ME 250A HEAT TRANSFER

COURSE OUTLINE

Fall Quarter, 1978

Week	Topic	Read	Problems
Sept. 27	Introduction	Chap 1	S-1
Oct. 2	Conduction	Chap 2,3	2.9, 20, 32, 38, 41 tried 3.10, 27, 28, 31, S-2
Oct. 9	Conduction	Chap 4	4.5, 8, 9, 18, 21
Oct. 16	Conduction	Chap 4	~~4.23,~~ 24, 27, 34, 41, S-3
Oct. 23	Convection	Chap 6,7	
Oct. 30	Convection	Chap 8,9	
Nov. 6	Change of Phase (Midterm Nov 8)	Chap 10	
Nov. 13	Heat Exchanger	Chap 11	
Nov. 20	Heat Exchangers	Chap 11	
Nov. 27	Radiation	Chap 5	
Dec. 4	Radiation	Chap 5	

Instructor: R. H. Eustis, Office 520F

Tutorial instructor: Pradip Parikh, Office 501D

Text: Kreith, Principles of Heat Transfer, 3rd Ed.

Problems due Wednesday of following week.

1 Midterm 50%
1 Final 50%

ME 250A - HEAT TRANSFER

TUTORIAL SESSION

MONDAY, 7:00 pm - Room 300

ARVIZU, Dan E.
BAZA, John R.
BOOLITROP, Francois, J-L
BUDZYNSKI, Daniel J.
CARUSO, Steven C.
CHIANG, Jin C.
DA PRAT, Giovanni
DEAN, William W.
DIMICELI, Emanuel V.
FURUHAMA, Kokichi
GOBRAN, Brian D.
HAUSKNECHT, Peter C.
LAMOREAUX, David G.
LEAR, William E.
LINNE, Mark A.
PRIOR, Christopher J.
ROBLES, James A.
ROOK, Brian P.
SCHULTZ, Andrew M.
SHINOHARA, Kiyoshi
SUN, Rickson
- EDBERG, Donald L. 23
MORRIS, Mark W.

WEDNESDAY, 4:15 pm - Terman 101

AGAN, Dwight D.
CATTELINO, Mark J.
- DAVIS, John B.
EVERETT, Louis J.
GLASS, Michael T.
HARPER, James R.
HOLCHMEYER, John O.
LAM, Lawrence K.
LONGFELLOW, Wayne L.
LOUNGE, Michel Y.
MAC LAREN, Vanessa
MANDELL, Jay A. 20
MASUTANI, Stephen M.
NAMCHAISIRI, Chan
O'BRIEN, Robert E.
PAM, Rick
POWELL, James R.
SCHEITRUM, Glenn P.
SCHMELING, David J.
STEWART, John B.
TRIBOLET, David C.
WAGGONER, Bruce E.
YOUSSEFMIR, Paul
MISRA, Mahanand

TUESDAY, 9:00 am - Terman 101

- BOSCH, Richard J.
BURKHART, Patrick J.
DAVILA, Jose B.
DEHEKKER, Glenn
ELSON, Thomas D.
FANORIANA, Lilian
FEARING, Harold A.
GREENE, David T.
HALPIN, John P.
HAMILTON, William E.
IINO, Toshiki
LAROCHE, Robert C.
LENDRUM, Donalds B.
- LEVY, Cesar
NIKOLCHEV, Julian
NOIROT-NERIN, Isabelle
PRESS, Weston J.
SPRINGHORN, Mary E.
SULLIVAN, Margaret M.
TEJADA, Godfredo S.
WALKER, Frances R.
WESTPHAL, Russell V.
YUEN, Ronald A.
FASSIHI, Mohammad R.
WRIGHT, Patrick A. 25

ME 250A

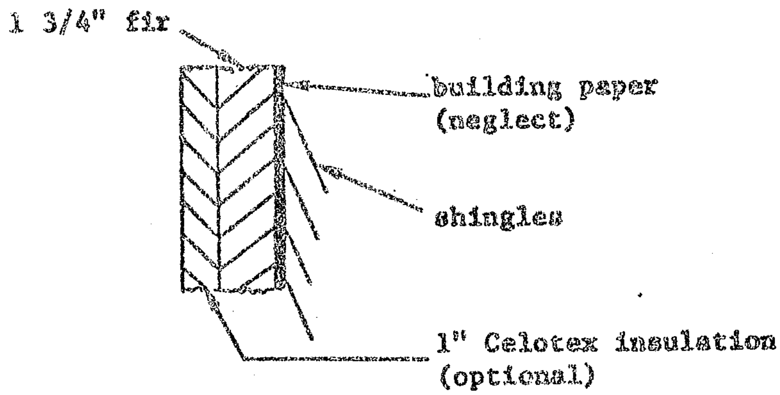
Heat Transfer

Fall, 1978

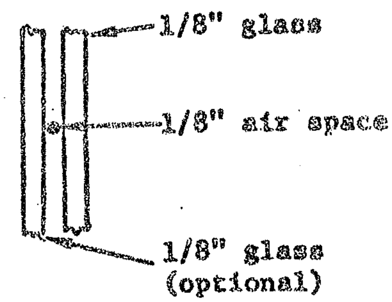
Problem S-1

The object of this problem is to see if insulation and double glass windows (Thermopane) are worth the cost for a mountain cabin. Consider an A-frame cabin with ends mostly windows and sides formed by the extended roof. The roof-side area is 2400 sq. ft. and the total area of the two ends is 600 sq. ft. The construction is shown below.

Roof-Side



End

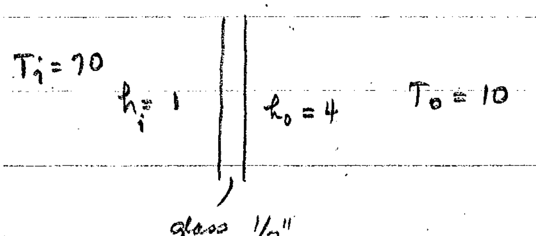


As design conditions, we will assume the inside temperature to be 70°F and the outside 10°F. The inside convection film coefficient is 1 Btu/hr ft^2 F and the outside is 4 Btu/hr ft^2°F. Shingles may be taken as equivalent to 1/8" air space plus 1/4" of pine.

The house is used 40 days per year during the winter and heat loss costs about \$2.00 per 10^6 Btu. Celotex costs about 25¢/sq. ft. installed and Thermopane windows cost about \$5.00/sq. ft. compared to \$1.75/sq. ft. for single pane glass.

Estimate the savings in fuel cost by installing insulation and by installing Thermopane windows. Compare this to the cost of the Celotex and to the extra window cost. What would you recommend?

	k, Btu/hr. ft°F
Glass	0.45
Pine	0.07
Celotex	0.028



cond $\quad q = -kA \dfrac{\Delta T}{\Delta x}$

conv $\quad q = hA\,\Delta T$

$$q = \frac{T_{in} - T_{out}}{\sum_{i=1}^{3} R_i}$$

$1/R_1 = h_i A_{wind} = 1 \times 600 = 600$ Btu/hr °F $\qquad R_1 = 1.667 \times 10^{-3}$

$\dfrac{1}{R_2} = \dfrac{k_{gl} A_{gl}}{\Delta x_{gl}} = \dfrac{.45 \times 600}{\frac{1/8}{12}} = 25920 \qquad R_2 = 3.86 \times 10^{-5}$

$1/R_3 = h_o A_{wind} = 4 \times 600 = 2400 \qquad R_3 = 4.167 \times 10^{-4}$

optional $1/R_4 = \dfrac{k_{air} A_{wind}}{\Delta x_{air}} = \dfrac{.0140\,(600)}{1/8/12} = 806.4 \qquad R_4 = 1.240 \times 10^{-3}$

optional $\dfrac{1}{R_5} = \dfrac{k_{gl} A_{gl}}{\Delta x_{glass}} = \dfrac{.45 \times 600}{1/8/12} = 25920 \qquad R_5 = 3.86 \times 10^{-5}$

without insulation $q = \dfrac{\Delta T}{\sum R_i} = \dfrac{60}{2.1223 \times 10^{-3}} = 28271.22$ Btu/hr.

$q = \dfrac{60}{3.4009 \times 10^{-3}} = 17642.39$ Btu/hr. with extra glass

q without insul = 32931.01 + 28271.22 = 61202.23 Btu/hr.

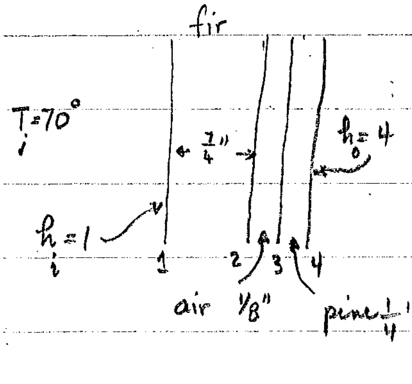
in a typical year $\quad$ 40 days × 24 hrs/day × q Btu/hr × cost/10^6 Btu = \$117.51

q with insul (extrawindow + etc) = 19588 + 17642.39 = 37230.39 Btu/hr

40 × 24 × q × cost = \$71.48

Savings of \$46.03 in heating bill / year

Compute q through roof without insulation and then with



cond $q = -kA \dfrac{\Delta T}{\Delta X} = -kA \dfrac{T_{hot} - T_{cold}}{\Delta x}$

conv $= q = hA(T_\infty - T_w)$

$$q = \frac{T_{in} - T_{out}}{\sum_{i=1}^{5} R_i}$$

$1/R_1 = h_i A_{roof} = 1 \times 2400 = 2400$ BTU/hr °F $\qquad R_1 = 4.167 \times 10^{-4}$

$1/R_2 = \dfrac{k_{pine} A_{roof}}{\Delta X_{pine}} = \dfrac{.07 \times 2400}{\frac{1/4''}{12''}} = 1152$ Btu/hr °F $\qquad R_2 = 8.681 \times 10^{-4}$

$1/R_3 = \dfrac{k_{air} A_{roof}}{\Delta X_{air}} = \dfrac{.0140 \times 2400}{\frac{1/8''}{12''}} = 3225.6$ Btu/hr °F $\qquad$ where k of air assumed @ T = 32°F

$R_3 = 3.100 \times 10^{-4}$

$1/R_4 = \dfrac{k_{pine} A_{roof}}{\Delta X_{pine}} = \dfrac{.07 \times 2400}{\frac{1/4''}{12''}} = 8064$ Btu/hr °F $\qquad R_4 = 1.240 \times 10^{-4}$

$1/R_5 = h_{out} A_{roof} = 4 \times 2400 = 9600$ Btu/hr °F $\qquad R_5 = 1.042 \times 10^{-4}$

$1/R_{6\,optional} = \dfrac{k_{cel} A_{roof}}{\Delta X_{cel}} = \dfrac{.028 \times 2400}{1''/12''} = 806.4 \qquad R_6 = 1.2401 \times 10^{-3}$

without insulation $q = \dfrac{70 - 10}{18.23 \times 10^{-4}} = \dfrac{60}{18.23 \times 10^{-4}} = \dfrac{60 \times 10^4}{18.23} = 32931.01$ BTU/hr

with insulation $q = \dfrac{70 - 10}{30.631 \times 10^{-4}} = \dfrac{60 \times 10^4}{30.631} = 19588$ Btu/hr

Cost of Celotex + thermopane

$$.25 \times 2400 + (3.25) \times 600 = 600 + 1950 = 2550$$

additional cost of install'g thermopane

Cost could be absorbed in $\frac{2550}{46.03}$ years $= 55.4$ years

Suppose we only added the roofing insulation but not the window insulation

① $q_{roof\ ins} + q_{wind\ un} = 19588 + 28271.22 = 47859.22$ Btu/hr cost $= 91.89$

② $q_{uninsulatn} = 61202.23$ Btu/hr cost of heating $= \$117.51$

Savings in heat is \$25.62

cost of roof insulation $= .25 \times 2400 = 600$ cost would be asorbed in 23.42 years

Would recommend no insulation unless the house is used for greater lengths of time

Fall, 1970

ME 250A - HEAT TRANSFER

Problem S-2

Computer Simulation of Steady-State Heat Conduction in an MHD Electrode

The Problem:

In proposed schemes for MHD power generation, a high temperature combustion gas, seeded with potassium, is passed through a channel in the presence of a high magnetic field, B (see Figure 1). The high temperatures cause the potassium to ionize (making a plasma), and the effect of the magnetic field is to cause the electrons and ions to migrate across the channel, creating a current, J. This current is taken off by electrodes which line the walls of the channel; when the electrodes are connected through a load.

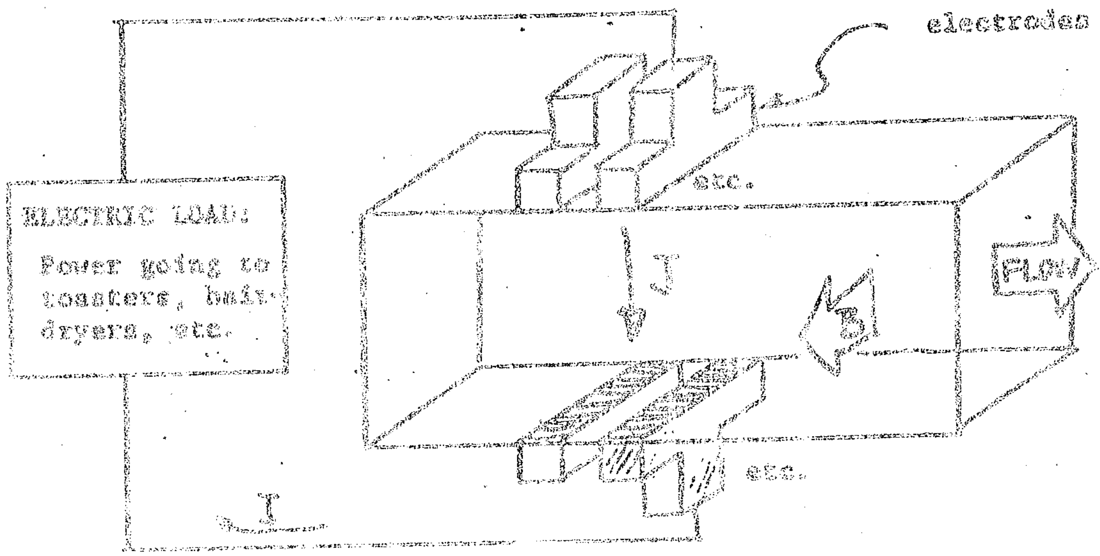


Figure 1.

Since the electrodes are exposed to the hot and corrosive plasma, they must be cooled.

The Physical Model:

Our task is to examine the heat transfer to a single electrode. A simple model of an electrode is shown in Figure 2. For corrosion resistance, stainless steel electrodes with thermal conductivity $k = 21$ W/mK are chosen. If there are many electrodes, then between each there is little variation in properties in the streamwise direction; hence we assume a two-dimensional model. The upper electrode surface is exposed to the hot flowing plasma and heat is transferred to the electrode by convection per the equation,

$$q''_{local} = \bar{h}\,(T_\infty - T_{wall,\ local}), \quad \text{heat transferred per unit area} \qquad (1)$$

We will use $T_\infty = 2300$ K. From the experiments at the Stanford HTGL facility we estimate $\bar{h} = 800$ W/m²K as the average convective coefficient. The lower electrode surface is water-cooled, with the coolant passing in sufficient quantity to maintain the lower surface at a uniform temperature of $T_c = 300$ K. The remaining sides of the electrode are in contact with ceramics which offer a poorer path for heat conduction. Hence, wishing to examine the most adverse conditions for the electrodes, we will consider these surfaces to be insulating.

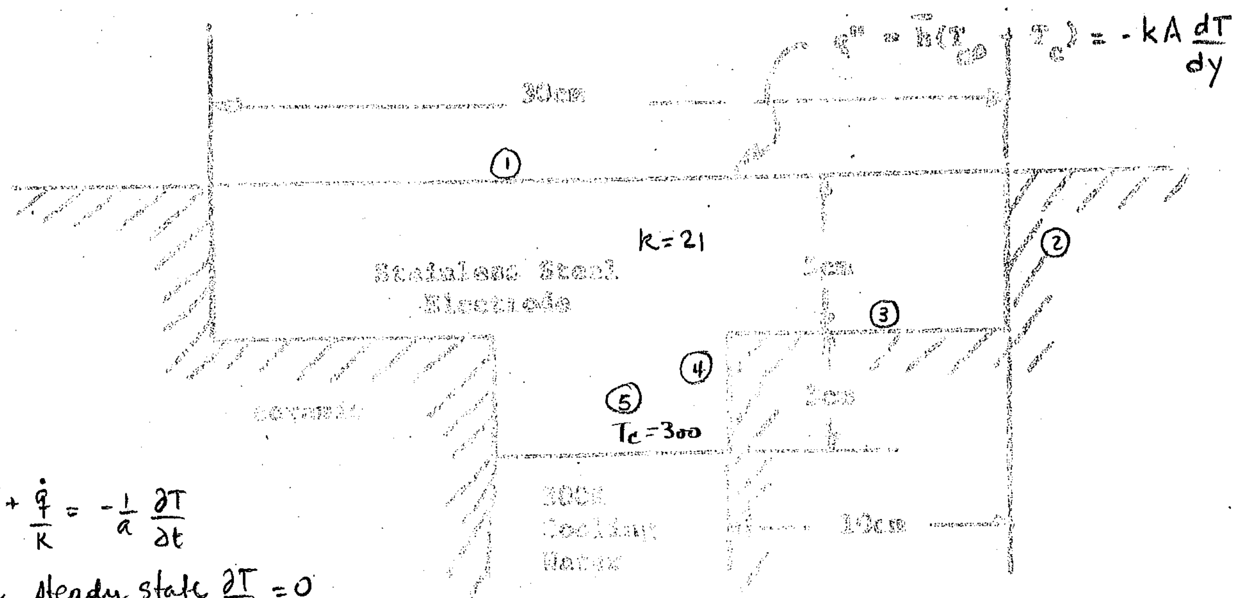


Figure 2

note $\Delta T + \dfrac{\dot{q}}{k} = -\dfrac{1}{a}\dfrac{\partial T}{\partial t}$

since steady state $\dfrac{\partial T}{\partial t} = 0$

and since heat generation $= 0$

$\Delta T = 0$ w/ BC

1 $-kA\dfrac{\partial T}{\partial y} = \bar{h}(T_\infty - T_c)$

2 $\dfrac{\partial T}{\partial x} = 0$ @ boundary

5 $T_{bound} = 300$

3 $\dfrac{\partial T}{\partial y} = 0$ @ bound

4 $\dfrac{\partial T}{\partial x} = 0$ @ bound

The Numerical Model:

Recognizing the symmetry of the electrode, an equally-spaced grid is laid out on half an electrode as shown in Figure 3.

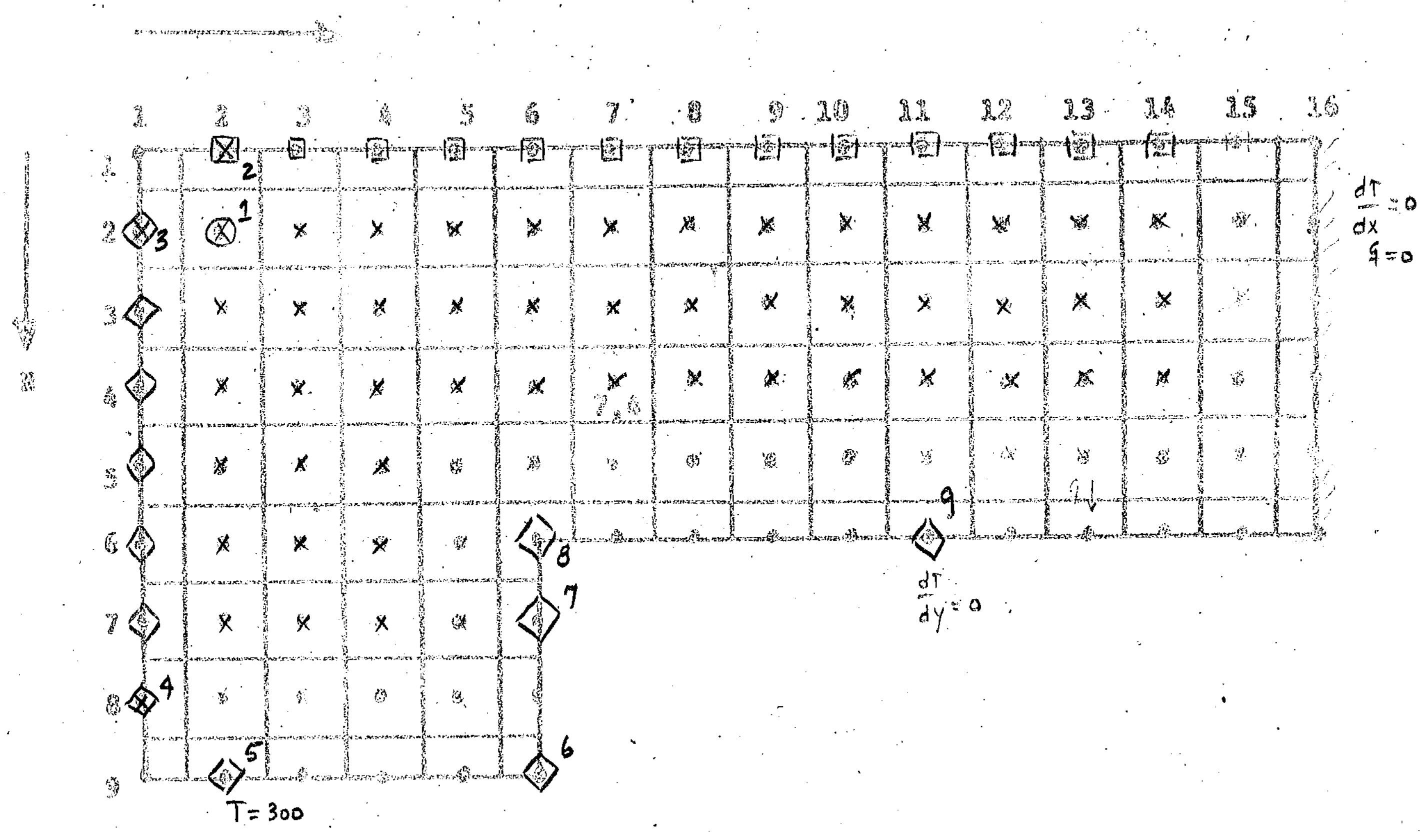


Figure 3.

Each node has a temperature T(M,N). By finite difference methods and use of some fundamental physics, the temperature at each node can be approximated as a linear function of the temperatures of the adjacent nodes, etc.,

$$T(M,N) = A_{MN}\,T(M+1,\,N) + B_{MN}\,T(M-1,\,N) + C_{MN}\,T(M,N+1) + D_{MN}\,T(M,\,N-1) + E_{MN} \qquad (2)$$

Given an initial estimate of the temperature distribution, the steady-state solution can be found by successive relaxation, similar to the program of Smith, Table 3.12.

The Assignment:

1. Identify each of the different types of nodes (not each node) and determine the coefficients, A,B,C,D,E, for each type of node. (Hint: there are 10 types here; T(1,3) is not the same type of node as T(16,3).

2. Designate each type of node by a number (1 through 10), and describe which node type corresponds to every node in the grid.

3. As initial conditions, the computer program assumes a linear temperature variation in the N-direction from T_∞ at the top surface to T_c at the bottom. As a possible improvement, by very simple means give a qualitative description of the actual surface temperature.

4. Questions 1 and 2 create input for a computer program. Create a data file in your LOTS account, execute the program and gather the results. (see below)

5. On the output, draw temperature isotherms (at equal increments). Then draw approximate heat flux lines. (Do the results seem qualitatively correct?)

6. Regarding the heat transfer, what design criteria must the electrode meet? Under these criteria, is the design satisfactory? If not, suggest an improved design. (Be concise.)

Using the Program:

1. Open an account if you have not already done so. Type "Open" and LOTS will prompt you.

2. Create a data file by typing:

   CREATE FOR21.DAT .

   then input the data line by line as described in the following format. The program reads data like this, from "unit" 21.

```
          READ (21, 1001) NTYP
          DO  1 J = 1, NTYP
1         READ (21, 1002) A(J), B(J), C(J), D(J), E(J)
          DO  2 J = 1,3
2         READ (21, 1001)  (ITYP(I,J),   I = 1, MMAX)
1001      FORMAT (25I3)
1002      FORMAT (5F10.3)
```

(NTYP = 10, the number of node types. MMAX = the number of nodes in each row, set internally).

Thus, your data file will look like this:

```
_10
A1     B1     C1     D1     E1
|      |                    |
|      |                    |
|                           |
A10 ----------------------- E10
ITYP(1,1)-------------------ITYP(16,1)
|                               |
|                               |
|                               |
ITYP(1,6) ---- ITYP(6,6) ---- ITYP(16,6)
|                 |
|                 |
|                 |
ITYP(1,9) ---- ITYP(6,9)
```

Read the "EDIT USER's CARD" available at LOTS, which tells all about creating and editing files.

3. Check that FOR21.DAT is in your library by typing DIR. You can check the contents of any file by typing "TYPE (file name, whatever it is)". (Forget the brackets.)

4. To use the program, simply type

   < P.PRUFROCK>ME2501.EXE.1 (don't forget brackets)

   When called, the program automatically will read FOR21.DAT, and create a file of results called FOR20.DAT.

