SJK
Autumn 78/79

ME 255

Course Conduct

1. Text: A. H. Shapiro, The Dynamics and Thermodynamics of Compressible Fluid Flow, Ronald Press, 1953.

   Need Also: Gas Tables, J.H. Keenan and J. Kaye, Wiley.

2. We will do both exercises and problems.

   Exercises: Not to be turned in. To check understanding of concepts, raise questions for further discussion, clarification.

   Problems: To be turned in. Will be graded and returned.

   One Hour Quiz: See schedule.

3. Grading: Grades will depend on total score, S, as follows:

   A $S \geq 950$
   B $949 \geq S \geq 800$
   C $799 \geq S \geq 650$
   D/NC $649 \geq S$

Problem Sets	- 5 Sets -	100 pts. each	500	874
Quiz		100 pts.	100	91
Final			500	

   Total Possible S = 1,100
   without Bonus

   Bonus Points: You can earn up to a maximum of 100 bonus points by turning in extra problems. Each problem counts 5 points. Up to a maximum of 50 bonus points will be assigned for class participation indicating good preparation and/or thought -- at discretion of instructor.

4. Movies: If at all possible, arrange to see movie by Bryson on "Waves" and Coles on "One-Dimensional Channel Flow." As many others as you can manage. (See schedule sheet attached.)

5. Some Useful References:
   1. H. W. Liepmann and A. Roshko, Elements of Gasdynamics, Wiley, 1957. (Standard text, continuum approach emphasizing external flow.)
   2. J. E. A. John, Gas Dynamics, Allyn and Bacon, 1969. (Bare bones essentials in easy-to-read form.)
   3. W. G. Vincenti and C. H. Kruger, Jr., Introduction to Physical Gasdynamics, Wiley, 1965. (Standard reference on molecular approach.)
   4. L. Howorth (Editor), Modern Developments in Fluid Dynamics -- High Speed Flow. Two volumes, Oxford Press, 1953. (Monograph on classic analyses.)

SJK

Class Schedule

ME 255, Fall 1977-78

Dates	Topic	Assignments	Emphasis
Sept. 27	Class organization		
Sept. 29	Review foundations	pp. 3-21	Study
- Oct. 6	plus control volume theory	pp. 23-43	Study
Oct. 9-13	Wave motion	pp. 45-54	Study
	Weak compressibility waves	pp. 55-69	Useful
Oct. 16-25	Isentropic flow (simple area change)	pp. 73-81	Study
	Converging nozzles	pp. 81-82	Useful
		§4.4	Study
		§4.5	Illustrative
		§4.6, 4.7	Study
		§4.8, 4.9, 4.10	Illustrative
Oct. 27	Hour Quiz X		
Oct. 30	Shock waves	§5.1, 5.2	Useful
- Nov. 8	Supersonic nozzles	§5.3	Study
		§5.4, 5.5	Illustrative
		§5.6	Useful
		§5.7	Useful
		§5.8	Illustrative
		§5.9, 5.10	Study
		§5.11, pp. 143-148	Study
		pp. 149-152	Illustrative
		§5.12	Study
Nov. 10-20	Simple friction	pp. 159-168	Study
	Fanno line	pp. 169-172	Illustrative
		pp. 173-180	Study
		pp. 181-186	Illustrative
Nov. 22-27	Simple energy exchange	pp. 190-196	Study
	Rayleigh line	pp. 197-200	Illustrative
		pp. 201-203	Study
		pp. 203-213	Illustrative
Nov. 29	Combined processes	pp. 219-240	Study
- Dec. 6		pp. 241-260	Illustrative
Dec. 8	Review and summary		

ME 251A/255
Autumn 1978
J. P. Johnston
S. J. Kline

ME 251A/255

SCHEDULE OF FLUID MECHANICS FILMS

Time: Films start at 12:15 p.m. and run, on the average, for 30 minutes.

Place: Room 300

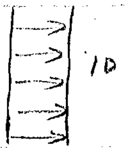
Oct. 9	Eulerian and Lagrangian Description in Fluid Mechanics
Oct. 16	Surface Tension in Fluid Mechanics
Oct. 23	Pressure Fields and Fluid Acceleration
Oct. 31	Flow Visualization
Nov. 6	Waves in Fluids
Nov. 13	Channel Flow of a Compressible Fluid
Nov. 20	Vorticity, Parts 1 and 2 (longer than average film)
Nov. 27	Deformation of Continuous Media
Dec. 4	Rheological Behavior of Fluids

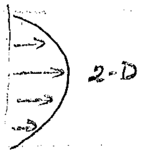
NOTE: A paperback book which discusses all these, and other films of the NCFMF series is available. It is entitled, "Illustrated Experiments in Fluid Mechanics," A. Shapiro, Editor; M.I.T. Press, (Price $7.50).

If there is enough interest, we could place a group order later in the quarter.

ME 255 Gas Dynamics Steve Kline

1-D we mean in velocity profile





Compressibility is appreciable wrt mach no when $M > .2$

Shapiro Ch 1-8 will be covered.
Need to use all equations CONT, MOMENT, EQUATION OF STATE, ENERGY, ENTROPY, HEAT TRANSFER.

Read pg 3-21 and 23-43 by Friday

9/29/78

Fluid Something which deform under shearing forces
Continuum - treat fluid as a continuous substance, $\frac{\lambda}{\delta x} << 1$ mean free path << any physical dimension in problem.
density $\rho \triangleq \lim_{\delta V \to 0} \frac{\delta m}{\delta V}$

$\bar{V}_p \triangleq \lim_{\delta t \to 0} \frac{\delta \bar{j}}{\delta t}$: $\bar{j}$ position vector LAGRANGIAN wrt inertial frame of reference follow particle as it move through space. a function of time only.

Eulerian - look at a point in space & see what happens to properties of flow as they pass through it $V = V(x, y, z, t)$ When you get derivative you get the substantive derivative

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}\frac{dx}{dt} + \frac{\partial V}{\partial y}\frac{dy}{dt} + \frac{\partial V}{\partial z}\frac{dz}{dt}$$

$$= \underset{\text{time}}{\frac{\partial V}{\partial t}} + \underset{\text{convective}}{\underline{u\frac{\partial V}{\partial x} + v\frac{\partial V}{\partial y} + w\frac{\partial V}{\partial z}}}$$

Streamline locus of all pts tangent to velocity vector.

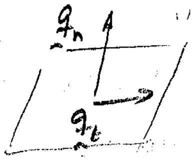
Steady flow $(\ )_{\partial t} = 0$

Boundary Layer conditions R_N must be high, viscous flow, flow must be unseparated

If one solves complete Navier Stokes and then sets $\mu = 0$, we will not get same solution as if we first ~~set~~ $\mu = 0$ then solve

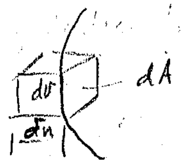
Control Volume

System: a fixed and identifiable quantity of mass (CM)
Flux: flow of an extensive value thru a surface (CV)
↳ a value proportional to mass or volume



during δt $\quad \delta m_{in} = \rho \delta V = \rho \delta n \, \delta A$

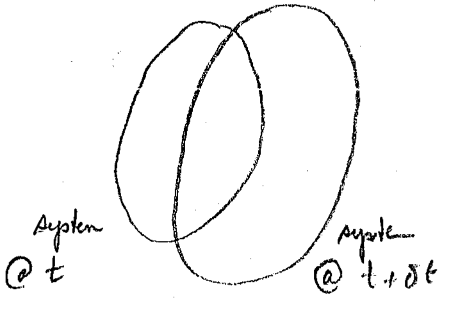
and $\dot{m} = \lim_{\delta t \to 0} \frac{\delta m}{\delta t} = \rho \frac{dn}{dt} dA$
$= \rho \underset{\sim}{q} \cdot d\underline{A}$



Conservation of mass

$$\sum_{i=1}^{N} \delta \dot{m}_i = \sum \rho \underset{\sim}{q}_i \cdot d\underset{\sim}{A}_i$$ rate of change of control mass across boundary

elemental volume gives differential equations
Control volume approach gives integral equations



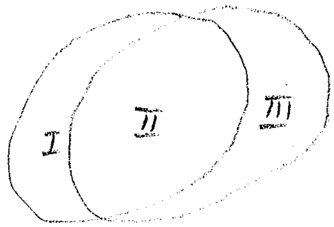
$$\left(\frac{dm}{dt}\right)_{in} = \left(\frac{dm}{dt}\right)_{inside} + \left(\frac{dm}{dt}\right)_{out} \quad \text{or} \quad \dot{m}_{in} = \left(\frac{dm}{dt}\right)_{inside} + \dot{m}_{out}$$

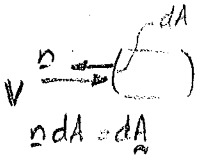
$$\text{ie} \int_{in} \rho \underset{\sim}{q} \cdot d\underset{\sim}{A} = \frac{\partial}{\partial t} \int_V \rho \, dV + \int_{out} \rho \underset{\sim}{q} \cdot d\underset{\sim}{A}$$

$$\text{or} \quad 0 = \frac{\partial}{\partial t} \int_V \rho \, dV + \int_{CS} \rho \underset{\sim}{q} \cdot d\underset{\sim}{A}$$ positive vectors out of control vol.

For steady 1-D flow $\rho_i A_i V_i = \text{const.}$

C.V. Momentum





Syste @ t I+II
Syste @ t+dt II, III

$$\mathbb{F}_{on_{cv}} + \mathring{M}_{in} = \underset{\text{of } M \text{ inside}}{\text{rate of change}} + \mathring{M}_{out}$$

$$\underset{+\text{ right}\rightarrow}{\mathbb{F}_{on\ cv}} = \frac{d}{dt}(M\mathbb{V}) = \lim_{\delta t \to 0}\left\{ -\frac{\mathbb{V}\delta m_{I}}{\delta t} + \frac{\mathbb{V} m_{II_{t+\delta t}} - \mathbb{V} m_{II_t}}{\delta t} + \frac{\mathbb{V}\delta m_{III}}{\delta t}\right.$$

$$= -\int_{in} \mathbb{V} \cdot (\rho\, \mathbb{V}\cdot d\mathbb{A}) + \int_{out} \mathbb{V}(\rho\, \mathbb{V}\cdot d\mathbb{A}) + \frac{\partial}{\partial t}\int_{\upsilon} \rho \mathbb{V}\, d\upsilon$$

relative to cv

relative to inertial frame

energy equation

$$q + \int h_f(\rho \underset{\nwarrow \text{relative to cv}}{\mathbb{V}} \cdot d\mathbb{A}_{in}) = \frac{\partial}{\partial t}\int_{\upsilon} e\rho\, d\upsilon + \int h_f(\rho \mathbb{V}\cdot d\mathbb{A}_{out}) + P_x$$

e involves all energy / unit mass $\quad e = u + \frac{V^2}{2} + gz + \dots$

h_f all energy crossing boundary

internal energy $u = u(p,T)$ can be written as this if the list below

Simple substance, one reversible work mode$^{(pdv)}$. Equil, one pure substance & reference rest frame

one reversible work mode excludes effects of

1. Electro magnetism
2. surface tension
3. fluids stresses or anisotropic stress in solids
4. motion
5. gravity

$h \triangleq u + pv$, $h_f \triangleq e + pv$ (flow enthalpy), $h_0 \triangleq u + pv + \frac{V^2}{2}$

which is obtained from $Q = \Delta E + W$

$$\delta W = p\, \delta n A$$
$$= p\, \delta v$$

Where do we get Equations of state? comes from definition of system (primarily) + 1st & 2nd law of Thermo + how many independent variables you assume.

no other governing equations

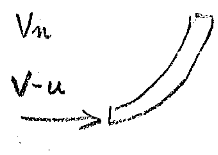
mass $$\int_{cs} \rho \overset{relative}{\boxed{\mathbb{V}}} \cdot dA + \frac{\partial}{\partial t}\int_{c.v} \rho\, dv = 0$$

mom $$\mathbb{F} = \oint_{cs} \overset{inertial}{\boxed{\mathbb{V}_i}} (\rho \overset{relative}{\boxed{\mathbb{V}_n}} \cdot dA) + \frac{\partial}{\partial t}\int_{cv} \rho \mathbb{V}_i\, dv$$ $\mathbb{r} \times$ mom = angular mom.

energy $$q = \oint_{cs} h_f (\rho \overset{relative}{\boxed{\mathbb{V}}} \cdot dA) + \frac{\partial}{\partial t}\int_{cv} \rho e\, dv + P_x$$

$$P_x = \dot{w} - pv \qquad h_f \text{ (includes } pv)$$

Prob 1.4 work/heat are transfers across the boundary



Assume
Steady state
1-D

1. get CV
2. inertia frame
3. Steady State flow what parts of equs can be dropped.
   1-D

steady flow – no dependence of property on time $\frac{\partial}{\partial t} = 0$

steady state flow – $\frac{\partial}{\partial t} = 0$ & flow is in thermodynamic equil

$$\mathbb{F} = -V_i (\rho \mathbb{V}_n A)$$

on left

$$F_x = -\rho A V^2 + V_{Ex}(\rho A V)$$

where $\overset{absolute}{V_{Ex}} = u + (V - u)\cos\beta$

$V_{ABS} = V_{veh} + V_{rel}$

$\overset{\circ}{work} = F \times u$

$\frac{d\dot{W}}{du} = 0$ give's you max.

for moving blade

$F_x = \left[u + (V-u)\cos\beta\right]\rho(V-u)A - V\left[(V-u)\rho A\right]$

$\overset{\circ}{W} = F_x u$

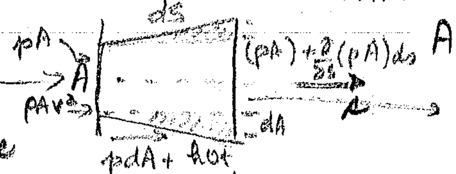
10/4/78

1) Mom Theorem ⇒ Euler "s" eqn. newton's 2nd law on controlvolume
2) Analysis of "weak" pressure waves

For complete Eulers Eqns see p273 ff

take mom theorem applied to stream tube

Along a Streamline



Ads is volume

$\rho AV^2 + \frac{\partial}{\partial s}(\rho AV^2)ds$ mom$_{out}$

derive - Steady State flow w/o friction, w/o bodyforce (SSF w/o frict w/o gravity)

$$\mathbb{F} + \int_{in} \mathbb{V}_1(\rho\, \mathbb{V}_n \cdot dA) = \underbrace{\frac{\partial}{\partial t}\int_{V} \rho\, \mathbb{V}_1\, dV}_{\text{zero SSF}} + \int_{out} \mathbb{V}_2(\rho\, \mathbb{V}_n \cdot dA)$$

$$\mathbb{F}_s = -\frac{\partial}{\partial s}(pA)ds + pdA = \frac{\partial}{\partial s}(\rho AV^2)ds$$

$\dot{m}_{out} - \dot{m}_{in}$

$$= -A\frac{\partial p}{\partial s}ds - p\frac{\partial A}{\partial s}ds + \cancel{pdA} = \underbrace{V\frac{\partial}{\partial s}(\rho AV)ds}_{} + \rho AV\frac{\partial V}{\partial s}ds$$

= 0 since mom theorem $\underline{sst}$ ⇒ $\nabla \cdot q = 0$
or $\dot{m}$ = const

∴ drop Ads ⇒ $\boxed{-\frac{\partial p}{\partial s} = \rho V_1\frac{\partial V}{\partial s}}$ or $dp = -\rho VdV$

w/gravity $dp = -\rho VdV - \rho g dz$

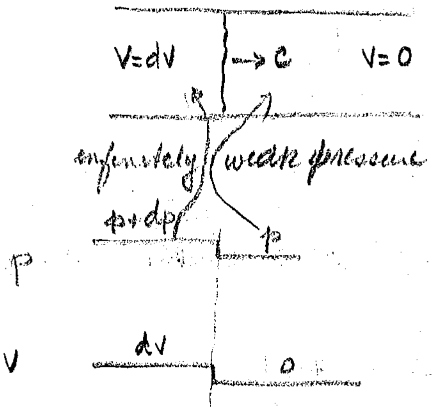
$$\frac{dp}{dn} = \frac{\rho V^2}{R_{streamline}}$$ p ↑ outward from the center of curvature in absolute space

s equation can also be obtained if flow is irrotational $\nabla \times \underline{V} = 0$

$$(\underline{V}\cdot\nabla)\underline{V} = \nabla\frac{\underline{V}^2}{2} - \underline{V}\times\nabla\times\underline{V}$$

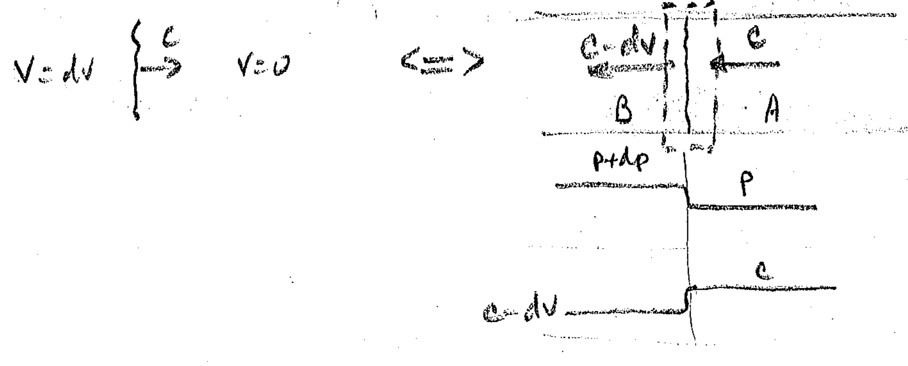
for incompressible flow: $\frac{p}{\rho} + \frac{V^2}{2} = const$ (doesn't require a specific process path.)

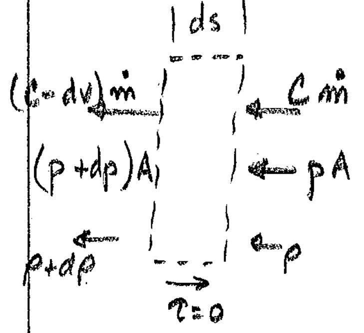
Barotropic fluid is one in which we have $\rho = \rho(p)$ only



steady wave @ velocity c ; plane wave w/o friction

this is an unsteady problem unless you ~~at~~ transform problem ∋ wave is standing still





$$(p+dp)A - pA = -(c-dv)\dot{m} + C\dot{m}$$
$$dpA = +dv\,\dot{m}$$
$$dp\cdot\cancel{A} = \rho\cancel{A}c\,dv$$
$$dp = \rho c\,dv$$

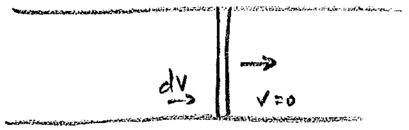
from continuity $\frac{d}{ds}(\rho AV)=0 \Rightarrow \rho AV = \text{const.}$

or $(\rho+d\rho)A(c-dV) - \rho Ac = 0 \Rightarrow c\,d\rho - \rho dV = 0$

$$c^2 d\rho = \rho c\,dv = dp \qquad c^2 = \left(\frac{dp}{d\rho}\right)_{\text{isent.}}$$

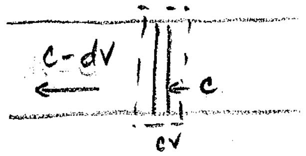
$\dot{q}=0$ (negligible heat transfer), reversible $\Rightarrow$ isentropic

10/12/78



$$dp = \rho c\,dV_{\text{particle}}$$
$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_{\text{wave}}$$

make transformation



an adiabatic reversible process = isentropic

Assuming isentropic process $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$

note that $\left(\frac{\partial p}{\partial \rho}\right)_s \neq \left(\frac{\partial p}{\partial \rho}\right)_T$ the path is different

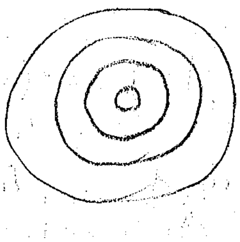
$\frac{P}{\rho^k}$ = constant for isentropic process in perfect gas

$\ln p - k\ln\rho = 0$ differentiating gives $\frac{dp}{p} - k\frac{d\rho}{\rho} = 0$

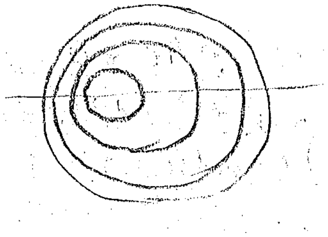
or $\left(\frac{dp}{d\rho}\right)_s = \frac{kp}{\rho} = kRT$ (using $p=\rho RT$) $\therefore$ $c = \sqrt{kRT}$ since $c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$

for air $R = 53.3 \frac{ft\#}{\#_{mass}\,^\circ R}$ & $k = 1.4$ $\therefore$ $c = 49.02\sqrt{T}$ where $T = {}^\circ R$

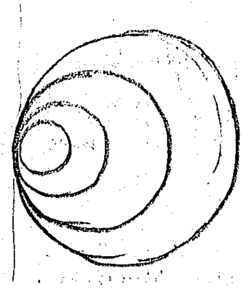
if a stationary wave is propagated



if wave moves at some velocity $V < c$



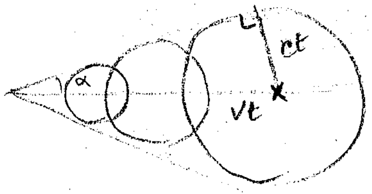
if wave moves at speed of sound



if velocity is supersonic $V > c$

$\sin^{-1}\frac{1}{M}$?

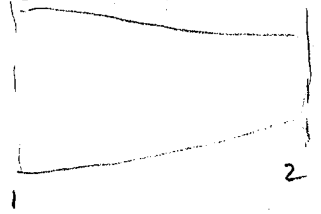
$$\sin\alpha = \frac{ct}{Vt} = \frac{1}{M}$$



10/13/78

Exercises due 18 October Text 3.1, 3.6, 3.7

Isentropic flow



1) find CV

2) Assumptions 1-D, steady state, adiabatic, assume $\Delta grav \approx 0$ drop EM etc

3) Equations

<u>Continuity</u> S.S. $\Rightarrow$ $\dot m = \rho A V = \text{const.}$

<u>Energy</u> - no work, adiabatic, normally $\cancel{\dot q} + \sum_{in} h_f \dot m = \cancel{\frac{\partial E_{cv}}{\partial t}} + \cancel{\dot W_x} + \sum_{out} h_f \dot m$

$$h_{f_1} = h_{f_2} \qquad h_f \overset{\Delta}{=} e + pv$$

$$e = u + gz + \frac{V^2}{2} + EM + \ldots$$

system + equation of state gives you what e is made up from.

now $h_0 = u + pv + \frac{V^2}{2}$

$\therefore h_f \Rightarrow h_0$

$\therefore$ define $u + pv = h$ $\therefore$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = \text{const} \qquad h = \text{energy/unit mass} \qquad (\text{isoenergetic}$$

<u>equation of state</u> $h = h(\rho, s)$

if you look at a Δx slice since SS & we are looking at 1-D then we can see that all equations are functions of x $\therefore$ take total diff

$\therefore$ $dh + V dV = 0$

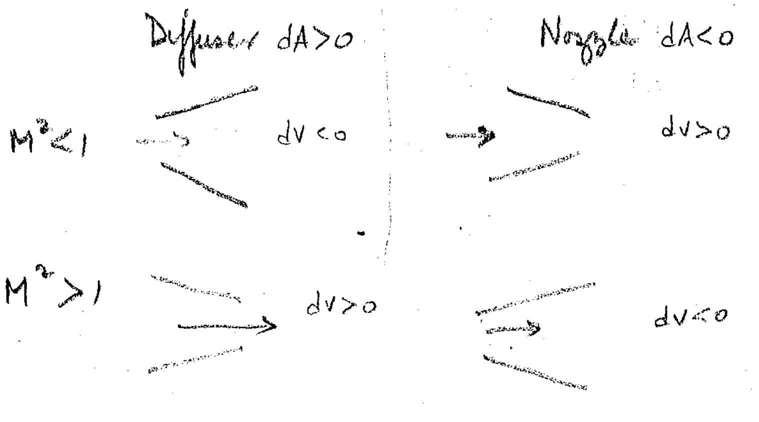
Thermo: we dont care about time $\therefore$ the only types of energy transport are work n heat

$$\frac{dA}{A} + \frac{d\rho}{\rho} + \frac{dV}{V} = 0 \quad \text{from continuity}$$

For isentropic flow <u>momentum eq</u> says $dp = -\rho V dV$

$$\frac{dA}{A} = -\left[\frac{dV}{V} + \frac{d\rho}{\rho}\right] = -\frac{dp}{\rho}\left[\frac{1}{V^2} - \frac{1}{c^2}\right) = \frac{dp}{\rho V^2}\left[1 - M^2\right] \text{ \& using } dp = -\rho V dV$$

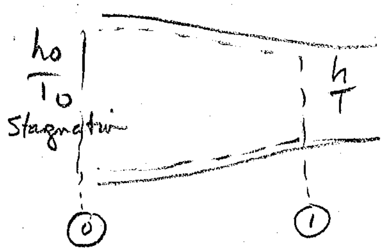
$$\frac{dA}{\rho A V} = \frac{dA}{\dot m} = -\left(\frac{1 - M^2}{\rho V^2}\right) dV$$

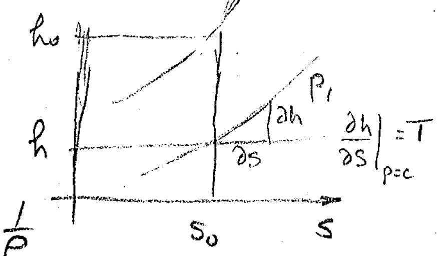


10/16/78

Review of Quasi 1-D Steady, Adiabatic isentropic

Assumptions	Basic Eq
Quasi 1-D flow	$w = \rho V A = \text{const}$
Steady	$h_0 = h + \frac{V^2}{2}$
Isentropic	$\longrightarrow s = s_0$
Adiabatic	$h = h(s, p)$





we will be looking at area change

Gibbs Relation holds $dh = T ds + \frac{dp}{\rho}$

$$\left(\frac{\partial h}{\partial s}\right)\Bigg|_p = T \text{ since isentropic} \qquad \left(\frac{\partial h}{\partial p}\right)\Bigg|_s = \frac{1}{\rho}$$

note that slope of p=const line increase as p increases.

from $\underset{\text{isentropic}}{T\overset{\to 0}{ds}} = dh - \frac{dp}{\rho}$ & $dh = \frac{dp}{\rho}$ and from energy eq $dh = -VdV$

using the 2 gives $\boxed{-VdV = \frac{dp}{\rho}} = \left(\frac{dp}{d\rho}\right)\frac{d\rho}{\rho} = c^2 \frac{d\rho}{\rho}$

$$-V^2 \frac{dV}{V} = c^2 \frac{d\rho}{\rho} \text{ or } \boxed{-M^2 \frac{dV}{V} = \frac{d\rho}{\rho}}$$

Using $w = \rho V A = \text{const} \implies \boxed{\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0}$

put result involving $(d\rho, dV) \implies \frac{dA}{A} = (M^2 - 1)\frac{dV}{V}$ (4)

for $M < 1$ $dV \propto -dA$ $V\uparrow$ $A\downarrow$

$M > 1$ $dV \propto dA$ $V\uparrow$ $A\uparrow$

$M = 1$ $\Rightarrow dA = 0$ $A = \text{constant}$ or area is min

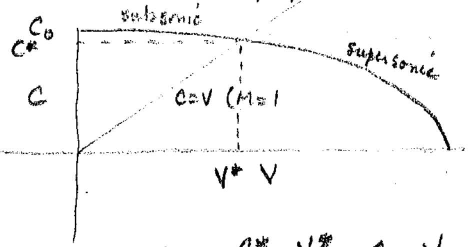
now if we add that the fluid is a perfect gas then $h = c_p T$

perfect gas $\Rightarrow \left(\frac{dh}{dT}\right) = \text{const}$ & $p = \rho R T$ must be satisfied, $\frac{du}{dT} = \text{const}$.

$\therefore$ from energy $h_0 = h + \frac{V^2}{2} \Rightarrow C_pT_0 = C_pT + \frac{V^2}{2}$ where $C_p = \frac{k}{k-1}R$

and $\frac{C_0^2}{k-1} = \frac{V^2}{2} + \frac{C^2}{k-1}$ where $C, C_0 = \sqrt{kRT}, \sqrt{kRT_0}$

divide by $\frac{C_0^2}{k-1} \Rightarrow 1 = \frac{V^2}{\frac{2C_0^2}{k-1}} + \frac{C^2}{C_0^2}$ gives an ellipse (steady flow adiabatic ellipse)



when $T \to 0°R$ $V = \sqrt{2C_pT_0} = V_{max}$

(critical speed) $\longrightarrow M=1 \longrightarrow C^* = V^* = C = V$

for an isentropic process. $\frac{kRT}{k-1}(T_0/T - 1) = C_pT(T_0/T - 1) = \frac{V^2}{2} = \frac{C^2}{k-1}(T_0/T - 1)$

from $C_pT_0 = C_pT + \frac{V^2}{2} \Rightarrow C_p(T_0 - T) = \frac{V^2}{2} \Rightarrow$ using $C_p = \frac{kR}{k-1}$, $\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$ (1

using $\frac{C_0^2}{k-1} = \frac{V^2}{2} + \frac{C^2}{k-1}$ and $V^* = C^* \Rightarrow \left(\frac{C^*}{C_0}\right)^2 = \frac{2}{k+1} = \frac{T^*}{T_0}$ (2)

$$\frac{C^2}{k-1} + \frac{V^2}{2} = \left(\frac{C_0}{C^*}\right)^2 \frac{C^{*2}}{k-1} = \left[\frac{k+1}{2}\right]\frac{C^{*2}}{k-1}$$

$$\frac{1}{k-1}\left(\frac{C^2}{C^{*2}}\right) + \frac{1}{2}\left(V/C^*\right)^2 = \frac{k+1}{2(k-1)}$$

$$\frac{1}{k-1}\frac{M^2}{M^{*2}} + \frac{1}{2}M^{*2} = \frac{k+1}{2(k-1)}$$

$$M^{*2} = \frac{\frac{k+1}{2}M^2}{1 + \frac{k-1}{2}M^2} \quad (3)$$

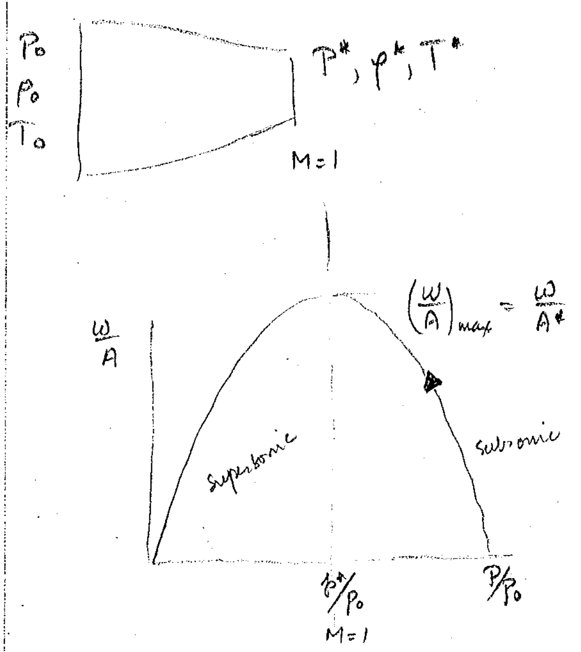
M	M^*
0	0
1	1
∞	$\sqrt{\frac{k+1}{k-1}}$
>1	>1
<1	<1

10/18/78

Isentropic flow

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}} \quad (5)$$

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}} = \left(1 + \frac{k-1}{2}M^2\right)^{\frac{1}{k-1}} \quad (6)$$



$$\left(\frac{T^*}{T_0}\right)_{air} = \frac{2}{k+1} = .8333$$

$$\left(\frac{P^*}{P_0}\right)_{air} = .5283$$

$$\left(\rho^*/\rho_0\right)_{air} = .6339$$

$(\ )^*$ is critical value @ M=1

$\left(\frac{\omega}{A}\right)_{max}$ occurs at a minimum area

Flow rate/unit area

$$G = \frac{\omega}{A} = \rho V = \frac{P}{RT} V = \frac{kp}{kRT} V = \frac{kpV}{c^2} = \frac{kp}{c_0} \frac{V}{c} \frac{c_0}{c}$$

But $\frac{c_0}{c} = \sqrt{\frac{T_0}{T}}$ from $c_0^2 = kRT_0$ etc. also $k/c_0 = k/\sqrt{kRT_0} = \sqrt{\frac{k}{RT_0}}$

$$\frac{\omega}{A} = \sqrt{\frac{k}{R}} \frac{P}{\sqrt{T_0}} M \left(1 + \frac{k-1}{2} M^2\right)^{1/2} \quad (7) \quad = \sqrt{\frac{kW}{\mathcal{R}}} \frac{P}{\sqrt{T_0}} M \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{2}}$$

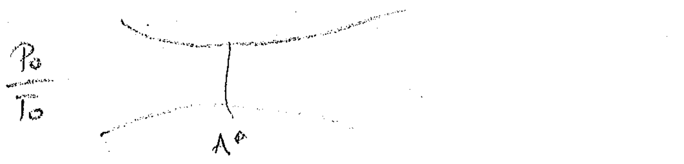
$$\frac{\omega}{A} \cdot \frac{\sqrt{T_0}}{P} \frac{1}{\sqrt{W}} = f(M)$$ plotted on pg 82

$\uparrow$ molecular wt

$$\frac{\omega}{A} = \sqrt{\frac{K}{R}} \frac{P_0}{\sqrt{T_0}} \frac{M}{\left(1 + \frac{k-1}{2} M^2\right)^{\frac{k+1}{2(k-1)}}}$$

$$@\ M=1 \quad \frac{\omega}{A} = \frac{\omega}{A^*} = \left(\frac{\omega}{A}\right)_{max} = \sqrt{\frac{K}{R}} \frac{P_0}{\sqrt{T_0}} \frac{1}{\left(\frac{k+1}{2}\right)^{\frac{k+1}{2(k-1)}}} \quad (8)$$

$$\frac{A}{A^*} = \frac{\omega/A^*}{\omega/A} = \frac{1}{M} \left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2} M^2\right)\right]^{\frac{k+1}{2(k-1)}}$$



for a given gas $\frac{w}{A} \propto \frac{P_0}{\sqrt{T_0}}$

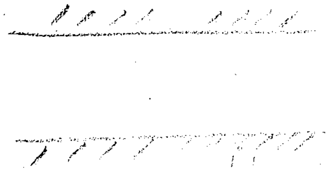
Given $P_0, T_0\ A^*$ $\quad \frac{w}{A} \propto \sqrt{\frac{W}{\mathcal{R}}}$

for air $k = 1.4$ $\quad R = 53.3$ $\quad w = \frac{lbm}{sec}$ $\quad A^* = ft^2$ $\quad p = \frac{lb}{ft^2}$

$$\boxed{\text{Fliegner} \quad \frac{w\sqrt{T_0}}{A^* p_0} = .532}$$

Example 4.5 pg 106

$d = 4\,in$ $\quad w = 2.20\ \frac{lbm}{sec}$ $\quad T_0 = 100°F$ $\quad P_1 = 6\,psia$



find M_1, V_1, P_{0_1}

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}\left(\frac{4}{12}\right)^2 = .0873\,ft^2$$

$$\frac{w}{A}\frac{\sqrt{T_0}}{p}\frac{1}{\sqrt{W}} = \frac{2.2}{.0873}\frac{(560)}{6\times144}\frac{1}{\sqrt{28.9}} = 3.987\times10^{-3}$$ W for air is 28.9

M obtained from charts (pg 82) for $\frac{w}{A}\frac{\sqrt{T_0}}{p}\frac{1}{\sqrt{W}} \Rightarrow M_1 = .73$

go to table B2 pg 615 $\quad M = .73 \Rightarrow P/P_0 = .70155$, $\frac{T}{T_0} = .90368$

$\therefore \frac{6}{P_0} = .70155 \quad P_0 = \frac{6}{.70155} \cong 8.55\,psia$

$\therefore \frac{T}{560} = .90368 \quad T = 506\,°R$

use now $c_1 = \sqrt{kRT_1}$ and then $V = M_1 c_1$ or use $T_{0_1} = T_1 + \frac{V^2}{2c_pJ}$.

10/20/78

Last time

$w = \rho AV$

$h_0 = h + \frac{V^2}{2}$

$s_0 = s$

$P = \rho RT$

$c_v = const$

$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2$

$P_0/p = f_1(M)$

$\rho_0/\rho = f_2(M)$

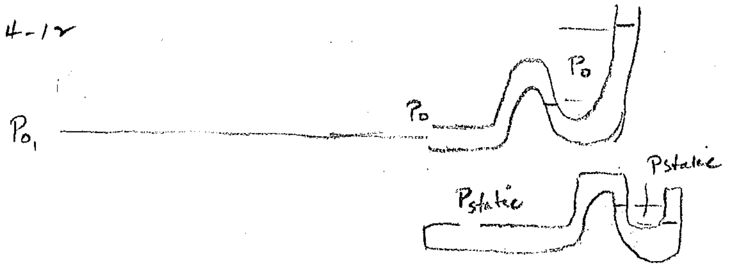
$\frac{w}{A} = \sqrt{\frac{k}{R}}\frac{P_0}{\sqrt{T_0}} f_3(M)$

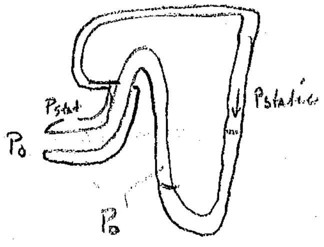
$A/A^* = f_4(M)$

$\frac{T_0}{T^*} = \frac{k+1}{2}$

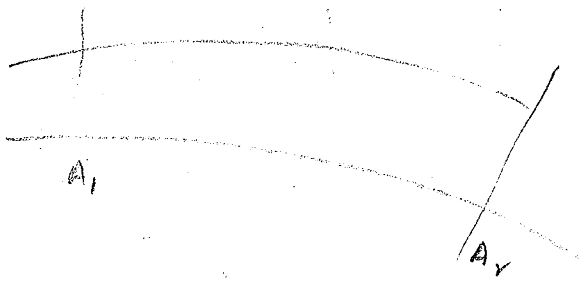
$\frac{P_0}{P^*} = \left(\frac{2}{k+1}\right)^{k/k-1}$

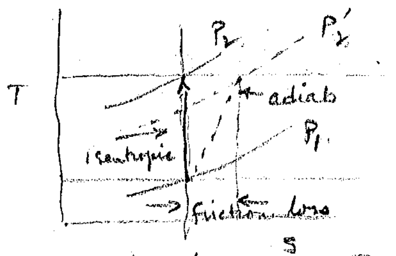
4-12





4-13





$\frac{A_2}{A_1} = .85$

adiabatic ⟺ $T_{o_1} = T_{o_2}$

Isentropic ⟺ $P_{o_1} = P_{o_2}$

not isentropic ⟹ $P_{o_2}' < P_{o_2}$
$s_2' > s_2$

$V_1 = 500$ ft/sec
$P_1 = 10$ psia
$T_1 = 500°R$

find $P_{o2}, T_{o2}, P_2, T_2, V_2, M_2, M_2^*$

since we are adiab & isentropic

$V_1 \, \& \, (T_1 \to C_1) \to M_1 \xrightarrow{\text{Table B2}} \frac{P_{o_1}}{P_1} \to P_{o_1} = P_{o_2}$

$\frac{T_{o_1}}{T_1} \to T_{o_1} = T_{o_2}$

$M_1 = \frac{V_1}{C_1} = \frac{500}{49.02\sqrt{500}} = .452$

Table B2 gives → $\frac{P_1}{P_{o_1}} = .869 \Rightarrow P_{o_1} = 11.5$ psia $= P_{o_2}$

$\frac{T_1}{T_{o_1}} = .96 \Rightarrow T_{o_1} = 520.8°R$

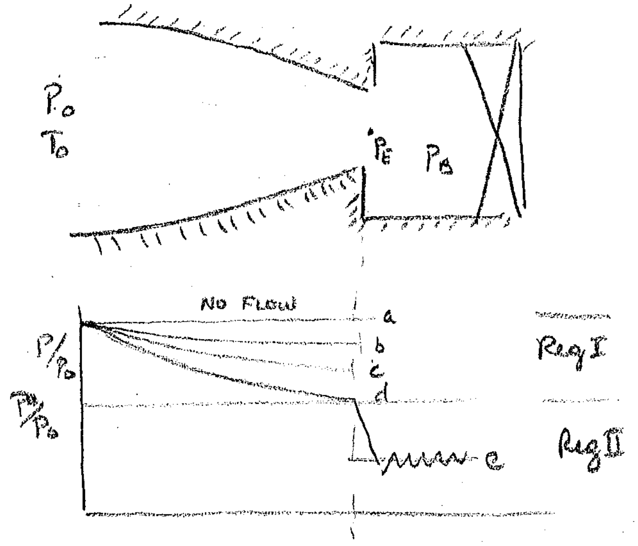
from $\frac{A_2}{A_1} = \frac{A_2/A^*}{A_1/A^*} = .85$ $\quad A_1/A^*$ (for $M = .452$) $= 1.436$

$\therefore A_2/A^* = \frac{A_2}{A_1} \cdot \frac{A_1}{A^*} = .85(1.436) = 1.22$

for $\frac{A_2}{A^*} = 1.22$ go to B2 & get M_2(for $\frac{A_2}{A^*} = 1.22$) $\left\langle \begin{matrix} M_2 = .575 \\ M_2 = 1.56 \end{matrix} \right.$

using $M_2 \xrightarrow{B2} \frac{P_2}{P_{o2}} \to P_2$; $\frac{T_2}{T_{o_2}} \to T_2 \to C_2 \xrightarrow{\& M_2} V_2$ & M_2^* is found from M_2

Part 2 $A_{min} = A^*$ $\frac{A_1}{A^*} = 1.436 \Rightarrow A^* = \frac{A_1}{1.436}$ or 30.4% reduction in area $= \frac{1-1.436}{1.436}$



Region I

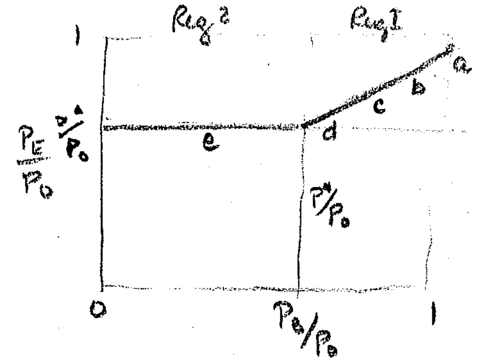
for $M \leq 1$ $P_B = P_e$, $P_B > P^*$, $M_E \leq 1$, $\frac{\omega}{A} \propto \frac{1}{P_B}$

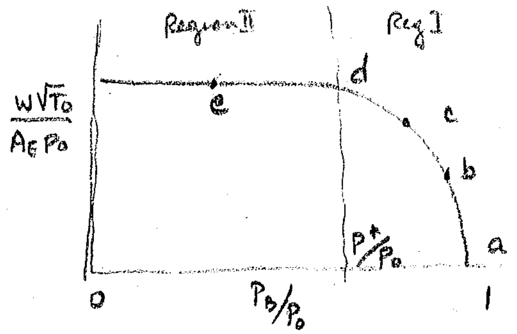
if $P_B \neq P_E$ look at stream tube

since pressure is a point function $\Rightarrow \Delta p$ is set up on ( ) $\downarrow P_B$ $\uparrow P_e$ causing the stream tube to expand or contract depending on whether $p_e - p_B \gtrless 0$

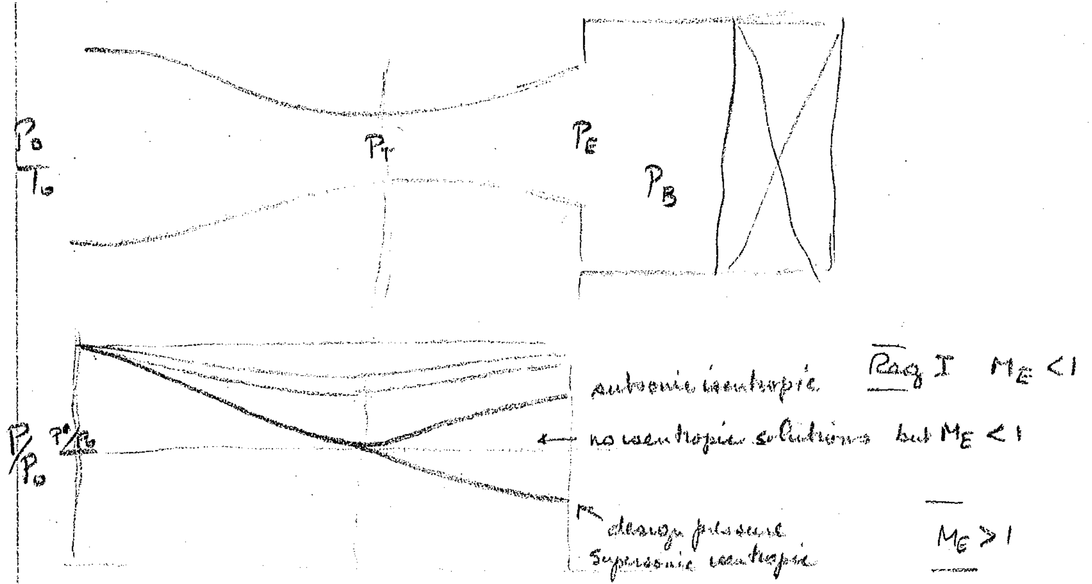
for Reg II $P_E = P^*$ $M_E = 1$, choked flow

$P_B < P^*$, ω is indep of P_B





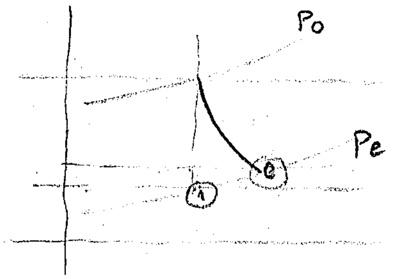
Index pressure $= P^*$ controlling parameter.



performance of real nozzles

Nozzle Efficiency $\eta = \dfrac{\text{actual KE/unit mass at exit}}{\text{isentropic KE/unit mass at exit}}$

expanded to the same pressure in some hypothetical nozzle



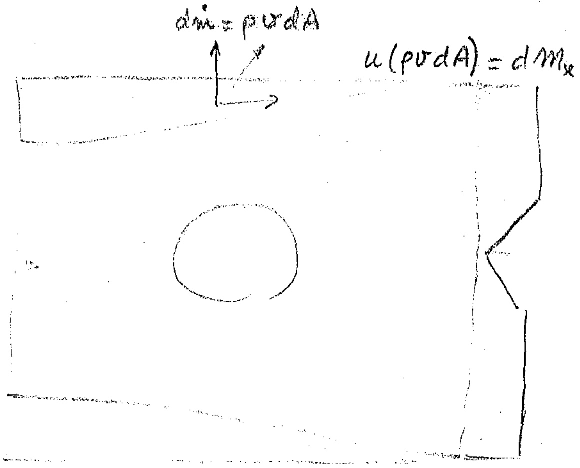
$$\eta = \frac{V_e^2}{V_1^2} = \frac{h_0 - h_e}{h_0 - h_1}$$

straight nozzles .94
.99

curved .90
.95

discharge coeff $C_W = \dfrac{\text{actual mass flow}}{\text{isentropic mass flow}}$

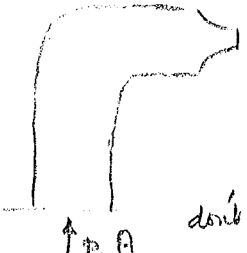
10/23/78



1st use conservation of mass to find the displacement of the streamline

2nd since streamline no $\dot{m}$ through top & bottom

3rd use momentum to find F on cylinder



don't forget p, A

physical way to describe choking

choking: $\frac{\dot{m}}{A}$ = max for given P_0, T_0

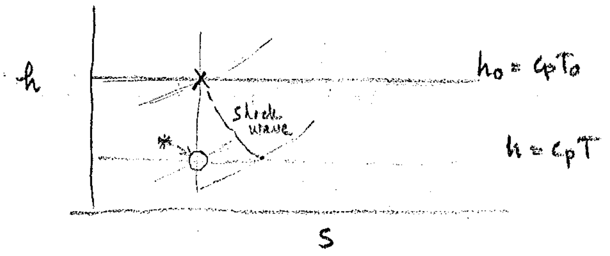
$$\frac{\dot{m}}{A} = f(k, R; T_0, p_0)$$

index press is a back pressure that tells you in what flow regime you in for n flow regimes ⇒ n-1 index pressures. (It is geometry dependent.

choked flow: the flow rate becomes independent of the back pressure for given P_0, T_0

— we will look at the effects of these simple flows and talk about them.
energy exchange, area change, shock waves, viscous effects

define
$$\begin{cases} P^* = P_{M=1} \\ T^* = T_{M=1} \\ A^* = A_{M=1} \\ M^* = \frac{V}{C^*} \quad \text{at } C^* = \sqrt{kRT^*} \end{cases}$$



$$\boxed{\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2}$$

10/25/78

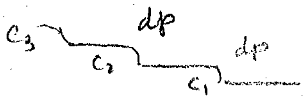
for 1-D flow we need to fix state of substance which is done by means of 2 state variables + 1 to describe motion of flow.

reason we nondim - to reduce no of indep variable
- provide solution for similar flows
- generalize

Gas tables links motion of fluid to state of fluid by means of the equations (Cont, mass, Mom) + process path (adiab, isentropic

SHOCKWAVES

for pure substance there are 2 methods of irreversibility friction and heat loss over a finite temperature drop.

