Florida International University
CWR 3201 Fluid Mechanics, Fall 2020
Final Exam

Instructor: Arturo S. Leon, Ph.D., P.E., D.WRE

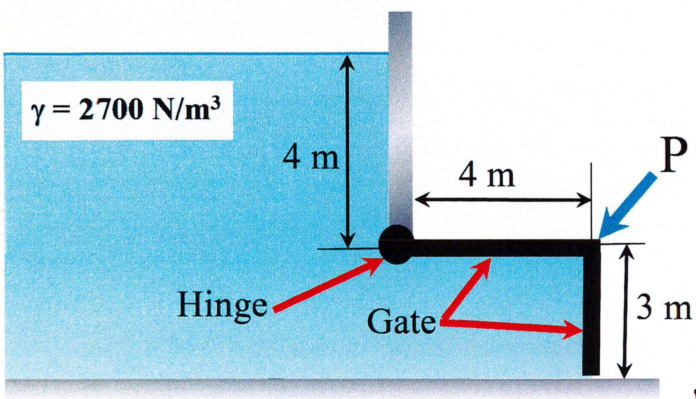
Student Name: Arturo Leon **Date:**

Panther ID:

- ✓ You will have 2 hours to complete the exam. You will have an extra 10 minutes to scan your solution and upload it to Canvas [**Assignment "Upload your Final Exam Solution HERE"**].
- ✓ The exam is closed book and closed notes. You can use the two-page formula sheet provided via Canvas. Only the two pages (front and back) with handwritten equations are allowed.

Put your full name on **ALL** pages of your scratch paper containing your solution **AND** upload your solution as a **SINGLE** PDF file **(2 points).**

1. **(18 points)** The gate below is closed, as shown in the figure below. What is the horizontal and vertical force of the liquid acting on the gate below? The gate width is 5 m. The **liquid has a specific weight of 2700 N/m³**.



Horizontal Force

$F_h = \gamma \bar{h} A$

$\bar{h} = \left(4 + \frac{3}{2}\right)$

$\bar{h} = 5.5\ m$

$A = 3 \times 5 = 15 m^2$

$F_h = 2700 \times 5.5 \times 15$

$F_h = 222,750 N$

$\boxed{F_h = 222.8\ kN}$

Vertical Force

$F_V = P \cdot A = (2700 \times 4)(4 \times 5)$

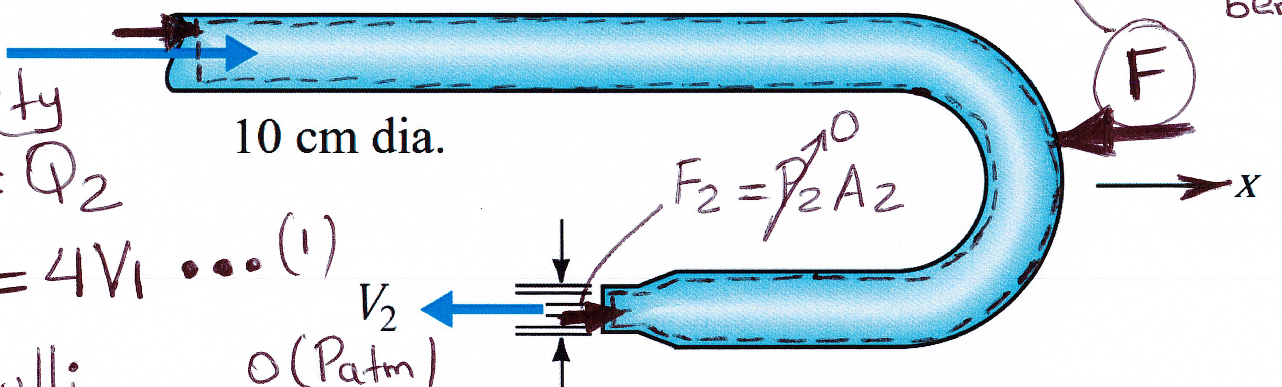
$F_V = 216,000\ N$

$\boxed{F_V = 216\ kN}$

2. **(20 points)** Find V_1 (upstream velocity) if the x-direction force of the **water** on the **horizontal bend** shown below is 20 kN. Neglect head losses. **Hint:** The bend is horizontal, which means that the entire pipe has the same elevation.

