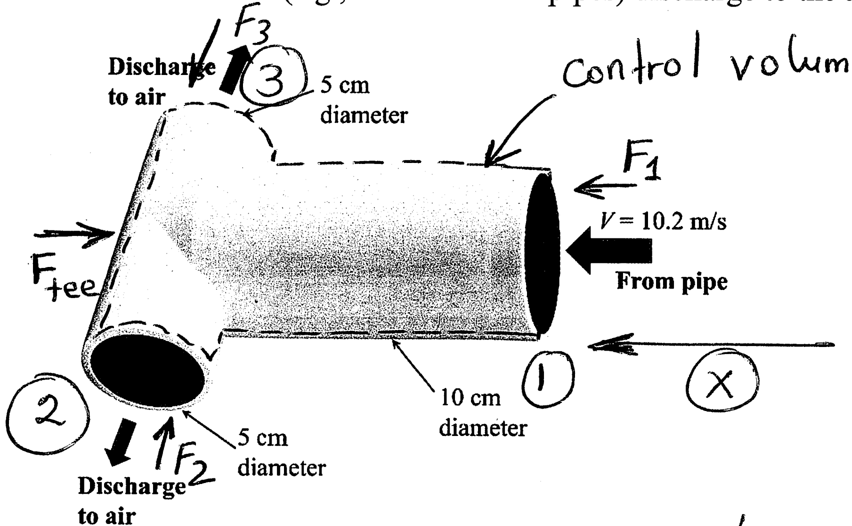
Florida International University
CWR 3201 Fluid Mechanics, Fall 2023
Mid-term # 2

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

Student Name: Arturo Leon **Panther ID:** ____________

✓ **You will have 1 h 15 minutes to complete the exam. The exam is closed book and closed notes. Only one page (front and back) with handwritten equations are allowed**

1. **(35 points)** Water flows at 10.2 m/s in a 10-cm-diameter stem of a horizontal T-section that branches into 5-cm diameter pipes as shown in the figure below. Find the force of the water on the T-section if the branches (e.g., 5-cm diameter pipes) discharge to the atmosphere (e.g., air). Neglect viscous effects.



Momentum eq:

$$\sum F_x = \dot{m}(V_{2x} - V_{1x})$$ ($V_{2x} = 0$)

$$F_1 - F_{tee} = -\dot{m}(10.2)$$

$$F_{tee} = P_1 A_1 + \dot{m}(10.2) \quad \cdots (1)$$

Bernoulli ① – ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$ ($P_2 = 0$ (atmosphere))

$z_1 = z_2$ (horizontal T)

$$\frac{P_1}{\gamma} + \frac{10.2^2}{19.6} = \frac{V_2^2}{2g} \quad \cdots (2)$$

Continuity

$$A_1 V_1 = 2 A_2 V_2$$

$$\frac{\pi \times 0.1^2}{4} V_1 = 2 * \frac{\pi \times 0.05^2}{4} V_2$$

$$V_2 = 2V_1 \rightarrow V_2 = 20.4 \frac{m}{s}$$

In ②

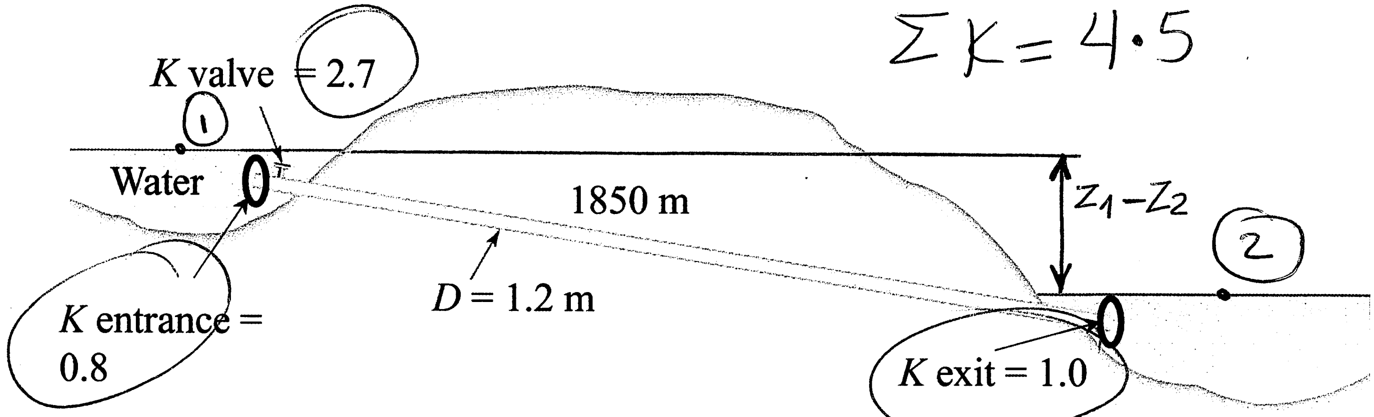
$$\frac{P_1}{9800} + \frac{10.2^2}{19.6} = \frac{20.4^2}{19.6} \rightarrow P_1 = 156{,}060 \text{ Pa}$$