   Execution should end with a STOP and a prompt, @. (If you program fails, you probably have the data messed up. Check this by looking at the results in FOR20.DAT, where inputs have been written after being read originally. If you still have problems, see a TA)

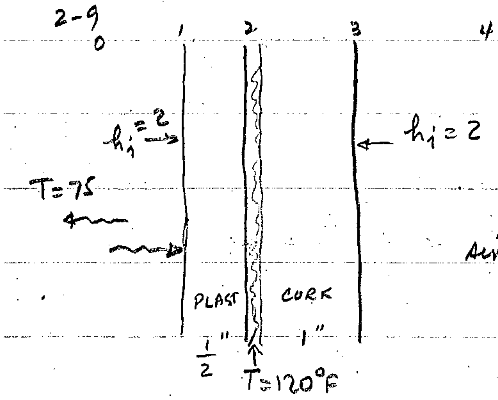
5. You can get the output by typing either

   TYPE FOR20.DAT (for display at terminal)

   or

   PRINT FOR20.DAT (for line-printer)

2-9



$k = 1.3$ $k = .025$

$$q_{conv} = h_i A (T_1 - T_0)$$

T=75

Since we know $T_2 - T_4$ find

$$q_{cond\ 2-4} = k_p \frac{A}{\Delta x_p}(T_1 - 120) = k_c \frac{A}{\Delta x_c}(120 - T_3) = h_i A (T_3 - 75)$$

$$\Delta T_{2-4} = (120 - T_3) + (T_3 - 75) = \frac{q_{cond}}{\frac{k_c A}{\Delta x_c}} + \frac{q_{cond}}{h_i A}$$

$$120 - 75 = q_{cond}\left[\frac{\Delta x_c}{k_c A} + \frac{1}{h_i A}\right]$$

or $$\frac{q_{cond}}{A} = \frac{120-75}{\left[\frac{1/12}{.025} + \frac{1}{2}\right]} = \frac{45}{3.33 + .5} = \frac{45}{3.83} = 11.74 \text{ Btu/hr}$$

$\therefore$ $\frac{q_{cond}}{A} = 11.74$ Btu/hr between 2-4

@ interface $\quad q_{conv} \leftarrow \Big| \rightarrow q_{cond\ plastic}$, $\ q_{supp} \rightarrow$ $\quad \therefore \quad \frac{q_{supp}}{A} = \frac{q_{cond}}{A} + \frac{q_{conv}}{A}$

$$= 11.74 + \frac{q_{conv}}{A}$$

① to find q_{conv} need T_1 (this will then give $\frac{q_{supp}}{A}$)

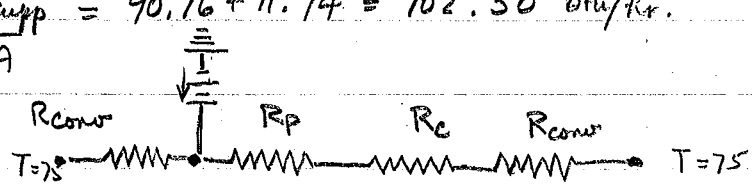
$$\therefore \quad \frac{q_{cond}}{A} = \frac{k_p}{\Delta x_p}(T_1 - 120) = 11.74 \qquad \therefore \quad T_1 = 11.74 \frac{\Delta x_p}{k_p} + 120$$

$$= 11.74 \frac{\left(\frac{1/2}{12}\right)}{1.3} + 120$$

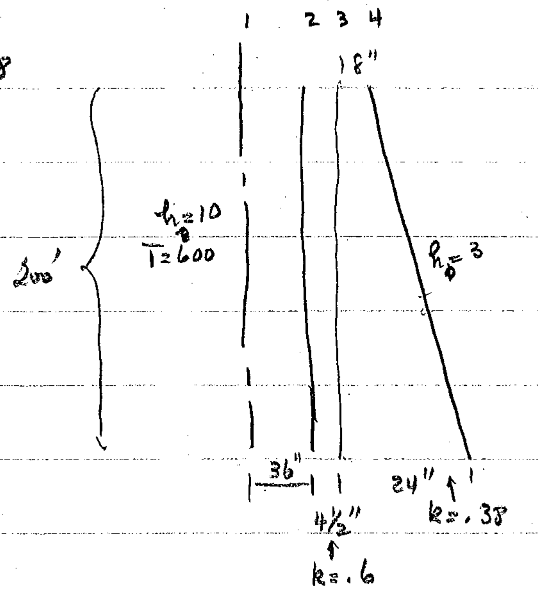
$$= 120.38°$$

$$\therefore \quad \frac{q_{conv}}{A} = h_i (T_1 - T_0) = 2(120.38 - 75) = 2(45.38) = 90.76$$

$$\frac{q_{supp}}{A} = 90.76 + 11.74 = 102.50 \text{ Btu/hr.}$$



2-28



$$T_1 - T_5 = (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + (T_4 - T_5)$$

$$= q\left[\frac{1}{h_i A_1} + \frac{\ln(r_3/r_2)}{2\pi k_{23}\, l} + \frac{\ln(r_4/r_3)}{2\pi k_{34}\, l} + \frac{1}{h_o A_o}\right]$$

$$A_1 = 2\pi r_2 l$$

$k \; \frac{\Delta y}{\Delta x} = m \quad b \quad \bar{x} L = \int x \, dL \qquad dL = \sqrt{1 + x'^2}\, dy$

$$= \int_0^b y \left(\sqrt{1 + \tfrac{1}{m^2}}\right) dy = \sqrt{1 + \tfrac{1}{m^2}} \, \frac{y^2}{2}\Big|_0^b$$

$$\sqrt{1 + \tfrac{1}{m^2}}\, \frac{b^2}{2} = \bar{y} L$$

$$2\pi \sqrt{1 + \tfrac{1}{m^2}}$$

$$\frac{2\pi}{m}\sqrt{m^2 + 1}\, \frac{b^2}{2} = 2\pi \bar{y} L = A$$

$$\frac{2\pi L}{b\, L}\sqrt{b^2 + L^2} \cdot \frac{b^2}{2}$$

$$\pi b \sqrt{b^2 + L^2}$$

$$2\pi b L \sqrt{\left(\tfrac{b}{L}\right)^2 + 1}$$

since $y = mx + b \qquad dy = m\,dx \qquad \therefore \frac{dy}{dx} = m = \text{const}$

$$2\pi \bar{y} L = 2\pi \int y \, dL = 2\pi \int_0^b y \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \pi \int_0^b 2y\,dy \left(\sqrt{1 + \tfrac{1}{m^2}}\right)$$

$$= \frac{\pi}{m}\int_0^b 2y\,dy\,\sqrt{m^2 + 1}$$

$$m = \frac{b_2 - b_1}{L} \qquad = \frac{\pi}{m}\sqrt{m^2 + 1}\;\; y^2\Big|_{b_1}^{b_2}$$

$$= \frac{\pi L}{b_2 - b_1}\, \frac{\sqrt{(b_2 - b_1)^2 + L^2}}{L}\left(b_2^2 - b_1^2\right)$$

$$= \pi (b_2 + b_1)\sqrt{\delta^2 + L^2}$$

$$A_o = \pi (r_4 + r_5)\sqrt{(r_5 - r_4)^2 + L^2}$$

$2\pi(r + dr)\,ds$

$ds \quad \frac{dr}{\cos\alpha} \quad ds'$

$$q = -k\, 2\pi \qquad -k(2\pi r\, ds)$$

$$\Delta T = \frac{-q}{2\pi r l k}\, dr \qquad r = r(l) \quad r - r_4 = \frac{r_5 - r_4}{L}\, l$$

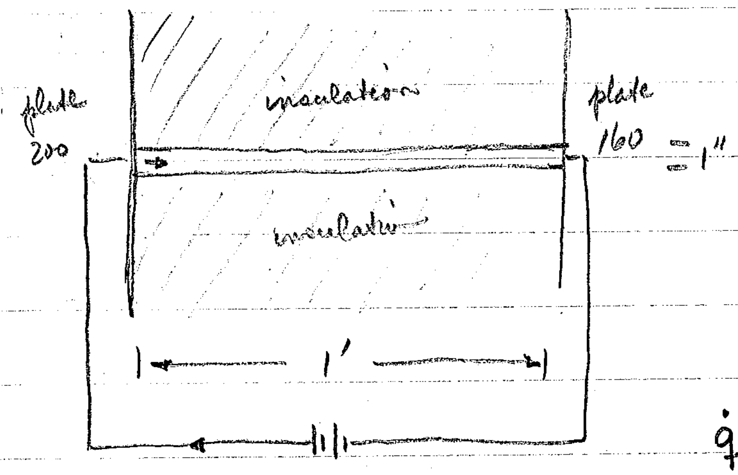
$$r = r_4 + \frac{r_5 - r_4}{L}\, l$$

$$dr = \frac{r_5 - r_4}{L}\, dl$$

$$\int_0^{200} \frac{T_1 - T_5 \quad dl'}{\dfrac{1}{h_i\, 2\pi r_{1\text{-}2}} + \dfrac{\ln(r_{3\text{-}1}/r_{2\text{-}1})}{2\pi k_{23}} + \dfrac{\ln(r_{4\text{-}1}(l')/r_{3\text{-}1})}{2\pi k_{3\text{-}4}} + \dfrac{1}{h_o \cdot 2\pi\, r_{4\text{-}1}(l')}} = q$$

$$r_{4\text{-}1}(l') = \frac{-1.33}{200}\, l + \frac{64.5}{12} \qquad \frac{1}{h_o\left[\pi (r_{4\text{-}1} + r_{5\text{-}1})\sqrt{(r_5 - r_4)^2 + L^2}\right]}$$

2-32



$$q_{in} \longrightarrow \boxed{\ \ }_{dx} \longrightarrow q_{out}, \quad \downarrow \dot{q}_{source}$$

$$-kA\left.\frac{dT}{dx}\right)_x + \dot{q}A\,dx = -kA\left.\frac{dT}{dx}\right)_{x+dx}$$

$$= \left(-kA\frac{dT}{dx}\right)_x + \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)dx$$

$$\dot{q}A\,dx = \frac{\partial}{\partial x}\left(-kA\frac{\partial T}{\partial x}\right)dx$$

if k, A are not fns of x

$$\dot{q} = -k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = -k\frac{\partial^2 T}{\partial x^2}$$

$$-\dot{q}\frac{x}{k} + C_1 = \frac{dT}{dx} \quad \& \quad -\dot{q}\frac{x^2}{2k} + C_1 x + C_2 = T$$

@ $x=0$ $T=200$ $\therefore$ $C_2 = 200$

@ $x = 1'$ $T = 160$ $\therefore$ $C_1 = 160 + \frac{\dot{q}}{2k} - 200 = -40 + \frac{\dot{q}}{2k}$

$$\therefore \quad T = -\frac{\dot{q}}{2k}(x^2 - x) - 40x + 200$$

$$\dot{q} = \frac{Q}{Al} = \frac{40}{\pi\left(\frac{.5}{12}\right)^2 \cdot 1} = 7333.86 \text{ Btu/hr cu ft.}$$

$\therefore$ for steel use $k = 25.9$ Btu/hr ft °F

$$\therefore \quad T = -141.58(x^2 - x) - 40x + 200 = -141.58x^2 + 101.58x + 200$$

$$\frac{dT}{dx} = 0 = -141.58(2x - 1) - 40 \quad \& \quad x = \frac{1}{2}\left(\frac{-40}{141.58} + 1\right) = .359' \text{ from left end}$$

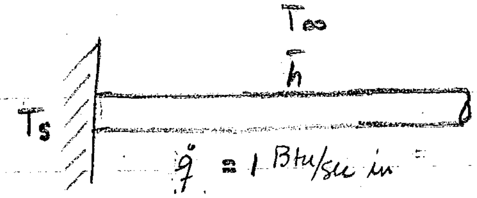
$$T_{max} = -18.25 + 36.47 + 200 = 218.22\,°F$$

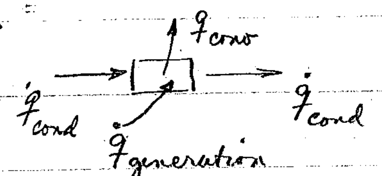
To find q @ both ends:

$$\left.\frac{dT}{dx}\right|_{x=0} = 141.58 - 40 = 101.58 \qquad -kA\frac{dT}{dx} = q = -(25.9)\left(\pi\left(\frac{.5}{12}\right)^2\right)(101.58) = 14.3$$

$$\left.\frac{dT}{dx}\right| = -181.58 \qquad -kA\frac{dT}{dx} = q = -(25.9)\left(\pi\left(\frac{.5}{12}\right)^2\right)(-181.58) = 25.65$$

2-38





we assume a 1-D profile since x ≥ y dim

$$-kA\frac{\partial T}{\partial x}\Big|_x + \dot{q}A\,dx = -kA\frac{\partial T}{\partial x}\Big|_{x+dx} + \bar{h}\,(T-T_\infty)\cdot 2\pi r\,dx$$

$$+\dot{q}\,(\pi r^2 dx) = -k\,(\pi r^2)\,\frac{d^2T}{dx^2}dx + \bar{h}(T_s-T_\infty)\cdot 2\pi r\,dx$$

$$\dot{q}r = -kr\frac{d^2T}{dx^2} + 2\bar{h}(T-T_\infty) \quad \text{or} \quad \frac{d^2T}{dx^2} - \frac{2\bar{h}}{kr}(T-T_\infty) + \frac{\dot{q}}{k} = 0$$

$$\dot{q} = 1\ \text{Btu/sec in} = 12(3600)\ \text{Btu/hr ft.}$$

let $\Theta = T - T_\infty - \dfrac{\dot{q}r}{2\bar{h}}$ $\quad \dfrac{d^n\Theta}{dx^n} = \dfrac{d^nT}{dx^n}$ $\quad \therefore$

$$0 = \frac{d^2\Theta}{dx^2} - \frac{2\bar{h}}{kr}\Theta$$

solution to this ODE of the form $\Theta'' - \dfrac{2\bar{h}}{kr}\Theta = 0$: if we let $p^2 = \dfrac{2\bar{h}}{kr}$

then $\Theta = C_1 e^{px} + C_2 e^{-px}$ is a solution since

@ $x = \infty$ $\quad \dfrac{dT}{dx} = \dfrac{d\Theta}{dx} = p\left[C_1e^{px} - C_2e^{-px}\right] = 0$ since $T\big|_{x=\infty} \longrightarrow T_\infty$

$$\therefore\ C_1 = 0$$

@ $x = 0$ $\quad T = T_s \quad \therefore\ \Theta = T_s - T_\infty - \dfrac{\dot{q}r}{2\bar{h}}$

$\therefore\ \left[T_s - T_\infty - \dfrac{\dot{q}r}{2\bar{h}}\right] = C_2$ $\quad$ hence

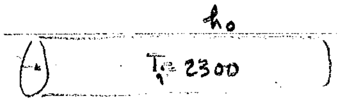
$$T - T_\infty - \frac{\dot{q}r}{2\bar{h}} = \left[T_s - T_\infty - \frac{\dot{q}r}{2\bar{h}}\right]e^{-\sqrt{\frac{2\bar{h}}{kr}}\,x}$$

$$T - T_\infty = (T_s - T_\infty)e^{-\sqrt{\frac{2\bar{h}}{kr}}\,x} + \left(1 - e^{-\sqrt{\frac{2\bar{h}}{kr}}\,x}\right)\frac{\dot{q}r}{2\bar{h}}$$

$$\frac{T-T_\infty}{T_s-T_\infty} = e^{-\sqrt{\frac{2\bar{h}}{kr}}\,x}\left[1 - \frac{1}{T_s-T_\infty}\frac{\dot{q}r}{2\bar{h}}\right] + \left\{\frac{\dot{q}r}{2\bar{h}}\frac{1}{T_s-T_\infty}\right\}$$

2-41 $P = 10\,kW$, $R = \frac{\rho L}{A}$, $\rho = 32\times10^{6}\,\Omega/in/sq\,in$, $T_i = 2300\,°F$, $h_o = 300$ Btu/hr sqft

$V = 9$ or 12 volts



$T_\infty = 200$ let $\dot q$ = heat generated / unit vol

heat in + heat gen = heat out : $-k A_r\left(\frac{dT}{dr}\right)_r + \dot q\, 2\pi r l\, dr = -k\left(A\frac{dT}{dr}\right)_{r+dr} = -k\left[\left(A\frac{dT}{dr}\right)_r + \frac{d}{dr}\left(A\frac{dT}{dr}\right)dr\right]$

$$\cancel{-kA_r\frac{dT}{dr}\Big|_r} + \dot q\, 2\pi r l\, dr = \cancel{-k A\frac{dT}{dr}\Big|_r} - k(2\pi dr\, l)\frac{dT}{dr} - k\, 2\pi r l\frac{d^2T}{dr^2}dr$$

$$\dot q \cancel{2\pi} r \cancel{l}\cancel{dr} = -k\cancel{2\pi}\cancel{l}\cancel{dr}\left[\frac{dT}{dr} + r\frac{d^2T}{dr^2}\right]$$

Rate of Creation of heat : $-\frac{\dot q r}{k} = \frac{d}{dr}\left(r\frac{dT}{dr}\right)$

or $\int_0^{r_0} -\dot q\frac{r' dr'}{k} = \int d\left(r\frac{dT}{dr}\right)$

$$-\dot q\frac{r^2}{2k}\Big|_0^{r_0} = r\frac{dT}{dr} + C \qquad \text{or} \qquad -\frac{\dot q}{2k}r^2 = r\frac{dT}{dr} + C_1$$

applying BC @ $r=0$

since $\frac{dT}{dr} = 0$ at $r=0 \Rightarrow C_1 = 0$ or $-\frac{\dot q}{2k}r = \frac{dT}{dr}$

$$\therefore \int_0^r -\frac{\dot q r}{2k}dr = \int^T dT$$

$$-C - \frac{\dot q r^2}{4k} = T \qquad \text{since } T = T_0 \text{ at } r = r_0$$

using heat cond = heat convect at surface $q_{cond} = q_{conv} = h_o 2\pi r_o l(T_0 - T_\infty)$

and heat cond = $q_{cond} = -kA\frac{dT}{dr}\Big|_{r_0} = -k\,2\pi r_0 l\left(-\frac{\dot q r_0}{2k}\right) = \dot q\pi r_0^2 l$

this gives explicitly T_0

$$\therefore \dot q\pi r_0^2 l = h_o 2\pi r_0 l(T_0 - T_\infty) \qquad \text{or} \qquad \frac{\dot q r_0}{2h_0} + T_\infty = T_0$$

and now solve for C & put back into equation

$$-\frac{\dot q r_0^2}{4k} = T_0 + C = \frac{\dot q r_0}{2h_0} + T_\infty + C \qquad -C = +\dot q\left(\frac{r_0}{2h_0} + \frac{r_0^2}{4k}\right) + T_\infty$$

$$T = +\frac{\dot q r_0^2}{4k}\left[1 - \left(\frac{r}{r_0}\right)^2\right] + T_\infty + \frac{\dot q r_0}{2h_0}$$

$\therefore \quad T(0) - T(r_0) = \dfrac{\dot{q} r_0^2}{4k}$ Solve for ΔT center $\rightarrow$ surface

and $T(0) - T_\infty = \dfrac{\dot{q} r_0^2}{4k} + \dfrac{\dot{q} r_0}{2h_0}$ ΔT center $\rightarrow$ medium

but $\dot{q}$ = heat generated/unit vol

$\dot{q} = \dfrac{P}{V}$ where $V = \pi r_0^2 l$ $\quad \therefore \quad T(0) - T(r_0) = \dfrac{P}{\pi r_0^2 l} \cdot \dfrac{r_0^2}{4k} = \dfrac{P}{4\pi k l} \neq f(r_0)$

and $T(0) - T_\infty = \dot{q}\left[\dfrac{r_0^2}{4k} + \dfrac{r_0}{2h_0}\right] = \dfrac{P}{\pi r_0^2 l}\left[\dfrac{r_0^2}{4k} + \dfrac{r_0}{2h_0}\right]$

We must somehow get rid of the length term

but $P = I^2 R = \dfrac{V^2}{R} = \dfrac{V^2 A}{\rho l}$ or $l = \dfrac{V^2 A}{\rho P} = \dfrac{\pi V^2 r_0^2}{\rho P}$

Cesar!

$$T(0) - T_\infty = \rho \frac{P^2}{\pi r_0^2 \, \pi V^2 r_0^2}\left[\frac{r_0^2}{4k} + \frac{r_0}{2h_0}\right] = \frac{\rho P^2}{\pi^2 V^2 r_0^3}\left[\frac{r_0^2}{4k} + \frac{1}{2h_0}\right]$$

solve for cubic in terms of r_0 let $\Theta = T(0) - T_\infty$

$$r_0^3 - \frac{\rho P^2}{\pi^2 V^2 \Theta}\left[\frac{r_0^2}{4k} + \frac{1}{2h_0}\right] = 0 \quad \text{or} \quad r_0^3 + A r_0^2 + B = 0$$

where $A = \dfrac{-\rho P^2}{4\pi^2 V^2 \Theta k}$ $\quad B = \dfrac{-\rho P^2}{2\pi^2 V^2 h_0 \Theta}$ $\quad$ note: $r_0 = r_0(\text{voltage})$

once r_0 is found, l can be found; knowing P and V gives R and hence I

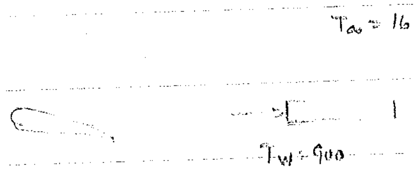
note that if P = constant $\quad$ to discuss heat transfer coeff look at $\dfrac{\rho P^2 r_0}{2V^2 (\pi r_0^2)^2} \dfrac{1}{h_0}$

if h_0 decreases $\Rightarrow$ A must increase (larger wire)

if h_0 increases $\Rightarrow$ A must decrease (smaller wire)

3-10 is a graphical

3-27



@ front end $hA(T_\infty - T_w) = q\, A \frac{dT}{dx}$

① $q_{c,\infty \to 1} + q_{k,2 \to 1} = 0$

②, ③, ④, ⑤ $q_{k,i-1 \to i} + q_{c,\infty \to i} + q_{k,i+1 \to i} = 0$

⑥ $q_{k,5 \to 6} + q_{c,\infty \to 6} + q_{c,\infty \to 6} = 0$

$q_{k\ i-1 \to i} = \frac{kA}{\Delta x}(T_{i-1} - T_i) \qquad q_{c,\infty \to i} = hA(T_\infty - T_i)$

$q_{k\ i+1 \to i} = \frac{kA}{\Delta x}(T_{i+1} - T_i)$

②, ③, ④, ⑤ $\frac{kA}{\Delta x}(T_{i-1} - T_i) + h\overset{p\Delta x}{A'}(T_\infty - T_i) + \frac{kA}{\Delta x}(T_{i+1} - T_i) = Q_i$

① $\frac{kA}{\frac{\Delta x}{2}}(T_W - T_1) + h\overset{p\Delta x}{A'}(T_\infty - T_1) + \frac{kA}{\Delta x/2}(T_2 - T_1) = Q_1$

⑥ $\frac{kA}{\Delta x/2}(T_5 - T_6) + h\overset{p\Delta x/2}{A'}(T_\infty - T_6) = Q_6$

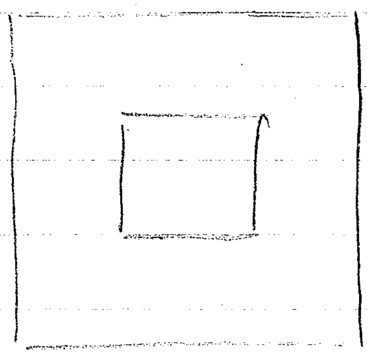
$k = 15 \quad A = .005 \quad \Delta x = \frac{1}{24}' \quad h = 80 \quad T_\infty = 1600 \quad T_W = 900 \quad p = .4$

① $1196 + T_2 - T_1(2.185) = Q'$

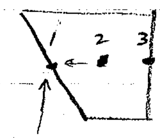
2,3,4,5 $1184 + T_{i-1} + T_{i+1} - 2.740\,T_i = Q'$

⑥ $296 + T_5 - T_6(1.185) = Q'$

3-28



$=$



$$\Delta T + \frac{\dot q}{K} = \frac{1}{a}\frac{\partial T}{\partial t} \quad \text{where} \quad \frac{1}{a} = \frac{c\rho}{k}$$

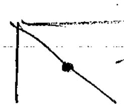
we assume

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{n-1,m} - 2T_{nm} + T_{n+1\,m}}{(\Delta x)^2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{n\,m-1} - 2T_{nm} + T_{n\,m+1}}{\Delta y^2}$$

$$T_{n-1,m} + T_{n+1,m} + T_{n,m-1} + T_{n,m+1} - 4T_{n,m} = 0$$

$\boxed{T_{n,m+1} = T_{n+1,m}}$ 0 $\quad T_{n+1,m} + (+T_{n+1,m}) - 4T_{n,m} = 0$



$\rightarrow \quad T_{n+1,m} - 2T_{nm} = 0 \qquad T_{n+1,m} = 2T_{nm}$

$$T_{nm} + T_{n+2,m} - 4T_{n+1,m} = -100\ T$$

$\boxed{T_{n+3,m} = T_{n+1,m}}$ 1 $\quad 2T_{n+1,m} - 4T_{n+2,m} = -100$

1 -2 50

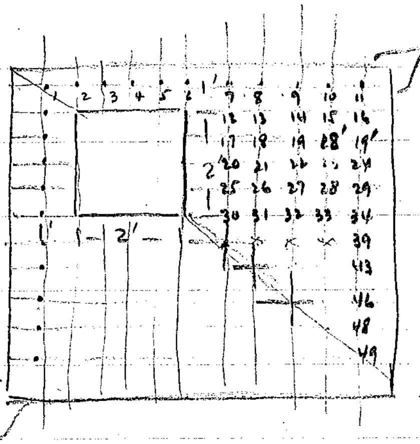
$$\begin{pmatrix} -2 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & -2 \end{pmatrix}\begin{pmatrix} T_{nm} \\ T_{n+1m} \\ T_{n+2m} \end{pmatrix} = \begin{pmatrix} 0 \\ -100 \\ -50 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 1 & -4 & 1 & -100 \\ 0 & 1 & -2 & -50 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & -7 & 2 & -200 \\ 1 & -4 & 1 & -100 \\ 0 & 1 & -2 & -50 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & 0 & -12 & -550 \\ 1 & 0 & -7 & -300 \\ 0 & 1 & -2 & -50 \end{array}\right)$$

$T_{n+2,m} = 45.83\,°F \qquad T_{n+1,m} = 41.66\,°F \qquad T_{nm} = 20.81\,°F$

$$q = -kA\frac{dT}{dx} = kA\frac{\Delta T}{\Delta x} = 1\,\frac{Btu}{hr\,ft\,°F}\cdot\frac{(100\,°F)}{}$$

3-31



Symmetric about axis $\therefore$ $T_{ij} = T_{ji}$

@ 1 $\quad 200 + 2T_2 - T_1 = 0$

@ 2-6 $\quad 600 + T_{i-1} + T_{i+1} - 4T_i = 0$

@ 7-10 $\quad 100 + T_{i-1} + T_{i+1} - 4T_i + T_{i+5} = 0$

$500 + T_{i+5} + T_{i+4} + T_{i+1} - 4T_i = 0$ @ 12, 17, 20, 25, 30

@ 13-15 $\quad T_{i+5} + T_{i-5} + T_{i-1} + T_{i+1} - 4T_i = 0$
18, 19, 18'
21-23
26-28
31-33, 36, 37, 38, 41, 42, 45

@ 16, 19', 24, 29, 34

$T_{i-1} + T_{i-5} + T_{i+5} + 100 - 4T_i = 0$ for 15, 16 $T_{i+5} = T_{18'}, T_{19'}$

1, 2-6, 7-10, 11, 12, 13-15, 16, 17, 18-19,' 20
21-23, 24, 25-28, 29, 30, 31-34, 35, 36-38, 39
40, 41, 42, 43, 44-49

51 points
since its symmetric

@ 11 $\quad 200 + T_{10} + T_{16} - 4T_{11} = 0$

@ 35 $\quad 2T_{30} + 2T_{36} - 4T_{35} = 0$

@ 40 $\quad 2T_{35} + 2T_{41} - 4T_{40} = 0$

@ 44 $\quad 2T_{41} + 2T_{45} - 4T_{44} = 0$

@ 47 $\quad 2T_{45} + 2T_{48} - 4T_{47} = 0$

@ 49 $\quad 2T_{48} + 200 - 4T_{49} = 0$

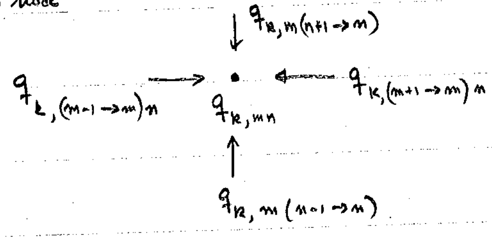
@ 39 $\quad T_{38} + T_{34} + T_{43} + 100 - 4T_{39} = 0$

@ 43 $\quad T_{42} + T_{39} + T_{46} + 100 - 4T_{43} = 0$

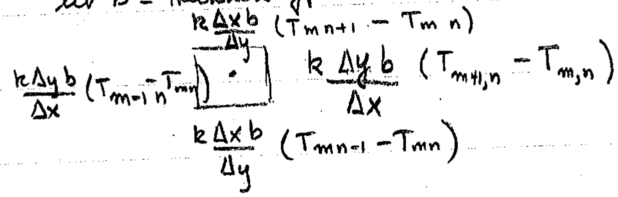
@ 46 $\quad T_{45} + T_{43} + T_{48} + 100 - 4T_{46} = 0$