Force of water on horizontal bend

$F_1 = P_1 A_1$



$F_2 = \cancel{P_2}^{\,0} A_2$

*Continuity

$Q_1 = Q_2$

$V_2 = 4V_1 \quad \cdots (1)$

*Bernoulli

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{\cancel{P_2}^{\,0\,(P_{atm})}}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} \quad (z_1 = z_2)$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{V_2^2}{2g} \longrightarrow \text{From (1)} \quad P_1 = \left(\frac{16V_1^2}{2g} - \frac{V_1^2}{2g}\right)\gamma$$

$$P_1 = \frac{15}{2} V_1^2 \rho \quad \cdots (2)$$

*Momentum "x" direction

$\dot{m} = \rho A_1 V_1$

$$\Sigma F = \dot{m}(V_{2x} - V_{1x})$$

$$F_1 + \cancel{F_2}^{\,0} - F = \dot{m}(-4V_1 - V_1)$$

$$P_1 A_1 - F = \rho A_1 V_1 (-5V_1)$$

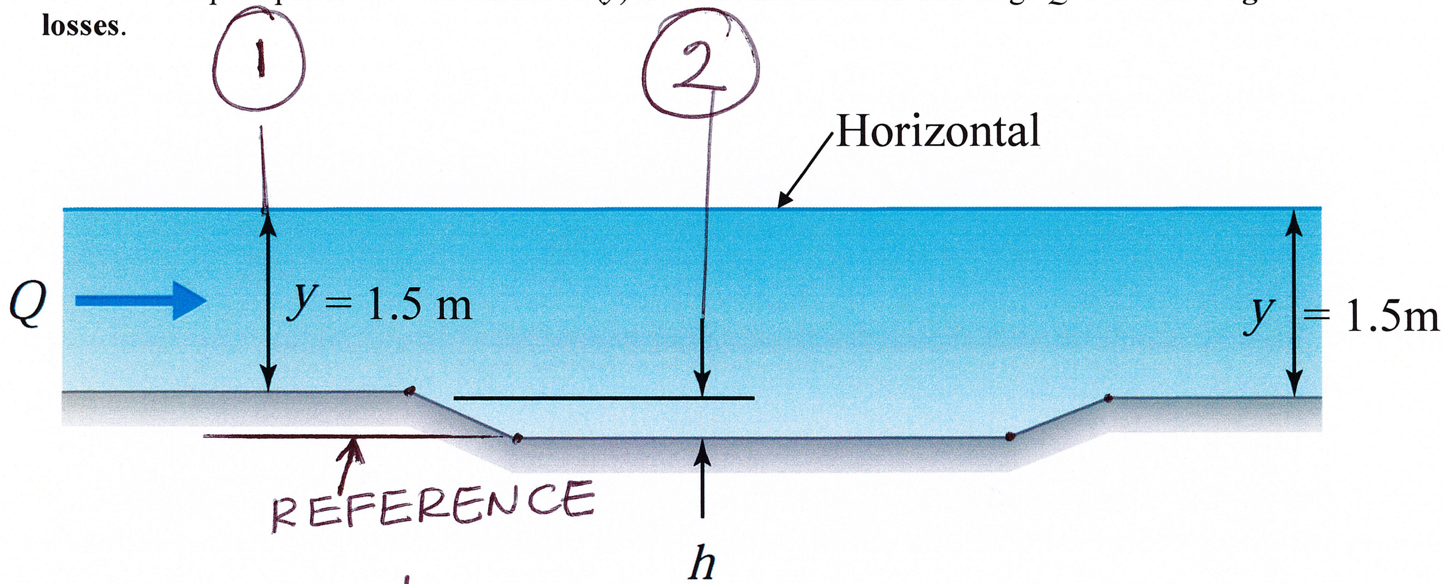
From (2)

$$\frac{15}{2} V_1^2 (1000)\, \pi \times \frac{0.1^2}{4} - 20{,}000 = 1000 \times \pi \times \frac{0.1^2}{4}(-5V_1^2)$$