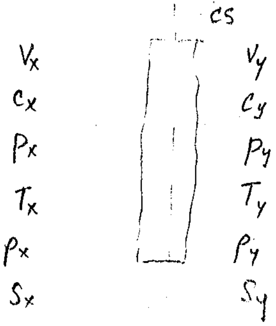


$c_3 > c_2 > c_1$

$dp \Rightarrow dT > 0 \Rightarrow c_2 \sim \sqrt{T+dT} > \sqrt{T} \sim c_1$

the waves move forward and stack up until the Δp is balanced by the frictional & heat loss effects within

Look at a control volume around the shock. Energy balance + momentum balance on either side of CV doesn't tell you what way the shock goes; 2nd law will



We will need 5 eqs (Cont, Mom, Energy, 2nd law, Eq of State due to irreversibility))

10/31/78

"Jump Analysis"

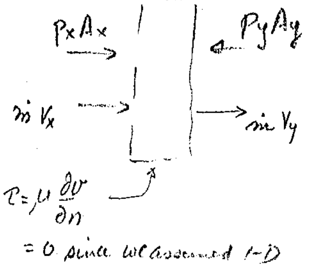
Assumption - steady state $\frac{\partial}{\partial t}(\ ) = 0$

1-D

x, y are far upstream & downstream

Conserv of Mass $(\rho_x V_x A_x) = \rho_y V_y A_y$ since $A_x = A_y$ $\boxed{\rho_x V_x = \rho_y V_y}$

Conserv of Mom

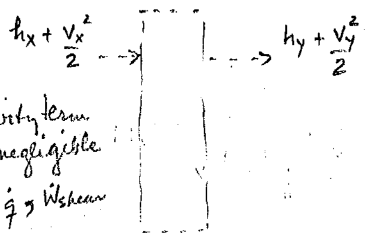


$P_x A + \rho_x A V_x^2 = P_y A + \rho_y A V_y^2$

or $\boxed{P_x + \rho_x V_x^2 = P_y + \rho_y V_y^2}$

Energy

for 1 Btu/lb change occurs for V ~ 223 ft/sec



gravity term is negligible

no $\dot{q}$; $\dot{W}_{shear}$ $\dot{W}_{shaft}$

$$\therefore \rho_x A V_x \left(h_x + \frac{V_x^2}{2}\right) = \rho_y A V_y \left(h_y + \frac{V_y^2}{2}\right)$$

Equation of State $h = h(\rho, s)$ outside the shock wave

2nd law $(\rho_x A V_x) S_x = (\rho_y A V_y) S_y - (\text{entropy flow from surroundings}) \therefore \boxed{S_x < S_y}$

entropy must be measured in the system + surroundings ∴ the entropy gain is in the surroundings since process is highly irreversible

Summarizing

$$p_x - p_y = \frac{\dot{m}}{A}(V_x - V_y)$$

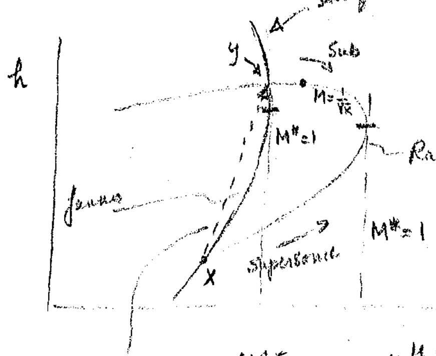
mom ⟹ or $\boxed{p + \rho V^2 = \text{const}}$ (impulse)

Energy $\dot{m}_x \left(h_x + \frac{V_x^2}{2}\right) = \dot{m}_y \left(h_y + \frac{V_y^2}{2}\right)$

or $\boxed{h + \frac{V^2}{2} = \text{constant}}$ ⟹ T_0 is const

2nd law $S_1 < S_2$

"Four" Simple Processes 1-D steady	constant
isentropic, Adiab, area change	energy impulse $p + \rho V^2$ no shock
shock	area, impulse energy
Energy exchange (Rayleigh)	area, impulse & no shock
Simple friction (Fanno)	area, energy no shock



satisfies (cont + energy but not mom)

Rayleigh (satisfies mom + cont but not energy)

shock process - dotted since h, s represents only equilib states - however shock does not go through x→y via equilib.

$\boxed{\text{Exercises Text 5.1, 5.7, 5.9, 5.19 by monday}}$

11/1/78

We continue with the shock wave

note that if the Fanno line has same T_o as isentropic process then * conditions would be the same (since both processes are isoenergetic)

we obtained

Cont. $\rho_x V_x = \rho_y V_y$

Mom $p_x + \rho_x V_x^2 = p_y + \rho_y V_y^2 = const$

Energy $h + \frac{V^2}{2} = h_o = const$

Eqn of state $h = h(p, s)$

2nd law : $s_y > s_x$

<u>Aside</u> : To show * conditions for fanno is at $M=1$

take d(Energy) $\therefore$ $dh + v\,dv = 0 \Rightarrow dh = -VdV$

take d(cont) $d(\rho V) = \rho dV + Vd\rho = 0 \Rightarrow -VdV = V^2\frac{d\rho}{\rho}$

take d(eq of state) $Tds = du + pdv \Rightarrow dh = Tds + v\,dp \Rightarrow h(p,s)$

For isentropic process or in the fanno flow near to *cond.

$$\underbrace{\frac{1}{\rho}dp = dh}= v\,dp = -VdV = V^2\frac{d\rho}{\rho} \Rightarrow \frac{1}{\cancel{\rho}}dp\Big|_s = V^2\frac{d\rho}{\cancel{\rho}}\Big|_s \Rightarrow V^2 = \frac{dp}{d\rho}\Big|_s \equiv c^2$$

$$c^2 = \frac{dp}{d\rho}\Big)_s = V^2 \quad \therefore \quad M^2 = 1$$

$$\Rightarrow \frac{V^2}{c^2} = M^2 = 1$$

<u>Return</u> to <u>shock</u>

if we use the equation of state ie $p = \rho RT$ & $h = C_pT$ then $h_o = C_pT_o$

Thus $h_{o_x} = h_{o_y}$ and $\boxed{T_{o_x} = T_{o_y}}$

$$M_y^2 = \frac{M_x^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_x^2 - 1}$$

Isoenergetic Work = 0
$q = 0$

adiabatic $q = 0$.

adiabatic but not isoenergetic $s_y - s_x = -R\ln\frac{P_{oy}}{P_{ox}}$

by second law $s_y - s_x > 0 \Rightarrow \ln\frac{P_{oy}}{P_{ox}} < 0 \Rightarrow \frac{P_{oy}}{P_{ox}} < 1 \Rightarrow \boxed{P_{oy} < P_{ox}}$

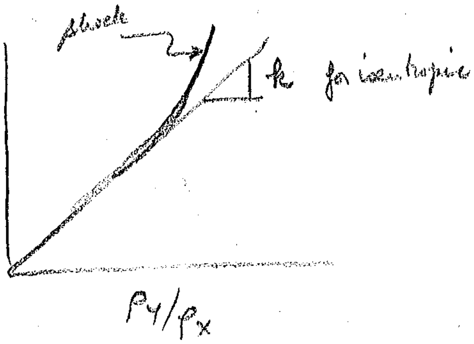
if we derive $\frac{s_y - s_x}{c_v} = \ln\left[\left(\frac{P_y}{P_x} - 1\right) + 1\right] - k\ln\rho_y/\rho_x$

and use the taylor series

$$\frac{s_y - s_x}{c_v} = \frac{2}{3}\frac{k}{(k+1)^2}(M_x^2 - 1)^3 - \frac{2k^2}{(k+1)^2}(M_x^2-1)^4 + \ldots$$

∴ strength of shock is $O(M_x^2-1)^3$ thus for weak shocks $\Delta s \approx 0$ (essentially isentropic)

If we plot P_y/P_x



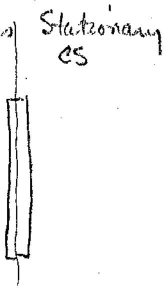
11/3/78

need 3 properties here (2 for state + 1 motion)

$M_x = 2$

$P_{0x} = 100$ psia

$T_x = 520°R$



From Table 48 in Gas Tables

$M_x = 2 \quad M_y = .5774$

$P_y/P_x = 4.5$

$\rho_y/\rho_x = 2.66\bar{6}$

$T_y/T_x = 1.6875$

$P_{0y}/P_{0x} = .72088$

$P_{0y}/P_x = 5.6405$

$\bar{R} = 1545.32 \ \frac{ft\ \#}{lb\ mole}$

$M = 28.970$

Gibbs eq for a moving shock where observer is not on the shock is $Tds = \overbrace{du + pdv}^{dh - vdp} + VdV$

- $T_y = \frac{T_y}{T_x} T_x = 1.6835 (520) = \underline{877°R}$
- using isentropic $\frac{P_{0x}}{P_x} = \left(1 - \frac{k-1}{2} M_x^2\right)^{\frac{k}{k-1}}$ then $P_y = \frac{P_y}{P_x} \cdot P_x = \frac{P_y}{P_x} \cdot \frac{P_x}{P_{0x}} \cdot P_{0x}$

$= 4.5(.1278)(100) = \underline{57.2}$

table 30

$$\rho_x = \frac{P_x}{RT_x} = \frac{P_x/P_{0x} \cdot P_{0x}}{RT_x} = \frac{.1278 \times 100 \times 144}{53.3 \frac{lb}{lbm} \times 520} = \underline{.1771\ lbm/ft^3}$$

$$\rho_y = \frac{\rho_y}{\rho_x} \cdot \rho_x = 2.666(.1771\ lb/ft^3)$$

$$P_{0y} = \frac{P_y}{P_y} \cdot P_{0y} \quad \left[\left(\frac{P_{0y}}{P_y}\right) \text{from table 30}\right] = P_y \cdot \frac{P_{0y}}{P_y} = 57.21 \cdot \frac{1}{.79760} = 71.72\ psi$$

$$\Delta S = S_y - S_x = C_p \ln\left(\frac{T_y}{T_x}\right) - R \ln P_y/P_x = -R \ln \frac{P_y}{P_x}\left(\frac{T_x}{T_y}\right)^{\frac{k-1}{k}}$$

using $\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$, $\frac{P_0}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{k/k-1}$

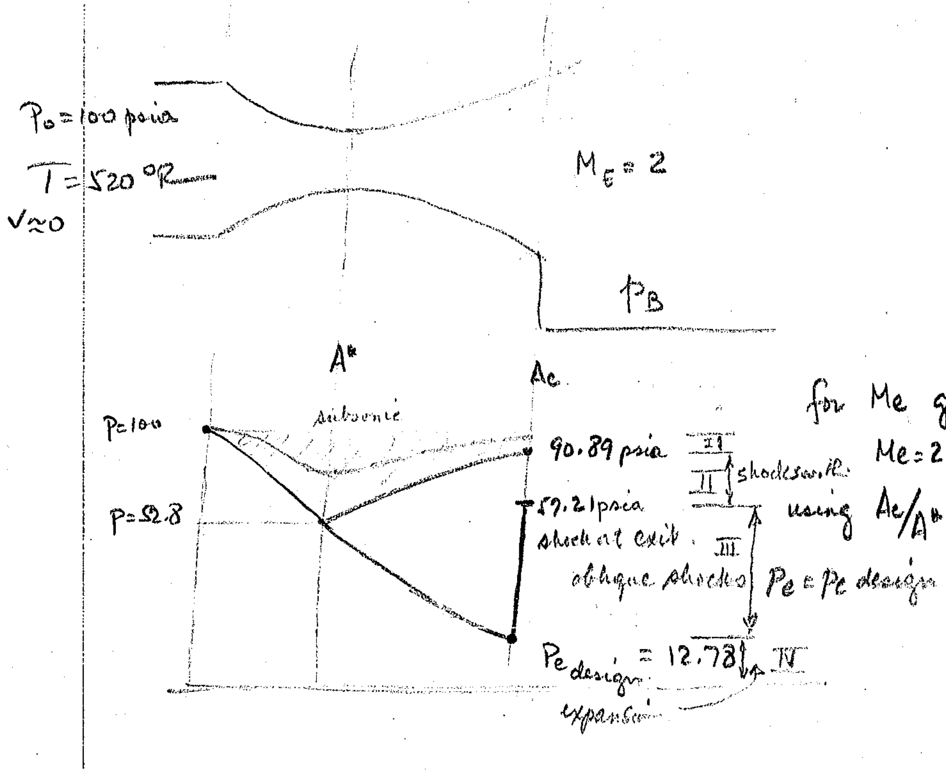
$$\therefore\ S_y - S_x = -R \ln \left[\overbrace{\frac{P_y}{P_{oy}} \cdot \frac{P_{oy}}{P_{ox}} \cdot \underbrace{\frac{P_{ox}}{P_x} \cdot \left(\frac{T_x}{T_{ox}}\right.}_{1} \ \underset{\overset{\uparrow}{1}}{\frac{T_{ox}}{T_{oy}}} \ \left.\frac{T_{oy}}{T_y} \right)^{\frac{k-1}{k}}}^{1} \right]$$

$$= -R \ln \left(\frac{P_{oy}}{P_{ox}} \right)$$

since stag temp = const across sho

$$\Delta S = 17.72 \ \text{ft}\ \#/\text{lb}_m {}^\circ R$$

in any iso energetic flow $\Delta S = -R \ln \left(P_{oy}/P_{ox} \right)$



for Me get A_e/A^*

$M_e = 2$ $A_e/A^* = 1.6875$

using A_e/A^* get $M_e = .373$, $P/P_o = .9089$

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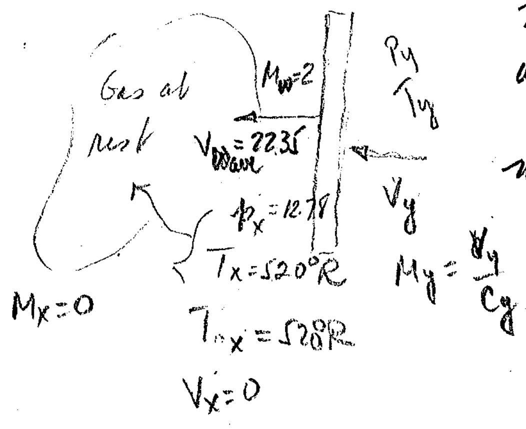
Blast wave



need to transform this problem to one where CV moves with 0 veloc. Galilean transf

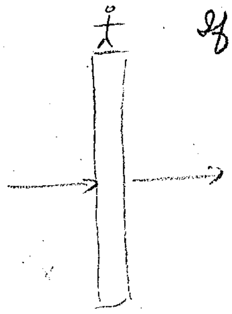
now add, to the right, a velocity = V_x

If $(\;)'$ = quantity with fixed wave

$\therefore$

$P_x' = P_x$

$M_x' = \dfrac{V_W}{c_x}$



$P_y' = P_y$

$T_y' = T_y$

$M_y' = \dfrac{V_y'}{c_y} = \dfrac{V_W - V_y}{c_y}$

— since, P, T are not func/velocity

$(P)' = P$ only depend on fluid

NOTE stagnation press/temp depends on velocity and thus change w/ transformation,

since $\dot{m}$ = const $\quad V_y' = V_W - V_y$

$$T_{oy} = T_y\left[1 + \frac{k-1}{2} M_y^2\right]$$

$$T_{oy}' = T_y\left[1 + \frac{k-1}{2} (M_y')^2\right]$$

$$P_{ox} = P_x\left[1 + \frac{k-1}{2} M_x^2\right]^{k/k-1}$$

$$\neq\left\{\begin{array}{l} P_{oy} = P_y\left[1 + \frac{k-1}{2} M_y^2\right]^{k/k-1} \\ P_{oy}' = P_y\left(1 + \frac{k-1}{2} M_y'^2\right)^{k/k-1} \end{array}\right.$$

Using the previous results.

$P_y = P_y' = 57.51$ psia

$T_y = T_y' = 877.5$ °R

now $|V_w| = V_x$

$V_y' = 2235 - 838.6 = 1397$ ft/sec

$C_y = 1452$

$\therefore M_y' = .96$

now $P_{oy}' = 57.21\left[1 + .2(.96)^2\right]^{3.5} = 103.4$ psi

Soln: go from $A \xrightarrow{\text{Galil Trans}} X \xrightarrow{\text{Shock Table}} Y_, \xrightarrow{\text{Inverse Galil Trans}} B$

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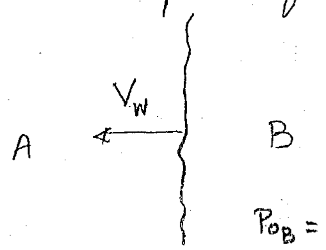
Moving Shock

"A" fluid before "B" fluid after shock



$$P_{O_B} = P_B \left(\frac{T_{OB}}{T_B}\right)^{\frac{K}{K-1}}$$

$P_B = P_y$

$T_B = T_y$

$V_A = 0$

$P_A = P_X$

$T_A = T_X$

$T_{O_A} = T_A$ } since $V_A=0$

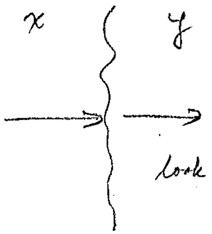
$P_{O_A} = P_A$

$V_B = V_X - V_y = V_W - V_y$

$M_B = \frac{V_B}{c_B} = \frac{V_B}{\sqrt{kRT_B}}$

$T_{OB} = T_B + \frac{k-1}{2} M_B^2 T_B$

Stationary Shock



Given

$P_X =$

$T_X =$

$M_X =$

derive T_{OX}

$V_X = |V_W|$

look up M_y, P_y/P_x, T_y/T_X, P_{oy}/P_x

derive

$P_y = P_y/P_x \cdot P_x$

$T_{oy} = T_{OX}$

$P_{oy} = P_{oy}/P_x \cdot P_X$

$V_y = M_y \cdot c_y = M_y \cdot \sqrt{kRT_y}$

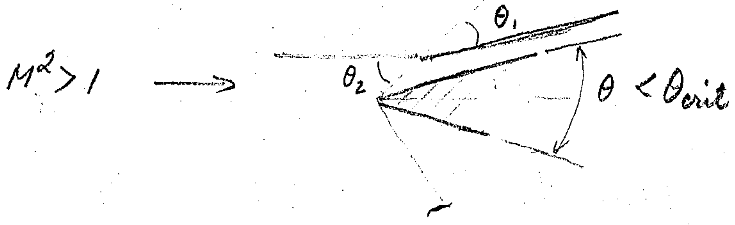
Transformation from moving to Stationary add $|V_W| = V_X$ rightward

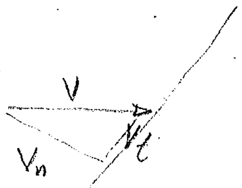
stationary to moving " " " leftward

Since energy change is very large across wave, as wave propagates forward the wave will dissipate.

This analysis of the moving shock is at a particular instant - This whole process is really unsteady but we will look at the quasi steady proble.

$M^2 > 1$

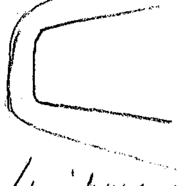




$|V_n| < |V|$

effected mach $= \frac{V_n}{c} < \frac{V}{c}$ ∴ shock strength is decreased when going through shock that is inclined at some angle

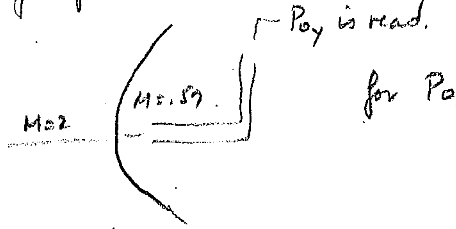
for $\theta > \theta_{crit}$



curved shock layer generates a vorticity layer, since at ∞ h_o is const thus $A_{before} - A_{after} \sim \Delta S \neq 0$ ∴ $\Delta A \neq$ constant. By Crocco's thm flow ∴ is rot.

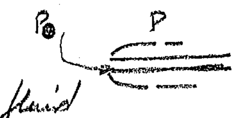
$A = h_o - \overset{T_{atmosphere}}{T_A} S$ availability fn/unit mass

Crocco said that if A = const flow is irrotational for pitot tubes



for P_{ox} = const & M_x ↑ $\frac{P_{oy} - P_{ox}}{P_{ox}}$ ↑

impact pressure = stag press = total pressure.



static pressure = thermodynamic press w/ observer on fluid

dynamic = impact − static

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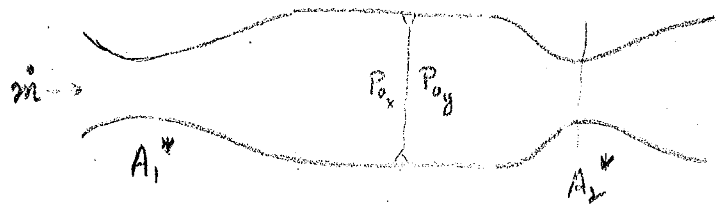


ṁ = const throughout this double nozzle

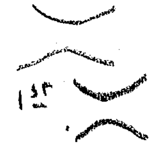
for $\dot{m} = f(k, R)\frac{P_0 A_1^*}{\sqrt{T_0}}$ since T_0 = const and $P_{0x} > P_{0y}$

then $\frac{P_{0x} A_1^*}{\sqrt{T_0}} = \frac{P_{0y} A_2^*}{\sqrt{T_0}} \Rightarrow A_2^* > A_1^*$

$s_y - s_x = -R \ln \frac{P_{oy}}{P_{ox}}$ for isoenergetic perfect gas flow.

if $A_2 > A_2^*$ flow will be subsonic in next

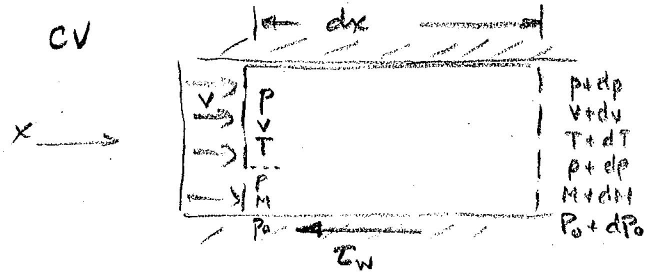
if $A_2 < A_2^*$ will drive shock back down 1st



− Fanno line ≡ simple friction : Steady 1-D flow without energy (iso eng exchange, or area exchange

− Shocks are allowable as discontinuities.

Look at CV



1-D velo profile
all viscosity is at the wall (no bdry layer effect on V will be allowed)
we will neglect body forces

Governing Eqs $p = \rho RT$ equations of State (1)

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$$

$$\frac{dM^2}{M^2} = \frac{dV^2}{V^2} - \frac{dc^2}{c^2} = \frac{dV^2}{V^2} - \frac{dT}{T}$$ def of mach no. (2)

$$\frac{dT}{T} + \frac{K-1}{2} M^2 \frac{dV^2}{V^2} = 0$$ conser of energy (3)

$$\dot{m} = \rho AV = \text{const} \qquad \frac{d\dot{m}}{\dot{m}} = \frac{d\rho}{\rho} + \frac{dV}{V} = 0$$
$$= \frac{d\rho}{\rho} + \frac{1}{2}\frac{dV^2}{V^2} = 0$$ cont (4) since A=const

$$\overset{(p_{er} \cdot dx)}{}$$
$$-A\,dp - \tau_W\, dA_W = \dot{m}\, dV$$ (5) no body forces steady flow since cv is fixed

\# var $p; \rho; T; V, \tau_w, M^2$ only 5 eqs $\therefore$ need 1 more eq must correlate τ_w w/something

let $$D = \frac{4A}{P} = \frac{4A}{dA_W/dx} = 4A\frac{dx}{dA_W} \qquad f \triangleq \frac{\tau_W}{\frac{1}{2}\rho V^2}$$

$$p_o/p = \left(1 + \tfrac{K-1}{2}M^2\right)^{K/K-1}$$ can be written (even though process is not isentropic) because we can imagine that if the flow were brought to rest isentropically this would define p_o

$$\frac{dp_0}{p_0} = \frac{dp}{p} + \frac{\frac{1}{2}KM^2}{T_0/T}\frac{dM^2}{M^2}$$

and now since isoenergetic $T_0 = \text{const} \Longrightarrow -\frac{ds}{R} = \frac{dp_o}{p_o}$ (see shock relation)

thus using all the above

$$\frac{dp}{p} = -\frac{KM^2}{2}\frac{[1+(K-1)M^2]}{(1-M^2)} 4f\frac{dx}{D}$$

$$\frac{dV}{V} = \frac{KM^2}{1-M^2} 4f\frac{dx}{D}$$

$$\frac{dM^2}{M^2} = \frac{KM^2}{1-M^2}\left(1 + \frac{K-1}{2}M^2\right) 4f\frac{dx}{D}$$

$$\frac{dT}{T} = \frac{1}{2}\frac{dc}{c} = -\frac{K(K-1)M^4}{2(1-M^2)} 4f\frac{dx}{D}$$

$$\frac{d\rho}{\rho} = -\frac{KM^2}{2(1-M^2)} 4f\frac{dx}{D}$$

$$\frac{ds}{R} = -\frac{dp_0}{p_0} = \frac{KM^2}{2} 4f\frac{dx}{D}$$ as $x \uparrow$ $s \uparrow$ $p_o \downarrow$

as M goes through 1 inequality signs flows over.

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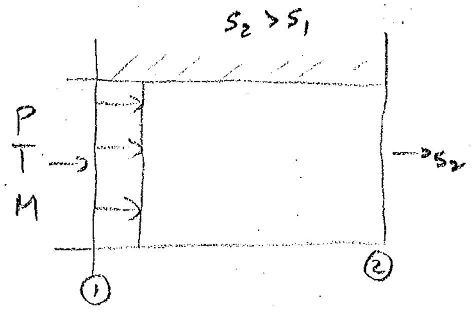


$$\frac{dp}{p} = -\frac{KM^2\left[1+(K-1)M^2\right]}{2(1-M^2)}\,4f\frac{dx}{D}$$

$$\frac{dV}{V} = \frac{KM^2}{2(1-M^2)}\,4f\frac{dx}{D}$$

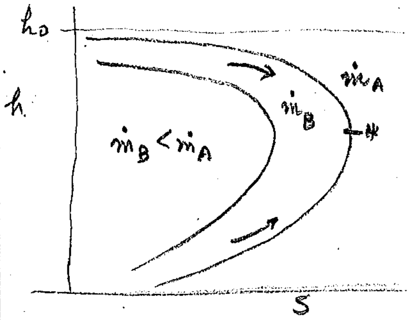
$$\frac{dT}{T} = \frac{1}{2}\frac{dc}{c} = -\frac{K(K-1)\,M^4}{2(1-M^2)}4f\frac{dx}{D}$$

$$\frac{d\rho}{\rho} = \frac{-KM^2}{2(1-M^2)}\,4f\frac{dx}{D}$$

$$\frac{ds}{R} = -\frac{dp_0}{p_0} = \frac{KM^2}{2}\,4f\frac{dx}{D}$$

$D = \dfrac{4 \times \text{cross sectional area}}{\text{perimeter}}$

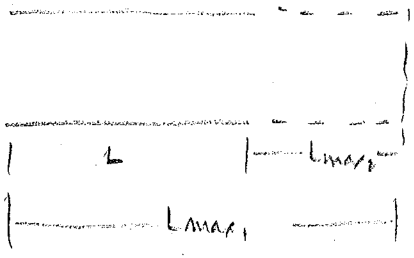
Variables: M^2, V, P, T, ρ, p_0



$$\int_0^{L_{max}} 4f\frac{dx}{D} = \int_{M_1^2}^{1}\frac{1-M^2}{KM^4\left(1+\frac{K-1}{2}M^2\right)}\,dM^2 \quad \text{using } \frac{dM^2}{M^2} = g(K,1)\,4f\frac{dx}{D}$$

$$= 4\bar{f}\frac{L_{max}}{D} = \frac{1-M^2}{KM^2} + \frac{K+1}{2}\ln\frac{(K+1)M^2}{2\left(1+\frac{K-1}{2}M^2\right)}$$

$$\bar{f} = \frac{1}{L_{max}}\int_0^{L_{max}} f\,dx$$



$$L = L_{max_1} - L_{max_2}$$

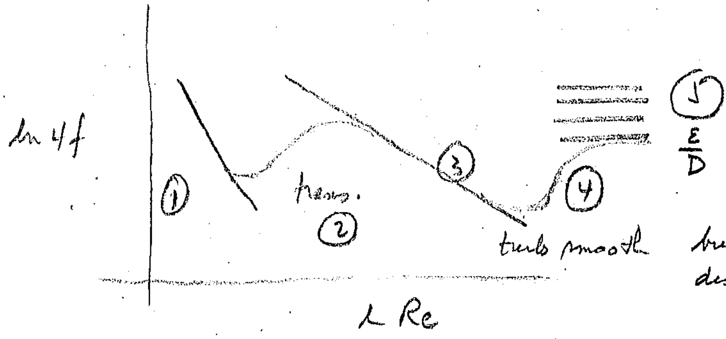
$$4\bar{f}\frac{L}{D} = 4\bar{f}\frac{L_{max_1}}{D} - 4\bar{f}\frac{L_{max_2}}{D}$$

∃ an L_{max} ∋. $M_{L_{max}} = 1$ $p_{L_{max}} = p^*$ etc. for a given $\dot{m}$,
if $L > L_{max}$ flow will choke and $\dot{m}$, will decrease (you will jump the fanno lines). for $L < L_{max}$ compute for a given M_1 the L_{max_1} then get $L_{max_2} = L_{max_1} - L$ for this length go to tables to find M_2

we define 4f from $\Delta p \triangleq 4f \frac{L}{D} \frac{\rho V^2}{2}$

Assume fully-established flow <u>ie</u> velocity profile is the same at any cross section.

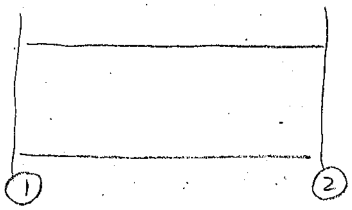
Assume square profile.



⑤ turbulent rough
independent of R_N - bumps on wall disturb sublayer

bumps on wall are not big enough to disturb sublayer.

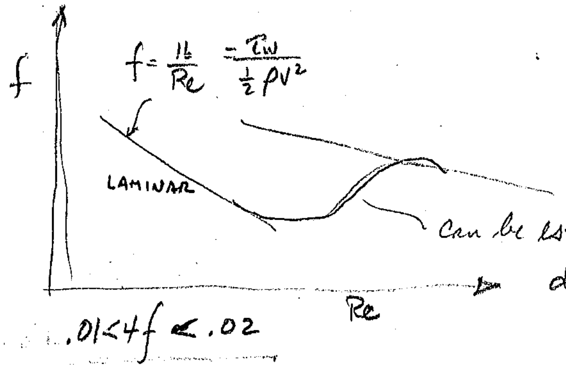
11/15/78



$$\int_1^* 4f\frac{dx}{D} = \int_1^2 4f\frac{dx}{D} + \int_2^* 4f\frac{dx}{D}$$

$$4\bar{f}\frac{L_{max_1}}{D} = 4\bar{f}\frac{L_{12}}{D} + 4\bar{f}\frac{L_{max_2}}{D}$$

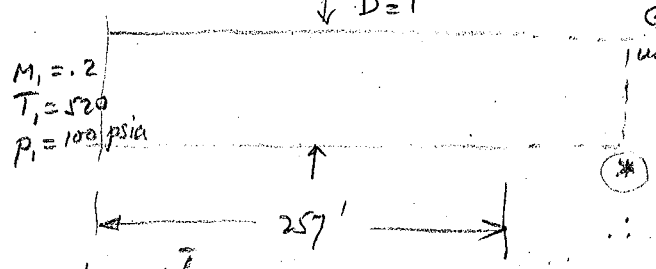
$$\therefore 4f\frac{L_{12}}{D} = 4\bar{f}\frac{L_{max_1}}{D} - 4\bar{f}\frac{L_{max_2}}{D}$$



can be estimated by erf fn. the flow here is gaussian distribution of laminar & turbulent flow.

$.01 < 4f < .02$

Air



Go to table using $M_1 = .2$ $\Rightarrow$ $\left.\frac{4\bar{f}L_{max}}{D}\right|_{M_1=.2} = 14.533$

$\frac{4\bar{f}L_{12}}{D} = .020 \times 257 = 5.143$

$\therefore$ Case I $\quad \frac{4\bar{f}L_{12}}{D} < \frac{4\bar{f}L_{max}}{D}$

take $4\bar{f} = .02$

$$\frac{4\bar{f}L_{max_1}}{D} - \frac{4\bar{f}L_{12}}{D} = \therefore \frac{4\bar{f}L_{max_2}}{D} = 14.533 - 5.143 = 9.39$$

find $P_2, T_2, M_2, \dot{m}$

$$p^* = \frac{P^*}{P_1}P_1 = \frac{1}{5.4555} \times 100 = 18.33 \text{ psia}$$

$$T^* = \frac{T^*}{T_1} \cdot T_1 = \frac{1}{1.1905} \cdot 520 = 436\ {}^\circ R$$

$M_2 = .24$ $\qquad P_2 = P_2/p^* \cdot p^* = 4.5383 \times 18.33 = 83.187$ psia

$\qquad T_2 = T_2/T^* \cdot T^* = 436 \times 1.1863 = 517\ {}^\circ R$

$$\dot{m} = \rho A V = \frac{P_2}{RT_2} A\, M_2 \sqrt{kRT_2}$$

$$= .4347\ \text{lb}_m/\text{ft}^3 \cdot \frac{\pi}{4}(0.24)\left(\sqrt{(1.4 \times 1716)(517^\circ R)}\right) = 91.328$$

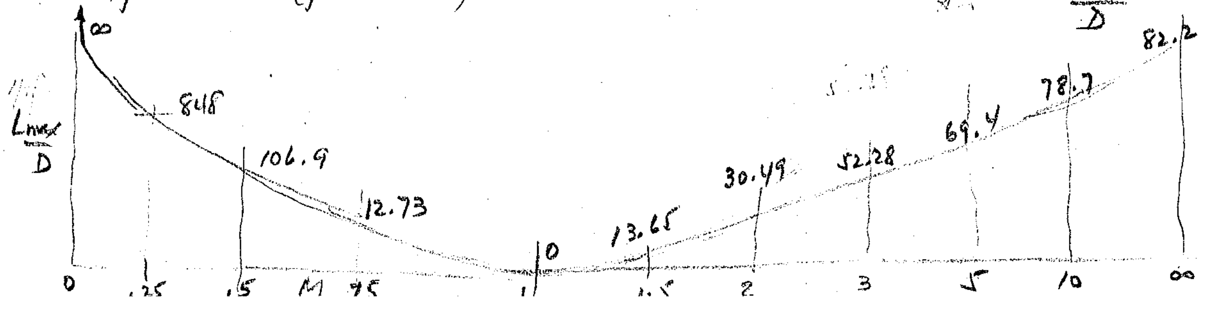
$$P_{o_1} = \left(\frac{P_o}{P_1}\right)_{M_1=.2} P_1 = \frac{1}{.9725} \times 100 = 102.88 \text{ psia}$$

$$P_{o_2} = \left(\frac{P_o}{P_2}\right)_{M_2=.24} P_2 \qquad \frac{1}{.9607} \times 83.187 = 86.468 \text{ psia}$$

$\Delta P_o = -16.412$ psi

from isentropic tables

let $4f = .01$ $(f = .0025)$ and look at $k = 1.4$ M vs. $\frac{L_{max}}{D}$



case 1 $\quad 4f\frac{L_{12}}{D} < 4f\frac{L_{max}}{D}$

a. $\quad p_B > p^*$ unchoked $\qquad$ this p^* is not same as isentropic p^*

b $\quad p_B \leq p^*$ choked $\quad M_e = 1$

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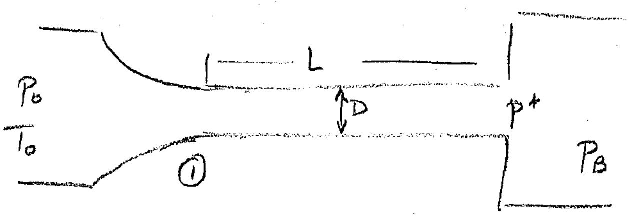
$p^* = fn\,(f, L, D, p_0)$



$L = 2270$ ft $\quad D = 1'$

$4\bar{f} = .02$

$$4f\frac{L}{D} = 45.40$$

$\left.\begin{matrix}P_0\\T_0\end{matrix}\right\}$ given as $\begin{matrix}100\text{ psi}\\520\,°R\end{matrix}$

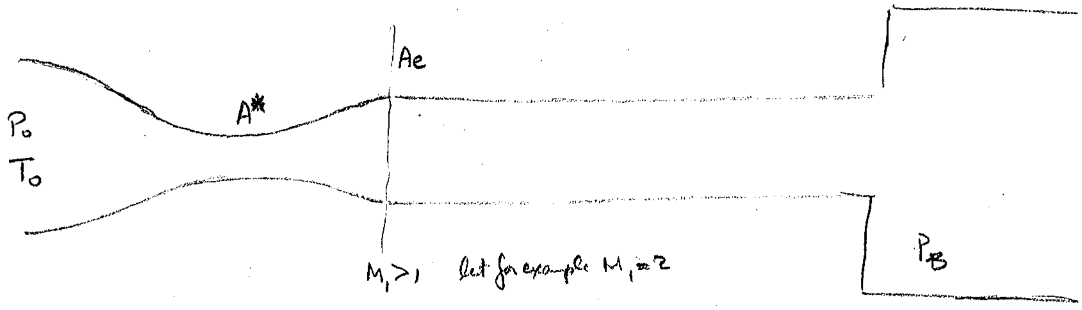
if we ask will $M_1 = .2$? $\Rightarrow 4f\frac{L_{max_1}}{D}\Big|_{M_1=.2} = 14.533 \Rightarrow$ pipe is too long for $M_1 = .2$

$\Rightarrow$ somewhere the pipe will reach $M = 1 \Rightarrow M_e = M^* = 1 \Rightarrow$ flow will choke

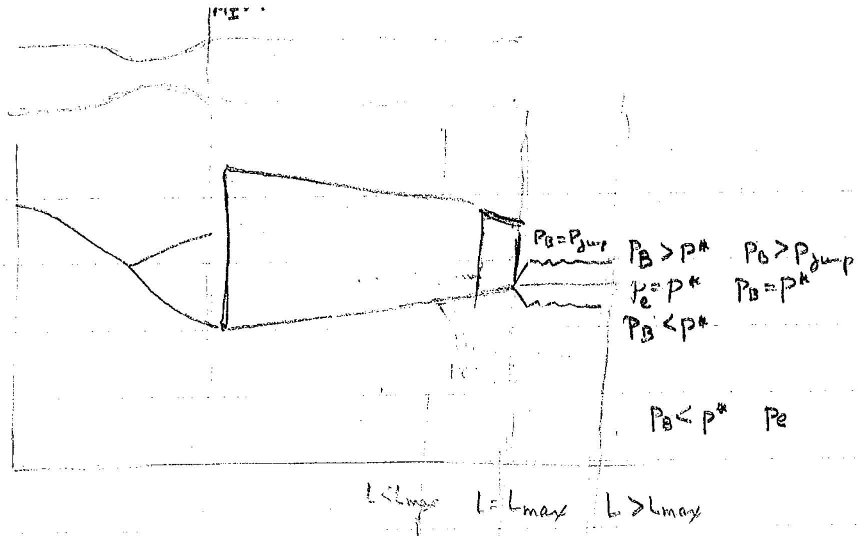
thus $\dot{m} < \dot{m}_{M=.2}$. To find $\dot{m}$ use $\frac{4fL}{D} = \frac{4fL_{max}}{D}$

$\frac{4fL}{D} = \frac{4fL_{max_2}}{D} \Rightarrow M_1 = .12 \;\therefore$ we can then get $\dot{m}_{M=.12}$

thus no matter how much you drop $p_B < p^*$ you cannot get $M_1 > .12$

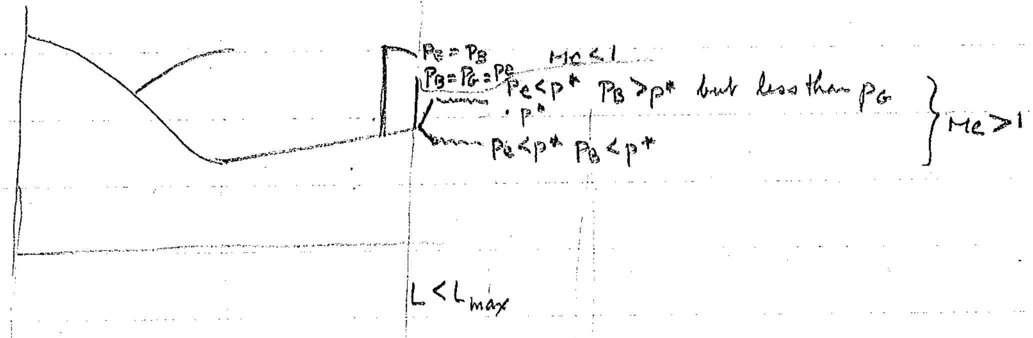


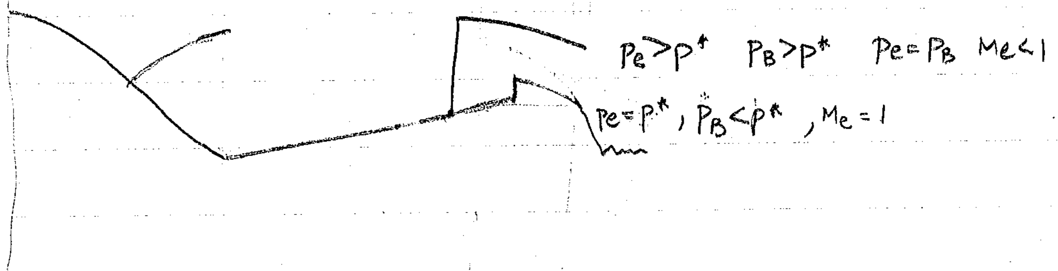
for a given Ae/A^* $\exists$ only 1 $Me > 1$



$\int \frac{dQ}{T} < ds$

$\therefore\ Tds - dQ > 0 \implies$





~~$Tds - dQ = dh$~~

$$dQ = c_p\, dT + d\left(\frac{V^2}{2}\right) = c_p\, dT_0$$

$$Tds = dh - dp/\rho$$

$$\therefore\ Tds - dQ = \cancel{dh} - dp/\rho - c_p \cancel{dT} - d\left(\frac{V^2}{2}\right)$$

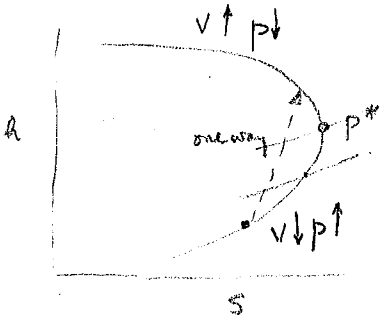
$$= -\,dp/\rho - d\left(\frac{V^2}{2}\right)$$

$$= -\,\frac{dp}{kp}\cdot \frac{kp}{\rho}$$

$$=$$

$P_E' \triangleq$ exit pressure for shock free flow in nozzle & duct
$P_G \triangleq$ Pressure behind normal shock at exit plane.
P^* flow ∋. $M_{exit} = 1$
} 3 index pressures

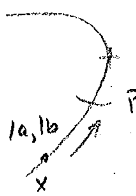
5 flow regimes exist and we have possibilities of the shock.



for $\frac{4fL}{D}$ and $M_1 = 2$

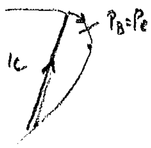
Class 1a. $P_B \leq P_e' < P^*$ Supersonic through out $L < L_{max}$
since $P_e' < p^*$ $L < L_{max}$

Class 1 $P_e' < P^*$



(a) $P_B \leq P_e' < P^*$ Supersonic to end of duct w/ Expansion waves outside nozzle

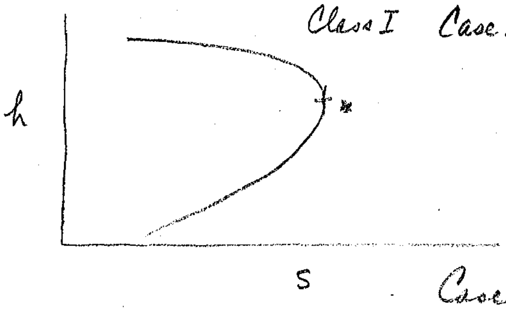
(b) $P_G \geq P_B > P_E'$ Supersonic to end of duct w/ compression waves outside nozzle



(c) $P_B > P_G$; $P_E' < P^*$ shock in duct. $4\bar{f}\frac{L}{D} < 4\bar{f}\frac{L_{max}}{D}$

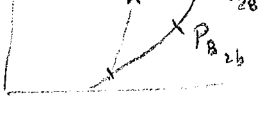
11/20/78

Supersonic nozzle feeding a const. area duct w/friction but w/o energy exch.
SSF



Class I Case I $P_E' < P^* \Rightarrow L < L_{max}$
P_G

Case IV Class II $P_e' > p^*$
$L > L_{max}$

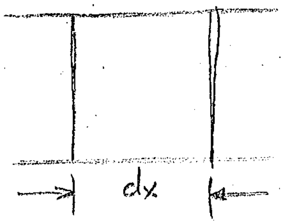


2a $P_B > p^*$ shock w/ $P_E = P_B$

2b $P_e' > p^*$ and $P_B < P^*$ shock w/ $P_e = P^*$ expansion outside

Isothermal flow

long ducts are isothermal rather than adiabatic



Momentum remains same

Cont. remains same

energy change $\quad dQ = c_p \overset{0}{\cancel{dT}} + d\left(\frac{V^2}{2}\right) = c_p dT_0$

for a perfect gas, isothermal cond

$$T_0 = T\left(1+\frac{k-1}{2}M^2\right) \qquad \frac{dT_0}{T_0} = \frac{(k-1)M^2}{2\left(1+\frac{k-1}{2}M^2\right)}\frac{dM^2}{M^2}$$

from perfect gas eq of state $p = \rho R T \Rightarrow \frac{dp}{p} \overset{\text{EQ of state}}{=} \frac{d\rho}{\rho} \overset{\text{cont}}{=} -\frac{dV}{V}$

and from def of Mach no. $\frac{dM^2}{M^2} = 2\frac{dV}{V}$ (since $dc^2 = 0$)

Cont. can be rewritten $\quad \frac{d\rho}{\rho} = -\frac{1}{2}\frac{dV^2}{V^2}$

Mom. " " " $\quad \frac{dp}{p} + \frac{kM^2}{2}\,4f\frac{dx}{D} + \frac{kM^2}{2}\frac{dV^2}{V^2} = 0$

we can get a relation from $\frac{P_0}{P} = \left(1+\frac{k-1}{2}M^2\right)^{k/k-1}$ for dp_0, dp, dM^2

hence we finally show $\quad \overbrace{dp/p = d\rho/\rho}^{\text{STATE}} \overbrace{= -\frac{dV}{V}}^{\text{Continuity}} \overbrace{= -\frac{dM^2}{2M^2}}^{\text{MACH}} \overbrace{= -\frac{kM^2}{2}\frac{1}{(1-kM^2)}4f\frac{dx}{D}}^{\text{MOM}}$

$$\frac{dT_0}{T_0} = \frac{k(k-1)M^4}{2(1-kM^2)\left(1+\frac{k-1}{2}M^2\right)}\,4f\frac{dx}{D}$$

$$\frac{dP_0}{P_0} = \frac{kM^2\left(1-\frac{k+1}{2}M^2\right)}{2(1-kM^2)\left(1+\frac{k-1}{2}M^2\right)}\,4f\frac{dx}{D}$$

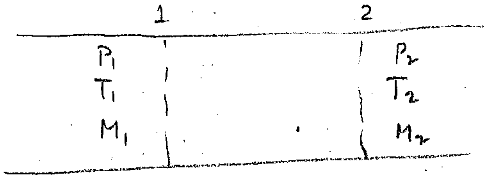
Singular @ $M = \frac{1}{\sqrt{k}}$ or choking occurs when $M = \frac{1}{\sqrt{k}}$

$$\boxed{4f\frac{L_{max}}{D} = \frac{1-kM^2}{kM^2} + \ln kM^2}$$

11/22/78

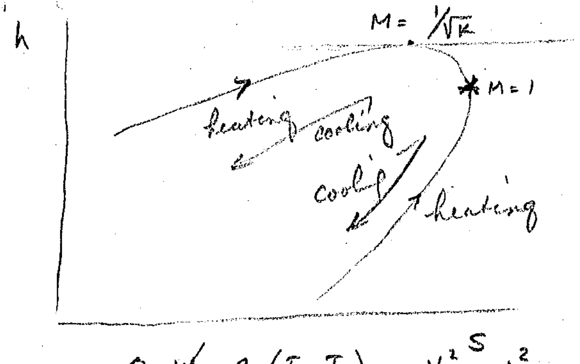
Simple Energy Exch – Rayleigh Line   reversible process unlike fanno

Assume : SSF   1-D flow , constarea , $\tau_W = 0$ Shocks allowed



Since ~~now~~ no wall shear   mom becomes $p + \rho V^2 = \text{const.} = F/A$

cont. becomes $\rho_1 V_1 = \rho_2 V_2 \triangleq G$



as you add heat you move to the right so that entropy ↑

Energy $$\frac{Q}{\dot m} - \cancelto{0}{W} = C_p(T_2 - T_1) + \frac{V_2^2}{2} - \frac{V_1^2}{2} : c_p(T_{0_2} - T_{0_1})$$

Using perfect gas law $\rho V^2 = kpM$ $$P_2/P_1 = \frac{\rho_2 T_2}{\rho_1 T_1} \quad \text{and} \quad \frac{M_2}{M_1} = \frac{V_2}{V_1}\frac{C_1}{C_2} = \frac{V_2}{V_1}\sqrt{\frac{T_1}{T_2}}$$

$$\frac{P_{0_2}}{P_{0_1}} = \frac{P_2}{P_1}\left[\frac{1 + \frac{K-1}{2}M_2^2}{1 + \frac{K-1}{2}M_1^2}\right]^{K/K-1} \qquad \frac{T_{0_2}}{T_{0_1}} = \frac{T_2}{T_1}\left[\frac{1 + \frac{K-1}{2}M_2^2}{1 + \frac{K-1}{2}M_1^2}\right] \qquad \frac{S_2 - S_1}{R} = \ln \frac{(T_2/T_1)^{\frac{K}{K-1}}}{(P_2/P_1)}$$

Tabulate everything wrt * conditions

$$\frac{T}{T^*} = \frac{(K+1)^2}{(1+KM^2)^2} M^2 \qquad \frac{V}{V^*} = \frac{(K+1)M^2}{1+KM^2} = \frac{\rho^*}{\rho}$$

$$\frac{T_0}{T_0^*} = \frac{2(K+1)M^2\left(1 + \frac{K-1}{2}M^2\right)}{(1+KM^2)^2} \qquad \frac{S - S^*}{Cp} = \ln M^2\left(\frac{K+1}{1+KM^2}\right)^{\frac{K+1}{K}}$$

$$\frac{P}{P^*} = \frac{K+1}{1+KM^2}$$

if you add more heat than what is necessary to bring it to star you choke flow. If you do $\dot m$ decreases and you jump to another Rayleigh Line

If you cool the flow you will never choke it.

