Florida International University
Department of Civil and Environmental Engineering

CWR 3201 Fluid Mechanics
Fall 2018
Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE
TA: Thao Do, CEE Undergraduate
Homework Assignment 4 Solutions
*Mechanics of Fluids (**Fifth edition**), by M.C. Potter, D.C. Wiggert and B.H. Ramadan.*

1. 4.35 (same number in *Fourth edition*)

Continuity Equation: $A_1V_1 = A_2V_2$

$$\pi(0.025^2)\left(\frac{10m}{s}\right) = (2\pi * 0.6 * \sout{0.03})V_2 \quad \text{0.003m}$$

$$V_2 = \sout{0.1736}\ m/s \quad \text{1.736 m/s}$$

Mass flux: $\dot{m} = \rho AV = 1000 * \pi * 0.025^2 * \left(\frac{10m}{s}\right) = 19.63\ kg/s$

Flow Rate: $Q = AV = (2\pi * 0.6 * 0.03) * \sout{0.1736}\ \frac{m}{s} = 0.0196\ \frac{m^3}{s}$ (1.736)

2. 4.52 (same number in *Fourth edition*)

Maximum velocity is at the center of the tube in section 1: $V_{max} = 10(4 - 0^2) = 40\ m/s$

Average velocity at section 1: $\bar{V_1} = \frac{V_{max}}{2} = 20\ m/s$

Conservation of mass: $\dot{m_{in}} = \dot{m_2} + \dot{m_3}$

$$1000 * \pi * 0.02^2 * 20\frac{m}{s} = 10\frac{kg}{s} + 1000 * \pi * 0.02^2 * \bar{V_3}$$

$$\bar{V_3} = 12.04\ m/s$$

3. 4.79 (same number in *Fourth edition*)

a) Across nozzle, continuity equation: $\pi * 0.07^2 * V_1 = \pi * 0.025^2 * V_2$

$$V_2 = 7.84V_1$$

Bernoulli: $\frac{V_2^2}{2g} + \frac{p_2}{\gamma} = \frac{V_1^2}{2g} + \frac{p_1}{\gamma}$ $\qquad p_1 = 9810\frac{7.84^2-1}{2*9.81}V_1^2$

For contraction: $\pi * 0.07^2 * V_1 = \pi * 0.05^2 * V_3$

$$V_3 = 1.96V_1$$

$Bernoulli: \frac{V_3^2}{2g} + \frac{p_3}{\gamma} = \frac{V_1^2}{2g} + \frac{p_1}{\gamma}$

Manometer: $\gamma * 0.15 + p_1 = 13.6\gamma * 0.15 + p_3$ $\qquad \frac{p_1}{\gamma} = 12.6 * 0.15 + \frac{p_3}{\gamma}$

Sub all known values into $Bernoulli: \frac{V_3^2}{2g} + \frac{p_3}{\gamma} = \frac{V_1^2}{2g} + \frac{p_1}{\gamma}: \frac{V_1^2}{2g} + 12.6 * 0.15 = \frac{V_3^2}{2g} = 1.96^2\frac{V_1^2}{2g}$

$$V_1 = 3.612\ m/s$$

$$p_1 = 394\ kPa$$

Reservoir surface to section 1: $\frac{V_0^2}{2g}+\frac{p_0}{\gamma}+z_0=\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1$

$$H=\frac{V_1^2}{2g}+\frac{p_1}{\gamma}=40.8\ m$$

b) Follows same method as part a.

4. 4.82 (same number in *Fourth edition*)

$$\frac{V_0^2}{2g}+\frac{p_0}{\gamma}+z_0=\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1$$

$$\frac{80000Pa}{9810}+4=\frac{V_1^2}{2*9.81}$$

$$V_1=15.44\ m/s$$

$$Q=A_1V_1=\pi*0.025^2*19.04=0.0303\ m^3/s$$

Part b and c only have different diameters for the flow rate equation.

