Florida International University
CWR 3201 Fluid Mechanics, Fall 2024
Mid-term # 2

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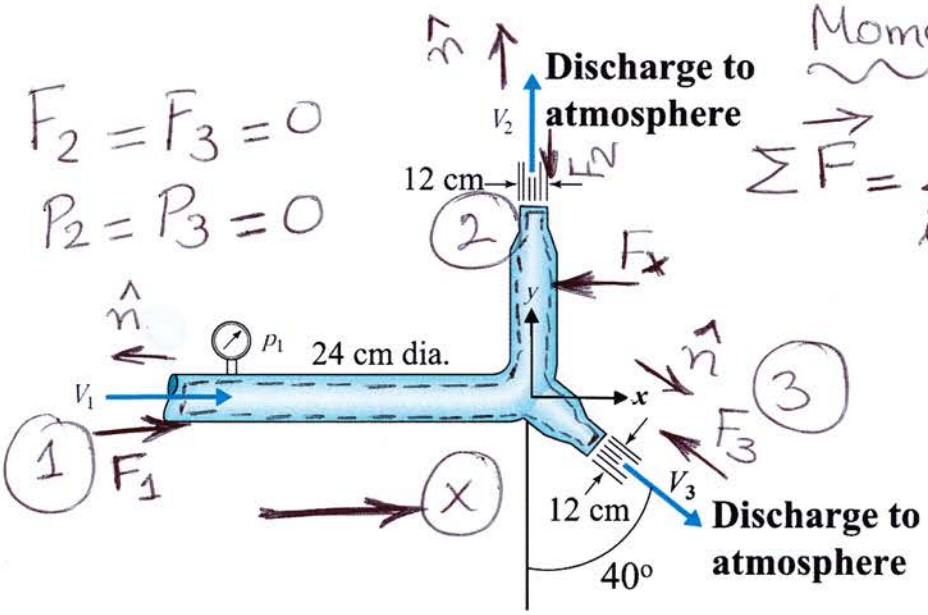
✓ **You will have 1 h 15 minutes to complete the exam. The exam is closed book and closed notes. Only one page (front and back) with handwritten equations are allowed**

1. **(35 points)** Determine the force in the "*x* direction (F_x) of the water on the **horizontal** bifurcation shown in the figure below if the pressure P_1 is 450 kPa. **Neglect head losses**.



Momentum equation

$$\sum \vec{F} = \sum_{i=1}^{N} \rho_i A_i \vec{V_i}(\vec{V_i} \cdot \hat{n})$$

$$\sum F_x = \rho_1 A_1(+V_1)(-V_1) + \rho_3 A_3(+V_3 \sin 40^\circ)(V_3)$$

$$F_1 - F_x = -\rho A_1 V_1^2 + \rho A_3 V_3^2 \sin 40^\circ \quad \ldots (1)$$

* Bernoulli between (1) and (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad (z_1 = z_2)$$

$$\frac{450{,}000}{9800} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \quad \ldots (2)$$

* Bernoulli between (2) and (3)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad (z_2 = z_3)$$

$$V_2 = V_3 \quad \ldots (3)$$

* Continuity $Q_1 = Q_2 + Q_3$

Because $A_2 = A_3$ and $V_2 = V_3$, $Q_2 = Q_3$

$\therefore Q_1 = 2Q_2$

$$V_1 \frac{\pi \times 0.24^2}{4} = 2 V_2 \frac{\pi \times 0.12^2}{4}$$

$$\boxed{V_1 = \frac{V_2}{2}}$$

In (2) $45.92 + \frac{V_1^2}{19.6} = \frac{V_2^2}{19.6}$

$$45.92(19.6) = 4V_1^2 - V_1^2$$

$$V_1 = 17.32\ m/s, \quad V_2 = 34.64\ m/s$$

$(F_1 = A_1 P_1)$

In (1)

$$450{,}000\left(\frac{\pi \times 0.24^2}{4}\right) - F_x = -1000\left(\frac{\pi \times 0.24^2}{4}\right) \times 17.32^2$$