	Add energy		Extract energy	
	$M^2<1$	$M^2>1$	$M^2<1$	$M^2>1$
T_o	↑	↑	↓	↓
M	↑	↓	↓	↑
T	↑ to $M = \frac{1}{\sqrt{k}}$ then ↓	↑	↓ for $M < \frac{1}{\sqrt{k}}$ then ↑	↓ $M = \frac{1}{\sqrt{k}}$ then ↓
p	↓	↑	↑	↓
p_o	↓	↓	↑	↑
$\frac{1}{\rho}$ & V	↑	↓	↓	↑

I can pump a gas by heating or cooling.

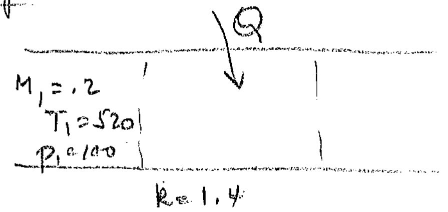
11/27/78

How to raise To

External energy add
a. add Q
b. and work

Change in form of internal energy
c. Combustion (Chem. Rearrangement)
d. Phase Change

Example



$$T_{o_1} = T_1\left(1 + \frac{k-1}{2}M^2\right) = 524^\circ R$$

$$\frac{T_o}{T_o^*} = .17355$$

$$T_o^* = \frac{T_o^*}{T_{o_1}} \cdot T_{o_1} = 3019^\circ R$$ can be unrealistic since tube will melt.

Steady combustor, adiabatic at const p_0
we will treat it as a simple process since $T_0 \uparrow$ due to Q
but in reality K, W of reaction gases will change.

Rayleigh flow will not tell you where a shock will stand since $T_{0x} = T_{0y}$ for shock, whereas T_0 changes in Rayleigh flow, Friction will

To find ΔT_0 for this combustor

we define $-\Delta h_{\hat{RP}}$ (heat needed to go from reactant to products)

$\cong 20000 \pm 10\%$ Btu/lb

$\hat{} \Rightarrow$ @ 1 atm $25°C \cong 77°F$

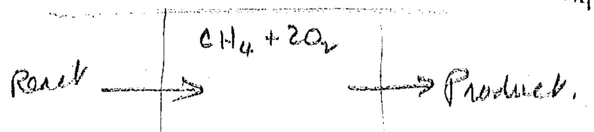
$$\Delta T_0 \cong \frac{20{,}000}{.27 \times 16} = \frac{\dot m_f}{\dot m_p}\frac{\Delta h_{\hat{RP}}}{c_p} \cong 4630°F \quad T_{0_2} - T_{0_1}$$

$$\begin{array}{r} 524°R \quad + T_{0_1} \\ \hline T_{0_2} = 5154°R \cong T_{0_2} \end{array}$$

This temp $\Rightarrow$ choking

Example Pg 220 Keenan

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$



Write energy eqn for adiabatic Steady State Combustor (2)

$$h_f \dot m_f + h_o \dot m_o = H_R = H_p = \underbrace{\dot m_p h_p}_{\text{products}} = h_{H_2O}\dot m_{H_2O} + h_{CO_2}\dot m_{CO_2}$$

fuel oxidizer

$$0 = \underset{\dot m(h_p - h_R)}{H_p - H_R} = \dot m_p(h_p - h_{\hat p}) - \dot m_R(h_R - h_{\hat R}) + \underbrace{\dot m_p h_{\hat p} - \dot m_R h_{\hat R}}_{\dot m(h_{\hat p} - h_{\hat R})}$$

$$h_p - h_R = (h_p - h_{\hat p}) - (h_R - h_{\hat R}) + (\Delta h_{\hat{RP}})$$

get this from table   get this from table

$$\dot m(h_p - h_{\hat p}) = (\underset{\text{at }(2)}{h_{CO_2}} - \underset{\text{at }25°C}{h_{\hat{CO_2}}})\dot m_{CO_2} + (\underset{\text{at }(2)}{h_{H_2O}} - \underset{\text{at }25°C}{h_{\hat{H_2O}}})\dot m_{H_2O}$$

if T at (2) ≠ 77°F then $\overset{\text{for adiab}}{\wedge}$ $0 = H_P - H_R = (h_P - \hat{h_P}) - (h_R - \hat{h_R}) + \Delta \hat{h}_{RP}$

we can find $h_p \Rightarrow T_p \Rightarrow T_0$ of products which can be checked w T_0^* to see if choking occurs

for non adiab $H_P - H_R = Q$ and if we know RHS of eqn. then we can get Q needed to keep flow from choking or getting very high Temperatures.

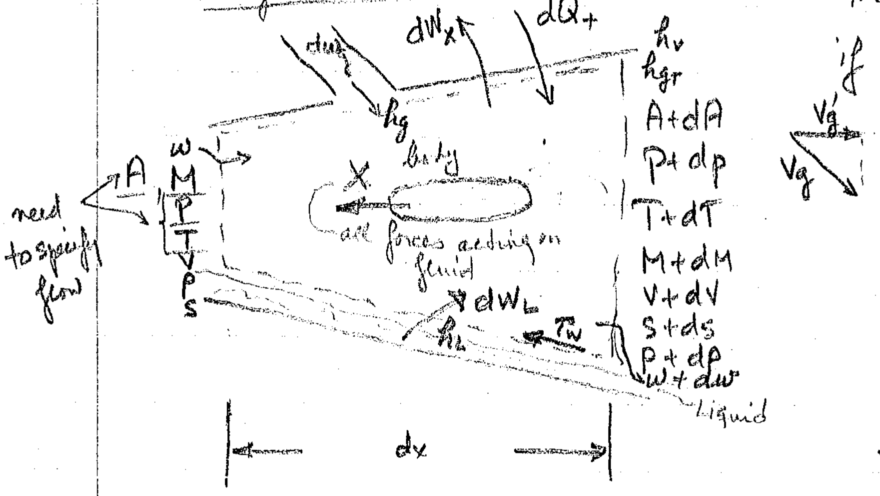
11/29/78

Final this room Monday 11 Dec @ 8:30 AM –

perfect mixing, 1-D, Steady State flow are assumptions.

$V_g = V_\ell = V$

The general case



since we are working with 1-D flow the equations will reduce to ODE's if 2 or 3 D we would need PDE's.

$$p = \rho R T = \rho \frac{\mathcal{R}}{W} T \qquad ①$$

$$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T} - \frac{dW}{W} \qquad ② \text{ from above}$$

$$\frac{dc}{c} = \frac{1}{2}\left(\frac{dK}{K} + \frac{dT}{T} - \frac{dW}{W}\right) \quad \text{from } c^2 = \frac{K\mathcal{R}T}{W}$$

$$\frac{dM^2}{M^2} = \frac{dV^2}{V^2} + \frac{dW}{W} - \frac{dT}{T} - \frac{dK}{K} \quad \text{defn of } M^2$$

continuity $\rho A V = w \quad \therefore \quad \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = \frac{dw}{w} = \frac{dw_g + dw_L}{w}$

if liquid evaporates dw_L is + if vapor condenses dw_L is −

all work except flow work — since perfect mixing

$$w(dQ - dW_X) = w\left[(h + dh) + h_{gr}\, dw_g + \overset{\text{vapor}}{h_v}\, dw_L\right] + \left[(w + dw_g + dw_L)\left(V^2 + \frac{dV^2}{2}\right)\right]$$
$$- \left[w h + h_g\, dw_g + \overset{\text{liquid}}{h_L}\, dw_L\right] - \left[w\frac{V^2}{2} + \frac{V_g^2}{2}\, dw_g + \frac{V_L^2}{2}\, dw_L\right]$$

first two terms will be effluxes last two are effluxes. Since we are talking about energies we use total velocities and we don't worry about direction (since it is a dot product)

define $h_{g_r} - \left[h_g + \frac{V_g^2}{2} \right] = \bar{C}_{pg} \left(T - T_{0g} \right)$

$$\therefore \; dH \overset{\Delta}{=} dh_{PR} - \left[\bar{C}_{pg} (T - T_{0g}) + \frac{V_g^2}{2} \right] \frac{dw_g}{w} - \left[h_L - h_V + \frac{V^2 - V_L^2}{2} \right] \frac{dw_L}{w}$$

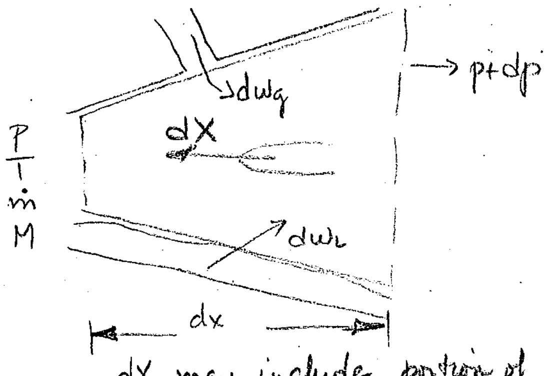
$$\therefore \; \frac{dQ - dW_x + dH}{C_pT} = \frac{dT}{T} + \frac{k-1}{2} M^2 \frac{dV^2}{V^2}$$

Using the mom. eqn.

$$pA + \left(p + \frac{dp}{2}\right) dA - (p + dp)(A + dA) - \tau_w dA_w - dX = A dp - \tau_w dA_w - dX =$$

$$\overset{\text{efflux of mom.}}{(w + dw)(V + dV)} - \left(V_g' dw_g + \overset{\text{Inflow of mom.}}{V_L'} dw_L + wV \right)$$

12/1/78



dX may include portion of gravity force

indep variables	dep. var
1) dA/A	dM^2/M^2
2) $dQ - dW_H + dH$	dV/V
3) $4f \frac{dx}{D} + \frac{dX}{\frac{1}{2} K p M^2} - 2y \frac{dw}{w}$	dc/c
4) dw/w	$d\rho/\rho$
5) dW/W	dp/p
6) dK/K	dA/c_p

we also have parameters f, D, dH, y which are defined as

$f = \tau_w / \left(\frac{1}{2} \rho V^2\right)$

$D = 4A/(\text{perimeter})$

$dH = dh_{pr} - \left[\bar{C}_{pg} (T - T_{0g}) + V_g^2/2 \right] \frac{dw_g}{w} - \left(h_L - h_V + \frac{V^2 - V_L^2}{2} \right) \frac{dw_L}{w}$

$y \frac{dw}{w} = y_g \frac{dw_g}{w} + y_L \frac{dw_L}{w} \qquad y_g = V_g'/V_g \qquad y_c = V_c'/V_c$

Eqns

1) Mom

2) Cont

3) Perfect Gas eqn of state

4) $M^2 = V^2/C^2$

5) $c = \sqrt{KRT/W}$

for constant properties $dW = dK = dh_{pr} = 0$

$$\frac{dM^2}{M^2} = \overset{\text{isent}}{a_1 \frac{dA}{A}} + \overset{\text{rayleigh}}{a_2\left(\frac{dQ - dw_x + dH}{c_p T}\right)} + \overset{\text{FANNO}}{a_3\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}KpM^2} - 2y\frac{dw}{w}\right)} + \overset{\text{mass transfer}}{a_4 \frac{dw}{w}} + \underset{\text{CHEM DECOMP}}{a_5 \frac{dW}{W}}$$

$$+ a_6 \frac{dK}{K} \longleftarrow$$

now $a_1 = \dfrac{\partial \ln M^2}{\partial \ln A}$ hence a_i are the logarithmic derivatives of the dependent variables wrt indep variables.

note that we can get simple processes by looking at the basics of the process & applying it to influence coeff.

ie Fanno no energy X-change, no area change, no body force, no mass transfer no chemical changes $\therefore$ $dA = (dQ - dw_x + dM) = dX = dw = dW = dK = 0$

$$\therefore \quad \frac{dM^2}{M^2} = a_3\, 4f\frac{dx}{D}$$

Choking effects can be obtained from all changes except dK.

12/4/78

Eqn 8.40 Shapiro Pg 230

$$(1-M^2)\,dM^2 = M^2\left(1+\frac{k-1}{2}M^2\right)\left[-2\frac{dA}{A} + (1+kM^2)\frac{dT_0}{T_0} + kM^2\left(4f\frac{dx}{D} + \frac{dX}{\frac{1}{2}kpM^2} - 2y\frac{dw}{w}\right) + 2(1+kM^2)\frac{dw}{w}\right]$$

$$= \underbrace{M^2\left(1+\frac{k-1}{2}M^2\right)}_{\text{positive definite}} \; K(x)$$

↳ a fn of x (since this is 1-D eqn.)

let us look at $K(x) > 0$ if $M^2>1$ $dM^2<0 \Rightarrow M \to 1$

CHOKING here $M^2<1$ $dM^2>0 \Rightarrow M \to 1$

NO CHOKING → $K(x) < 0$ if $M^2>1$ $dM^2>0 \Rightarrow M \to \infty$

if $M^2<1$ $dM^2<0 \Rightarrow M \to -\infty$

$K(x) = 0$ if no change in M = M=const.

if body force is upstream (w/all other effects = 0 we may have choking

" " " " downstream (" " " " we may not have choking)

$\underline{K(x) < 0}$ changing to >0

↳ no choking here but if you go far enough after $K(x) > 0$ we can get choking

$\underline{K(x) > 0}$ changing to <0

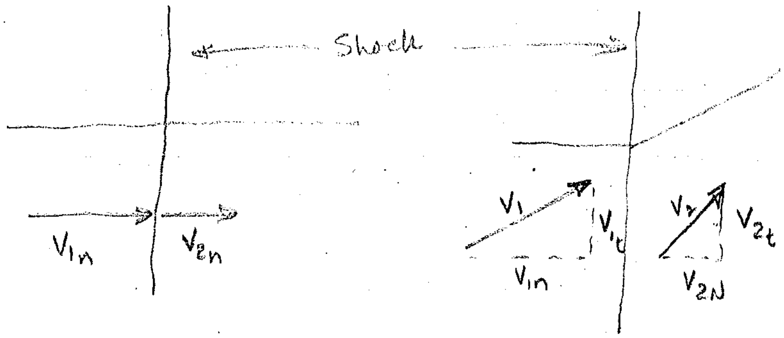
↳ depends on whether we reach $M=1$ before or after sign changing.

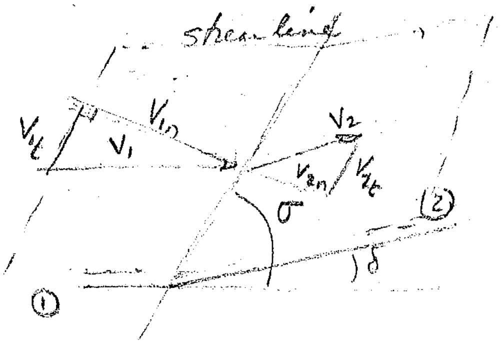
for continuous M change at pt where $K(x)=0$ $M=1$

Cont. passage from sub to super is normally stable

" " " supersonic to subs is normally unstable.

12/6/78





V_2 can be $> C$ even if $V_{2N} < C$ note that $V_{1t} = V_{2t}$

1. Assume steady flow SSF
2. (Straight) Shock - oblique
3. Adiabatic (changes occur in short distance ∴ $\dot q = 0$) & no work.

Eqns

Cont $\dot m_1$ (upstream) $= \dot m_2$ (downstream)

$\rho_1 V_{1n} A = \rho_2 V_{2n} A \Rightarrow \boxed{\rho_1 V_{1n} = \rho_2 V_{2n}}$ note CS has streamline bdy ∴ no mass flow rate through top.

t mom. extend CS to ∞ ∴ no pressure forces

$\dot m_1 = \dot m_2$

$\underbrace{\rho_1 V_{1n} V_{1t}}_{\dot m_1} A = \rho_2 \underbrace{V_{2n} V_{2t} A}_{\dot m_2} \Rightarrow \rho_1 V_{1n} V_{1t} = \rho_2 V_{2n} V_{2t} \Rightarrow \boxed{V_{1t} = V_{2t}}$

n mom

$-\rho_1 V_{1n}^2 A + \rho_2 V_{2n}^2 A = p_1 A - p_2 A$ or $p_1 - p_2 = \rho_2 V_{2n}^2 - \rho_1 V_{1n}^2 = \rho_1 V_{1n}(V_{2n} - V_{1n})$

bernoulli is not applicable here since process for bernoulli is reversible & shock requires $\Delta s > 0$.

for perfect gas

$p = \rho R T$

Energy we assumed adiabatic w/no work

$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$ since energy is scalar

and $V^2 = V_t^2 + V_n^2$

we can rewrite energy

$$c_pT = \frac{k}{k-1}\, p/\rho \quad \therefore$$

$$\frac{k}{k-1}\left(p_2/\rho_2 - p_1/\rho_1\right) = \frac{V_{1n}^2 - V_{2n}^2}{2}$$

we can solve for

$$\boxed{\frac{p_2}{p_1} = \frac{\frac{k+1}{k-1}\,\rho_2/\rho_1 - 1}{\frac{k+1}{k-1} - \rho_2/\rho_1} \qquad \text{Rankine Hugoniot}}$$

Prandtl Relation becomes

$$\boxed{V_{n1}V_{n2} = c^{*2} - \frac{k-1}{k+1}V_t^2}$$

again we don't have enough eqs for the unknowns ∴ we must find a relation between σ and other variables

now

$$\boxed{\begin{array}{ll} V_{t_1} = V_1\cos\sigma & V_{t_2} = V_2\cos(\sigma-\delta) \\ V_{n_1} = V_1\sin\sigma & V_{n_2} = V_2\sin(\sigma-\delta) \end{array}}$$

$$\boxed{\frac{V_1}{V_2} = \frac{\cos(\sigma-\delta)}{\cos\sigma}}$$ since $V_{t_1} = V_{t_2}$

put into cont. we get that $\rho_1V_1\sin\sigma = \rho_2V_2\sin(\sigma-\delta)$

$$\therefore\ \rho_2/\rho_1 = \frac{V_1\sin\delta}{V_2\sin(\sigma-\delta)} = \frac{\cos(\sigma-\delta)}{\cos\delta}\,\frac{\sin\delta}{\sin(\sigma-\delta)} = \frac{\tan\delta}{\tan(\sigma-\delta)}$$

now $p_2/p_1 - 1 = kM_2^2\left(1 - \rho_2/\rho_1\right)\sin(\sigma-\delta)$

now for every $\delta \Leftrightarrow \exists!\ \sigma$.

Pg 532/541 will have analysis of oblique shocks & mach waves

– in an oblique shock mach waves run into shock wave

$\sin\sigma = \frac{V_{n1}}{V_1}$

$\sin\alpha_1 = c_1/V_1 \implies V_{n_1} > c_1$ by entropy cond $\implies \sigma > \alpha_1$

$$\sin(\sigma-\delta) = \frac{V_{2n}}{V_2}$$

$$\sin\alpha_2 = \frac{c_2}{V_2} \qquad c_2 > V_{2n} \Rightarrow \alpha_2 > \sigma-\delta$$

a weak shock is one where $\frac{V_2}{c_2}$ is still > 1 $\quad (\underline{\text{total}}\ V_2 > c_2)$

a strong shock is one where $\frac{V_2}{c_2} < 1$ $\quad (\underline{\text{total}}\ V_2 < c_2)$

$\overset{\text{wt=2}}{\overbrace{2H_2}} + O_2 \longrightarrow 2H_2O$ 24 lb H_2 76 lb O_2

4 32

12 moles $= \frac{24}{\text{wt of } H_2}$ 2.375 moles $= \frac{76}{\text{wt of } O_2}$

$\underbrace{2 \cdot 2.375}$

$\therefore$ 12 moles of H_2 + 2.375 moles of O_2 $\longrightarrow$ 7.25 H_2 4.75 moles of H_2O

$\underset{\text{left over}}{(12 - 4.75)H_2}$ + 2 (2.375 moles of O_2)

$\therefore$ $7.25 \, (\underbrace{H_2}_{\text{2 mole wt}}) + 4.75 \, (\underbrace{H_2O}_{\text{18 molecular wt}})$ = # moles of prod (wt of product)

12 (8.33)

$\therefore$ $W = 8.33$

Question	Score
1	20
2	25
3	22
4	24
5	
6	
7	
8	
Total	91

STANFORD UNIVERSITY

OFFICIAL EXAMINATION BOOK

(Coordinate Book – 8 pp.)

Name of student CESAR LEVY

Date of examination 11 DEC 78

Course ME 255 A Gas Dyn.

THE STANFORD UNIVERSITY HONOR CODE

A. The Honor Code is an undertaking of the students, individually and collectively:

(1) that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;

(2) that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.

B. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.

C. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

I acknowledge and accept the Honor Code.

(Signed) Cesar Levy

Interpretations and applications of the Honor Code appear on the back cover of this examination book.

grade = A

ME 255 Final Exam Monday, Dec. 11, 1978

Open Book & Notes

1. An impact tube in a wind tunnel with air at 70°F reads 20 psia. If a wall static tap at the same section, where the flow streamlines are straight, reads 4 psia, what is the Mach number in the tunnel at this section? What is the change in Mach number if the temperature rises to to 95°F? Assume Po, P remain the same.

2. A rocket motor is to be tested on a horizontal thrust stand. The design Mach number at the nozzle exit is 5. The test is to be run at sea level. The area of the nozzle throat is 2 ft^2. The stagnation pressure and temperature in the combustion chamber just ahead of the nozzle are P_o = 2000 psia and T_o = 3000°R. You can assume the combustion gases act like a perfect gas with K = 1.40 and a mol weight of 100. Find:
   - (a) The mass flow rate.
   - (b) The pressure, temperature, and velocity in the exit plane of the nozzle.
   - (c) The force the nozzle exerts on the test stand. Define direction of force by a sketch. You can neglect forces of connector hoses and inflows which are at low velocity.

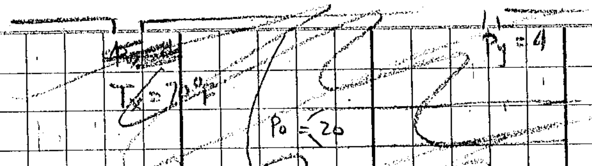
3. Air enters a duct of round cross seciton at M = 0.60, T_o = 600°R, and p_o = 100 psia. The friction factor is 4f = 0.0150. Heat is added through the walls at a rate such that dT_o/T_o = 0.020 dx/D. Find an expression for A(x) that will maintain constant Mach number in the channel.

4. (a) Describe at least four different physical effects that can cause choking.

   (b) Give a general word description of the choking phenomenon (this can include more than one kind of qualitative view if you desire).

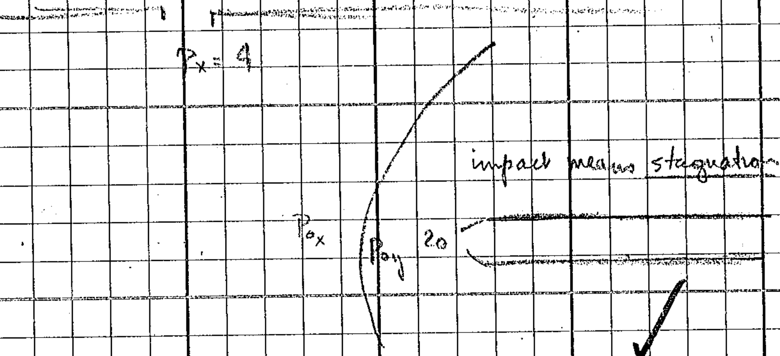
   (c) Apply your description to the examples in 4(a) to indicate (physically) how choking arises in each cases. Please try to keep answers brief.

1.



~~$\frac{P_{oy}}{P_x} = 5 \Rightarrow M_x = 1.87 \quad M_y = .60159$~~

~~$\frac{P_{oy}}{P_y} = 5 \Rightarrow \frac{P_y}{P_{oy}} = .2$~~



impact means stagnation

✓

$\frac{P_{oy}}{P_x} = 5 \Rightarrow M_y = .6016 \quad M_x = 1.87 \qquad T_x/T_o = .58845$ and $T_x = 530°F$

$\therefore T_o = \frac{530}{.58845} = 900.7°R$

if everything else is unchanged

~~$\frac{P_o}{P_x} = \frac{900.7}{4.5} = 1.97956 = 1 + .2 M_x^2 \therefore \sqrt{\frac{.97956}{.2}} = M \quad 2.213$~~

~~$\frac{P_x}{P_{ox}} \quad \frac{P_{oy}}{P_{ox}} = 0.91572 \therefore P_{ox} = 43.68 \qquad \frac{P_o}{P_{ox}}$~~

20/25

$$\frac{\dot m}{A} = \rho V = \sqrt{\frac{k}{R}}\,\frac{p}{\sqrt{T_0}}\,M\sqrt{1+\frac{k-1}{2}M^2}$$

$$= \frac{p}{RT}\cdot M\sqrt{kRT} = \frac{p\,M\sqrt{k}}{\sqrt{RT}}$$

$P_{oy}/P_x = f(M_x)$

$M_x^2 \sim \frac{T_{ox}}{T_x}$

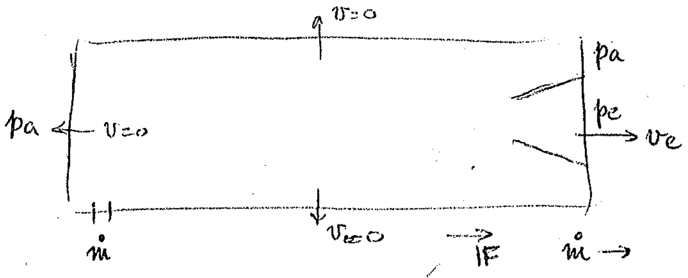
$$\frac{\dot m}{A} = \frac{4 \times 144 \times 1.87 \times \sqrt{1.4}}{\sqrt{1715 \times 530°}} = 1.337 \frac{\text{slugs}}{\text{sq.ft - sec}} = \frac{p}{RT}\cdot M\sqrt{k}\sqrt{RT} = \frac{P\,M\sqrt{k}}{\sqrt{RT}}$$

$$\therefore \frac{\dot m}{A}\cdot\frac{\sqrt{RT}}{P\sqrt{k}} = M = 1.91$$

what's this

$M = f(P_{ox}/P_y)$ ONLY

so no change!



$$\sum F_{on_{cv}} = \cancel{\frac{\partial}{\partial t}\int \rho w \, d\vartheta} + \int d\dot{m} \, w$$

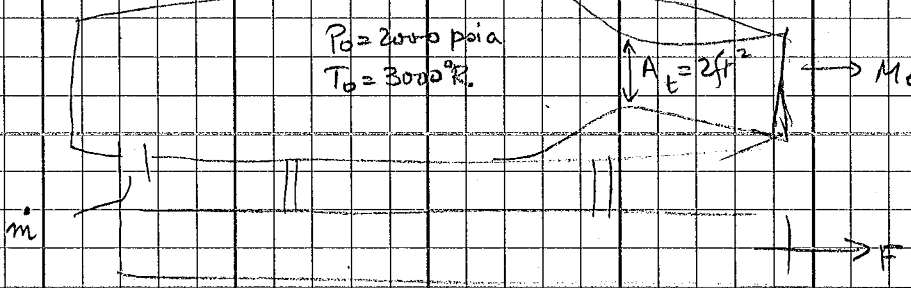
steady state.

$d\dot{m}_{in} = 0$ $\quad$ $d\dot{m}_{out} = \rho v_e A = \dot{m}$

$$F + p_a A_e - p_e A_e = \sum F_{on_{cv}} =$$

$$F + (p_a - p_e) A_e = \dot{m} V_e$$

2



$P_e/P_0 = .00189$
$T_e/T_0 = .16667$
$A_e/A^* = 25.000$

$F + (p_a - p_e) A_e = \dot{m} V_e$ where F is on the CV (or the nozzle)

$\dot{m} = A_t p_0 \sqrt{\frac{k}{g_c R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \frac{1}{\sqrt{T_0}}$ since flow chokes $A_t = A^*$ ✓

$= 2 \cdot 2000 \cdot 144 \sqrt{\frac{1.40}{15.45 \cdot g_c}\left(\frac{2}{2.4}\right)^{\frac{2.4}{.4}}} \frac{1}{\sqrt{3000}} = \frac{1831.78}{\sqrt{g_c}} = 322.94$ slugs/sec

$= 10,371$ lbs/sec OK

for $M_e = 5$ $\frac{T_e}{T_0} = .16667$ $\therefore$ $T_e = .16667 T_0 = .16667(3000) = 500°R$ ✓

$P_e/P_0 = .00189$ $\therefore$ $P_e = .00189 P_0 = .00189(2000) = 3.78$ psia ✓

$A_e/A^* = 25.000$ $\therefore$ $A_e = 25 \times A^* = 25 \times 2 = 50$ sq ft.

$V_e = M_e = 5 \cdot \sqrt{k R g_c T_e} = 5\sqrt{(1.4)(15.45 \times 32.174)(500)} = 589.88$ ft/sec

$\therefore$ $F = \dot{m} V_e + A_e(p_e - p_a)$

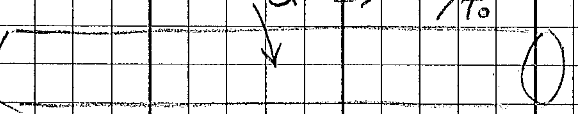
$= (322.94)(589.88) + 50(3.78 - 14.7)(144) = 84659.63$ lb ✓

the force that the nozzle exerts on the stand is 84659.63 lb ⟵

very neatly done! efficient

25/25

$Q \Rightarrow dT_0/T_0 = .020 \frac{dx}{D}$



$M_i = .6$

$T_0 = 600°R$

$P_0 = 100$ psia

$4f = .0150$

we assume constant molecular weight and constant specific heat
we assume no other flows ($y=0$) we assume no body forces ($dX=0$)
and no change in flow rates ($dw=0$)

hence

$$\frac{dM^2}{M^2} = \frac{-2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} + \frac{1+kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dT_0}{T_0}$$

$$+ \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2} 4f\frac{dx}{D}$$

$$= \frac{\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\left[-2\frac{dA}{A} + (1+kM^2)\frac{dT_0}{T_0} + kM^2 \cdot 4f\frac{dx}{D}\right]$$

we want $dM^2 = 0$ for $M = \text{const}$. if $M \neq 1$ we can write

$$\therefore \quad 2\frac{dA}{A} = (1+kM^2)\left(.02\frac{dx}{D}\right) + kM^2 \cdot \left(.015\frac{dx}{D}\right)$$

$$\frac{dA}{A} = \left[\underset{ok}{.01} + .0175kM^2\right]\frac{dx}{D} \qquad \frac{\pi D^2}{4} = A \quad \therefore D = \sqrt{\frac{4}{\pi}A}$$

$$\therefore \quad \frac{dA}{A} = \left[.01 + .0175kM^2\right]\frac{dx}{\sqrt{\frac{4}{\pi}A}} \quad \text{or}$$

$$\frac{dA}{A} \cdot \sqrt{A} = \sqrt{\pi/4}\left[.01 + .0175kM^2\right]dx$$

$$\frac{dA}{\sqrt{A}} = \frac{\sqrt{\pi}}{2}\left[.01 + .0175kM^2\right]dx$$

$$\therefore \quad A_{(x)}^{1/2} = \frac{\sqrt{\pi}}{2}\left[.01 + .0175kM^2\right]x + C$$

$$@\ x=0 \quad A(0)^{1/2} = \frac{\sqrt{\pi}}{2}\left[.01 + .0175(1.4)(.6)^2\right] \cdot 0 + C \qquad \therefore A(0)^{1/2} = C$$

$$\therefore \quad A(x) = \frac{\pi}{4}\left(\left[.01 + .0175kM^2\right]x + \sqrt{A(0)}\right)^2 = .7854\left(.01882x + \sqrt{A(0)}\right)^2$$

$.01 + .0175kM^2 = .01$

method okay (A)
algebra ok
23/25

okay to here

constant wrong a bit

~~A(0) is obtained at the inlet conditions~~

~~Since that crossection has p_0, T_0 then from isentropic tables get~~

~~$P/P_0 = .784 \quad T/T_0 = .93284 \quad \rho/\rho_0 = .84045$~~

~~$\frac{dp_0}{p_0} = -\frac{kM^2}{2}(.035)\frac{dx}{D}$~~

~~$\therefore \ln p_0 = -\frac{kM^2}{2}(.035)$~~

~~$\therefore T = (.93284)T_0 = 559.7\ °R \quad V_{in} = M_{in}\sqrt{kRT_{in}} = 695.83\ ft/sec$~~

~~$\rho = (.84045)\frac{P_0}{RT_0} = .0118\ \frac{slug}{ft^3}$ but $\frac{\dot{m}}{A} = \rho V = \sqrt{\frac{k}{R}}\frac{p}{\sqrt{T_0}} M \sqrt{1+\frac{k-1}{2}M^2}$~~

~~$\therefore \rho V =$~~

in order to define the flow completely A(0) must be obtained or be given simple Processes

4. a) Case of area change : C-D nozzle $.\ni.$ $A_t = A^*$ and $M/_{throat} = 1$

✓ b) Case of friction : in a length of pipe (of const. area) $.\ni.$ $L_{pipe} \geq L_{max}$ $.\ni.$ $M_{exit\ L_{pipe}} = 1$

c) Case of heat addition : if $Q \geq Q_{max}$ needed to bring M_{exit} of pipe $= 1$

then the flow will be choked

d) Isothermal flow in long ducts if $L_{pipe} \geq L_{max}$ $.\ni.$ $M_{exit\ L_{pipe}} = 1/\sqrt{K}$

in a length of pipe of constant area.

e) If we write

✓ $$\frac{dM^2}{dx} = \frac{G(x)}{1-M^2}$$

where $G(x) \equiv M^2\left(1+\frac{k-1}{2}M^2\right)K(x)$

and $K(x) = (1+kM^2)\left(\frac{dT_0}{T_0}+\frac{dw}{w}\right)$

now if $K(x)$ is positive $\Rightarrow \frac{dT_0}{T_0}+\frac{dw}{w} > 0$ flow will choke

i.e. for combined effect of heat addition and increased total mass flow rate

b. You can't stuff any more mass flow through your pipe w/o changing some initial conditions.

✓ c. for a) the isentropic flow for a given A_t & initial p_0 T_0 $\exists$. a unique p^* $.\ni.$ at $A(x)$ where $p = p^*$ $M=1$. That $A(x) = A^*$ & $\dot{m} = \max$ if you try to stuff more mass flow in initial conditions must readjust

✓ for b) In case of friction flow it usually appears when $L > L_{max}$; the flow would normally reach an $M=1$ at $L_1 = L_{max}$ - but if it tries to reach $L = L_1+\epsilon \Rightarrow$ violation of entropy law. hence something happens

if $M_{in} < 1$ $\dot{m}\downarrow$; if $M_{in} > 1 \Rightarrow$ shock and accel. of flow to exit if L from shock to exit $> L_{max} \Rightarrow M_e = 1$ $\dot{m}\downarrow$ and shock moves in tube

✓ for c) In case of heat addition the independent variable here is dQ in friction it was $4fL_{max}/D$. however the same effect occurs if $dQ > dQ_m$

then flow will readj when $M_{in} < 1$ by changing initial cond.
if $M_1 > 1$ flow cannot readj by means of shock (since $T_{0x} = T_{0y}$) in the tube hence readj must be made in the supplying nozzle hence shock will be in nozzle and will adjust its initial conditions through tube

d. for an isothermal relation again there will be a max length for which $L > L_{max}$ flow will choke. Since isothermal is usually connected w/long ducts and low M's the flow pattern for $L > L_{max}$ will be for given inlet conditions $M = \frac{1}{\sqrt{k}}$ at $L = L_{max}$; when flow tries to continue to increase speed the effects would be the same as violating 2nd law of thermo ∴ flow must readjust initial conditions.

e. If the heating and mass flow injection increased for a constant area tube there's only so much you can stuff in there in terms of $\dot{m}$ before flow would choke. Again this would depend on if both $dT_0, dw > 0$

$dT_0 < 0$ $dw > 0$ but $|dW| > |dT_0|$

$dT_0 > 0$ $dw < 0$ but $|dw| < |dT_0|$

in case ① tube is heated and mass flow is being pumped into CV.

② tube is cooled and " " " " " " but $|dW| > |dT_0|$

③ tube is heated " " " " " pumped out of CV but $|dW| < dT_0|$

in ① readjustment would be same as Rayleigh flow for T_0 and for dw change would be to decrease dW.

in ② dw would have to adj downward hence $\dot{m}$ would decrease

in ③ readj would be same as Rayleigh flow for T_0.

24/25

Autumn Quarter
S. J. Kline

ME 255

Due Monday 13 Nov

1. Air flows steadily through a pipe of constant cross-sectional area and at a certain section has properties $p_x = 10$ psia, $T_x = 1240°F$, and $U_x = 3{,}000$ fps.

   a. Make clear plots, choosing appropriate scales, of the Fanno and Rayleigh lines corresponding to point x on the P-v, T-v, and T-s diagrams. Find from these diagrams the pressure, temperature, and velocity downstream of a normal shock occurring at condition x.

   b. Compare your results with the corresponding results found from the shock tables.

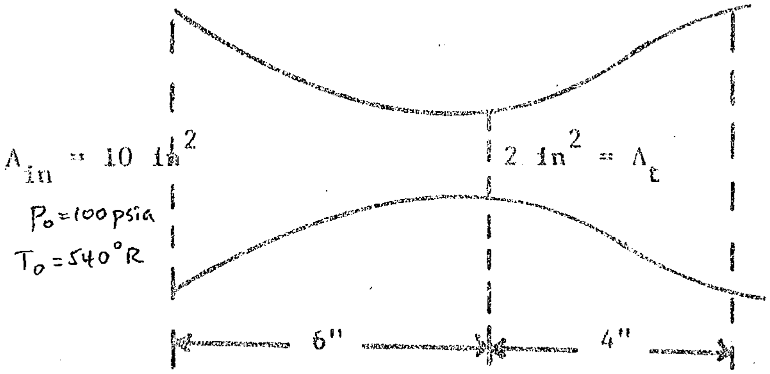
2. Suppose that a blast wave, which might have been initiated by an explosion, is traveling through air at standard atmospheric conditions with a speed of 200,000 fps.

Estimate the changes in pressure (atm), temperature (°F), stagnation pressure (atm), stagnation temperature (°F), and velocity (f/sec) produced by the wave with respect to an observer who is stationary with respect to the undisturbed air. (Assume air acts as a perfect gas with constant specific heats for simplicity).

3. A converging-diverging nozzle is designed such that the inlt area is 10 in^2 and the throat area is 2 in^2.

The nozzle length is 10 inches. The nozzle is designed to produce a linear increase in Mach number fron inlet to exit.

The inlet conditions are helium with $k = 1.66$ at 100 psia, 540°R.



   a. Compute the index pressures.

   b. For operation at design Mach Number compute: (i) the mass flow rate; (ii) the exit Mach number.

   c. Plot the area, pressure, and temperature as functions of x for design Mach number.

   d. Keeping the same exit area, calculate p(x) for operation at 0.5 lbm/sec.

Same

Cesar Levy
ME255A
Prof. Kline

100
very good

Problem Set #3

1. air flows steadily through a pipe of constant cross-sectional area; at a certain section $p_x = 10$ psia, $T_x = 1240°F$, $U_x = 3,000$ fps.

a. Plot the P-v, T-v, T-s diagrams for Fanno and Rayleigh flow and find the y-cond. for a normal shock having the x-conditions as described

b. Compare the y-conditions found with those of the shock table.

I will outline what was done, give tables of results and finally provide the plots to part a.

(1) Since flow is steady, for Fanno flow the following equations are satisfied

Energy: $c_pT + \frac{U^2}{2} = c_pT_0$; continuity $\rho U =$ constant ; the equation of state for a perfect gas (in this case): $\Delta S = \frac{R}{k-1} \ln \left(\frac{T_2}{T_1}\right)\left(\frac{v_2}{v_1}\right)^{k-1}$

So given p_x, T_x we find $\rho_x = \frac{P_x}{RT_x}$; define $\rho_x g_c = \tilde{\rho}_x \left(\frac{lbm}{ft^3}\right)$ and $\frac{1}{\tilde{\rho}_x} = v_x \left(\frac{ft^3}{lbm}\right)$, giving $\tilde{\rho}_x U_x = \dot{m} \left(\frac{lbm}{sec/ft^2}\right)$. Also $T_{0x} = T_{0y} = T_x + \frac{U_x^2(k-1)}{2kR}$.

Now we do the following repetitively: for a given U_y define $\tilde{\rho}_y = \frac{\dot{m}}{U_y}$ and $v_y = \frac{1}{\tilde{\rho}_y}$

now $T_y = T_{0y} - \frac{U_y^2(k-1)}{2kR}$ and $S_y = S_x + \frac{R}{k-1} \ln \left(\frac{T_y}{T_x}\right)\left(\frac{v_y}{v_x}\right)^{k-1}$

100

[we define $S_x = \phi_x - R \ln p_x$ where p_x is in atmospheres and $\phi_x = c_p\int_{T^0}^{T_x} \frac{dT'}{T'}$, $T^0 = 77°F$

hence $S_x = .88758 - 53.34 \ln (10/14.7) = 21.437$ Btu/lbm°R]

Also $p_y = \frac{\tilde{\rho}_y}{g_c} RT_y$, $M_y = \frac{U_y}{\sqrt{kRT_y}}$ and $P_{0y} = P_y\left(\frac{T_{0y}}{T_y}\right)^{\frac{k}{k-1}}$.

Using the values of $R = \frac{1545.32}{28.97} \times 32.174 = 1716.23 \frac{ft^2}{sec^2\,°R} \left(= \frac{\bar{R}}{m_{air}} g_c\right)$; $k = 1.4$ the values of $U_y, T_y, S_y, p_y, v_y, M_y, T_{0y}, P_{0y}$ were printed for $200 \le U_y \le 4000$ ft/sec. Results are attached and are plotted.

(2) Since the flow is steady, for Rayleigh Flow the following equations are satisfied

Momentum: $p + \rho U^2 =$ const ; continuity $\rho U =$ const ; the equation of state for a perfect gas (in this case): $\Delta S = \frac{R}{k-1} \ln \left(\frac{P_2}{P_1}\right)\left(\frac{v_2}{v_1}\right)^k$

So again given p_x, T_x we find $\rho_x = \frac{P_x}{RT_x}$; define $\rho_x g_c = \tilde{\rho}_x$ and $\frac{1}{\tilde{\rho}_x} = v_x$, giving $\tilde{\rho}_x U_x = \dot{m}$ also $T_{ox} = T_x \left(1 + \frac{k-1}{2kR} \frac{U^2}{T_x^2}\right)$

Now we do the following repetitively: for a given U_y define $\tilde{\rho}_y = \frac{\dot{m}}{U_y}$ and $v_y = \frac{1}{\tilde{\rho}_y}$ then $p_y = p_x + \rho_x U_x^2 - \tilde{\rho}_y/g_c\, U_y^2$. Also $T_y = \frac{g_c P_y}{\tilde{\rho}_y R}$, $M_y = \frac{U_y}{\sqrt{kRT_y}}$, $T_{oy} = T_y \left(1 + \frac{k-1}{2} M_y^2\right)$ $S_y = S_x + \frac{R}{k-1} \ln \left(\frac{P_y}{P_x}\right)\left(\frac{v_y}{v_x}\right)^k$ and $P_{oy} = P_y \left(\frac{T_{oy}}{T_y}\right)^{\frac{k}{k-1}}$

Using the values of $R = 1716.23 \frac{ft^2}{sec^2\,°R}$ and $k = 1.4$ the values of $U_y, T_y, S_y, P_y, v_y, M_y, T_{oy}, P_{oy}$ were printed for $200 \leq U_y \leq 3900$ ft/sec. Results are attached and plotted.

FROM THE PLOT : Y SIDE 24 psia = P_y $\Delta S = .005$ Btu/lbm°R $T_y = 2225$ °R

The actual solution:

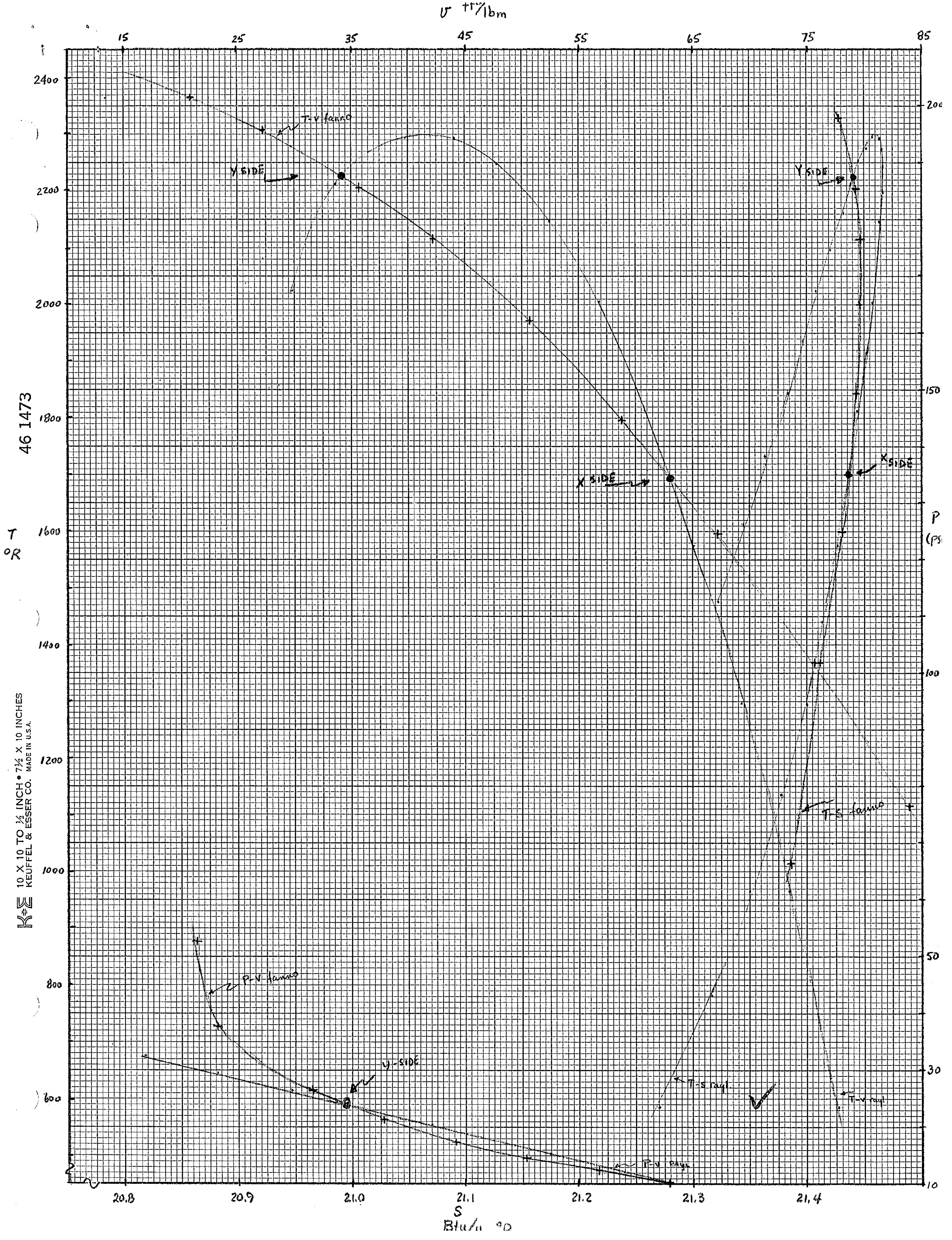
$C = 49.02 \sqrt{1700} = 2021.5$ ft/sec $M_x = \frac{U_x}{c} = 1.484$ ✓

From shock tables $M_y = .7067$ ✓ $P_y/P_x = 2.403$ $T_y/T_x = 1.3096$ ✓

for $P_y = \frac{P_y}{P_x} \cdot P_x = 24.03$ psia ✓; $T_y = \frac{T_y}{T_x} T_x = 2226.4$ °R ✓ and for $P_{ox} = \frac{P_{ox}}{P_x} \cdot P_x = \frac{10}{.2787} = 35.88$ psia

$P_{oy} = \frac{P_{oy}}{P_y} \cdot P_y = \frac{1}{.7166} \cdot 24.03 = 3353$ psia (note: P_{ox}/P_x, P_{oy}/P_y were found from the isentropic tables)

$$\Delta S = -R \ln P_{oy}/P_{ox} = 116.263 \frac{ft^2}{sec^2\,°R} = \frac{116.263}{778.16 \frac{ft\text{-}lb}{Btu}} \cdot \frac{1}{32.174 \frac{lbm}{lb\text{-}sec^2/ft}} = .0046 \frac{Btu}{lbm\,°R}$$


v ft³/lbm
15
25
35
45
55
65
75
85
2400
2200
2000
1800
1600
1400
1200
1000
800
600
T
°R
200
150
100
50
30
10
P
(psi
T-v fanno
Y SIDE
Y SIDE
X SIDE
X SIDE
T-s fanno
P-v fanno
y-side
T-s rayl
T-v rayl
P-v Rayl
20.8
20.9
21.0
21.1
21.2
21.3
21.4
S
Btu/lbm °R

2. Suppose we have a wave traveling through a standard atmosphere (59°F, 1 atm) with a speed of 200,000 fps.

Estimate changes in p (atm), temperature (°F), P_o (atm), T_o (°F) and velocity (ft/sec) with respect to an observer who's standing on the ground.