In ①

$$F_{tee} = 156{,}060 \times \frac{\pi \times 0.1^2}{4} + 1000\, \frac{\pi \times 0.1^2}{4} \times 10.2\,(10.2)$$

$$F_{tee} = 1225.6 + 817.1 = 2042.7 \text{ N}$$

2. (**30 points**) For the pipeline below, the friction factor f is 0.029, the pipe diameter is 1.2 m, and the flow rate through the pipe is 4.4 m^3/s. Determine the reservoirs elevation difference.



$Z_1 - Z_2 = ??$

Energy eq. ① - ②

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2 + \left(\frac{fL}{D} + \Sigma K\right)\frac{V^2}{2g}$$

$$Z_1 - Z_2 = \left(\frac{0.029 \times 1850}{1.2} + 4.5\right)\frac{V^2}{19.6}$$

$$Z_1 - Z_2 = 2.51\,V^2 \qquad V = \frac{4.4}{\pi \times 1.2^2/4} = 3.89\ \frac{m}{s}$$

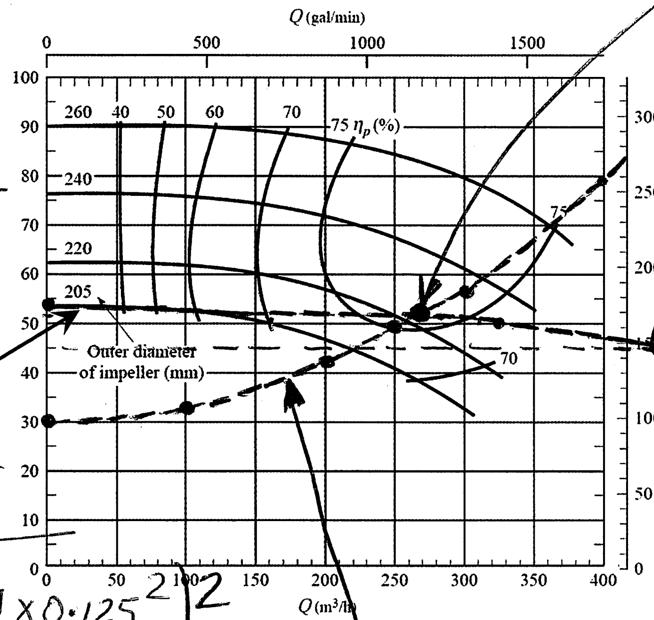
$$Z_1 - Z_2 = 2.51 \times 3.89^2$$

$$\boxed{Z_1 - Z_2 = 37.98\ m}$$

3. (35 points) The **205-mm**-outer diameter of impeller pump represented in the figure below is used to move water between two reservoirs through a pipeline with the following characteristics: $D = 125$ mm, $L = 70$ m, $f = 0.018$, $\Sigma K = 1.7$. Determine the actual discharge and pump head when **two pumps in parallel** are used (**each pump is 205-mm outer diameter of impeller**). The elevation difference between the reservoirs is 30 m ($z_2 - z_1 = 30$ m).

System curve:

$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$



Use the 205-mm outer diameter of impeller

intersection

pump curve (two pumps in parallel)

system curve

$$H_p = 30 + \left(\frac{0.018 * 70}{0.125} + 1.7\right)\frac{Q^2}{19.6\left(\frac{\pi \times 0.125^2}{4}\right)^2}$$

$$H_p = 30 + 3990.9\,Q^2 \quad \ldots (1)$$

→ Q is in m³/s

The chart is in m³/h but the system curve is in m³/s

We need to make the conversion as follows:

Q (m³/h)	Q (m³/s)	Hp (Eq. 1) [m] ← system curve
0	0	30
100	0.0277	33.1
200	0.056	42.3
300	0.083	57.71
400	0.111	79.3
250	0.0694	49.2

From chart:

The intersection of both curves give

$Q = 270$ m³/h (0.075 m³/s)

$H_p = 52$ m