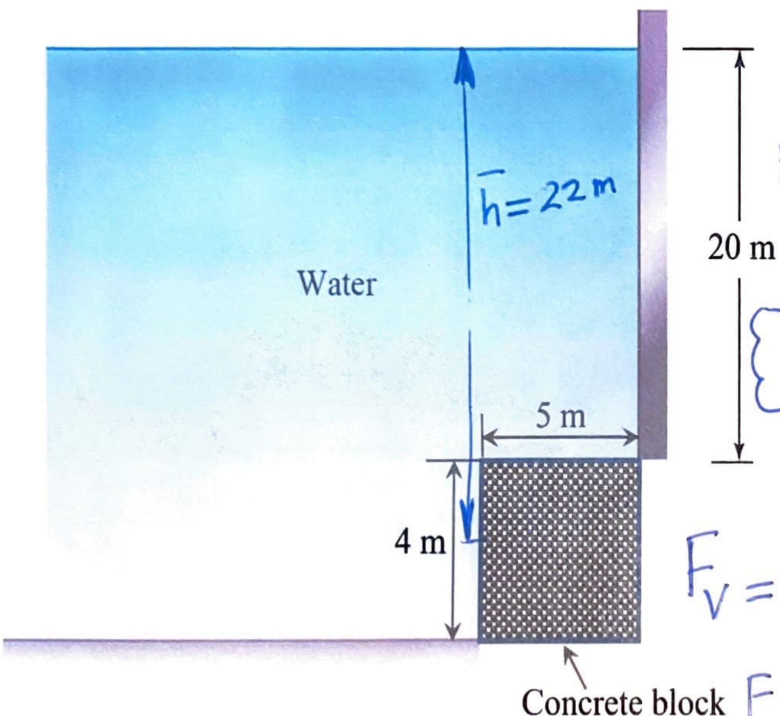
Florida International University
CWR 3201 Fluid Mechanics, Fall 2023
Final Exam

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

Student Name: Arturo Leon **Panther ID:** ____________

- ✓ You will have 2 hours to complete the exam. The exam is closed book and closed notes
- ✓ Only two pages with handwritten equations are allowed (no photocopies or artificially reduced text will be allowed)
- ✓ No cell phones or any type of communication device will be allowed.

1. **(25 points)** Determine the magnitude of the horizontal and vertical components of the hydrostatic force acting on the concrete block in the figure below. The concrete block has a width of 6 m perpendicular to the plane of the paper.