A

$V_A = 0$
$P_A = 1\text{atm}$
$T_A = 59°F = 519°R$
$T_{0A} = 59°F = 519°R$
$P_{0A} = 1\text{atm}$

$\xleftarrow{V_W}$

B

Finally Find
$P_B = P_y$
$T_B = T_y$
P_{oB}
T_{oB}
$V_B = V_W - V_y$

$\xRightarrow{\text{transform}}$

x

$P_x = P_A = 1\text{atm}$
$T_x = T_A = 519°R$
$V_x = |V_W|$
$c = \sqrt{kRT_x}$
$M_x = \frac{V_x}{c}$
$T_{ox} = T_x\left(1 + \frac{k-1}{2}M_x^2\right)$
$P_{ox} = P_x\left(T_{ox}/T_x\right)^{k/k-1}$

$\longrightarrow$

y

Find first
$P_y = P_B$
P_{oy}
T_{oy}
$T_y = T_B$
V_y

Now $c_x = \sqrt{kRT_x} = 49.02\sqrt{T_x} = 1116.7$ fps

$$M_x = \frac{V_x}{c_x} = \frac{2 \times 10^5}{1116.7} = 179.1$$

since $M_x \gg 1$ we can use $T_{ox} \approx T_x\left(\frac{k-1}{2}M_x^2\right) = 3{,}329{,}540.6\,°R$ or $3.329 \times 10^6\,°F$

$$P_{ox} \simeq P_x\left(T_{ox}/T_x\right)^{k/k-1} = 2.11475 \times 10^{13}\text{atm}$$

$$M_y^2 = \frac{M_x^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_x^2 - 1}\ ; \text{ since } M_x^2 \text{ is very large } M_y^2 \approx \frac{M_x^2}{\frac{2k}{k-1}M_x^2} = \frac{k-1}{2k} = \frac{.4}{2.8} = \frac{1}{7} \quad \therefore \underline{\underline{M_y = .37796}}$$

$$\frac{T_y}{T_x} = \frac{\left(1 + \frac{k-1}{2}M_x^2\right)\left(\frac{2k}{k-1}M_x^2 - 1\right)}{\frac{(k+1)^2}{2(k-1)}M_x^2} = \frac{k\,M_x^4 + M_x^2(\text{fn of } k) - 1}{\frac{(k+1)^2}{2(k-1)}M_x^2} \approx \frac{2k(k-1)}{(k+1)^2}M_x^2 = 6237.1575$$

$$T_y = T_y/T_x \cdot T_x = 6237.1575\,(519°R) = 3237084.7\,°R \text{ or } \underline{\underline{3{,}236{,}624.7\,°F = T_B}}$$

$$c_y = 49.02\sqrt{T_y} = \underline{\underline{88196.3 \text{ ft/sec}}} \qquad V_y = M_y c_y = (.37796)(88196.3) = \underline{\underline{33334.7 \text{ ft/sec}}}$$

$$\frac{P_y}{P_x} = \frac{2k}{k+1}M_x^2 - \frac{k-1}{k+1} \simeq \frac{2k}{k+1}M_x^2 = 37422.95 \quad \therefore P_y \simeq \underline{\underline{37423 \text{ atm} = P_B}}$$

$\frac{T_{oy}}{T_y} = 1 + \frac{k-1}{2}M_y^2 = 1.0286$ or $T_{oy} = \underline{\underline{3329570.6}}\,°R$ note that T_{oy} should equal T_{ox} but because of the approximation $T_{oy} - T_o \cong 30°$, but at this temperature what is a difference of 30° -- it's still very hot.

$$\frac{P_{oy}}{P_y} = \left(\frac{T_{oy}}{T_y}\right)^{\frac{k}{k-1}} = (1.0286)^{3.5} = 1.104 \implies P_{oy} \simeq \underline{\underline{41301 \text{ atm}}}$$

over

✓

$V_B = V_w - V_y = 2 \times 10^5 - \frac{1}{3} \times 10^5 = 1.67 \times 10^5$ or $166,667$ ft/sec

$M_B = \frac{V_B}{c_y} = \frac{166,667}{49.02\sqrt{T_y}} = \frac{166,667}{88196.3} = 1.89$ ✓

$\frac{T_{oB}}{T_B} = 1 + \frac{K-1}{2} M_B^2 = 1.7142$ $\quad T_{oB} = 3237084.7\,°R\,(1.7142) = 5549010.6\,°R,\ 5548550.6\,°F$

$\frac{P_{oB}}{P_B} = \left(1 + \frac{K-1}{2} M_B^2\right)^{\frac{K}{K-1}} = 6.595$ $\quad P_{oB} = 37423\,(6.595)$ $\quad P_{oB} = 246805$ atm.

100

3. Given a converging-diverging nozzle whose $A_i = 10\ in^2$ and $A_t = 2 in^2$. The nozzle length is 10 inches and the mach no. is a linear function of the length at design conditions ($P_E = P_{\substack{supersonic \\ isent.}}$). Inlet conditions are $k = 1.66$, helium ($\bar{m} = 4.004$) at $P_0 = 100$ psia, $T_0 = 540°R$

a. Compute $p_{\substack{subsonic \\ isentropic}}$, $p_{\substack{shock \\ at exit}}$, $p_{\substack{supersonic \\ isentropic}}$

b. for $p_{\substack{supersonic \\ isentropic}}$ find $\dot{m}$ and Me

c. Plot A, p, T as a fn of x for design mach number ($p_{\substack{supersonic \\ isentropic}}$)

d. If $\dot{m} = 0.5$ lbm/sec and for the designed nozzle find p(x)

a&b. Since for all three cases the flow is choked and $\dot{m} = \frac{A_t p_0}{\sqrt{T_0}} \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}$

where $R = 12417.4 \frac{ft^2}{sec^2 \, °R}$

$$\dot{m} = \frac{2 \times 100}{\sqrt{540}} \sqrt{\frac{1.66}{12417.4}\left(\frac{2}{2.66}\right)^4} = \underline{\underline{1.8046\ lbm/sec}} \checkmark$$

100

Next since we know $A_{in}/A^* = 5$ then if we solve

$$A_{in}/A^* = \frac{1}{M_{in}}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2} M_{in}^2\right)\right]^{\frac{k+1}{2(k-1)}}$$ for M_{in} we can then say that $M = aL + b$ when @ $L = 6''$ $M = 1$, $L = 0''$ $M = M_{in}$. Solving we get $M_{in} = .1135$ and $M = .1135\left(1 - \frac{L}{6}\right) + \frac{L}{6}$. Hence $\underline{\underline{Me = 1.591}}$ $\checkmark$

- now picking values of L we use $M(L) = .1135\left(1 - \frac{L}{6}\right) + \frac{L}{6}$ to get M. We use this in $\frac{A(L)}{A^*} = \frac{1}{M(L)}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2} M(L)^2\right)\right]^{\frac{k+1}{2(k-1)}}$ to get A(L). We also use M(L) in $\frac{T_0}{T(L)} = 1 + \frac{k-1}{2} M(L)^2$ to get T(L) and also in $\frac{P_0}{P(L)} = \left(\frac{T_0}{T(L)}\right)^{\frac{k}{k-1}}$ to get p(L)
  The plots are attached.

- we find for a $M_e = 1.591$ $A_e/A^* = 1.202$ $P_e/P_0 = .2166$ or $P_e = \underline{\underline{21.66\ psia}}$ (design pres) for $A_e/A^* = 1.202$ we find $Me = .5781$ and $P_e/P_0 = .7679$ or $P_e = \underline{\underline{76.79\ psia}}$ (subsonic exit with choked flow). for the shock at exit $Me = M_x = 1.591$ and $M_y = .6889$ and $P_y = \underline{\underline{63.13\ psia}}$ $\checkmark$

d.) Since $\dot{m} < \dot{m}_{\text{CHOKED FLOW}}$ therefore the flow is totally subsonic. However we can find a hypothetical throat such that M = 1 there hence $\checkmark$

$$A^*_{HYP} = \frac{\dot{m}\sqrt{T_0}}{P_0} \frac{1}{\sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}} = .5542\ in^2$$

assume same
$P_0 . T_0$ inlet

We can now use the $A(L)$'s found in part b. and solve

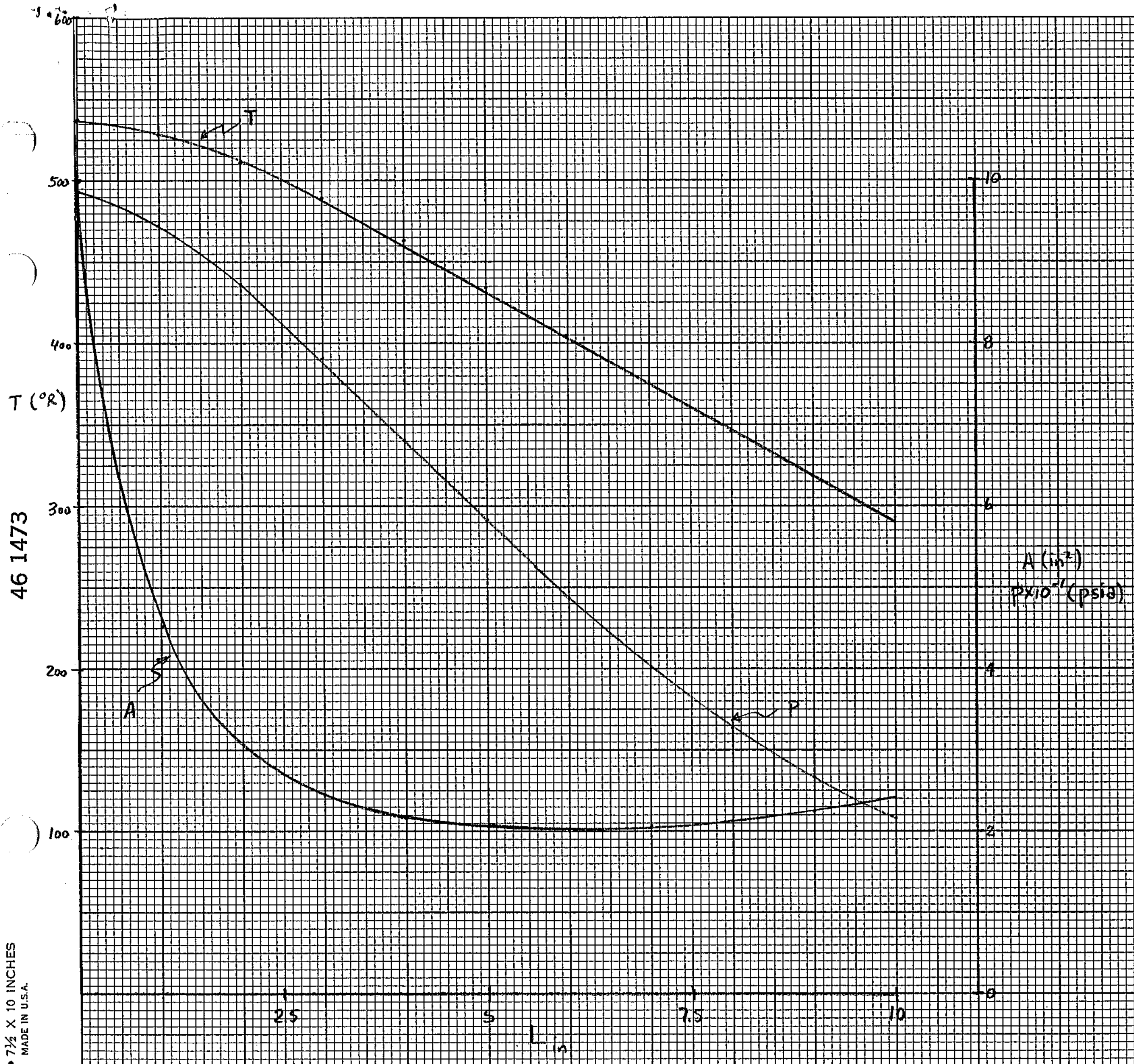
$$\frac{A(L)}{A^*_{Hyp}} = \frac{1}{M(L)}\left[\frac{2}{K+1}\left(1+\frac{K-1}{2}M(L)^2\right)\right]^{\frac{K+1}{2(K-1)}} \quad \text{for } M(L)$$

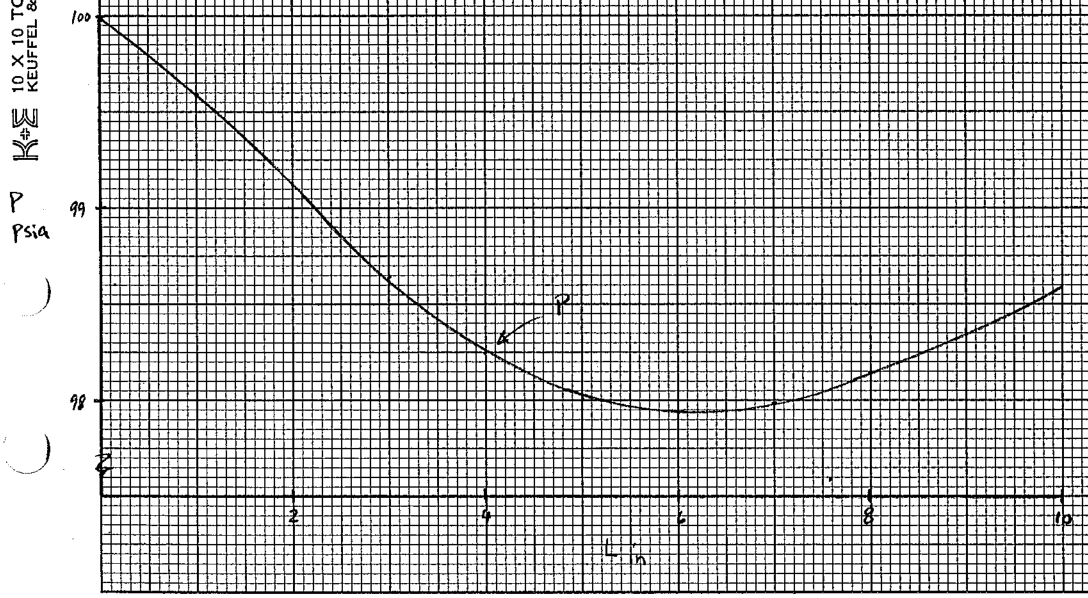
✓

We can then use $\frac{P_0}{P(L)} = \left[1+\frac{K-1}{2}M(L)^2\right]^{\frac{K}{K-1}}$ to find $P(L)$; this is also plotted.

```
                M                    P                    T                    A
   1    0. 3119132E-01     0. 9991897E+02     0. 5392249E+03     0. 100000
   2    0. 6941468E-01     0. 9959959E+02     0. 5391311E+03     0. 450500
   3    0. 1023787E+00     0. 9913186E+02     0. 5381197E+03     0. 306600
   4    0. 1281018E+00     0. 9864548E+02     0. 5370623E+03     0. 246000
   5    0. 1457544E+00     0. 9825135E+02     0. 5362079E+03     0. 216900
   6    0. 1555013E+00     0. 9801303E+02     0. 5354823E+03     0. 203700
   7    0. 1584759E+00     0. 9793739E+02     0. 5355169E+03     0. 200000
   8    0. 1560553E+00     0. 9799905E+02     0. 5356517E+03     0. 203000
   9    0. 1498069E+00     0. 9815404E+02     0. 5359901E+03     0. 211200
  10    0. 1411368E+00     0. 9835913E+02     0. 5364381E+03     0. 223800
  11    0. 1311547E+00     0. 9858080E+02     0. 5369214E+03     0. 240400
 0. 1241736E+05    0. 5541529E+00   and subsonic isent flow
                      A*HYP
```

✓


600
500
400
300
200
100
T (°R)
T
A
P
2.5
5
7.5
10
L in
10
8
6
4
2
0
A (in²)
P×10⁻¹ (psia)


100
99
98
P
psia
P
2
4
6
8
10
L in

```
                 0. 1804556E+01 ṁ       0. 1134677E+00  Min
          0. 0000000E+00     0. 1134677E+00      0. 9893510E+02        0. 5376924E+03
          0. 1000000E+01     0. 2612231E+00      0. 9453241E+02        0. 5279904E+03
          0. 2000000E+01     0. 4089785E+00      0. 8731577E+02        0. 5114826E+03
          0. 3000000E+01 L   0. 5567338E+00      0. 7820756E+02        0. 4894330E+03
          0. 4000000E+01     0. 7044892E+00 M    0. 6819929E+02 P      0. 4633463E+03 T
          0. 5000000E+01     0. 8522446E+00      0. 5815707E+02        0. 4347452E+03
          0. 6000000E+01     0. 1000000E+01      0. 4871393E+02        0. 4050000E+03
          0. 7000000E+01     0. 1147755E+01      0. 4024963E+02        0. 3752308E+03
          0. 8000000E+01     0. 1295511E+01      0. 3292849E+02        0. 3462761E+03
          0. 9000000E+01     0. 1443266E+01      0. 2676081E+02        0. 3187083E+03
          0. 1000000E+02     0. 1591022E+01      0. 2166334E+02        0. 2928761E+03
Shock -> 0. 6313102E+02 Py        0. 6888535E+00  My
subsonic || 0. 1804556E+01 ṁ      0. 5780868E+00  M
isentropic || 0. 7679432E+02 P    0. 4858760E+03  T ||
           0. 0000000E+00
```

```
00000E+02
04809E+01
66041E+01
59835E+01
168974E+01 Λ
36602E+01
00000E+01
29989E+01
111806E+01
237725E+01
403788E+01
```

Set #5

ME 255

(Kline)

Due Wed., Dec. 6, '78

$\dot{q}=0$

R=const, W=const

Fanno + area change.

1. Air flows adiabatically in a conical duct of circular cross section, the included angle between the walls being 2θ.) 2θ

If f is the coefficient of friction, find a relation between θ, D/D^*, M and f, where D is the diameter at Mach Number M_x and D^* is the diameter at Mach Number unity. Plot the angle θ needed to keep the Mach Number M constant versus the Mach Number M for 0 < M < 5 with f as a parameter with the values:

- i) 4f = .010
- ii) 4f = .015
- iii) 4f = .020

$\dot{q}\neq 0$

2. Consider the compressible flow of steam in an electrically heated tube. It is desired to keep the flow at a constant Mach Number of 0.5, and changes in cross-sectional area will therefore be necessary.

$\frac{kR}{k-1}=C_p$

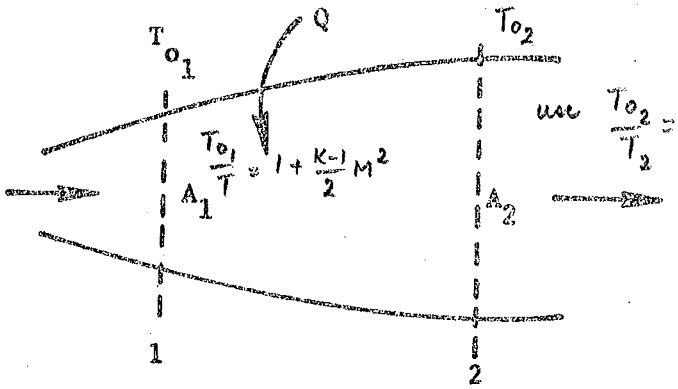
R=const

To simplify the analysis, wall friction will be neglected and constant specific heats will be assumed. Rayleigh + area change.

a) Express the area ratio (A_2/A_1) as a function of the heat flux per unit mass Q and the stagnation temperature T_{o_1}.

b) Compute the area ratio for a heat flux of 810 Btu/lbm and T_{o_1} = 1000°R.

c) Compute the velocities V_1 and V_2 for the conditions given in b).



$\frac{T_{o_1}}{T}=1+\frac{k-1}{2}M^2$

use $\frac{T_{o_2}}{T_2}=1+\frac{k-1}{2}M^2$

$\dot{q} = 0 \Rightarrow T_0 = \text{const} \quad (dT_0 = 0)$

air we assume $k = \text{const}$ & $dk = 0$
$W = \text{const}$ & $dW = 0$

since no external work or other type influx (gas/fluid)
and there is no other forces acting on body $dX = 0$
and since air will not change phase $dw = 0$

$\Rightarrow$ that we can rewrite

$$\frac{dM^2}{M^2} = \frac{-2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} + \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\cdot 4f\frac{dx}{D}$$

$$= \frac{-2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\cdot\frac{2dr}{r} + \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\cdot\frac{4f\,dr}{2r\tan\theta}$$

$$= \frac{-4\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dr}{r} + \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{4f}{2\tan\theta}\frac{dr}{r}$$

$$\frac{dM^2}{M^2} = \frac{1+\frac{k-1}{2}M^2}{1-M^2}\left[-4 + \frac{kM^2\cdot 4f}{2\tan\theta}\right]\frac{dr}{r}$$

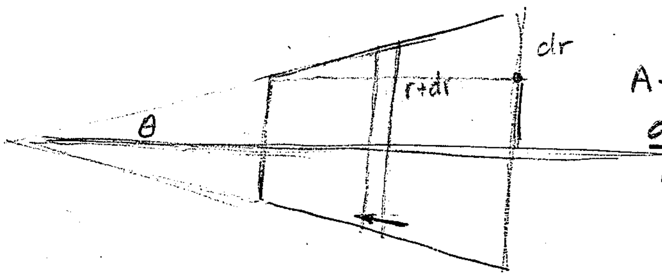
$$\text{or} \quad \frac{dM^2\cdot(1-M^2)}{1+\frac{k-1}{2}M^2\quad M^2}\left[\frac{2\tan\theta}{-8\tan\theta+4fkM^2}\right] = \frac{dr}{r}$$

of the form $\dfrac{du\,(1-u)}{(1+\alpha u)u}\left(\dfrac{\beta}{\gamma+\delta u}\right)$

$$\frac{A}{u} + \frac{\overset{B}{\cancel{A}}}{1+\alpha u} + \frac{\overset{C}{\cancel{B}}}{\gamma+\delta u} = \frac{du}{(1+\alpha u)(\gamma+\delta u)}$$

$$(\cancel{\gamma+\delta u})A + (1+\alpha u)B$$

now $\quad 2r = D \qquad \dfrac{2dr}{2r} = \dfrac{dD}{D} = \dfrac{dr}{r}$



$\frac{\triangle}{dx}dr$ $\tan\theta = \frac{dr}{dx}$

$A + dA = \pi(r+dr)^2 - \pi r^2$

$\frac{dA}{A} = \frac{2\pi r\,dr}{\pi r^2} = 2\frac{dr}{r}$

$$\frac{dM^2}{M^2} = -\frac{2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dr}{r} + \frac{(1+kM^2)\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dT_0}{T_0} + kM^2$$

$dx =$

$\frac{dr}{dx} = \tan\theta$

$\frac{1}{1-x} =$

$\frac{5K}{8\tan\theta}$

$\frac{25KM^2}{8\tan\theta}$

$\left(1 - \frac{25KM^2}{8\tan\theta} - \left(\frac{}{\tan\theta}\right)\right)$

$\dot{q} = 0 \Rightarrow$

$-8\tan\theta\left[1 - \frac{5KM^2}{8\tan\theta}\right]$

$-8\tan\theta\left[1 - \frac{5K}{8\tan\theta}\right]$

$\left(1 - \frac{5KM^2}{8\tan\theta}\right)\left[\left(\frac{5K}{8\tan\theta}\right) \cdot \left(\frac{5K}{8\tan\theta}\right)\right]$

$\varepsilon - \frac{\varepsilon^2}{2} + \frac{\varepsilon^3}{3} - \varepsilon^4$

$\ln(1+\varepsilon) = \varepsilon - \frac{\varepsilon^2}{2!} + \frac{2\varepsilon^3}{3}$

$$\frac{du}{(1+\alpha u)u}\frac{(1-u)}{}\frac{\beta}{(\gamma+\delta u)}$$

$$\frac{A\,du}{u}+\frac{B\,du}{1+\alpha u}+\frac{C\,du}{\gamma+\delta u}=\frac{\beta(1-u)\,du}{(1+\alpha u)u(\gamma+\delta u)}$$

$$A(1+\alpha u)(\gamma+\delta u)+B(\gamma+\delta u)u+C(u)(1+\alpha u)=\beta-\beta u$$

$\therefore$ $\cancel{(\gamma+\gamma\alpha}$

$$A(\underline{\gamma}+\underline{\delta u}+\underline{\gamma\alpha u}+\overline{\alpha\delta u^2})+B(\underline{\gamma u}+\overline{\delta u^2})+C(\underline{u}+\overline{\alpha u^2})=\underline{\beta}-\underline{\beta u}$$

$$\therefore\quad A\gamma=\beta\quad\therefore\quad\boxed{A=\beta/\gamma}$$

$$A(\delta+\gamma\alpha)u+(B\gamma+C)u=-\beta u\qquad\therefore\quad \beta/_{\gamma}(\delta+\gamma\alpha)+B\gamma+C=-\beta$$

$$A(\alpha\delta)+B\delta+C\alpha=0\qquad\qquad \beta/_{\gamma}\,\alpha\delta+B\delta+C\alpha=0$$

$$\therefore\quad -\beta\left(\frac{\delta}{\gamma}+\alpha\right)-\beta=B\gamma+C=-\beta\left[\frac{\delta}{\gamma}+\alpha+1\right]$$

$$-\frac{\beta\alpha\delta}{\gamma}=B\delta+C\alpha$$

$$\therefore\quad -\beta\delta\left[\frac{\delta}{\gamma}+\alpha+1\right]=B\gamma\delta+C\delta$$

$$+\frac{\beta\alpha\delta\gamma}{\cancel{\gamma}}=-B\delta\gamma+C\alpha\gamma$$

$$-\beta\delta\left[\frac{\delta}{\gamma}+\alpha+1\right]+\beta\alpha\delta=C(\delta-\alpha\gamma)$$

$$-\frac{\beta\delta^2}{\gamma}-\beta\cancel{\alpha\delta}+\beta\cancel{\alpha\delta}-\beta\delta=C(\delta-\alpha\gamma)$$

$$C=\frac{-\beta\delta\left[\frac{\delta}{\gamma}+1\right]}{\delta-\alpha\gamma}=-\frac{\beta\delta[\delta+\gamma]}{\gamma(\delta-\alpha\gamma)}$$

$$-\beta\left[\frac{\delta}{\gamma}+\alpha+1\right]=B\gamma-\frac{\beta\delta}{\gamma}\frac{(\delta+\gamma)}{(\delta-\alpha\gamma)}$$

$$-\frac{\beta[\delta+\alpha\gamma+\gamma]}{\gamma(\delta-\alpha\gamma)}(\delta-\alpha\gamma)=B\gamma-\frac{\beta\delta(\delta+\gamma)}{\gamma(\delta-\alpha\gamma)}$$

$$\therefore\quad -\frac{\beta[\delta+\alpha\gamma+\gamma](\delta-\alpha\gamma)+\beta\delta(\delta+\gamma)}{\gamma^2(\delta-\alpha\gamma)}=B$$

$$\frac{1-u}{ku^2(1+\gamma u)}\,du$$

$$\frac{A\,du}{ku^2}+\frac{B\,du}{1+\gamma u}=\frac{du}{ku(1+\gamma u)}$$

$$\frac{\beta\,du}{(1+\alpha u)(\gamma+\delta u)} \qquad \frac{-\beta u\,du}{(1+\alpha u)(\gamma+\delta u)} \qquad \cancel{-\beta u\,du}$$

$$-\beta\,[$$

$$\gamma+(\gamma\alpha+\delta)u+\alpha\delta u^2$$

$$\therefore\ (\gamma\alpha+\delta)+2\alpha\delta u$$

$$\frac{A}{1+\alpha u}+\frac{B}{\gamma+\delta u}=\quad A\gamma+A\delta u+B+B\alpha u=\beta$$

$$\therefore\ A\gamma+B=\beta \quad\text{and}\quad (A\delta+B\alpha)u=0\ \therefore$$

$$B-B\frac{\alpha\gamma}{\delta}=\beta \qquad A=-\frac{B\alpha}{\delta}$$

$$B\left(\frac{\delta-\alpha\gamma}{\delta}\right)=\beta \quad\therefore\quad B=\frac{\delta\beta}{\delta-\alpha\gamma}\quad\text{and}\quad A=\frac{-\alpha\beta}{\delta-\alpha\gamma}$$

$$\cancel{\frac{A\,du}{\gamma+\alpha u}}\qquad \cancel{A\ln(\gamma+\alpha u)+B\ln}$$

$$\frac{-\alpha\beta\,du}{(\delta-\alpha\gamma)(1+\alpha u)}\ \text{or}\ \frac{-\beta}{\delta-\alpha\gamma}\,\frac{\alpha\,du}{1+\alpha u}\ \text{or}\ \frac{-\beta}{\delta-\alpha\gamma}\ln(1+\alpha u)$$

$$\frac{\delta\beta}{\delta-\alpha\gamma}\,\frac{du}{\gamma+\delta u}\ \text{or}\ \frac{\beta}{\delta-\alpha\gamma}\,\frac{\delta\,du}{\gamma+\delta u}\ \text{or}\ \frac{\beta}{\delta-\alpha\gamma}\ln(\gamma+\delta u)$$

$$\text{or}\ \frac{\beta}{\delta-\alpha\gamma}\ln\left(\frac{1+\alpha u}{\gamma+\delta u}\right)$$

$$\frac{-\beta u}{(1+\alpha u)(\gamma+\delta u)}=\frac{\check{A}}{1+\alpha u}+\frac{\check{B}}{\gamma+\beta u}\ \text{or}\ \check{A}\gamma+\check{B}+u(\check{B}\alpha+\delta\check{A})=-\beta u$$

$$\therefore\ \check{A}\gamma+\check{B}=0\quad\therefore\quad \check{B}=-\check{A}\gamma$$

$$\text{and}\ \check{B}\alpha+\delta\check{A}=-\beta=-\check{A}\alpha\gamma+\delta\check{A}$$

$$\therefore\ \check{A}=\frac{\beta}{\alpha\gamma-\delta}\qquad \check{B}=\frac{-\gamma\beta}{\alpha\gamma-\delta}$$

$$\therefore\ \frac{\beta\,du}{(1+\alpha u)(\alpha\gamma-\delta)}\ \text{or}\ \frac{\beta}{\alpha(\alpha\gamma-\delta)}\cdot\frac{\alpha\,du}{1+\alpha u}=\frac{\beta}{\alpha(\alpha\gamma-\delta)}\ln(1+\alpha u)$$

$$\frac{-\gamma\beta}{\alpha\gamma-\delta}\,\frac{du}{\gamma+\delta u}\ \text{or}\ \frac{-\gamma\beta}{\delta(\alpha\gamma-\delta)}\,\frac{\delta\,du}{\gamma+\delta u}=\frac{-\gamma\beta}{\delta(\alpha\gamma-\delta)}\ln(\gamma+\delta u)$$

$$-\beta\left[\cancel{\delta^2}-\alpha^2\gamma^2+\cancel{\gamma\delta}-\alpha\gamma^2-\cancel{\delta^2}-\cancel{\delta\gamma}\right] = +\frac{\beta\alpha\gamma^2(\alpha+1)}{\cancel{\gamma^2}(\delta-\alpha\gamma)} = \beta\frac{\alpha(\alpha+1)}{\delta-\alpha\gamma} = B$$

$$\therefore\quad \frac{\beta}{\gamma}\frac{du}{u}+\frac{\beta\,(\alpha+1)\alpha\,du}{(\delta-\alpha\gamma)\,(1+\alpha u)}-\frac{\beta(\delta+\gamma)}{\gamma(\delta-\alpha\gamma)}\frac{\delta\,du}{\gamma+\delta u}$$

$$\frac{\beta}{\gamma}\ln u\Big|_1^u+\frac{\beta(\alpha+1)}{\delta-\alpha\gamma}\ln(1+\alpha u)\Big|_1^u-\frac{\beta(\delta+\gamma)}{\gamma(\delta-\alpha\gamma)}\ln(\gamma+\delta u)\Big|_1^u=\ln D\Big|_{D^*}^{D}$$

$$\delta-\alpha\gamma = 5K+4(K-1)\tan\theta$$

$$\beta/\gamma = -\frac{1}{4}$$

$$\delta+\gamma = 5K-8\tan\theta$$

$$\alpha+1 = \frac{K+1}{2}$$

$$\beta = 2\tan\theta$$

$$\ln D/D^* = -\frac{1}{4}\ln M^2+\frac{\cancel{2}\tan\theta\left(\frac{K+1}{\cancel{2}}\right)}{5K+4(K-1)\tan\theta}\ln\left(\frac{1+\frac{K-1}{2}M^2}{\frac{K+1}{2}}\right)+\frac{1}{4}\frac{(5K-8\tan\theta)}{5K+4(K-1)\tan\theta}\ln\left(\frac{-8\tan\theta+5KM^2}{5K-4\tan\theta}\right)$$

$$\frac{1}{4(K-1)\tan\theta}\left\{(K+1)\tan\theta\ln\left\{2+\qquad\right\}-2\tan\theta\ln 1\right\}-\frac{1}{4}\ln M^2$$

$$\frac{K+1}{4(K-1)}\ln\left\{\qquad\right\}-\cancel{\frac{1}{2(K-1)}\ln 1}\xrightarrow{}0\qquad\Big\}-\frac{1}{4}\ln M^2$$

$\ln\frac{D}{D^*}$ since $\tan\theta\longrightarrow\infty$ faster than $\ln\left(\frac{5KM^2-8\tan\theta}{5K-8\tan\theta}\right)\longrightarrow 0$

$\beta = 2\tan\theta$

$\frac{K-1}{2} = \alpha\,, \quad \gamma = -8\tan\theta \quad \delta = 4fK$

$$\therefore \quad \frac{1-u}{1+\alpha u}\cdot\frac{\beta}{\gamma+\delta u}\,du = \frac{dM^2(1-M^2)}{1+\frac{K-1}{2}M^2}\left[\frac{2\tan\theta}{-8\tan\theta+4fKM^2}\right] = \frac{dr}{r}$$

$$\therefore \quad \frac{\beta}{\delta-\alpha\gamma}\ln\left(\frac{1+\alpha u}{\gamma+\delta u}\right) \quad \frac{-\beta}{\alpha(\delta-\alpha\gamma)}\ln(1+\alpha u) + \frac{\gamma\beta}{\delta(\delta-\alpha\gamma)}\ln(\gamma+\delta u) = \ln r$$

$$\check{C} + \frac{(\alpha-1)\beta}{\alpha(\delta-\alpha\gamma)}\ln(1+\alpha u) + \frac{(\gamma-\delta)\beta}{\delta(\delta-\alpha\gamma)}\ln(\gamma+\delta u) = \ln r$$

when $M = 1 \quad u = 1 \quad r = r^*$

$$\check{C} + \frac{(\alpha-1)\beta}{\alpha(\delta-\alpha\gamma)}\ln(1+\alpha) + \frac{(\gamma-\delta)\beta}{\delta(\delta-\alpha\gamma)}\ln(\gamma+\delta) = \ln r^*$$

$$\check{C} = \ln r^* - \frac{(\alpha-1)\beta}{\alpha(\delta-\alpha\gamma)}\ln(1+\alpha) - \frac{(\gamma-\delta)\beta}{\delta(\delta-\alpha\gamma)}\ln(\gamma+\delta)$$

$$\frac{\alpha-1}{\alpha(\delta-\alpha\gamma)}\ln\left(\frac{1+\alpha u}{1+\alpha}\right) + \frac{(\gamma-\delta)\beta}{\delta(\delta-\alpha\gamma)}\ln\left(\frac{\gamma+\delta u}{\gamma+\delta}\right) = \ln r/r^* = \ln D/D^*$$

$\delta - \alpha\gamma = 4fK + 4(K-1)\tan\theta \qquad \gamma - \delta = -8\tan\theta - 4fK \qquad \gamma + \delta = 4fK - 8\tan\theta$

$\frac{\alpha-1}{\alpha} = \frac{\frac{K-1}{2}-1}{\frac{K-1}{2}} = \frac{\frac{K-3}{2}}{\frac{K-1}{2}} = \frac{K-3}{K-1} \qquad \alpha+1 = \frac{K-1}{2}+1 = \frac{K+1}{2}$

$$\frac{1}{4fK+4(K-1)\tan\theta}\left\{\frac{K-3}{K-1}\cdot\ln\left[\frac{1+\frac{K-1}{2}M^2}{\frac{K+1}{2}}\right] - \frac{2(8\tan\theta+4fK)\tan\theta}{4fK}\ln\left[\frac{-8\tan\theta+4fKM^2}{-8\tan\theta+4fK}\right]\right\} = \ln D/D^*$$

We assume $\dot{q}=0$, $\tau_w=0$ $(f=0)$, $dk=0$, $dW_x=0$, $dx=0$, $dW=0$, and since nothing is added or removed from the flow and no phase change will be assumed then $dH=0$. Thus

$$\frac{dM^2}{M^2} = -\frac{2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} + \frac{\left(1+kM^2\right)\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dT_0}{T_0}$$

for the case where $M=\text{const} \Rightarrow dM^2=0$

$\therefore$

$$-2\frac{dA}{A} + \left(1+kM^2\right)\frac{dT_0}{T_0} \quad \text{or} \quad -2\ln A_2/A_1 + \left(1+kM^2\right)\ln\frac{T_{0_2}}{T_{0_1}}$$

$$\left(\frac{A_2}{A_1}\right)^2 = \left(\frac{T_{0_2}}{T_{0_1}}\right)^{1+kM^2} \quad \therefore \quad c_p\left(T_{0_2}-T_{0_1}\right) = Q \quad \therefore \quad \frac{Q}{c_pT_{0_1}} + 1 = \frac{T_{0_2}}{T_{0_1}}$$

$$\therefore \left(\frac{A_2}{A_1}\right) = \left(\frac{Q}{c_pT_{0_1}}+1\right)^{\frac{1+kM^2}{2}} = \left(\frac{(k-1)Q}{kRT_{0_1}}+1\right)^{\frac{1+kM^2}{2}}$$

$Q = c_p\left(T_{0_2}-T_{0_1}\right)$ since Q is added $\Rightarrow T_{0_2} > T_{0_1}$

~~$\ln\frac{V_2}{V_1} = -\frac{1}{1-M^2}\ln\frac{A_2}{A_1} + \frac{1+\frac{k-1}{2}M^2}{1-M^2}\ln\frac{T_{0_2}}{T_{0_1}}$~~

$\therefore$ ~~$\frac{V_2}{V_1}$~~ since W, $c_p = \text{const} \Rightarrow \frac{T_0}{T} = 1+\frac{k-1}{2}M^2$ gives $T_1, T_2 \Rightarrow c_1, c_2$
$\Rightarrow V_1, V_2$

100

Cesar Levy
ME255
Prof. Kline

Problem Set #5

1. Air flows adiabatically in a conical duct of circular cross-section, the included angle between the walls being 2θ.

a. If f is the friction coefficient, D is the diameter at mach no. M, D^* is the diameter for mach number $M=1$ find $g(\theta, D/D^*, M, f)=0$

Assumptions: 1-D Steady State flow, adiabatic ($\dot{q}=0$) air ($dk = dW = dw = 0$) in a simple duct ($dX=0$, $dW_x=0$) with no phase changes ($y=0, dH=0$)

Hence from table 8.1 we get that with $\zeta = 4f$

$$\frac{dM^2}{M^2} = -\frac{2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} + \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\cdot \zeta\frac{dx}{D} \quad ✓ \quad (1)$$

we note that since $A = \pi r^2$ & $dA = 2\pi r\,dr$ then $\frac{dA}{A} = \frac{2dr}{r}$. Also we note that $\tan\theta = \frac{dr}{dx}$ hence $\frac{dx}{D} = \frac{dr}{2r\tan\theta}$. We thus transform (1) to

$$\frac{dM^2}{M^2} = \frac{-4\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{dr}{r} + \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\frac{\zeta}{2\tan\theta}\frac{dr}{r} = \frac{1+\frac{k-1}{2}M^2}{1-M^2}\left\{\frac{-8\tan\theta + kM^2\zeta}{2\tan\theta}\right\}\frac{dr}{r}$$

$$\text{or} \quad \frac{dr}{r} = \frac{dM^2(1-M^2)}{M^2\left(1+\frac{k-1}{2}M^2\right)}\left\{\frac{2\tan\theta}{-8\tan\theta + kM^2\zeta}\right\} = \frac{dD}{D} \quad (2) \quad ✓$$

We note that the middle term can be transformed to $\frac{du}{u}\,\frac{(1-u)}{1+\alpha u}\,\frac{\beta}{\gamma+\delta u}$ where

$u = M^2$, $\alpha = \frac{k-1}{2}$, $\beta = 2\tan\theta$, $\gamma = -8\tan\theta$, $\delta = k\zeta$. By means of partial fractions we can integrate (2) with respect to $D = D^*$ and $M(D^*) = 1$ to give

$$\ln\frac{D}{D^*} = \frac{1}{\zeta k + 4(k-1)\tan\theta}\left\{(k+1)\tan\theta\,\ln\left[\frac{2+(k-1)M^2}{k+1}\right] + \left(\frac{\zeta k}{4} - 2\tan\theta\right)\ln\left[\frac{\zeta kM^2 - 8\tan\theta}{\zeta k - 8\tan\theta}\right]\right\} - \frac{1}{4}\ln M^2 \quad ✓ \quad (3)$$

Note that for $\theta = 0°$ the RHS gives you $\frac{1}{\zeta k}\left[\frac{\zeta k}{4}\ln M^2\right] - \frac{1}{4}\ln M^2 = 0 \Rightarrow \frac{D}{D^*} = 1$ as is expected. Note also that as $\theta \to 90°$ the RHS tends towards

$$\frac{k+1}{4(k-1)}\ln\left[\frac{2+(k-1)M^2}{k+1}\right] - \frac{1}{4}\ln M^2$$

since $\dfrac{\frac{5K}{4} - 2\tan\theta}{5K + 4(K-1)\tan\theta} \longrightarrow -\dfrac{1}{2(K-1)}$ faster than $\ln\left[\dfrac{5KM^2 - 8\tan\theta}{5K - 8\tan\theta}\right] \longrightarrow 0$.

Thus $\lim\limits_{\theta\to\pi/2} \ln\dfrac{D}{D^*} = \dfrac{K+1}{4(K-1)} \ln\left[\dfrac{2 + (K-1)M^2}{K+1}\right] - \dfrac{1}{4}\ln M^2$. Note that physically we will never have this happen since flow separation and boundary layer effects will become pronounced long before θ reaches $\pi/2$. This condition of $\theta \to \pi/2 \implies D^* \to 0$ for fixed D.

b. Plot θ (for M = const) vs. M in the range $0 < M < 5$ for

i) $4f = .010$

ii) $4f = .015$ ✓

iii) $4f = .02$

We go back to equation (1) and for constant M set $dM^2 = 0$ thus

$$\frac{2\left(1 + \frac{k-1}{2}M^2\right)}{1-M^2}\frac{dA}{A} = \frac{kM^2\left(1+\frac{k-1}{2}M^2\right)}{1-M^2}\zeta\frac{dx}{D} \qquad (4)$$

if $1 + \frac{k-1}{2}M^2 \neq 0$ and $M \neq 1$ eq. (4) reduces to

$$2\frac{dA}{A} = kM^2\zeta\frac{dx}{D} \quad \text{or} \quad 4\frac{dr}{r} = kM^2\zeta\frac{dr}{2r\tan\theta}$$

or
$$\boxed{\tan\theta = \frac{kM^2}{8}\zeta} \qquad M \neq 1 \ ✓ \qquad (5)$$

Now I will remove the restriction on M = 1 (since it is a removable singularity) We will note that we can rewrite (4) so that we get it in the form

$$\frac{dA}{A} = \frac{kM^2}{2}\frac{(1-M^2)}{(1-M^2)}\zeta\frac{dx}{D} \quad \text{which} \rightarrow \frac{0}{0} \text{ when } M \to 1$$

Using L'Hôpital's Rule $\dfrac{dA}{A} = \left(kM^2 - \dfrac{k}{2}\right)\zeta\dfrac{dx}{D} = \left(\dfrac{k}{2}\zeta\dfrac{dx}{D}\right)$ which is determinate and hence the fn. given by eq. 5 is continuous at M=1.

Because k, M are constant we get that

$$A_2/A_1 = \left(T_{o_2}/T_{o_1}\right)^{\frac{1+kM^2}{2}} \qquad (3)$$

and $Q = c_p(T_{o_2} - T_{o_1})$ hence $T_{o_2}/T_{o_1} = \frac{Q}{c_p T_{o_1}} + 1$. Thus (3) becomes

$$\boxed{\frac{A_2}{A_1} = \left(\frac{Q}{c_p T_{o_1}} + 1\right)^{\frac{1+kM^2}{2}}} \quad ✓ \qquad (4)$$

We are also given that the fluid is steam: $\bar{M} = 18.016$, $k = 1.3$ for $T = 1000°R$ with $c_p = 8.554 \frac{Btu}{lb\text{-}mole\,°F} = .4748 \frac{Btu}{lbm\text{-}°F}$ and $Q = 810$ Btu/lbm

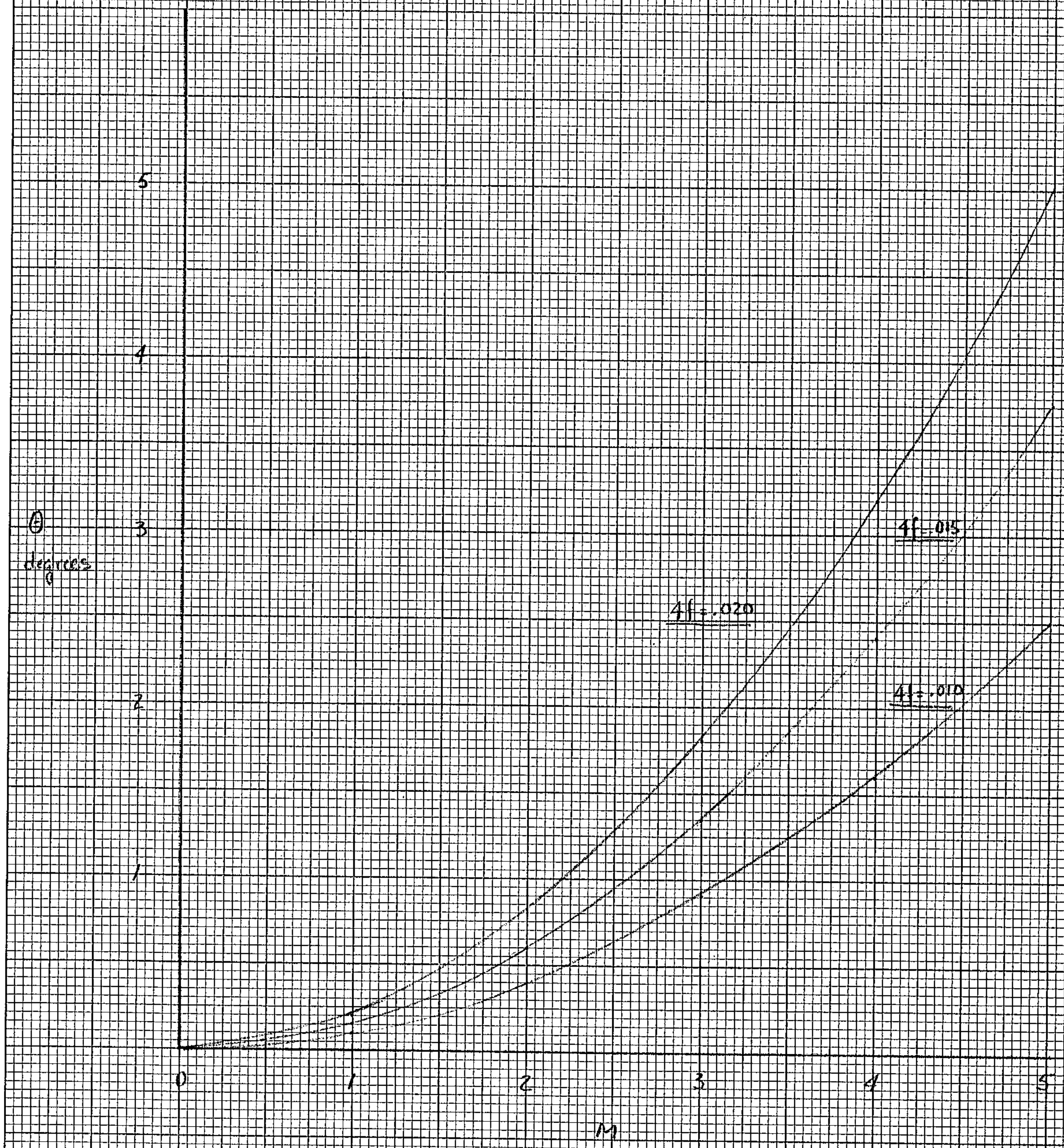
$$\frac{A_2}{A_1} = \left(\frac{810}{.4748\,(1000)} + 1\right)^{\frac{1+1.3\,(.25)}{2}} = (\;2.706\;)^{.6625} = 1.934 \; ✓$$

Also $R = \frac{(k-1)}{k} c_p J = 85.26 \frac{ft\text{-}lb_f}{lbm\text{-}°R} = 2743.23 \frac{ft\text{-}lb}{slug\ °R}$. But we also know that $Q = c_p(T_{o_2} - T_{o_1})$ or $T_{o_2} = \frac{Q}{c_p} + T_{o_1} = \frac{810}{.4748} + 1000 = \underline{2706\,°R}$ ✓

We also know that $\frac{T_{o1}}{T_1} = 1 + \frac{k-1}{2}M^2$ and $T_1 = 763.86°R$ since W, cp are const. ✓

Also $c_1 = \sqrt{kRT_1} = 1854$ ft/sec and $V_1 = 927$ ft/sec. We can also get T_2 for $\frac{T_{o2}}{T_2} = 1 + \frac{k-1}{2}M^2$; hence $T_2 = 2608.2\,°R$ and $c_2 = \sqrt{kRT_2} = 3050$ ft/sec

hence $V_2 = 1524.9$ ft/sec ✓


θ
degrees
5
4
3
2
1
0
1
2
3
4
5
M
4f = .020
4f = .015
4f = .010

Assumption: perfect gas c_p & c_v are const & $p = \rho RT$

Isentropic $s = \text{const}$, $T_o = \text{const}$, $p_o = \text{const}$ Governing : $\rho VA = \text{const}$, $h_o = h + \frac{V^2}{2}$, $dp = -\rho V dV$

Fanno flow - flow w/ Friction $T_o = \text{const}$: $\rho VA = \text{const}$, $h_o = h + \frac{V^2}{2}$, $-Adp - \tau_w dA_w = \rho VA dV$

Rayleigh flow - flow w/Heat or w/cooling $\overset{p_o = \text{const}}{}$: $\rho VA = \text{const}$, $dQ = c_p dT + VdV$, $dp = -\rho V dV$

Adiabatic : $T_o = \text{constant}$ $h_o = h + \frac{V^2}{2}$

Isothermal : $dQ = VdV = c_p dT_o \Rightarrow T_{o_2} - T_{o_1} = \frac{1}{c_p} \frac{V_2^2 - V_1^2}{2}$

Isentropic: means adiabatic frictionless, adiab reversible, crit changes at $M^2 = 1$

$$p/p_o = (\rho/\rho_o)^k \qquad \frac{T}{T_o} = (p/p_o)^{\frac{k-1}{k}} = (\rho/\rho_o)^k$$

$T/T_o = 1 + \frac{k-1}{2} M^2$ is true for any adiabatic flow isentropic; fanno,

* for non isentropic flow $p < p_{isent}$ for same initial condition $p_{isent}/p_o = \left(\frac{T_{isent}}{T_o}\right)^{\frac{k}{k-1}}$ $p_{\substack{non\\isent}} = \left(\frac{T_{\substack{non\\isent}}}{T_o}\right)^{\frac{k}{k-1}} e^{\frac{s-s_o}{c_v}}$

since for adiab flow $T_{isent} = T_{\substack{non\\isent}}$ $\therefore$ $\frac{p_i - p_{ni}}{p_o} = \left[1 - e^{-\left(\frac{s-s_o}{c_v}\right)}\right]\left(\frac{T_{isent}}{T_o}\right)^{\frac{k}{k-1}} > 0$

- Pg 83 $\frac{T_o}{T}$, p_o/p, $\frac{\rho_o}{\rho}$ vs. M. Pg 84. $\dot{m} = \dot{m}(A, p_o, T_o, M)$
- LOOK AT RUBINS notes for differential form of eqns.
- Pg 106 p/p^*, T/T^*, ρ/ρ^* vs. M & Pg 109 $V, M^2, \omega, A^*/A$ vs. p/p_o

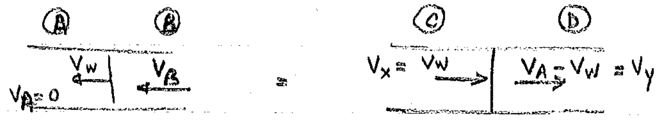
Normal Shock : adiabatic process assumed $\therefore$ $T_o = \text{const}$

$\rho V = \text{const}$, $p + \rho V^2 = \text{const}$, $h + \frac{V^2}{2} = \text{const}$. $\Delta S > 0$

diff eqns. pg 115 - 116

Eqns. pg 117-120 y vs. x eqs.