@ 48 $\quad T_{47} + T_{46} + T_{49} + 100 - 4T_{48} = 0$

1 Interior node



let b = thickness of plate



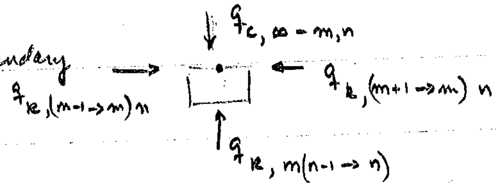
if b = const & $\Delta x = \Delta y = \Delta l$ then

$$kb(T_{m-1\,n}-T_{mn})+kb(T_{m+1\,n}-T_{mn})+kb(T_{m\,n-1}-T_{mn})+kb(T_{m\,n+1}-T_{mn})=0$$

or $$(T_{m+1\,n}+T_{m-1\,n}+T_{m\,n+1}+T_{m\,n-1}-4T_{mn}) = Q'_{mn}$$

$\forall$ $m=2,\dots,9$ $n=2,\dots,7$ & $m=6,\dots,14$ $n=2,\dots,4$

2 at convective boundary



$$\frac{kb}{2}(T_{m-1\,n}-T_{mn})+\frac{kb}{2}(T_{m+1\,n}-T_{mn})+kb(T_{m\,n-1}-T_{mn})+\bar{h}_c b\Delta l(T_\infty-T_{mn})=Q_n$$

$$T_{m-1\,n}+T_{m+1\,n}+2T_{m\,n-1}+\frac{2\bar{h}}{k}\Delta l\,T_\infty-\left(2+\frac{2\bar{h}_c\Delta l}{k}\right)T_{mn}=Q'_{mn}$$

$\forall$ $m=1$ $n=2,\dots,14$

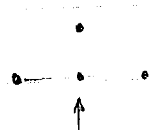
3. at left edge (note that sym $\Rightarrow$ $T_{m-1\,n}=T_{m+1\,n}$)

$$\therefore\quad 2T_{m+1,n}+T_{m\,n+1}+T_{m\,n-1}-4T_{mn}=Q'_{mn}$$

$\forall$ $m=2,7$ & $n=1$

4 at 8,1 $$2T_{m+1,n}+T_{m,n+1}+300-4T_{mn}=Q'_{mn}$$

5 at bottom bound



ME 250A - Heat Transfer

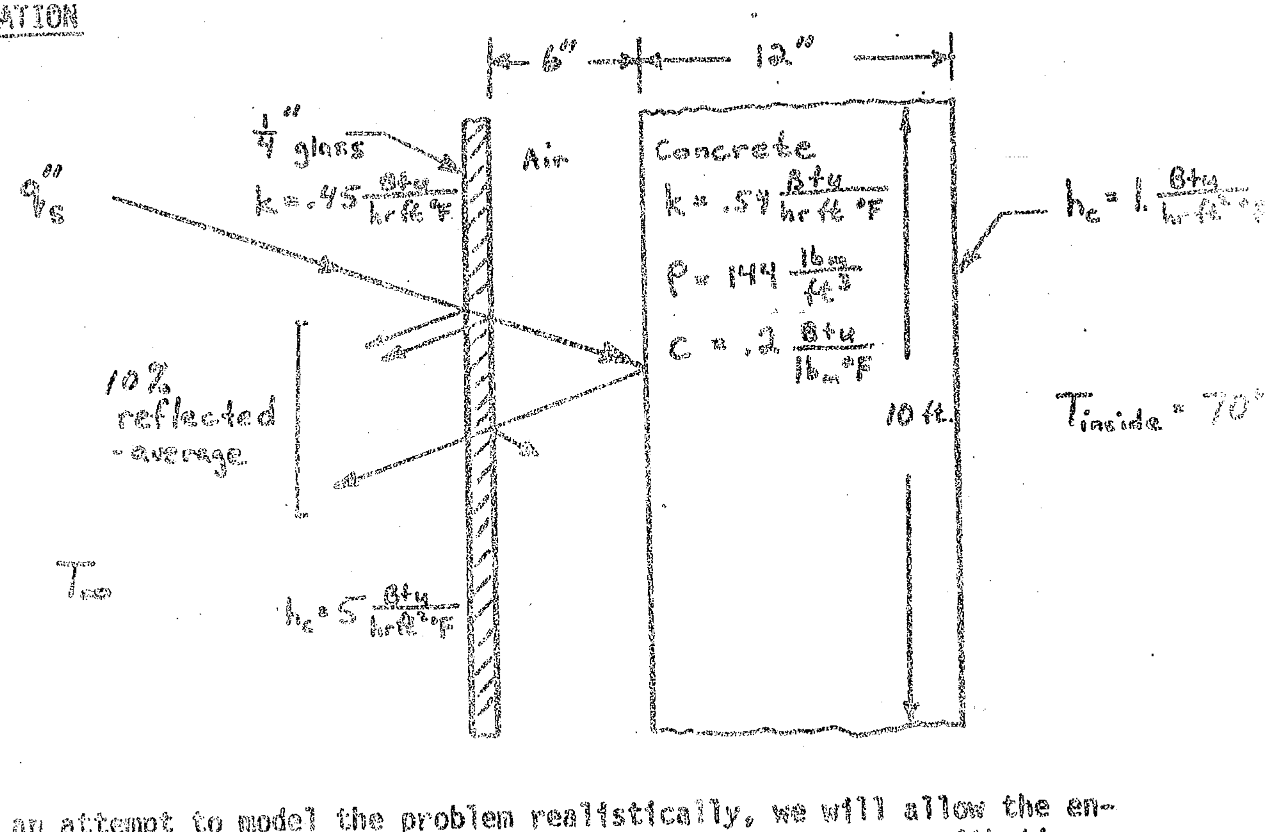
Problem S-3

A popular solar heating scheme features a south-facing wall which serves as a collector and storage medium. Typically the outside wall is covered with one or two panes of glass. The glass is nearly transparent to the incoming solar radiation (very short wavelength) and opaque to thermal radiation (relatively long wavelength): a combination which produces the "greenhouse" effect. In the evening, light-weight insulating panels are placed on the outer face of glass to reduce night losses.

The wall itself is usually constructed of concrete or a composite framing with water bags between the joints. One unique design simply uses 55 gallon drums stacked horizontally with the ends forming the inner and outer wall surfaces. In any case, the outer wall surface is coated in such a way that most of the incident solar radiation is absorbed.

The present problem will examine the time-dependent behavior of such a system. The wall below is similar to one used in France.

DAY OPERATION



In an attempt to model the problem realistically, we will allow the environmental temperature T_∞ and the solar energy influx to vary with time. The time scale will be defined in the following way:

mid.	6am	noon	6pm	mid.	6am	
0	6	12	18	24	30	. . .

With this time base let

$$T_\infty = 50 + 20 \sin \frac{\pi(t-8)}{12} \ {}^\circ F$$

and the solar influx, q_s'', be approximated by

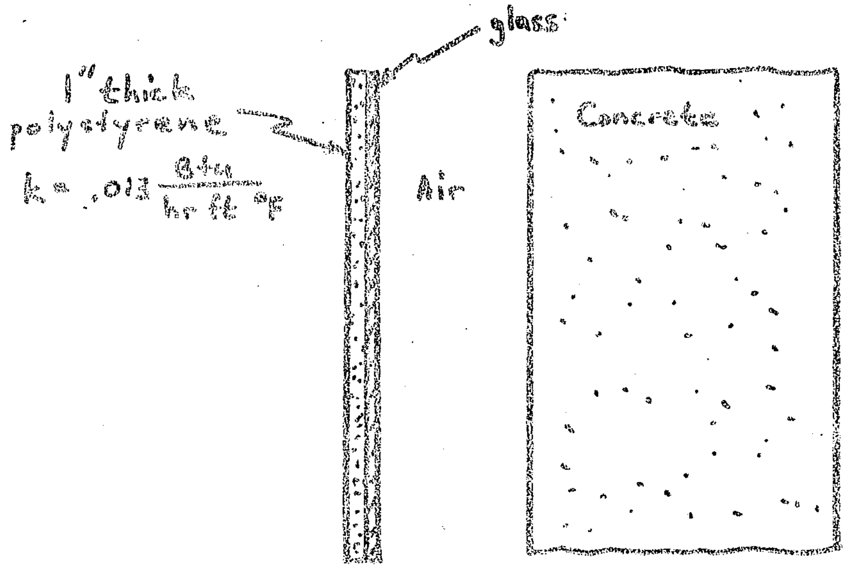
$$q_s'' = \begin{cases} 175 \sin \dfrac{\pi(t-6)}{12} & \text{Btu/hr-ft}^2 \text{ between 6am and 6pm ,} \\ 0 & \text{for all other times .} \end{cases}$$

The latter formulation is very rough, inasmuch as the exact formulation involves the time of year, latitude, and the exact wall orientation. This information is available in chart and equation form and could be used in a more detailed analysis.

The temperature within the house will be maintained at 70°F and the inside face of the concrete will have a nominal convection coefficient of 1 Btu/ft²-hr-F°. The outside face of the glass will be subject to a convective coefficient of 5 Btu/hr-ft²-F°. From 6pm in the evening until 6am the following morning, a one-inch-thick polystyrene cover will be placed over the outside surface of the glass.

NIGHT OPERATION

6 PM - 6 AM



Heat transfer in the air gap will be calculated using an empirical correlation for the heat transfer coefficient due to free convection and an additional radiative coefficient which has been calculated for this problem. The empirical correlation is presented in Kreith on page 402 and is applicable for free convection in an enclosed space between isothermal vertical walls. The correlation is presented as a function of the Grashof number and the ratio of the wall height to the space thickness:

$\overline{Nu} = 1.0$ for $Gr < 2{,}000$

$\overline{Nu} = .18\ Gr^{.25}\ (L/\delta)^{-1/9}$ for $2{,}000 < Gr < 20{,}000$

$\overline{Nu} = .065\ Gr^{.33}\ (L/\delta)^{-1/9}$ for $20{,}000 < Gr < 11 \times 10^6$

where $Gr = \dfrac{\rho^2\ g\ \beta\ \Delta T\ \delta^3}{\mu^2}$

and $\overline{Nu} = \dfrac{\bar{h}\ \delta}{k}$

where δ = thickness of air gap

L = wall height

ΔT = temperature difference between walls

ρ = density, g = gravitational constant

β = thermal expansion coefficient, μ = viscosity

k = thermal conductivity of air

$\bar{h}$ = average overall convective heat transfer coefficient for heat transfer across the gap.

A good curvefit for the group $g\ \beta\ \rho^2/\mu^2$ over the temperature range of $0 < T < 200^oF$ is

$\dfrac{(T_{air} + T_{wall})}{2} = T$

$$\frac{g\ \beta\ \rho^2}{\mu^2} = 4.167E6 - 3.221E4 * T + 78.25 * T^2 \quad (1/ft^3\ {}^oF)$$

In this problem, the Grashof number may slightly exceed the upper limit of the correlation for $\overline{Nu}$. Since Gr should not be much greater than 11×10^6, assume that the correlation for the upper range of Gr will still hold.

Radiation heat transfer between the sides of the air gap can be treated in the following simplified manner. The glass plate can be considered to be opaque to low temperature thermal radiation, and obviously the concrete wall is opaque to all radiation. Thus the heat transfer by radiation between the glass and the wall can be idealized as radiation between two infinite opaque plates. If the heat transfer rate equation is linearized, a radiative heat transfer coefficient for radiation between the glass and the wall is obtained. This is approximately

$$h_r = 1.1\ Btu/hr\text{-}ft^2\ {}^oF$$

A few words should be said about the incoming solar radiation. The amount transmitted through the glass is dependent upon the incident angle and, to a lesser extent, upon the absorptivity of the glass. For a single pane, nearly 10 percent of the incoming radiation is reflected at either the outer or inner glass-air interface. This value is fairly constant until the angle of incidence exceeds 60^0, when a strong dropoff in transmissivity is experienced. Of the radiation getting through the window, a certain fraction is reflected back off the absorber surface and out through the glass again. Special coatings keep this reflected fraction below 3 to 5%. Thus, we can expect a 10 or 15% loss due to reflective processes alone. (This is over and above thermal radiation and convection losses from the collector once it heats up.) For this analysis, assume the expression for q_s'' incorporates these reflective losses.

THE PROBLEM

1. Neglect the thermal capacity of the air, glass, and insulating cover, and assume that no variations in variables or properties occur in the plane of the wall. Write the differential equation and boundary conditions which apply to the problem of heat conduction in the wall. Mention one or two characteristics of the problem which make an analytical solution intractable.

2. Model the equivalent thermal circuits for both day and night operation. Neglect the thermal capacity of the air, glass, and insulating cover. Treat the wall by considering it to be divided into six slabs, each two inches thick. Let the heat capacity of the concrete be .2 Btu/lbm ^{0}F. Treat the solar radiation influx as a current source at the absorbing wall surface. Write the results in a system of six simultaneous, first order ODE's.

3. A routine to solve the system of ODE's has been written, and uses the 4th order Runge-Kutta method. The driver program contains an initial temperature profile for the wall and solves the system of equations through a period of six days to reach a periodic steady state temperature distribution in the wall. It then solves the system through two more days, writing out results for every hour. A time step of .25 hr is used in the routine. Two subroutines must be written to accompany the driver program. The first subroutine must calculate the values of dY/dT as a function of the independent variable T (time), and the dependent variables, Y_i (temperature). (The subscript i denotes the i^{th} node.) The second subroutine must calculate the heat lost to the outside (QLOSS), the heat transferred to the inside of the house (QIN), the thermal energy stored in the wall, referenced to 70^{0}F (ESTOR), and the temperature of the inner wall (TIW).

The first subroutine should be called F, and its arguments are time (T), the vector of nodal temperatures (Y), and the vector of nodal temperature derivatives, dY_i/dT, (YP). Thus, for example, the first line of the subroutine should be:

```
SUBROUTINE  F(T,Y,YP)
```

The vectors Y and YP should have a dimension of 6. Some quantities which you will need to determine in your calculation of YP are also of interest and will be printed out by the output routine in the driver program. These quantities are the solar radiative heat flux (QS), the temperature of the environment (TINF), the temperature of the outer surface of the concrete wall, (TOW), and the temperature of the inner surface of the glass, (TIG). These should be transferred to the driver by means of a COMMON statement. The second subroutine should be called CALC and have as its arguments the variables: Y, QIN, QLOSS, ESTOR, and the temperature of the inner surface of the wall (TIW). It may be easier to transfer certain variables into CALC, which will be needed in calculating QIN, QLOSS, ESTOR, and TIW, rather than recalculating them again. This can be done by placing these variables in COMMON. Space in COMMON has been reserved for 20 real variables in addition to the variables previously mentioned.

Thus, your two subroutines should look something like the following:

```
SUBROUTINE   F(T,Y,YP)
COMMON TOW, TIG, TINF, QS, (and up to 20 other variables if you
                                                          choose)
DIMENSION  Y(6), YP(6)
.
.
.
        main body of subroutine
YP(1) =    . . .
.
.
.
YP(6) =    . . .
RETURN
END
SUBROUTINE CALC (Y, QIN, QLOSS, ESTOR, TIW)
COMMON TOW, TIG, TINF, QS, . . .
DIMENSION  Y(6)
.
.
.
        main body of subroutine
.
.
.
RETURN
END
```

After you have written the two subroutines, the program can be run by typing the following:

EXECUTE SUB.FOR, <P.PRUFROCK>MAIN

where SUB.FOR is the name of the file containing your two subroutines. (Actually you can name that file whatever you want.) A message will tell you when the program has finished execution -- it should take around 9 CPU seconds. The results are written to a file on the disk called FOR21.DAT, and it is added to your directory upon completion of the program. To look at the results on the terminal, type TYPE FOR21.DAT; to get a copy of the results, type PRINT FOR21.DAT. The units of the output variables are: TIME (hours, starting with midnight as 0 hrs); QS, QNET, QLOSS (Btu/hr.ft^2); TINF, TXG, TOW, T1, T2, T3, T4, T5, T6, TIW (^oF); ESTOR (Btu/ft^3). If you want to see the contents of the driver program and Runge-Kutta routine, look at the file <P.PRUFROCK>MAIN.FOR.

4. Plot the steady state temperature distribution in the wall for the 4 times of midnight, 6 AM, noon, and 6 PM. Plot QIN and QLOST vs. time over 24 hours.

5. If a well-insulated home with a 10 ft x 40 ft south wall had a heating requirement of 2 x 10^5 Btu/day, what percentage of the heating load could be provided by heat transfer from the solar wall?

DATE: October 19, 1978

TO : ME 250A students

FROM : Bob Eustis

SUBJECT: Homework style

Heat transfer is a subject that is quite simple in concept and the major difficulties come in trying to adapt the concepts to real-life problems. We have to generate an idealized model and apply our concepts to the model rather than the real case. If we invest enough time and money in computer solutions we often can make our model approach the real situation rather closely. Experience is necessary to choose appropriate models and experience requires working problems.

In this class we have two types of problems - exercises from the book and the special problems. The required style is different for the two cases.

<u>Exercises</u>

It is not necessary to copy the problem nor to write a discussion. But it is necessary to define your model with a sketch if appropriate and to write the pertinent equations before substituting numerical values. Your work should be neat and done in a professional manner.

<u>Special problems</u>

These problems are intended to be closer to real life than the exercises and more demanding in terms of time. They are counted as equal in value to the total of the exercises in the problem set. You should describe the assumptions in your model and also write a discussion section which indicates some thinking about the results. For example, how could the design you analyzed be improved, or how could your analysis be improved if you had time, etc.

As you look back on the class you will find that the problems were useful. Good luck!

BE

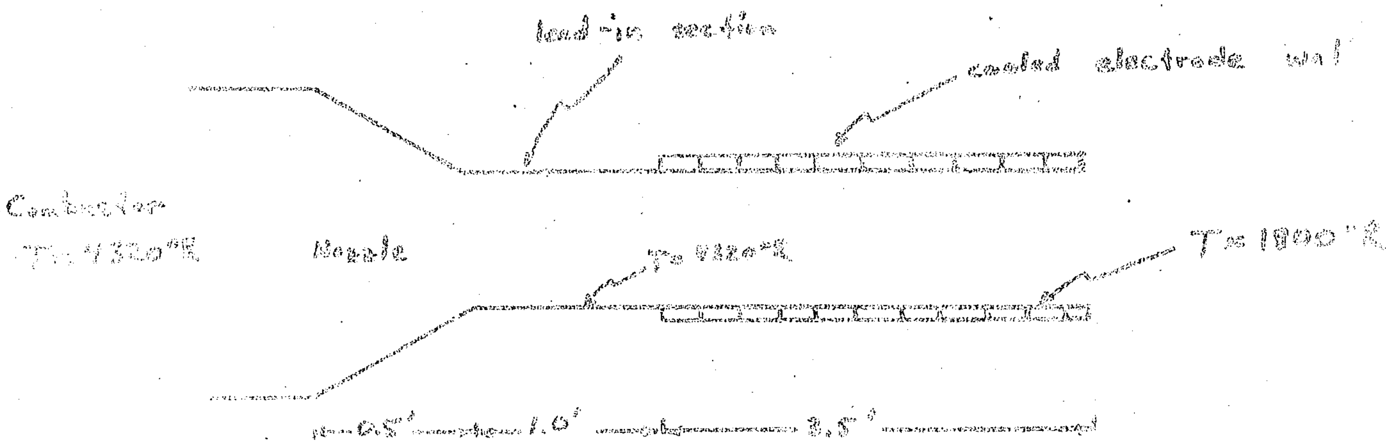
RHE/es

ME 250A - Heat Transfer

Problem S-4

The intent of this problem is to provide some exposure to a typical numerical routine for boundary layer heat transfer. The routine which is used is called STAN5 and is the product of much analytical and experimental work at Stanford, directed largely by Prof. Kays. Stan5 solves the time-averaged boundary layer equations of continuity, momentum, and stagnation enthalpy by a finite difference technique. The model for eddy diffusivity which STAN5 uses in this problem is the Prandtl mixing-length model.

The problem involves the analysis of convective heat transfer in an MHD generator channel. In our MHD experiments at Stanford, both low velocity and high velocity flows are used, depending on the phenomenon we are trying to study. Also, a lead-in section is often used to allow the boundary layer to develop before entering the test section. As a rough approximation for a small lab-scale MHD generator (and a good approximation on a larger one), a flat-plate model can be used in analyzing the boundary layer. Due to turbulence in the combustor and roughness in the flow train, the flow entering the lead-in section can be assumed to be turbulent. The lead-in section is lined with MgO, which is a common refractory ceramic used at high temperatures and therefore this section is uncooled. For the analysis of heat transfer in the MHD channel then, the situation is that of a high temperature turbulent flow over a flat plate with an unheated starting length. Three cases will be run with STAN5, two at a high velocity (M = 0.8), and one at a low velocity (M = 0.2). For all three cases, the stagnation temperature of the fluid leaving the combustor is 4320°R (2400 K). To make matters simple (and also because the STAN5 version on LOTS does not handle variable properties for combustion products), the fluid is assumed to be air. Variable properties will be used for a case of both low and high velocities, while constant



properties are used in the second high velocity case. STAN5 can also include the affect of viscous dissipation, however, due to certain problems it is not included in any of the cases.

Analytical solutions to the problem of turbulent flow over a flat plate with an unheated starting length (and for constant properties) are available and will be presented here. This problem will consist of running the three cases of STAN5 and comparing the results with the analytical solution. You will also be asked to analyze the output of STAN5 to get a picture of the details of the convection process.

THE PROBLEM

1. Running STAN5

STAN5 exists as an exec. module in the directory of T.TAME250A. The input data file also exists in the same place. Since STAN5 is not the easiest program to learn to use this input file has been created for you. All that is required to run STAN5 is for you to copy the input file into your directory and then run the program. The results are printed in the file FOR21.DAT which is added to your directory upon completion of the program. List this file to get a copy of the results. The commands you need to use are then:

```
COPY <T.TAME250A>FOR20.DAT

RUN  <T.TAME250A>STAN5.EXE

     computer prints a message when the program has finished
     (~ 40 cpu seconds)

PRINT FOR21.DAT
```

The printed output will contain the results of the three cases. For each case, the input data is printed first. Under the heading of BOUNDARY CONDITIONS, X is axial distance (ft.), UG is the free stream velocity (ft/sec), and FJ is the enthalpy of the fluid at the wall (Btu/lbm). Under the heading of INITIAL PROFILES (initial velocity and enthalpy profiles in the boundary layer), Y is vertical distance from the wall (ft), U is velocity (ft/sec), and F is the fluid stagnation enthalpy (Btu/lbm).

The output format chosen for this problem prints boundary layer parameters of interest and boundary layer profiles at the first and last steps, and at intervals of 21 steps in the middle. For this output, XU is the axial distance (ft), PRESSURE is in lb/ft^2, REM is the enthalpy thickness Reynolds number, CF2 is the friction factor ($C_f/2$), RHO(1) is the fluid density at the wall (lbm/ft^3) and RHO(NP3) os tje density at the free stream, TAUWALL is the wall shear stress, ST is the Stanton number, QWALL is the heat flux from the wall into the fluid (Btu/ft^2sec). In the boundary layer profiles, the only new variable of interest is T, the fluid temperature (^oR).

2. Comparison of Numerical and Analytical Solutions.

The analytical solution for the local Stanton number for constant properties is, in a simplified version,

$$St_x \, Pr^{0.4} = \frac{0.0295}{Re_x^{0.2}(1 - x_o/x)^{0.12}}$$

where Re_x is the local Reynolds number and x_o is the unheated starting length.

A correction for variable properties for cooling with a turbulent external boundary layer for gases is,

$$St/St_{cp} = (T_o/T_\infty)^{-0.25}$$

where St_{cp} is the Stanton number for the constant property case, T_o is the wall temperature, and T_∞ is the static free stream temperature. This correction is approximate in nature, and is based on both analytical and experimental investigations.

Based on the above equations, calculate the local Stanton number at the two or three axial locations in the cooled wall region where STAN5 has also printed out results. Do this for all three cases, and compare the agreement between the numerical and analytical values. For all cases, use $Pr = .916$, $\mu = 4.6 \times 10^{-5}$ lbm/ft sec.

3. Temperature Profiles.

On a graph, plot the temperature profile of the boundary layer in the unheated lead-in section and at the end of the cooled wall section. Superimpose the plots so that they can be easily compared. Do this for the variable property high velocity case and the variable property low velocity case. Give a succinct explanation for the shape of the profiles and a comparison between them.

4. Combustion Gas Properties.

A combustion gas would have properties which are close to those of air except that it would have a much higher specific heat. How would the Stanton number and the convective heat transfer coefficient be effected if the specific heat is 30% greater? How do the heat transfer coefficients calculated in this problem compare with the one used in the first computer problem. Give one or two considerations which might cause the heat transfer coefficient calculated in this problem to be different from that in an actual MHD generator.

ME 250A

Heat Transfer

Midterm

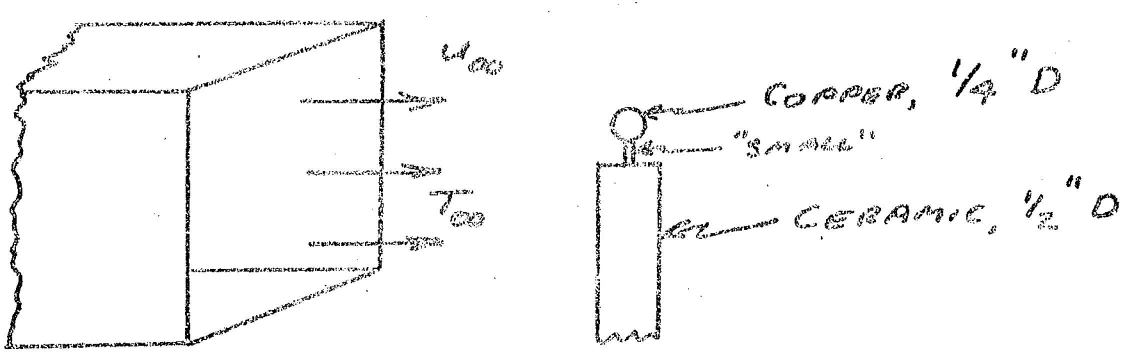
November 8, 1978

Open Book

1. A high temperature flow tube reactor used for coal combustion studies is insulated with a material for which the conductivity is a function of temperature. The insulation approximates a long cylinder of inside radius r_i and outside radius, r_o, where r_o is considerably larger than r_i. If the temperature at r_i is T_i and at r_o is T_o and if $k = a + bT + cT^2$, find an expression for the steady-state heat loss through the insulation.

2. A Langmuir probe is a device used to measure the concentration of electrons in an ionized gas (plasma). In MHD generator experiments it must be swept through the hot plasma to avoid overheating. The design as sketched below shows a copper sphere attached to a ceramic-encased support

1 atm
perfect gas



a) For the following gas conditions, calculate how long the sphere, originally at 100°F, can remain in the plasma before its center temperature exceeds 800°F.

u_∞ = 2000 ft/sec

T_∞ = 4000°F

C_p = 0.35 Btu/lbm °F

μ = 4.2×10^{-5} lbm/ft sec

k = 0.08 Btu/hr ft °F

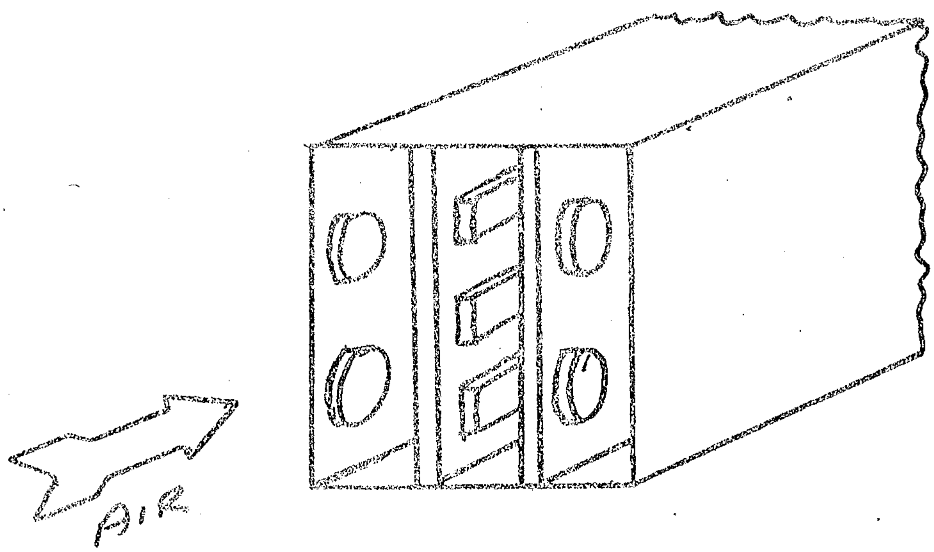
M (molecular weight) = 28

Use gas properties above rather than at T_f to save your time when calculating $\bar{h}_c$.

b) The ceramic support has low thermal conductivity and may fail in thermal shock. Describe how you would estimate the temperature gradient, dT/dr, for several values of time after insertion. Do not work out the numerics.

