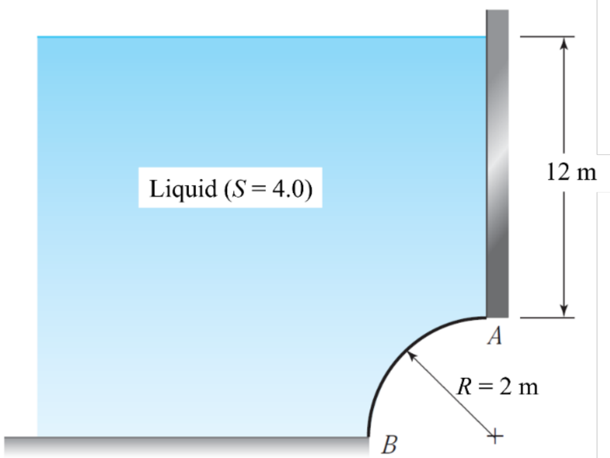
Florida International University
CWR 3201 Fluid Mechanics, Fall 2022
Final Exam

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

Student Name: ____**Arturo Leon**__________________________ **Panther ID:** ___________________

- ✓ You will have 2 hours to complete the exam. The exam is closed book and closed notes
- ✓ Only two pages with handwritten equations are allowed (no photocopies or artificially reduced text will be allowed)
- ✓ No cell phones or any type of communication device will be allowed.
- ✓ The final exam consists of five questions; however, the **grading will be based on four questions only**. If five problems are solved, the grading will consider the 4 solutions with the highest scores.

1. **(25 points)** Determine the magnitude of the horizontal and vertical components of the hydrostatic force acting on the curved surface ***AB*** shown in the figure below, which has a radius of 2 m and a **width of 4 m**. The specific gravity of the liquid is 4.0.

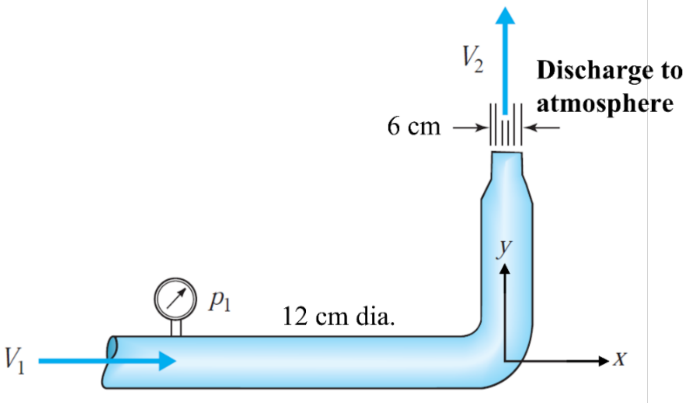


$$F_H = \gamma \bar{h} A = 4 \times 9800 \times (12+1)(2\times 4) = 4{,}076{,}800 \text{ N}$$
$$= 4{,}076.8 \text{ KN}$$

$$F_V = \gamma \forall$$
$$\forall = 4\left[2\times 14 - \frac{\pi R^2}{4}\right] = 4\left(28 - \frac{\pi (2)^2}{4}\right)$$
$$\forall = 4(28 - \pi) = 99.43 \text{ m}^3$$

$$F_V = 4 \times 9800 \times 99.43 = 3{,}897{,}656 \text{ N}$$
$$= 3{,}897.7 \text{ KN}$$

2. **(25 points)** Determine the horizontal force components (F_x and F_y) of the water on the **horizontal** bend shown in the figure below if the pressure P_1 is 450 kPa. Neglect head losses.



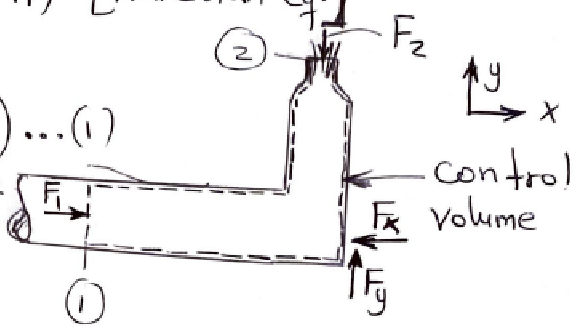
$\Sigma F = \dot{m}(V_2 - V_1)$ [Momentum eq.]

