

Mid-term # 1

Student Name: _____

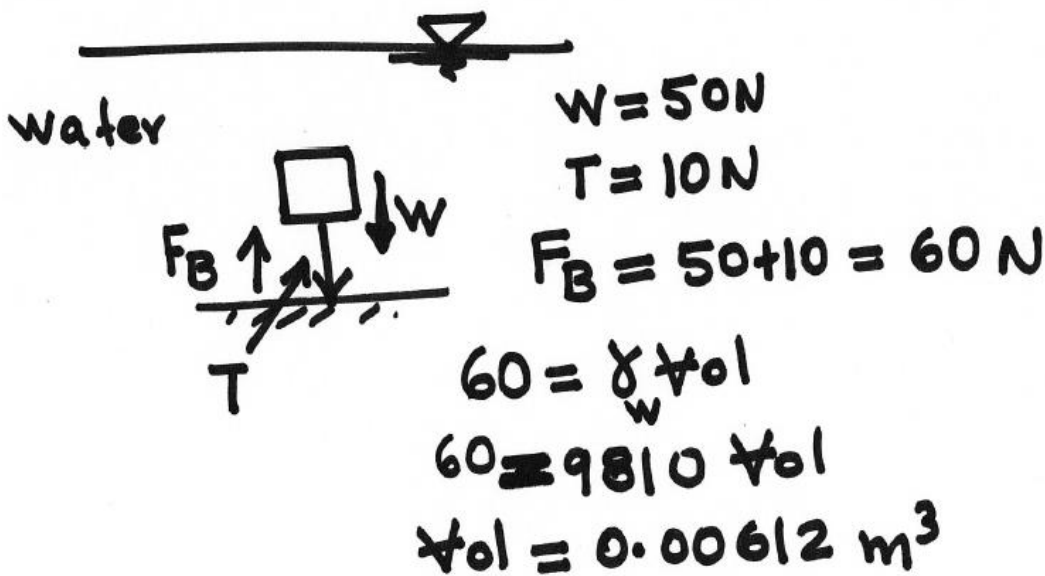
Date: 09/26/2018

✓ You will have 1h 15 minutes to complete the exam. The exam is closed book and closed notes.

Only one page (front and back) with handwritten equations are allowed (no photocopies or artificially reduced text will be allowed).

1. (20 points) An object is constructed of a material lighter than water. It weighs 50 N in air and a force of 10 N is required to hold it under water. What is its density, specific weight, and specific gravity?

Because object is lighter than water :



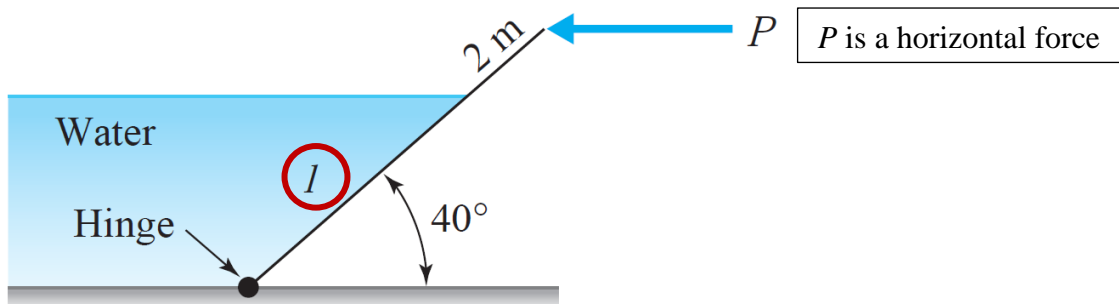
In air

$$\rho_{\text{object}} \times g \forall_{ol} = W$$

$$S = \frac{833.3}{1000} \quad \left. \begin{array}{l} \rho_0 \times 9.81 \times 0.00612 = 50 \\ \rho_{\text{object}} = 833.3 \text{ kg/m}^3 \end{array} \right\}$$

$$\boxed{S = 0.833} \quad \left. \begin{array}{l} \rho_{\text{object}} = 833.3 \text{ kg/m}^3 \\ \gamma_{\text{object}} = \rho_0 \times g = 8174.7 \text{ N/m}^3 \end{array} \right\}$$

2. (20 points). Find the force P needed to hold the 3-m-wide rectangular gate as shown below if $l = 5$ m

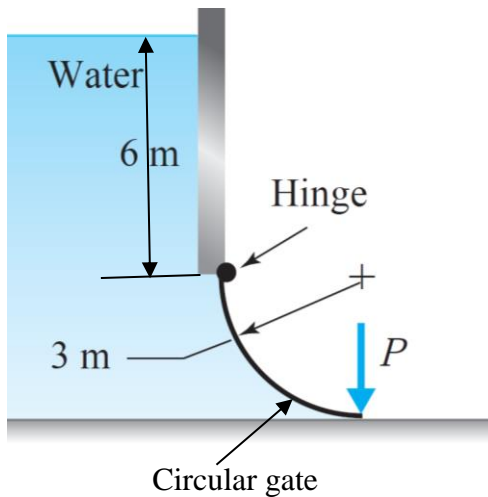


$$F = \gamma \frac{1}{2} l \sin 40^\circ \times 3l. \quad F \times \frac{l}{3} = (l + 2)P \sin 40^\circ. \quad \therefore \gamma l^3 = 2(l + 2)P.$$

$$9810 \times 5^3 = 2(5 + 2)P.$$

$$P = 87600 \text{ N}$$

3. (30 points) What force P is needed to hold the 10-m-wide gate shown in the figure below closed?



