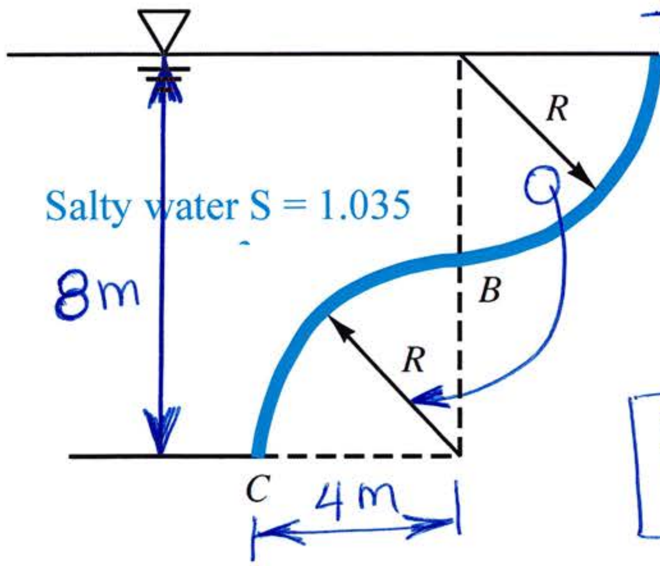
Florida International University
CWR 3201 Fluid Mechanics, Fall 2024
Final Exam

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

Student Name: Arturo Leon **Panther ID:** ____________

- ✓ You will have 2 hours to complete the exam. The exam is closed book and closed notes
- ✓ Only two pages with handwritten equations are allowed (no photocopies or artificially reduced text will be allowed)
- ✓ No cell phones or any type of communication device will be allowed.

1. **(25 points)** Calculate the **horizontal** and **vertical** forces of salty water (S = 1.035) acting on the curved gate ABC. Assume R = 4 m and the gate width is 12 m.



$* \; F_H = \gamma \bar{h} A$

$\bar{h} = \frac{8}{2} = 4\text{ m}$

$A = 8 \times 12 = 96$

$F_H = 1.035 \times 9800 \times 4 \times 96$

$F_H = 3{,}894{,}912 \text{ N}$

$$\boxed{F_H = 3{,}895 \text{ KN}}$$

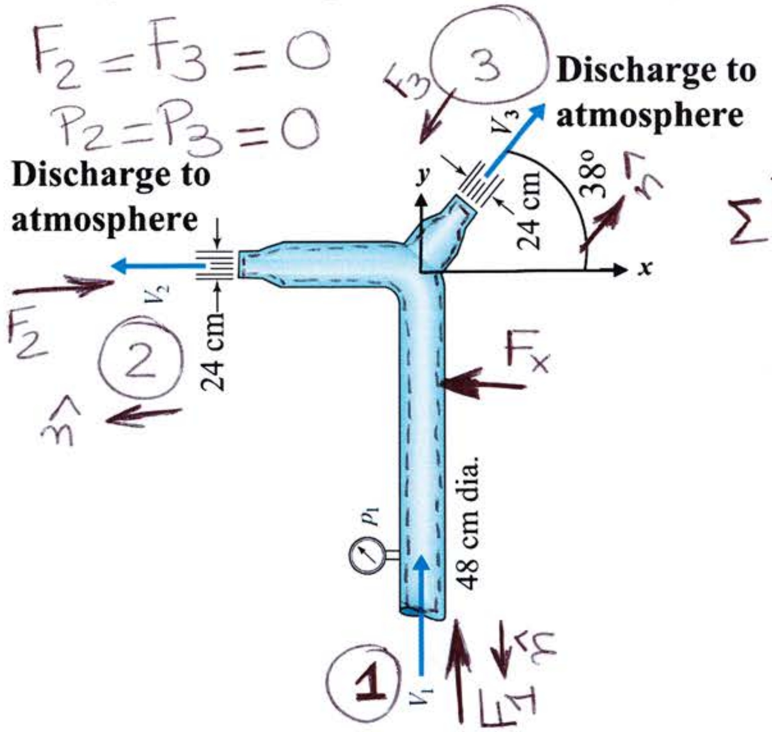
$* \; F_V = \gamma \forall = 1.035 \times 9800 \times (4 \times 8 \times 12)$

$$\boxed{F_V = 3{,}895 \text{ KN}}$$

2. **(25 points)** Determine the force in the "x direction ($\boldsymbol{F_x}$) of the water on the **horizontal** bifurcation shown in the figure below if the pressure P_1 is 300 kPa. **Neglect head losses**.