Moving Shock waves.



go from A → C → D → B note that all static quantities A = C B = D pg 138 - 139

$$\frac{P_{oy}}{P_x} = \frac{P_{oy}}{P_y} \cdot \frac{P_y}{P_x} = \left(\frac{k+1}{2} M_x^2\right)^{\frac{k}{k-1}} \Big/ \left(\frac{2k}{k+1} M_x^2 - \frac{k-1}{k+1}\right)^{\frac{1}{k-1}}$$

Fanno flow Crit Changes $M^2 = 1$

if a is pt of max entropy $\frac{s - s_a}{c_p} = \ln\left(\frac{2}{k-1}\right)^{\frac{k-1}{2k}} \left(\frac{T}{T_a}\right)^{\frac{1}{k}} \left(\frac{k+1}{2} - \frac{T}{T_a}\right)^{\frac{k-1}{2k}}$

diff eqs Pg 163-165

$p^*_{\text{for choking}} = \frac{w}{A}\sqrt{\frac{R}{K}}\sqrt{\frac{2T_0}{K+1}}$

Qualitative Changes P, M, V, T, ρ, p_0, F Pg 166

Eqs Pg 168

__Isothermal__ $dQ \neq 0$, $dT = 0$, $\tau_w \neq 0$ Remember __relate 4f to R_N__ using graph

diff eq Pg 178-179

Qualitative Change Pg 180 $p, \rho, V, M, T_0, p_0, Q$

Eqs Pg 180-182

__Rayleigh line__ $\tau_w = 0$ $dQ \neq 0$ Critical Changes at $M^2 = 1$ and $M = \frac{1}{\sqrt{K}}$

$$\frac{s - s_b}{c_p} = \ln \frac{T}{T_b}\left[\frac{2}{(k+1) - \sqrt{(k+1)^2 - 4k\,T/T_b}}\right]^{\frac{k-1}{k}}$$

where b is max entropy pt on Rayleigh line.

eqs pg 193-195 relating ① to ②

__Note in book__ * conditions are at section __1__ not downstream at __section 2__

eqs pg 196 relate * to any section

Qualitative Changes Pg 199 T_0, M, T, P, p_0, V

changes in

↓ Supersonic choking occurs in the C-D nozzle

__Generalized__ Flows

diff eqs Pg 221 - 227

Pg 228 influence coeffs Table 8.1 Pg 228 Generalized flow for semi perfect gas

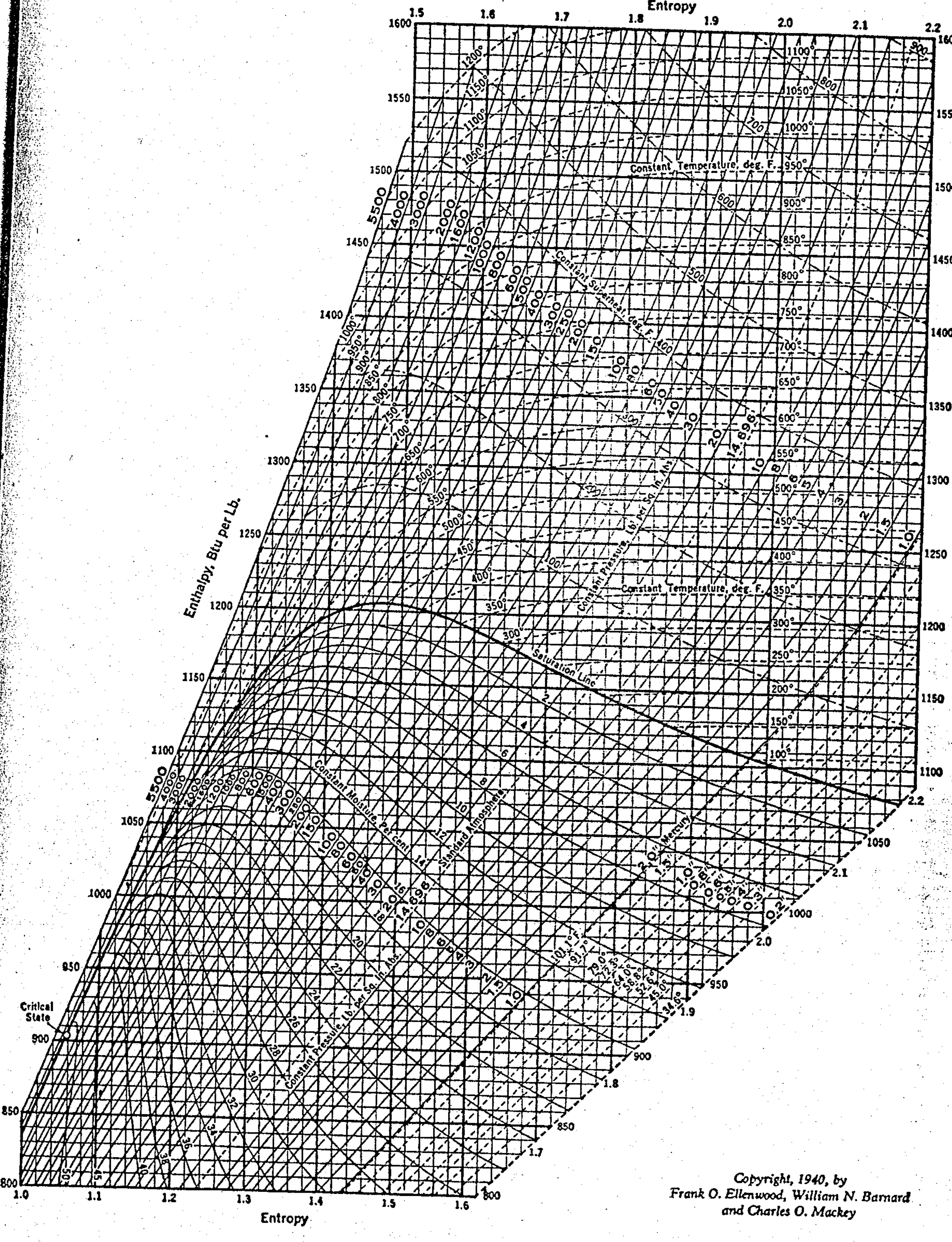
for a __perfect gas__ $dW, dc_p, dk = 0$ Table 8.2 pg 231

__Recap of all type flows__ Pg 239 __Table 8.3__

Qualitative pgs 255-258 discussion of __G(x)fn__

Mollier Chart for Steam ITEM B 16

Modified and greatly reduced from Keenan and Keye's *Thermodynamic Properties of Steam,* published (1936) by John Wiley and Sons, Inc. Reproduced by permission of the publishers.



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Frank O. Ellenwood, William N. Barnard
and Charles O. Mackey

properties, this equation could be useful in checking the compatibility data, perhaps of different origins.

a fourth variable v (standing for *any* property), we may first write the (v, y);

$$dx = \left(\frac{\partial x}{\partial v}\right)_y dv + \left(\frac{\partial x}{\partial y}\right)_v dy$$

his value of dx into (11-1). This gives

$$dz = \left(\frac{\partial z}{\partial x}\right)_y \left[\left(\frac{\partial x}{\partial v}\right)_y dv + \left(\frac{\partial x}{\partial y}\right)_v dy\right] + \left(\frac{\partial z}{\partial y}\right)_x dy$$

$$= \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial v}\right)_y dv + \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_v + \left(\frac{\partial z}{\partial y}\right)_x\right] dy.$$

differential of $z(v, y)$;

$$dz = \left(\frac{\partial z}{\partial v}\right)_y dv + \left(\frac{\partial z}{\partial y}\right)_v dy.$$

cients of dv in **(f)** and **(g)** must be equal, we find

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial v}\right)_y \left(\frac{\partial v}{\partial z}\right)_y = 1,$$

ship, useful in changing a variable, for any four properties; that is, the dependent variables. In these various equations, any two of the properties interchange positions, but the patterns shown must be maintained.

ns of obtaining a possibly useful relation is to let $dz = 0$ in equation (11-1)

s

$$\left(\frac{\partial y}{\partial x}\right)_z = -\frac{M}{N} \quad \text{or} \quad \left(\frac{\partial x}{\partial y}\right)_z = -\frac{N}{M}.$$

ELATIONS*

roperty relations, known as the *Maxwell relations*, are found by applying est to the functions for the properties internal energy u, enthalpy h, ction A, and Gibbs function G (§ 7.22). The results apply to a pure substance ical reaction) and the symbols will be as for unit mass, but the relations

---

Maxwell (1831–1879), another of the colossal minds of the nineteenth century, was y parents near Edinburgh. At the age of fifteen, he presented a paper to the Royal burgh on the calculation of the refractive index of a material. He graduated from Cam- the time he was twenty-nine, he was a professor of natural philosophy at Kings College, rote voluminously on scientific matters, his greatest contributions being in electro-

hold as well for 1 mole. From the familiar $T\,ds = du + p\,dv$, we have

$$du = T\,ds - p\,dv, \tag{3-14}$$

where $M = T$ and $N = -p$. Application of the exactness test, equation (11-2), gives

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v, \tag{a}$$

the Maxwell relation I, Table V. In equation (3-14), let $v = C$ ($dv = 0$), or use equation (11-2), and get

$$T = \left(\frac{\partial u}{\partial s}\right)_v, \tag{b}$$

an expression often considered to be the definition of thermodynamic temperature. Similarly, with $s = C$ or by equation (11-2),

$$\left(\frac{\partial u}{\partial v}\right)_s = -p. \tag{c}$$

Maxwell Relations and Others, Pure Substance TA

Function: $du = T\,ds - p\,dv$; Maxwell relation I: $\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$.

By (11-2): $\left(\frac{\partial u}{\partial v}\right)_s = -p = \left(\frac{\partial A}{\partial v}\right)_T$ and $\left(\frac{\partial u}{\partial s}\right)_v = T = \left(\frac{\partial h}{\partial s}\right)_p$.

By (11-6): $\left(\frac{\partial s}{\partial v}\right)_u = \frac{p}{T}$. Basic:—$du = \left(\frac{\partial u}{\partial s}\right)_v ds + \left(\frac{\partial u}{\partial v}\right)_s dv$

Function: $dh = T\,ds + v\,dp$; Maxwell relation II: $\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$.

By (11-2): $\left(\frac{\partial h}{\partial s}\right)_p = T = \left(\frac{\partial u}{\partial s}\right)_v$ and $\left(\frac{\partial h}{\partial p}\right)_s = v = \left(\frac{\partial G}{\partial p}\right)_T$.

By (11-6): $\left(\frac{\partial s}{\partial p}\right)_h = -\frac{v}{T}$. Basic:—$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp$

Function: $dA = -p\,dv - s\,dT$; Maxwell relation III: $\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$.

By (11-2): $\left(\frac{\partial A}{\partial T}\right)_v = -s = \left(\frac{\partial G}{\partial T}\right)_p$ and $\left(\frac{\partial A}{\partial v}\right)_T = -p = \left(\frac{\partial u}{\partial v}\right)_s$.

By (11-6): $\left(\frac{\partial v}{\partial T}\right)_A = -\frac{s}{p}$. Basic:—$dA = \left(\frac{\partial A}{\partial v}\right)_T dv + \left(\frac{\partial A}{\partial T}\right)_v dT$

Function: $dG = v\,dp - s\,dT$; Maxwell relation IV: $\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$.

By (11-2): $\left(\frac{\partial G}{\partial T}\right)_p = -s = \left(\frac{\partial A}{\partial T}\right)_v$ and $\left(\frac{\partial G}{\partial p}\right)_T = v = \left(\frac{\partial h}{\partial p}\right)_s$.

thermodynamic properties, this equation could be useful in checking the compatibility of experimental data, perhaps of different origins.

To introduce a fourth variable v (standing for *any* property), we may first write the differential of $x(v, y)$;

$$dx = \left(\frac{\partial x}{\partial v}\right)_y dv + \left(\frac{\partial x}{\partial y}\right)_v dy \tag{e}$$

and substitute this value of dx into (11-1). This gives

$$\begin{aligned} dz &= \left(\frac{\partial z}{\partial x}\right)_y \left[\left(\frac{\partial x}{\partial v}\right)_y dv + \left(\frac{\partial x}{\partial y}\right)_v dy\right] + \left(\frac{\partial z}{\partial y}\right)_x dy \\ &= \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial v}\right)_y dv + \left[\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_v + \left(\frac{\partial z}{\partial y}\right)_x\right] dy. \end{aligned} \tag{f}$$

Next write the differential of $z(v, y)$;

$$dz = \left(\frac{\partial z}{\partial v}\right)_y dv + \left(\frac{\partial z}{\partial y}\right)_v dy. \tag{g}$$

Since the coefficients of dv in **(f)** and **(g)** must be equal, we find

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial v}\right)_y \left(\frac{\partial v}{\partial z}\right)_y = 1, \tag{11-5}$$

a valid relationship, useful in changing a variable, for any four properties; that is, the system has 3 independent variables. In these various equations, any two of the properties x, y, z, or v may interchange positions, but the patterns shown must be maintained.

Another means of obtaining a possibly useful relation is to let $dz = 0$ in equation (11-1) or **(c)**; this gives

$$\left(\frac{\partial y}{\partial x}\right)_z = -\frac{M}{N} \quad \text{or} \quad \left(\frac{\partial x}{\partial y}\right)_z = -\frac{N}{M}. \tag{11-6}$$

MAXWELL RELATIONS*

Significant property relations, known as the *Maxwell relations*, are found by applying the exactness test to the functions for the properties internal energy u, enthalpy h, Helmholtz function A, and Gibbs function G (§ 7.22). The results apply to a pure substance (no chemical reaction) and the symbols will be as for unit mass, but the relations hold as well for 1 mole. From the familiar $T\,ds = du + p\,dv$, we have

$$du = T\,ds - p\,dv, \tag{3-14}$$

where $M = T$ and $N = -p$. Application of the exactness test, equation (11-2),

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v, \tag{a}$$

the Maxwell relation I, Table V. In equation (3-14), let $v = C$ $(dv = 0)$, or use e
(11-2), and get

$$T = \left(\frac{\partial u}{\partial s}\right)_v, \tag{b}$$

an expression often considered to be the definition of thermodynamic temp
Similarly, with $s = C$ or by equation (11-2),

$$\left(\frac{\partial u}{\partial v}\right)_s = -p. \tag{c}$$

Maxwell Relations and Others, Pure Su

Function:	$du = T\,ds - p\,dv;$		Maxwell relation I:	$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v.$
By (11-2):	$\left(\frac{\partial u}{\partial v}\right)_s = -p = \left(\frac{\partial A}{\partial v}\right)_T$	and		$\left(\frac{\partial u}{\partial s}\right)_v = T = \left(\frac{\partial h}{\partial s}\right)_p.$
By (11-6):	$\left(\frac{\partial s}{\partial v}\right)_u = \frac{p}{T}.$		Basic:—	$du = \left(\frac{\partial u}{\partial s}\right)_v ds +$
Function:	$dh = T\,ds + v\,dp;$		Maxwell relation II:	$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p.$
By (11-2):	$\left(\frac{\partial h}{\partial s}\right)_p = T = \left(\frac{\partial u}{\partial s}\right)_v$	and		$\left(\frac{\partial h}{\partial p}\right)_s = v = \left(\frac{\partial G}{\partial p}\right)_T.$
By (11-6):	$\left(\frac{\partial s}{\partial p}\right)_h = -\frac{v}{T}.$		Basic:—	$dh = \left(\frac{\partial h}{\partial s}\right)_p ds +$
Function:	$dA = -p\,dv - s\,dT;$		Maxwell relation III:	$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T.$
By (11-2):	$\left(\frac{\partial A}{\partial T}\right)_v = -s = \left(\frac{\partial G}{\partial T}\right)_p$	and		$\left(\frac{\partial A}{\partial v}\right)_T = -p = \left(\frac{\partial u}{\partial v}\right)_s$
By (11-6):	$\left(\frac{\partial v}{\partial T}\right)_A = -\frac{s}{p}.$		Basic:—	$dA = \left(\frac{\partial A}{\partial v}\right)_T dv +$
Function:	$dG = v\,dp - s\,dT;$		Maxwell relation IV:	$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T.$
By (11-2):	$\left(\frac{\partial G}{\partial T}\right)_p = -s = \left(\frac{\partial A}{\partial T}\right)_v$	and		$\left(\frac{\partial G}{\partial p}\right)_T = v = \left(\frac{\partial h}{\partial p}\right)_s.$
By (11-6):	$\left(\frac{\partial p}{\partial T}\right)_G = \frac{s}{v}.$		Basic:—	$dG = \left(\frac{\partial G}{\partial p}\right)_T dp +$

---

* James Clerk Maxwell (1831–1879), another of the colossal minds of the nineteenth century, was born of wealthy parents near Edinburgh. At the age of fifteen, he presented a paper to the Royal Society of Edinburgh on the calculation of the refractive index of a material. He graduated from Cambridge, and by the time he was twenty-nine, he was a professor of natural philosophy at Kings College, London. He wrote voluminously on scientific matters, his greatest contributions being in electromagnetic theory. In thermodynamics, he contributed the Maxwell relations, which are mathematical relationships essential to advanced study of properties. He aided significantly in promulgating kinetic theory and the new science of thermodynamics.

resented and see its relation to the fundamental laws. Although the beginner is prone to epend unduly on memory for details, it is easier and more profitable to recognize the epetitive utility of basic ideas. Concentrate on learning the definitions of words and laws r better understanding and communication. Thus, the summary of Table III is not ntended as a crutch on which to lean, but as an example of review activity.

deal Gas Formulas

or constant mass systems undergoing internally reversible processes.

Process →	*Isometric* $V = C$	*Isobaric* $p = C$	*Isothermal* $T = C$	*Isentropic* $S = C$	*Polytropic* $pV^n = C$
p, V, T relations	$\frac{T_2}{T_1} = \frac{p_2}{p_1}$	$\frac{T_2}{T_1} = \frac{V_2}{V_1}$	$p_1V_1 = p_2V_2$	$p_1V_1^k = p_2V_2^k$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$ $= \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$	$p_1V_1^n = p_2V_2^n$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ $= \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$
$\int_1^2 p\,dV$	0	$p(V_2 - V_1)$	$p_1V_1 \ln \frac{V_2}{V_1}$	$\frac{p_2V_2 - p_1V_1}{1 - k}$	$\frac{p_2V_2 - p_1V_1}{1 - n}$
$-\int_1^2 V dp$	$V(p_1 - p_2)$	0	$p_1V_1 \ln \frac{V_2}{V_1}$	$\frac{k(p_2V_2 - p_1V_1)}{1 - k}$	$\frac{n(p_2V_2 - p_1V_1)}{1 - n}$
$U_2 - U_1$	$w\int c_v\,dT$ $wc_v(T_2 - T_1)$	$w\int c_v\,dT$ $wc_v(T_2 - T_1)$	0	$w\int c_v\,dT$ $wc_v(T_2 - T_1)$	$w\int c_v\,dT$ $wc_v(T_2 - T_1)$
Q	$w\int c_v\,dT$ $wc_v(T_2 - T_1)$	$w\int c_p\,dT$ $wc_p(T_2 - T_1)$	$w\int T\,ds$ $p_1V_1 \ln \frac{V_2}{V_1}$	0	$w\int c_n\,dT$ $wc_n(T_2 - T_1)$
n	∞	0	1	k	$-\infty$ to $+\infty$
Specific heat, c	c_v	c_p	∞	0	$c_n = c_v\left(\frac{k - n}{1 - n}\right)$ $[k = C]$
$H_2 - H_1$	$w\int c_p\,dT$ $wc_p(T_2 - T_1)$	$w\int c_p\,dT$ $wc_p(T_2 - T_1)$	0	$w\int c_p\,dT$ $wc_p(T_2 - T_1)$	$w\int c_p\,dT$ $wc_p(T_2 - T_1)$
$S_2 - S_1$	$w\int \frac{c_v\,dT}{T}$ $wc_v \ln \frac{T_2}{T_1}$	$w\int \frac{c_p\,dT}{T}$ $wc_p \ln \frac{T_2}{T_1}$	$\frac{Q}{T}$ $wR \ln \frac{V_2}{V_1}$	0	$w\int \frac{c_n\,dT}{T}$ $wc_n \ln \frac{T_2}{T_1}$
			$w\int \frac{c_v\,dT}{T} + wR \ln \frac{v_2}{v_1}$;	$w\int \frac{c_p\,dT}{T} - wR \ln \frac{p_2}{p_1}$	

We might sneak in here another form of the second law, to wit, that in course of events entropy increases, things become more disorderly, and that measure of disorder. Having achieved an orderly state of logical understan principles of thermodynamics, one is in a position from which, without tryi proceed to a more disorderly state of greater entropy.

SJK
Autumn 78/79

ME 255

Exercises

September 29 - October 6

Text 1.1, 1.3, 1.4

Also for a perfect gas with constant specific heats derive the equation of state for $p = p(\rho, s)$. Start from first principles, and list all assumptions using this equation, examine under what condition the equation

$$p/\rho k = \text{constant}$$

can be correct.

Solutions will be posted Monday, October 9 on the board outside Room 501H.

$p = p(\rho, s)$
$= p(\frac{1}{v}, s)$

~~$dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho ds = \partial$~~

~~since $\rho = \frac{1}{v}$ then $d\rho = -\frac{1}{v^2} dv$~~ or

~~& $\frac{\partial}{\partial \rho} = \frac{\partial}{\partial v} \cdot \frac{\partial v}{\partial \rho} = \frac{\partial}{\partial v} \cdot \left(-\frac{1}{\rho^2}\right)$~~

Given C_p & C_v are const and first principle

P/ρ^k = const for ~~isentropic~~ process, C_p, C_v are const (perfect gas), homogenous chemical comp & invariable (pure substance), calorically perfect.

$dS = \frac{du + p\,dv}{T}$ satisfies $pv = RT$ $\left(\frac{\partial u}{\partial v}\right)_T = 0$

~~$h = u + P/\rho$~~ $= \frac{du}{dT} \cdot \frac{dT}{T}$

~~$dh = du + \frac{dp}{\rho}$~~ $- \frac{R}{\rho^2} d\rho$ $c_v \frac{dT}{T} + \frac{P}{\rho^2 T} d\rho = C_v \frac{dT}{T} - R \frac{d\rho}{\rho}$

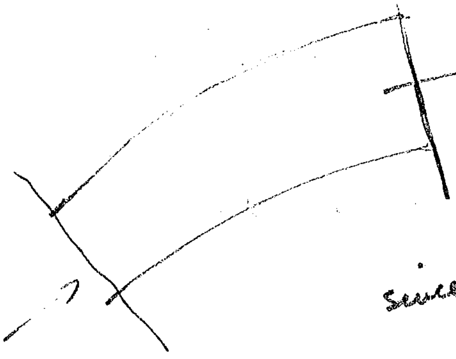
$\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$ $\therefore \rightarrow dS = C_v \left(\frac{dp}{p} - \frac{d\rho}{\rho}\right) - R \frac{d\rho}{\rho}$

$= c_v \frac{dp}{p} - (c_v + R) \frac{d\rho}{\rho} = c_v \frac{dp}{p} - c_p \frac{d\rho}{\rho}$

$$\frac{dp}{p} = \frac{1}{c_v} ds + \frac{c_p}{c_v}\frac{d\rho}{\rho}$$

$$\ln p = \ln e^{\frac{s}{c_v}} + \frac{c_p}{c_v}\ln\rho$$

$$p/p_0 = e^{\frac{s-s_0}{c_v}} \rho/\rho_0^{\ \gamma}$$



$(\rho+d\rho)(A+dA)(v+dv) = \dot m_{out}$

since steady no flow change within $\dot m_i = \dot m_{out}$

$\rho AV = \dot m_{in}$

$$\therefore \rho AV = \cancel{(\rho AV)} + \rho\left(1+\frac{d\rho}{\rho}\right)A\left(1+\frac{dA}{A}\right)\left(1+\frac{dV}{V}\right) \qquad \left(1+\frac{d\rho}{\rho}\right)\left(1+\frac{dA}{A}+\frac{dV}{V}+\frac{dAdV}{AV}\right)$$

$$= \rho AV + \cancel{\rho}\left(\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} \qquad 1+[\ ] + \frac{dAdV}{AV} + d.\right.$$

$$= \rho AV + \rho AV\left[\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V}\right] + O(d)^2 + O(d^3)$$

$$\lim_{dA,dV,d\rho\to 0} \text{ is } \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0 \quad \text{Cont.}$$

Look at forces acting of fluid using mom.

3a. $\rho_1V_1A_1 + \rho_2V_2A_2 = \rho_3V_3A_3$

since $V_1, V_2 < .2a \Rightarrow \rho_1, \rho_2, \rho_3 =$ ~~const~~

$V_1A_1 + V_2A_2 \doteq V_3A_3$

$(100)(.1) + 10(.9) = V_3 \cdot 1$

$10 + 9 = V_3 = 19$ ft/sec

$p_1A_1 + p_2A_2 - p_3A_3 = \rho V_3^2A_3 - \rho V_2^2A_2 - \rho V_1^2A_1$ } add pressure terms

$-\rho V^2 \underset{\sim}{i} + \rho V^2\cos\beta\, \underset{\sim}{i} \doteq -R_x \underset{\sim}{i}$

$\rho V^2 \sin\beta\, \underset{\sim}{j} = R_y \underset{\sim}{j}$

3b.

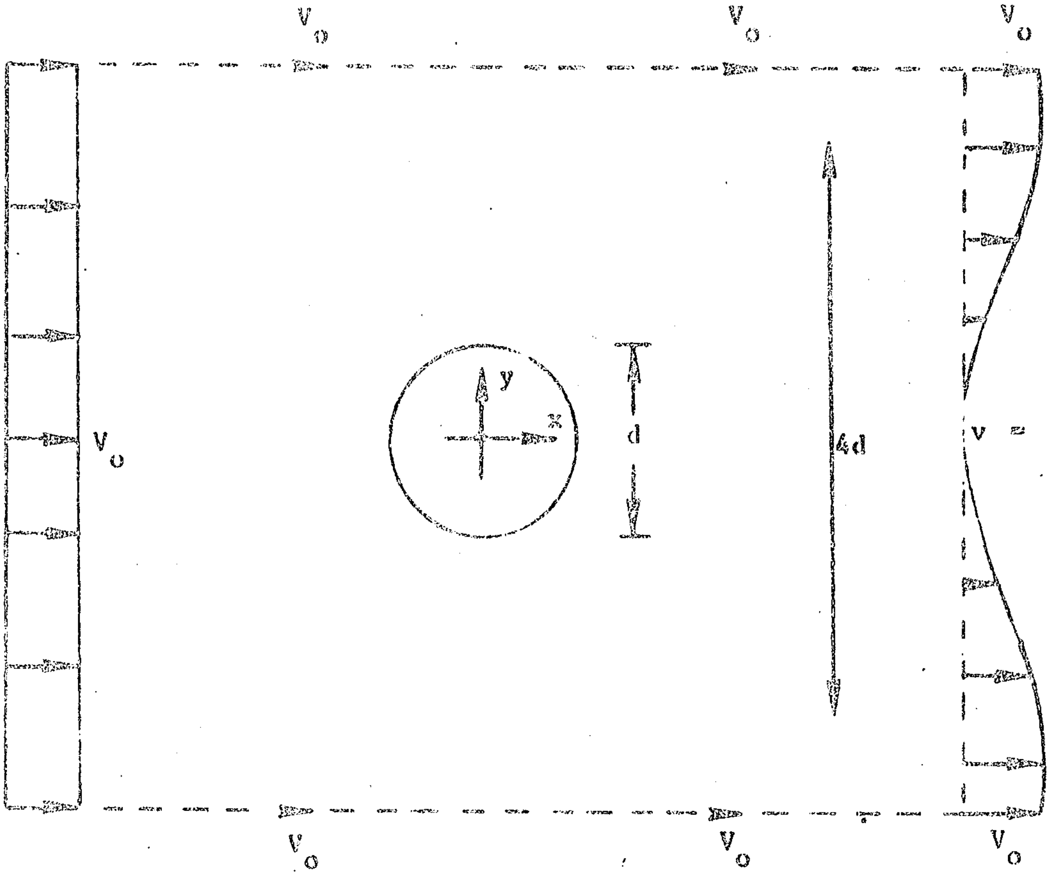
1.4

S. J. Kline
Problem Set No. 1 ME 255 Autumn 1978/79
DUE Friday OCT 13

1. In an experiment to determine drag, a circular cylinder of diameter d was immersed in a steady-two-dimensional, incompressible flow.

Measurements of velocity and pressure were made at the boundaries of the control surface shown. The pressure was found to be uniform over the entire control surface. The x-component of velocity at the control surface boundary was approximately as indicated by the sketch.

From the measured data, calculate the drag force per unit of length of the cylinder.



$$v = \begin{cases} V_o \text{ if } |y| > 2d, \\ \dfrac{V_o}{2}\left(1 - \cos \dfrac{\pi y}{2d}\right) \\ \text{for } |y| \leq 2d. \end{cases}$$

2. Water issues from a nozzle which is connected to a 90° elbow. A flexible hose connects the elbow to the rest of the piping. The elbow and nozzle are held in place by two forces, as shown. The flexible connection cannot take any restraining force.

The following data are given:

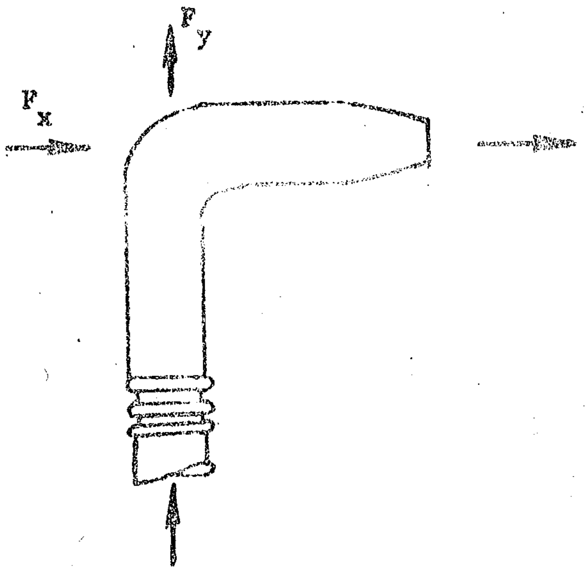
Jet area = A_j = 0.25 in^2. Flow rate = Q = 0.06 ft^3/sec.

Cross-sectional area of elbow = 1.00 in^2.

a) Find the pressure at the entrance of the elbow.

b) Find the forces F_x and F_y.

For parts (a) and (b) neglect all losses and assume steady-state conditions.



3. Steam is accelerated by a nozzle in an isentropic process. The inlet velocity is 50 ft/sec. The inlet state is 400°F, 25 psia. The area of the nozzle decreases linearly from 30 in^2 at the inlet to 10 in^2 at the exit. The nozzle length is 18 in. The back pressure is ~~14.7~~ 20.1 psia.

a) Using the steam tables, compute the mass flow rate and plot the velocity and density along the nozzle.

b) Compute the mass flow rate and plot the velocity and density along the nozzle, assuming the steam to be a perfect gas with $k = 1.33$ and $R = 85.58\ lb_f\ ft/lb_m\ °R$.

$$\frac{\partial}{\partial t}\int \rho V \, d\forall + \int_A V\,(\rho \underset{\sim}{V}_{rel} \cdot \underset{\sim}{n})\, dA = -\int p \underset{\sim}{n}\, ds +$$

since the flow is steady and the density incompressible and the flow is two dimensional we can rewrite the continuity equation as

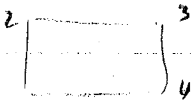
$$\nabla \cdot \underset{\sim}{V} = 0 \quad \text{or in integral form} \quad \int \rho \underset{\sim}{V} \cdot \underset{\sim}{n}\, ds = 0 \implies (\rho V A)_{in} = (\rho V A)_{out}$$

yes but compute to find divergence of streamlines

$$\text{or } \rho V_0 (4d \cdot 1) = \rho \int v(y)\, 1 \cdot dy = \rho\left[\int_0^{2d} + \cancel{\int_d^{2d}} + \int_{-2d}^{0} + \cancel{\int_{-2d}^{-d}}\right]$$

$$\rho\left[\int_{-2d}^{2d} \frac{V_0}{2}\left(1 - \cos\frac{\pi y}{2d}\right) dy + \cancel{\int_d^{2d} V_0\, dy} + \cancel{\int_{-2d}^{-d} V_0\, dy}\right]$$

$$\frac{V_0}{2}\left(y - \frac{2d}{\pi}\sin\frac{\pi y}{2d}\right)\Bigg|_{-2d}^{2d} \qquad V_0(d) + V_0(d)$$



$$2d - \; - [-2d + 0] = 4 V_0 d$$ ✓

we can write the mom eq ⇒ since, pressure is uniform pressure terms cancel & F_x (drag) is

$$F_x = \int_{bound} \rho \underset{\sim}{V} (\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA$$ note that $\underset{\sim}{V} \cdot \underset{\sim}{n}$ at top & bottom $= 0$ since $\underset{\sim}{V} \perp \underset{\sim}{n}$

$$\therefore \int_{bound} = \int_{1-2} + \int_{3-4}$$

@ bound 1-2 $\quad \underset{\sim}{V} \cdot \underset{\sim}{n} = -V_0 \quad dA = 4d \cdot$ unit width $\quad$ now since CV is stationary $V_{rel} = V_{abs}$ $\quad \therefore$

$$\int_{1-2} \rho \underset{\sim}{V} (\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA = -\rho V_0^2 \cdot 4d$$

@ bound 3-4 $\quad \underset{\sim}{V} \cdot \underset{\sim}{n} = + \frac{V_0}{2}\left(1 - \cos\frac{\pi y}{2d}\right) \quad \therefore$

$$\int \rho \underset{\sim}{V} (\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA = \int_{-2d}^{2d} \rho \left(\frac{V_0}{2}\right)^2 \left(1 - \cos\frac{\pi y}{2d}\right)^2 dy \cdot 1 =$$

$$2\int_0^{2d} \rho \left(\frac{V_0}{2}\right)^2 \left[1 - \cos\frac{\pi y}{2d}\right]^2 dy$$ since $f(-y) = f(y)$ & interval is sym

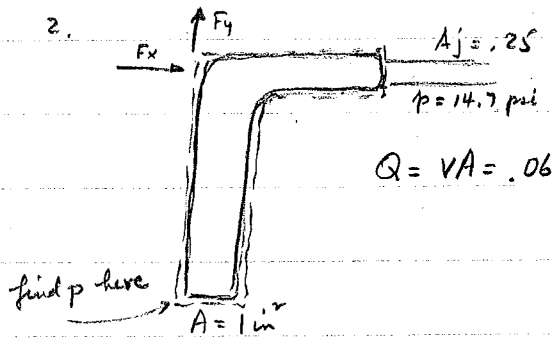
$$\text{or}\quad \rho\frac{V_0^2}{2}\int_0^{2d}\left(1-2\cos\frac{\pi y}{2d}+\cos^2\frac{\pi y}{2d}\right)dy = \rho\frac{V_0^2}{2}\int_0^{2d}\left(1-2\cos\frac{\pi y}{2d}+\frac{1+\cos \pi y/d}{2}\right)dy$$

$$\rho\frac{V_0^2}{2}\left[\int \frac{3}{2}-2\cos\frac{\pi y}{2d}+\frac{1}{2}\cos\frac{\pi y}{d}\right]dy = \rho\frac{V_0^2}{2}\left[\frac{3}{2}y-\frac{4d}{\pi}\sin\frac{\pi y}{2d}+\frac{d}{2\pi}\sin\frac{\pi y}{d}\right]_0^{2d}$$

$$\rho\frac{V_0^2}{2}\left[3d-\frac{4d}{\pi}\cdot 0+\frac{d}{2\pi}(0)\right] = \frac{3d}{2}\rho V_0^2$$

$$\therefore\quad F_x = +\rho V_0^2\left[\frac{3d}{2}-4d\right] = -\rho V_0^2\cdot\frac{5}{2}d$$ (– means it is retardant forces

2.



Assume steady state conditions

$Q = VA = .06\ ft^3/sec$

ρ is that of water

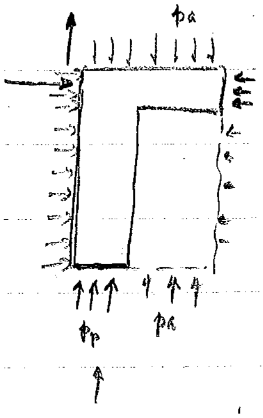
since steady state & water $\Rightarrow \nabla\cdot\underline{V} = 0$ or

$\int \rho\underline{V}\cdot\underline{n}\,dA = 0 \Rightarrow$ since ρ=const $\int \underline{V}\cdot\underline{n}\,dA = 0$ or

$(VA)_{in} = (VA)_{out}$ $\therefore$ since $Q = .06\ ft^3/sec$ & $A_{in} = 1\ in^2$ or $\frac{1}{144}\ ft^2$

$\Rightarrow \frac{1}{144}\cdot V = .06$ or $V_{in} = .06(144) = 8.64\ ft/sec$

$V_{out} = 4\times V_{in} = 34.56\ ft/sec$



neglect body forces and neglect the height difference of the inlet & outlet

$F_y + (p_p - p_a)\times A_{in} = -\rho V_i(Q)$ or $F_y = -\rho V_{in}Q + \frac{\rho}{2}A_{in}(V_i^2 - V_{out}^2)$

$F_x + (\cancel{-p_a A_{out} + p_a A_{in}}) = \rho V_{out}(Q)$

$p_a(A_{in} - A_{out})$

$F_x = \rho V_{out}Q + (A_{out} - A_{in})p_a$

$\int \rho\underline{q}(\underline{q}\cdot\underline{n})dA \qquad q = v\underset{\sim}{j}$

or using Bernoulli $p_i + \rho\frac{V_i^2}{2} = p_a + \rho\frac{V_0^2}{2}$ @ streamline

$p_a - p_p = \frac{\rho}{2}(V_i^2 - V_0^2)$

$= \frac{62.4}{32.2}\frac{1}{2}(8.64^2 - 34.56^2)$

$\therefore\ p_i = p_a + \frac{62.4}{64.4}(\overline{34.56}^2 - 8.64^2)$

$\downarrow$

$15\cdot 8.64^2$

using $p = 25$ and $T = 400$ $s = 1.8145$ $v = \frac{1}{\rho g} = 20.307$ $\rho = 1.529 \times 10^{-3}$

$p = 20.1$ $T = 358$ $s = 1.8145$ $v = \frac{1}{\rho g} = 24.063$ $\rho = 1.291 \times 10^{-3}$

$R g_c \times J = (h_o - h) g_c J$

now since $778.16 \frac{ft-lb}{Btu} = J$

$h = \frac{BTU}{lbm} \cdot 32.2 \frac{LBm}{Slug} = \frac{BTU}{slug} = \frac{BTU}{\frac{\# sec^2}{ft}} \times \frac{ft\,lb}{BTU} = \frac{ft^2}{sec^2}$

$$h_o = h + \frac{V^2}{2g} \Rightarrow h_o = h + \frac{V^2}{2gJ}$$

$\frac{ft^2 sec^2}{ft\,sec^2} = \frac{ft\,Btu}{ft\,lbm}$

$\frac{H_o}{mg} = \frac{H}{mg} + \frac{V^2}{2gJ}$ $ft \times \frac{Btu}{ft \cdot lb}$

h @ (p,T) given is 1238.5 Btu/# $\therefore$ $h_o = 1238.5 + \frac{(50)^2}{2(32.2)(778.16)} = 1238.5 + .05 = 1238.55$

$h_o = 1238.55$

$\sqrt{2gJ\left(\frac{H_o}{mg} - \frac{H}{mg}\right)} = V$

use $\sqrt{(h_o - h)\, 2 g_c J} = V$

$\dot{m} = \rho v A = (1.49 \times 10^{-3})(50)\left(\frac{30}{144}\right) = 1.593 \times 10^{-2}$ slugs/sec

P	T	V	S	A	ρ	h
25	400 °F	50 ft/sec	1.8145	30 in²	1.529×10^{-3}	1238.5
24	393 °F	421.6		3.75	1.45×10^{-3}	1235
22.5	383 °F	654.3		2.44	1.436×10^{-3}	1230
21.5	373 °F	823.70		2.01	1.385×10^{-3}	1225
20.5	363 °F	963.8		1.78	1.337×10^{-3}	1220
20.1	360 °F	1014.4		1.785	1.267×10^{-3}	1218

① using h compute V get p, T from Mollier Table also

② using T to get ρ

③ using ρ, V get A

$h_o - h = C_p T = \frac{V^2}{2}$

$h \frac{Btu}{lb_m} \cdot 32.2 \frac{lb_m}{slug} \frac{ft}{\# sec^2}$ $32.2\,h$

$\frac{\gamma}{\gamma - 1} RT$

$32.2\,h$ $\frac{ft^2}{sec^2}$ $\frac{Btu}{ft \cdot lb_m}$

$\frac{lb\,ft}{lbm\,°R}\,°R\,\frac{32.2\,lbm}{1\,slug} = R g_c = \frac{lb \cdot ft \cdot 32.2}{\frac{slugs}{\# sec^2}}$

$(h_o - h) J$

$h g_o$ $\frac{V^2}{J}$ $\frac{ft\,lb}{lb} \cdot \frac{Btu}{lb} \cdot \frac{ft-lb}{Btu}$

$\frac{V^2}{2\,J}$ $2\frac{V^2}{J}$ $\frac{ft^2}{sec^2} \cdot \frac{Btu}{ft\,lb} = \frac{Btu}{\frac{lb-sec^2}{ft}} = \frac{Btu}{slug}$

$\frac{H}{m} = h \qquad \frac{V^2}{2} \qquad \frac{2k}{k-1} R (T-T) \cdot g_c \frac{lbm}{slug} ft$

$\frac{\# ft}{lbm\ °R} \ °R \qquad \frac{\#}{sec^2}$

3b, $\dot{m} = \rho V A \qquad p = \rho R T \quad \therefore \ \rho = \frac{P}{RT} = \frac{25(144)}{(85.58)(32.2)(859)} = 1.521 \times 10^{-3}$

$\therefore \ \dot{m} = .0015208 \times 50 \times \frac{30}{144} = .015842$ slugs/sec $= \rho V A = \frac{p VA}{RT} = .015842$

now we know that $\rho V A =$ constant throughout

and for isentropic flow we have from the energy equation that

$$\frac{V^2}{2} + \frac{k}{k-1}\frac{P}{\rho} = \text{constant} \quad \text{and that} \quad (P_1/P_0) = (\rho_1/\rho_0)^k = \left(\frac{T_1}{T_0}\right)^{\frac{k}{k-1}}$$

since $\frac{V_1^2}{2} + C_p T_1 = \frac{V_2^2}{2} + C_p T_2$ knowing that $P_2/P_1 = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$

$\frac{ft^2}{sec^2} \qquad C_p = \frac{ft}{sec}$

if we pick $V_2 \Rightarrow T_2 \Rightarrow P_2 \Rightarrow \rho_2$ because $p_2 = \rho_2 R T_2 \Rightarrow A_2$

$\therefore \ T_2 = \frac{V_1^2 - V_2^2}{2}\left(\frac{\gamma-1}{\gamma R g}\right) + T_1$ and $P_2 = P_1 \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \quad \frac{4/3}{1/3} = 4$

$\frac{V_1^2}{2g} + \frac{\gamma}{\gamma-1} RT = V_1 \qquad \frac{ft^2}{sec^2}\ \frac{V_1^2}{2} + \frac{ft\ lb}{lb\ °R}\ °R = \frac{ft\ lb}{lbm}\ \frac{32.178\ lb}{lb_f}$

and $\rho_2 = \frac{P_2}{RT_2}$ and $\frac{\dot{m}}{\rho_2 V_2} = A_2$

$\frac{k-1}{k} = \frac{1/3}{4/3}$

	V_2	T_2	P_2	ρ_2	A_2
$\therefore$	50	400	25	1.521×10^{-3}	30 in^2
	440.6	391.3	24	1.474×10^{-3}	3.51 in^2
	626.4	382.3	23	1.43×10^{-3}	2.55
	773.5	372.95	22	1.38×10^{-3}	2.14
			21	1.333×10^{-3}	1.9
$\frac{\dot{m}}{\rho A} = V$	900.5	363.3	20.1	1.289×10^{-3}	1.76
	1003.5	354.4			

using $T_1\left(\frac{P_2}{P_1}\right) = T_2$

using $T_2 \Rightarrow V_2 = \sqrt{2\frac{kR}{k-1}g(T_1 - T_2) + V_1^2}$

using $P_2, T_2 \Rightarrow \rho_2 = \frac{P_2}{RT_2}$

using $\dot{m}, \rho_2, V_2 \Rightarrow A_2 = \frac{\dot{m}}{\rho_2 V_2}$

$\frac{T_1^{\frac{k}{k-1}}}{P_1} = \frac{T_2^{\frac{k}{k-1}}}{P_2}$

$\left(\frac{T_1}{T_2}\right)^{\frac{k}{k-1}} = \frac{P_1}{P_2}$

~~$\frac{P_1}{\rho_1^k} = \frac{P_2}{\rho_2^k}$~~ $\therefore$ ~~$\frac{P_1}{P_2} = \left(\frac{\rho_1}{\rho_2}\right)^k$~~

~~$\frac{P_1}{P_2} = \frac{\rho_1}{\rho_2} \cdot \frac{T_1}{T_2}$~~

$\therefore$ ~~$\frac{P_1}{P_2} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{k}} \frac{T_1}{T_2}$~~

~~$\left(\frac{P_1}{P_2}\right)^{\frac{k-1}{k}} = \frac{T_1}{T_2}$~~

$\frac{V_1^2}{2} + \frac{\gamma}{\gamma-1} RT \qquad V = \sqrt{2(h_0 - h)} \qquad$ lbf

$2 C_p (T_0 - T$

at $\quad C_p(T_0) = C_p T + \frac{V^2}{2} \qquad P = \rho = \frac{slugs}{ft^3} \cdot R \cdot °R$

$c_p \Delta T \cdot °R \qquad \therefore \ C_p(T_0 - T) = \frac{V^2}{2} \qquad$ so $\qquad \frac{\#}{ft^2} = \frac{\#\ ft}{slug\ °R} \cdot \frac{1\ slug}{32.2\ lbm} =$

$\frac{85.58\ \#\ ft}{32.2\ lbm\ °R}$

$\frac{\gamma}{\gamma-1} R \qquad \frac{V_1^2}{2} + C_p T_1 = C_p T_2 + \frac{V_2^2}{2}$

$\frac{lb\ ft}{\#/lb\ °R} \times \frac{32.174\ lbm}{1\ slug} \qquad \sqrt{V_1^2 + 2C_p(T_1 - T_2)} = V_2 \qquad 85.58\ \frac{\#\ ft}{lbm\ °R} \cdot \frac{32.2\ lbm}{1\ slug}$

$\frac{lb \cdot ft}{ft\ °R\ slug} = \frac{lb}{ft} \cdot R\ \frac{ft}{\#\ sec^2} \quad \sqrt{V_1^2 + \frac{2\gamma}{\gamma-1} R(T_1 - T_2)} = V_2 \qquad \therefore \sqrt{\frac{V_1^2}{2} + \frac{2\gamma}{\gamma-1} R_{give}\, g(T_1 - T_2)} = V_2^2 \qquad 85.58\ \frac{\#\ ft}{slug\ °R}$

$$\int \rho \underline{V}\cdot\underline{n}\, dA = \quad -\rho_0 V_0 \cdot 4d + \int \rho_0 \underline{V}\cdot\underline{n}\, dA = 0$$

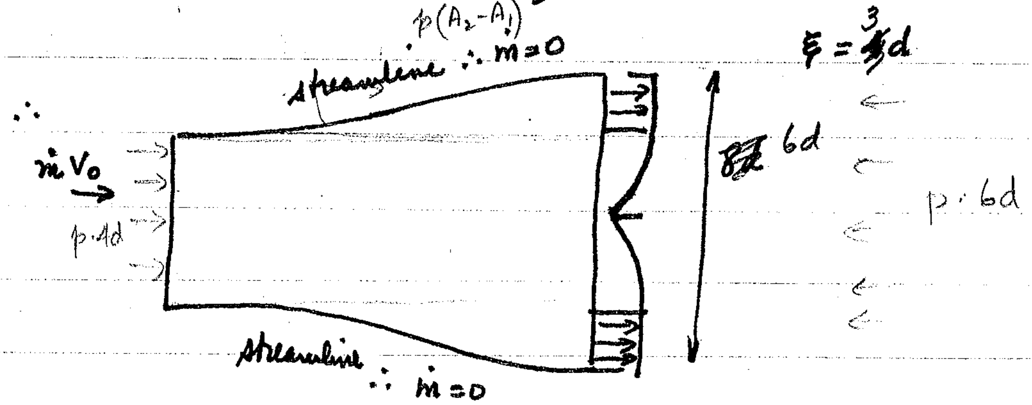
$$2\rho_0\left[\int_0^{2d} \frac{V_0}{2}\left(1-\cos\frac{\pi y}{2d}\right)dy + \int_{2d}^{\xi} V_0\, dy\right] = \rho_0 V_0 \cdot 4d$$

$$2\rho_0\left[\frac{V_0}{2}\left(y-\frac{2d}{\pi}\sin\frac{\pi y}{2d}\right)\Big|_0^{2d} + V_0\left[\xi-2d\right]\right]$$

$$2\rho_0\left[\frac{V_0}{2}\left\{2d\right\} + V_0\left[\xi-2d\right]\right] = \rho_0 V_0 4d$$

$$\cancel{2\rho_0 V_0 d} + 2\rho_0 V_0\left[\xi-2d\right] = \rho_0 V_0\cdot 2d$$

$$\xi = \overset{3}{\cancel{4}}d$$



$$-\dot m V_0 + \rho\frac{V_0^2}{4}\int_{-2d}^{2d}\left(1-\cos\frac{\pi y}{2d}\right)^2 dy + \int_{2d}^{3d}\rho\underline{V}(\underline{V}\cdot\underline{n})dA + \int_{-3d}^{-2d}\rho\underline{V}(\underline{V}\cdot\underline{n})dA$$

$$+ 2\rho\frac{V_0^2}{4}\int_0^{2d}\left(1-\cos\frac{\pi y}{2d}\right)^2 dy + 2\int_{2d}^{3d}\rho_0 V_0(V_0)\,dy$$

$$-\dot m V_0 + \frac{2\rho V_0^2}{\cancel{4}}\cdot 3d + 2\rho_0 V_0^2 \cancel{2}d = -\cancel{2}\rho_0 V_0^2 4d + \overset{2}{\cancel{4}}\rho_0 V_0^2 d + \frac{3}{2}\rho V_0^2 d = F_{\cancel{x}\,\text{fluid}}$$

on fluid

$$\left(-\overset{8/2}{\cancel{4}} + \frac{\overset{+\ 1/2}{\cancel{4}}}{2} + \frac{3}{2}\right)\rho_0 V_0^2 d = -\frac{1}{2}$$

∴ on cylinder same magnitude oppositive direction $-\frac{\cancel{3}}{2}\rho V_0^2 d = F_{\text{cylinder}}$

This should be answer ⟶

Cesar Levy
ME255
Prof Kline
10/13/78

87

1. Measurements of velocity & pressure were made to determine the drag on a circular cylinder of diameter d, immersed in a steady 2-dimensional, incompressible flow, at the boundaries of the control surface shown. The pressure was found to be uniform over the control surface and the x-component of velocity at the control surface boundary is as indicated. Using the data calculate the drag force/unit area.