3. For the new Boeing 757 a cockpit display is under consideration which shows the plane superimposed on a map of the ground below with other planes in the area also shown. The electronics is sophisticated and packaging in a small volume is a serious thermal problem. Our job is to estimate the convection film coefficient for the electronics which will be arranged in a duct with turbulent air passing over them. A wooden mock-up of the electronics in the duct is available and tests show a pressure drop of 0.1 psi when the air velocity is 50 ft/sec. The duct may be considered a "rough tube" with an L/D of 100.

For air at 70°F and 14.7 psia estimate $\bar{h}_c$. Note that this value would then be used for preliminary design of the electronics package.



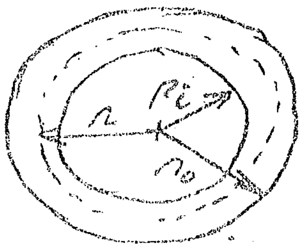
ME 950 A

MIDTERM

Nov. 8, 1978

SOLUTION

1.



$$q = -kA\frac{dT}{dr}$$

$$= -(a+bT+cT^2)\,2\pi r l\,\frac{dT}{dr}$$

$$q\int_{r_i}^{r_o}\frac{dr}{r} = -\int_{T_i}^{T_o}(a+bT+cT^2)\,2\pi l\,dT$$

$$q\left(\ln\frac{r_o}{r_i}\right) = +\Big[a(T_i-T_o) + \frac{b}{2}(T_i^2-T_o^2) + \frac{c}{3}(T_i^3-T_o^3)\Big]\,2\pi l$$

$$q = \Big[a(T_i-T_o) + \frac{b}{2}(T_i^2-T_o^2) + \frac{c}{3}(T_i^3-T_o^3)\Big]\frac{2\pi l}{\ln\frac{r_o}{r_i}}$$

2. First find h and check Biot number

From p. 487 in Kreith, (or eq. 9-4a)

$$\frac{\bar{h}D}{k} = 2 + (0.4\,Re_D^{0.5} + 0.06\,Re_D^{0.67})\,Pr^{0.4}\left(\frac{\mu_\infty}{\mu_0}\right)^{0.25}$$

$$Re = \frac{\rho U_\infty D}{\mu}\;;\quad \rho = \frac{P}{RT} = \frac{(14.7)(144)}{\frac{1545}{28}\,4460}$$

$$= 0.00860\ \text{lbm/cu ft}^3$$

2. (Cont.)

$$Re = \frac{(0.00860)(2000)\left(\frac{1}{4} \times \frac{1}{12}\right)}{4.2 \times 10^{-5}} = 8533$$

$$Pr = \frac{c_p \mu}{k} = \frac{(0.35)(4.2 \times 10^{-5})(3600)}{0.08}$$

$$= 0.662$$

$$\frac{hD}{k} = 2 + \Big[\overset{36.96}{(0.4)92.4} + \overset{25.8}{0.06(430)}\Big](.848) \cdot 1$$

$$= 55.2$$

$$h_{sphere} = (0.08)\, 55.2 \frac{1}{\left(\frac{1}{4} \times \frac{1}{12}\right)} = 212 \text{ Btu/hr ft}^2 \, {}^\circ F$$

$$\text{Biot No.} = \frac{hr}{k} = \frac{212\left(\frac{1}{8} \cdot \frac{1}{12}\right)}{215} = 0.0102$$

OK for lumped parameter

Driving potential for heat transfer is:

$$T_{aw} = T_\infty + R(T_0 - T_\infty)$$

$$T_0 = T_\infty + \frac{V_0^2}{2 g_c J c_p} = 4000 + \frac{(2000)^2}{(50000)(0.35)} \quad (228)$$

$$= 4228^\circ F$$

Assume turbulent, $R = Pr^{\frac{1}{3}} = 0.87$

$T_{as} = 4000 + (.87)(228)$
$= 4198°F$

Now $\dfrac{T - T_{as}}{T_o - T_{as}} = e^{-(hA/c\rho V)\theta}$

(Here T_o is base temp)

$$\frac{800 - 4198}{100 - 4198} = e^{-\frac{(212)\,4\pi r^2}{(.091)\,558\,\frac{4}{3}\pi r^3}\theta}$$

$$\ln\frac{3398}{4098} = -\frac{(212)\,3}{(.091)(558)(\frac{1}{8}\cdot\frac{1}{12})}\theta$$

$$-.1873 = -1202\,\theta$$

$\theta = 0.000156$ hrs $= 0.56$ sec

b) Use the Heisler charts for an infinite cylinder. Find the axis temperature from Fig 4-11 and the local temperature from Fig 4-12. T_∞ is same as above (T_{as}) and $T_i = 100°F$.

3. From Reynolds analogy

$$St = \frac{Nu}{Re\,Pr} = f/8$$

Also we can write

$$p_1 - p_2 = f\,\frac{L}{D}\,\rho\,\frac{V^2}{2g_c} \quad ; \quad \rho_{in} = \frac{p}{RT} = 0.0749\,lb_m/ft^3$$

$$f = \frac{(0.1)(144)(64.4)}{(100)(0.0749)(2520)} = 0.0495$$

$$St = \frac{h}{\rho c_p V} = \frac{0.0495}{8}$$

$$h = \frac{0.0495}{8}(.24)(.0749)(50 \times 3600)$$

$$\underline{= 20.0} \quad Btu/hr\,ft^2\,°F$$

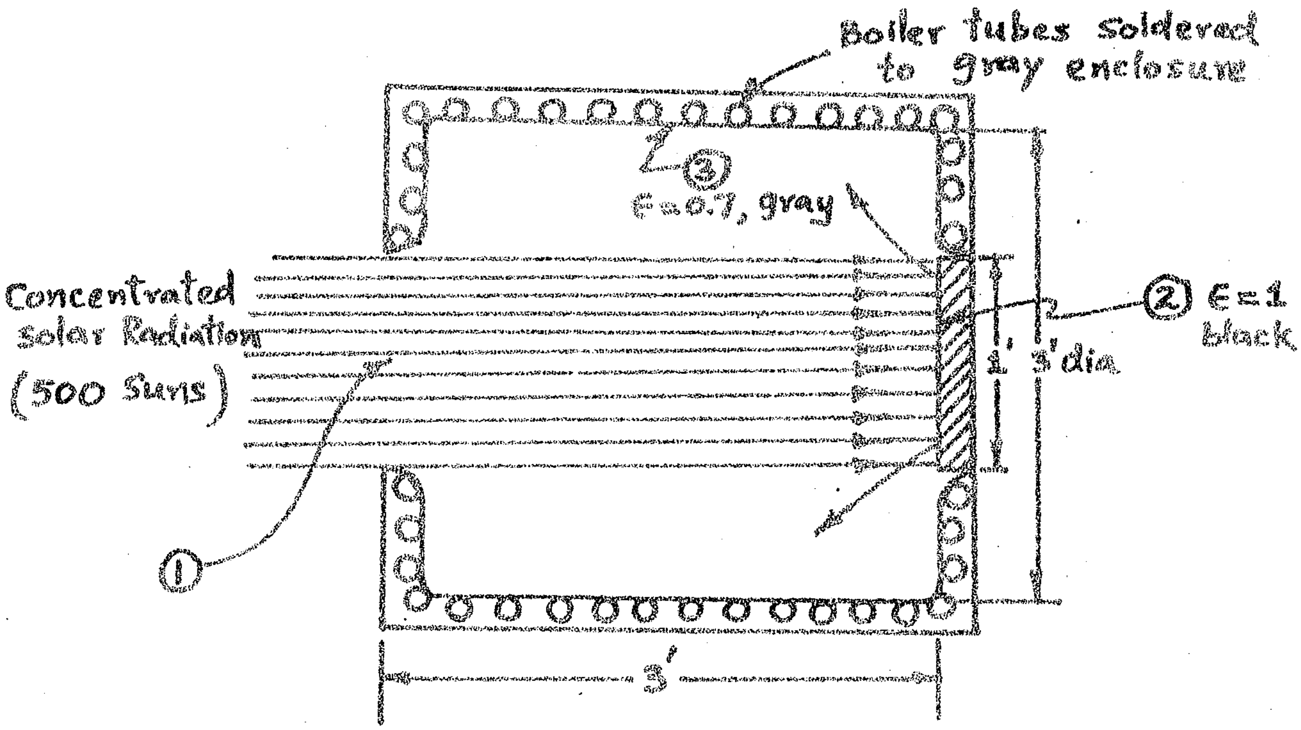
ME 250A HEAT TRANSFER

Final Exam

December 13, 1978

Open book and notes

1. A design for a solar radiation collector and boiler is sketched below. This collector would receive radiation from a large number of mirrors which would be focussed on the opening (1). This radiation falls on a "disperser" (2)



which is very well insulated and may be treated as a black surface. The disperser radiates to the boiler tubes (3) and also back out the opening (1).

Find 1) the temperature of the disperser

2) the efficiency of the boiler

$$\eta_{boiler} = \frac{Q_{water}}{Q_{in} \text{ at } (1)}$$

To save your time, we do not want numerical answers to (1) and (2) above. But we would like you to show a radiation network diagram and

a) determine the numerical values of the pertinent radiation shape factors

b) clearly indicate the unknowns

c) write independent equations equal in number to the unknowns

d) assume radiation into the opening at 1 and T_3 are known.

(Please keep your work organized so that it can be followed without the aid of numerics.)

2. Consider a straight, single-pipe heat exchanger which has imposed on its surface a constant heat flux of 120,000 Btu/hr ft^2. The pipe is 1.3" O.D. (1.0" I.D.) and is 40 ft long. Water enters at a mixed mean temperature of 100°F and passes through the pipe in fully developed turbulent flow without boiling. If the pipe temperature at the water entrance is 200°F, find

   a) the water mass flow rate

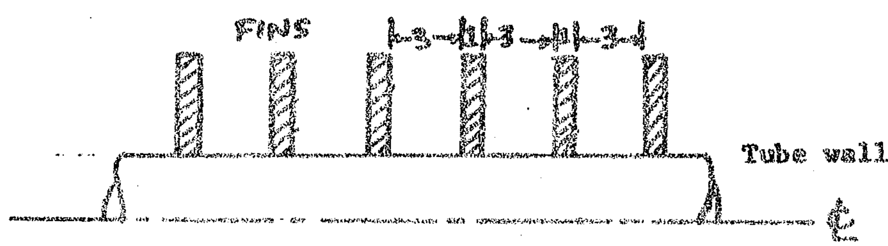
   b) the water exit mixed mean temperature

   c) the pipe temperature at the exit end.

Assume steady flow with constant properties (use data for water at 100°F). Neglect the thermal resistance of the pipe wall.

3. An economizer for home hot water heating systems is being considered by a manufacturer. The design goal is an effectiveness of 0.70 for a simple unmixed cross-flow heat exchanger. Water flows inside 1/2" tubes which are finned on the outside where hot gas flows. The fins are 75% efficient with a ratio of fin area to exposed tube area of 3. The fins cover one-fourth of the tube surface (see sketch). The water-side heat transfer coefficient is 300 Btu/hr ft^2°F and the gas side is 15 Btu/hr ft^2°F. The tube wall thickness may be neglected. The temperature rise of the water is expected to be 10°F and the gas temperature decrease is expected to be 150°F when the water flowrate is 800 lbm per minute.

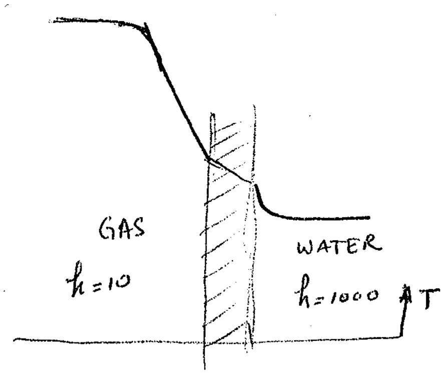
How many square feet of surface would be required for this economizer?



ME 250A Prof. Eustis Bldg 520F

$q/A = h\Delta T$ (convection) $q/A = k\dfrac{\Delta T}{L}$ (conduction)

we can define a resistance as $\dfrac{\Delta T}{q/A}$ $\dfrac{\text{(volt different)}}{\text{(current)}}$ $\dfrac{\text{Temp Diff}}{\text{heat flux}}$ $\sim \dfrac{1}{h}, \dfrac{1}{k/L}$



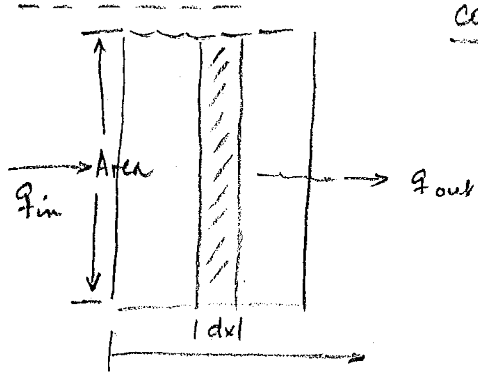
LOTS CERAS / Terman

Getting Started Classes M, T 1200pm, 730pm } in Terman Audit
Th 1200pm

type open to start a lots account. Terman Basement

TAD SIMONS / KURT ANNEN 521F X-3188

CONDUCTION



$$q_{in} = -kA\frac{\partial T}{\partial x} \quad \text{if}$$

$$q_{out} = -kA\frac{\partial T}{\partial x} + A\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right]dx \quad \text{1-D case}$$

1st law of thermo.

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

work done by the control mass is +

$$\frac{dQ}{dt} = -kA\frac{\partial T}{\partial x} - \left[-kA\frac{\partial T}{\partial x} + A\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\right]$$

$$\frac{dE}{dt} = \overset{m}{\rho A\,dx}\, c_v\frac{\partial T}{\partial t} \quad \text{where} \quad c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

u = internal energy for fluids $c_p \neq c$ for incomp $c_p = c_v$

specific heat

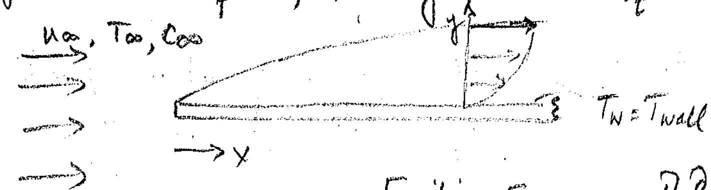
$$\frac{dW}{dt} = -\dot{q}_{gen} A\,dx$$

– sign since done on control mass

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q}_{gen} = \rho C_v \frac{\partial T}{\partial t}$$

if $k = \text{const}$ $\dot{q} = 0$, steady state $\Rightarrow {}_{,t} = 0$ or $T_{,xx} = 0$

CONVECTION



look at $T - T_W$

$C - C_W$ concentration

Friction Force $= \mu A \frac{\partial u}{\partial y}$

Energy $\quad q = -kA\frac{\partial T}{\partial y}$

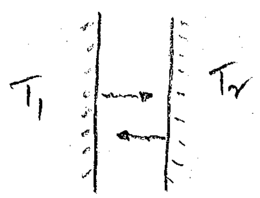
mass transfer $\quad \dot{m} = -DA\frac{\partial C}{\partial y}$

$\frac{\partial T}{\partial y}$, $\frac{\partial C}{\partial y}$ are hard to measure, then Energy equation $\Rightarrow q = hA(T_\infty - T_W)$

mass transfer $\Rightarrow \dot{m} = h_D A (C_\infty - C_W)$

$h = h(V, k, \text{b.l. thickness}, \mu, x, C_p, \rho)$

RADIATION

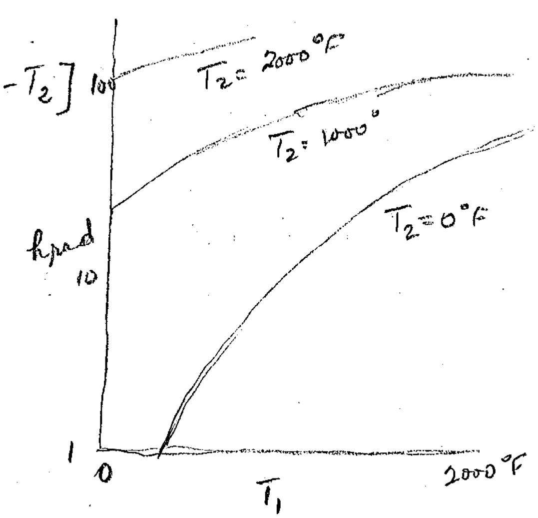


$q = \sigma A [T_1^4 - T_2^4]$ Black body, // plates

$= \sigma A \mathcal{F}_{12} [T_1^4 - T_2^4]$

$\mathcal{F}_{12}$ form factor for skewed plates, non black bodies

$\sigma A \mathcal{F}_{12}[T_1^4 - T_2^4] = h_{radiation} A [T_1 - T_2]$



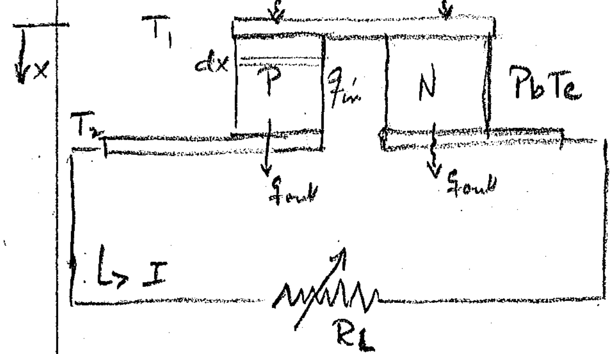
"h" Btu/hr ft² °F

air, free convection		1 – 5
air, steam, forced convection		5 – 50
Oil,	" "	10 – 300
Water,	" "	50 – 2000

Water Boiling 500 – 10000

Steam Condensing 1000 – 20000

Thermoelectric generators



Steady State 1-D
interesting because of work term

10/2/78

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q}_{gen} = \rho c \frac{\partial T}{\partial \theta}$$ θ = time

efficiency $\eta = \frac{\text{Power out}}{\text{heat flow in}}$

since steady state $\frac{\partial T}{\partial \theta} = 0$

since k = const and $\dot{q}_{gen} = \frac{I^2 R}{\text{Volume}}$ $R = \rho\frac{dx}{A}$ where ρ = resistivity of PbTe

$$k\,T_{,xx} = \frac{-I^2\rho\frac{dx}{A}}{A\,dx} = -\frac{I^2\rho}{A^2}$$

B.C. at $x=0$ $T=T_1$
at $x=l$ $T=T_2$

and $T = -\frac{I^2\rho}{A^2 k}\frac{x^2}{2} + C_1 x + C_2$ @ $x=0$ $T=T_1 = C_2$
@ $x=l$ $T=T_2 = -\frac{I^2\rho}{A^2k}\frac{l^2}{2} + C_1 l + T_1$

or $C_1 = \frac{T_2 - T_1}{l} + \frac{I^2\rho l}{2A^2k}$

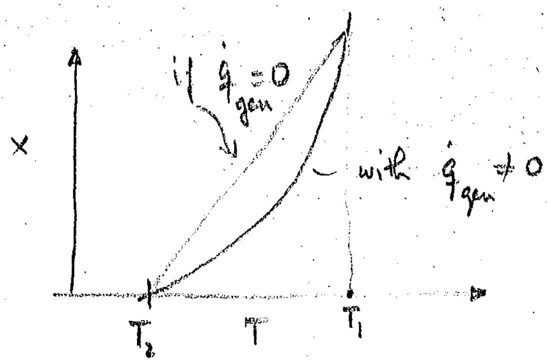
$$\therefore\quad T = -\frac{I^2\rho}{A^2k}\frac{x^2}{2} + \frac{T_2-T_1}{l}x + \frac{I^2\rho l}{2A^2k}x + T_1$$

$$T = T_1 + \frac{I^2\rho}{A^2k}\frac{x(l-x)}{2} - \frac{T_1-T_2}{l}x$$

@ $x=0$ $q_1 = -kA\frac{dT}{dx} = -kA\left[\frac{I^2\rho}{A^2k}(l-2x) - \frac{(T_1-T_2)}{l}\right]\Bigg|_{x=0}$

$$q_1 = -kA\left[\frac{I^2 \rho l}{A^2 k} - \frac{(T_1 - T_2)}{l}\right] = -\frac{I^2 \rho l}{A} + \frac{kA}{l}(T_1 - T_2)$$

Plot of T



if areas of poles P, N are not same $\quad r = r_a + r_b = \left(\frac{\rho_a l}{A_a}\right) + \left(\frac{\rho_b l}{A_b}\right)$

and $\quad K = \left(\frac{kA}{l}\right)_a + \left(\frac{kA}{l}\right)_b$

$$q_{cond} = K(T_1 - T_2) - \frac{I^2 r}{2}$$

voltage (open circuit) $= S(T_1 - T_2) \qquad S -$ constant

$$I = \frac{Vol}{R_L + r} = \frac{S(T_1 - T_2)}{r(m+1)} \qquad m = \frac{R_L}{r}$$

Peltier effect $\quad q = AI \quad$ where $A = ST$

$$q_{TOT} = q_{COND} + q_{PELT}$$

$$\frac{\text{Power out}}{\text{TOTAL } q} = \eta = \frac{\left(\frac{S^2 (T_1 - T_2)^2}{r^2 (m+1)^2}\right) R_L}{\frac{S^2 T_1 (T_1 - T_2)}{r(m+1)} + K(T_1 - T_2) - \frac{1}{2} \frac{S^2 (T_1 - T_2)^2}{r(m+1)^2}} = \frac{I^2 R}{q_{PELT} + q_{COND}}$$

$$= \left[\frac{m/m+1}{1 + \frac{Kr}{S^2}\left(\frac{m+1}{T_1}\right) - \frac{1}{2}\frac{T_1 - T_2}{T_1}\left(\frac{1}{m+1}\right)}\right] \frac{T_1 - T_2}{T_1}$$

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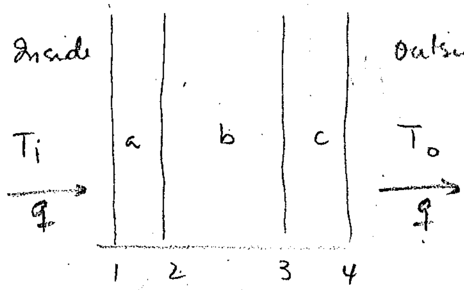
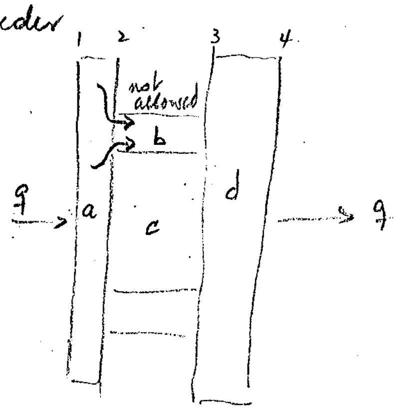


$$q = h_i A(T_i - T_1) = k_a A(T_1 - T_2)/l_a = \cdots\cdots$$

$$T_i - T_o = (T_i - T_1) + (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4) + (T_4 - T_o)$$

$$= q\left(\frac{1}{h_i A} + \frac{l_a}{k_a A} + \cdots \qquad\right) = q(R_i + R_a + R_b + R_c + R_o)$$

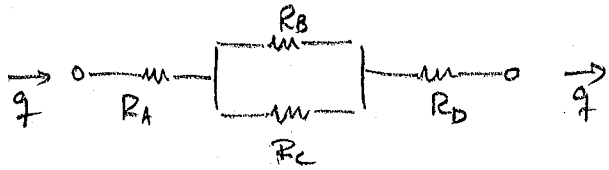
Consider



$$q_a = k_a A_a \frac{(T_1 - T_2)}{l_a}$$

$$\left.\begin{array}{l} q_b = k_b A_b \dfrac{(T_2 - T_3)}{l_b} \\ q_c = k_c A_c \dfrac{(T_2 - T_3)}{l_c} \end{array}\right\} q_a = q_b + q_c = q_d$$

We assumed that 1-D heat flow; we assumed ΔT across b = ΔT across c and did not allow heat flow in 2-D



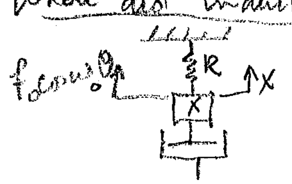
$$R_A + \left(\frac{R_B R_C}{R_B + R_C}\right) + R_D = \frac{T_1 - T_4}{q}$$

<u>Thermal circuits</u>	Potential Diff	<u>Thermal Res</u>
Conduction	$\Delta T = (T_{hot} - T_{cold})$	$\frac{\Delta x}{kA}$
Convection	$(T_{fluid} - T_{surface})$	$\frac{1}{hA}$
Radiation	$\sigma(T_1^4 - T_2^4)$	$R_{radiation}$

Electrical Circuits

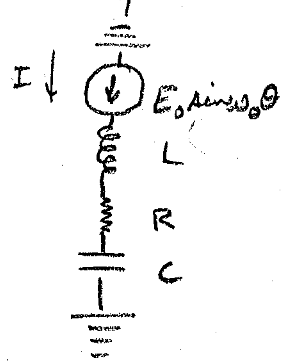
<u>Quantity</u>	<u>Thermal Symbol</u>	<u>Electrical</u>
Current	q, Btu/hr	I amps; coulombs/sec
Potential	ΔT, °F (conduct / convect)	ΔE volts
resistance	R_{th} °F/Btu/hr	R ohms, volts/coul/sec
capacitance	C Btu/°F	C farads coul/volt
Inductance	<u>none</u>	L henrys
"time constant"	RC hrs	RC sec

<u>Where did inductance from?</u>



$$\frac{m}{g_c}\ddot{x} + b\dot{x} + kx = F_o \cos\omega_o\theta$$

Electrical equiv

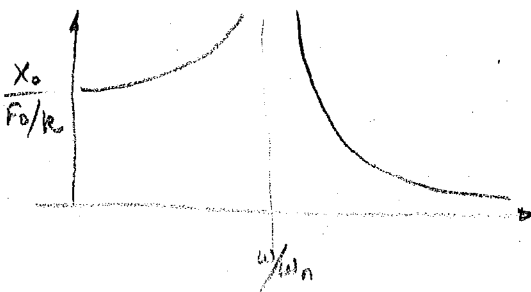


$$L\dot{I} + RI + \frac{1}{C}\int I\,d\theta = E_0 \sin \omega_0 \theta$$

diff wrt time, θ,

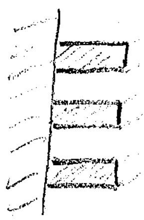
$$L\ddot{I} + R\dot{I} + \frac{I}{C} + \omega_0 E_0 \cos \omega_0 \theta$$

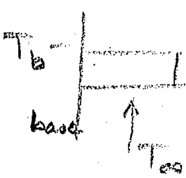
Electrical inertia term



this $L\ddot{I}$ does not appear in analogy for heat transfer and such a plot does not appear either.