5. 3.54 (same number in *Fourth edition*)

Using Bernoulli's Equation: $\frac{V^2}{2}=\frac{p}{\rho}$, where the density of air at 3000 ft is 0.0021 slugs/ft^3

$$v=\sqrt{\frac{2p}{\rho}}=\sqrt{\frac{2(0.3\ psi)*\frac{144\ in^2}{1\ ft^2}}{0.0021\ slugs/ft^3}}=203\ ft/sec$$

Part b and c are the same steps but with different pressures.

6. 3.68 (same number in *Fourth edition*)

Bernoulli: $\frac{V_2^2}{2g}+\frac{p_2}{\gamma}=\frac{V_1^2}{2g}+\frac{p_1}{\gamma}$, where the pressure at point 2 is the open end of the manometer so p_2 is equal to 0.

Manometer: $p_1+\gamma z+\gamma_{Hg}H-\gamma H-\gamma z=\frac{V_2^2}{2g}\gamma+p_2$

Substitute Bernoulli into the manometer equation: $p_1+\gamma_{Hg}H-\gamma H=\frac{V_1^2}{2g}\gamma+p_1$

H=0.01 m: $\frac{V_1^2*9800}{2*9.81}=(13.6)(9800)-(1)(9800)*0.01$

$$V_1=1.572\ m/s$$

Plugging into Bernoulli: $p_1=\frac{V_2^2-V_1^2}{2g}\gamma=\frac{20^2-1.572^2}{2*9.81}*9800=198562\ Pa$

Other parts follow same solution

7. 13.9 (same number in *Fourth edition*)
   a. Manometer: $p_1+\gamma_w h=p_2+\gamma_w*SG_{mmHg}*h=p_1+(9810)(0.12)=p_2+13.6(9810)(0.12)$

$$p_1-p_2=14832.7\ Pa$$

$$\frac{D_0}{D} = \frac{6}{12} = 0.5$$

$$Head\ loss\ h_1 - h_2 = \frac{p_1 - p_2}{\gamma_w} = \frac{14832.7\ Pa}{9810} = 1.51\ m$$

Try $Re = 10^5 \rightarrow K \approx 1.0\ (from\ graph\ 13.10)$

$$Q = KA\sqrt{2g(h_1 - h_2)} = (1.0)(\pi/4 * 0.06^2)\sqrt{2(9.81)(1.51)} = 0.0154\ \frac{m^3}{s}$$

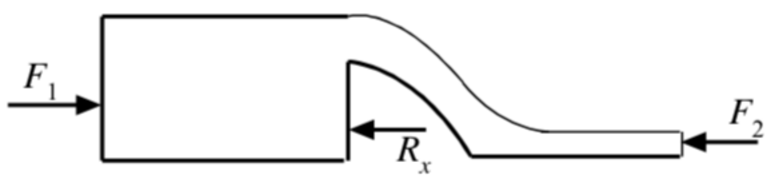
Check: $V = \frac{Q}{A} =$ ~~5.45~~ m/s 1.362 m/s $\qquad Re = \frac{vD}{\nu} = \frac{(\text{~~5.45~~})(0.12)}{0.658*10^{-6}} =$ ~~993920~~ 2.47 *10^5

K= ~~1.01 OK~~

Recalulate Q based on K=1.12, Q=0.0172 m/s. Another iteration will show that this guess is correct

b. Follows same procedure as part A except using curve for orifice on 13.10 graph: $Q = 0.0096\ \frac{m^3}{s}$

8. 4.123 (same number in *Fourth edition*)



Continuity: $0.7V_1 = 0.1V_2 \qquad V_2 = 7V_1$

Bernoulli (Energy): $\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + h_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + h_2$