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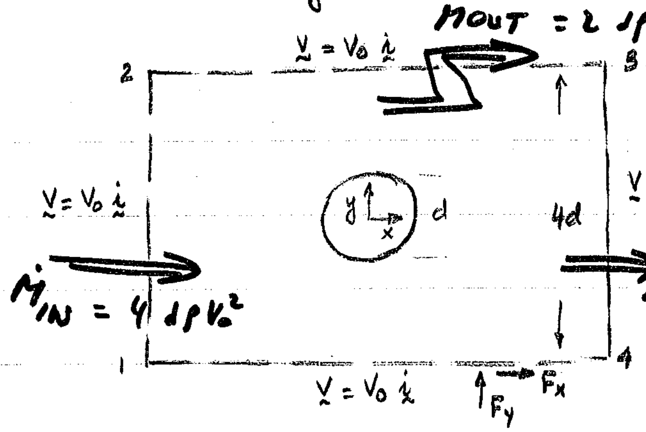


$\dot{M} = \dot{m}_{OUT} V_{abs}$

$\dot{m}_{IN} \Rightarrow -\dot{m}_{OUT}$

$\underset{\sim}{V} = \frac{V_0}{2}\left(1-\cos\frac{\pi y}{2d}\right)\underset{\sim}{i}$ for $|y| \le 2d$

$= V_0 \underset{\sim}{i}$ for $|y| > 2d$

$\dot{M}_{OUT} = 1.5\ d\rho V_0^2$

$\Sigma \dot{M} = -\frac{1}{2}\rho d V_0^2 = F_D$

The P.L.M states that $\frac{\partial}{\partial t}\int_{\vartheta} \rho \underset{\sim}{V}\, d\vartheta + \int_{cs} \rho \underset{\sim}{V}(\underset{\sim}{V}\cdot\underset{\sim}{n})\,dA = \underset{\sim}{F}$ (1) where $\underset{\sim}{F}$ are all the external forces acting on the volume. Since flow is steady ($\frac{\partial}{\partial t}=0$), incompressible ($\rho$ = const), and the pressure was found to be uniform (the contributions cancel each other out) and since we are going to neglect body forces then the PLM equation (1) reduces to $F_x = \int_{cs} \rho \underset{\sim}{V}(\underset{\sim}{V}\cdot\underset{\sim}{n})\,dA$ where F_x is the drag force. (2)

@ boundary 1-2 $\underset{\sim}{V}\cdot\underset{\sim}{n} = V_0 \underset{\sim}{i}\cdot(-\underset{\sim}{i}) = \overset{?}{\boxed{-}} V_0$; @ boundaries 2-3 & 1-4 $\underset{\sim}{V}\cdot\underset{\sim}{n} = V_0\underset{\sim}{i}\cdot(\pm \underset{\sim}{j}) = 0$

@ boundary 3-4 $\underset{\sim}{V}\cdot\underset{\sim}{n} = \frac{V_0}{2}\left(1-\cos\frac{\pi y}{2d}\right)\underset{\sim}{i}\cdot(\underset{\sim}{i}) = \frac{V_0}{2}\left(1-\cos\frac{\pi y}{2d}\right)$ (3)

Velocity is a vektor and we ~~are interested~~ in its x-component and it happens that in this problem all velocities are in the "plus-x-direction".

Note that since control surface is stationary $\underset{\sim}{V}_{rel} = \underset{\sim}{V}_{absolute}$ thus placing (3) into (2)

$$\frac{F_x}{\text{unit length}} = \int_{1-2} \rho \underset{\sim}{V}(\underset{\sim}{V}\cdot\underset{\sim}{n})\,dA + \int_{3-4} \rho \underset{\sim}{V}(\underset{\sim}{V}\cdot\underset{\sim}{n})\,dA = -\rho V_0^2 \cdot 4d + \rho \frac{V_0^2}{4}\int_{-2d}^{2d}\left(1-\cos\frac{\pi y}{2d}\right)^2 dy \quad (4)$$

See solution posted!

since $(1-\cos\frac{\pi y}{2d})$ is symmetric $\int_{-2d}^{2d}(\ \ )\,dy = 2\int_{0}^{2d}(\ \ )\,dy$ and thus we need to evaluate

$$\int_0^{2d}\left(1-\cos\frac{\pi y}{2d}\right)^2 dy = \int_0^{2d}\left(1 - 2\cos\frac{\pi y}{2d} + \frac{1+\cos\frac{\pi y}{d}}{2}\right)dy = 3d$$

$$\therefore\ F_x = -\rho V_0^2\cdot 4d + \rho\frac{V_0^2}{4}\cdot 2\cdot 3d = \rho V_0^2 d\left[\frac{3}{2}-4\right] = -\frac{5}{2}\rho V_0^2 d \quad X$$

minus sign indicates a retardant force on the fluid. Force on the cylinder is same in magnitude but opposite in direction.

2. Water issues from a nozzle which is connected to a 90° elbow. A flexible hose connects

100 the elbow to the rest of the piping. The elbow and nozzle are held in place by two forces as shown, the flexible connection cannot take any restraining force.

Given that: jet area $A_j = .25\,in^2$

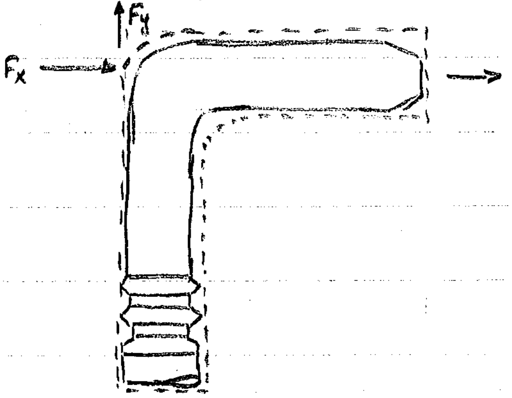
cross sectional area of the elbow $= 1.00\,in^2$

and the flow rate $Q = .06\ ft^3/sec$

a) Find the pressure at the entrance of the elbow

b) Find the forces F_x and F_y.

For parts (a) and (b) neglect all losses and assume steady state conditions



Take the control surface as shown. Since flow is steady-state and we are to neglect all losses, I will also make one more assumption; that the elevation head is small in comparison

to the other heads (pressure & velocity). Thus we can write the Bernoulli's equation along a streamline since the flow is inviscid, steady-state.

$$p_{in} + \rho \frac{V_{in}^2}{2} = p_o + \rho \frac{V_o^2}{2} \quad \text{where } (\;)_{in} = \text{inlet} \quad (\;)_o = \text{outlet} \qquad (1)$$

we can also write that from continuity $(\rho VA)_{in} = (\rho VA)_o$ and for water $\rho =$ constant for $M < .2$ and

$$(VA)_{in} = (VA)_o = Q \qquad (2)$$

Thus from (2) $Q = .06 = V_{in}(\frac{1}{144}\ ft^2)$ or $V_{in} = 8.64$ ft/sec and $V_o = 34.56$ ft/sec (3)

To solve for p_i we use (1) with V_{in} & V_o from (3) and the fact that $p_o = p_a = 2116$ psfa

$$\therefore \quad p_{in} = p_o + \frac{\rho}{2}(V_o^2 - V_{in}^2) = 2116 + \frac{62.4}{2(32.2)}(\overline{34.56}^2 - \overline{8.64}^2)$$

$$p_{in} = 2116 + \frac{62.4}{64.4}(15)(8.64)^2 = \underline{\underline{3200.96}} \text{ psfa or } 22.23 \text{ psia or } 7.53 \text{ psig} \checkmark$$

To find F_x and F_y we know that $\frac{\partial}{\partial t}\int_{\mho} \rho \underset{\sim}{V}\, d\mho + \int_{cs} \rho \underset{\sim}{V}(\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA = \underset{\sim}{F}$ where $\underset{\sim}{F}$ (4) is the vector sum of all external forces acting on the cv; therefore

$$\underset{\sim}{F} = F_x \underset{\sim}{i} + F_y \underset{\sim}{j} + \int_{cs} p\, dA = F_x \underset{\sim}{i} + F_y \underset{\sim}{j} + (p_{in} - p_a) A_{in} \underset{\sim}{j} \qquad (5)$$

since flow is steady, only term one needs to look at is 2nd term on left side of (4)

$$\therefore \quad \int_{cs} \rho \underset{\sim}{V}(\underset{\sim}{V} \cdot \underset{\sim}{n})\, dA = \rho V_{in} \underset{\sim}{j}(-V_{in}) A_{in} + \rho V_o \underset{\sim}{i}(V_o) A_o$$

$$\therefore \quad F_x = \rho V_o^2 A_o \quad \text{and} \quad F_y = A_{in}(p_a - p_{in}) - \rho V_{in}^2 A_{in} = \frac{A_{in}}{2}(\rho V_{in}^2 - \rho V_o^2) - \rho V_{in}^2 A_{in}$$

$$F_x = \rho Q V_o \qquad F_y = -\frac{1}{2}\rho\left[A_{in} V_{in}^2 + V_o^2 A_{in}\right] = \left[17 V_{in}^2 A_{in}\right] = -\frac{17}{2}\rho Q V_{in}$$

$$F_x = \left(\frac{62.4}{32.2}\right)(.06)(34.56) \qquad F_y = \frac{-17}{2}\left(\frac{62.4}{32.2}\right)(.06)(8.64)$$

$$F_x = \underline{\underline{4.02 \text{ lb}}} \ \checkmark \qquad F_y = \underline{\underline{-8.54 \text{ lb}}} \ \checkmark$$

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3. Using the mollier tables for $p = 25$ psia & $T = 400\,^\circ F$ $\quad s = 1.8145$

h at given $p, T = 1238.5$

							Questionable	
P	T	V	S	A	ρ	h	V	A
25	400	50	1.8145	30 in²	1.53×10^{-3}	1238.5	50	30
24	393		↓		1.45×10^{-3}	1235	73.8	20.74
22.5	383				1.436×10^{-3}	1230	115.4	13.5
21.5	373				1.385×10^{-3}	1225	145.2	11.1
20.5	363				1.337×10^{-3}	1220	169.9	10.2
20.1	360				1.267×10^{-3}	1218	178.8	10
psia	°F	ft/sec			slugs/ft³	Btu/lb		

$V = \sqrt{2(\text{factor})(h_0 - h)}$

$\dot{m} = \rho A V$; factor depends on units

$\rightarrow \checkmark\ \dot{m} = \rho V A = (1.53 \times 10^{-3})(50)(30/144) = .01593$ slugs/sec

b. Using isentropic equation $\checkmark\ T_1 \cdot \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = T_2$, $P_2/RT_2 = \rho_2$,

P	T	V	A	ρ
25	400	50	30 in	1.521×10^{-3}
24	391.3			1.474×10^{-3}
23	382.3			1.43×10^{-3}
22	373			1.38×10^{-3}
21	363.3			1.333×10^{-3}
20.1	354.4			1.289×10^{-3}

$\checkmark\ \dot{m} = \rho V A = (1.521 \times 10^{-3})(50)(30\text{ in}^2/144) = .015842$ slugs/sec

reason why V, A are not listed is that the units of h got to me and I was getting areas & velocities that made no sense. the results I listed as questionable are exactly that, as the units of $h_0 = h + \frac{V^2}{2}$ were unclear.

Autumn 1978/79
SJK

ME 255

Exercise Set 2 --- Due Wednesday, October 18

Text: 3.2, 3.6, 3.7

3.1 $\beta = \rho \frac{dp}{d\rho}$ $\beta/\rho = \frac{dp}{d\rho}$ $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$

if $p = f(\rho)$ then $\left(\frac{\partial p}{\partial \rho}\right)_s = \frac{dp}{d\rho} \Rightarrow \beta/\rho = c^2$ or $\sqrt{\beta/\rho} = c$

3.2 $c = \sqrt{\gamma R g T}$ for Air $\gamma = 1.4$ $R = 53.342$ $\frac{ft-lb}{lbm-°R}$ $c = \sqrt{(1.4)(32.174)(53.342)530}$

$T = 70°F = 530°R$ H_2 $\gamma = 1.4$ $R = 766.54$ $c = \sqrt{}$

UF_6 $\gamma =$ $R =$

mercury $\gamma = 1.666$ $R = 7.703$

water vapor $\gamma = 1.329$ $R = 85.77$

liquid water at 14.7 psia.

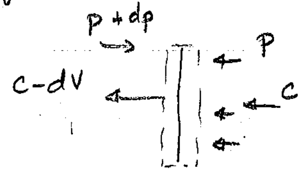
$c_{air} = 1128.47$

$c_{H_2} = 4277.81$

$c_{merc} = 467.80$

$c_{H_2O\,vapor} = 1394.19$

3.6 for a perfect gas $p = \rho R T$



$\therefore$ by mom $\Sigma F = \left[(p+dp)A - pA \right]$

$\int \rho \underline{v} (\underline{v} \cdot \underline{n}) dA = \left[-(c-dV)^2 A\rho + c^2 \rho A \right]$

$= \omega \left[c - (c-dV) \right]$

from mom. or $dp\,A = \omega\, dV = \rho A c\, dV$ or $dp = \rho c\, dV$

now $\frac{dp}{p} = \frac{\rho}{p} c\, dV$ $\frac{kp}{\rho} = c^2$ $\therefore \left(\frac{c^2}{k}\right)^{-1} = \left(\frac{p}{\rho}\right)^{-1}$

$\therefore \frac{dp}{p} = \frac{k c\, dV}{c^2} = \frac{k\, dV}{c}$ to write $\frac{kp}{\rho} = c^2$ we assumed $p/\rho^k = $ const and hence an isentropic flow but if we assume that flow is not isentropic but that for a perfect gas $p = f(s)\rho^k \Rightarrow c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = f(s)\, k \rho^{k-1} = \frac{k\rho^k f(s)}{\rho} = \frac{kp}{\rho}$

ie $p/p_0 = e^{\frac{s-s_0}{c_v}} \left(\rho/\rho_0\right)^k$

$\therefore$ even for a non-isentropic process <u>but</u> for a perfect gas and baratropic flow $c^2 = \frac{kp}{\rho}$

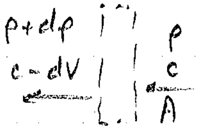
$$\therefore \quad \frac{dp}{p} = \frac{\rho}{p} c\, dV = \frac{k}{c^2} c\, dV = \frac{k\, dV}{c}$$

now since $p = \rho R T$ then $\frac{dp}{p} = \frac{d\rho}{\rho} + \frac{dT}{T}$

$$\downarrow$$

$$\frac{k}{c} dV = \frac{d\rho}{\rho} + \frac{dT}{T}$$

from continuity



$$\rho c A = (\rho + d\rho)(c - dV) A$$

$$\cancel{\rho c A} = \cancel{\rho c A} - \rho dV A + c d\rho A + O(\delta^2)$$

$$\rightarrow \quad \rho dV = c d\rho + O(\delta^2)$$

$$\lim_{dV, d\rho \to 0} \frac{dV}{c} = \frac{d\rho}{\rho}$$

by subs into $\therefore$ $\frac{k}{c} dV = \frac{dV}{c} + \frac{dT}{T}$ or $(k-1)\frac{dV}{c} = \frac{dT}{T}$

3.7 since continuity $\frac{dV}{c} = \frac{d\rho}{\rho}$ for a compression wave since $d\rho > 0 \Rightarrow dV > 0$

for a rarefaction wave $d\rho < 0 \Rightarrow dV < 0$ rightward

for 1-D the governing equation for small pulse is $\frac{\partial^2 \rho}{\partial t^2} = c^2 \frac{\partial^2 \rho}{\partial x^2}$

or $\rho' = f(x - ct) + g(x + ct)$ and $p' = c^2 \rho'$

and $\frac{\partial u'}{\partial t} = -\frac{c^2}{\rho_\infty} \frac{\partial \rho'}{\partial x}$ $\therefore$ $u' = \frac{c}{\rho_\infty}\left[f(x - ct) - g(x + ct) \right]$

if closed off must say that for a wave moving to right the fluid behind it is moving with speed $u' = \frac{c}{\rho_\infty} f(x - ct)$ at wall $u' = 0 \Rightarrow f = g$

$\therefore$ ρ' is greater and hence compression (ρ' in fluid = 2 × ρ' in fluid before)

see Rubin's notes on compression & linearized momentum

Question	Score
1	40
2	50
3	1
4	
5	
6	
7	
8	
9	
10	
Total	91

STANFORD UNIVERSITY

OFFICIAL EXAMINATION BOOK

(The Coördinate Book—8 pp.)

Name of Student CESAR LEVY

Date of Examination Oct 27 1978

Subject GAS DYNAMICS ME255

HONORABLE CONDUCT
in academic work is the spirit of conduct in this University.

In recognition of and in the spirit of the Honor Code, I certify that I will neither receive nor give unpermitted aid on this examination and that I will report, to the best of my ability, all Honor Code violations observed by me.

(signed) Cesar Levy
Name

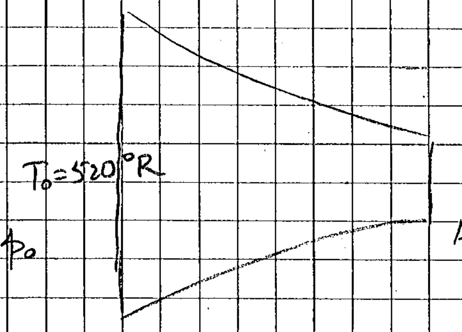
Suggestions For Conduct

1. Occupy alternate seats where possible.
2. When in doubt as to the meaning of a question, consult the instructor, who will be found in his or her office.

Gas = N_2

gas $\underline{\underline{k = 1.4}}$ $R = \frac{R}{m} = \frac{1545}{28.16}$

$= 55.15$



find $\dot{m}$ $\frac{P^*}{P_0} = .5283$ $p^* = 10.566$ psia ✓

if for $P_0 = 20$ psia $\nearrow$ $\therefore$ since $p^* < p_B$ ✓ $\Rightarrow$ M at exit ≠ 1 but M<1

$\therefore$ flow not choked $P_0 = 20$ psia $T_0 = 520°R$, $Pe = 14.7$ use $\dot{m} = \rho_e V_e A_e$

$\therefore$ $Pe = P_0 \left(\frac{Te}{T_0}\right)^{\frac{k}{k-1}}$ or $Te = T_0\left(\frac{Pe}{P_0}\right)^{\frac{k-1}{k}} = 520\left(\frac{14.7}{20}\right)^{\frac{.4}{1.4}}$

$Te = 476°R$ ✓

$\frac{Pe}{RTe} = \rho_e = \frac{14.7 \times 144}{55.15 \times 476 \times 32.17} = .0025$ slugs/sec

$Ve = \sqrt{\frac{2gkR}{k-1}(T_0 - Te)} = \sqrt{\frac{2(1.4)(55.15)}{.4}(44)(32.2)} = 738$ ft/sec ✓

$\dot{m} = \rho_e V_e A_e$
$= .0025 \times 738 \times \frac{10}{144} =$
$.128 \frac{slug}{sec}$ ✓

if $P_0 = 40$ psia $P^*/P_0 = .5283$ $\therefore$ $p^* = 21.132$ psia

since $p^* > p_B$ flow is choked and

$\dot{m} = \sqrt{\frac{k}{g_c R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\ \frac{P_0}{T_0}\ A^*$ ← need $\sqrt{T_0}$

$A^* = 10\ in^2$ $T_0 = 520°R$
$P_0 = 40$ psia $R = \frac{1545}{28.016} g_0$

$= \sqrt{\frac{1.4}{\frac{1545}{28.016} \times 32.2}\left(\frac{2}{2.4}\right)^{\frac{2.4}{.4}}}\ \frac{(40)}{520} .10$

$= \sqrt{\frac{1.4}{\frac{1545}{28.016}(32.2)} \times .335}\ \frac{40}{520} .10 = \sqrt{\frac{1.4}{55.15(32.2)} \times .335}\ \frac{40}{520} .10$

$\boxed{\dot{m} = .0125 \text{ slugs/sec}}$

X must be more than part (a)

?

Your missing a $\sqrt{T_0}$

40

$$p = A + \frac{B}{V} \exp\left(\frac{s-s_0}{c_v}\right)$$

now $c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = -V^2 \frac{\partial p}{\partial V}\Big|_s$

$$c^2 = -V^2 B \exp\left(\frac{s-s_0}{c_v}\right)\Big|_{s=c} - \frac{1}{V^2}$$

$$= + B \exp\left(\frac{s-s_0}{c_v}\right)\Big|_{s=c}$$

$c^2 = B$ ✓

$\therefore \quad c = \sqrt{B}$

since $V = \frac{1}{\rho}$ or $\rho = \frac{1}{V}$

$\partial\rho = -\frac{1}{V^2}\partial V$

or $\frac{\partial}{\partial\rho} = -V^2 \frac{\partial}{\partial V}$

$$\boxed{c = \sqrt{10^6 \text{ ft}^2/\text{sec}^2} = 10^3 \text{ ft/sec}}$$ ✓

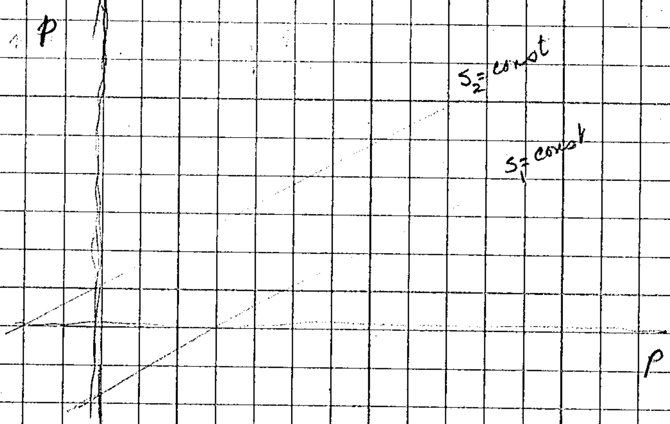
50

for Extra credit

since small compression & rarefraction waves are essentially isentropic ✓

$p = A + B\rho \times \text{const.}$

1



Autumn Quarter
S. J. Kline
Friday, Oct. 27, 1978

HOUR QUIZ -- ME 255A

Each problem is worth 50 points for parts (a) and (b). Question 2(c) is worth ten extra credit points.

1. A converging nozzle with an exit area of 10 in^2 discharges into a room at 14.7 psia. The nozzle is supplied with nitrogen flowing from a large reservoir held at 520°R. Compute the mass flow through the nozzle for reversible, adiabatic operation if:

   (a) tank P_o = 20 psia,

   (b) tank P_o = 40 psia.

2. A pure gas in a simple system has an equation of state

$$p = A + \frac{B}{v} \exp\left(\frac{S-S_o}{C_v}\right)$$

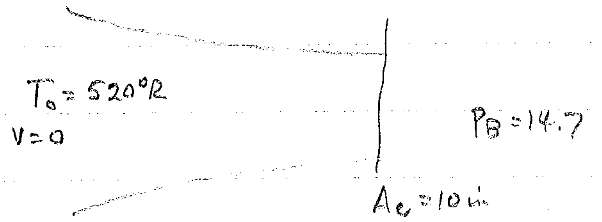
   (a) Derive the speed of sound (algebraically) in this gas if $S = S_o$.

   (b) Given A = 14.7 psia and $B = 10^6$ ft^2/sec^2, find the speed of sound numerically in ft/sec.

EXTRA

(c) Discuss the implications of the result 2(a) on wave forms arising from a series of small compression or rarefaction waves following each other.

1.



gas = N_2 $\qquad R = 55.15 \frac{\text{ft-lb}}{\text{lb}_m \text{ °R}} = 1774.3 \frac{\text{ft-lb}}{\text{slug °R}}$

$k = 1.4$

for $P_0 = 20$ psia $\qquad P^*/P_0 = .5283 \qquad P^* = 10.566$ psia

since $P^* < P_B$

$\Rightarrow$ flow is <u>not</u> choked $\therefore P_e = P_B$

and $\dot{m} = \rho_e V_e A_e = \dfrac{P_e}{RT_e} \cdot \sqrt{2c_p(T_0 - T_e)} \cdot A_e = \dfrac{P_e}{RT_e} \sqrt{\dfrac{2kR}{k-1}(T_0 - T_e)}\, A_e$

$T_e = T_0 \left(\dfrac{P_e}{P_0}\right)^{\frac{k-1}{k}} = 476.2$

$\rho_e = .00251$ slugs/ft³ $\qquad V_e = 737.6$ ft/sec

$\therefore \dot{m} = .128$ slugs/sec

for $P_0 = 40$ psia $\qquad P_B/P_0 < .5283 \quad \therefore \quad P_B < P^* \Rightarrow$ flow is choked & $P_e = P^*$ $A_e = A^*$

$P_e = 21.132 \qquad \dot{m} = \sqrt{\dfrac{k}{R}\left(\dfrac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \; \dfrac{P_0 A^*}{\sqrt{T_0}} = \sqrt{\dfrac{1.4}{1774.3}\left(\dfrac{1}{1.2}\right)^{6}} \; \dfrac{40 \times 10}{\sqrt{520}}$

$= \sqrt{\dfrac{1.4}{1774.3}(.335)} \; \dfrac{400}{22.804} = .01626 \times \dfrac{400}{22.8} = .285$ slugs/sec

2 a. $p = A + \dfrac{B}{V} \exp\left(\dfrac{s - s_0}{c_v}\right)$ $\qquad$ note $p = A + B\rho \exp\left(\dfrac{s - s_0}{c_v}\right)$

Specific volume $\quad V = \frac{1}{\rho}$

$\left(\dfrac{\partial p}{\partial \rho}\right)_s = B \exp\left(\dfrac{s - s_0}{c_v}\right)$; @ $s = s_0$ $\left(\dfrac{\partial p}{\partial \rho}\right)_s = B$

$c = \sqrt{\left(\dfrac{\partial p}{\partial \rho}\right)_s} = \sqrt{B}$

b. $c = \sqrt{10^6}$ ft/sec $= 1000$ ft/sec

c. note that for entropy always increasing

Autumn Quarter
S. J. Kline

ME 255

Problems on isentropic flow -- due Wednesday, October 25th.

Text: 4.6, 4.9, 4.12, 4.19, 4.25.

4.6

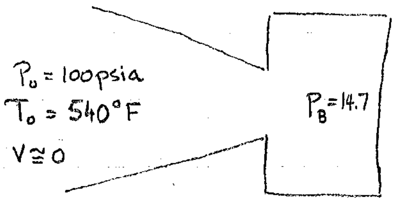
frictionless adiabatic flow

$k = 1.4$ $R = 100 \frac{ft\ lb}{lbm\ °R}$

Cesar Levy
ME255A
Prof Kline

Homework #2

Problem #4.6 100



Find: P_E, V_E, A_E

Given: frictionless adiabatic flow ≡ isentropic flow

$k = 1.4 \quad R = 100 \frac{ft-lb_f}{lb_m-°R}$

$\dot{m} = 1 \ lb_m/sec$

a. Since the flow depends on P^*/P_o, we know that $\left.\frac{P^*}{P_o}\right|_{K=1.4} = .528$ for conditions to become critical. This implies that $P^* = 52.8$ psia.

Note for $P_B \geq P^*$ $P_E = P_B$ and if $P_B < P^*$ $P_E = P^*$. For this problem $P_B < P^*$ ∴

$$\boxed{P_E = P^* = 52.8 \text{ psia}}$$ ✓

b. Since $P_E = P^*$, this implies conditions at exit are critical and $M_E = M^* = 1$. Hence $V^* = V_E = C_E = \sqrt{kRg_cT_E}$. However we know that $\frac{T^*}{T_o} = \frac{2}{K+1} = .833\bar{3}$. Thus $T^* \equiv T_E = .833\bar{3}\,(540+460) = 833.3°R$

and $V_E = \sqrt{(1.4)\left(100 \frac{ft-lb_f}{lb_m-°R}\right)\left(32.174 \frac{lb_m-f}{lb_f\,sec^2}\right)(833.\bar{3} \ °R)} = 1937.43$ ft/sec

$$\boxed{V_E = 1937.43 \text{ ft/sec}}$$ ✓

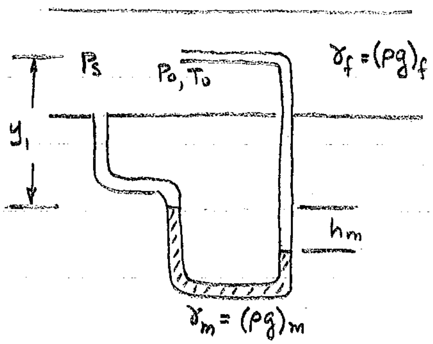
c. Since $P_B < P^*$, then the flow is choked then we can say that $A_E = A^*$ and

$$\frac{\omega}{A^*}\frac{\sqrt{T_o}}{P_o} = 0.532\left(\frac{R_{air}}{R_{fluid}}\right)^{1/2} \quad \underline{\text{if}} \quad k = 1.4 \text{ for the fluid}$$

$$\text{or } A_E = A^* = \frac{\omega\sqrt{T_o}}{P_o}\left(\frac{R_{fluid}}{R_{air}}\right)^{1/2}\frac{1}{.532} = \frac{\left(1\frac{lbm}{sec}\right)\left(\sqrt{1000°R}\right)}{(100\,psia)\left(144\frac{si}{ft^2}\right)}\left(\frac{100}{53.3}\right)^{1/2}\frac{1}{.532} = .00565 \ ft^2$$

$$\boxed{A_E = .00565 \ ft^2}$$ ✓

Problem #4.9 100



Find : V when the flow is incompressible & then when it is compressible

Given : $p_s = 5.2$ psig ; $h_m = 19.42''$ Hg

$p_\infty = 29.73$ inches Hg ; $T_0 = 80°F$

a) From the pitot-static tube set up and the incompressible case, we can use the hydrostatic equation to finally get

$$p_0 - p_s = P_{o_g} - p_{s_g} = h_m(\gamma_m - \gamma_f) \qquad \text{where } (\ )_g = \text{gage quantity} \qquad (1)$$

since we assume that the density of air is incompressible then we can solve for p_0

$$\checkmark \quad P_{o_g} = \frac{h_m \gamma_m + p_s}{\left[1 + \frac{h_m g}{R T_0}\right]} = \frac{(19.42''/12\tfrac{''}{ft})(847 \tfrac{lb}{ft^3}) + (5.2 \text{ psi} \times 144 \tfrac{si}{ft^2})}{\left[1 + \frac{(19.42/12)(32.2)}{(1715 \frac{ft-lb}{slug-°R})(540°R)}\right]} \qquad (2)$$

$$P_{o_g} = \frac{1370.50 + 748.8}{1 + .000056} = 2119.18 \text{ psfg}$$

Now since $P_a = 29.73''$ Hg this converts to a pressure $= 14.61$ psia or 2103.36 psfa

thus $P_0 = P_{o_g} + P_a = 4222.54$ psfa assuming that $\rho_{merc}\big|_{T=60°F} = \rho_{merc}\big|_{any\,temp}$;

and $\rho_0 = \dfrac{P_0}{RT_0} = \dfrac{4222.54}{(1715)(540)} = .00456$ slugs/ft^3 $= \rho_s$ (by incompressibility)

thus $V_s = \sqrt{\dfrac{2(P_{o_g} - P_{s_g})}{\rho}}$ using Bernoulli's equation along a streamline and (3)
assuming : no body forces ; height z above some reference frame at s & 0 to be the same ; no external forces acting on the fluid except for the normal pressure (frictionless) ; no area variations (ie 1-D approximation of flow)

$$\boxed{V_s = \sqrt{\frac{2(2119.2 - 748.8)}{.00456}} = 775.27 \text{ ft/sec}} \checkmark$$

if one looks at $T_s = \dfrac{P_s}{\rho_s R} = \dfrac{19.61 \times 144}{.00456 \times 1715} = 364.7°R$ and $c_s = 49.02\sqrt{T_s} = 936.14$ ft/sec

$M_s = \frac{V_s}{C_s} = \frac{775.27}{936.14} = .83$ hence this approximation is not very good since compressibility starts to take affect at $M \approx .3$.

b. We can again use the same principles as before to derive

$$p_o - p_s = p_{o_g} - p_{s_g} = (\gamma_m - \gamma_{f_o}) h_m + (\gamma_{f_s} - \gamma_{f_o}) y_1$$

since the density will nolonger be a constant. Here we will assume that since $\gamma_m >> \gamma_{f_o}$ & γ_{f_s} that we can approximate this by

$$p_{o_g} - p_{s_g} = (\gamma_m - \gamma_{f_o}) h_m$$ or as before

$$p_{o_g} = \frac{p_{s_g} + \gamma_m h_m}{\left[1 + \frac{h_m g}{RT_o}\right]} = 2119.18 \text{ psfg}$$ as before

p_a as before $= 2103.36$ psfa

and $P_o = P_a + P_{o_g} = 4222.54$ psfa

$P_s = p_a + p_{s_g} = 2852.16$ psfa

Now using the compressible bernoulli equation assuming frictionless adiabatic flow we get that

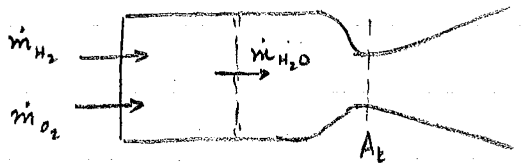
$$\checkmark \quad V_s = \sqrt{\frac{2k}{k-1}\frac{P_o}{\rho_o}\left(1 - \left[\frac{P_s}{P_o}\right]^{\frac{k-1}{k}}\right)}$$

or $$\boxed{V_s = \sqrt{\frac{2(1.4)}{.4}(1715)(540)\left(1 - \left[\frac{2852.16}{4222.54}\right]^{\frac{.4}{1.4}}\right)} = 829.14 \text{ ft/sec}} \checkmark$$

Let us look at the term $(\gamma_{f_s} - \gamma_{f_o}) y_1 = \gamma_{f_o}\left(\frac{\rho_s}{\rho_o} - 1\right) y_1 = .147(.894 - 1) y_1 = -.016\, y_1$

This will be very small wrt $P_o - P_s = (\gamma_m - \gamma_{f_o}) h_m = 1370.38$ psf unless $y_1 \geqq 8565$ ft in which case that term will be $\geqq .1(P_o - P_s)$. But normally y_1 is much less and this term can be neglected.

Problem 4-12 90



Find A_t

Given: $p_0 = 23$ atm $\quad T_0 = 4960\ °F$

$\dot{m}_{H_2} = 24$ lbm/sec $\quad \dot{m}_{O_2} = 76$ lbm/sec

$k = 1.25$

Assumption: friction and dissociation are neglected

In order to find A_t we will use (Eq 4-17) for choked flow since we have p_0, T_0, w, k.

In order to find $R(= \bar{R}/W)$ we need to know the molecular weight of the product

Since $2H_2 + O_2 \rightarrow 2H_2O$ with no dissociation $\quad W_{H_2O} = 18 \quad \therefore \quad R = \frac{\bar{R}}{W} = \frac{1545}{18} = 85.83 \frac{ft^2}{sec^2\ °R}$

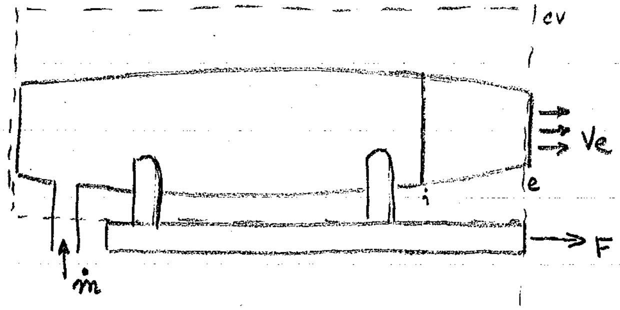
Rxn is not stoichiometric, ∴ R [illegible]

H2 in products

$$\therefore \checkmark\ A_t = \frac{w\sqrt{T_0}}{p_0 f(K,R)} = \frac{w}{p_0}\frac{\sqrt{T_0}}{\left[\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{1/2}} = \frac{100}{32.174}\ \frac{1}{(23)(2116.8)}\ \frac{\sqrt{5420\ °R}}{\left[\frac{1.25}{85.83}\left(\frac{2}{2.25}\right)^{9}\right]^{1/2}}$$

$$\boxed{A_t = .066\ ft^2 = 9.53\ in^2}$$

Problem # 4-19 100



Find pe

Given: Thrust = 1845 lb $\quad \dot{m} = 30$ lbm/sec

$T_i = 1400\ °F$, $V_i = 300$ ft/sec, $p_a = 14.7$ psi

no heat loss in gas; frictionless nozzle

working fluid is air.

by using the above control volume (and noting that the flow is steady) the momentum equation yields that

$$\checkmark\ F + p_a A_e - p_e A_e = \dot{m}(V_e) \quad \text{if the inlet hose is flexible}$$

where F is the force on the control volume exerted by the reaction stand.

we also note that to find pe we must find Ae and Ve first.

Method of solution:

1) Use of the conservation of mass throughout leads to $\dot{m}$ = constant

2) Since the flow is frictionless adiabatic in the nozzle we may use the isentropic equations. We will find Ve first.

3) Since the stream stabilizes at $p_a = 14.7$ psia outside the convergent nozzle, this implies that the flow must be choked (otherwise the flow would have to stabilize at the exit plane) and $V_e = C^*$, $A_e = A^*$.

To find C^*, find M^* at entrance to nozzle.

a) $c_i = 49.02\sqrt{T_i} = 49.02\sqrt{1860} = 2114.12$ ft/sec

$$M_i = \frac{V_i}{C_i} = \frac{300}{2114.12} = .142 \Rightarrow \text{from the table } M_i^* = .155 = \frac{V_i}{C^*}$$

$$\therefore \; C^* = V_e = \frac{V_i}{.155} = 1932.52 \text{ ft/sec} \Rightarrow T_e = (C^*/49.02)^2 = 1554.17\ ^\circ R$$

Now we note that $\dot{m} = \rho_e V_e A_e = \frac{P_e}{RT_e} V_e A_e \Rightarrow \frac{\dot{m} RT_e}{V_e} = p_e A_e$;

also $F - \dot{m}V_e = (1 - P_a/P_e) A_e p_e$ thus we can get the ratio P_a/P_e

$$P_a/P_e = 1 - \frac{F - \dot{m}V_e}{A_e P_e} = 1 - \left(\frac{F - \dot{m}V_e}{\dot{m}RT_e}\right) V_e = 1 - \left(\frac{1845 - \frac{30}{32.174} \cdot 1932.52}{\frac{30}{32.174} \cdot 1715 \cdot 1554.17}\right) 1932$$

$$P_a/P_e = .9665 \quad \therefore \quad \boxed{P_e = 15.21 \text{ psia}} \checkmark$$

Problem #4-25

Derive relationships for Δz vs. ΔM, Δc, ΔV, $\Delta \rho$, Δp, ΔT_o, Δp_o for a constant area duct for a perfect gas that flows adiabatically and without friction for changes of state that occur due to changes in elevation in the earth's gravity field. Start from basic principles.

Problem # 4-25 (cont)

(a) Look at a vertical constant area duct, wherein a perfect gas flows adiabatically and without friction. The governing equations are:

a. Energy: If we pick a fixed control volume, the flow through the CV can be considered steady state not doing shaft work (such as turning a turbine shaft etc), and since we are given frictionless flow there is no shear work term. Also $\dot{q} = 0$. Thus energy law reduces to

100 $h + \frac{V^2}{2} + gz$ = constant or in differential form $dh + VdV + gdz = 0$ (1)

b. Since flow is frictionless adiabatic then s = constant from 2nd law of Thermo (ie isentropic)

c. Continuity: For steady state flow, continuity reduces to $\rho dV + Vd\rho = 0$ (2)

d. Equation of State: $h = h(s, p)$ which gives from the gibb's equation ✓ the differential

$dh = dp/\rho$ (3)

e. Defining $M = \frac{V}{c}$ where $c^2 = \left.\frac{dp}{d\rho}\right|_s = \frac{kp}{\rho} = kRT$ (4)

There are other assumptions involving g that we will discuss later

Develop dz vs. dV

Using (1) and (3) $dh = dp/\rho = \frac{d\rho}{\rho}\frac{dp}{d\rho} = c^2 d\rho/\rho = -VdV - gdz$

Using (2) for $d\rho/\rho$ we get that

$$\boxed{\frac{dV}{V} = \frac{-g}{c^2}\left[\frac{1}{M^2-1}\right]dz}$$ ✓ (5)

Using from (2) that $d\rho/\rho = -dV/V$ we can get dz vs. $d\rho$

Hence $$\boxed{d\rho/\rho = \frac{g}{c^2}\left[\frac{1}{M^2-1}\right]dz}$$ ✓ (6)

Using (3) we can develop dp vs. dz

$$dh = \frac{dp}{\rho} = \frac{dp}{p}\frac{p}{\rho} = \frac{dp}{p}\frac{c^2}{K} = -VdV - gdz = -V^2\frac{dV}{V} - gdz$$

Using (5), rearranging terms and consolidating terms we get

$$\boxed{\frac{dp}{p} = \frac{kg}{c^2}\left[\frac{1}{M^2-1}\right]dz}$$ ✓ (7)

Using $c^2 = \frac{kp}{\rho}$ we differentiate to get $2cdc = k\left[\frac{\rho dp - pd\rho}{\rho^2}\right] = c^2\left[\frac{dp}{p} - \frac{d\rho}{\rho}\right]$

Substituting (6) and (7) and consolidating, we obtain

$$\boxed{\frac{dc}{c} = \frac{(k-1)g}{2c^2}\left[\frac{1}{M^2-1}\right]dz}$$ ✓ (8)   This gives dc vs. dz

Using (4) we will obtain dM vs. dz

Differentiating (4) $dM = \frac{cdV - Vdc}{c^2} = M\left[\frac{dV}{V} - \frac{dc}{c}\right]$

Now using (5) and (8) we obtain

$$\boxed{\frac{dM}{M} = -\frac{(k+1)}{2c^2}g\left[\frac{1}{M^2-1}\right]dz}$$ (9) ✓

We also know that at stagnation condition V=0 we can therefore say that in the limit of $dh = -VdV - gdz = C_p dT$ as we approach stagnation condition

$C_p dT_0 = -g dz_0$ and $\boxed{dT_0 = -\frac{g}{C_p} dz_0}$ (10) ✓

and again we get from (3) ✓

$dh_0 = \frac{dp_0}{\rho_0} = -g dz_0$ and $\boxed{dp_0 = -\rho_0 g dz_0}$ (11)

In obtaining these equations we have assumed that z << Radius of the earth so that g ≈ const, and that properties in the y, x directions are approximately uniform ✓ ie 1-D flow.

(b) Choked flow is possible ✓. Since dM>0 for M<1 and dM<0 for M>1, this implies that the flow tends towards M=1. So for a given set of inlet condition there is a given length for which M=1 at exit. Suppose we have a length of duct $L < L_{max}$ and let's look at conditions such that inlet M<1 (we can show the same results occur if M inlet >1). The flow will tend to M=1. Now if we and length to the duct the flow will accelerate until for $L = L_{max}$ $M_{exit} = 1$. If we try to add more duct the flow will try to go supersonic but yet it can't since eq (9) will tend to drive the M back down to 1 so the flow must somehow readjust but at the point where we add a length (an infinitesimal length) there will be a $\frac{dp}{dz} = -\infty$ which represents a discontinuity. Since a discontinuity cannot exist because the flow is subsonic, the flow must readjust through choking.

In a perfect gas shock wave is only possible from supersonic to subsonic.

SUBSONIC: For $dz>0$

$dV>0, d\rho<0, dp<0$

$dc<0, dM>0, dT_0<0, dp_0<0$

SUPERSONIC: For $dz>0$

$dV<0, d\rho>0, dp>0$

✓ $dc>0, dM<0, dT_0<0, dp_0<0$

(c) Aircraft systems has large dz in comparison to the other two. c in ventilating systems is normally $>$ c for aircraft at altitude but $<<$ c for fluid machinery

Thus with these observations and the above inequalities we note that

(i) change in dz make no difference for small M except in fluid machinery where $\frac{dp}{dz} \sim -\rho g$

(ii) Same as (i)

(iii) yes in all three systems changes in dz produce significant effects

(iv) changes in dz make no difference for $M \sim 2$ except in fluid machinery since $\frac{dp}{dz} \sim \frac{\rho g}{3}$

(v) changes in dz are too small to be felt in any of the three systems. ✓

very good!