Finned Surface





h = const bet air & fin

$$\eta_{fin} = \frac{\text{heat conducted of actual fin}}{\text{heat conducted of fin @ } T = T_b}$$

such a problem depends on BC @ end of fin

- you always assume 1-D heat flow within fin
- constant conductivity with
- h is assumed const which is not true since h is a funct of bl and bl is not same near base as at the end

- a rectangular fin is more efficient than triangular fin since the extra metal will still conduct.

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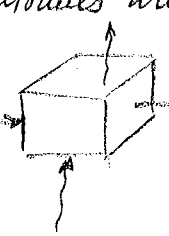
All tutorials are in room 501A



$$kA\frac{\partial T}{\partial x} \qquad -kA\frac{\partial T}{\partial x} + \frac{d}{dx}\left(-kA\frac{\partial T}{\partial x}\right)dx$$

in absence of heat sources & sinks

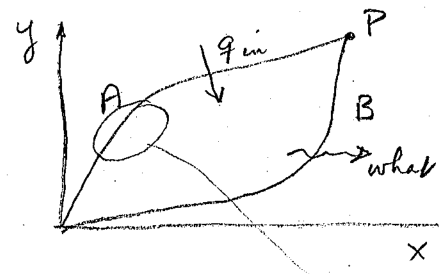
and $k \neq k(x,y,z)$

Thus for steady state, no sources, $k = $ const

$$\Delta T = \partial_x^2 T + \partial_y^2 T + \partial_z^2 T = 0$$

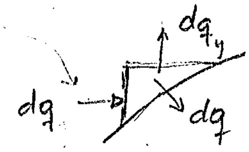
We can also write

$$(\partial_x^2 + \partial_y^2) q = 0$$



what is heat transfer $q_{out} = q_{in}$ if no heat source, steady state

$$dq = \frac{\partial q}{\partial x}\, dx + \frac{\partial q}{\partial y}\, dy \quad (1)$$



$$dq = dq_x - dq_y$$

$$= -k\, dy \frac{\partial T}{\partial x} + k\, dx \frac{\partial T}{\partial y} \quad (2)$$

comparing (2) $\Rightarrow$ $\dfrac{\partial q}{\partial x} = k \dfrac{\partial T}{\partial y}$ $\qquad \dfrac{\partial q}{\partial y} = -k \dfrac{\partial T}{\partial x}$

thus $\dfrac{\partial}{\partial x}\left(\dfrac{\partial q}{\partial x} = k \dfrac{\partial T}{\partial y}\right) + \dfrac{\partial}{\partial y}\left(\dfrac{\partial q}{\partial y} = -k \dfrac{\partial T}{\partial x}\right)$

shows that $\boxed{q_{,xx} + q_{,yy} = 0}$ if $k = $ const because $T_{,xy} = T_{,yx}$

Using $\boxed{dq = -k\, dy \dfrac{\partial T}{\partial x} + k\, dx \dfrac{\partial T}{\partial y}}$ if $q = $ const $dq = 0$ and

$$\left(\frac{dy}{dx}\right)_q = \frac{\partial T/\partial y}{\partial T/\partial x}$$

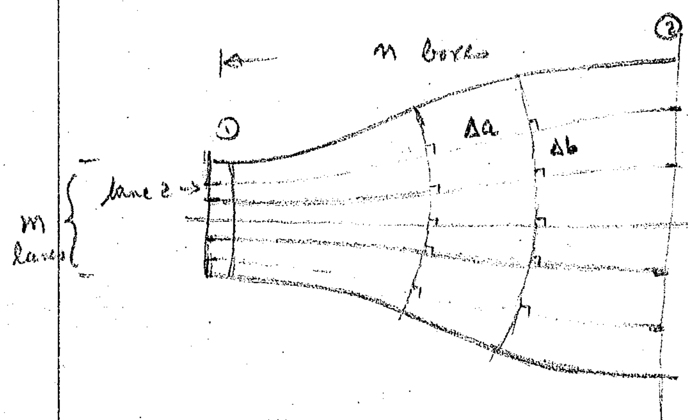
Using $dT = \dfrac{\partial T}{\partial x}\, dx + \dfrac{\partial T}{\partial y}\, dy$ if $T = $ const $dT = 0$ and

$\left(\dfrac{dy}{dx}\right)_T = -\dfrac{\partial T/\partial x}{\partial T/\partial y}$ note that $\left(\dfrac{dy}{dx}\right)_q \left(\dfrac{dy}{dx}\right)_T = -1$

or that $q = \text{const line} \perp T = \text{const line}$

- To solve $\Delta q = 0$
  1) analysis (Carslaw & Jaeger
  2) numerical
  3) analog w/electric
  4) graphical

A. graphical solution of $\Delta T = 0$ which have BC
@ $x = 0$ $T = T_1$
@ $x = L$ $T = T_2$
@ $y = \text{Top}$ $\frac{\partial T}{\partial y} = 0$
$y = \text{bottom}$ $\frac{\partial T}{\partial y} = 0$



1 draw in isotherms & q streamlines by eye, keep them ⊥, use squares if possib

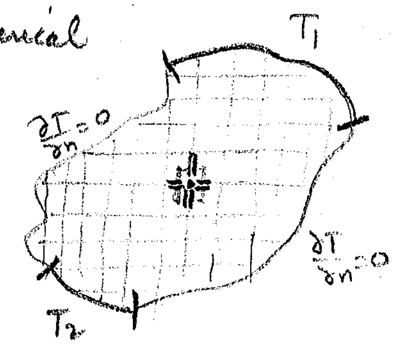
2. $q_{(\text{lane } 2)} = -k\,\Delta b \cdot 1\,\frac{\Delta T}{\Delta a}$

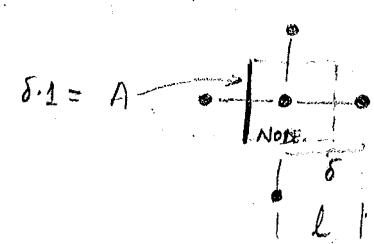
$\Delta T = \frac{T_1 - T_2}{n}$ n = no. of intervals

for square boxes then $\Delta a = \Delta b$ & $q_2 = -k\,\frac{T_1 - T_2}{n}$

$$q_{TOT} = \sum_{i=1}^{m} q_i = -k\frac{M}{N}(T_1 - T_2)$$

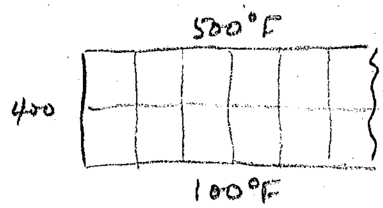
B. numerical

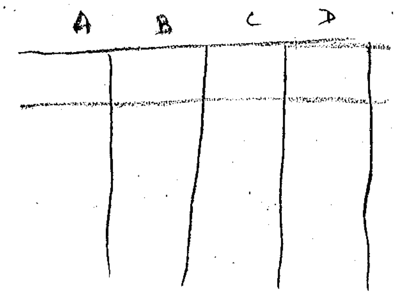




$$q_{1-0} = k\frac{A}{\ell}(T_1 - T_0)$$
$$= k\frac{(\delta \cdot 1)}{\delta}(T_1 - T_0)$$
$$q_{2-0} = k\,(T_2 - T_0)$$
$$\vdots$$
$$q_{4-0} = k(T_4 - T_0)$$

or $q_{net} = k(\Sigma T_i - 4T_0) =$





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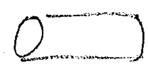
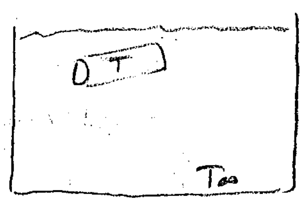
Look at body w/uniform temp which varies w/time.
suppose we want internal resistance < external resistance



$$\frac{l}{kA} < \frac{1}{hA}$$

Biot no. $\frac{hl}{k}$ < arbitrarily picked no. ie .1 ΔT in bar is less than 5%

look at bar placed in tank of oil



bar is originally at T_o
now we write heat transfer eq

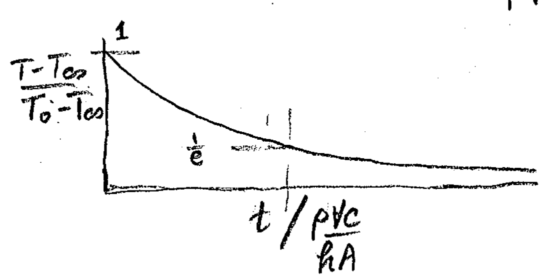
$$q = hA(T-T_\infty) = -\rho V c \frac{dT}{dt}$$

specific heat

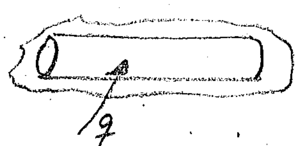
$$\frac{dT'}{(T-T_\infty)'} + \frac{hA}{\rho V c}\, dt' = 0$$

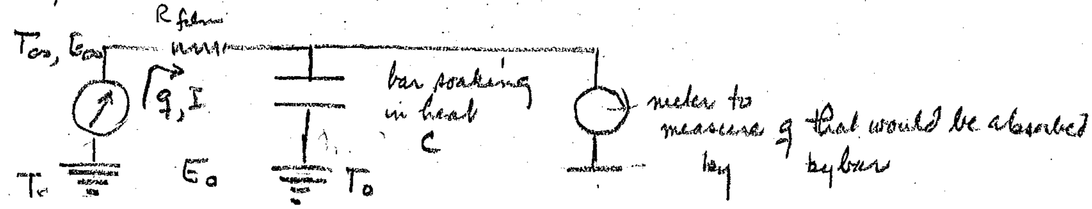
$$\ln(T-T_\infty)\Big|_{T_o-T_\infty}^{T-T_\infty} + \frac{hA}{\rho V c}\, t'\Big|_0^t = 0$$

$$\ln\left(\frac{T-T_\infty}{T_o-T_\infty}\right) = -\frac{hA}{\rho V c}\, t \quad \text{or} \quad e^{-\frac{hA}{\rho V c} t} = \frac{T-T_\infty}{T_o-T_\infty}$$



the fact that h should vary w/T is a bad assumption unless the oil were agitated to keep temp near surface interface more even





$$E = E_\infty - IR = E_0 + \frac{1}{C}\int I\,dt$$

$$\frac{dE}{dt} = -\frac{dI}{dt}R = \frac{1}{C}I$$

$$\int_{E_\infty - E_0}^{E_\infty - E} \frac{dV}{V} = \frac{dI}{I} = -\frac{1}{RC}\int_0^t dt'$$

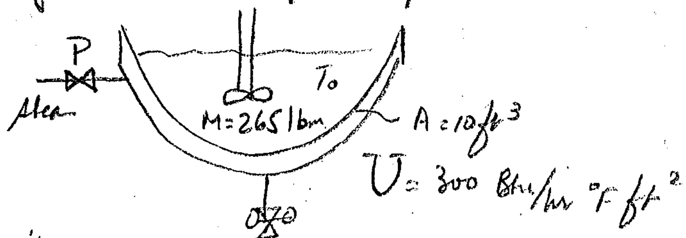
V across resistor

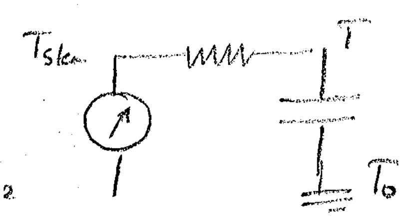
$$\frac{E - E_\infty}{E_0 - E_\infty} = e^{-\frac{1}{RC}t} \qquad R = \frac{1}{hA} \qquad C = \rho V c$$

thermocouples in a jet engine k of metal is ≫ , h gas is ≪ , ∴ Biot number is small hence we can use the preceding analysis. How big should the thermocouple be Note that $RC = \frac{\rho V c}{hA}$

note that $RC = f(V/A)$ thus we want bead of thermocouple to be small & hence small wires.

Prof. London's soup kettle problem.

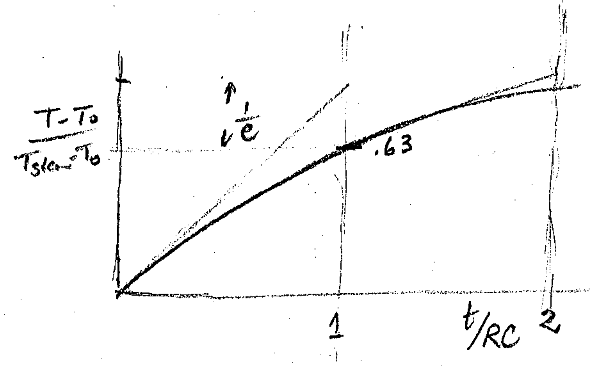




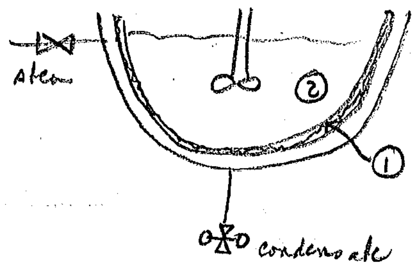
U is resistance of steam film to kettle, kettle to kettle cond. kettle to soup film interfaces

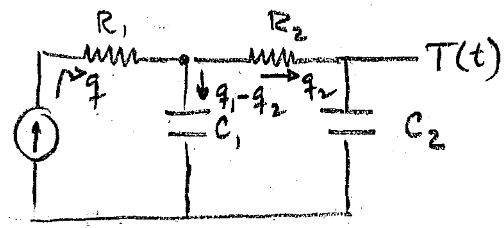
$$\frac{T - T_0}{T_{steam} - T_0} = 1 - e^{\frac{-t}{RC}} \qquad R = \frac{1}{UA} \qquad C = Mc \text{ of soup}$$

$$RC = \frac{Mc}{UA} = \frac{265 \cdot 1}{300\,(10)} = .0883 \text{ hrs.} = 5.3 \text{ min}$$



lumping of parameter may be a problem thus lets look at a two parameter type problem





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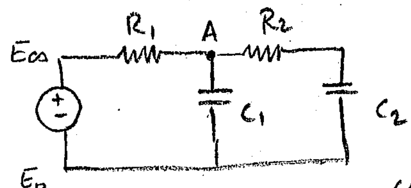


$$E_\infty = I_1 R_1 + \frac{1}{C_1}\int (I_1 - I_2)\,dt + E_0$$

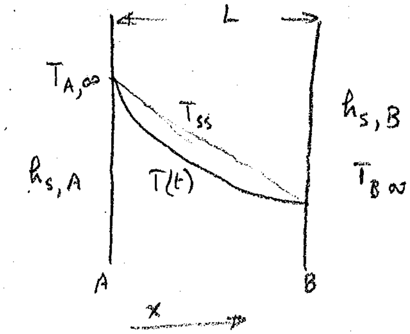
$$E_A = E_\infty - I_1 R_1 = I_2 R_2 + \frac{1}{C_2}\int I_2\,dt + E_0$$

$$\frac{dE_\infty}{dt} = R_1 \dot{I}_1 + \frac{1}{C_1}(I_1 - I_2) = 0$$

$$\frac{dE_A}{dt} = -\dot{I}_1 R_1 + \frac{dE_\infty}{dt} = \dot{I_2} R_2 + \frac{I_2}{C_2} \Rightarrow R_1\dot{I}_1 + R_2\dot{I}_2 - \frac{I_2}{C_2} = 0$$

$$\dot{E}_1 + \frac{E_1}{R_1 C_1} - \frac{R_1}{R_2} E_2 = 0 \qquad (?)$$

$$\dot{E}_1 - \dot{E}_2 - \frac{E_2}{R_2 C_2} = 0$$

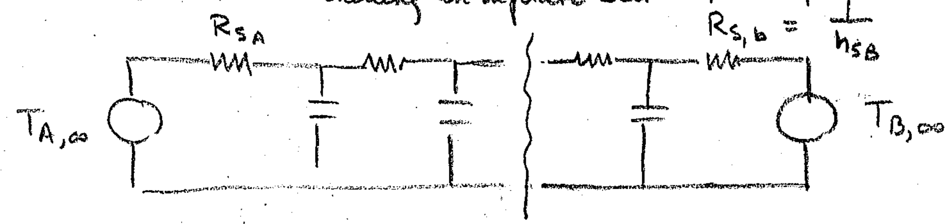


for $\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$ $\quad \alpha = \frac{k}{\rho c}$

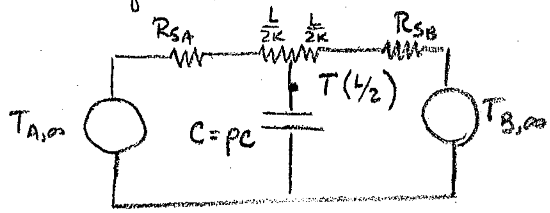
for ss $\alpha \frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow ax + b = T_{ss}$

for time dep let $T = T(t + \frac{1}{\sqrt{\alpha}}x)$

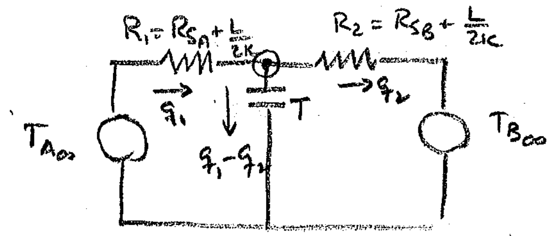
Thermal circuits modelling an infinite slab by dividing it into n such sections



Tough to solve - instead use 1 lump system



transform system



$q_1 = \frac{T_{A,\infty} - T}{R_1}$ $\qquad q_2 = \frac{T - T_{B,\infty}}{R_2}$

Energy equation is $q_1 - q_2 = C \frac{dT}{dt}$

$$C \frac{dT}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) T = \frac{T_{A,\infty}}{R_1} + \frac{T_{B,\infty}}{R_2}$$ using Kirchoff's law at $\odot$

Boundary and initial conditions 1. @ $t=0$ $T=0$

2. $C\left(\frac{dT}{dt}\right)_{t=0} = \frac{T_{A,\infty}}{R_1} + \frac{T_{B,\infty}}{R_1}$

where $q_1 = \frac{T_{A,\infty}}{R_1}$ $\qquad q_2 = -\frac{T_{B\infty}}{R_2}$

let $T^* = T/T_{ss}$; $q_1^* = q_1/q_{ss}$; $q_2^* = q_2/q_{ss}$

Then $T^* = 1 - \exp\left[-t/R_{eq}C\right]$ $\quad R_{eq} = \dfrac{R_1 R_2}{R_1 + R_2}$

$$q^*_{\substack{1\\2}} = 1 + \frac{T_{ss}}{R_{\substack{1\\2}}\, q_{ss}} \exp\left(-t/R_{eq}C\right)$$

and $T_{ss} = \left(\dfrac{T_{A,\infty}}{R_1} + \dfrac{T_{B,\infty}}{R_2}\right) \Big/ \left(\dfrac{1}{R_1} + \dfrac{1}{R_2}\right)$

hence $q_{ss} = \dfrac{T_{A,\infty} - T_{B,\infty}}{R_1 + R_2} = \dfrac{T_{A,\infty} - T_{ss}}{R_1} = \dfrac{T_{ss} - T_{B,\infty}}{R_2}$

estimate $T @ x = 0$ $\quad q_1 = \dfrac{T_{A,\infty} - T_{x=0}}{R_{SA}}$

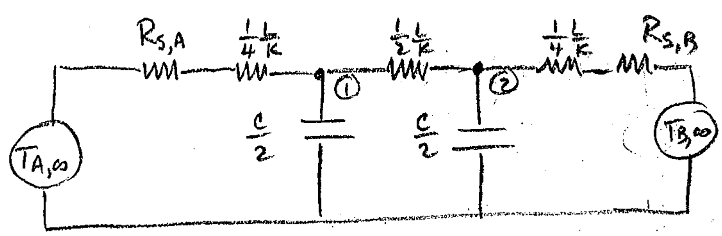
$$q_2 = \frac{T_{x=L} - T_{B,\infty}}{R_{SB}}$$

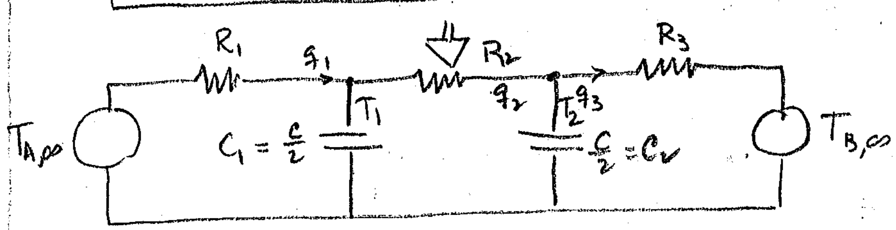
$$\left.\frac{dT}{dx}\right|_{x=0} = -\frac{1}{K} q_1$$

$$\left.\frac{dT}{dx}\right|_{x=L} = -\frac{1}{K} q_L$$

with these you can estimate temp distrib in the wall if $R^*_{max} = \left(\dfrac{hL}{K}\right)_{max} < \dfrac{1}{4}$ results are good.

10/13/78





rate eqns

$q_1 = \dfrac{(T_{\infty,A} - T_1)}{R_1}$ $\quad q_2 = \dfrac{T_1 - T_2}{R_2}$ $\quad q_3 = \dfrac{T_2 - T_{B,\infty}}{R_3}$

Energy balance $q_1 - q_2 = C_1 \frac{dT_1}{dT}$ $\qquad q_2 - q_3 = C_2 \frac{dT_2}{dt}$

BC/IC @ $t=0$

$T_1 = T_2 = 0$

$q_1 = \frac{T_{A,\infty}}{R_1} = C_1 \frac{dT_1}{dt}\Big|_{t=0}$; $q_2 = 0$; $q_3 = -\frac{T_{B,\infty}}{R_3} = -C_2 \frac{dT_2}{dt}\Big|_{t=0}$

after to algebra

$\ddot{T}_1 + A_1 \dot{T}_1 + B_1 T_1 = E_1 T_{A,\infty}$

where

$A_1 = [C_1(G_2+G_3) + C_2(G_1+G_2)]/C_1C_2$; $G_i = 1/R_i$

$B_1 = [G_1G_2 + G_2G_3 + G_3G_1]/C_1C_2$

$E_1 = [(G_2+G_3)G_1 + G_2G_3 \frac{T_{B,\infty}}{T_{A,\infty}}]/C_1C_2$

$\ddot{T}_2 + A_2 \dot{T}_2 + B_2 T_2 = E_2 T_{B,\infty}$

$A_2 = A_1 \qquad B_2 = B_1$

$E_2 = [(G_2+G_1)G_3 + G_1G_2 \frac{T_{A,\infty}}{T_{B,\infty}}]/C_1C_2$

we would expect that $T_i = T_{i_{ss}} + T_{i_{transient}}$

for steady state no current into capacitance

$$\frac{T_{A,\infty} - T_{1,ss}}{T_{A,\infty} - T_{B,\infty}} = \frac{R_1}{R_1+R_2+R_3}$$

$T_{1,trans} = K_1' \exp m_1't + K_2' \exp m_2't$

where $m_{1,2}' = -\frac{A_1}{2} \pm \frac{1}{2}\sqrt{A_1^2 - 4B}$

$\therefore \; K_1' = (m_2' T_{1ss} + \frac{G_1}{C_1} T_{A,\infty}) / (m_1' - m_2')$

$K_2' = -T_{1ss} + K_1'$

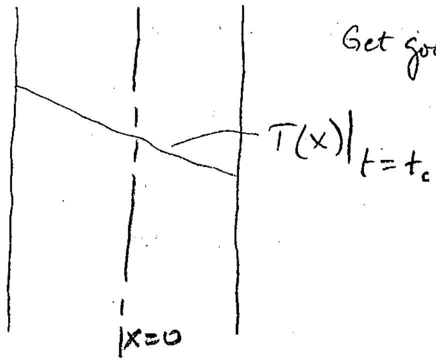
for $T_2 = T_{2ss} + K_1'' \exp m_1''t + K_2'' \exp m_2''t$

Same as before for ss... $\frac{T_{2ss} - T_{B,\infty}}{T_{A,\infty} - T_{B,\infty}} = \frac{R_3}{R_1+R_2+R_3}$

$\Rightarrow m_{1,2}' = m_{1,2}''$

$\therefore \quad K_1'' = \left(m_2' T_{2ss} + \frac{G_3}{C_2} T_{B,\infty}\right) / \left(m_1'' - m_2''\right)$

$\quad\;\; K_2'' = -\left(T_{2ss} + K_1''\right)$



Get good results for $R^* = \frac{h_{max} L}{K} < \sim 2$

Transient charts $T = T(x, t, \rho, c, k, h, L, T_\infty)$

we can reduce this to $T = T(l, t, \text{energy})$

Buckingham Π theorem $\Rightarrow$ # dimensionless groups = # Variables − # dimensions

let $x^* = \frac{x}{L}$ $\quad T^* = \frac{T - T_0}{T_\infty - T_0}$ or $T^* = \frac{T}{T_\infty}$

use equations for others $\quad \alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad \alpha = \frac{k}{\rho C}$

using dimensionless param $\quad \partial x^* = \frac{1}{L} \partial x$; $\partial T^* = \frac{1}{T_\infty} \partial T$

$\therefore \quad \alpha \frac{T_\infty}{L^2} \frac{\partial^2 T^*}{\partial x^{*2}} = T_\infty \frac{\partial T^*}{\partial t}$ or $\left(\frac{\alpha}{L^2}\right) \cdot T^*_{,x^*x^*} = T^*_{,t}$

let $t^* = \frac{L^2}{\alpha} t$ then $\partial t^* = \frac{L^2}{\alpha} \partial t$

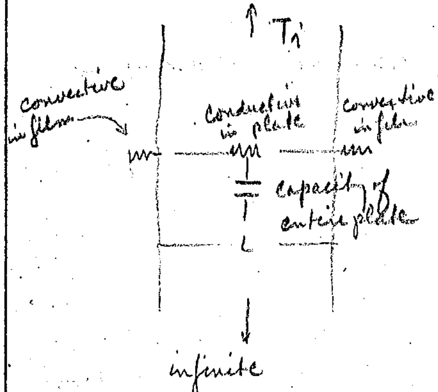
causing $\alpha T_{,xx} = T_{,t} \Rightarrow T^*_{,x^*x^*} = T^*_{,t^*}$

and $K \frac{\partial T}{\partial x} = h(T_\infty - T) \Rightarrow K \frac{T_\infty}{L} T^*_{,x^*} = h T_\infty (1 - T^*)$

or $T^*_{,x^*} = \frac{hL}{K}(1 - T^*)$ thus let $h^* = \frac{hL}{K}$

10/15/78

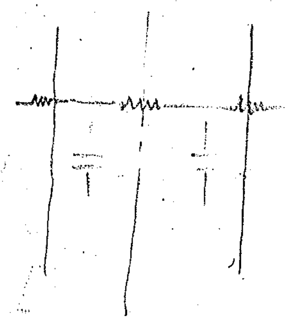
No Problem session This wed It will be on Tues instead @ 3:15 pm in 501A



suppose @ $t=0$ we immerse this infinite plate in medium of $T = T_\infty$

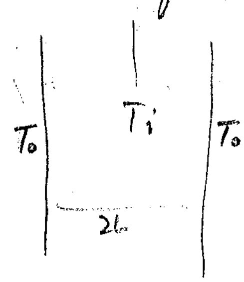
1 lumped parameter method approx.

$$\dot{q}''_{conv} = h\,(T_\infty - T_0)$$



2 lumped parameter approximation if problem is messy

For infinite plate 1^{ST} type of problem



Governing equation $T_{xx} = \frac{1}{\alpha} T_t$

IC $T(x,0) = T_i$ @ $t=0;\ -L< x < L$

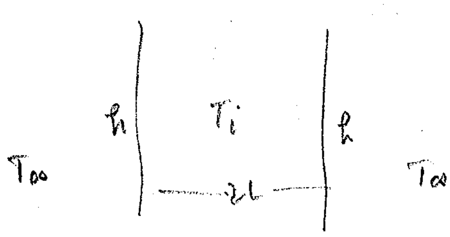
BC $T(\pm L, t) = T_0$

Method of Separation of Variables: assume $T(x,t) = X(x)\,\varphi(t)$ $\Rightarrow$ will give 2 ODE

$\rightarrow$ $X''\varphi = \frac{1}{\alpha}\varphi' X$ or $\underset{(f(x)\text{ only})}{\frac{X''}{X}} = \underset{(f(t)\text{ only})}{\frac{\varphi'}{\alpha\varphi}} = \eta^2$

$X'' - \eta^2 X = 0$ & $\varphi' = \eta^2\alpha\varphi$

$X = C_1 e^{\eta x} + C_2 e^{-\eta x}$ $\qquad \varphi = C_3 e^{\eta^2 \alpha t}$ $\qquad$ Eq 4-35b



2nd type
Solution – eq 4.41

$T = f(\underbrace{x, t}_{\text{indep variables}}; \underbrace{\rho, c, k, l, T_i, T_\infty, h}_{\text{independent parameters}})$

T is not a funct. of all those but on some non dimensional groups involving these items

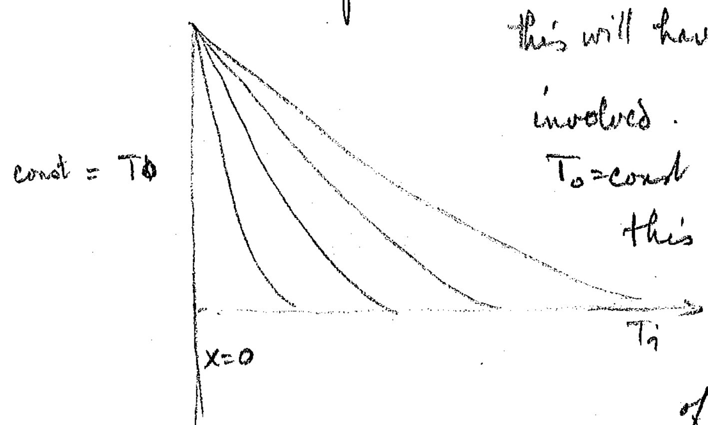
define

$$\theta \triangleq \frac{T - T_\infty}{T_i - T_\infty} \; ; \quad x^* = \frac{x}{L} \; ; \quad t^* = \frac{\alpha t}{L^2} \; ; \qquad \alpha = \frac{k}{\rho c} \qquad \frac{hL}{k} = Bi$$

$\therefore$ we can write $\theta = f[x^*, t^*; Bi]$

we can collapse x^*; t^* into a new variable of $\frac{x^*}{\sqrt{\alpha t^*}}$ if there is no characteristic length. In this problem there is namely l.