X-direction

$F_1 - F_x = \dot{m}(V_{2x}^{\,0} - V_{1x}) \quad \ldots(1)$

$F_1 = P_1 A_1 = 450\times10^3 \times \frac{\pi(0.12)^2}{4}$

$= 5{,}089.4 \text{ N}$



* Bernoulli equation (1) − (2)

$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \ldots (2)$ (P_2 = 0 (atmospheric)) ($Z_1 = Z_2$) because of horizontal bed

* Continuity $A_1 V_1 = A_2 V_2$

$\frac{\pi \times 0.12^2}{4} V_1 = \frac{\pi \times 0.06^2}{4} V_2$

$V_2 = 4V_1$

In (2)

$\frac{450{,}000}{1000\times9.8} + \frac{V_1^2}{19.6} = \frac{16V_1^2}{19.6} \rightarrow \frac{15V_1^2}{19.6} = 45.918$

$V_1 = 7.75 \text{ m/s}$

$V_2 = 30.98 \text{ m/s}$

* $\dot{m} = \rho A V = \rho_1 A_1 V_1 = 1000 \times \frac{\pi \times 0.12^2}{4} \times 7.75 = 87.65$

In ①

$5089.4 - F_x = 87.65(-7.75)$

$F_x = 5769 \text{ N} = 5.8 \text{ KN}$

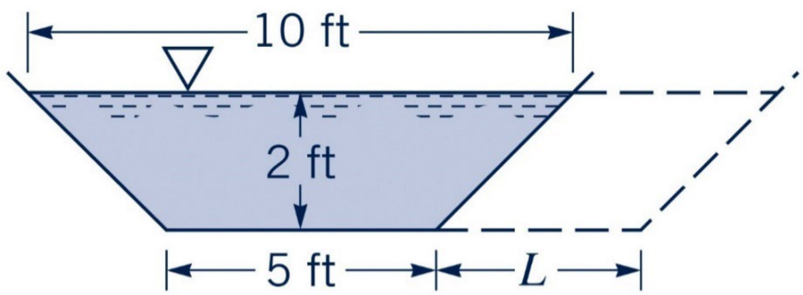
y-direction

$F_y - \cancel{F_2}^{0} = \dot{m}(V_{2y} - \cancel{V_{1y}}^{0})$ $\quad F_2 = \cancel{P_2}^{0} A_2 = 0$

$F_y = 87.65(30.98)$

$F_y = 2715.4 \text{ N} = 2.7 \text{ KN}$

3. **(25 points)**. The canal shown below is to be widened so that the water flow discharge can be increased. If the additional width, ***L*** is 3 ft, by what percentage will the flow discharge increase with respect to the initial flow discharge. **All other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same**.



Manning's eq: $Q = \frac{C_1}{n} A R^{2/3} S_o^{1/2}$

Original canal [1]

Widened canal [2]

$$\frac{Q_1}{Q_2} = \frac{\frac{C_1}{n} A_1 R_1^{2/3} S_o^{1/2}}{\frac{C_1}{n} A_2 R_2^{2/3} S_o^{1/2}}$$

$$\frac{Q_1}{Q_2} = \frac{15 \times 1.316^{2/3}}{21 \times 1.458^{2/3}}$$

$$\frac{Q_1}{Q_2} = 0.667$$

$$\frac{Q_2}{Q_1} = 1.499 \rightarrow \boxed{Q_2 = 1.5\, Q_1}$$

Original [1]

$A_1 = \left(\frac{10+5}{2}\right) 2 = 15\ ft^2$

$P_1 = 5 + 2\sqrt{2^2 + 2.5^2} = 11.4\ ft$

$R_1 = \frac{A_1}{P_1} = 1.316$

Widened [2]