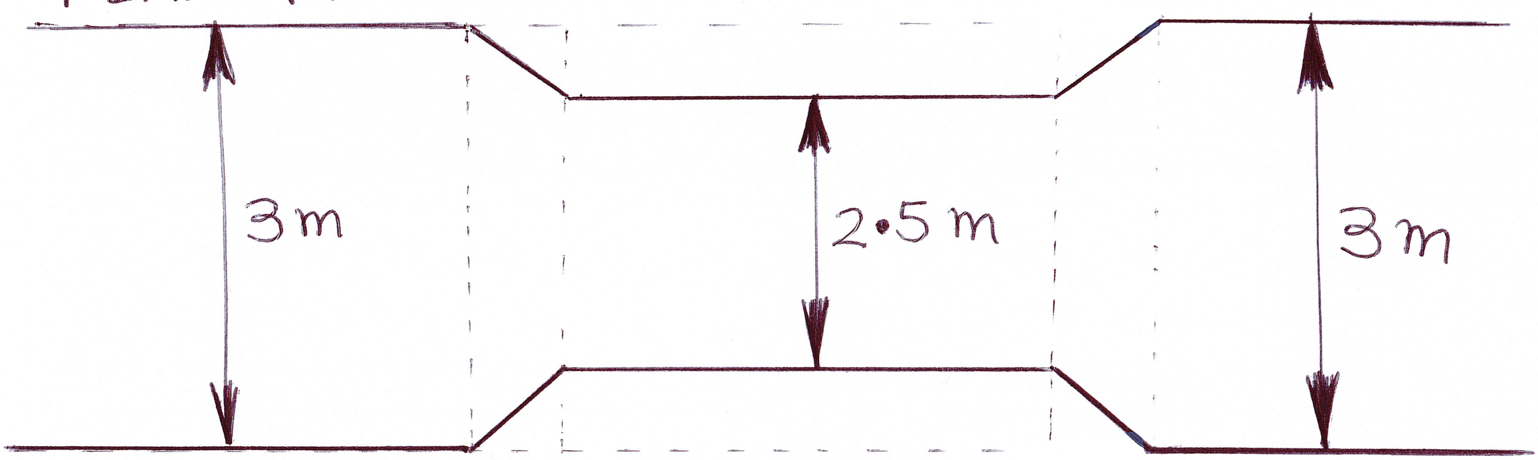
$$98.17\, V_1^2 = 20{,}000$$

$$\boxed{V_1 = 14.27 \text{ m/S}}$$

3. **(20 points) Water** flows in a 3 m wide rectangular channel. At a transition section, the **channel width** is decreased to 2.5 m for a short distance, and then is increased back to the original **channel width** of 3 m. Find "h" (channel bottom elevation drop) in the figure below to maintain a horizontal water surface through the transition. **Hint:** The water elevation through the transition is horizontal, as shown in the figure below. The water depth upstream of the transition (y) is 1.5 m and the flow discharge Q is 6 m^3/s. **Neglect head losses**.



PLAN VIEW



* Energy equation neglecting head losses.

$$y_1 + \frac{V_1^2}{2g} + h = y_2 + \frac{V_2^2}{2g} + 0$$

$$\cancel{y} + \frac{V_1^2}{2g} + \cancel{h} = \cancel{y} + \cancel{h} + \frac{V_2^2}{2g}$$

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} \longrightarrow V_1 = V_2 \quad \ldots (1)$$

$$Q = 6\,\frac{m^3}{S} = A_1 V_1 = A_2 V_2$$

$$6 = (3 \times 1.5)\, V_1 \rightarrow \boxed{V_1 = 1.33\ m/s}$$

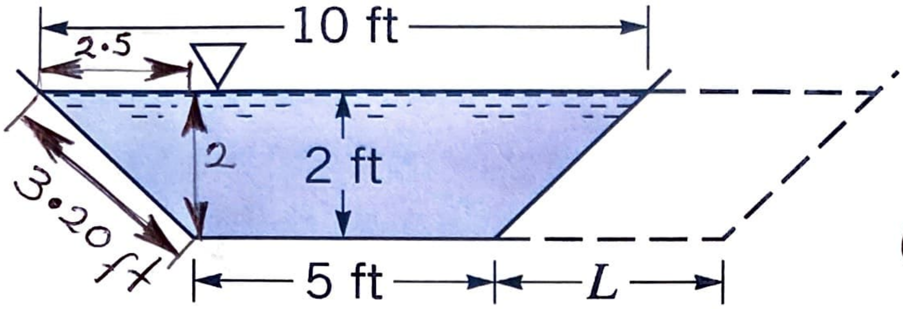
From (1) $\boxed{V_2 = 1.33\ m/s}$

$$\therefore\ 6 = A_2 V_2$$

$$6 = 2.5 \times (1.5 + h)(1.33)$$

$$\boxed{h = 0.30\ m}$$

4. **(20 points).** The canal shown below is to be widened so that the **water flow discharge can be tripled** (i.e., flow discharge after widening is three times the initial flow discharge). Determine the additional width, ***L***, required if all other parameters (i.e., flow depth, bottom slope, surface material, side slope) are to remain the same.