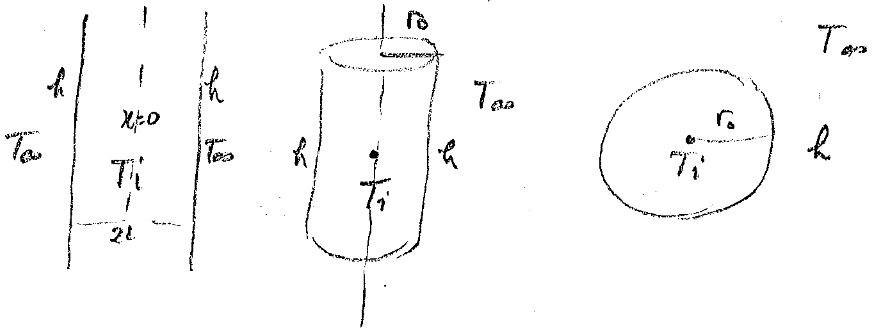
Consider a semi infinite slab



this will have the $\frac{x^*}{\sqrt{\alpha t^*}}$ variable ~~of~~ involved. If BC is changed from T_0 = const to $h(T_0 - T_\infty)$ we lose this $\frac{x^*}{\sqrt{\alpha t^*}}$ because reintroduction of h will require the addition of B_i as an indep variable

Heisler Chart (convective cooling/heating only)

$\theta = f(x^*, t^*; Bi)$



10/18/78

Consider a long bar $2L \times 2L \times \infty$ Initially at T_i, surfaces changes to T_∞

Eqn. $\Delta T = \frac{1}{\alpha}\frac{\partial T}{\partial t}$

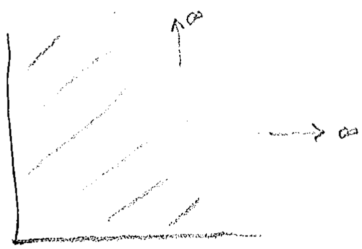
BC $t=0 \quad T(x,y)=T_i$

$t=0^+, \quad T(x=\pm L, y=\pm L) = T_\infty$

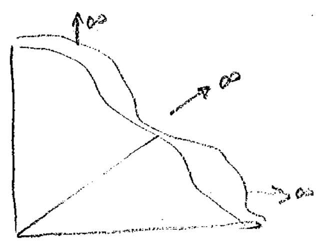
$$T(x,y,t) = T(x,t)\cdot T(y,t) = T_x \cdot T_y$$

$$\Delta T = T_x\left[\frac{\partial^2 T_y}{\partial y^2}\right] + T_y\left[\frac{\partial^2 T_x}{\partial x^2}\right] = \frac{1}{\alpha}\left[T_x\frac{\partial T_y}{\partial t} + T_y\frac{\partial T_x}{\partial t}\right]$$

$$T_y\left[\frac{\partial^2 T_x}{\partial x^2} - \frac{1}{\alpha}\frac{\partial T_x}{\partial t}\right] + T_x\left[\frac{\partial^2 T_y}{\partial y^2} - \frac{1}{\alpha}\frac{\partial T_y}{\partial t}\right] = 0$$

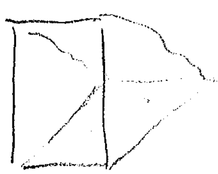


2-D corner = 2 semi infinite slabs intersecting



3-D corner = 3 semi infinite slabs intersecting

thick plate (semi infinite)



1 thick plate = 1 infinite slab

Restrictions of intersection method

1. Uniform initial temp
2. " BC
3. Constant Properties

Heisler charts for thick plates

1. Find Biot numbers $\frac{ha}{k}$, $\frac{hb}{k}$, $\frac{hc}{k}$

2. Set θ and calculate Fourier numbers $\frac{\alpha\theta}{a^2}$, $\frac{\alpha\theta}{b^2}$, $\frac{\alpha\theta}{c^2}$

How to find temp in a brick if you know solution for thick semi infinite plates

3. Find $\left(\frac{T_{\text{¢}} - T_\infty}{T_i - T_\infty}\right)_{2a}$, $\left(\frac{T_{\text{¢}} - T_\infty}{T_i - T_\infty}\right)_{2b}$, $\left(\frac{T_{\text{¢}} - T_\infty}{T_i - T_\infty}\right)_{2c}$
using fourier & Biot numbers for 1st chart

4. Find $\left(\frac{T_A - T_\infty}{T_{\text{¢}} - T_\infty}\right)_{2a}$, $\left(\frac{T_A - T_\infty}{T_{\text{¢}} - T_\infty}\right)_{2b}$, $\left(\frac{T_A - T_\infty}{T_{\text{¢}} - T_\infty}\right)_{2c}$
using $\frac{x}{a}$, $\frac{x}{b}$, $\frac{x}{c}$ and fourier numbers

5. Find $\left(\frac{T_A - T_\infty}{T_i - T_\infty}\right)_{2a} = [3]\cdot[4]_{2a}$, $[3]\cdot[4]_{2b}$, $[3]\cdot[4]_{2c}$

6 Mult $[5]_{2b}\cdot[5]_{2a}\cdot[5]_{2c} = \left(\frac{T_A - T_\infty}{T_i - T_\infty}\right)$ Brick temp at pt A.

	k Btu/hr ft °F	ρ lbm/cu ft.	c Btu/lbm (specific heat)	ρc Btu/ft³ °F	α ft³/hr (thermal diffusivity)
Silver	240	657	0.57	37.4	6.4
Aluminum	118	169	0.213	36.1	3.3
Copper	220	552	0.093	51.3	4.3
Steel	26	493	0.115	56.8	0.46
Concrete	0.54	133	0.21	28	0.019
Cork board	0.025	10	0.45	4.5	0.0055

high $\alpha = \frac{k}{\rho c}$ → rapid responsive for temp change

10/20/78

Omit problem 4.23

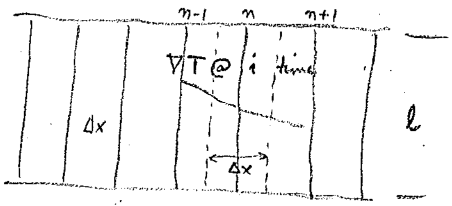
Fire Resistance

Steel ½" I-beam, $\alpha_{steel} = .319$

Wood 6"D timber, $\alpha_{wood} = .003$

time to failure ~ 6 minutes for steel, 3 hrs wood.

Numerical 1-D



unit depth

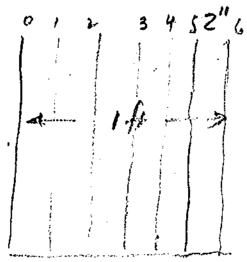
heat flow into slab $\left[\dfrac{T_{n-1}^{i} - T_n^{i}}{R_{n-1,n}} + \dfrac{T_{n+1}^{i} - T_n^{i}}{R_{n+1,n}}\right]\Delta t = C_n\left(T_n^{i+1} - T_n^{i}\right)$ change of internal energy of slab

$$T_n^{i+1} = T_n^{i}\left[1 - 2\frac{\Delta t k}{\rho c \Delta x^2}\right] + \frac{\Delta t k}{\rho c \Delta x^2}\left(T_{n-1}^{i} + T_{n+1}^{i}\right) \quad ; \quad T_j^{i} \Rightarrow \begin{array}{l} i \text{ time} \\ j \text{ segment \#} \end{array}$$

$$R_n = \frac{L}{kA} = \frac{\Delta x}{k\,l \cdot 1} = \frac{1}{k} \qquad C_n = mc = \rho V c = \rho\, l\, \Delta x c \cdot 1$$

if we let $2\dfrac{\Delta t k}{\rho c \Delta x^2} = 1 \Rightarrow T_n^{i+1} = \frac{1}{2}\left(T_{n-1}^{i} + T_{n+1}^{i}\right)$ Schmidt's Method

Example

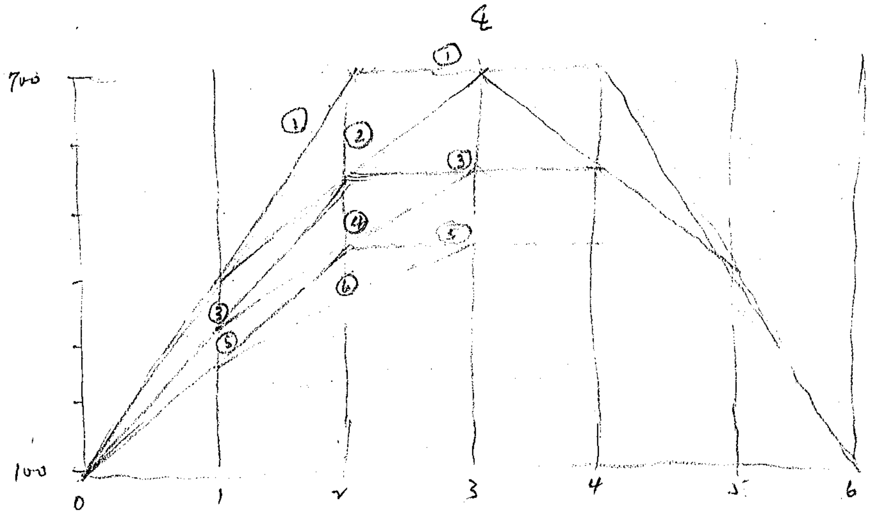


$T_{initial} = 700\,°F$

$T_{surface}$ 100 °F

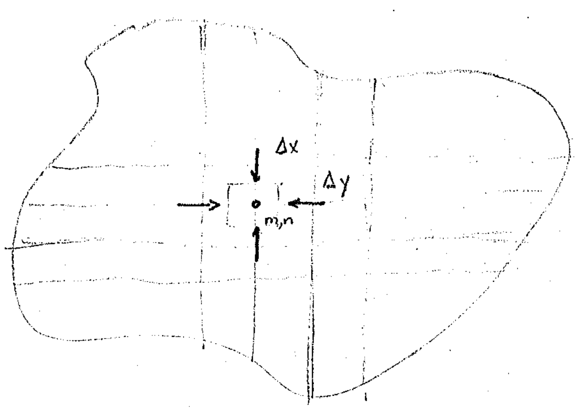
what is $T(x) = ?$ after 14.85 min = 7ΔΘ

ΔΘ	T_0	T_1	T_2	T_3	T_4	T_5
0	100	700	700	700	700	700
1	100	400	700	700	700	400
2	100	400	550	700	550	400
3	100	325	550	550	550	325
4	100	325	437	550	437	
5	100	269	437	437	437	
6	100	269	354	437	354	

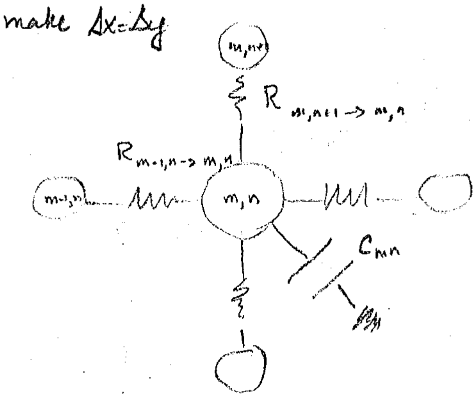


2-D

heat into



make $\Delta x = \Delta y$



$$T_{m,n}^{i+1} = \frac{k\Delta t}{\rho c(\Delta x)^2}\left(T^i_{m+1,n} + T^i_{m-1,n} + T^i_{m,n+1} + T^i_{m,n-1}\right) + \left[1 - 4\frac{k\Delta t}{\rho c(\Delta x)^2}\right]T^i_{m,n}$$

Explicit form: if we pick out coeff we may find that Δt is ≥ 0 in fact too small

we can rewrite an implicit form

$$T_n^{i+1} = \frac{T_n^i + \frac{k\Delta t}{\rho c(\Delta x)^2}\left(T_{n-1}^{i+1} + T_{n+1}^{i-1}\right)}{1 + 2\frac{k\Delta t}{\rho c(\Delta x)^2}}$$

note that we don't have the coeff problem that we had before, but we must solve a matrix form & get all pts simultaneously.

10/23/78

Conduction topics we have studied.

A Steady

1. 1-D Composite structures
 parallels

2. 2-D
 Laplaces { Hand drawn curvilinear squares
 Analyses (math)
 Analogy
 Computer

3. 3-D
 Computer

B. Transient

1. Lumped parameter
 a) 1 Lump System (Biot No < 0.1)
 b) More Lumps
2. 1-D Cases - Heisler Charts
3. Extend Heisler charts to 2-D, 3-D h is assumed uniform over body
4. Computer. (allows h to vary)

CONVECTION

Goal - is to find h for a variety of situations

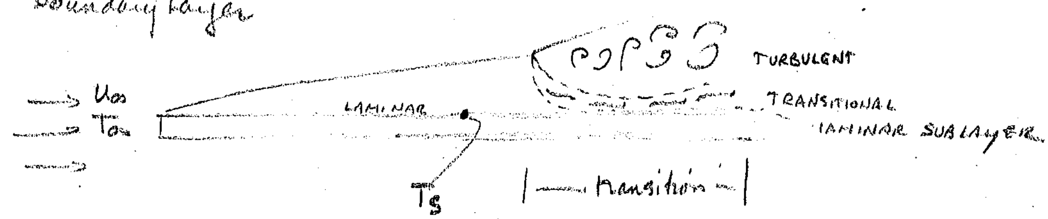
I. Talk about BL fundamentals
 1. Dimensional analysis
 2. Exact Solutions to laminar case (Blasius, Pohlhausen)
 3. Integral approximation for simpler solution.
 4. Reynolds Analogy - turbulent flow similarity between Fluids / Heat Transfer
 5. Computer solution for turbulent flow (STAN-5)

II Free convection.
 1. Dimensional analysis
 2. Experimental correlations

III Forced Convection inside of tube
 1. Reynold's analogy
 2. Variation of properties.

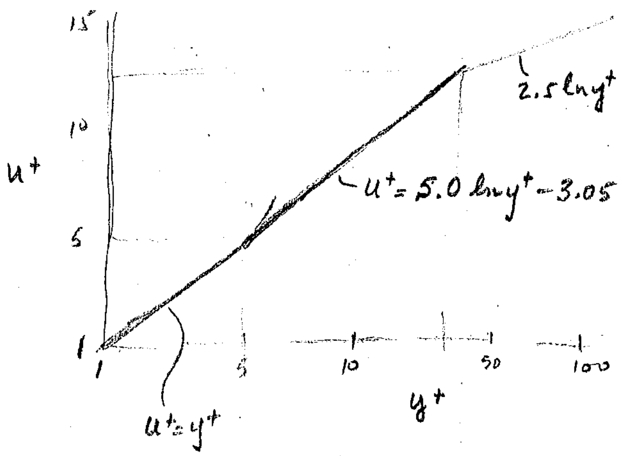
IV Flow over Exterior surfaces
 1. Experimental correlations

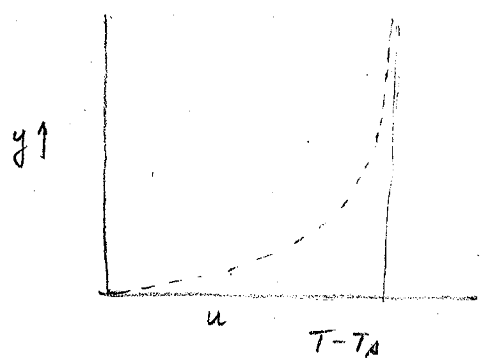
Boundary layer



develop b.l. parameters u^+, y^+ $\quad u^+ \triangleq \dfrac{\bar{u}}{U^*}$ $\quad u^* \triangleq \sqrt{\dfrac{\tau_s}{\rho}}$

$$y^+ \triangleq \frac{U^*}{\nu} y$$





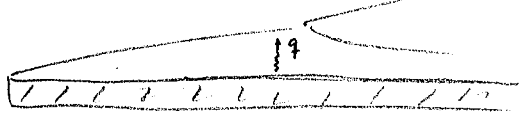
$$q = -k_{fluid} A \left.\frac{\partial T}{\partial y}\right)_s$$

we have problems finding $\left.\dfrac{\partial T}{\partial y}\right)_{surface}$ we can use Blasius & Pohlhausen or use

$q = hA(T_\infty - T_s)$ which lumps all uncertainty into h

10/25/78

homework 6-21, 35, 36, 46, 54



$$-k_f A \frac{\partial T}{\partial y} = h_f A (T - T_\infty)$$

$$\left.\frac{\partial (T_s - T)}{\partial y}\right|_{y=0} \Big/ \frac{T_\infty - T_s}{L} = \frac{hL}{K_f} = N_u \quad \text{(nusselt no.)}$$

and $h = h(u_\infty, \rho, C_p, K, L, \mu)$

Dimensional analysis shows we must use L, M, t, T

$Re = \dfrac{\rho L u}{\mu}$, P_r (Prandtl No) $= \dfrac{C_p \mu}{K} = \dfrac{\mu/\rho}{\dfrac{K}{C_p \rho}} = \dfrac{\text{Diffusion of vorticity}}{\text{Dissipation of heat}}$

$$Re = \frac{\text{inertia forces}}{\text{viscous forces}}$$

we should be able to write $Nu = f(Re, Pr)$

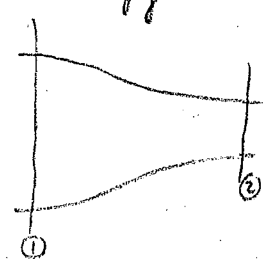
$Nu_x = .332\, Re_x^{1/2}\, Pr^{1/3}$ Laminar flow over a flat plate

$Nu_x = .023\, Re^{.8}\, Pr^{.4}$ Turbulent flow in a pipe

$Nu_x = .2\, Re^{.6}\, Pr^{.31}$ Flow over a tube

Let i = enthalpy

For steady flow w/no shaft work, adiabatic

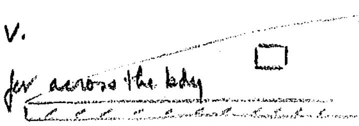


$$i_1 + \frac{V_1^2}{2} = i_2 + \frac{V_2^2}{2} \quad \text{using 1-D flow}$$

$$\therefore \quad c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$$

$$\text{if } V_1 = 0 \implies T_1 = T_2 + \frac{V_2^2}{2c_p}$$

lets look at a C.V.



calculate the energy transfer across the bdy

= heat flux in + work terms

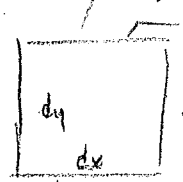
$$Q = \dot{m}(h_{out} - h_{in}) + \dot{w}_{shaft} + \dot{w}_{shear}$$

for most fluids we can neglect the temp gradient in the x-direction

$\rho v\, dx \left(c_p T_0 + \frac{\partial}{\partial x}(c_p T_0)\, dx\right)$

$-k\, dx \left(\frac{dT}{dy} + \frac{d}{dy}\left(\frac{\partial T}{\partial y}\right) dy\right)$

$\rho u\, c_p T_0\, dy$



$\rho u\, dy \left(c_p T_0 + \frac{\partial}{\partial x}(c_p T_0)\, dx\right)$

$-k\, dx \frac{dT}{dy}$ $\quad \rho v\, dx\, c_p T_0$

$\mu \frac{\partial u}{\partial y} dx \cdot u + \frac{\partial}{\partial y}\left(\mu u \frac{\partial u}{\partial y}\right) dy\, dx$

what about $\dot{w}_{shear}$

$\mu \frac{\partial u}{\partial y} dx \cdot u$

Performing the energy balance if we assume c_p = const, ρ = const and u, v are const

$$\rho c_p u \frac{\partial T_0}{\partial x} + \rho c_p v \frac{\partial T_0}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + \frac{\partial}{\partial y}\left(\mu u \frac{\partial u}{\partial y}\right)$$

for low speed flow we can say:

① $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

to solve we need the relationships between u & v ie continuity

② using steady incomp flow $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

using the momentum in the x direction $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} - \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial x}}_{0 \text{ for an infinite flat plate}}$

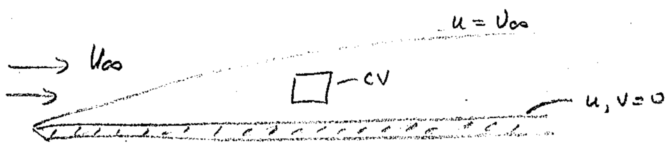
① & ② are similar if $\alpha = \mu/\rho$ & u is replaced by T
and the temp profile is identical to the velocity profile.

If we define $\eta = y/\delta$ $\qquad \frac{\delta}{x} \approx \frac{1}{\sqrt{Re_x}}$

We can also use the Blasius solution $\eta \equiv y\sqrt{\frac{u_\infty}{x\nu}}$ and $\psi = \sqrt{\nu x u_\infty}\, f(\eta)$

10/27/78

Consider a steady, incompressible, constant property, low velocity flow and laminar flow.



Cont $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Mom $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{d^2 u}{dy^2} - \cancel{\frac{\partial p}{\partial x}}^{\,0}$$ since plate is ∞

Energy $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Solution: see Schlichting Boundary Layer Theory Pg 117 4th ed
Blasius solution
let $\eta = y\sqrt{\frac{U_\infty}{\nu x}}$ and Define stream function $\psi = \sqrt{\nu x u_\infty}\, f(\eta)$

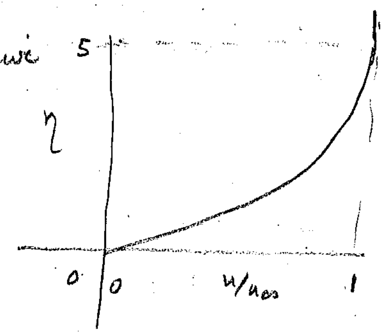
$$u = \frac{\partial\psi}{\partial y} = U_\infty f' \qquad v = -\frac{\partial\psi}{\partial x} = -\sqrt{\frac{\nu u_\infty}{x}}\left[\eta f'(\eta) - f(\eta)\right]$$

These solve continuity. Plug these in to momentum eq thus

$$f f'' + 2f''' = 0$$

w/BC $\eta = 0$ $f = 0$ $f'(0)$, $\eta = \infty$

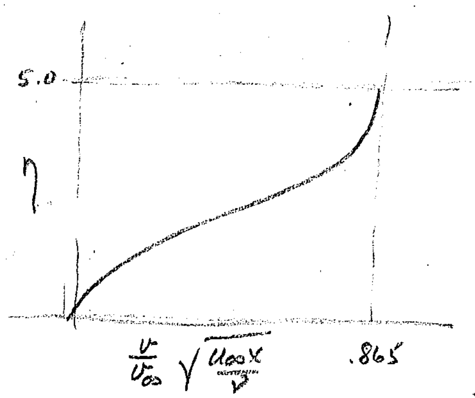
Blasius solved this to give



$y\sqrt{\frac{u_\infty}{\nu x}} \cong 5.0$

at $\frac{U}{U_\infty} = .99$

$\therefore$ if at $y = \delta$ $\frac{u}{U_\infty} = .99$ $\Rightarrow$ $\frac{\delta}{x} = 5.0 \Big/ \sqrt{\frac{x u_\infty}{\nu}} = \frac{5.0}{\sqrt{R_{N_x}}}$



To obtain a relation of v vs. η

$$\frac{V_\infty}{u_\infty} = \left(\frac{.865}{5.0}\right)\frac{\delta}{x}$$

Defining the shear at the wall

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = .332\, \mu \frac{u_\infty}{x}\sqrt{R_{N_x}}$$

Define a friction coeff (local)

$$C_{f,x} = \frac{\tau_s}{\frac{1}{2}\rho u_\infty^2} = .664 \Big/ \sqrt{R_{N_x}}$$

Pohlhausen used these results to find the temp grad.