$$+ 1000\left(\frac{\pi \times 0.12^2}{4}\right)(34.64^2) \sin 40^\circ$$

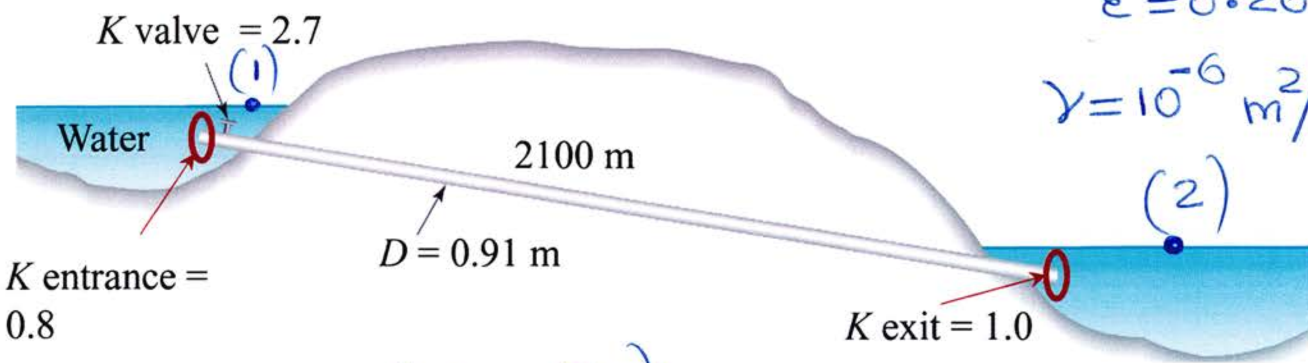
$$20{,}357.5 - F_x = -13{,}570.9 + 8723.2$$

$$F_x = 25{,}205\ N$$

2. (**30 points**) For the pipeline below, the reservoirs elevation difference is 30 m and the pipe diameter is 0.91 m. Determine the flow rate through the pipe. The pipeline is of cast iron material

$\varepsilon = 0.26$ mm

$\nu = 10^{-6}$ m²/s



$\Delta z = 30$ m $(z_1 - z_2)$

$D = 0.91$ m

$Q = ?$

$\sum K = 2.7 + 1 + 0.8 = 4.5$

* Energy equation between (1) and (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left(\frac{fL}{D} + \sum K\right)\frac{V^2}{2g}$$

$$30 = \left(\frac{f \times 2100}{0.91} + 4.5\right)\frac{V^2}{19.6} \quad \dots (1)$$

$$\frac{\varepsilon}{D} = \frac{0.26/1000}{0.91} = 0.00029$$

For completely turbulent regime, $f = 0.015$ (Moody diagram) for this pipe

We will use this "f" as our initial guess for the value of "f".

f	V (m/s) (from Eq. (1))	Re (Reynolds number)	f (from Moody chart)
0.015	3.88	3,530,800	0.0152 close enough to 0.015