$F_H = \gamma \bar{h} A$

$F_H = 9800 \times 22 \times (4 \times 6)$

$F_H = 5{,}174{,}400$ N

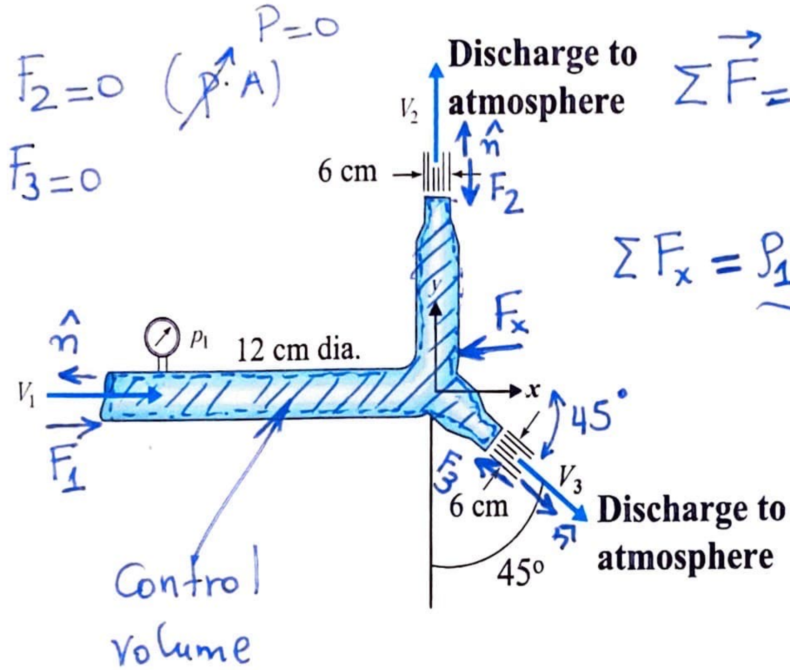
$F_H = 5{,}174$ KN

$F_V = \gamma \forall = 9800 \times (5 \times 20 \times 6)$

$F_V = 5{,}880{,}000$ N

$F_V = 5{,}880$ KN

2. **(25 points)** Determine the force in the "x direction ($\boldsymbol{F_x}$) of the water on the **horizontal** bifurcation shown in the figure below if the pressure P_1 is 225 kPa. **Neglect head losses.**



$$\Sigma \vec{F} = \sum_{i=1}^{N} \rho_i A_i V_i \left(V_i \cdot \hat{n} \right)$$

$$\Sigma F_x = \underbrace{\rho_1 A_1 V_1}_{\dot{m}_1} (-V_1) + \underbrace{\rho_3 A_3 V_3}_{\dot{m}_3} \cdot \underbrace{V_3 \cos 45^\circ}_{V_{3x}}$$

$$F_1 - F_x = -\rho A_1 V_1^2 + \rho A_3 V_3^2 \cos 45^\circ \quad \ldots (1)$$

Bernoulli (1) and (2) (atmospheric)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \qquad (z_1 = z_2)$$

$$\frac{225\,000}{9800} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \quad \ldots (2)$$

Bernoulli (2) and (3) (atmospheric)

$$\frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \qquad (z_2 = z_3)$$

$$\boxed{V_2 = V_3} \quad \ldots (3)$$

Continuity $\quad Q_1 = Q_2 + Q_3 \quad \ldots (4)$

Because $A_2 = A_3$ and $V_2 = V_3$, $Q_2 = Q_3$

$$V_1 A_1 = 2 V_2 A_2$$

$$V_1 \frac{\pi \times 0.12^2}{4} = 2 V_2 \frac{\pi \times 0.06^2}{4}$$

$$V_1 = \frac{V_2}{2}$$

In (2)

$$22.96 + \frac{V_1^2}{19.6} = \frac{4 V_1^2}{19.6} \rightarrow V_1 = 12.25 \text{ m/s}$$

$$\therefore V_2 = 24.5 \text{ m/s}$$

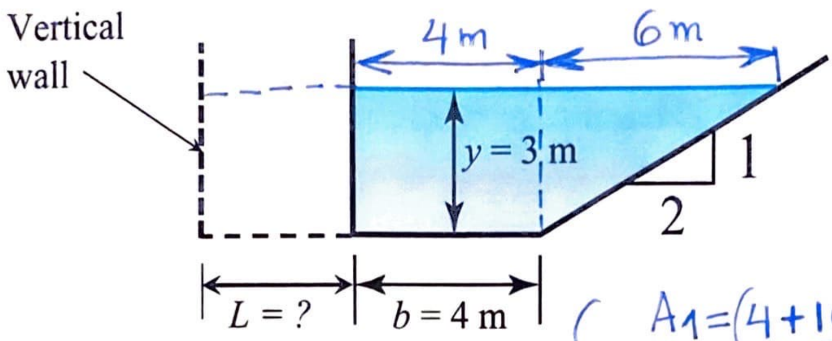
In (1)

$$225000 \times \frac{\pi \times 0.12^2}{4} - F_x = -1000 \times \frac{\pi \times 0.12^2}{4} \times 12.25^2 + 1000 \times \frac{\pi \times 0.06^2}{4} \times 24.5^2 \cos 45^\circ$$

$$2544.7 - F_x = -1697.2 + 1200.1$$

$$F_x = 3{,}041.8 \text{ N}$$

3. **(25 points)**. The canal shown below is to be widened by "L" so that the initial water flow discharge can be doubled. Determine the additional width "L" if all other parameters (i.e., flow depth, bottom slope and surface material) are to remain the same.