$\underline{F} = \nabla\phi$ $\qquad |F| = \sqrt{(F_x)^2+(F_y)^2+(F_z)^2}^{\,2} = const$

$\underline{F} = F_x \underline{i} + F_y$

$(F\cdot i)\, i$

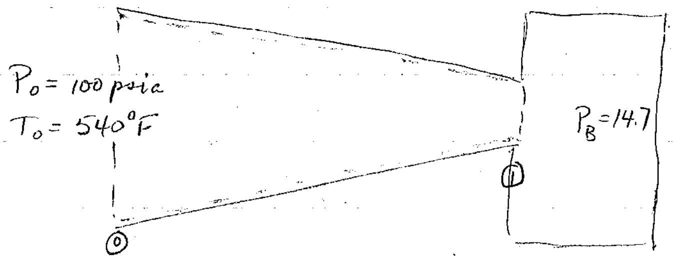
$|F| \cos(i,F)$

$\underline{F} =$

$$n = \frac{\underline{F}}{|\underline{F}|} = \frac{\nabla\phi}{|\nabla\phi|}$$

4.6, 4.9, 4.12, 4.19, 4.25

4.6



Since the flow is frictionless adiabatic and we can take the CV fixed in space then the governing energy equation is

$$\cancelto{0}{\frac{\partial}{\partial t}\int \rho e\, d\mho} + \oint_{cs} h_f\, \rho\, \underline{V}\cdot\underline{n}\, ds + \cancelto{0}{P_x} - \cancelto{0}{q} = 0$$

since the flow can be taken as steady $(\partial/\partial t)$ no external work is being performed, $(P_x = 0)$ it is frictionless and it is adiabatic $(q = 0)$. Then

$$\oint h_f\, \rho\, \underline{V}\cdot\underline{n}\, ds = 0$$

we now will make the 1-D approx meaning that the velocity profile is unifo at any crossection ⊥ to the flow. Hence the equation can be reduced to $h_{f_0} \dot{m}_0 = h_{f_1} \dot{m}_1$ since we can also assume from the momentum equation that ρ, p, u, T are <u>not</u> fns of y but only x. From continuity we obtain for steady flow $\dot{m}$ = constant

$\therefore\ h_{f_0} = h_{f_1}$ or for a perfect gas $h_{f_0} = c_p T_0$ and $h_{f_1} = c_p T_1 + \dfrac{V_1^2}{2}$

thus $c_p T_0 = c_p T_1 + \frac{V_1^2}{2}$ energy

for isentropic flow $\frac{T^*}{T_0} = \frac{2}{k+1} = .8333$ for $k=1.4$

the exit area for a converging nozzle can either be $\leq$ to A^*

let us assume that it is equal to $A^* \Rightarrow P^*/P_0 = \frac{P_e}{P_0} = .5283 = \frac{P_B}{P_0}$

a) $\boxed{\text{but } \frac{P_B}{P_0} < \frac{P^*}{P_0}} \Rightarrow \boxed{\text{the flow is choked ; the } M_E = 1 \text{ and } P_E = P^* = 52.8 \text{ psia}}$

$\boxed{\overset{\circ}{m} = 1 \text{ lbm/sec throughout}}$

now since $M_E = 1 = M^* \Rightarrow T_E = T^* = .8333\,(T_0) = .8333\,(540+460) = 833.3°R$

~~now from~~ $p = \rho RT$ (for perfect gas)

now since $C^* = \sqrt{kRT} = \sqrt{k g_c R T} = \sqrt{(1.4)\left(32.174 \frac{\text{lbm ft}}{\text{lbf sec}^2} \times 100 \frac{\text{ft lbf}}{\text{lbm}\,°\cancel{R}}\right)(833.3°\cancel{R})}$

$= \frac{\text{ft}}{\text{sec}}\sqrt{(1.4)(32.174)(100)(833.3)} = 1937.4$ ft/sec

b). $\therefore$ since $M_E = M^* = 1 \Rightarrow V^* = C^* = V_E = 1937.4$ ft/sec

now since $p = \rho RT \Rightarrow \rho_E = \frac{P_E}{g_c R T_E} = \frac{52.8 \times 144 \text{ psfa}}{100 \frac{\text{Ft lbf}}{\text{lbm}\,°\cancel{R}} \times 32.174 \frac{\text{lbm ft}}{\text{lbf sec}^2} \times 833.3°\cancel{R}} = 2.836 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}$

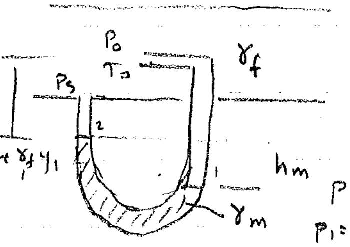
$\frac{m}{V} = \frac{\text{slugs}}{\text{ft}^3} = \frac{\#\,\text{sec}^2}{\text{ft}^4}$

$\frac{\#\,\text{sec}^2}{\text{ft}^4}$

$\overset{\circ}{m} = 1 \frac{\cancel{\text{lbm}}}{\text{sec}} \times \frac{1}{32.174 \frac{\cancel{\text{lbm}}\,\text{ft}}{\text{lbf sec}^2}} = \frac{1}{32.174} \frac{\text{lbf sec}^2/\text{ft}}{\text{sec}} = \frac{1}{32.174} \frac{\text{slugs}}{\text{sec}} = .0311$ slugs/sec

c. $\overset{\circ}{m} = \rho V A \Rightarrow A_E = \frac{\overset{\circ}{m}}{\rho V} = \frac{.0311 \text{ slugs/sec}}{2.836 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^{\cancel{3}2}} \cdot 1937.4 \frac{\text{ft}}{\cancel{\text{sec}}}} = \frac{.0311}{(2.836)(1.9374)} \text{ft}^2 = 5.66 \times 10^{-3}$ ft

4.9



③ $P_2 = p_s + \gamma_{f_1} y_1$

$p_1 = p_0 + \gamma_f (y_1 + h_m)$ ①

$p_1 = p_2 + \gamma_m h_m$ ②

using ①, ②, ③ $\Rightarrow p_0 - p_s = (\gamma_m - \gamma_f) h_m$

Given $p_a = 29.73$ in Hg $\quad p_s = 5.2$ psig

$h_m = 19.42''$ Hg

$\gamma_m = (\rho g)_{Hg} = 847$ lb/ft^3 @ 80°

$\gamma_f = (\rho g)_{air}$

now $P_{o_g} - P_{s_g} = h_m (\gamma_m - \gamma_f)$ for subsonic

$$P_{o_g}\left[1 + \frac{h_m g}{RT_o}\right] = P_{s_g} + h_m \gamma_m \quad \Rightarrow \quad P_{o_g} \approx 2119.18 \text{ psfg}$$

$P_a = 29.73''$Hg $= 14.61$ psia and 2103.36 psf.

$$\therefore \quad P_o = P_a + P_{o_g} = 4222.54 \text{ psfa}$$

$$\rho_o = \frac{P_o}{RT_o} = .00456 \text{ slugs/ft}^3$$

$\therefore$

$$V_s = \sqrt{\frac{2(P_o - P_s)}{\rho}} = \sqrt{\frac{2(P_{o_g} - P_{s_g})}{\rho_o}} = 775.27 \text{ ft/sec}$$

from the fact that $\rho_o = \rho_s$

$$T_s = \frac{P_s}{\rho_s R} = \frac{(5.2 + 14.61) \times 144}{.00456 \times 1715} = 364.7 \text{ °R}$$

$\Rightarrow \quad c_s = 49.02\sqrt{T_s} = 936.14 \quad M_s = .83$ approx is incorrect

~~$P_{o_g} - P_{s_g} = h_m (\gamma_m)$~~

$P_2 = p_s + \gamma_1 y_1$

$P_1 = p_o + \gamma_{f_2}(y_1 + h_m)$

$P_1 = p_2 + \gamma_m h_m$

$$\therefore \quad p_1 - p_2 = \gamma_m h_m = p_o + \gamma_{f_2}(y_1 + h_m) - (p_s + \gamma_1 y_1)$$

$$= p_o - p_s + (\gamma_{f_2} - \gamma_{f_1}) y_1 + \gamma_{f_2} h_m$$

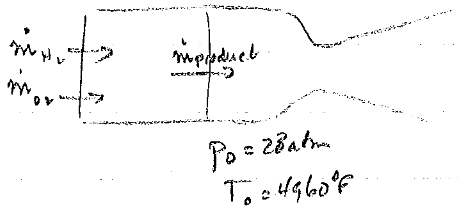
$$\text{or } (\gamma_m - \gamma_{f_2}) h_m + \underset{\text{neglect}\uparrow}{(\gamma_{f_1} - \gamma_{f_2}) y_1} = p_o - p_s$$

$\therefore$

p_0 is sames as before p_s is same's before

but
$$V_s = \left[\frac{2k}{k-1} \frac{P_0}{\rho_0} \left(1 - \left(\frac{P_s}{P_0} \right)^{\frac{k-1}{k}} \right) \right]^{1/2}$$
$$= \left[\frac{2k}{k-1} RT_0 \left(1 - \left(\frac{P_s}{P_0} \right)^{\frac{k-1}{k}} \right) \right]^{1/2} = 829.14 \text{ ft/sec}$$

4.12



$$\frac{w}{A_t\, p_0} = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\ \frac{1}{\sqrt{T_0}} = f(k)\frac{1}{\sqrt{T_0}}$$

we know p_0, w

$$\sqrt{\frac{1}{\frac{ft^2}{sec^2\ °R}}}\quad °R \qquad \frac{sec}{ft} = \frac{w}{lb}$$

$$\frac{x}{lb} = \frac{sec}{ft} \qquad x = \frac{lb\ sec^2}{ft}\ \frac{}{sec} = \frac{slug}{sec}$$

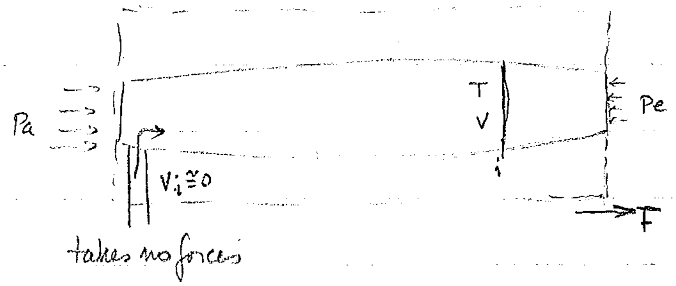
Since it is frictionless and we are neglecting dissociation we will assume that

$2H_2 + O_2 \rightarrow 2H_2O$ $\quad\therefore$ molecular weight of $H_2O \cong 18.$ $\quad\therefore\ R = \frac{\bar{R}}{W} = \frac{1545}{18} = 85.83$

$\therefore$ we will use the fact that the flow is choked to find A_t

$$\therefore\quad A_t = \frac{w}{p_0}\frac{\sqrt{T_0}}{f(k)} = \frac{w}{p_0}\frac{\sqrt{T_0}}{\left[\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\right]^{1/2}} = \frac{100}{32.174}\cdot\frac{1}{23(2116.8)}\ \frac{\sqrt{5420°R}}{\left[\frac{1.25}{85.83}\left(\frac{2}{2.25}\right)^{9}\right]^{1/2}}$$

$$= .066\ ft^2 = 9.53\ in^2$$



$$\Sigma F = F + p_A A_e - p_e A_e$$
$$\Sigma \dot{m} V = \dot{m} V_e$$

$$\therefore \quad \Sigma F = \Sigma \dot{m} V \Rightarrow F + (p_A - p_e) A_e = \dot{m} V_e$$

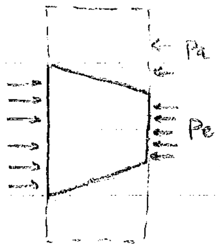
must find Pe :

Given: F, $\dot{m}$, p_a need to find Ae & Ve first

Method: given that we have a converging nozzle and that the stream stabilizes outside the nozzle $\Rightarrow p_e = p^*$ and $V_e = V^* = c^*$

since we know that $c = 49.02\sqrt{T}$ & since the mixture is assumed to be air we can form $M = \frac{V}{c}$ at the inlet. Going to table B2 we can get $M^* = \frac{V}{c^*} = \frac{V_{in}}{V_e}$ and thus Ve. Since $V_e = 49.02\sqrt{T_e}$ we can find $T_e = T^*$

$c_i = 2114.12$, $c_* = 1932.52$
$M_i = .142$ $M_i^* = .155$
$T^* = 1554.17°R$
$p_a/p_e = .9665$

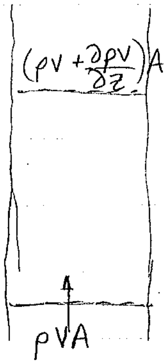


$$\cancel{-p_e A_e - p_a (A_{in} - A_e) + p_{in} A_{in} = \dot{m}(V_e - V_{in})}$$
$$\cancel{A_e(p_a - p_e) + A_{in}(p_{in} - p_a) = \dot{m}(V_e - V_{in})}$$

$$\dot{m} = \rho_e V_e A_e = \frac{p_E}{RT_e} V_e A_e \Rightarrow \frac{\dot{m} R T_e}{V_e} = p_e A_e$$

$$F - \dot{m} V_e = -\left(\frac{p_a}{p_e} - 1\right) A_e p_e \Rightarrow \frac{F - \dot{m} V_e}{p_e A_e} = -\frac{p_a}{p_e} + 1$$

$$\therefore \quad -\frac{F - \dot{m} V_e}{p_e A_e} + 1 = \frac{p_a}{p_e} \Rightarrow p_e$$



find dz vs. $\check{M}, \check{V}, \check{c}, \check{\rho}, \bar{p}, T_0, p_0$

Energy with no work, no heat transfer, steady flow, frictionless

$$h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2$$

Thermodynamics Second Law $s = s_0$ find frictionless, adiabatic

Continuity equation: steady flow gives $\rho V =$ constant

Equation of state $h = h(s_0, p)$, $\rho = \rho(s_0, p)$

define $M = \frac{V}{c}$ where $c = \left.\frac{\partial p}{\partial \rho}\right)_s = \frac{dp}{d\rho}$ or for a adiab flow $c^2 = \frac{kp}{\rho}$

Using the differential forms $dh + VdV + gdz = 0$ Energy

$ds = 0$ 2nd law

$\rho dV + V d\rho = 0$ continuity

$$dh = \left.\frac{\partial h}{\partial s}\right|_p ds + \left.\frac{\partial h}{\partial p}\right|_s dp \quad \text{or} \quad dh = \frac{1}{\rho} dp \text{ for a perfect gas}$$

From the equation of state & energy $Tds + \frac{1}{\rho} dp$

$$dh = \frac{dp}{\rho} = -VdV - gdz = \frac{d\rho}{\rho}\frac{dp}{d\rho} = c^2 \frac{d\rho}{\rho} \Rightarrow -gdz = \frac{c^2 d\rho}{\rho} + \frac{V^2 dV}{V}$$

using continuity for $\frac{d\rho}{\rho}$ $\quad -gdz = -c^2 \frac{dV}{V} + V^2 \frac{dV}{V} = \frac{dV}{V}\left[-1 + M^2\right] c^2$

$$\therefore \boxed{\frac{dV}{V} = \frac{-g}{c^2}\left[\frac{1}{M^2-1}\right] dz}$$

since $\frac{dV}{V} = -\frac{d\rho}{\rho}$ for continuity $\quad \boxed{\frac{d\rho}{\rho} = \frac{g}{c^2}\left[\frac{1}{M^2-1}\right] dz}$

Using equation of state $\frac{dp}{\rho} = -VdV - gdz = -V^2\frac{dV}{V} - gdz = V^2\left[\frac{g}{c^2}\frac{1}{M^2-1}\right]dz - gdz$

$$\text{or } \frac{dp}{\rho} = \frac{dp}{p}\cdot\frac{p}{\rho} = \frac{dp}{p}\cdot\frac{c^2}{k} = gdz\left[\frac{M^2}{M^2-1} - 1\right] = gdz\left[\frac{1}{M^2-1}\right] \Rightarrow \boxed{\frac{dp}{p} = \frac{kg}{c^2}\left[\frac{1}{M^2-1}\right]dz}$$

$$\text{now since } c^2 = k\frac{p}{\rho} \Rightarrow 2cdc = k\left[\frac{\rho dp - p d\rho}{\rho^2}\right] = k\left[\frac{dp}{\rho} - \frac{p}{\rho}\frac{d\rho}{\rho}\right] = c^2\left[\frac{dp}{p} - \frac{d\rho}{\rho}\right]$$

now since $\frac{dp}{p} = k\frac{d\rho}{\rho} \Rightarrow \frac{dp}{p} - \frac{d\rho}{\rho} = (k-1)\frac{d\rho}{\rho} = \frac{(k-1)}{c^2}g\left[\frac{1}{M^2-1}\right]dz$

thus $\overset{dc^2}{2c\,dc} = \frac{(k-1)g}{M^2-1}\,dz$ or $\boxed{\frac{dc}{c} = \frac{(k-1)g}{2c^2}\left[\frac{1}{M^2-1}\right]dz}$

now since $M = \frac{V}{c}$ $dM = \frac{c\,dV - V\,dc}{c^2} = \frac{dV}{c} - M\frac{dc}{c} = M\left[\frac{dV}{V} - \frac{dc}{c}\right]$

or $\frac{dM}{M} = \left[-\frac{g}{c^2}\left(\frac{1}{M^2-1}\right)dz - \frac{(k-1)g}{2c^2}\left(\frac{1}{M^2-1}\right)dz\right] = \frac{g\,dz}{2c^2(M^2-1)}\left\{-2-(k-1)\right\}$

$$\boxed{\frac{dM}{M} = \frac{-(k+1)g}{2c^2}\left(\frac{1}{M^2-1}\right)dz}$$

also we know that that @ stagnation $V=0 \Rightarrow dh_0 = -g\,dz = c_p\,dT_0$

hence $\boxed{dT_0 = \frac{-g}{c_p}\,dz}$

$\ell p_0 = \ell \rho_0 + \ell T_0$

$\frac{dp_0}{p_0} = \frac{d\rho_0}{\rho_0} + \frac{dT_0}{T_0}$

and $\frac{p_0}{\rho_0^k} = c$ $\therefore \ln p_0 - k\,\ell\rho_0 = \ell c$

$\frac{dp_0}{p_0} - k\frac{d\rho_0}{\rho_0} = 0$

$\frac{dp_0}{p_0}\left(\frac{k-1}{k}\right) = \frac{dT_0}{T_0}$ $\therefore \frac{dp_0}{p_0} = k\frac{d\rho_0}{\rho_0}$

and again at stagnation conditions $dh_0 = dp_0/\rho_0 = -g\,dz$

hence $\boxed{dp_0 = -\rho_0 g\,dz}$

$\frac{dp_0}{p_0} = -RT_0 g\,dz$

$= -1\frac{c_0^2 g}{k}dz$

Subsonic $dz > 0$	Supersonic
$dV>0$, $d\rho<0$, $dp<0$	$dV<0$, $d\rho>0$, $dp>0$,
$dc<0$, $dM>0$, $dT_0<0$, $dp_0<0$	$dc>0$, $dM<0$, $dT_0<0$, $dp_0<0$

$\frac{dM}{dz} = -\frac{(k+1)g}{2c^2}\frac{M}{M^2-1}$ note that since $\frac{dM}{dz} = 0 \overset{for}{\Rightarrow} M>0 \Rightarrow \boxed{M=1}$

and this extrema is a maximum since $\frac{dM}{dz} > 0$ for $M<1$ & $dz\uparrow$

and $\frac{dM}{dz} < 0$ for $M>1$ & $dz\uparrow$

$h_0 + gz_0 = h + \frac{V^2}{2} + gz \Rightarrow \sqrt{2\left\{c_p(T_0 - T^*) + g(z_0 - z^*)\right\}} = V^* = \sqrt{kRT^*}$

or $\frac{2k}{k-1}R(T_0 - T^*) + g(z_0 - z^*) = kRT^*$

$\rho_f > \rho_g$

~~c) Aircraft dz > 0~~ ~~fluid machinery~~ ~~ventilating systems~~

(i) for $M << 1$ $\quad \overset{NO}{dV = \frac{gM}{c}\frac{dz}{(M^2+1)}}$, $\quad \overset{NO}{dp = \frac{-\rho g\,dz}{c^2(M^2+1)}}$, $\quad \overset{NO}{dp = -\frac{\rho g}{c^2}\left[\frac{2M^2+1}{M^2+1}\right]dz}$, $\quad dc = \frac{g}{2c}\left\{\frac{\overset{1-K}{1 - k[2M^2+1]}}{M^2+1}\right\}dz$, $\quad dM = \frac{g\,M(k+1)\overset{1+K}{[1+k(2M^2+1)]}}{2c^2(M^2+1)}dz$

(ii) for $M \sim .5$ $\quad \overset{no}{dV \sim \frac{g\,\overset{1}{.5}\,dz}{c(1.25)}}$

Flow is choked see discussion of pg 166 and p 171

for fluids in machinery $\quad c \gg c_{air}$

$$\frac{dV}{dz} = \frac{-g}{c}\frac{M}{M^2-1}; \quad \frac{dp}{dz} = \frac{\rho g}{c^2}\frac{1}{M^2-1}; \quad \frac{dp}{dz} = \rho g\frac{1}{M^2-1}$$

$$\frac{dc}{dz} = \frac{(k-1)g}{2c}\frac{1}{M^2-1}; \quad \frac{dM}{dz} = -\frac{(k+1)g}{2c^2}\frac{M}{M^2-1}$$

$.5\left(\frac{g}{c}\right)$

ventilating syst Δz small, $c|_{z=0} > c|_{z\ \text{aircraft}}$

(i) ~~no~~ yes water

(ii) no. $\quad$ for water $\frac{dp}{dz} = \frac{64}{}$ yes

(iii) yes $\quad$ all 3

(iv) Yes water

(v) no

$$\cancel{\frac{dp_0}{p_0} = \frac{dp}{p} + \frac{k}{k-1}\left[d \ln\left(1 + \frac{k-1}{2} M^2\right)\right]}$$

$$\cancel{\frac{\frac{k-1}{2} \cdot 2M\,dM}{1 + \frac{k-1}{2} M^2}}$$

$$= \frac{dp}{p} + \frac{k\,M\,dM}{1 + \frac{k-1}{2} M^2}$$

$$\frac{2V\,dV}{V^4}$$

$$\cancel{\frac{p}{\rho^k}}$$

$$\frac{d\rho}{\rho} + \frac{dV}{V} = \frac{d\rho}{\rho} + \frac{1}{2}\frac{dV^2}{V^2} = 0$$

$$M^2 = \frac{V^2}{c^2}$$

$$\frac{dp}{\rho} = -\frac{1}{2}dV^2 - g\,dz = c^2\frac{d\rho}{\rho}$$

$$\therefore \quad -g\,dz = c^2\frac{d\rho}{\rho} + \frac{1}{2}dV^2$$

$$= c^2\frac{d\rho}{\rho} + \frac{1}{2}V^2\left(\frac{dV^2}{V^2}\right)$$

$$= c^2\frac{d\rho}{\rho} + V^2\frac{1}{2}\left(\frac{dV^2}{V^2}\right)$$

$$= c^2\frac{d\rho}{\rho} + V^2\left(-\frac{d\rho}{\rho}\right)$$

$$-g\,dz = c^2\frac{d\rho}{\rho}\left[1 - M^2\right]$$

$$\frac{g\,dz}{c^2(M^2-1)} = d\rho$$

$\cancel{k_p}$

define $z=0$ @ pt where $V=0$

$\Rightarrow$

$$h_0 = h + \frac{V^2}{2} + gz \qquad U^2 = \frac{V^2}{2} + gz$$

$$\therefore \cancel{k_p}\ c_p(T_0 - T) = \frac{V^2}{2} \qquad U \qquad \text{define} \quad \frac{U^2}{2} = \frac{V^2}{2} + gz$$

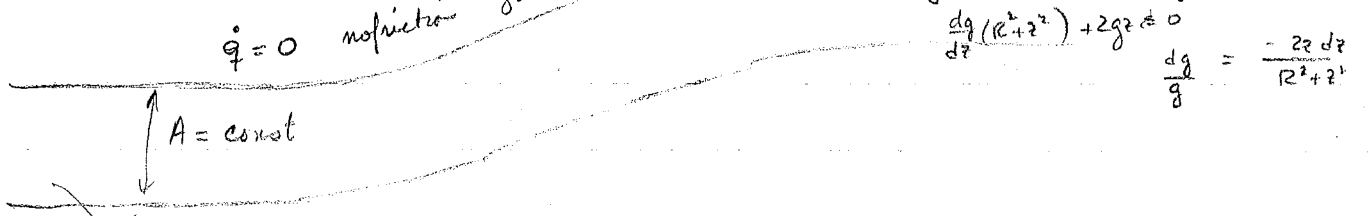
$$c_p(T_0 - T) = \frac{U^2}{2}$$

$$M^2 = \frac{U^2}{c^2} \qquad U^2 < 0 \Rightarrow \boxed{T > T_0} \text{ impossible}$$

$$= \frac{1}{2}M^2 + \frac{gz}{c^2}$$

$\dot{q} = 0$ nofriction flow isentropic

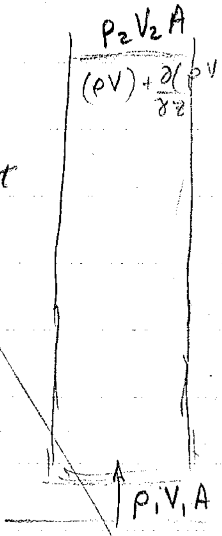
$$g(R^2+z^2) = R^2 g_0$$
$$\frac{dg}{dz}(R^2+z^2) + 2gz = 0$$
$$\frac{dg}{g} = \frac{-2z\,dz}{R^2+z^2}$$



find dz vs. M, V, c, ~~ρ~~, p

T_0, p_0,

we assume g = constant



no work done by fluid

Energy $h_1 + \frac{V_1^2}{2} + gz_1 = h_2 + \frac{V_2^2}{2} + gz_2$

Thermo second law $s = s_0$

Eq of Cont: $\rho_1 V_1 = \rho_2 V_2$

Equation of state $h = h(s, p)$

$\rho = \rho(s, p)$

define $M = \frac{c}{V}$ & $c = \left.\frac{dp}{d\rho}\right)_s$

$dh = -VdV - gdz$ Energy

$\frac{d\rho}{\rho} + \frac{dV}{V} = 0$ Continuity

from the Gibbs equation $T\overset{0}{\cancel{ds}} = dh - dp/\rho$ or $dh = dp/\rho$

$$\therefore \quad dh = \frac{dp}{\rho} = -VdV - gdz = \frac{dp}{d\rho} \cdot \frac{d\rho}{\rho} = c^2 \frac{d\rho}{\rho} \qquad \text{since } \left.\frac{dp}{d\rho}\right|_{s=c} = \frac{c^2}{k}$$

$$\boxed{-\frac{c^2 d\rho}{\rho g} - \frac{VdV}{g} = dz}$$

$$\therefore \quad dz = \frac{-VdV}{g} - \frac{c^2 d\rho}{\rho g} = \frac{-V^2}{g}\frac{dV}{V} - \frac{c^2 d\rho}{\rho g} = \frac{c^2}{g}\left[-M^2 \frac{dV}{V} - \frac{d\rho}{\rho}\right]$$

$$\boxed{dz = \frac{-c^2}{g}\left[(M^2-1)\frac{dV}{V}\right]} \quad \text{or} \quad \boxed{\frac{-g\,dz}{c^2(M^2-1)} = \frac{dV}{V}} \quad ✓$$

also

$$\boxed{dz = \frac{+c^2}{g}\left[(M^2-1)\frac{d\rho}{\rho}\right]} \quad \text{or} \quad \boxed{\frac{+g\,dz}{c^2(M^2-1)} = \frac{d\rho}{\rho}} \quad ✓$$

$$\frac{dp}{\rho} = -VdV - gdz$$

$$-\left(\frac{dp}{\rho g} + \frac{V^2 dV}{gV}\right) = dz = -\left(\frac{dp}{\rho g} + \frac{dz}{c^2}\frac{V^2}{M^2+1}\right) = dz$$

$$-\frac{dp}{\rho g} = dz\left[1 + \frac{M^2}{M^2+1}\right] = dz\left[\frac{2M^2+1}{M^2+1}\right]$$

$$\frac{p}{\rho} = \frac{c^2}{k} \qquad -\frac{dp}{p}\,\frac{p}{\rho g} = -\frac{dp}{p}\,\frac{c^2}{kg}$$

$$\boxed{\frac{dp}{p} = -\frac{kg}{c^2}\left[\frac{2M^2+1}{M^2+1}\right]dz}$$

$h_0 + gz_0 = h + \frac{V^2}{2} + gz$ or $h_0 - h = C_p(T_0 - T) = \frac{V^2}{2} + g(z - z_0) = \frac{\Omega^2}{2}$ or $\Omega^2 = V^2 + 2g(z - z_0)$

divide by C_pT $\Rightarrow$ $\left(\frac{T_0}{T}\right) = 1 + \frac{\Omega^2}{2C_pT} = 1 + \frac{K-1}{2}\frac{\Omega^2}{c^2}$ $\quad \frac{h_0 - h}{C_pT} + 1$

Take ln & differen

$$\frac{dT_0}{T_0} - \frac{dT}{T} = \frac{\frac{K-1}{2}\, d\left(\frac{\Omega^2}{c^2}\right)}{1 + \frac{K-1}{2}\frac{\Omega^2}{c^2}} = \frac{\frac{K-1}{2}\left[\frac{c^2 d\Omega^2 - \Omega^2 dc^2}{c^4}\right]}{1 + \frac{K-1}{2}\frac{\Omega^2}{c^2}}$$

now $d\Omega^2 = 2VdV + 2g(dz - dz_0)$ and $dc^2 = \frac{K-1}{M^2-1} g\, dz$; $\frac{dT}{T} = \frac{dp}{p} - \frac{d\rho}{\rho} = \frac{K-1}{c^2} g \left[\frac{1}{M^2-1}\right] dz$

$$\therefore \frac{dT_0}{T_0} = \frac{K-1}{c^2} g \frac{1}{M^2-1} dz + \frac{\frac{K-1}{2}\frac{c^2(2VdV + 2g(dz - dz_0)) - (V^2 + 2g(z - z_0))\left(\frac{K-1}{M^2-1} g\, dz\right)}{c^4}}{1 + \frac{K-1}{2}M^2 + \mathbb{P}}$$

where $\mathbb{P} = g(z - z_0)\frac{K-1}{c^2}$

$$= \frac{\frac{K-1}{c^2} g \frac{1}{M^2-1} dz \left(1 + \frac{K-1}{2}M^2 + \mathbb{P}\right) + \frac{K-1}{2}\left\{2\frac{V^2}{c^2}\frac{dV}{V} + \frac{2g(dz - dz_0)}{c^2}\right\} - \frac{(K-1)}{2c^4}\left\{V^2\frac{(K-1)}{M^2-1} g\, dz + \frac{(K-1)}{M^2-1} g \cdot 2g(z- \overset{dz}{\ } \right\}}{1 + \frac{K-1}{2}M^2 + \mathbb{P}}$$

$$= \frac{\overset{A}{\underline{\left(\frac{K-1}{c^2} g \frac{1}{M^2-1} dz\right)}}\left(1 + \frac{K-1}{2}M^2 + \cancel{\mathbb{P}}\right) + \frac{K-1}{2}\left\{2M^2\frac{dV}{V} + \frac{2g(dz - dz_0)}{c^2}\right\} - \frac{K-1}{2c^2}\left\{M^2\frac{K-1}{M^2-1} g\, dz + \cancel{\frac{2g}{M^2-1}\mathbb{P}\, dz}\right\}}{1 + \left(\frac{K-1}{2}M^2\right) + \mathbb{P}}$$

$$= \left(\overset{A}{\frac{K-1}{c^2} g \frac{1}{M^2-1} dz}\right) + \cancel{A\left(\frac{K-1}{2}M^2\right)} + \frac{K-1}{2}\left\{-\frac{2M^2 g}{c^2}\left(\frac{1}{M^2-1}\right) dz + \frac{2g(dz - dz_0)}{c^2}\right\} - \cancel{\frac{K-1}{2}M^2 A}$$

$$A \quad + \frac{K-1}{2} \cdot \frac{2g\, dz}{c^2}\left\{\frac{-M^2}{M^2-1} + 1\right\} - \frac{(K-1)}{c^2} g\, dz_0$$

$$\frac{K-1}{c^2} g \frac{1}{M^2-1} dz \quad - \frac{K-1}{c^2} g\, dz \left\{\frac{-M^2 + \cancel{M^2} + 1}{M^2-1}\right\} = \frac{K-1}{c^2} g\, dz_0$$

$$\frac{dT_0}{T_0} = \frac{K-1}{c^2} g \frac{1}{M^2-1} dz \quad \frac{-1}{M^2-1}\left(\frac{K-1}{c^2} g\, dz\right) - \frac{K-1}{c^2} g\, dz_0$$

$$\frac{dT_0}{T_0} = \frac{-\frac{(K-1)}{c^2} g\, dz_0}{1 + \frac{K-1}{2}M^2 + \frac{K-1}{c^2} g(z - z_0)}$$

for positive $dz_0 \Rightarrow dT_0 \leq 0$

$$\frac{-\frac{(K-1)}{kRT} g\, dz_0}{\frac{kRT_0}{kRT}}$$

$$\frac{-\frac{(K-1) g\, dz_0}{kRT}}{c_p}$$

$\frac{dp}{p} = k\frac{d\rho}{\rho}$ $\qquad \frac{dp}{p} - \frac{d\rho}{\rho} = \frac{K-1}{K}\frac{dp}{p} = \frac{dT}{T}$

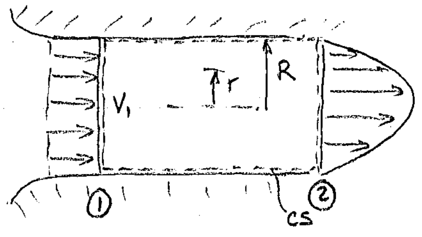
$$\frac{dp_0}{p_0} = \frac{-\frac{K}{c^2} g\, dz_0}{1 + \frac{K-1}{2}M^2 + \frac{K-1}{c^2} g(z - z_0)}$$

$\frac{h_0 - h}{C_pT} + 1 = \frac{h_0}{h} = \frac{T_0}{T}$

Cesar Levy
ME255
Prof Kline

Extra Problems. Ch #1-4

Pg 21 #1-2 a) Show $V_1/V_{max} = \frac{1}{2}$



$V_2 = V_{max}\left(1-\frac{r^2}{R^2}\right)$

If we assume 1-D flow, frictional effects, and steady state conditions, the continuity equation states $\int_{cs} \rho \underline{V}\cdot\underline{n}\, dA = 0$ where ρ = const here.

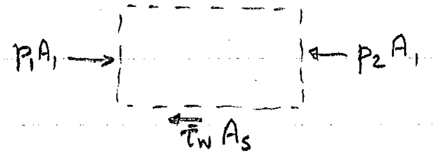
@ 1 $\underline{V}\cdot\underline{n} = -V_1$; @ 2 $\underline{V}\cdot\underline{n} = V_{max}\left(1-\frac{r^2}{R^2}\right)$; on the walls $\underline{V}\cdot\underline{n} = 0$

Thus $\int \rho \underline{V}\cdot\underline{n}\, dA = -\rho V_1 \pi R^2 + \rho\int_0^R V_{max}\left(1-\frac{r^2}{R^2}\right)\cdot 2\pi r dr = 0$

$$\therefore \rho\left[-V_1\pi R^2 - V_{max}\pi\frac{R^2}{2}\left(1-\frac{r^2}{R^2}\right)^2\Big|_0^R\right] = 0 \Rightarrow V_1 = \frac{V_{max}}{2}$$

b) Find $p_1 - p_2 = f(V_{max}, \rho, L, r, \bar{\tau}_w)$

Using $\Sigma F_x = \int \rho V_x(\underline{V}\cdot\underline{n})dA$ since the system is steady, then ΣF_x = shear + pressure forces



$$\therefore (p_1-p_2)A_1 - \tau_w A_s = -\rho V_1^2 A_1 + \rho\int_0^R \left(V_{max}\left[1-\frac{r^2}{R^2}\right]\right)^2 2\pi r dr$$

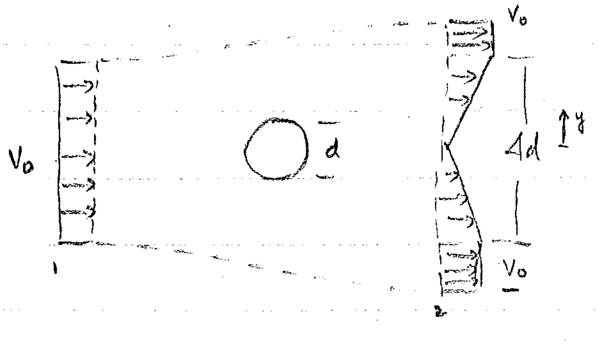
$$(p_1-p_2)\pi R^2 - \tau_w 2\pi RL = -\rho V_1^2\cdot\pi R^2 - \rho V_{max}^2 \frac{\pi R^2}{3}\left(1-\frac{r^2}{R^2}\right)^3\Big|_0^R$$

$$= -\rho\frac{V_{max}^2}{4}\pi R^2 + \rho V_{max}^2\pi\frac{R^2}{3}$$

$$(p_1-p_2)\pi R^2 - \tau_w 2\pi RL = \rho\frac{V_{max}^2}{12}\pi R^2$$

or $\boxed{p_1 - p_2 = \rho\frac{V_{max}^2}{12} + \tau_w\cdot\frac{2L}{R}}$

Pg 22 #1-5 I will be using the streamlines method. We will assume steady state conditions and 1-D flow. The control volume is defined as follows



By continuity $\int_{cs} \rho\underline{V}\cdot\underline{n}\, dA = 0$ per unit length

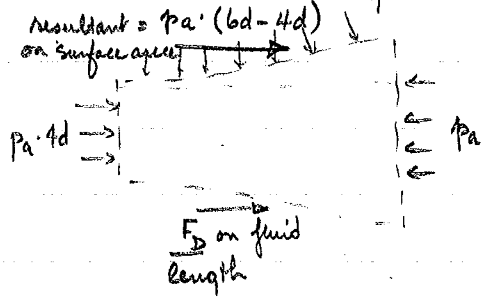
@ 1 $\underline{V}\cdot\underline{n} = -V_0$ @ 2 $\underline{V}\cdot\underline{n} = \begin{cases} V_0 & |y| \geq 2d \\ V_0\frac{|y|}{2d} & |y| \leq 2d \end{cases}$

$$\therefore \int \rho\underline{V}\cdot\underline{n}\, dA = -\rho V_0 4d + 2\int_0^{2d}\rho V_0\frac{y}{2d}\, dy + 2\int_{2d}^{\xi}\rho V_0\, dy = 0$$

$$= -4\rho V_0 d + \frac{\rho V_0}{d}\frac{y^2}{2}\Big|_0^{2d} + 2\rho V_0 y\Big|_{2d}^{\xi} = 0$$

$$\therefore \quad -4\rho V_o d + \rho \frac{V_o}{d} \cdot \frac{4d^2}{2} + 2\rho V_o(\xi - 2d) = -2\rho V_o d + 2\rho V_o(\overset{d}{\overline{\xi - 2d}}) = 0 \implies \xi = 3d$$

for the momentum equation ΣF_x = pressure + drag forces = $\int \rho V_x (\underline{V} \cdot \underline{n})\, dA$



pressure forces will cancel : $p_a \cdot 4d + p_a(6d-4d) - p_a(6d) = 0$

$$\frac{F_D}{\text{length}} = -\rho V_0^2 \cdot 4d + 2\rho \int_0^{2d} V_0^2 \cdot \frac{y^2}{4d^2}\, dy + 2\int_{2d}^{3d} \rho V_0^2\, dy$$

$$= -\rho V_0^2 \cdot 4d + \frac{2\rho V_0^2}{4d^2} \left. \frac{y^3}{3} \right|_0^{2d} + 2\rho V_0^2 y \Big|_{2d}^{3d}$$

$$= -\rho V_0^2 \cdot 4d + \frac{2\rho V_0^2}{4d^2} \frac{8d^3}{3} + 2\rho V_0^2 d$$

$$F_D = \rho V_0^2 d\left[-4 + \frac{4}{3} + 2\right] = -\frac{2}{3}\rho V_0^2 d$$

$\frac{F_D}{\text{length}}$ on cylinder = $-F_D$ on fluid = $\frac{2}{3}\rho V_0^2 d$

now $\boxed{C_D = \frac{\frac{2}{3}\rho V_0^2 d}{\frac{1}{2}\rho V_0^2 d} = \frac{1}{3}}$

Pg 69 # 3-8

a) since $dp = \rho c\, dV$ For water @ $p = 14.7$ psi and $T = 70°F$, $K = 3.19 \times 10^5$ psi

and $\rho = 1.94 \frac{\text{lb-sec}^2}{\text{ft}^4}$ $\therefore$ $c = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{3.19 \times 10^5 (144\ \text{in}^2/\text{ft}^2)}{1.94}} = 4866$ ft/sec

$$dp = 1.94 \frac{\text{lb-sec}^2}{\text{ft}^4} \times 4866 \frac{\text{ft}}{\text{sec}} \times 10 \frac{\text{ft}}{\text{sec}} = 9.44 \times 10^4 \frac{\text{lb}}{\text{ft}^2} = 655 \text{ psia}$$

b) since $dp = \rho c\, dV$ for ~~water~~ air @ $p = 14.7$ psi and $T = 70°F$ $\rho = 2.375 \times 10^{-3} \frac{\text{slugs}}{\text{ft}^3}$

$c = 49.02\sqrt{T} = 1128.53$ ft/sec

$$\therefore \quad dp = 2.375 \times 10^{-3} \frac{\text{lb-sec}^2}{\text{ft}^4} \times 1128.53\ \text{ft/sec} \times 10\ \text{ft/sec} = 26.8 \frac{\text{lb}}{\text{ft}^2} = .0186 \text{ psia}$$

Pg 106 #4-5

Given $\dot{m} = 2.2$ lbm/sec , $d_{duct} = 4''$, $T_0 = 100°F = 560°R$, $p_1 = 6$ psia

Find M_1, V_1, p_0 we assume isentropic flow w/ $k=1.4$

Since $\frac{w}{A} = \rho V = \sqrt{\frac{k}{R}} \frac{p_1}{\sqrt{T_0}} M_1 \sqrt{1+\frac{k-1}{2}M_1^2} = \left(\frac{2.2}{32.174} \frac{\text{slugs}}{\text{sec}}\right) \Big/ \left(\frac{\pi \cdot 4^2}{4} \text{in}^2 \cdot \frac{1\,\text{ft}^2}{144\,\text{in}^2}\right)$

$$= \frac{.0684}{.0873} \frac{\text{slugs}}{\text{ft}^2\text{sec}} = .7836 \frac{\text{slugs}}{\text{ft}^2\text{sec}}$$

Using chart on pg82 we need to find $\frac{w}{A} \frac{\sqrt{T_0}}{p} \frac{1}{\sqrt{W}} = .7836 \frac{\text{slugs}}{\text{ft}^2\text{sec}} \times \frac{\sqrt{560\,°R}}{6 \times 144\,\text{psf}} \times \frac{1}{\sqrt{28.97}}$

$= .00399 \sim .004$ which provides $\underline{M_1 \cong .72}$

Now $\frac{T_0}{T_1} = 1 + \frac{k-1}{2} M_1^2 = 1 + .2(.72)^2 = 1.1037$ or $T_1 = \frac{560}{1.1037} = 507.4\,°R$

$c_1 = 49.02\sqrt{T_1} = 1104.19$ ft/sec $\therefore$ $V_1 = M_1 c_1 = \underline{795.0 \text{ ft/sec}}$

Since $\frac{p_0}{p_1} = \left(\frac{T_0}{T_1}\right)^{\frac{k}{k-1}} = (1.1037)^{\frac{1.4}{.4}} = 1.4124$ or $p_0 = 1.4124 p_1 = 8.47$ psia

Pg 106 #4-10

Given $p = 20$ psia , $M = .6$ and $W = .5$ lbm/sec $A = 1\,\text{in}^2$ find T_0, $\frac{A-A^*}{A}$, V^*, p^*

we again are working with air $k=1.4$ in isentropic flow

$$\sqrt{T_0} = \sqrt{\frac{k}{R}} \frac{PA}{w} M \sqrt{1+\frac{k-1}{2}M^2} = \sqrt{\frac{1.4}{1716}} \cdot \frac{20 \times 1}{.5/32.174} \cdot (.6) \sqrt{1+.2(.6)^2}$$

$$22.836 = (.0286) \cdot (1286.96) \cdot (.6) \cdot (1.0354)$$

a) $\therefore$ $\underline{T_0 = 521.5\,°R}$

b) now for $M = .6$ $A/A^* = 1.188$ $\therefore$ $\frac{A-A^*}{A} = 1 - \frac{A^*}{A} = 1 - \frac{1}{1.188} = .158$

$\therefore$ a 16 % reduction is area can occur

c) now for $M = .6$ $T^*/T_0 = .8333$ $\therefore$ $T^* = 434.6\,°R$ $\Rightarrow$ $\underline{C^* = V^* = 49.02\sqrt{T^*} = 1022 \frac{f}{s}}$

$p^* = \rho^* R T^* = \frac{w}{A^* V^*} R T^* = \frac{.5/32.174}{(.842/144)(1022)} (1716)(434.6) = 1939.4$ psf $= \underline{13.5 \text{ psia} = p^*}$

XTRA

✓

5.1, 5.7, 5.9, 5.19

5.7 The equations that apply are

1. Continuity $d(\rho V) = 0$ for constant A $\qquad \rho dV = -V d\rho$ or $VdV = -V^2 \frac{d\rho}{\rho}$

2. $dp + \frac{\dot{m}}{A} dV = 0 \qquad dp = -\frac{\dot{m}}{A} dV = -\rho V dV$

3. equation of state $Tds = dh - dp/\rho$

at max entropy $ds = 0$ $\quad \therefore \underbrace{dh = dp/\rho}_{\text{from ③}}\Big|_s = -VdV = \underbrace{V^2 \frac{d\rho}{\rho}}_{\text{from ①}}$

from②

since $(dp/\rho) = V^2 \frac{d\rho}{\rho} \quad \therefore \left(\frac{dp}{d\rho}\right)_s = V^2$ but $\left(\frac{dp}{d\rho}\right)_s \overset{def}{=} c^2$

$\therefore$ since $V^2 = c^2 \Rightarrow M^2 = \frac{V^2}{c^2} = 1$ at pt where $ds = 0$

5.9 $T_1 = 70°F = 530°R \qquad p = 10\,\text{psia} = 1440\,\text{psfa} \qquad V = 400$ ft/sec

using $c = 49.02\sqrt{T_1} = 1128.53$ ft/sec $\quad \therefore M = \frac{V}{c} = .354$

$h = c_p T = 6000 \frac{\text{ft lb}}{\frac{\text{lb sec}°\text{R}}{\text{ft}}} \times 530°R = 3.18 \times 10^6 \frac{\text{ft}^2}{\text{sec}^2}$

$h_0 = C_p T_0 = \frac{V^2}{2} + C_p T = 8\times10^4 + 3.18\times10^6 = 3.26\times10^6 \quad \therefore T_0 = \frac{3.26\times10^6}{6000} = 543.3\,°R$

$c_0 = 1142.6$ ft/sec this is the speed of the wave

since $\frac{P_1}{P_0} = \left(\frac{T_1}{T_0}\right)^{\frac{K}{K-1}} \Rightarrow P_0 = P_1\left(\frac{T_0}{T_1}\right)^{\frac{K}{K-1}} = 10\left(\frac{543.3}{530}\right)^{\frac{1.4}{.4}} = 10.91$ psia

we do this under the assumption that no heat transfer occurs if the process of shutting the valve is done in an infinitesimal time.

