Florida International University Department of Civil and Environmental Engineering
CWR 3201 Fluid Mechanics,
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Homework Assignment 5
Mechanics of Fluids (Fifth edition), by M.C. Potter, D.C. Wiggert and B.H. Ramadan.

1. 6.20 (same number in Fourth Edition)
   The problem states that Velocity will be dependent on gravity g, height H, and density ρ.

   Repeating variables are chosen based on whether they can be reduced further in dimensions or if they are commonly recurring in fluids equations. Ex: Height H has dimensions of L only, which means this cannot be reduced further and should be chosen as a repeating variable. Density is a term that is commonly found in many fluid mechanics questions and should be considered a repeating variable.

   Shifting all four variables into dimensions:

$$M^0L^0T^0 = L^a(LT^{-2})^b(ML^{-3})^c(LT^{-1})^1$$

$$M: 0 = 1c \rightarrow c = 0$$
$$T: 0 = -2b - 1 \rightarrow b = -1/2$$
$$L: 0 = 1a + 1b - 3c + 1 \rightarrow a = -1/2$$

   Substituting exponents back into first equation:

$$\pi = \frac{V}{\sqrt{gH}}$$

   Since no conditions are given for equation, must consider a constant to account for all possibilities: $V = C\sqrt{gH}$

2. 6.22
   There are 7 unique variables and 3 fundamental dimensions so 7-3 indicates 4 unique relationships.

   Choose ρ, V, D as repeating variables.

$$\pi_1 = \rho^a V^b D^c \Delta p$$
$$M^0L^0T^0 = (ML^{-3})^a(LT^{-1})^b(L)^c(ML^{-1}T^{-2})^1$$
$$M: 0 = 1a + 1 \rightarrow a = -1$$
$$T: 0 = -1b - 2 \rightarrow b = -2$$
$$L: 0 = -3a + 1b + 1c - 1 \rightarrow c = 0$$

$$\pi_1 = \frac{\Delta p}{\rho V^2}$$

$$\pi_2 = \rho^a V^b D^c \nu$$
$$M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L)^c (L^2 T^{-1})^1$$
$$M: 0 = 1a \rightarrow a = 0$$
$$T: 0 = -1b - 1 \rightarrow b = -1$$
$$L: 0 = -3a + 1b + 1c + 2 \rightarrow c = -1$$
$$\pi_2 = \frac{\nu}{VD}$$

$$\pi_3 = \rho^a V^b D^c L$$
$$M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L)^c (L)^1$$
$$M: 0 = 1a \rightarrow a = 0$$
$$T: 0 = -1b \rightarrow b = 0$$
$$L: 0 = -3a + 1b + 1c + 1 \rightarrow c = -1$$
$$\pi_3 = \frac{L}{D}$$

$$\pi_4 = \rho^a V^b D^c e$$
$$M^0 L^0 T^0 = (ML^{-3})^a (LT^{-1})^b (L)^c (L)^1$$
$$M: 0 = 1a \rightarrow a = 0$$
$$T: 0 = -1b \rightarrow b = 0$$
$$L: 0 = -3a + 1b + 1c + 1 \rightarrow c = -1$$
$$\pi_4 = \frac{e}{D}$$

3. 6.45

   Dynamic similarity: $Re_m = Re_p$

$$\frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

   $Since\ water\ temperature\ is\ the\ same, \nu_m = \nu_p\ and\ can\ cancel\ them\ out$

   Moving equation around: $\frac{V_p}{V_m} = \frac{L_m}{L_p} = \frac{1}{7}$

   Flow Rate: $Q_m = Q_p (\frac{L_m}{L_p})^2 (\frac{V_m}{V_p}) = 1.5 * \frac{1}{7}^2 * 7 = 0.214\ \frac{m^3}{s}$

   Power:

   $\frac{P_m}{P_p} = \frac{\rho_m L_m^2 V_m^3}{\rho_p L_p^2 V_p^3}$

$$P_m = P_p (\frac{\rho_m}{\rho_p})^1 (\frac{L_m}{L_p})^2 (\frac{V_m}{V_p})^3 = 200 * 1 * \frac{1}{7}^2 * 7^3 = 1400\ kW$$

   - Part B follows the same steps except the viscosities are now different for the model and prototype.

- $\nu_m = 8.917 * 10^{-7} \frac{m^2}{s}$
- $\nu_p = 0.0000013 \frac{m^2}{s}$

4. 6.56
   Dynamic similarity and Force is given: Froude numbers must be equal

$$Fr_m = Fr_p$$

$$\frac{V_m}{\sqrt{L_m g_m}} = \frac{V_p}{\sqrt{L_p g_p}}$$

$$\textit{Gravity is the same on both sides and can be canceled}$$

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{\frac{1}{10}}$$

Flow Rate: $Q_m = Q_p (\frac{L_m}{L_p})^2 (\frac{V_m}{V_p}) \quad = 2 * \left(\frac{1}{10}\right)^2 * \sqrt{\frac{1}{10}} = 0.0063 \frac{m^3}{s}$

Force: $\frac{F_m}{F_p} = \frac{\rho_m L_m^2 V_m^3}{\rho_p L_p^2 V_p^3}$

$$F_p = F_m (\frac{\rho_p}{\rho_m})^1 (\frac{L_p}{L_m})^2 (\frac{V_p}{V_m})^2 = 12 * 1 * 10^2 * \sqrt{10}^2 = 12000\ N$$
$$= 12\ kN$$