$F_2 = F_3 = 0$

$P_2 = P_3 = 0$



Momentum equation

$$\sum \vec{F} = \sum_{i=1}^{N} \rho_i A_i V_i \left(\vec{V_i} \cdot \hat{n}\right)$$

$$\sum \vec{F_x} = \rho_2 A_2 (-V_2)(V_2) + \rho_3 A_3 (V_3 \cos 38^\circ)(V_3)$$

$$-F_x = -\rho A_2 V_2^2 + \rho A_3 V_3^2 \cos 38^\circ \quad \ldots (1)$$

* Bernoulli between (1) and (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \cancel{\frac{P_2}{\gamma}}^{\,0} + \frac{V_2^2}{2g} + \cancel{Z_2} \quad (Z_1 = Z_2)$$

$$\frac{300{,}000}{9800} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \quad \ldots (2)$$

* Bernoulli between (2) and (3)

$$\cancel{\frac{P_2}{\gamma}}^{\,0} + \frac{V_2^2}{2g} + \cancel{Z_2} = \cancel{\frac{P_3}{\gamma}}^{\,0} + \frac{V_3^2}{2g} + \cancel{Z_3} \quad (Z_2 = Z_3)$$

$$\left\{ V_2 = V_3 \right\} \quad \ldots (3)$$

* Continuity $Q_1 = Q_2 + Q_3$

Because $A_2 = A_3$ and $V_2 = V_3$, $Q_2 = Q_3$

$\therefore Q_1 = 2Q_2$

$$V_1 \cancel{\pi} \times \frac{0.48^2}{\cancel{4}} = 2V_2 \cancel{\pi} \times \frac{0.24^2}{\cancel{4}}$$

$$\boxed{V_1 = V_2/2}$$

In (2) $30.61 + \frac{V_1^2}{19.6} = \frac{V_2^2}{19.6}$

$$30.61(19.6) = 4V_1^2 - V_1^2$$

$$\boxed{V_1 = 14.14 \text{ m/s}, \quad V_2 = 28.28 \text{ m/s}}$$

In (1)

$$-F_x = -1000\left(\frac{\pi \times 0.24^2}{4}\right)\left(28.28^2\right) + 1000\left(\frac{\pi \times 0.24^2}{4}\right) \times 28.28^2 \times \cos 38^\circ$$

$$-F_x = -36,180.2 + 28,510.4$$

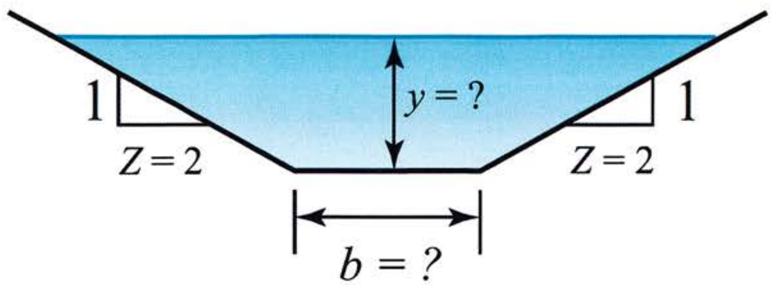
$$\boxed{F_x = 7669.8 \text{ N}}$$

3. **(25 points)** The trapezoidal channel below carries a discharge of 120 m^3/s of **water** with a velocity of 3.5 m/s. If the channel is designed for **maximum hydraulic efficiency** conditions, what should be the channel bottom (***b***) and the water height (***y***) of the trapezoidal channel?