5.19 $P_x = 2.34$, $P_{oy} = 10.02 \qquad P_{oy}/P_x = 4.282 \qquad M_x = 1.715$

Copy FANNO. FOR. 5 b RAYLEIGH. FOR. 1

```
      AJ = 778.16
      GC = 32.174
      SXT = 21.437
      PX = 10
      TX = 1700
      UX = 3000
      AK = 1.4 ,  R = 1545.32/28.97 * 32.174
      RHOX = PX * 144/(R * TX)          1440/1700
      RHOXT = RHOX * GC
      VX = 1./RHOXT
      FLOR = RHOXT * UX
      HX = AK * R * TX /(AK-1.)
      ~~HXT = HX/(GC*AJ)~~
      HOX = HX + UX*UX/2            ft²/sec²
      HOY = HOX                       ft²/sec²
      ~~HOXT = HOX/(GC*AJ) temp~~ TOX = (AK-1)*HOX/(AK*R)
      UY = 4100.
10    UY = UY-100.                    ft/sec
      IF (UY.LT. 200) GO TO 999
      RHOYT = FLOR / UY
      VY = 1./RHOYT                   ft³/lbm
      HY = HOY - UY*UY/2.
      ~~HYT = HY/(GC*AJ)~~
      TY = HY*(AK-1)/(AK*R)           °R
      AMY = UY/SQRT (AK*R*TY)
      PY = RHOYT * R*TY / (GC * 144.)    psia    POY= PY* (TOX/TY)**(AK/(AK-1.))
      DELS = R*( LOG ( TY/TX)/(AK-1.) + LOG ( VY/VX))
      DELST = DELS/(GC*J)
      SYT = SXT + DELST               Btu/lbm °R
      WRITE (3,15)  UY, TY, SYT, PY, VY, AMY, TOX, POY
15    FORMAT ( 6 (2X, E14.7), /2(2X, E14.7))
      GO TO 10
999   WRITE (3,20)
20    FORMAT ( 2X, *END FANNO FLOW* )
      STOP
      END
```

```
    UY = 4100
10  UY = UY-100.
    IF (UY.LT. 200) GOTO999
    RHOYT = FLOR/UY
    VY = 1./RHOYT
    PY = (PX*144. + ρHOX*UX*UX - ρHOYT*UY*UY)/GC /144.
    TY = GC*PY*144/(ρHOYT*R)
    AMY = UY/SQRT (AK*R*TY)
                ***
    DELS = R*(AK*LOG(TY/TX)/(AK-1) - LOG (PY/PX))
    DELST = DELS /(GC*J)
    SYT = SXT + DELST
    WRITE (3,15) UY,TY,SYT,PY,VY,AMY,TOY,POY
15  FORMAT (6(2X,E14.7)/, 2(2X,E14.7))
    GO TO 10.
999 WRITE (3,20)
 20 FORMAT (2X,* END RAYLEIGH FLOW*)
    STOP
    END

***
TOY = TY * (1. + (AK-1.)*AMY*AMY/2.)
POY = PY * ( TOY/TY)**(AK/(AK-1.))
```

21.437

actual solution $p_x = 10$ $T_x = 1700°R$ $\therefore$ $C = 49.02\sqrt{1700} = 2021.15$ $M_x = \frac{U_x}{C_x} = 1.484$

$M_y = .7067$ $P_y/P_x = 2.403$ $P_y = 24.03$ psia $\frac{T_y}{T_x} = 1.3096$ $T_y = 2226.4°R$ / 1766.4°F

ⓢ $M_x = 1.484$ $P_x/P_{0x} = .2787$ $P_{0x} = 35.88$ psia $T_x/T_{0x} = .6942$ $T_{0x} = 2448.8°R$

$M_y = .7067$ $P_y/P_{0y} = .7166$ $P_{0y} = 33.53$ psia $T_y/T_{0y} = .9092$ $T_{0y} = 2448.8°R$

$$\Delta S = -R \ln\left(\frac{P_{0y}}{P_{0x}}\right) = -1716.32 \ln\left(\frac{33.53}{35.88}\right) = 116.263 \frac{ft^2}{sec^2 °R} = 00464 \frac{Btu}{lbm\,°R}$$

given $p_x = 10$ psia $T_x = 1240°F$ and $U_x = 3000$ fps $k = 1.4$

Fanno satisfies energy, continuity, eq of state

Rayleigh satisfies mom, cont + eq of st

$$\frac{lb\,sec^2}{ft} \quad \frac{slugs}{ft^3} \qquad g_c\,\rho_x = \frac{lb\,sec^2}{ft^4}\cdot\frac{lb_m\,ft}{lb_f\,sec^2}$$

FANNO : Given p_x, T_x use $\rho_x = \dfrac{P_x}{RT_x}$ $\dfrac{lb/ft^2}{ft^2/sec^2}$ $\therefore$ $\rho_x V_x = \text{const} = \dot{m}$ (1)

$g_c\rho_x = \tilde{\rho}_x$ in $\dfrac{lb_m}{ft^3}$ $\tilde{\dot{m}} = \dfrac{lb_m}{sec}/ft^2$

also $h_x = c_pT_x = \dfrac{k}{k-1}RT_x$ $\therefore$ $h_{ox} = \left(\dfrac{kRT_x}{k-1} + \dfrac{V_x^2}{2}\right)$ in $\dfrac{ft^2}{sec^2}$. (2) $\dfrac{Btu}{lb_m}$

$$\bar{h}_{oy} = \bar{h}_{ox} = \frac{h_{ox}}{g_cJ}\cdot\frac{ft^2}{sec^2}\cdot\frac{lbf\,sec^2}{lb_m\,ft}\cdot\frac{Btu}{ft\,lbf} = \frac{Btu}{lb_m}$$

Choose V_y $\Rightarrow$ $\tilde{\rho}_y = \dfrac{\tilde{\dot{m}}}{V_y}$; $\overset{(2)}{h_{oy}} - \dfrac{V^2}{2} = h_y$ in $\dfrac{ft^2}{sec^2}$ $\dfrac{h_y}{g_cJ} = \bar{h}_y$ in $\dfrac{Btu}{lb_m}$

Given p_x, T_x use $\rho_x = \dfrac{P_x}{R\,T_x}^{(1)}$ and $\rho_x g_c = \tilde{\rho}_x^{(2)}$ $1/\tilde{\rho}_x = V_x^{(3)}$ $\dfrac{ft^3}{lb_m}$ $\tilde{\rho}_x U_x = \tilde{\dot{m}}^{(4)}$ $\dfrac{lb_m}{sec}/ft^2$

$9\times10^6\ 289\ (10^4$

$1700\ 1700$

$h_x = c_pT_x = \dfrac{k}{k-1}RT_x^{(5)}$ $\dfrac{ft^2}{sec^2\,°R}$, $\tilde{h}_x\ \dfrac{Btu}{lb_m} = \dfrac{h_x}{g_cJ}^{(6)}$ $h_{ox} = h_{oy} = \dfrac{kR}{k-1}T_x + \dfrac{U_x^2}{2}^{(7)}$, $\tilde{h}_{ox} = \tilde{h}_{oy} = \dfrac{h_{ox}}{g_cJ}^{(8)}$

~~pick~~ U_y use $\tilde{\rho}_y = \dfrac{(4)}{U_y}$ $V_y = \dfrac{1}{\tilde{\rho}_y}$ (9) $\dfrac{ft^3}{lb_m}$, $h_y = \overset{Hoy}{(7)} - \dfrac{U_y^2}{2}$ (10), $\tilde{h}_y = \dfrac{h_y}{g_cJ}$ $\dfrac{Btu}{lb_m}$ (11)

$s_y - s_x = \dfrac{R}{k-1}\ln\left(\dfrac{h_y}{h_x}\right)\left(\dfrac{U_y}{U_x}\right)^{k-1}$ $\overset{(13)}{\text{in}}$ $\dfrac{ft^2}{sec^2\ °R}$ $\tilde{S}_2 - \tilde{S}_1 = \dfrac{S_2 - S_1}{g_cJ} = \dfrac{Btu}{lb_m\,°R}$ (14)

$T_y = \dfrac{h_y(k-1)}{kR}$

$S_x = \phi_x - R\ln(10/14.7)$

N_2 $S_{cal/mole\,°K} = 38.171$ at $T = 100°K$

O_2 $S_{cal/mole\,°K} = 41.3967$ at $T = 100°K$

$s_1^a = \phi_1 - R\ln p$

$s_1^a = \overset{.88758}{\cancel{.90308}} - 53.34\ln(10/14.7)$

$s_1^a = \underbrace{c_p\ln T}_{\phi_1}$ $\dfrac{k}{k-1}R\ln T$ $53.\ \ln 1800 = 21.437$ Btu/lb$_m$ °R (12)

using $P_y = \rho_y RT_y = \dfrac{\tilde{\rho}_y}{g_c}RT_y$ (16) $M_y = \dfrac{U_y}{\sqrt{kRT_y}}$ (17) $T_y = h_y\dfrac{(k-1)}{kR}$ (15)

Given Rayleigh : moment, cont, equat of state $p_x = 10$ $T_x = 1240$ $\bar{U}_x = 3000$

p_x, T_x, U_x

using p_x, T_x $\rho_x = \frac{P_x}{RT_x}$ (1) $\tilde{\rho}_x = \rho_x g_c$ (2), $v_x = \frac{1}{\tilde{\rho}_x}$ (3) , $\tilde{\rho}_x U_x = \dot{m}$ (4)

$h_x = \frac{kRT_x}{k-1}$ (5) $\tilde{h}_x = \frac{h_x}{g_c J}$ (6)

pick ✓ U_y $\tilde{\rho}_y = \frac{(4)}{U_y}$ (5) ✓ $\tilde{V}_y = \frac{1}{\tilde{\rho}_y}$ (6) ✓ $P_y = p_x + \rho_x U_x^2 - \frac{\tilde{\rho}_y}{g_c} U_y^2$ (7)

$s_y - s_x = \frac{R}{k-1} \ln \left(\frac{P_y}{P_x}\right)\left(\frac{U_y}{U_x}\right)^k$ (8) $\tilde{s}_y - \tilde{s}_x = \frac{s_y - s_x}{g_c J}$ (9) and $\tilde{s}_x = 21.437 \frac{\text{Btu}}{\text{lbm}\,°R}$

✓ $T_y = \frac{P_y}{\rho_y R} = \frac{g_c P_y}{\tilde{\rho}_y R}$ (10) $h_y = \frac{kR}{k-1} T_y$ (11) $M_y = \frac{U_y}{\sqrt{kRT_y}}$

$h_{ox} = c_p T_{ox}$ $\frac{kR\,T_{ox}}{k-1}$

$\frac{k-1}{kR} h_{ox}$

$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$

$\frac{P_0}{P} = (\quad)^{k/k-1}$

Print RHOX , HX , HOX ; RHOYT, HY ,

$\rho_x = .4.94 \times 10^{-4}$ $\tilde{\rho}_x = .0159$ $\hat{\dot{m}} = 47.646$

$\tilde{\rho}_y = .01222$ $P_y =$

$\rho_y = .0003798$

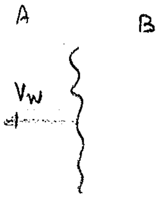
$V_A = 0$

$P_A = 14.7$ psia

$T_A = 59°F$

$T_{o_A} = 59°F$

$P_{o_A} = 14.7$ psia



X Y

$P_A = P_X = 14.7$ psia

$T_X = T_A = 59°F \longrightarrow | \longrightarrow$

✓ $V_X = |V_W|$

✓ $c = \sqrt{kRT_A}$

✓ $M_X = \frac{V_X}{c}$

✓ $T_{O_X} = T_X\left(1 + \frac{k-1}{2}M^2\right)$

✓ $P_{O_X} = P_X\left(1 + \frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}$

$M_y =$ ✓

$T_y =$ ✓

$P_y =$ ✓

$P_{oy} =$ ✓

$T_{oy} =$ ✓

$V_y =$ ✓

V_X: ✓ $V_X = 2\times10^5$ fps

c: ✓ $c = 49.02\sqrt{T_A} = 49.02\sqrt{519} = 1116.7$ fps

M_X: ✓ $M_X = 179.1$ since $\frac{k-1}{2}M_X^2 > 1$ we will approx $T_{O_X} \cong T_X\left(\frac{k-1}{2}M^2\right) = T_X(6.4153\times10^3$

T_{O_X}: ✓ $\therefore T_{O_X} \cong 3,329,540.6°R$ or $3329080.6°F$

P_{O_X}: ✓ $P_{O_X} \cong P_X\left(\frac{k-1}{2}M^2\right)^{k/k-1} = 2.11475\times10^{13}$ atm.

M_y: $M_y^2 = \frac{M_X^2 + \frac{2}{k-1}}{\frac{2k}{k-1}M_X^2 - 1}$ since $M_X^2 >> \frac{2}{k-1}$ and 1 $M_y^2 \approx \frac{M_X^2}{\frac{2k}{k-1}M_X^2} = \frac{k-1}{2k}$

$$\therefore M_y^2 \approx \frac{.4}{2.8} = \frac{1}{7} = .1428 \quad M_y = .37796$$

T_y: $$\frac{T_y}{T_X} = \frac{\left(1+\frac{k-1}{2}M_X^2\right)\left(\frac{2k}{k-1}M_X^2 - 1\right)}{\frac{(k+1)^2}{2(k-1)}M_X^2} = \frac{kM_X^4 + M_X^2\left(\frac{2k}{k-1} - \frac{k-1}{2}\right) - 1}{\frac{(k+1)^2}{2(k-1)}M_X^2}.$$

$$\doteq \frac{2k(k-1)}{(k+1)^2}M_X^2 + f(k) - \frac{g(k)}{M_X^2} \cong \frac{2k(k-1)}{(k+1)^2}M_X^2 = \frac{2.8(.4)}{(2.4)^2}(179.1)^2$$

$$\frac{T_y}{T_X} = 6237.1575 \quad \therefore T_y = 3237084.7°R, 3236624.7°F$$

P_y: $$\frac{P_y}{P_X} = \frac{2k}{k+1}M_X^2 - \frac{k-1}{k+1} \cong \frac{2k}{k+1}M_X^2 \doteq \frac{2.8}{2.4}(179.1)^2 = 37422.95$$

$P_y = 37423$ atm.

T_{oy}: $\frac{T_{oy}}{T_y} = 1 + \frac{k-1}{2}M_y^2 \cong 1.0286$ and $T_{oy} = T_{ox}$

P_{oy}: $\frac{P_{oy}}{P_y} = (1.0286)^{3.5} = 1.104 \Rightarrow P_{oy} = 41301$ atm.

$c_y = 88196.3$ ft/sec $\quad V_y = c_y M_y = 88196.3(.37796) = 33334.7$ ft/sec

$M_B = \frac{V_B}{C_y}$ $\quad V_B = V_W - V_y = 166{,}667$ ft/sec

$$M_B = \frac{166{,}667}{88196.3} = 1.89$$

6,474,169

$-$ 9,711,253

5503044

64742

$$T_{oB} = T_B \left(1 + \frac{K-1}{2} M_B^2\right)$$

$$P_{oB} = P_B \left(1 + \frac{K-1}{2} M_B^2\right)^2$$

This gives parts a, b, c only.

$\bar{R} = 1545.32$

$\bar{m} = 4.004$

$R = \bar{R} * gc/\bar{m}$

$AK = 5/3.$

$P_o = 100$ psi

$T_o = 540$ °R

$Ai = 10$ in^2

$A_S = 2$ in^2

```
      RAT = (AK+1.)/(AK-1.)
      FLOR = Po*As*gc * SQRT (AK * (2./(AK+1))** RAT   /(R*To))    lbm/sec      ARAT = Ai/As
                                                                              CALL AREA (ARAT, RAT, AM, AK)
      I=1
      AM=.3
-------------------------------------------------
 10   TRAT = 1. + (AK-1.)*AM*AM/2        IN
      ARAT = Ai/As
      FM =  (2.*TRAT/(AK+1.) )** (RAT/2.)/AM   - ARAT
      FPM = (2.*TRAT/(AK+1.))** (RAT/2. -1.)  -  (2.*TRAT/(AK+1.))** (RAT/2.)    in sub
      AMN = AM - FM/FPM
      IF (ABS (AMN - AM). LT. 1.0E-5) ~~GO TO 20~~ RETURN
      AM=AMN
      I= I+1
      IF (I. GT. 100) GO TO 999
      GO TO 10
-------------------------------------------------
 20   WRITE (3,15)  FLOR, AM
 15   FORMAT ( 2 (5X , E14.7))
      AMI = AM
      DO 30 I=1, 11
      AL = (I-1)
      AM = AMI* (1. - AL/6.) + AL/6.
      TRAT = 1. + (AK-1.) * AM * AM/2.
      PRAT = (TRAT) **(AK/(AK-1.))
      ARAT = (2. * TRAT /(AK-1.)) ** (RAT/2.)/AM
      P= PO /PRAT
      T= TO/TRAT
      A = ARAT * As
      WRITE (3,25)  AL, AM, P, T, A
 25   FORMAT ( 5 (2X , E14.7))
 30   CONTINUE
 
 999  WRITE (3,100)
 100  FORMAT ( 2X, 24HDIVERGENT ITER TECHNIQUE)
      STOP, END
```

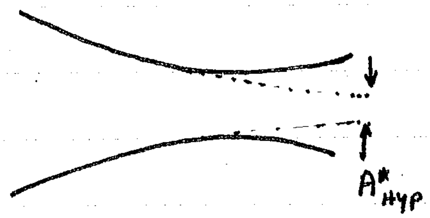
— final value of P = Pe design

```
                                 — C get Pe with shock at exit
                                 — Py exit standing shock
      PY= P* [2*AK*AM*AM/(AK+1.) - 1./RAT]
      MY= SQRT((AM*AM + 2./(AK-1.))/(2*AK*AM*AM/(AK-1.) -1.))
      WRITE (3,15)  PY,MY
      ARAT = A/As    — C get Pe with subsonic isentropic
      AM=.6
        CALL AREA (ARAT, RAT, AM, AK)
        WRITE (3,15) FLOR , AM
        TRAT = 1. + (AK-1.)* AM * AM/2
        PRAT = (TRAT) ** (AK/(AK-1.))
        P = PO/PRAT
        T = TO /TRAT
        WRITE (3,15)  P, T, M
```

in Sub

Areas 10.0, 4.504809, 3.066041, 2.459835, 2.168974, 2.036602, 2.0, 2.029989, 2.111806, 2.237725, 2.403788

∃ a hypothetical nozzle which chokes on $\dot{m} = .5$ lbm/sec.



$$\frac{\omega}{A^*_{HYP}} = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}}\ \frac{P_0}{\sqrt{T_0}}\, g_c \times 144$$

$$\frac{\omega}{A^*} \times \cancel{\frac{ft^2}{144\,in^2}} \sqrt{\frac{sec^2\ {}^\circ\cancel{R}}{ft^2}} \cdot \frac{\cancel{\sec}}{{}^\circ\cancel{R}}\ \frac{lb}{\cancel{ft}\ ft^2} \cdot \frac{lbm\ \cancel{ft}}{\cancel{lb}\ sec^2} = \frac{lbm}{sec}\ \frac{1}{ft^2}$$

$$\frac{\omega}{A^*_{in^2}} \cdot \frac{144\ in^2}{ft^2}$$

$$\frac{\omega}{F} \qquad \frac{\omega}{A^*_{hyp}} = flo\ \frac{lbm}{sec}\ \frac{1}{ft^2} \times \frac{ft^2}{144\,in^2}$$

$$\frac{\omega}{flow} \times 144 = A^*$$

$$\frac{\dfrac{lbm}{sec}\ \dfrac{ft^2}{ft^2} \cdot 144}{\dfrac{lbm}{sec\ ft^2}}$$

solve for A^*_{HYP} print A^*_{HYP}

use $A(I)/A^*_{HYP} = \frac{1}{M_i}\left[\frac{2}{k+1}\left(1+\frac{k-1}{2}M_i^2\right)\right]^{\frac{k+1}{2(k-1)}}$

define $\frac{T_0}{T_i} = 1 + \frac{k-1}{2} M_i^2$

```
      DIMENSION A(10)
      DATA A/
      R= 1545.32 * 32.174/4.004
      AK= 5./3.
      Po =100.
      To = 540.
      RAT= (AK+1.)/(AK-1.)
        W= .5
        FLOR = PO*GC* SQRT (AK* (2./(AK+1.))** RAT/(R*TO))
        A* = W/FLOR *144
      DO 10 I = 1,10
        AM=.1
DO 5 J = 1,100
      TRAT= 1+ (AK-1.)* AM * AM/2.
        ARAT = A(I)/A*
      FM =
      FPM=
      AMN = AM - FM/FPM
      WRITE (6,4) J,I, AMN,AM ,FM,FP, TRAT
    4 FORMAT ( 2(2X,I3), 5(2X, E14.7))
      IF (ABS (AMN-AM ).LE. 1.0E-5) GO TO 7
      AM= AMN
    5 CONTINUE
      WRITE (6,6)
    6 FORMAT (2X, 21H DIVERGENT ITER SCHEME )
        AM=.1*I
      GO TO 10
~~72 WRITE (3,8) I, AM~~
    7 TRAT = 1+(AK-1.)* AMN*AMN/2.
      PRAT = (TRAT)**(AK/(AK-1.))
      P = PO/PRAT
      T= TO/TRAT
      WRITE (3,8) I, AMN, P,T, A(I)
    8 FORMAT(2X,I3, 3(2X,E14.7))
   10 CONTINUE
      WRITE (3,15) R, A*
   15 FORMAT ( 2(2X,E14.7)/,2X,
     *END SUBSONIC ISENT FLOW*)
```

√100

$\bar{m} = 4.004$ $\bar{R} = \frac{1545.32}{4.004} \cdot g_c$ need $\left(\frac{\dot{m}}{A}\right)_{max} = \left(\frac{\dot{m}}{A^*}\right) = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \frac{p_0}{\sqrt{T_0}}$

$k = 1.66$

$$\dot{m} = \sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}} \frac{p_0}{\sqrt{T_0}} A_r \times g_c$$

∴ for $A/A^* = 5$ find $M_{in} < 1$

$\frac{sec}{ft} \cdot lb = \frac{\cancel{lb}\, \cancel{sec^2}}{\cancel{ft}\, sec} \cdot \frac{lb_m \cancel{ft}}{\cancel{lb_f}\, \cancel{sec^2}} = \frac{lb_m}{sec}$

now @ $L=0$ $M_{in} = M$ $L = 6$ $M = 1$

$$\therefore \text{slope } \frac{M_2 - M_1}{L_2 - L_1} = \frac{1 - M_{in}}{6'' - 0} \quad \therefore \quad \frac{1 - M_{in}}{6''}$$

$$\therefore \quad M - M_{in} = \left(\frac{1 - M_{in}}{6}\right)(L) \quad \therefore \quad M = M_{in} + \frac{L}{6} - \frac{M_{in} L}{6}$$

$$\boxed{M = M_{in}\left(1 - \frac{L}{6}\right) + \frac{L}{6}}$$

now at $L = 10$ $M_e = M_{in}\left(1 - \frac{2}{3}\right) + 1.67 \gg 1$

put back into $\frac{A}{A^*} = f(M, k) \Rightarrow A_e$, use $M = M_{in}\left(1 - \frac{L}{6}\right) + \frac{L}{6}$ for $0 \le L \le 10$

~~$\frac{p}{p_0} = ($~~

$$\tilde{p} = \frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}} \qquad p = \frac{1}{\tilde{p}} \times p_0$$

$$\tilde{p}(L) = \left(1 + \frac{k-1}{2} M(L)^2\right)^{\frac{k}{k-1}} \qquad p = \frac{1}{\tilde{p}} \times p_0 \qquad p = P(L)$$

$$= (\tilde{T})^{\frac{k}{k-1}}$$

ARAT = A/A_s

AM = .6

– IFL = –999

GO TO 10

$$\tilde{T} = \frac{T_0}{T} = \left(1 + \frac{k-1}{2} M(L)^2\right) \qquad T = \frac{1}{\tilde{T}} \times T_0 \text{ this gives } T = T(L)$$

~~$V = \sqrt{\frac{2kR}{k-1}(T_0 - T)}$ this gives $V = V(L)$~~

$$A(L) = \frac{1}{M(L)}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2} M(L)^2\right)\right]^{\frac{k+1}{2(k-1)}} \times A^* \quad \text{this gives } A = A(L)$$

$$F(M) = \frac{1}{M(L)}\left[(\quad)(\quad)\right]^{\frac{k+1}{2(k-1)}} - \frac{A}{A^*}$$

$$F'(M) = \frac{\cancel{M}\, \frac{k+1}{2(k-1)}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2} M^2\right)\right]^{\frac{k+1}{2(k-1)} - 1} \cdot \frac{2}{k+1} \cdot \frac{k-1}{\cancel{2}} \cdot \cancel{2}M - \left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2} M^2\right)\right]^{\frac{k+1}{2(k-1)}}}{M^2}$$

$$f' = \frac{\left[\left[(\quad)\right]\right]\left(\frac{1}{\quad} - 1\right)}{M} M^2$$

$$= \frac{M\, d(\quad) - (\quad)}{M^2}$$

$$\frac{M \frac{k+1}{2}(\quad)^{\frac{k+1}{2} - 1}\left(\frac{2}{k+1} \cdot \frac{k-1}{2}\right)}{M^2} - \left(\frac{\quad}{M^2}\right) \quad // \quad \frac{F_m + A_r}{m}$$

```
      dimension a(11)
      data a/10.,4.505,3.066,2.46,2.169,2.037,2.0,2.03,2.112,
      12.238,2.404/
      r=1545.32*32.174/4.003
      gc=32.174
      ak=5./3.
      po=100.
      to=540.
      rat=(ak+1.)/(ak-1.)
      u=0.5
      flor=po*gc*sqrt(ak*(2./(ak+1.))**rat/(r*to))
      as=u/flor
      write(6,100) as
100   format(2x,e14.7)
      am=0.05
      do 10 i=1,11
      arat=a(i)/as
      do 5 j=1,100
      trat=1.+(ak-1.)*am*am/2.
      fm=(2.*trat/(ak+1.))**(rat/2.)/am-arat
      fpm=(2.*trat/(ak+1.))**(rat/2.-1.)-(fm+arat)/am
      amn=am-fm/fpm
      write(6,4) j,i,amn,am,fm,fpm,trat
4     format(2(2x,i3),5(2x,e14.7))
      if(abs(amn-am).le.1.0e-5) go to 7
      am=amn
5     continue
      write(6,6)
6     format(2x,21hdivergent iter scheme)
      am=.1*i
      go to 10
7     trat=1.+(ak-1.)*amn*amn/2.
      prat=(trat)**(ak/(ak-1.))
      p=po/prat
      t=to/trat
      write(3,8) i,amn,p,t,a(i)
8     format(2x,i3,4(2x,e14.7))
10    continue
      write(3,15) r,as
15    format(2(2x,e14.7),25h  end subsonic isent flow)
      stop
      end
```

Autumn 78/79
SJK

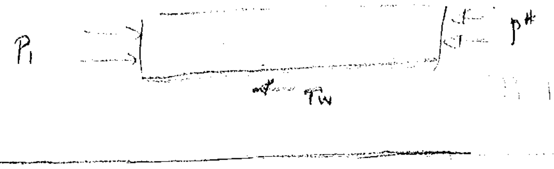
ME 255

Problem Set 4 -- Due Wednesday, November 23

Text problems 6.1, 6.3, 6.4, 6.8, 6.11

$$\frac{1 - kM_2^2\left(P_2/P_1\right)^2}{kM_2^2\left(P_2/P_1\right)^2} = \frac{\left(P_2/P_1\right)^2 - kM_2^2\left(P_2/P_1\right)^2}{kM_2^2\left(P_2/P_1\right)^2}$$

$$\frac{\left(P_1/P_2\right)^2 - 1}{kM_2^2} + 1$$



$$P_1 A - p^* A - \tau_w A_w = -\dot{m} V_1 + \dot{m} V_{out}$$
$$(p_1 - p^*)A - \tau_w A = \dot{m}(V_{out} - V_{in})$$
$$(10 - 4.68)\,1 - \tau_w A = \frac{29.6 \frac{lbm}{sec}}{32.174 \frac{lbm\,ft}{lbf\,sec^2}} (1025.65 - 548.24)$$
$$766.08 - (F\ )$$
$$326.86 \quad = 439.22$$

$A = 1$ sq ft

$P_1 = 10$

$T_1 = 500°R$

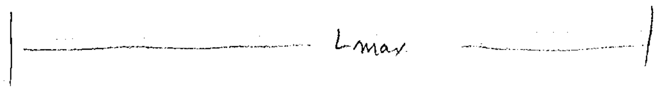
$\frac{\dot{m}}{A} = \rho V = 29.6 \frac{lbm}{sec\,ft^2}$

$M = 1 \quad Me = M^* = 1$

find $P_e = p^*$

$T_e = T^*$

find τ_w between $* = 1$



since we have $p_1, T_1 \Rightarrow \rho_1 = \frac{P_1}{RT_1} = \frac{10 \times 144}{53.34 \cdot 500°R} = \frac{\frac{lb}{ft^2}}{\frac{lbf\,ft}{lbm\,°R} \cancel{°R}} = \frac{lbm}{ft^3} = .053991 \frac{lbm}{ft^3}$

$$V_1 = \frac{\dot{m}}{A\rho_1} = \frac{29.6}{.05399} \frac{lbm}{sec\,ft^2} \cdot \frac{ft^3}{lbm} = 548.24 \frac{ft}{sec}$$

$$c_1 = \sqrt{kRT_1} = 49.02\sqrt{500} = 1096.07\ ft/sec$$

$$M_1 = \frac{V_1}{c_1} = .5$$

~~now if we find $\frac{4fL_{max}}{D}$~~

for $M_1 = .5 \quad P_1/p^* = 2.1381 \quad T_1/T^* = 1.1429 \quad \frac{4fL_{max}}{D} = 1.06908$

$V/V^* = .53453$

$$\therefore \quad P^* = \frac{P^*}{P_1} \cdot P_1 = \frac{1}{2.1381} \cdot 10 = 4.68\,psia$$

$$T^* = \frac{T^*}{T_1} \cdot T_1 = \frac{1}{1.1429} \cdot 500 = 437.48\ °R \qquad c^*$$

$$V^* = \frac{V^*}{V_1} \cdot V_1 = \frac{1}{.53453} \cdot 548.24 = 1025.65\ ft/sec$$

$$\sim -\tau_w\, dA_w = \dot{m}\, dV + A\, dp$$
$$-(F_w) = \dot{m}(V_2 - V_1) + A(p_2 - p_1)$$
$$= \frac{29.6}{32.174}\left(1025.65 - 548.24\right) + 1\,(4.68 - 10) \times 144$$
$$-766.08$$
$$= -326.86$$

$$\boxed{F_w = 326.86\ lb}$$

surface area

$$F = \tau_w \cdot L_{max} \cdot \cancel{2} \pi D$$

now $f = \frac{\tau_w}{\frac{1}{2}\rho_1 V_1^2}$ and $4f\frac{L_{max}}{D} = 4\frac{\tau_w}{\frac{1}{2}\rho_1 V_1^2}\frac{L_{max}}{D}\frac{\pi}{\pi}\frac{D}{D} = \frac{\cancel{4} F}{\frac{1}{2}\rho_1 V_1^2 \frac{\pi D^2}{4}} = \frac{F}{A_{cs} \cdot \frac{1}{2}\rho_1 V_1^2}$

$$\therefore \quad 4f\frac{L_{max}}{D} = \frac{F}{A\cdot\frac{1}{2}\rho_*V_*^2} = \frac{F}{A\cdot\frac{1}{2}\frac{\dot m}{A}V_*} = 1.06908 = \frac{F}{(1ft^2)\cdot\frac{1}{2}\left(\frac{29.6}{32.2}\frac{lbsec}{ft^2}\cdot\frac{ft}{sec}\right)\left(548.24\frac{ft}{sec}\right)}$$

$$\therefore \quad F = \frac{1}{2}\left(\frac{29.6}{32.2}\right)548.24\cdot 1.06908 = 269.611 \quad lb$$

$$A\cdot\frac{1}{2}\rho_*V_*^2\cdot 4f\frac{L_{max}}{D} = F$$

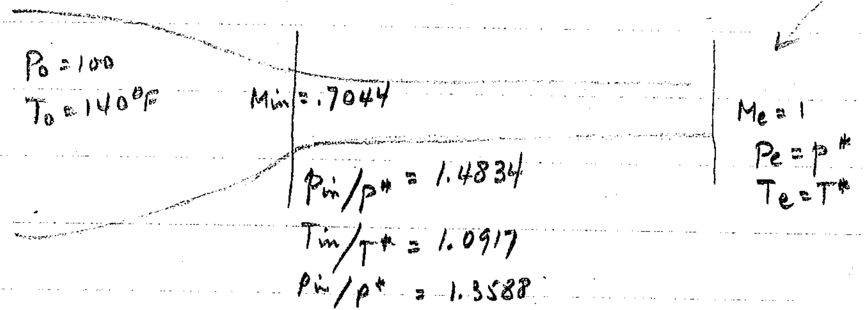
$$A\cdot\frac{1}{2}\frac{\dot m}{A}V_*\cdot 4f\frac{L_{max}}{D} = 1\cdot\frac{1}{2}\left(\frac{29.6}{32.174}\right)(1025.65)\cdot 1.06908 =$$

$$4f\frac{L_{max}}{D}$$

6.3 $L = 20'' = L_{max}$ $\qquad 4f\frac{L_{max}}{D} = 4(.005)\frac{20}{20} = .2$

$D = 2''$

$p_0 = 100$ psia
$T_0 = 140°F$) we assume these to be static



for $p_B < p^*$ flow will be choked & $\dot m = max$

but from isent tables for $M_{in} = .7044$ $\qquad \frac{p_i}{p_0} = .7181 \quad \frac{T_i}{T_0} = .9097 \quad \frac{\rho_i}{\rho_0} = .7894$

$$\therefore \quad p^* = \frac{p^*}{p_{in}}\cdot\frac{p_{in}}{p_0}\cdot p_0 = \frac{1}{1.4834}\;.7181\cdot 100 = 48.41 \text{ psi}$$

$$T^* = \frac{T^*}{T_i}\cdot\frac{T_i}{T_0}\cdot T_0 = \frac{1}{1.0917}\;.9097\cdot 600 = 500°R$$

$$\dot m = \rho VA = \frac{p^*}{RT^*}\cdot\frac{\pi D^2}{4}M_*\sqrt{kRg_cT^*} = \frac{48.41}{53.34\cdot 500}\cdot\frac{\pi\cdot(2)^2}{4}\cdot 1\cdot\sqrt{1.4\times 1716\times 500} = 6.25\frac{lbm}{sec}$$

6.4.

$\frac{4fL}{D}$	$\frac{p_{in}}{p^*}$	M_{in}	$\frac{p_{in}}{p_o}$	$p^*/p_o = p^*/p_{in} \cdot \frac{p_{in}}{p_o}$
10	4.6305	.2375	.9615	.2076
9	4.4487	.2457	.9589	.2155
8	4.2429	.2576	.9549	.2251
7	4.0119	.2733	.9494	.2366
6	3.7809	.2889	.9437	.2496
~~6~~				
5	3.5336	.3081	.9363	.2650
4	3.2484	.3352	.9252	.2850
3	2.9354	.3698	.9099	.3100
2	2.5672	.4208	.8853	.3449
1	2.0968	.5101	.8373	.3993
0	1.0	1.	.5283	1.0 × .5283 = .5283

2nd method

$$4f\frac{L}{D} = \frac{1-(P_2/P_1)^2}{kM_1^2} - \ln\left(\frac{P_1}{P_2}\right)^2 \qquad kM_1^2 = \frac{1-(P_2/P_1)^2}{4f\frac{L}{D} + \ln(P_1/P_2)^2} =$$

$$\cancel{\frac{1}{\sqrt{4f}} = -.8 + 2\log_{10}(Re\sqrt{4f})} \qquad \rho = \frac{P_1}{RT_1} = \frac{104.7\times 144}{\frac{1545\cdot 32}{18}\cdot 32.174\cdot 530} = .0102987\frac{\text{slugs}}{\text{ft}^3}$$

$$Re = \rho\frac{VD}{\mu} \qquad Re(V) = \frac{\rho VD}{\mu} = \frac{.0103\cdot V\cdot 3}{2.3\times 10^{-7}} = 1.343\times 10^5\,(V)$$

STO 7 = R = 2762.17

STO 6 = $\sqrt{k}$ ^{2} 1.1401754

STO 5 = $\ln(P_1/P_2)$

STO 4 = $1-(P_2/P_1)^2$

STO 3 = $\frac{\rho D}{\mu}$

STO 2 = $C = \sqrt{kRT}$

Assume $4f = .02$ $\Rightarrow$ $kM_1^2 = .000669$ $M = .0242289$

$V = 33.42$ $Re = 4.489\times 10^6 \Rightarrow 4f = .01$

Assume $4f = .01$ $kM_1^2 = .00134$ $M = .0342298$

$V = 47.28$

$$\left[4f\frac{L}{D} + \ln\left(P_1/P_2\right)^2\right]kM_1^2 = 1-\left(P_2/P_1\right)^2 \qquad 4f = .02$$

$$kM_1^2 = \frac{1-(P_2/P_1)^2}{4f\frac{L}{D} + \ln(P_1/P_2)^2} = .0006693 \qquad M_1 = .0226907$$

$V_1 = M_1 C_1 = M_1$ $V_1 = 31.302$ ft/sec $4.205\times 10^6 = Re$ $4f = .009$

Assume $4f = .015$ $\Rightarrow$ $kM_1^2 = .000892$ $M_1 = .026$

$V_1 = 36.$ $Re = 4.854\times 10^6$ $4f = .009$

Assume $4f = .01$ $\Rightarrow$ $kM_1^2 = .004$ $M_1 = .056$

$V_1 = 77.34$ $Re = 1.039\times 10^7$ $4f = .008$

Assume that ln kM^2 is small in comparison and $4f = .02$ find L_{max}

$$\therefore\ 4f\frac{L_{max}}{D} = \frac{1}{kM^2} - 1$$

.6

$$1/\sqrt{1.4\cdot 9\,(1.3)} \Rightarrow M = .0234$$

$$.4 = \frac{kM_1^2}{2}\left(4f\frac{L}{D}\right) \qquad \therefore\ P_1\sqrt{k}M_1 = p^{*t} \qquad p^{*t} = 2.789 \Rightarrow L < L_{max}$$

$$\frac{.4}{.03} = 13.33 = 4f\cdot 70400.$$

$$\frac{13.33}{70400} = 4f$$

Natural gas $\bar{m} = 18$ $k = 1.3$ $D = 36$ in $\overset{3 ft.}{}$ $L = 40$ miles $p_i = 90$ psig $p_e = 10$ psig

find $\dot{m}_{max}$ @ 70°F and $p_a = 14.7$ psi so that flow is isothermal

now $\rho VA = \dot{m}$ $p_1 = 104.7$ psi $T_1 = 70°F$ $R = \frac{1545.32}{18} \times 32.2 = 2762.2$

$\therefore \rho_1 = \frac{p_1}{RT_1}$ $A_1 = \frac{\pi D_1^2}{4}$ ~~$\rho_1 A_1 = \frac{\pi D_1^2 p_1}{4RT_1}$ = slugs/ft~~ =

$\rho_1 = .0103$ slugs/sec $A_1 = 7.0686\ ft^2$

$V_1 = M_1\sqrt{kRT_1} = 46.86$ ft/sec $\frac{24.7}{104.7}$

$\therefore$ Momentum equation relating P, M & $4f\frac{dx}{D}$ are valid

also p_0, p, M

The intersection of a tangent line to the p_2/p_1 scale at .11 on Fig 6.14 gives a value of $kM_1^2 = .0015$ $1 - \frac{kM_1^2}{2}\left(4f\frac{L}{D}\right) = .54$

$\therefore M_1 = \sqrt{\frac{.0015}{1.3}}$ and $V_1 = M_1 \cdot \sqrt{kRT_1} = 46.86$ ft.

$= .034$

$\therefore \dot{m} = 3.411$ slugs/sec slugs

$= 109.756$ lbm/sec

$= 395122$ lbm $\frac{\dot{m}}{\rho} = VA = ft^3/sec = 331,235\ ft^3/sec$

$\frac{hr}{day}\ 24 \times 3600\ \frac{sec}{hr} = \frac{sec}{day} = 2.8619 \times 10^7$ cuft/day

$4f\frac{(5280 \times 40)}{3} = \frac{1 - \cancel{81}\left(\frac{1}{9}\right)^2}{.0015} - \ln(81) =$

$4f(70400) = \overset{658.436}{\cancel{\frac{80}{.0015}}} - 4.394 = 654.042$

$4f = .00929$

$10.375 =$

Cannot use this since flow is not choking

$$dQ = c_p\,dT + d\left(\frac{V^2}{2}\right) = c_p\,dT_0$$

$$T\,ds = dh - v\,dp$$

$$\therefore \quad dQ - T\,ds = \cancel{c_p\,dT} + d\left(\frac{V^2}{2}\right) - \cancel{c_p\,dT} + v\,dp$$

$$= d\left(\frac{V^2}{2}\right) + v\,dp$$

$$= V^2\frac{dV}{V} + \frac{\cancel{k}\cancel{p}}{\rho\cancel{k}}\frac{dp}{\cancel{p}}$$

$$\frac{V^2}{M^2} \quad c^2$$

$$= V^2\left[\frac{kM^2}{2(1-kM^2)}\right]4f\frac{dx}{D} + \frac{kp}{\rho}\left[-\frac{M^2}{2(1-kM^2)}\,4f\frac{dx}{D}\right]$$

$$V^2 \cdot 4f\frac{dx}{D}\left[\frac{kM^2}{2(1-kM^2)} - \frac{1}{2(1-kM^2)}\right]$$

$$\left[\frac{\overset{-1}{\cancel{kM^2-1}}}{2(\cancel{1-kM^2})}\right] \qquad \therefore \quad dQ - T\,ds = -\frac{V^2}{2}\left[4f\frac{dx}{D}\right]$$

$$dQ - T\,ds \quad \Big\| \qquad \Delta S \geq \frac{dQ}{T}$$

$$\therefore \quad T\Delta S - dQ \geq 0$$

$$\text{or} \quad dQ - T\Delta S \leq 0 \qquad \Rightarrow f \geq 0$$

Cesar Levy
ME 255
Prof. S. J. Kline

Problem Set #4

1 Given a stream of air in Fanno flow with inlet conditions: $p_1 = 10$ psia, $T_1 = 40°F$, $\frac{\dot m}{A} = 29.6 \frac{lbm}{sec\,ft^2}$ and $A = 1$ sq ft. The tube exhausts with $p_B < p_e = p^*$.

Find M_1, $M_e = M^*$, $T_e = T^*$, $p_e = p^*$ and find the force due to shear acting on the tube.

method of solution

a. Use $p_1 = \rho_1 R T_1$ to find ρ_1 $\quad \rho_1 = \frac{P_1}{RT_1} = \frac{10 \cdot 144}{(53.34)(500)} = .05399 \frac{lbm}{ft^3}$

$$\dot m = \frac{\dot m}{A} \cdot A = 29.6 \cdot 1 = 29.6 \frac{lbm}{sec} \; ; \quad \text{since } \dot m = \rho_1 V_1 \Rightarrow V_1 = \frac{\dot m}{\rho_1} = \frac{29.6}{.05399} = 548.24 \text{ ft/sec}$$

$$c_1 = \sqrt{kRg_cT_1} = 49.02\sqrt{500} = 1096.07 \text{ ft/sec} \quad \text{and} \quad \boxed{M_1 = \frac{V_1}{c_1} = \frac{548.24}{1096.07} = .5} \checkmark$$

b. Since flow is choked $\boxed{M_e = M^* = 1} \checkmark$ and for $M_1 = .5$, $\frac{P_1}{p^*} = 2.1381$, $\frac{T_1}{T^*} = 1.1429$

$$\therefore \boxed{T_e = T^* = \frac{T^*}{T_1} \cdot T_1 = \frac{1}{1.1429} \cdot 500 = 437.48°R} \checkmark \qquad \boxed{V_e = V^* = 49.02\sqrt{T^*} = 1025.30 \text{ ft/sec}} \checkmark$$

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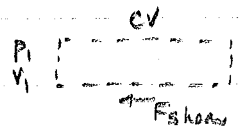
$$\boxed{P_e = P^* = \frac{P^*}{P_1} \cdot P_1 = \frac{1}{2.1381} \cdot 10 = 4.68 \text{ psia}} \checkmark$$

c. We use the conservation of momentum that $\sum F_{external} = \frac{\partial}{\partial t}\int_{\mho} \rho V \, d\mho + \int (\rho \underline{V} \cdot \underline{n}) \underline{V} \, dA$

this reduces for steady 1-D flow to



$$p_1 A - p^* A - F_{shear} = \dot m (V_{out} - V_{in})$$

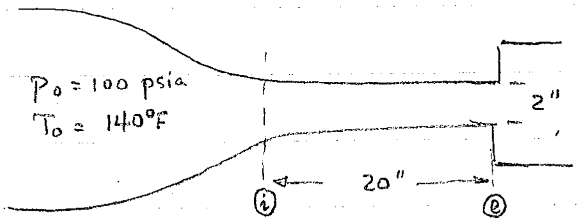
$$\therefore \; -F_{shear} = \dot m (V^* - V_1) + A(p^* - p_1) =$$

$$= \frac{29.6}{32.174}(1025.30 - 548.24) + 1 \cdot 144 (4.68 - 10)$$

$$= 438.894 \text{ lb} - 766.08 \text{ lb} = -327.19 \text{ lb}$$

$\therefore$ $F_{shear} = 327.19$ lb on fluid to left. Force on tube is 327.19 lb to the right. Force needed to hold tube in place must be in opposite direction to that on tube $\therefore$ $\boxed{F = 327.19 \text{ lb to the left}} \checkmark$

6.3 For the given flow find $\dot{m}_{max}$, if $\bar{f} = .005$. Also find the range of back pressures for which this $\dot{m}_{max}$ will be achieved.



For max flow rate $M_e = M^* = 1$, $p_e = p^*$ and $T_e = T^*$

$\dot{m}_{max}$ will be achieved for all $p_B \leq p^*$

$\therefore$ $L_{max} = 20''$ thus $4\bar{f}\frac{L_{max}}{D} = 4(.005)\left(\frac{20''}{2''}\right) = .2$

For this value of $4\bar{f}\frac{L_{max}}{D}$ we find $M_i = .7044$, $p_i/p^* = 1.4834$, $T_i/T^* = 1.0917$

To achieve an $M_i = .7044$ from the isentropic tables $\frac{P_i}{P_0} = .7181$, $\frac{T_i}{T_0} = .9097$

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$$\therefore \quad p^* = \frac{P^*}{P_i} \cdot \frac{P_i}{P_0} \cdot P_0 = \frac{1}{1.4834} \cdot .7181 \cdot 100 = 48.41 \text{ psia} \quad ✓$$

$$T^* = \frac{T^*}{T_i} \cdot \frac{T_i}{T_0} \cdot T_0 = \frac{1}{1.0917} \cdot .9097 \cdot 600°R = 500°R$$

$\therefore$ $\dot{m}_{max}$ will be achieved for all $p_B \leq 48.41$ psia

$$\dot{m}_{max} = \rho^* V^* A = \frac{p^*}{RT^*} \cdot \frac{\pi D^2}{4} \cdot M^* \sqrt{kRg_cT^*} = \frac{48.41}{(53.34)(500)} \cdot \frac{\pi (2)^2}{4} \cdot 1 \cdot \sqrt{(1.4)(1716.2)(500)}$$

$$\boxed{\dot{m}_{max} = 6.25 \frac{\text{lbm}}{\text{sec}}} \quad ✓$$

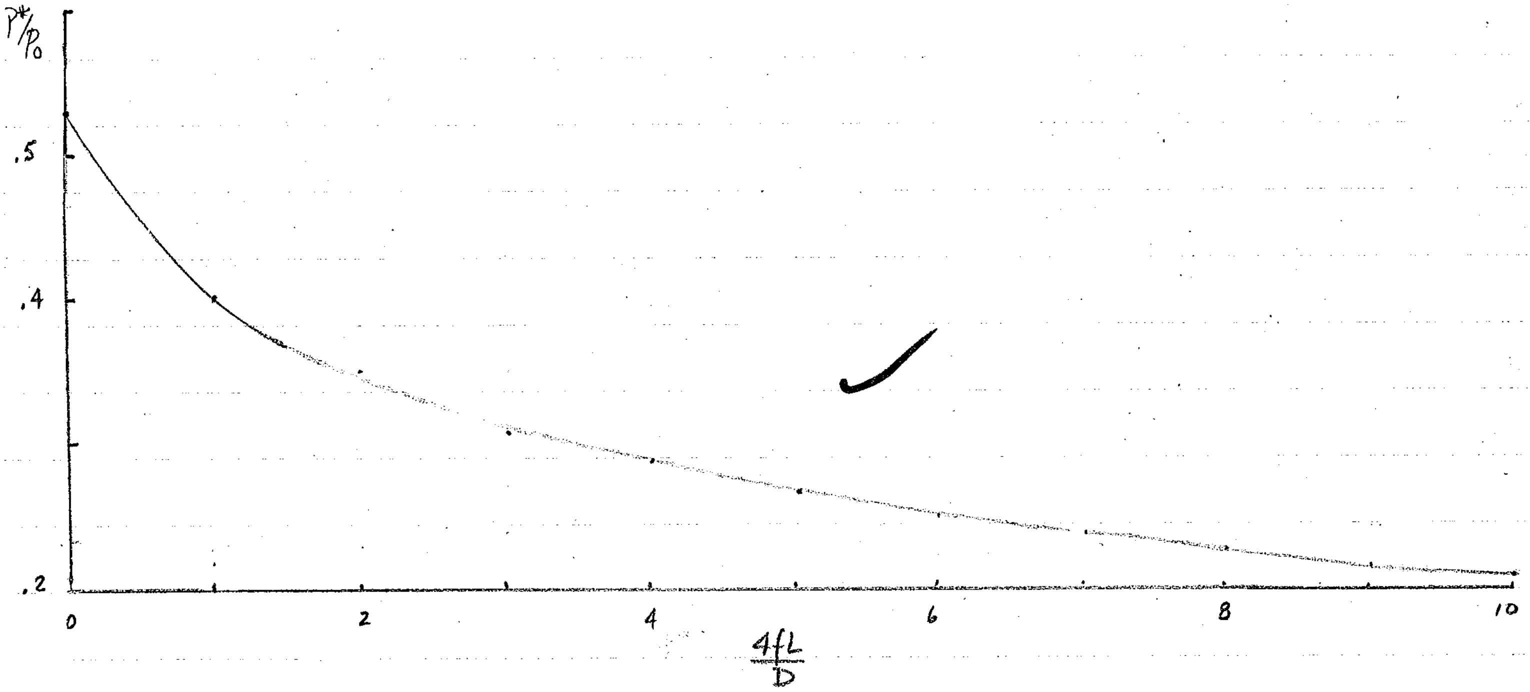
b) $P_{back} \leq P^* = 48.4$ psia

6.4 Consider a long round insulated tube fed with air by a frictionless nozzle. We define $CPR = P^*_{FANNO}/P_{0_{\text{isent nozzle}}}$. Plot $4fL/D$ vs CPR for $0 \leq CPR \leq 10$.

Method of Solution: using value of $\frac{4fL}{D}$ get $M = M$ at the outlet of converging nozzle $= M_{inlet\ of\ duct} = M_{in}$. For M_{in} find P/p^* using frictional tables and for M_{in} find P/P_0 using the isentropic tables. Now $CPR = \frac{P^*}{P_0} = \frac{P^*}{P} \cdot \frac{P}{P_0}$.

Thus

$4fL/D$	10	9	8	7	6	5	4	3	2	1	0
P_{in}/P^*	4.6305	4.4487	4.2429	4.0119	3.7809	3.5336	3.2484	2.9354	2.5672	2.0968	1.0
M_{in}	.2375	.2457	.2576	.2733	.2889	.3081	.3352	.3698	.4208	.5101	1.0
P_{in}/P_0	.9615	.9589	.9549	.9494	.9437	.9363	.9252	.9099	.8853	.8373	.5283
P^*/P_0 =CPR	.2076	.2155	.2251	.2366	.2496	.2650	.2850	.310	.3449	.3993	.5283



[illegible] $\times 10^{-6}$

6.8 Given a gas ($k=1.3$ $\bar{M}=18$) pumping through a 3 ft diameter pipe between 2 compressors 40 miles apart. At the upstream station $p = 90$ psig and at the downstream station $p_1 = 10$ psig. Calculate $\dot{m}$ (cu ft/day @ 70°F and 1 Atm) if enough heat transfer through the pipe walls maintain the gas at 70°F.

If we assume that $\ln kM^2$ is small in comparison with $\left(\frac{1}{kM^2} - 1\right)$ then for a range of values of $4f$ between $(\sim 0, .02)$ $p^{*t} < p_2$ $\therefore$ $L < L_{max}$; hence we will use

$$4f\frac{L}{D} = \frac{(P_1/P_2)^2 - 1}{kM_2^{\,2}} - \ln\left(P_1/P_2\right)^2 \quad \checkmark$$

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$$R = \frac{1545.32}{18} \times 32.174 = 2762.17 \ \frac{\text{lb}\cdot\text{ft}}{\text{slug}\ ^\circ\text{R}} \qquad \rho_2 = \frac{P_2}{RT_1} = \frac{24.7 \times 144}{R \cdot 530^\circ\text{R}} = .00243 \ \frac{\text{slugs}}{\text{ft}^3}$$

$\mu = 2.3 \times 10^{-7}$ slugs/ft-sec

Method: assume a value of $4f$; compute $kM_2^{\,2} = \dfrac{(P_1/P_2)^2 - 1}{4f\,L/D + \ln(P_1/P_2)^2}$ ✓ and find

$V_2 = M_2 c_1 = \frac{1}{\sqrt{k}} \cdot \sqrt{kM_2^{\,2}} \cdot c_1$; put this into $Re = \rho\frac{VD}{\mu}$ and check against value of $4f$ in figure 6.15 for smooth pipes

Assume $4f = .015$ $\quad kM_2^{\,2} = .0016$ $\quad V_1 = 153.2$ ft/sec $\quad Re = 4.85 \times 10^6$

for Commercial steel $\frac{\epsilon}{D} = .00015 \Rightarrow$ for $Re = 4.85 \times 10^6$ $\quad 4f = .016$

thus if we take $4f = .016$ $\quad kM_2^{\,2} = .0026$ $\quad V_2 = 61.2$ ft/sec $\quad Re = 1.94 \times 10^6$

for Commercial steel $\frac{\epsilon}{D} = .00015$ ✓ $\Rightarrow$ $Re \approx 1.94 \times 10^6$ ✓ $4f = .016$ ✓

hence we've found our value of 4f ✓

$$\dot{m} = \rho_2 V_2 A \times 86400 \frac{\text{sec}}{\text{day}} = 9.08 \times 10^4 \frac{\text{slugs}}{\text{day}} = 2.922 \times 10^6 \frac{\text{lbm}}{\text{day}} \quad \text{X}$$

6.11 Derive for isothermal flow a formula for $Tds - dQ$ as a function of $4f\frac{dx}{D}$

We have that $Tds = dh - \frac{dp}{\rho}$ and $dQ = c_p\,dT + d\left(\frac{V^2}{2}\right)$. Thus

$$Tds - dQ = dh - dp/\rho - dh - d\left(V^2/2\right) = -\left[\frac{dp}{\rho} + VdV\right] = -\left[\frac{kp}{\rho}\,\frac{dp}{kp} + V^2\frac{dV}{V}\right]$$

but for an isothermal process $\frac{dp}{kp} = -\frac{M^2}{2(1-kM^2)}\,4f\frac{dx}{D}$ and $\frac{dV}{V} = \frac{kM^2}{2(1-kM^2)}\,4f\frac{dx}{D}$

$$\therefore\ Tds - dQ = -\left[\frac{V^2}{\cancel{M^2}}\,\frac{-\cancel{M^2}}{2(1-kM^2)} + V^2\frac{kM^2}{2(1-kM^2)}\right]4f\frac{dx}{D} = \frac{V^2}{2}\left(4f\frac{dx}{D}\right) ✓$$

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now Corollary 6 on Pg. 34 for any infinitesimal system $dS \geq \frac{\delta Q}{T}$ or
$TdS - \delta Q \geq 0 \Rightarrow \frac{V^2}{2}\cdot 4f\frac{dx}{D} \geq 0$. Since dx is always taken positive in direction of flow and $V^2 \geq 0 \Rightarrow f \geq 0$.