$A_2 = \left(\frac{8+13}{2}\right) \times 2 = 21\ ft^2$

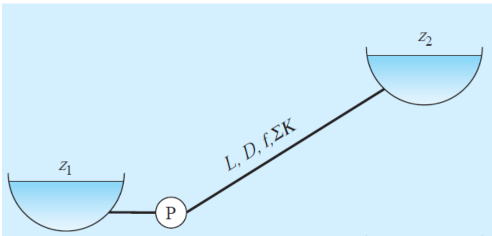
$P_2 = 11.4 + L = 14.4\ ft$

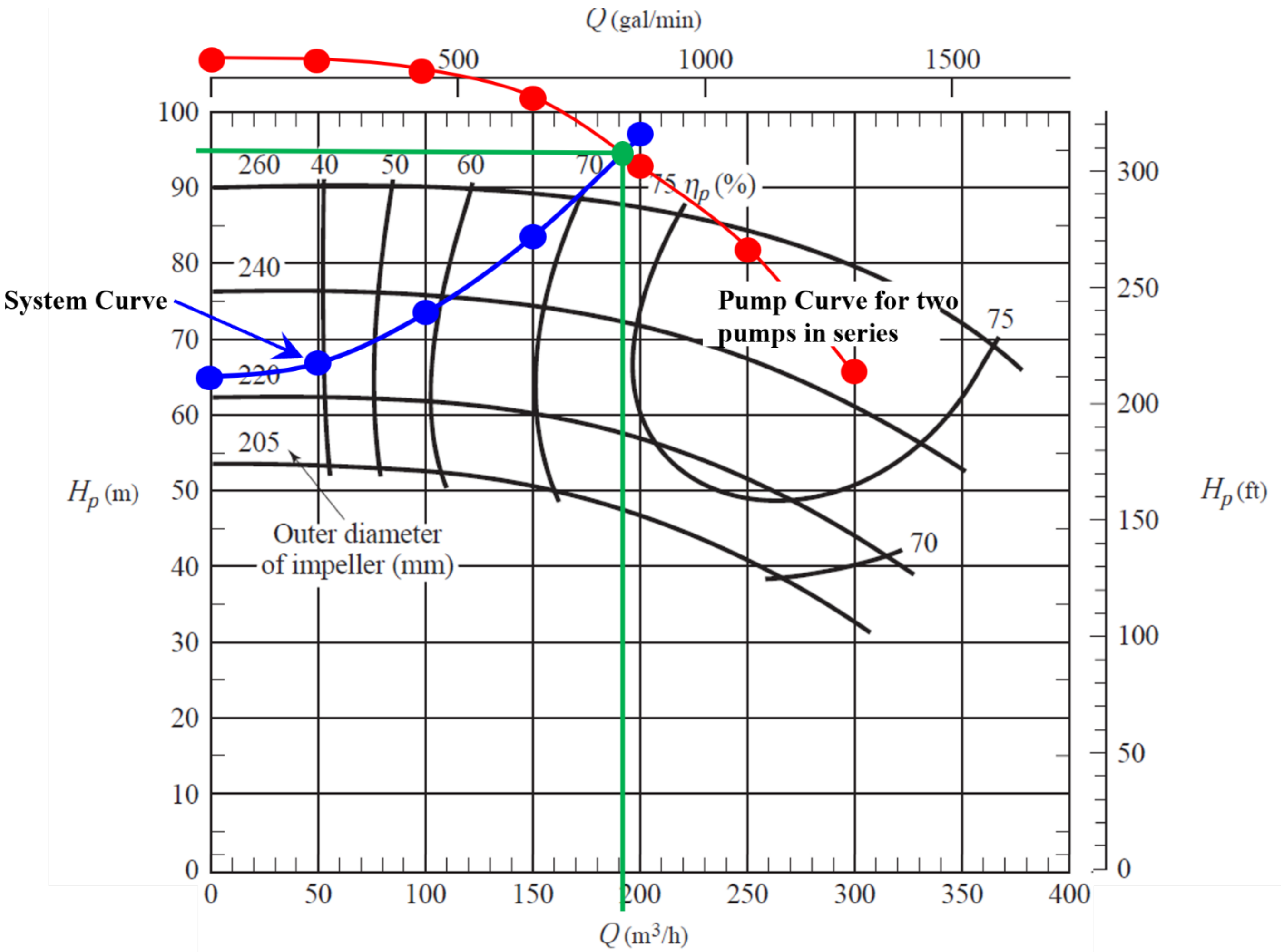
$R_2 = 1.458$

$1.5Q_1 - Q_1$
$= 0.5\, Q_1$

Flow will increase by 50%

4. **(25 points)** The **205**-mm-outer impeller diameter pump represented in the figure below is used to move **water** in a piping system. The pipeline has the following characteristics: $D = 100$ mm, $L = 50$ m, $f = 0.02$, $\Sigma K = 3.2$. Determine the actual flow discharge (m^3/s) and pump head (m) when **two pumps in series** (**205**-mm-impeller diameter pump) are used. The elevation difference between the reservoirs is 65 m ($z_2 - z_1 = 65$ m).