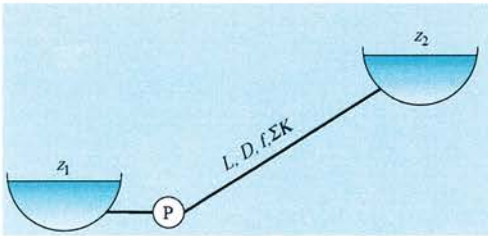
$\therefore f = 0.015$

$V = 3.88\ m/s$

$$Q = A \cdot V = \frac{\pi \times 0.91^2}{4} \times 3.88$$

$Q = 2.52\ m^3/s$

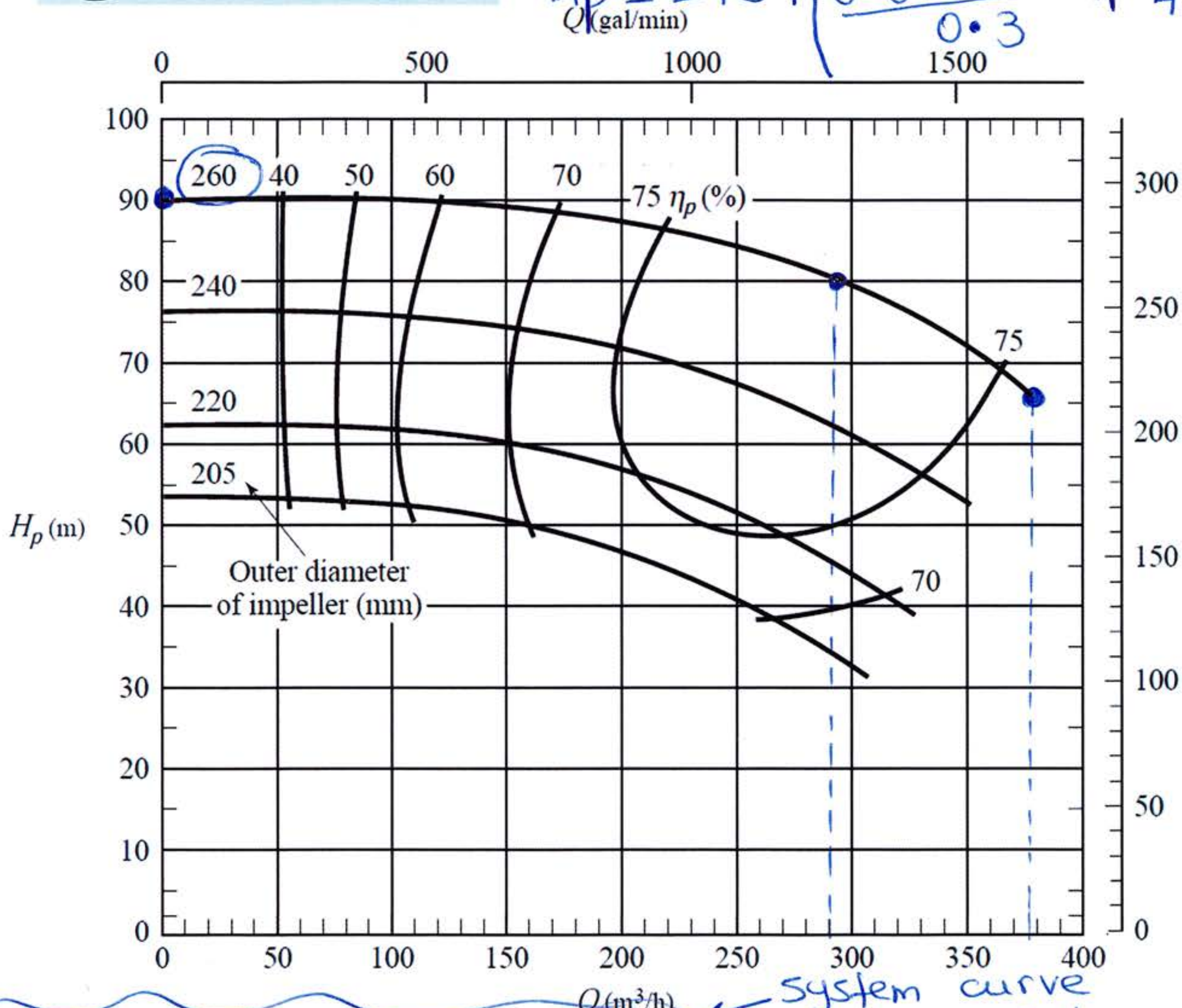
3. **(35 points)** The **260**-mm-outer impeller diameter pump represented in the figure below is used to move **water** in a piping system. The pipeline has the following characteristics: $D = 300$ mm, $L = 350$ m, $f = 0.02$, $\Sigma K = 4.2$. Determine the actual flow discharge (m^3/s) and pump head (m) when **four pumps in series** (**260** mm-impeller diameter pump) are used. The elevation difference between the reservoirs is 275 m ($z_2 - z_1 = 275$ m).



System curve

$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

$$H_p = 275 + \left(\frac{0.02 \times 350}{0.3} + 4.2\right)\frac{Q^2}{19.6\left(\frac{\pi \times 0.3^2}{4}\right)^2}$$



system curve

$H_p = 275 + 281.1\,Q^2$

Q is in m^3/s

* Finding pump curve

$H_p = aQ^2 + bQ + C$

Q needs to be in m^3/s

Hp (m)	Q (m^3/h)	Q (m^3/s)
90	0	0
80	295	0.082
66	380	0.1055

First point: $90 = a(0) + b(0) + c$

$$\boxed{c = 90}$$

Second point: $80 = a(0.082)^2 + b(0.082) + 90 \quad \cdots (1)$

$66 = a(0.1055)^2 + b(0.1055) + 90 \quad \cdots (2)$

In (1) $\frac{-10}{0.082^2} = a + 12.20b$

$-1487.2 = a + 12.20b \quad \cdots (3)$

In (2) $\frac{-24}{0.1055^2} = a + 9.48b$

$-2156.3 = a + 9.48b \quad \cdots (4)$

(3) - (4) $669.1 = 2.72b \rightarrow \boxed{b = 246.0}$

In (4) $\boxed{a = -4,488.3}$

Thus, the pump curve is $H_p = -4,488.3Q^2 + 246Q + 90$

For four pumps in series

$275 + 281.1Q^2 = 4(-4,488.3Q^2 + 246Q + 90)$

$18,234.3Q^2 - 984Q - 85 = 0$

$Q_1 = 0.100 \ m^3/s$

$Q_2 = -0.046 \ m^3/s$ (Not possible)

Flow discharge $Q = 0.100 \ m^3/s = \underline{360 \ m^3/h}$

$H_p = 275 + 281.1(0.1)^2$

$$\boxed{H_p = 277.81 \ m}$$