \frac{4.8}{2.62} = 1.84\ m$$

Part B:

$$E_2 = E_1 + h = 1.52\ m$$
$$y_{c2} = \frac{2}{3} * E_2 = \frac{2}{3} * 1.52 = 1.01$$
$$q_2 = \sqrt{1.01^3 * 9.81} = 3.18\ m^2/s$$
$$b_2 = \frac{Q}{q_2} = \frac{4.8}{3.18} = 1.50\ m$$

5. 10.16

$$E_1 = y_1 + \frac{q^2}{2gy_1^2} = 2.15 + \frac{5.5^2}{2 * 9.81 * 2.15^2} = 2.48\ m$$

Froude:

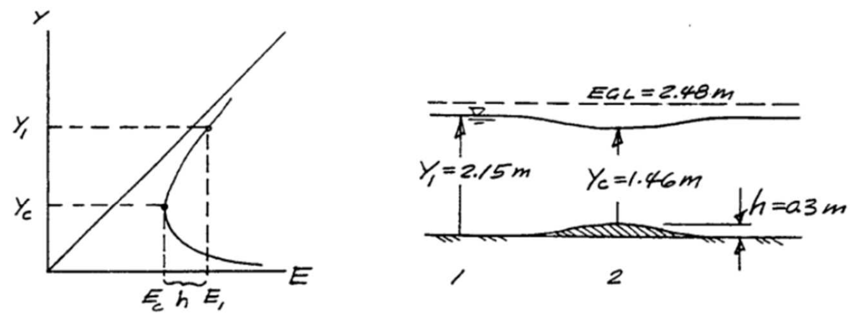
$Fr_1 = \frac{q}{\sqrt{gy_1^3}} = \frac{5.5}{\sqrt{9.81 * 2.15^3}} = 0.557$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{5.5^2}{9.81}} = 1.456\ m$$

$$E_c = \frac{3}{2} * y_c = 2.184\ m$$

Max height will be achieved when energy is at minimum:

$$h = E_1 - E_c = 0.3\ m$$



(b) (c)

Since Froude is less than 1, if h is greater than the max height (0.30 m), then subcritical non-uniform flow will occur upstream of the transition.

6. 10.40

   Energy Eqn between 1 and 2 and solve for q:

$$q = \sqrt{\frac{2g(y_2 - y_1)}{(y_1^{-2} - y_2^{-2})}} = \sqrt{\frac{2 * 9.81 * (0.10 - 2.5)}{(2.5^{-2} - 10^{-2})}} = 0.687\ m^2/s$$

$$Q = bq = 5 * 0.687 = 3.44\ m^3/s$$

   To get depth downstream, find Froude number at point 2:

$$Fr_2 = \frac{0.687}{\sqrt{9.81 * 0.10^3}} = 6.936$$

$$y_3 = \frac{y_2}{2}\left(\sqrt{1 + 8Fr_2^2} - 1\right) = 0.932\ m$$

   To get power lost, find head loss during jump:

$$h_j = \frac{(y_3 - y_2)^3}{4y_3y_2} = 1.544\ m$$

$$W = \gamma Q h_j = 9810 * 3.44 * 1.544 = 52\ kW$$

7. 10.53

$$q = \frac{Q}{b} = \frac{5}{3} = 1.67\ m^2/s$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{1.67^2}{9.81}} = 0.657\ m$$

   Since y_0 is less than yc, the channel upstream of A is steep. Find the depth conjugate of y_0:

$$Fr_0^2 = \frac{q^2}{gy_0^3} = \frac{1.67^2}{9.81 * 0.4^3} = 4.44$$

$$y_{cj} = \frac{y_0}{2}\left(\sqrt{1 + 8Fr_0^2} - 1\right) = 1.01\ m$$

The depth at the outfall is 1.6 ▯ yc ▯ 2.26 m. There is an H2 profile upstream of the weir up to location A; upstream of A an S1 profile exists. Between location A and the outfall the water depth is always greater than 2.26 m, therefore the jump will be upstream of A, where the depth on the S1 curve is equal to ycj.