④ $D = 0.10\,m$, $L = 120\,m$, $f = 0.02$, $\Sigma K = 3.2$

$Q = ?$, $H = ?$ $\quad Z_2 - Z_1 = 65\,m$

System curve

$$H_p = Z_2 - Z_1 + \left(\frac{fL}{D} + \Sigma K\right)\frac{Q^2}{2gA^2}$$

$$H_p = 65 + \left(\frac{0.02 \times 50}{0.10} + 3.2\right)\frac{Q^2}{19.6\left[\frac{\pi \times 0.1^2}{4}\right]^2}$$

$$H_p = 65 + 10917.9\,Q^2$$

Q is in m^3/s

* Two pumps in series

we draw pump curve for two pumps in series

$H_p = a + bQ + cQ^2$ (one pump)

$H_p = 2a + 2bQ + 2cQ^2$ Two pumps

Plot of system curve

$Q\,(m^3/h)$	$Q\,(m^3/s)$	$H_p\,(m)$
0	0	65
50	0.0138	67.1
100	0.0277	73.4
150	0.0416	83.9
200	0.0556	98.7

From the graph above

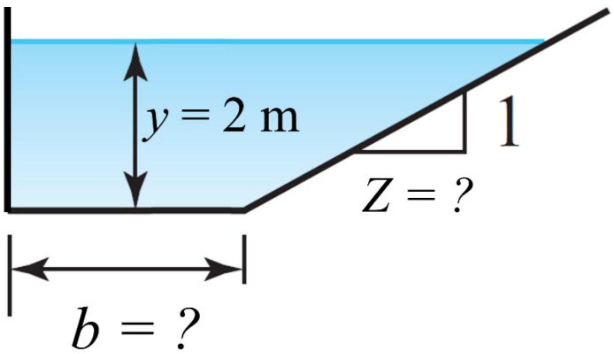
$H = 95\,m$

$Q = 190\,m^3/h$

$Q = 0.0528\,m^3/s$

5. **(25 points)** The channel below carries a discharge of 10 m^3/s of water with a velocity of 1.0 m/s. If the channel is designed for **maximum hydraulic efficiency** conditions, what should be the bottom (***b***) and the side slope (***Z***) of the channel?

Derivative rule for a power function: $\frac{dx^n}{dm} = nx^{n-1}\frac{dx}{dm}$



(5) $Q = 10\ m^3/s$ $\quad Q = AV$

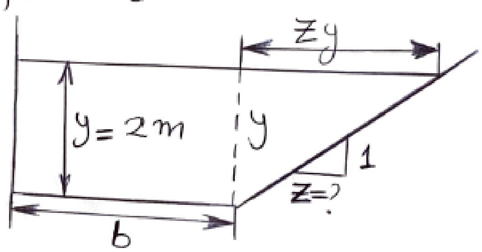
$V = 1\ m/s$ $\quad A = 10/1 = 10\ m^2$ (Constant)

$b = ?$ $\quad Z = ?$

Maximum hydraulic efficiency : $P_{minimum}$

$A = by + \frac{Zy^2}{2}$

$P = y + b + \sqrt{1+Z^2}\,y$



$A = 2b + 2Z$

$P = b + 2 + 2\sqrt{1+Z^2}$

$\frac{\partial P}{\partial b} = 0 \rightarrow 0 = 1 + 2\left(\frac{1}{2}\right)(1+Z^2)^{-1/2}(2Z)\frac{dZ}{db}$

$-1 = \frac{2Z}{\sqrt{1+Z^2}}\frac{dZ}{db} \rightarrow \frac{dZ}{db} = -\frac{\sqrt{1+Z^2}}{2Z}$ ...(1)

$* \frac{\partial A}{\partial b} = 0 \quad 0 = 2 + 2\frac{dZ}{db}$

$-2 = 2\left(-\frac{\sqrt{1+Z^2}}{2Z}\right) \rightarrow 2Z = \sqrt{1+Z^2}$

$4Z^2 = 1 + Z^2$

$3Z^2 = 1$

$Z = \sqrt{\frac{1}{3}}$

$Z = 0.577$

$A = 10\ m^2$

$10 = 2b + 2\sqrt{\frac{1}{3}}$

$b = 4.42\ m$