Florida International University
Department of Civil and Environmental Engineering

CWR 3201 Fluid Mechanics
Fall 2018
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Homework Assignment 2 Solutions

*Mechanics of Fluids (**Fifth edition**), by M.C. Potter, D.C. Wiggert and B.H. Ramadan.*

1. 2.33 (same number in *Fourth edition*)

$$\Delta p = \rho g \Delta h$$

$$p_{oil} - p_{water} = 1000\ \frac{kg}{m^3} * 0.9(SG\ of\ Oil) * 9.81\ \frac{m}{s} * (0.05\ m) + 1000\ \frac{kg}{m^3} * 13.6(\ SG\ of\ Mercury) * 9.81\ \frac{m}{s} * (0.10m) - 1000\ \frac{kg}{m^3} * 9.81 * (0.15\ m) = 12311.6Pa$$

2. 2.38 (same number in *Fourth edition*)

$$\rho_{gage} - 9810\ \frac{kg}{m^3} * (4m) + 9810\ \frac{kg}{m^3} * 13.6\ (SG\ of\ Mercury) * (0.16\ m) = 0\ atm\ (open\ to\ atmosphere)$$

$$\rho_{gage} = 17893.4\ Pa$$

3. 2.57 (same number in *Fourth edition*)

$$Area\ of\ gate, A = (4m)(5m) = 20\ m^2$$

We know that the vertical side of the gate is 4 m by trig. So, the height from the middle of the gate to the top of the water surface is : $\bar{h} = 2m + H = 2 + 6 = 8m$

Moment of inertia of the gate: $I = \frac{bh^3}{12} = \frac{4(5)^3}{12} = 41.67\ m^3$

Angle at the hinge: $\theta = \tan^{-1}\frac{4}{3} = 53.13\ deg$

The hydrostatic force F_H will act at a distance of:

$h_{cp} = \bar{h} + \frac{I*(\sin\theta)^2}{\bar{h}*A} = 8 + \frac{41.67*(sin53.13)^2}{8*20} = 8.16\ m$

$$F_H = \rho g \bar{h} A = 1000 * 9.81 * 8 * 20 = 1569\ kN$$

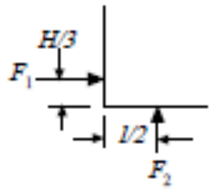
Length between hinge and wall: $\frac{8.16-2m}{\sin(53.13)} = 2.70m$ so distance of F_H from the hinge is 5-2.70=2.3 m

Taking the moment about the hinge: $P * 5 = F_H * 2.3$

$$P = 721.74\ kN$$

Note that this is only for part A, the others are the same except the wall height H, changes.

4. 2.63 (same number in *Fourth edition*)

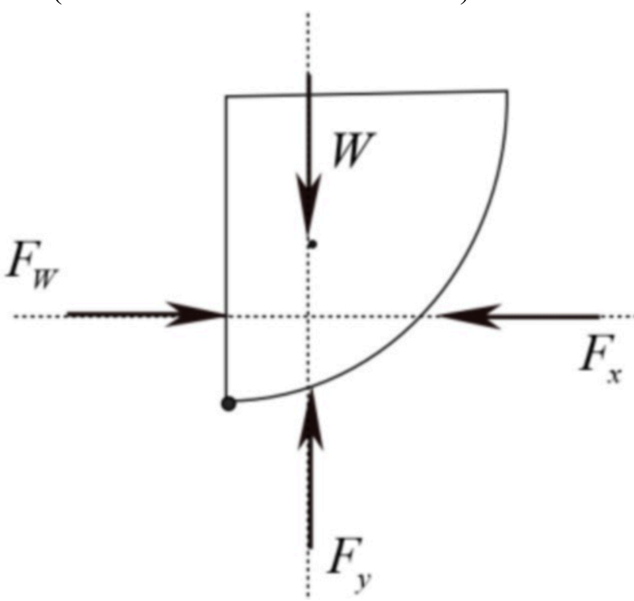


FBD of the gate

$$F_1 = \gamma \frac{H}{2} * bH = \frac{1}{2}\gamma bH^2$$
$$F_2 = \gamma H * lb$$
$$F_1 = F_2$$
$$\frac{1}{2}\gamma bH^2 * \frac{H}{3} = \gamma H * lb * \frac{l}{2}$$
$$H = \sqrt{3} * l$$

a) H=3.464 m b) H= 1.732 m c) H= 10.39 ft d) H= 5.196 ft

5. 2.74 (same number in *Fourth edition*)



FBD of gate

$F_{Water\ on\ side\ of\ gate} = \gamma_W A\bar{h} = 9810 * (R * 1\ unit\ width) * \frac{R}{2} = 4905R^2$, where R is the radius of the quarter circle gate.