Derivative rule for a power function: $\frac{dx^n}{dm} = nx^{n-1}\frac{dx}{dm}$

$Q = 120\ m^3/s$

$V = 3.5\ m/s$



$\boxed{A = 34.29\ m^2}$ ... (1)

* Max. hyd. effic → P is minimum $\left(\frac{dP}{dy} = 0\right)$

$P = b + 2\sqrt{5}\,y$

$\frac{dP}{dy} = 0 = \frac{db}{dy} + 2\sqrt{5} \rightarrow \boxed{\frac{db}{dy} = -2\sqrt{5}}$ (2)

* Area is constant. Thus $\frac{dA}{dy} = 0$

$A = \left(\frac{b + b + 4y}{2}\right)y = (b + 2y)y = by + 2y^2$ ... (3)

$\frac{dA}{dy} = 0 = b\cancel{\frac{dy}{dy}}^{1} + y\frac{db}{dy} + 4y\cancel{\frac{dy}{dy}}^{1}$

from (2)

$0 = b + y(-2\sqrt{5}) + 4y = b - 0.472y$

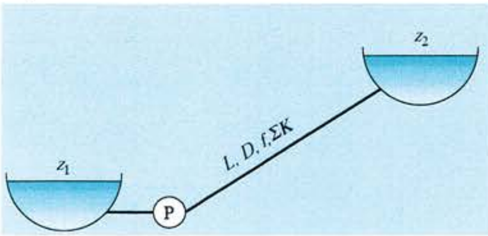
$\boxed{b = 0.472y}$

From (1) and (3) $0.472y^2 + 2y^2 = 34.29$

$y = 3.72\,m$

$b = 1.76\,m$

4. **(25 points)** The **220**-mm-outer impeller diameter pump represented in the figure below is used to move **water** in a piping system. The pipeline has the following characteristics: $D = 200$ mm, $L = 120$ m, $f = 0.025$, $\Sigma K = 3.8$. Determine the actual flow discharge (m^3/s) and pump head (m) when **two pumps in series (220** mm-impeller diameter pump) are used. The elevation difference between the reservoirs is 85 m ($z_2 - z_1 = 85$ m).