Q_1 : Initial

Q_2 : Final

$$\frac{Q_2}{Q_1} = 2$$

Manning's equation

$$Q = \frac{c}{n} A R^{2/3} S_o^{1/2}$$

$$\frac{Q_2}{Q_1} = 2 = \frac{\frac{c}{n_2} A_2 R_2^{2/3} S_o^{1/2}}{\frac{c}{n_1} A_1 R_1^{2/3} S_o^{1/2}}$$

$n_1 = n_2$

$$A_1 = \left(\frac{4+10}{2}\right) \times 3 = 21 m^2$$

$$P_1 = 3 + 4 + \sqrt{3^2 + 6^2}$$

$$P_1 = 13.70$$

$$R_1 = 1.532$$

$$A_2 = A_1 + 3L = 21 + 3L$$

$$P_2 = P_1 + L = 13.70 + L$$

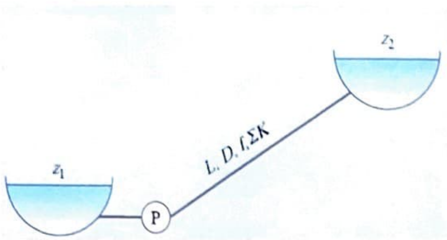
$$R_2 = \frac{21 + 3L}{13.70 + L}$$

$$2 = \frac{A_2 R_2^{2/3}}{A_1 R_1^{2/3}} \rightarrow 2 = \frac{(21+3L)\left(\frac{21+3L}{13.7+L}\right)^{2/3}}{21 (1.532)^{2/3}}$$

$$55.81 = (21+3L)\left(\frac{21+3L}{13.7+L}\right)^{2/3}$$

$$\boxed{L = 5.02 \text{ m}}$$

4. **(25 points)** The **260**-mm-outer impeller diameter pump represented in the figure below is used to move **water** in a piping system. The pipeline has the following characteristics: $D = 150$ mm, $L = 65$ m, $f = 0.022$, $\Sigma K = 2.6$. Determine the actual flow discharge (m^3/s) and pump head (m) when **four pumps in series (260 mm-impeller diameter pump)** are used. The elevation difference between the reservoirs is 280 m ($z_2 - z_1 = 280$ m).