$$V_{Water} = w * A = 1(\frac{\pi R^2}{4})$$
$$Weight\ of\ gate = \gamma_w * S * V_{Water} = 9810 * 0.2 * 1\left(\frac{\pi R^2}{4}\right) = 490.5\pi R^2$$
$$F_x = \gamma A\bar{h} = \gamma * (R * 1) * \frac{R}{2}$$
$$F_y = \gamma A\bar{h} = \gamma * \frac{\pi R^2}{4} * 1$$
$$Moment\ about\ hinge{:}\ -F_W\left(\frac{R}{3}\right) + F_x\left(\frac{R}{3}\right) - W\left(\frac{4R}{3\pi}\right) + F_y\left(\frac{4R}{3\pi}\right) = 0$$

Substitute all variables and solve for $\gamma = 4578\ N/m^3$

For English units, replace 9810 by 62.4 for the unit weight of water: $\gamma = 29.1 \frac{lb}{ft^3}$

6. 2.77 part b (same number in *Fourth edition*)

$$A = 4 * 8 = 32\ ft^2$$

$$\bar{h} = \frac{8}{2} = 4\ ft$$

$$F_H = \gamma A \bar{h} = 62.4 * 32 * 4 = 7987\ lb$$

The vertical force will be due to water inside of the parabolic gate: $F_W = \gamma * Volume\ of\ gate$

$$V = \int_0^H w * x\, dy = \int_0^8 4\left(\sqrt{\frac{y}{2}}\right) dy = 42.67\ ft^3$$

$$F_W = 62.4 * 42.67 = 2662.608\ lb$$

Centroidal distance from y-axis to parabola using differential strip dA:

$$d_w = \frac{\int_0^8 \bar{x} dA}{\int_0^8 dA} = \frac{\int_0^8 \frac{x}{2} x dy}{\int_0^8 x dy} = \frac{\int_0^8 \frac{y}{4} dy}{\int_0^8 \sqrt{\frac{y}{2}} dy} = 0.75\ ft$$

Moment about Hinge: $F_w * d_w + F_H * \frac{H}{3} = P * H$

Substitute all known values and solve for P: $P = 2911.95\ lb$

7. 2.81 (same number in *Fourth edition*)

$$100000\ kg * 9.81 \frac{m}{s^2} + 6000000\ N = (12 * 30m + 8h * 30m) 9810\ \frac{kg}{m^3}$$

$$h = 1.465\ m$$

Distance from the top = 2m – 1.465m = 0.535 m

8. 2.95 (same number in *Fourth edition*)

Calculate the horizontal length of the barge using Pythagoras equation: $C^2 = 6^2 + 6^2$

$$C = 8.485\ m$$

Calculate height of middle of barge: $h^2 = 6^2 + (8.485/2)^2$

$$h = 4.24\ m$$

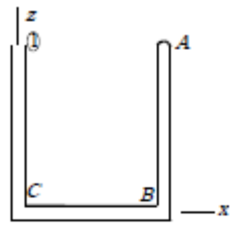
Neutral axis: $\bar{y} = \frac{\frac{1}{2}Ch*\left(l+\frac{h}{3}\right)+C*l*\frac{l}{2}}{\frac{1}{2}Ch+C*l} = \frac{\frac{1}{2}8.485*4.24*\left(2+\frac{4.24}{3}\right)+8.485*2*\frac{2}{2}}{\frac{1}{2}8.485*4.24+8.485*2} = 2.24\ m$

Distance from CG: $CG = 2.24 - 1.5 = 0.74\ m$

Metacentric height GM: $GM = \frac{I_o}{V} - CG = \frac{\frac{mC^3}{12}}{\left(\frac{1}{2}Ch+C*l\right)m} - CG = 0.71\ m$

Since GM is greater than zero, the barge is stable.

9. 2.102 parts a, b, c (same number in *Fourth edition*)



FBD of Figure. Position 1 is the open end

$$p_2 - p_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \qquad (2.5.2)$$

Use Eq:

a) $p_A = 0$ since there is no elevation difference between A and Position 1

$$p_B = 1000\frac{kg}{m^3} * 10\frac{m}{s^2} + 9.81\frac{m}{s^2} * 0.6\ m = 11886\ Pa$$

$$p_c = 1000\frac{kg}{m^3} * 10\frac{m}{s^2} + 9.81\frac{m}{s^2} * 0.6\ m = 11886\ Pa$$

b) $p_A = -1000\frac{kg}{m^3} * 20\frac{m}{s^2} * (0.6\ m * 1.5 - 0) - \left(-1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right) * (0m) = -18000\ Pa$

$$p_B = -1000\frac{kg}{m^3} * 20\frac{m}{s^2} * (0.6\ m - 0) - \left(-1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right) * (0 - 0.6m) = -6114\ Pa$$

$$p_C = -1000\frac{kg}{m^3} * 20\frac{m}{s^2} * (0) - \left(-1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2}\right) * (0 - 0.6m) = 5886\ Pa$$