System curve

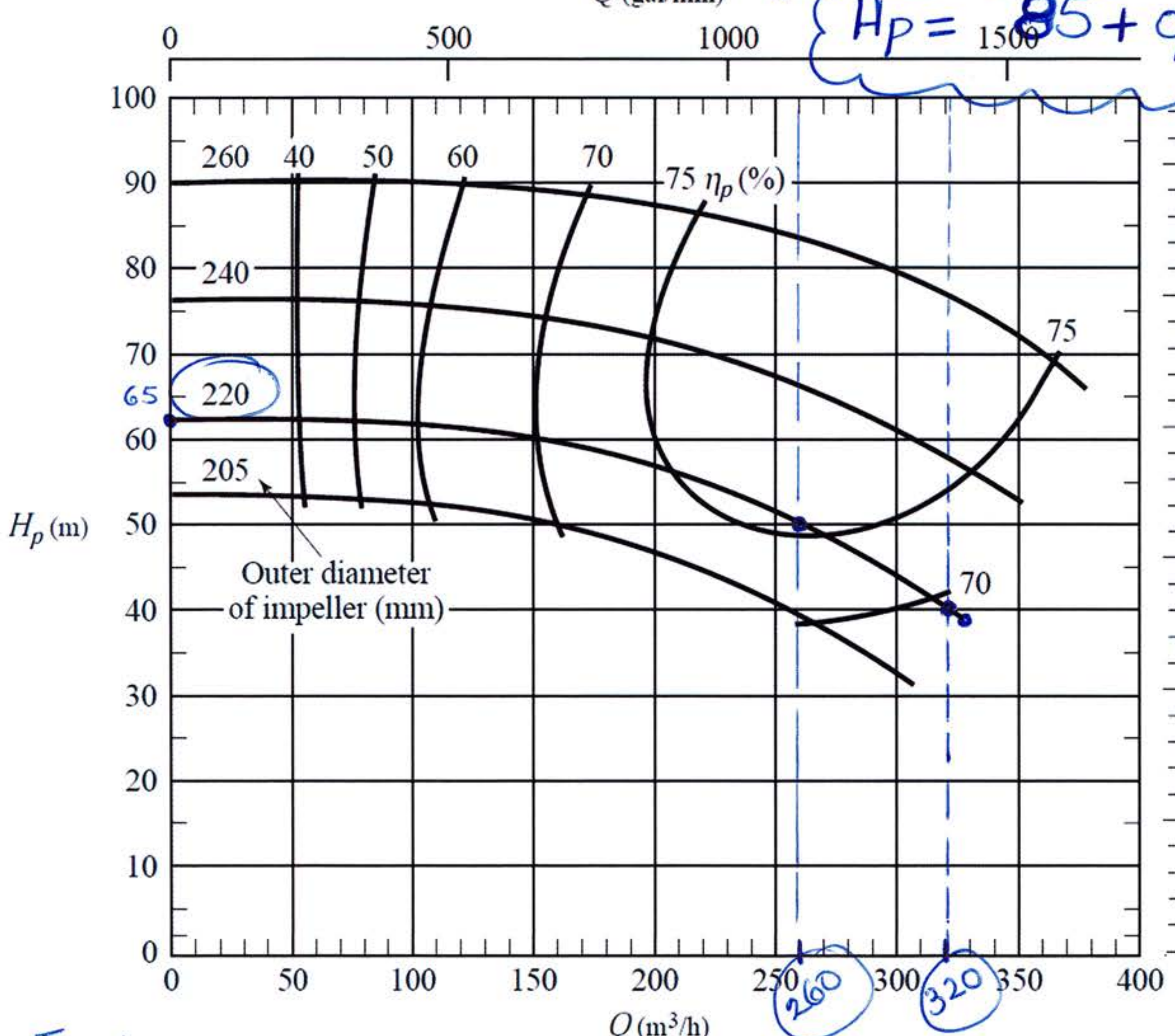
$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

$$H_p = 85 + \left(\frac{0.025 \times 120}{0.2} + 3.8\right)\frac{Q^2}{19.6\left(\frac{\pi \times 0.2^2}{4}\right)^2}$$

$$H_p = 85 + 972\,Q^2$$

system curve

Q is in $\frac{m^3}{s}$



* Finding pump curve

Hp(m)	Q(m^3/h)	Q(m^3/s)
62.5	0	0
50	260	0.072
40	320	0.089

$$H_p = aQ^2 + bQ + C$$

Q needs to be in m^3/s

First point: $62.5 = a(0) + b(0) + c$

$$\boxed{c = 62.5}$$

Second point: $50 = a(0.072)^2 + b(0.072) + 62.5 \quad \cdots (1)$

Third point: $40 = a(0.089)^2 + b(0.089) + 62.5 \quad \cdots (2)$

In (1)

$$\frac{-12.5}{(0.072)^2} = a + 13.89\,b$$

$$-2411.3 = a + 13.89b \quad \cdots (3)$$

In (2)

$$\frac{-22.5}{(0.089)^2} = a + 11.24b$$

$$-2840.6 = a + 11.24b \quad \cdots (4)$$

(3) − (4) $\quad 429.3 = 2.65\,b$

$$\boxed{b = 162}$$

In (3) $\longrightarrow$ $\boxed{a = -4661.5}$

Thus, the pump curve is $H_p = -4661.5\,Q^2 + 162Q + 62.5$

For two pumps in series

$$85 + 972Q^2 = 2(-4661.5\,Q^2 + 162Q + 62.5)$$

$$10{,}295Q^2 - 324\,Q - 40 = 0$$

$$Q = 0.080\ m^3/s\ (288\ m^3/h)$$

$\therefore$

$$H_p = 91.2\ m$$