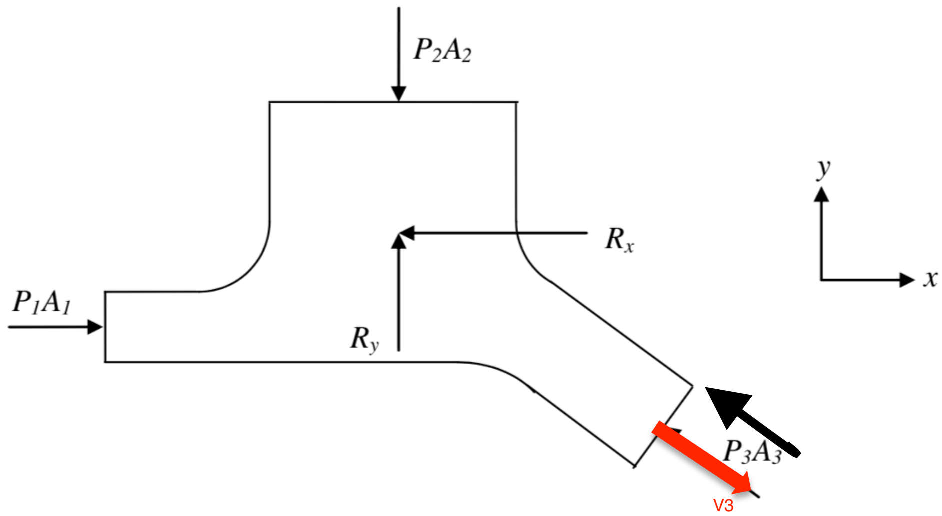
$$\frac{V_1^2}{2(9.81)} + 0.7 = \frac{(7V_1)^2}{2(9.81)} + 0.1$$

$$V_1 = 0.495, V_2 = 3.467\ m/s$$

Momentum: $F_1 - F_2 - R_x = \dot{m}(V_{out} - V_{in}) \rightarrow 9810 * \frac{0.7}{2}(0.7 * 1.5) - 9810 * \frac{0.1}{2}(0.1 * 1.5) - R_x = 1000(0.7 * 1.5)(0.495)(3.467 - 0.495)$

$$R_x = 1987\ N$$

9. 4.131 (same number in *Fourth edition*)



Mass flow rates: $\dot{m}_1 = \rho V_1 A_1 = 1000 * (15) * \left(\frac{\pi}{4} * 0.2^2\right) = 471\ kg/s$

$$\dot{m}_2 = \rho V_2 A_2 = 1000 * (5) * \left(\frac{\pi}{4} * 0.45^2\right) = 795\ kg/s$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 1266\ kg/s$$

$$V_3 = \frac{\dot{m}_3}{\rho A_3} = 25.8\ m/s$$

Conservation of momentum in the x: $p_1 A_1$ -p3A3 $\cos 40 - R_x = \dot{m}_3 V_3 \cos 40 - \dot{m}_1 V_1$

Solving for $R_x =$ -16495 N

Conservation of momentum in y: $R_y - p_2 A_2 + p_3 A_3 \sin 40 = \dot{m}_3(-V_3 \sin 40) - \dot{m}_2(-V_2)$

Solving for $R_y =$ -17612.92 N

10. 4.164 (same number in *Fourth edition*)

Velocity through the nozzle: $V_e = \frac{\dot{m}}{\rho A} = \frac{4\ kg/s}{1000*4[\frac{\pi}{4}*0.008^2]} = 19.89\ m/s$

$$M_{Inertia} \int_{c.v} r \times (2\Omega \times V) \rho d\, Volume = 4 \int_0^{0.3} r\hat{\imath} \times (-2\Omega\, \hat{k} \times V \hat{\imath}) \rho A dr$$

$$= 8 \rho A V \Omega \hat{k} \int_0^{0.3} r dr = -0.36 \rho A V \Omega \hat{k}$$

Sum of the moments must equal zero

For steady flow: $\sum M - (M_I)_Z = \int_{c.s.} (r \times V) V \cdot \hat{n} \rho A d = 0.3\, \hat{\imath} \times \left(0.707 V_e \hat{\jmath} + 0.707 V_e \hat{k}\right) V_e \rho A$

Solving for $-(M_I)_Z = 0.36\rho AV\Omega = 4 * (0.3 * 0.707{V_e}^2 A_e \rho$

Continuity: $AV = V_e A_e$

$$0.36\Omega = 4 * (0.707 * 0.3)(19.89)$$

$$\Omega = 46.9\ rad/s$$