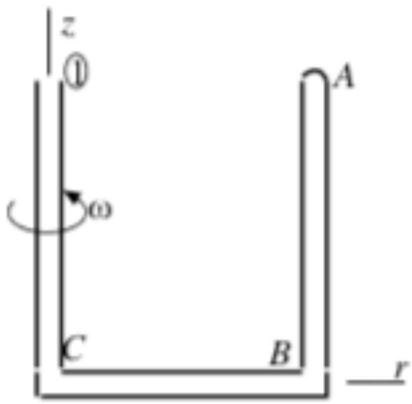
c) $p_A = -1000\frac{kg}{m^3} * 20\frac{m}{s^2} * (0.6\ m * 1.5 - 0) - \left(-1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2} + 10\frac{m}{s^2}\right) * (0m) = -18000\ Pa$

$$p_B = -1000\frac{kg}{m^3} * 20\frac{m}{s^2} * (0.6\ m * 1.5 - 0) - \left(-1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2} + 10\frac{m}{s^2}\right) * (0 - 0.6m) = 5886\ Pa$$

$$p_C = -1000\frac{kg}{m^3} * 20\frac{m}{s^2} * (0) - \left(-1000\frac{kg}{m^3}\right)\left(9.81\frac{m}{s^2} + 10\frac{m}{s^2}\right) * (0 - 0.6m) = 11886\ Pa$$

10. 2.103 (same number in *Fourth edition*)

$$p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1) \qquad (2.6.4)$$



FBD: Position 1 is open ended

$$\omega = \frac{50\ rpm * 2\pi}{60} = 5.236\ rad/s$$

a) $p_A = \left(\frac{1000*5.236^2}{2}\right) * (0.6\ m * 1.5)^2 = 11103.4\ Pa$

$$p_B = \left(\frac{1000 * 5.236^2}{2}\right) * (0.6\ m * 1.5)^2 + 1000 * 9.81 * 0.6m = 16990\ Pa$$

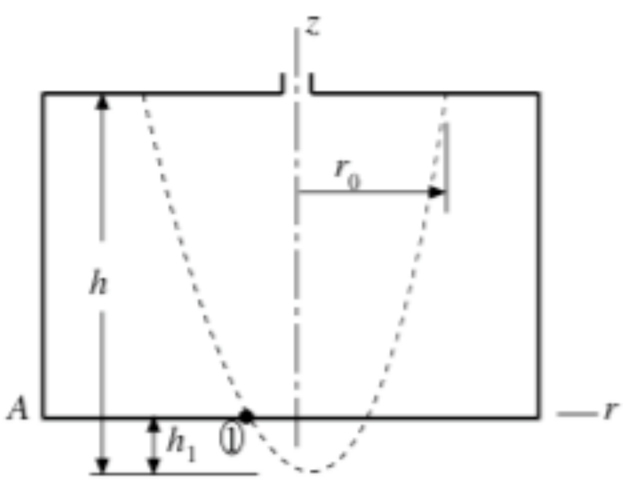
$$p_c = 1000 * 9.81 * 0.6m = 5886\ Pa$$

b) $p_A = \left(\frac{1000*5.236^2}{2}\right) * (0.4\ m * 1.5)^2 = 4935\ Pa$

$$p_B = \left(\frac{1000 * 5.236^2}{2}\right) * (0.4\ m * 1.5)^2 + 1000 * 9.81 * 0.4\ m = 8859\ Pa$$

$$p_C = 1000 * 9.81 * 0.4\ m = 3924\ Pa$$

11. 2.106 part d (same number in *Fourth edition*)



FBD

Equate the volumes of the tank: $\pi r_a^2 h_a = \frac{1}{2}\pi r_0^2 h - \frac{1}{2}\pi r_1^2 h_1$, Where r_a= 0.6m and h_a=0.2m

$$0.144 = r_0^2 h - r_1^2 h_1$$

Using equations 2.6.5: $\frac{\omega^2 r_0^2}{2g} = h \ and \ \frac{\omega^2 r_1^2}{2g} = h_1$ and subbing into equation above

$$0.144 = \frac{2gh^2}{\omega^2} - \frac{2gh_1^2}{\omega^2}$$

$$h^2 - h_1^2 = \frac{0.144 * (20\ rad/s)^2}{2 * 9.81} = 2.935$$

$$(h - h_1)(h + h_1) = 2.935$$

But we know that $(h - h_1) = 0.8\ m$ and so $(h + h_1) = 3.67m$

Solving system of equations gives:

$$h = 2.24\ m$$

$$h_1 = 1.43\ m$$

$$Using\ Eq\ 2.6.5, r_1 = 0.265\ m$$

$$p_2 - p_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1) \qquad (2.6.4)$$

$$p_A = \left(\frac{1000 * 20^2}{2}\right) * \left(0.6^2 - 0.265^2\right) = 57955\ Pa = 57.955kPa$$