③ Place the resultant force at the circular arc center. F_H passes thru the hinge, so $P = F_v$. Use the water that could be contained above the gate, it produces the same distribution and hence the same F_v .

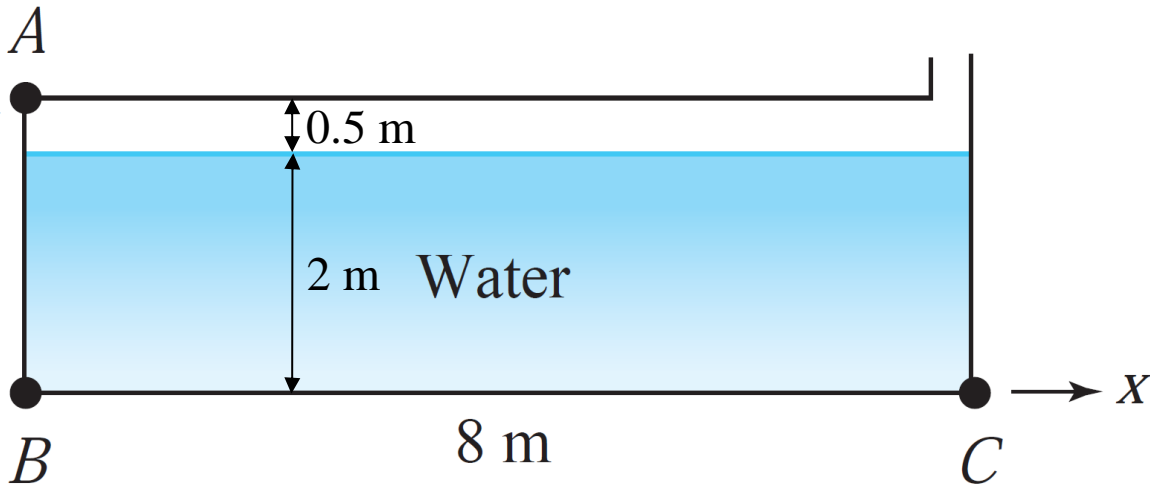
$$P = F_v = \gamma \text{Volume}$$

$$P = 9810 \left(6 \times 3 \times 10 + \frac{\pi \times 3^2}{4} \times 10 \right)$$

$$P = 2'459,228 \text{ N}$$

$$P = 2459.2 \text{ kN}$$

4. (30 points) The tank shown in the figure shown below is accelerated to the right at 10 m/s^2 . If the tank is 4 meters wide, find the force acting on the wall AB.

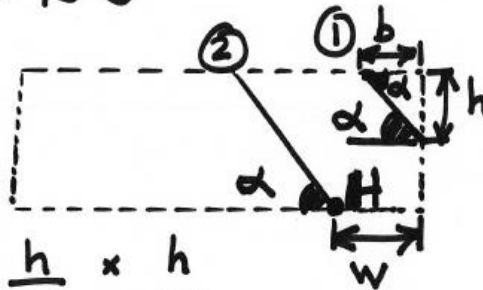


④ The air volume is the same before and after

$$\tan \alpha = \frac{10}{9.81} \quad \alpha = 45.5^\circ$$

$$\tan 45.5^\circ = \frac{h}{b}$$

$$0.5 \times 8 = \frac{bh}{2} \rightarrow 4 = \frac{h}{\tan 45.5^\circ} \times \frac{h}{2}$$



$$h = 2.856 \text{ m}$$

Because $h > 2.5$, air-water surface will look like ②

$$0.5 \times 8 = w \times 2.5 + \frac{2.5 \times 2.5}{2} \tan \alpha$$

$$w = 0.374 \text{ m}$$

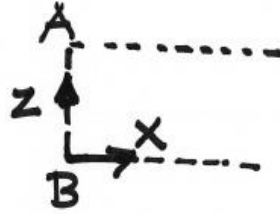
$$\tan \alpha = \frac{2.5}{x}$$

$$x = \frac{2.5}{\tan \alpha}$$

Note in the figure above that

$P_H = 0$. To calculate the force on the wall AB, we can relate the pressures on the wall AB to point H.

$$dp = -\rho a_x dx - \rho g dz \quad a_z = 0$$



$$P_z - P_H = -\rho(10)(x_z - x_H) - \rho \times 9.81 \times (z - 0)$$

$$P_z = -10000(0 - 7.626) - 9810z$$

$$P_z = 76260 - 9810z$$

$$F_{AB} = \int P_z dA \quad dA = b dz$$

$$F_{AB} = \int_0^{2.5} (76260 - 9810z) 4 dz$$

$$F_{AB} = 305040z - 39240 \frac{z^2}{2} \Big|_0^{2.5}$$

$$F_{AB} = 639975 \text{ N} = 640 \text{ kN}$$