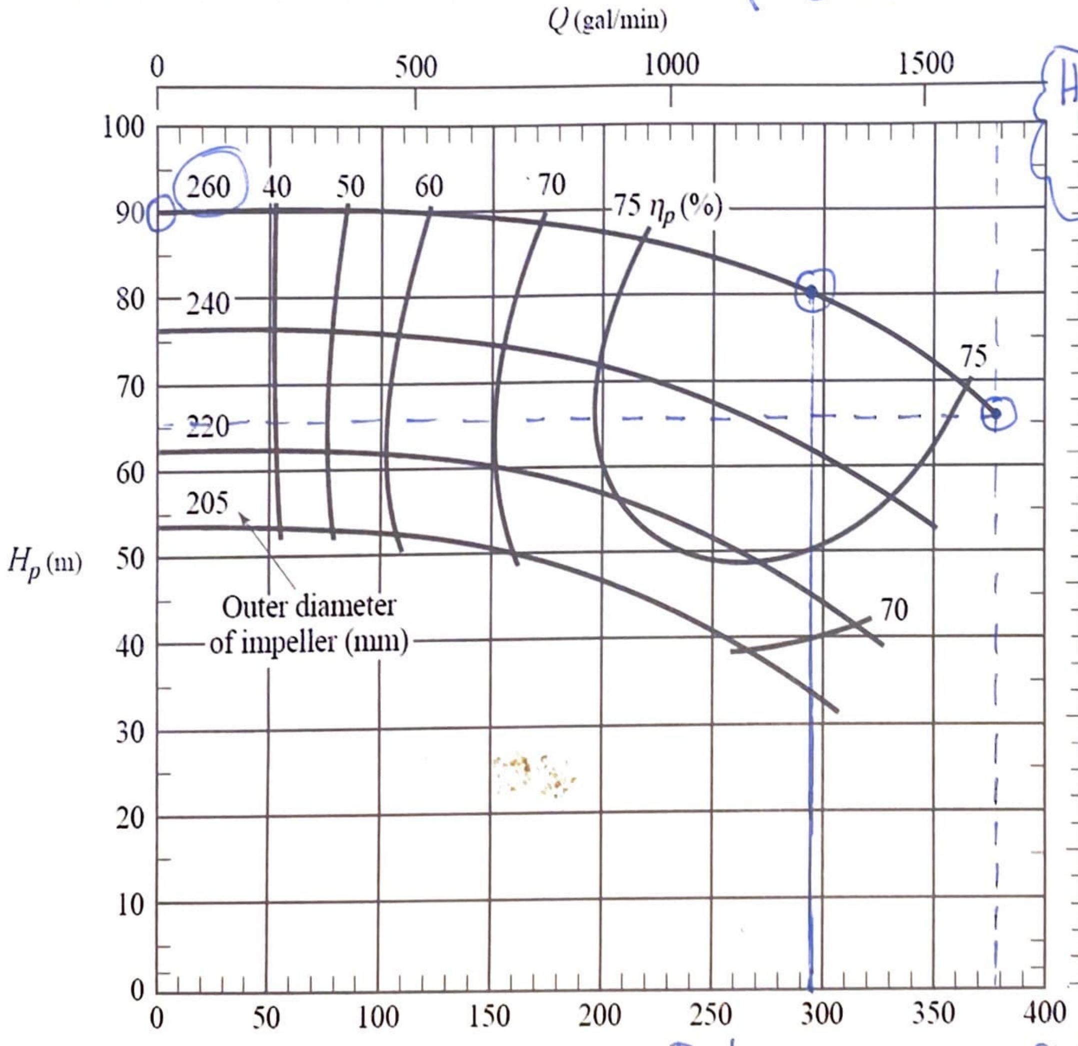
System curve:

$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

$$H_p = 280 + \left(\frac{0.022\times 65}{0.15} + 2.6\right)\frac{Q^2}{19.6\left(\frac{\pi\times 0.15^2}{4}\right)^2}$$

$$H_p = 280 + 1982.3\,Q^2$$

Q is in m^3/s



Determining pump curve: (Q also needs to be in m^3/s)

H_p (m)	Q (m^3/h)	Q (m^3/s)
90	0	0
80	295	0.082
66	380	0.1055

first point: $H_p = aQ^2 + bQ + C$

$$90 = a(0) + b(0) + C$$

$$C = 90$$

Second point: $80 = a(0.082)^2 + b(0.082) + 90 \quad \ldots (1)$

Third point: $66 = a(0.1055)^2 + b(0.1055) + 90 \quad \ldots (2)$

In (1)
$$\frac{-10}{0.082^2} = a + 12.20\,b$$
$$-1487.2 = a + 12.20\,b \quad \ldots (3)$$

In (2)
$$\frac{-24}{0.1055^2} = a + 9.48\,b$$
$$-2156.3 = a + 9.48\,b \quad \ldots (4)$$

(3) − (4) $\quad 669.1 = 2.72\,b \rightarrow \boxed{b = 246.0}$

In (4) $\quad \boxed{a = -4{,}488.3}$

Thus, the pump curve is $H_p = -4{,}488.3\,Q^2 + 246\,Q + 90$

For four pumps in series:
$$280 + 1982.3\,Q^2 = 4\left(-4{,}488.3\,Q^2 + 246\,Q + 90\right)$$
$$\boxed{19{,}935.5\,Q^2 - 984\,Q - 80 = 0}$$

$Q_1 = 0.0926\ m^3/s$

$Q_2 = -0.0433\ m^3/s$ (Not possible)

Flow discharge $Q = 0.0926\ m^3/s = 333.4\ m^3/h$

$$H_p = 280 + 1982.3\,(0.0926)^2$$
$$\boxed{H_p = 297.0}$$