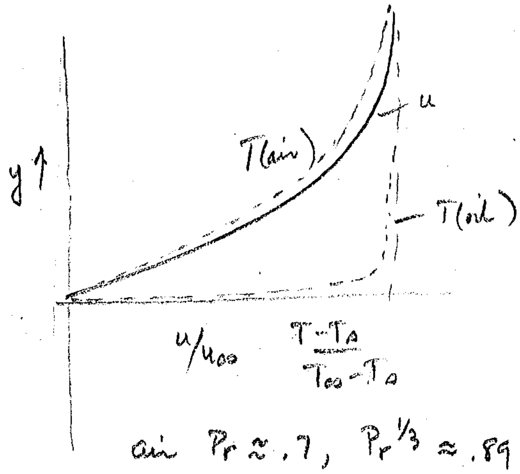
$$\frac{d\left(\frac{T - T_A}{T_\infty - T_A}\right)}{dy}\Bigg|_{y=0} \cong 0.332\, P_r^{1/3}$$

Now $-k \frac{\partial T}{\partial y}\Big|_{y=0} = h_x\,(T_s - T_\infty)$

$$h_x = .332\, k P_r^{1/3} \sqrt{\frac{u_\infty}{\nu x}}$$

define the Nusselt no. $\frac{h_x x}{k} = Nu_x = .332\, Re_x^{1/2}\, P_r^{1/3}$

as $x \uparrow$ $h_x \downarrow$

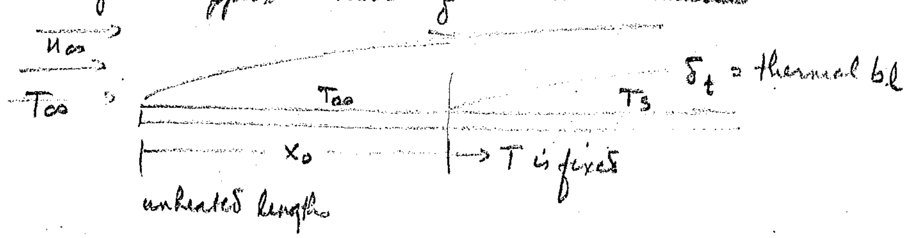


air $P_r \approx .7$, $P_r^{1/3} \approx .89$

Oil $P_r \approx 1000$, $P_r^{1/3} \approx 10$

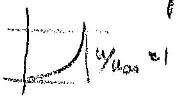
for turbulent flow we can no longer use $\tau = \mu \frac{\partial u}{\partial y}$ in momentum

Integral Approx Method , Karman - Pohlhausen



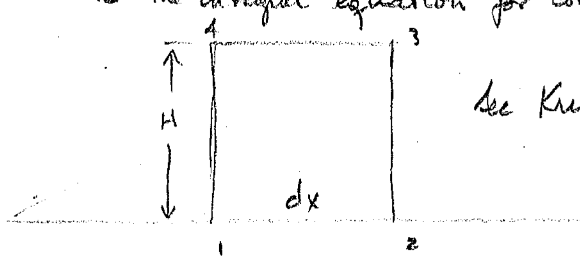
Method

1. Assume a boundary layer profile that satisfies B.C. and expect the slope at $\frac{u}{u_\infty} = 1$ $\approx \infty$



Take $T = a + by + cy^2 + dy^3$

2. Write the integral equation for cont, mass, momentum.



See Kreith pg 344 ff

3. Substitute assumed profile integral eqns ; integrate to get a 1st order ODE

4. Solve for δ & δ_t

5. We can then solve for h, τ_{wall} by using $q = h_A(T_A - T_\infty) = -kA \left.\frac{\partial T}{\partial y}\right)_{y=0}$

$$h = .323\, k P_r^{1/3} \left(\frac{u_\infty}{\nu x}\right)^{1/2} \left[1 - \left(\frac{x_0}{x}\right)^{3/4}\right]^{-1/3}$$

If we let $x_0 \to 0$ we should get Pohlhausen soln. $h = 3.332 \frac{R_N^{1/2}}{x} P_r^{1/3}$

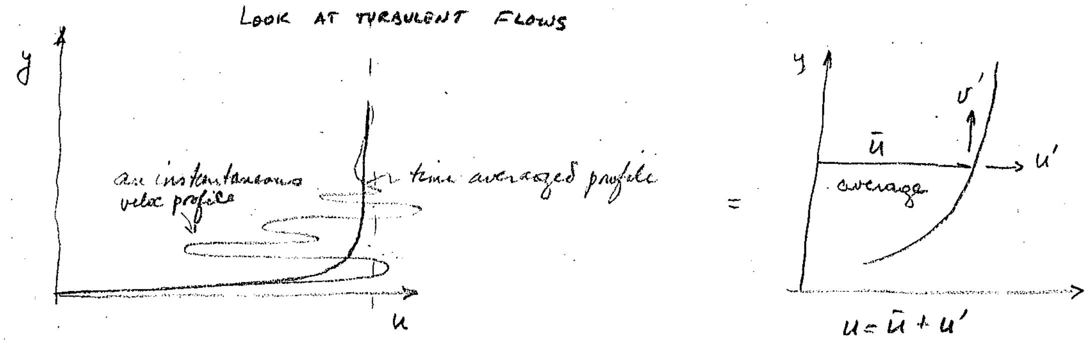
10-31-78

Prob. set #6 7.3, 7.18, 7.29, 8.16, 8.23 - Fri

S-4 - Mon

On STAN-5 [NASA CR-2742 Crawford and Kays December 1976.

LOOK AT TURBULENT FLOWS



$u = \bar{u} + u'$

$v = v'$

there will be an instantaneous momentum transport $\rho v'(\bar{u} + u')$

$\therefore$ shear $\tau_{turb} = -\frac{1}{t^*}\int_0^{t^*} (\rho v')(\bar{u} + u')\,dt = -\frac{1}{t^*}\int_0^{t^*} \rho v'u'\,dt = -\rho \overline{u'v'}$

due to $\int_0^{t^*} \frac{\rho \bar{u} v'\,dt}{t^*} = 0$ since $\bar{v}' = 0$

$\therefore \tau_{tot} = \tau_{lam} + \tau_{turb}$

Prandtl's mixing length theory $u' \approx l \frac{d\bar{u}}{dy}$

define an "eddy viscosity" ϵ_M thus $\tau_t = \rho \epsilon_M \frac{d\bar{u}}{dy} = -\rho \overline{u'v'}$

using Prandtl for $\frac{d\bar{u}}{dy}$ then

$\tau_t = \rho \epsilon_M \frac{u'}{l}$ and equating gives $\epsilon_M = -\frac{\overline{u'v'}\, l}{u'}$

$\therefore \tau_{tot} = \rho(\nu + \epsilon_M)\frac{d\bar{u}}{dy}$ for turb flow $\epsilon_M \gg \nu$ $\therefore \tau_{tot} \approx \rho \epsilon_M \frac{d\bar{u}}{dy}$

We can use a similar process for energy
the energy transfer that goes across bl is $(\rho v')C_pT$ if $T = \bar{T} + T'$
in the same manner as before if we time average

$$(q/A)_{turb} = \rho C_p \overline{v'T'} = -\rho C_p \epsilon_H \frac{d\bar{T}}{dy}$$

$$(q/A)_{TOT} = (q/A)_{turb} + (q/A)_{lam} = -\rho C_p (\underset{\frac{k}{\rho C_p}}{\alpha} + \epsilon_H) \frac{d\bar{T}}{dy}$$

if $\epsilon_M = \epsilon_H$ $\quad Pr_{turb} = \frac{\epsilon_M}{\epsilon_H} = 1$

if we ignore α in favor of ϵ_H and solve for dy and then placing it back into τ_{turb} we get

$$\boxed{(q/A)_{turb} = -\tau_{turb}\, C_p \frac{d\bar{T}}{d\bar{u}}} \qquad \text{Reynolds Analogy}$$

$C_{f,x} = C_{fric,x} = .0576\,(Re_x)^{-1/5}$ from experiment, turb flow, flat plate

where $C_{f,x} = \frac{\tau_{surface}}{\rho U_\infty^2/2}$ take this definition of $C_{f,x} \cdot \rho \frac{U_\infty^2}{2} = \tau_f$

$$\therefore \quad (q/A)_{turb} = -C_f C_p \rho \frac{U_\infty^2}{2} \frac{d\bar{T}}{d\bar{u}} = -C_f C_p \rho \frac{U_\infty^2}{2} \frac{d\left(\frac{\bar{T}-T_s}{T_\infty - s}\right)}{d\left(\bar{u}/U_\infty\right)} \cdot \frac{T_\infty - T_s}{U_\infty}$$

we also know that $(q/A)_{turb} = h(T_s - T_\infty) = -.0576\, Re^{-1/5} C_p \rho \frac{U_\infty^2}{2} \frac{T_\infty - T_s}{U_\infty}$

since for $Pr_{turb} = 1 \Rightarrow \frac{d\left(\frac{\bar{T}-T_s}{T_\infty - s}\right)}{d(\bar{u}/U_\infty)} = 1$

$$\therefore \quad \frac{hx}{k} = +.0576 \frac{x}{k} Re^{-1/5} C_p \frac{\rho U_\infty^2}{2 U_\infty} = .0288 \left(\frac{\rho x U_\infty}{\mu}\right)^{.8} \left(\frac{\mu C_p}{k}\right)$$

for $Pr \neq 1$ take $\left(\frac{\mu C_p}{k}\right)^{1/3}$ instead of $\left(\frac{\mu C_p}{k}\right)$ (by experimental observation)

What about bl thickness - experimental work shows that $\left(\frac{y}{\delta}\right)^{1/7} \sim \left(\frac{u}{U_\infty}\right)_{turb.}$

Kreith pg 367 ff

Summary

1. Momentum & energy are transported in a similar fashion by the v' component of fluid: (ρu) $(\rho c_p T)$
2. Prandtl: can express a mixing length before instantaneous mixing.
3. $\epsilon_M = \epsilon_H$ for $Pr = 1$

Look at CONT: $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0$

MOM: $\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu_{eff}\frac{\partial u}{\partial y}\right)$

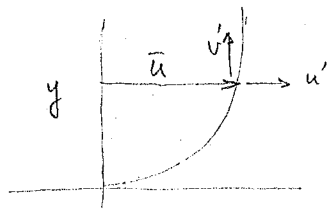
ENERGY: $\rho u \frac{\partial i^*}{\partial x} + \rho v \frac{\partial i^*}{\partial y} = \frac{\partial}{\partial y}\left[\left(\frac{k}{c_p}\right)_{eff}\frac{\partial i}{\partial y} + \frac{\partial}{\partial y}\mu_{eff}\frac{\partial}{\partial y}\frac{u^2}{2}\right]$ $\quad i^* = i + \frac{u^2}{2}$

$$\mu_{eff} = \mu + \mu_{turb} = \rho(\nu + \epsilon_M)$$
$$\left(\frac{k}{c_p}\right)_{eff} = \frac{k}{c_p} + \left(\frac{k}{c_p}\right)_{turb}$$

If $u' = v'$ & using prandtl mixing length then $\epsilon_M = l^2\left|\frac{d\bar{u}}{dy}\right|$

11/1/78

Midterm Reminder open book: 8 Nov 78



$$\tau_{turb} = -\rho\overline{v'u'} = \rho\epsilon_M\frac{d\bar{u}}{dy}$$

$$\tau = \rho(\nu + \epsilon_M)\frac{d\bar{u}}{dy} \quad \text{where } \epsilon_M \gg \nu$$

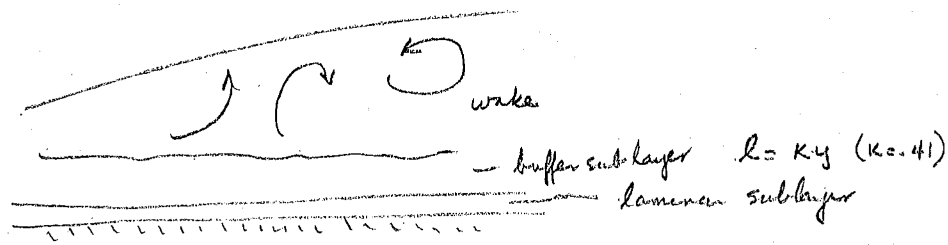
$$q/A = -\rho c_p(\alpha + \epsilon_H)\frac{d\bar{T}}{dy}$$

Reynolds: turbulent transport of energy is analogous to that of momentum

By Reynolds analogy $\Rightarrow$ $q/A = -\tau c_p \frac{d\bar{T}}{d\bar{u}}$

Prandtl: $u' = l\frac{d\bar{u}}{dy}$ and $u' = v'$ $\Rightarrow$ $\epsilon_M = l^2\frac{d\bar{u}}{dy}$

For turbulent bl.

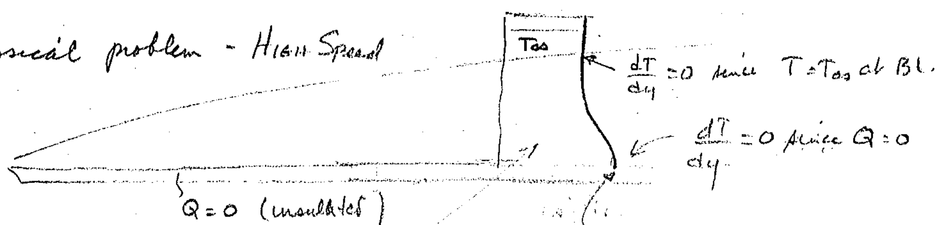


in laminar sublayer $D = 1.0 - \exp\left(\frac{y^+}{A^+}\right)$ where $y^+ = y\left(\sqrt{\frac{\rho\tau}{\mu}}\right)_{wall}$ $A^+ < 25$

$$\therefore\ l = KDy$$

in the wake $l = \lambda \delta_{.99}$; $\lambda \cong .085$ Wake region is defined for $y > \frac{\lambda}{K}\delta_{.99}$

Classical problem - High Speed



$i + \frac{V^2}{2} = i_0$ T profile - since $T \uparrow$ as $v \downarrow$ T_{as} $T_{as} \approx T_0$

$$\frac{T_{as} - T_\infty}{T_0 - T_\infty} = R = \text{recovery fn} = fn(P_r)$$

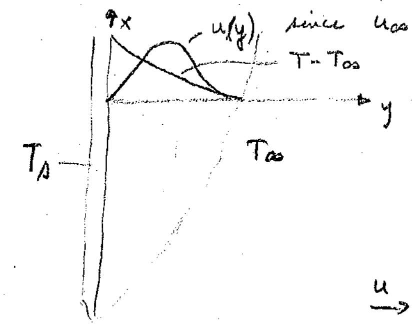
$$q/A = h(T_s - T_{as})$$

at surface since $T > T$ at layer above $\exists$ a conduction to the layer above since $\Delta T \neq 0$; however since T at surface is for one where $V = 0$ & no heat is conducted through plate (since it is insulated) $T_s > T_\infty$ (since plate is slowed down) but $T < T_0$ (by cond. from layer to layer)

for high speed flow where dissociation is present $q/A = \frac{h}{c_p}(i_s - i_{as})$ where the i (enthalpy) is the static enthalpy + chemical reaction enthalpy.

11/3/78

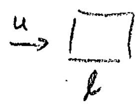
Free Convection - Natural convection.



since $u_\infty = 0$ $\quad Nu \sim f\left(Re = \frac{\text{inertia}}{\text{viscous}},\ Pr = \frac{\text{mom diff}}{\text{thermal diff}},\ \text{buoyant force}\right)$

$$\text{Buoyant force}/_{\text{unit vol}} \sim g(\rho_\infty - \rho)$$

$$\text{Viscous force / unit volume} \sim \frac{\tau \cdot \text{area}}{\text{volume}} \sim \frac{\mu \frac{u}{l} \cdot l^2}{l^3} = \frac{\mu u}{l^2}$$



$$\text{Momentum force / unit volume} \sim \frac{\rho u l^2}{l^3} u \sim \frac{\rho u^2}{l}$$

Δ veloc $\sim u$ $\quad$ volume $\sim l^3$ $\qquad \therefore Re \sim \frac{\rho u^2/l}{\mu u/l^2} \sim \rho u l/\mu$

$$\frac{\text{Buoyant force}}{\text{viscous force}} \sim \frac{g(\rho_\infty - \rho)}{\mu u/l^2} \sim \frac{g\beta\rho(T - T_\infty)l^2}{\mu u} \quad \text{where } \beta = \frac{\rho_\infty - \rho}{\rho(T - T_\infty)}$$

$$\text{Now } \frac{\text{Buoyant force}}{(\text{viscous forces})^2} \cdot \text{momentum force} \sim \frac{g\beta\rho^2(T - T_\infty)l^3}{\mu^2} \cong \text{Grashof No.}$$

$$\therefore \quad Nu = Nu(Gr, Pr)$$

From Heat Transmission - McAdams.

For air 1 atm, near room temp.

heated Horizontal plate facing upwards		$h = .38(\Delta T)^{1/4}$	Btu/hr-sq ft °F
" " " " downwards		$h = .2(\Delta T)^{1/2}$	$T_s - T_\infty = \Delta T$
Vertical plate	> 1 ft high	$h = .27(\Delta T)^{1/4}$	
"	< 1 ft high	$h = .28\left(\frac{\Delta T}{L}\right)^{.25}$	
Vertical pipes	> 1 ft high	$h = .27\left(\frac{\Delta T}{D}\right)^{1/4}$	

Horizontal pipes $\quad h = .27\left(\frac{\Delta T}{D}\right)^{.24}$

To be ready for exam

Ch #6 - fundamentals

1. Concept of B.L.

Laminar

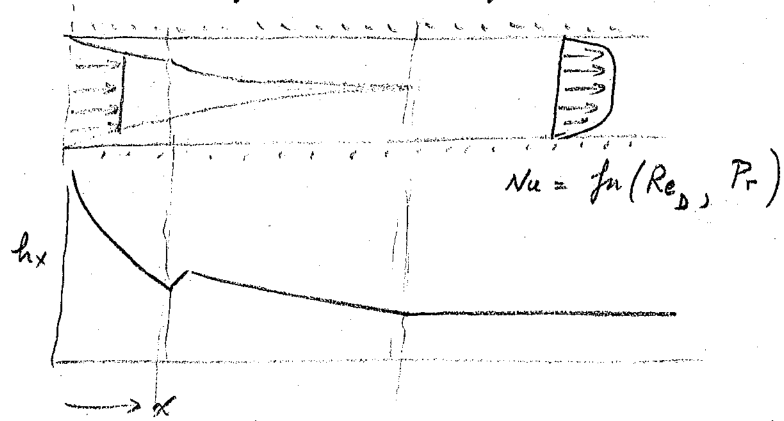
Turbulent $\begin{cases} \text{Laminar sublayer} \\ \text{Turbulent outer (wake) layer} \\ \text{buffer} \end{cases}$

2. Momentum & Heat Transfer Analogy
analogous mechanisms
3. Variation of h_x with x
Laminar $h_x \sim \frac{1}{x^{1/2}}$ as $\delta \uparrow$ $h_x \downarrow$ lower heat'g coeff
turbulent $h_x \sim \frac{1}{x^{.2}}$ better heat transfer coeff w/ turb than laminar

4. Variation of δ_x with x
laminar $\delta \sim x^{1/2}$
turb $\delta \sim x^{.8}$
5. Concept of Taw for high speed flows
6. Equations are in book.

Ch #7 free convections
1. Shape of $T(y)$, $u(y)$ curves
2. significance of Gr. No.
3. Orders of magnitude of h_c
4. Equations are in book

Ch #8 forced convection in tubes.
1. Applications of flat plate fundamentals

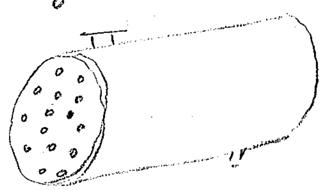


2. Entry length vs. fully developed flow.
3. effects of Prandtl No (Fig 8-4)
4. Equations in text.

11/6/78

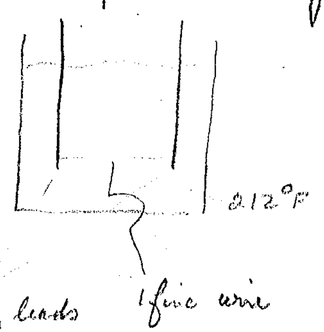
Flow through a pipe
Fully developed turb flow
$$Nu = 0.023\, Re^{.8}\, Pr^{.4}$$

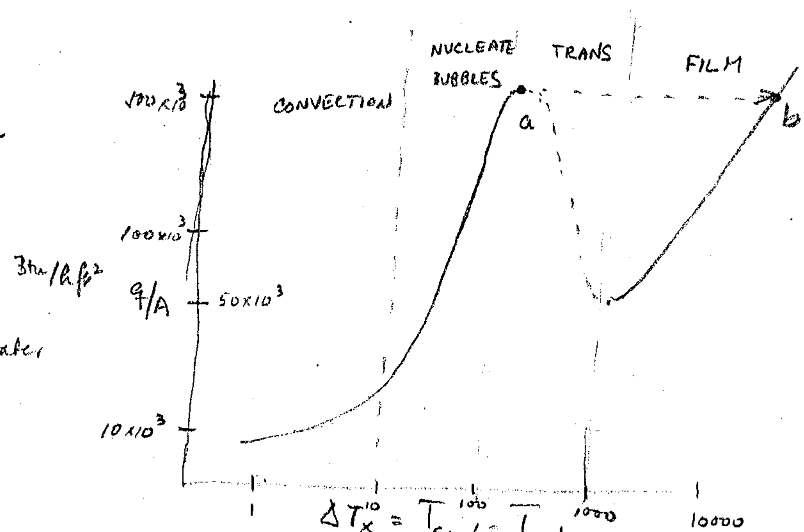
Look at heat exchanger



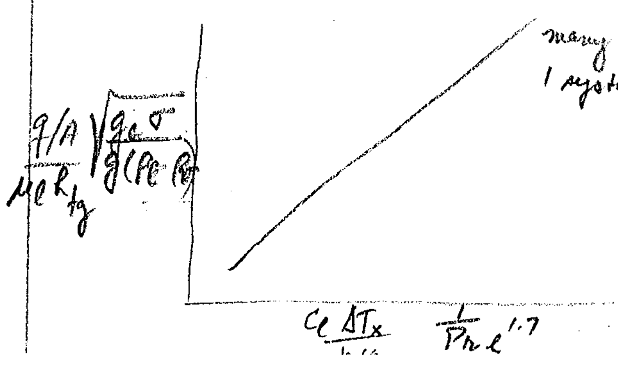
$h \sim D^{-.2}$ since $Nu = f(D)$
$Re = f(D^{.8})$
as $D \uparrow$ heat transfer $\downarrow$ for a given flow rate.

Boiling Heat Transfer





the more the ΔT the more bubbles come off the fine wire until we reach point a. At that time there will arise a film around the wire if more heat transfer is attempted and the transfer will be made to point b on the curve. Here $\Delta T \uparrow$ tremendously but q/A does not.



many pressures
1 system (water on platinum wire)

Rosinow correlated

h_{fg} - latent heat of vaporiz
σ - surface tension
μ_ℓ - viscosity of liquid

$$\frac{c_l \Delta T_x}{h_{fg}} \frac{1}{Pr^{.7}} = c_{sf}\left[\qquad \right]^{.33}$$

Burnout

$$\left(q/A\right)_{max} = 143\, \rho_v\, h_{fg} \left(g/g_c\right)^{1/4} \left(\frac{\rho_l - \rho_v}{\rho_v}\right)^{.6}$$

$$\left[Btu/hr\, ft^2\right]$$

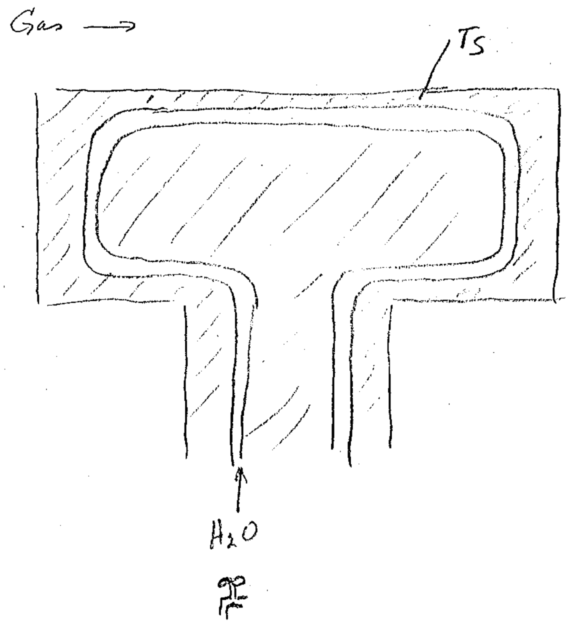
Forced convection through a tube (T high enough to vaporize fluid)

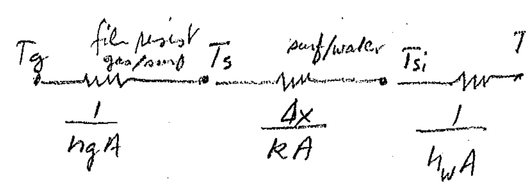
$$q_{TOT} = q_{boil} + q_{conv}$$

q conv turb.

$$\frac{\bar{h}D}{k_l} = .06 \left(\frac{\rho_l}{\rho_v}\right)^{.28} \left(\frac{DG\chi}{\mu_l}\right)^{.87} Pr_l^{.4}$$

11/11/78





if we want to control T_s by varying water rate.

since $k >>$ → $\frac{\Delta x}{kA} <<$

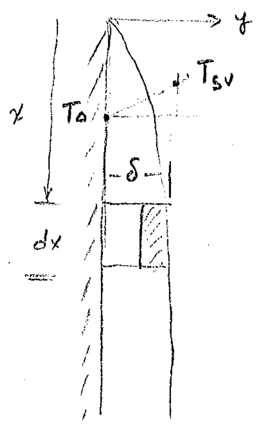
for water $h_w \sim 100's$ ∴ $\frac{1}{h_w A} <<$

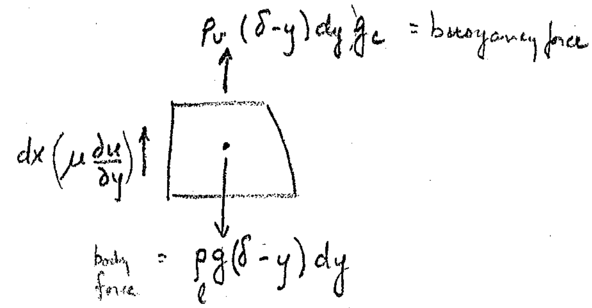
for a gas $h_g <<$ ∴ $\frac{1}{h_g A} >>$ this is the control resistance

no matter what we do if we are still in a liquid, $\frac{1}{h_w A}$ remains small.
if we turn down to flow until the $T_w > 212$ (until we reach burnout) and water ⟹ water vapor ⟶ $h_w <<$ ∴ $\frac{1}{h_w A} >>$ and becomes the

controlling factor	h Btu/hr ft^2 °R
Steam dropwise condensation	5000 - 20000
" filmwise "	1000 - 3000
water boiling	300 - 9000
organic vapors (F_2) condensing	200 - 400
water heating/cooling	50 - 3000
oils " / "	10 - 300
air " / "	.2 - 20 (100)

Condensation





assume T is linear for $T_{surface}$ (T_s) to $T_{saturated\ vapor}$ at boundary layer (T_{sv})

from pg 524 Kreith

$$\frac{hx}{k} = Nu_x = \left[\frac{\rho_l(\rho_l - \rho_v)\,g\,h'_{fg}\,x^3}{4\,\mu_l\,k\,(T_{sv} - T_s)}\right]^{1/4}$$

$$h'_{fg} = h_{fg} + \Delta h \text{ (subcooling to average film } T\text{)}$$

To check whether it is a laminar / turb film $\exists$ a R_N $\Rightarrow$ we can define the type of film

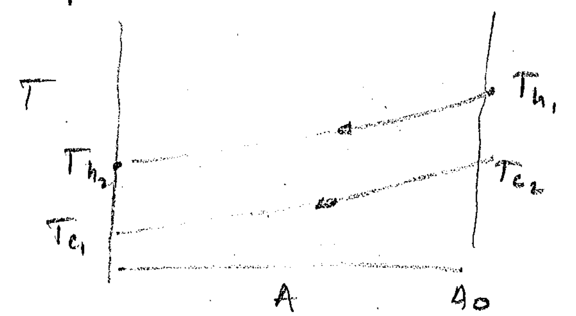
$$\Gamma = \frac{\dot{m}}{\text{width}} \qquad Re_{film} = \frac{4\Gamma}{\mu_{film}}$$

if $Re_f > 1800$ film is turb.