initial (o)

Widened (w)

Manning's eq.

$$Q = \frac{k}{n} A R^{2/3} S_o^{1/2}$$

$$Q_W = 3 Q_o$$

$$\frac{\cancel{\frac{k}{n}} A_W R_W^{2/3} \cancel{S_o^{1/2}}}{\cancel{\frac{k}{n}} A_o R_o^{2/3} \cancel{S_o^{1/2}}} = 3 \quad \ldots (1)$$

Initial

$$A_o = \frac{(10+5)}{2} \times 2 = 15\ ft^2$$

$$P_o = 5 + 2 \times 3.2 = 11.4\ ft$$

$$R_o = 1.316\ ft$$

Widened

$$A_W = \frac{(10+L+5+L)}{\cancel{2}} \times \cancel{2}$$

$$A_W = 15 + 2L\ ft^2$$

$$P_W = 11.4 + L\ ft$$

$$R_W = \frac{15+2L}{11.4+L}$$

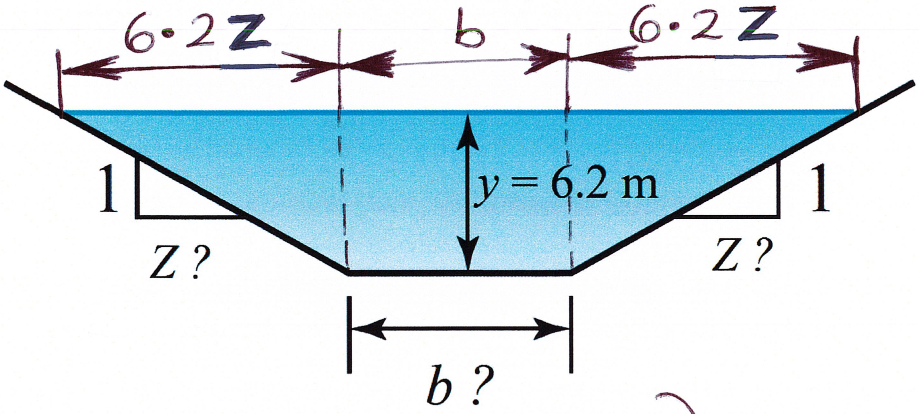
In (1)

$$(15+2L)\left(\frac{15+2L}{11.4+L}\right)^{2/3} = 3\left(15 \times 1.316^{2/3}\right)$$

$$\frac{(15+2L)^{5/3}}{(11.4+L)^{2/3}} = 54.04$$

$$\boxed{L = 11.8\ ft}$$

5. **(20 points)** The trapezoidal channel below carries a discharge of 90 m^3/s of water with a velocity of 2 m/s. The water height of the channel must be 6.2 m. If the channel is designed for **maximum hydraulic efficiency** conditions, what should be the channel bottom (***b***) and the side slopes (*z*) of the trapezoidal channel? **Hint:** The left and right side slopes of the channel (*z*) are the same.



$Q = AV$

$A = \frac{90}{2} = 45 \text{ m}^2$

$A = \left(\frac{b + b + 2(6.2Z)}{2}\right) \times 6.2$

$A = 6.2(b + 6.2Z) = 6.2b + 38.44Z \quad \ldots (1)$

$P = b + 2y\sqrt{1+Z^2} = b + 12.4\sqrt{1+Z^2} \quad \ldots (2)$

* A is constant. Thus $\frac{dA}{dZ} = 0$

In (1)

$0 = 6.2\frac{db}{dZ} + 38.44 \rightarrow \boxed{\frac{db}{dZ} = -6.2} \quad \ldots (3)$

* Max. hydraulic efficiency (P is minimum)

$\frac{dP}{dZ} = 0$

$d(x^n) = nx^{n-1}$

In (2)

$\frac{db}{dZ} + 12.4 \times \frac{1}{2}(1+Z^2)^{-1/2}(2Z) = 0$

$-6.2 + \frac{12.4Z}{\sqrt{1+Z^2}} = 0$

$$12.4Z = 6.2\sqrt{1+Z^2}$$

$$2Z = \sqrt{1+Z^2}$$

$$4Z^2 = 1+Z^2 \longrightarrow 3Z^2 = 1$$

$$\boxed{Z = 0.577}$$

Also: $A = 45\ m^2$

In 1

$$45 = 6.2b + 38.44(0.577)$$

$$\boxed{b = 3.68\ m}$$