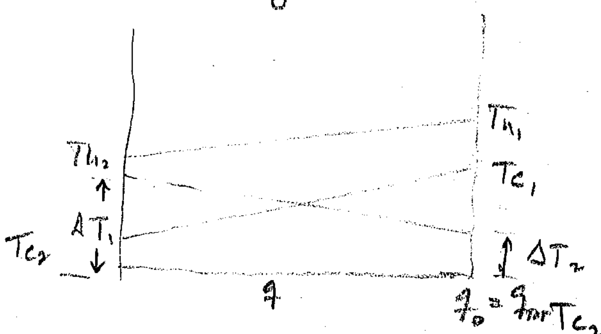
11/13/78

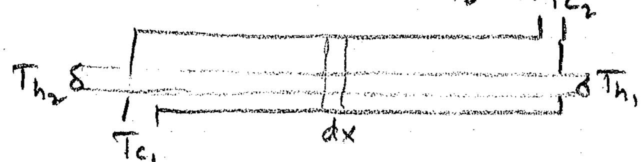
Due Nov 22   11.2, 11.3, 11.10, 11.26   5-5

$$q = A \, \Delta T_m \cdot U$$



We will assume steady state, constant fluid properties, const u, adiabatic heat exchange





1st law $q = \dot{m}_h C_h (T_{h_1} - T_{h_2})$
$= C_h \, \Delta T_h$

$$q = \dot{m}_c C_c (T_{c_2} - T_{c_1}) = C_c \, \Delta T_c$$

$$\frac{dT_h}{dq} = \frac{1}{\dot{m}_h c_h} = \frac{1}{C_h} \qquad \frac{dT_c}{dq} = \frac{1}{C_c}$$

from the graph $\dfrac{d(\Delta T)}{dq} = \dfrac{\Delta T_2 - \Delta T_1}{q_o}$

but $dq = u \, dA \, \Delta T$

$$\rightarrow \frac{d(\Delta T)}{u \, dA \, \Delta T} = \frac{\Delta T_2 - \Delta T_1}{q_o}$$

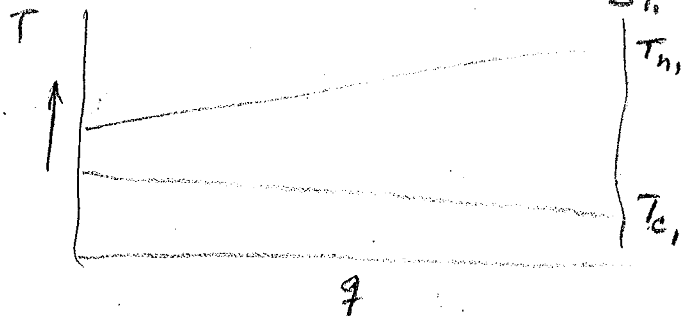
$$\rightarrow \int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \frac{\Delta T_2 - \Delta T_1}{q_o} \int_0^A u \, dA$$

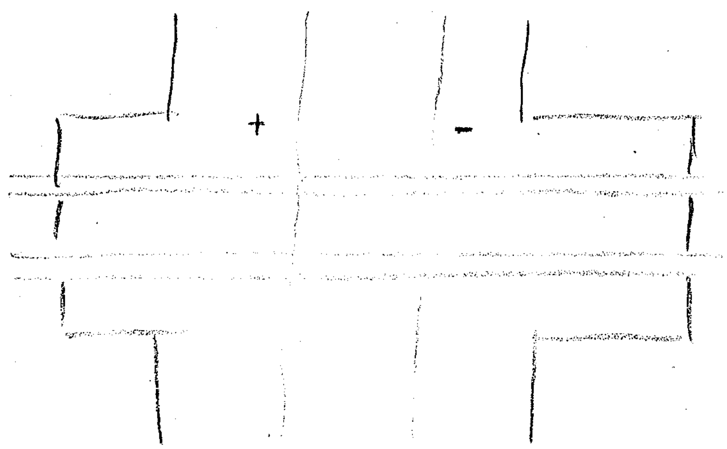
$$\rightarrow q_o = uA \left(\frac{\Delta T_2 - \Delta T_1}{\ln\left(\Delta T_2 / \Delta T_1\right)} \right) = uA \, \Delta T_{mean}$$

we see that

$$\Delta T_{mean} = (\Delta T_2 - \Delta T_1)/\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = LMTD$$

Parallel flow





Cross flow
mixed & unmixed types
to find the ΔT_{mean} for this is difficult

use to correction factor F in figure 11-12, 13, 14, 15

Example Problem 11-8

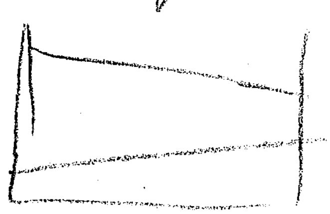
$T_{h_1} = 800$ $\quad$ $T_{c_1} = 100°F$ $\quad$ $u = 10$

$T_{h_2} = 400$ $\quad$ $T_{c_2} = 300°F$

$$q = \dot{m} c \Delta T = 100{,}000\ \frac{lbm}{hr} \times 1\ \frac{Btu}{lbm} \times 200$$

$$= 2 \times 10^7\ Btu/hr$$

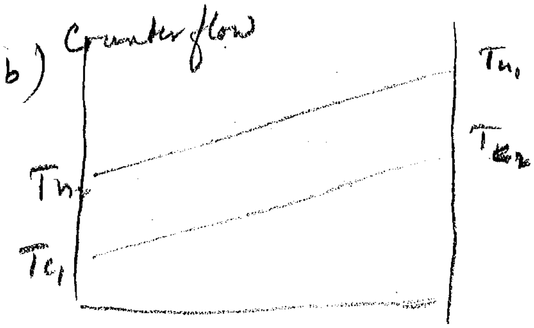
a) Parallel flow



$$LMTD = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{100 - 400}{\ln 1/4} = 308\ °F$$

$$A = 6500\ ft^2$$

b) Counterflow



$$LMTD = \frac{(800-300)-(400-100)}{\ln\left(\frac{500}{300}\right)} = 390\,^{\circ}F$$

$A = 5130\ ft^2$

Heat Exchangers. 11-15-78

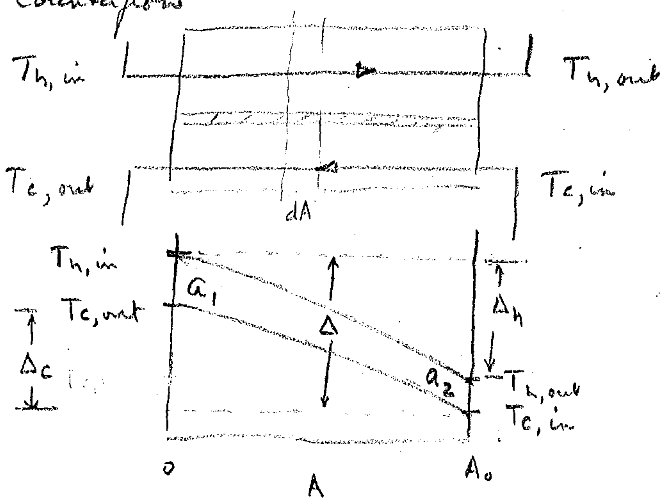
$\dot{q} = UAF \cdot (LMTD)$

Log Mean Temp Diff measured at hot temp inlet, hot temp outlet, cold temp inlet, cold temp outlet.

parallel & counterflow $F = 1.0$

For other flow arrangements $F < 1.0$

Counterflow



for a finite heat exchanger $T_{h,out} > T_{c,in}$;
for infinite $T_{h,out} \to T_{c,in}$

let $a_1 = T_{h,in} - T_{c,out}$ $\Delta = T_{h,in} - T_{c,in}$

$a_2 = T_{h,out} - T_{c,in}$

Now the amt of heat lost by hot fluid = am't gained by cold fluid (assuming no loses) = heat flow rate

$C_H = \dot{m}_h \, c_{p,h}$ $C_h \delta_h = C_h \delta_c$

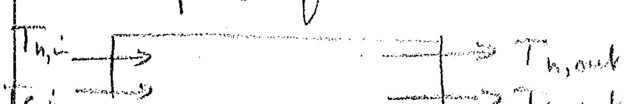
$C_c = \dot{m}_c \, c_{p,c}$ if $\delta_c > \delta_h$

$C_h > C_c$

$$LMTD = \frac{a_1 - a_2}{\ln a_1/a_2}$$

Effectiveness - NTU approach (superior to LMTD approach)

parallel flow



Limitation

If you increase the area then for any given $C_H = C_c$ Then $T_{h,out} = T_{c,out} = (T_{h,in} + T_{c,in})/2$

If also $C_H = \infty$ then cold fluid temp $T_{c,out} \to T_{h,in}$

$$\dot{q}_{max} \text{ (counterflow)} = C_{min} \cdot \Delta$$

$$C_{min} = \min(C_c, C_H)$$

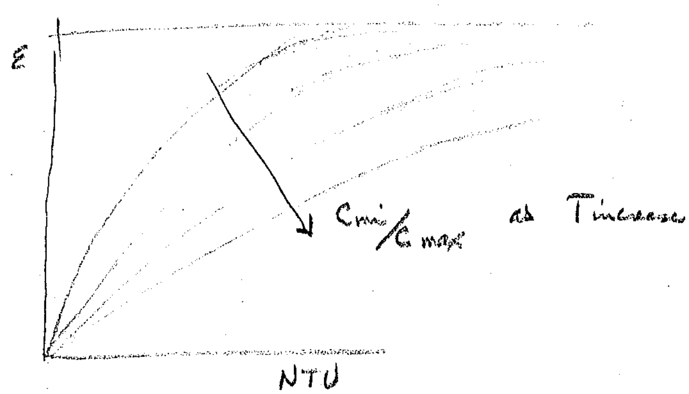
Effectiveness $\varepsilon \triangleq \dfrac{\dot{q}_{actual}}{\dot{q}_{max}} \leq 1$

$$= \frac{C_h \delta_h}{C_{min} \Delta} = \frac{C_c \delta_c}{C_{min} \Delta}$$

$$\dot{q}_{act} = \varepsilon C_{min} \Delta$$

$$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}, \text{Flow arrangement}\right)$$

$$NTU \triangleq \frac{UA_o}{C_{min}}$$



$$d\dot{q} = U\,dA\,(T_h - T_c) = -C_h(dT_h) = -C_c\,dT_c$$

$$d(T_h - T_c) = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)d\dot{q} = \left(\frac{1}{C_c} - \frac{1}{C_h}\right)U\,dA\,(T_h - T_c)$$

$$\frac{d(T_h - T_c)}{T_h - T_c} = \frac{U}{C_c}\left(1 - \frac{C_c}{C_h}\right)dA$$

$$\ln\frac{a_2}{a_1} = \ln(T_h - T_c)\Big|_{a_1}^{a_2} = \int_0^{A_o} \frac{U}{C_c}\left(1 - \frac{C_c}{C_h}\right)dA = NTU\left[1 - \frac{C_{min}}{C_{max}}\right]$$

we assumed $U = \text{const}$ if $U = U(A)$ use $U_{eff} = \frac{1}{A_0}\int U\,dA$

Now $\frac{a_2}{a_1} = \frac{\Delta - \delta_h}{\Delta - \delta_c}$ from diag and $C_h\delta_h = C_c\delta_c$

$$\frac{a_2}{a_1} = \frac{\Delta - \delta_h}{\Delta - \frac{C_{max}}{C_{min}}\delta_h} = \frac{\Delta - \delta_h}{\Delta(1-\varepsilon)} = \frac{1 - \frac{\delta_h}{\Delta}}{1-\varepsilon} = \frac{1 - \frac{C_{min}}{C_{max}}\varepsilon}{1-\varepsilon}$$

but $\varepsilon = \frac{C_h\delta_h}{C_{min}\Delta}$

inverting for $\varepsilon = \dfrac{1 - a_1/a_2}{1 - \frac{C_{min}}{C_{max}}\left(\frac{a_1}{a_2}\right)}$

$\therefore$ since $\ln\frac{a_2}{a_1} = NTU\left[1 - \frac{C_{min}}{C_{max}}\right]$ $\therefore$ $\ln\left[\dfrac{1 - \frac{C_{min}}{C_{max}}\varepsilon}{1-\varepsilon}\right] = NTU\left[1 - \frac{C_{min}}{C_{max}}\right]$

or

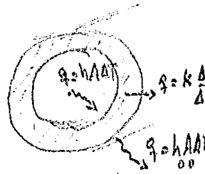
$$\varepsilon = \frac{1 - \exp\left[-NTU\left(1 - \frac{C_{min}}{C_{max}}\right)\right]}{1 - \frac{C_{min}}{C_{max}}\exp\left[-NTU\left(1 - \frac{C_{min}}{C_{max}}\right)\right]}$$

$\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}, \text{flow arrangements}\right)$

NTU = no. of transfer units

for parallel flow see equat 11-21

11/17/78



$q = UA\,\Delta T_m$

$= U_i A_i\,\Delta T_m$ i = inside   m = mean

$= U_o A_o\,\Delta T_m$ o = outside

$R_{Tot} = R_i + R_{wall} + R_o$

if deposit on inside & outside

$$\frac{1}{U_i A_i} = \frac{1}{h_i A_i} + \frac{1}{\frac{2\pi k l}{\ln(r_o/r_i)}} + \frac{1}{h_o A_o}$$

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{2\pi r_i L}{2\pi k l/\ln(r_o/r_i)} + \frac{A_i}{h_o A_o}}$$

LMTD   actual heat exchanger

$q = UAF_g\,\Delta T_{lm}$ ΔT_{lm} = log mean temp diff

$F_g = F_g(Z, X,$ flow arrangement$)$

$Z \triangleq \dfrac{\Delta T_{hot}}{\Delta T_{cold}}$ $X \triangleq \dfrac{\Delta T_{cold}}{T_{h,in} - T_{c,in}}$

ε-NTU

$q = \varepsilon\, C_{min}\,(T_{h,in} - T_{c,in})$

$\varepsilon = \varepsilon(NTU, C^*,$ flow arr.$)$

$C^* = C_{min}/C_{max}$

$$\varepsilon = q/q_{max} \quad \frac{C_h(T_{h,in} - T_{h,out})}{C_{min}(T_{h,in} - T_{c,in})} = \frac{C_c(T_{c,out} - T_{c,in})}{C_{min}(T_{h,in} - T_{c,in})}$$

$NTU = \dfrac{UA}{C_{min}}$ ("heat transfer size" of the exchanger)

$.5 \leq NTU \leq 1$ for auto radiator

$5 \leq NTU \leq 10$ for a vehicular gas turbine

$50 < NTU$ Stirling engine regenerator

$NTU < 200$ helium liquefier

1) as $\varepsilon \uparrow$ NTU $\uparrow$ for $C^* =$ fixed. It reaches an asymptotic value depending on flow arrangement
2) For a specified NTU, $\varepsilon \uparrow$ when $C^* \uparrow$
3) For $\varepsilon < 40\%$, C^* has little effect on ε.
4) when $C^* = 0$, ε is the same for all flow arrang. $\varepsilon = 1 - e^{-NTU}$ (when fluid changes phase $C^* = 0$)

$\therefore X = \varepsilon$ for $C_c = C_{min}$
$X = \varepsilon C^*$ for $C_h = C_{min}$
$Z = \frac{C_c}{C_h}$ = either C^* or $\frac{1}{C^*}$

$$F_q = \frac{\varepsilon}{NTU} \frac{T_{h,in} - T_{c,in})}{\Delta T_{lm}}$$

There are 2 type of heat exchangers problems

1) Sizing Problems - given $T_{h,in}$, $T_{c,in}$, $T_{h,out}$, $T_{c,out}$, C_c, C_h, U
find A

2) Rating problem - given A; U; C_h; C_c; $T_{c,in}$; $T_{c,out}$
find $T_{h,out}$; $T_{c,out}$; ε

Longitudinal heat conduction $Pe > 10$ and $X^* > .005$
where $Pe = R_N P_r$ $X^* = X/D_h^{Pe}$ D_h = hydraulic radius

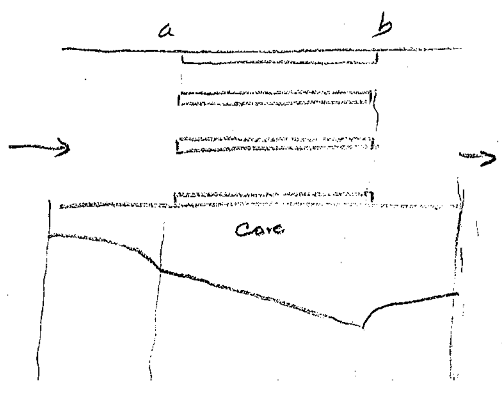
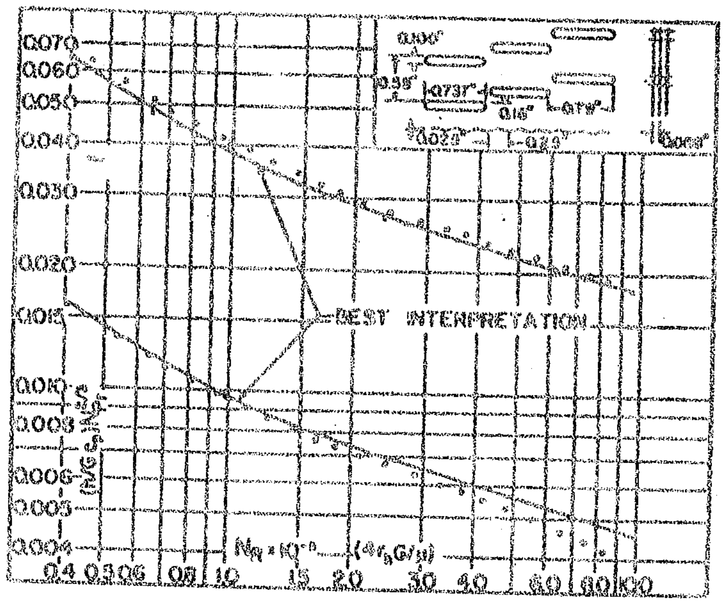


Fig. 10-89. Finned flat tubes, surface 11.32-0.737-SR.



Fin pitch = 11.32 per in.

Flow passage hydraulic diameter, $4r_h$ = 0.01192 ft

Fin metal thickness = 0.004 in., copper

Free-flow area/frontal area, σ = 0.780

Total heat transfer area/total volume, α = 270 ft^2/ft^3

Fin area/total area = 0.845

$$\Delta p_{1-a} = \frac{G^2}{2g_c}\left[\overbrace{\frac{1}{\rho_1}(1-\sigma^2)}^{\text{accel of fluid}} + \frac{K_c}{\rho_1}\right]$$

$$\sigma = \frac{\text{flow area}}{\text{frontal area}} \qquad K_c = \text{constant dependent on core.}$$

$$G = \frac{\dot{m}}{\text{flow area}} = \rho V$$

$$\Delta p_{a-b} = \frac{G^2}{2g_c \rho_1}\left[\underbrace{2\left(\frac{\rho_1}{\rho_2}-1\right)}_{\text{accel of fluid}} + \underbrace{\frac{f}{r_h}\,\rho_1\left(\frac{1}{\rho_m}\right)L}_{\text{friction loss}}\right] \qquad r_h = \text{hydraulic radius}$$

$$\Delta p_{b-2} = \frac{G^2}{2g_c}\left[\underbrace{\frac{1}{\rho_2}(1-\sigma^2)}_{\text{decel of fluid}} - K_c/\rho_2\right]$$

pressure rise due to deceleration & friction loss

11/20/78

Quantities used in compact heat exchangers

1. $Nu = \dfrac{h D_h}{K} = St\ Re\ Pr$

2. $St = \dfrac{h}{G c_p}$ depends on Re, Pr, geometry $(G = \rho V = \dot{m}/A)$

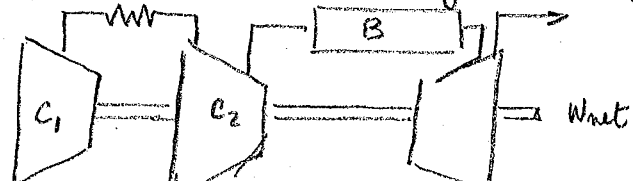
3. $j = St\ Pr^{2/3} = (Nu\ Pr^{-1/3})/Re$ (Colburn Factor j)

   use when $0.5 \lesssim Pr \lesssim 10$

4. $f = \dfrac{\tau}{\rho u_m^2/2g_c} = \dfrac{\Delta p}{\rho u_m^2/2g_c}\ \dfrac{D_h}{L}$

5. $Re = G D_H/\mu \qquad Pr = C_p \mu/K$

Let's look at a heat exchanger in a gas turbine



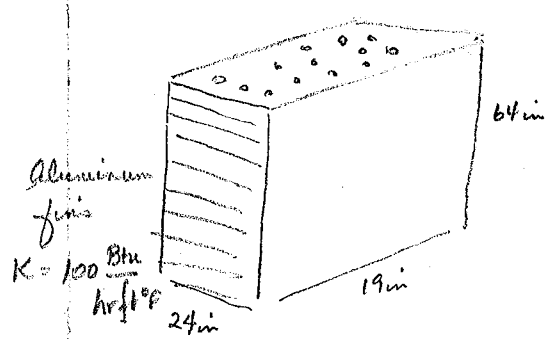
Air side: $\dot{m} = 2\times10^5$ lbm/hr

$T_{h_1} = 260°F = 720°R$

$P_1 = 39.7$ psi

Water side: $\dot{m} = 4\times10^5$ lbm/hr

$T_{c_1} = 60°F = 520°R$



Fluid properties : Assume $T_{h_2} = 75°F$, $T_{c_2} = 80°F$

$T_{ave} = 135°F$ $T_W \cong 70°F$

at average temp, we get for air : $P_r = .7$

$\mu = .0482$ lbm/hr ft $c_p = .244$ Btu/lbm°F

for water $k = .0347$ Btu/h ft °F

$\mu = 2.36$ lbm/hr ft

$c = 1.00$, $P_r = 6.8$, $\rho = 62.3$ lbm/ft^3

Use as first guess. If off, iterate a second time

?, R_N : air $G = \dot{m}/A = \dfrac{\dot{m}}{A_{frontal}\,\sigma} = \dfrac{2\times10^5}{10.67\times.78} = 24050 \dfrac{lbm}{hr\,ft^2}$

$$R_{N_{air}} = \frac{4 r_h G}{\mu} = \frac{(4\times.00288)\times 24050}{.0482} = 5760$$

?, water : $G = \dfrac{4\times10^5}{3.17\times(.129)} = 978000 \dfrac{lbm}{hr\,ft^2}$ calculate by looking at tube

? $R_{N_W} = \dfrac{4\,(.00306)\times 978000}{2.36} = 5080$

Now compute St and f

air $St\,P_r^{2/3} = .0054 \Longrightarrow St = .00685$

? $f = .021$ from ditto

water (circular tube data)

$Nu \sim 55$ $f \sim .0094$

Film Coeffs

air $h = St\,G\,c_p = .00685 \times 24050 \times .244 = 40.2 \dfrac{Btu}{hr\,ft^2\,°F}$

water $h = \dfrac{Nu\,k}{4 r_H} = 55 \dfrac{.347}{4\times 3.06\times10^{-3}} = 1560 \dfrac{Btu}{hr\,ft^2\,°F}$

Calculate fin effectiveness : $m = \left(\dfrac{2h}{K\,\delta}\right)^{1/2}$ thickness of fin, ft.

$$m = \left(\frac{2\times 40.2}{100\times .000333}\right)^{1/2} = 49.2\,ft^{-1}$$

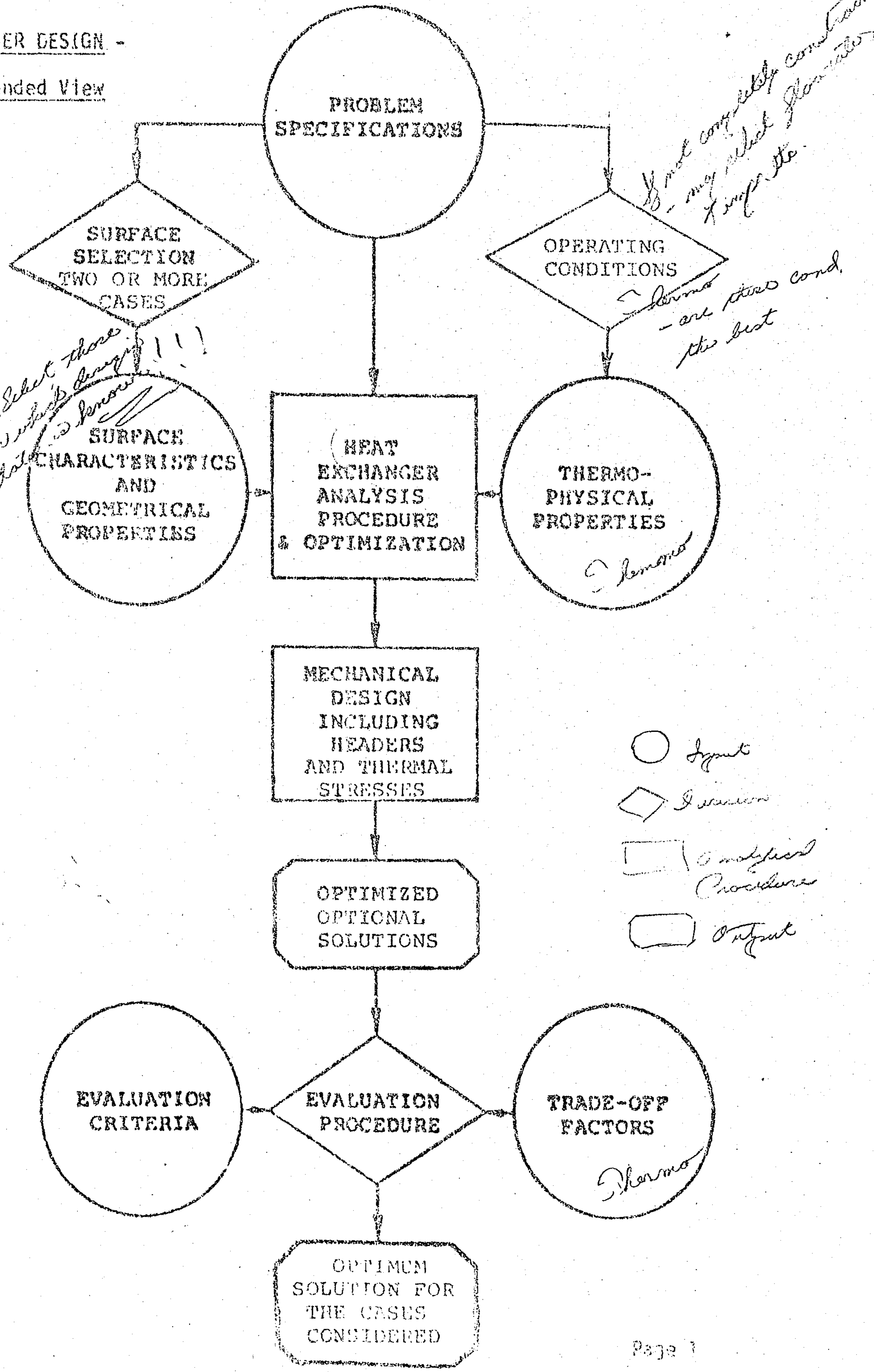
$$m\,l = 49.2\left(\frac{.225}{12}\right) = .923 \Longrightarrow \eta_f = .79$$

ME 250A

HEAT EXCHANGER DESIGN -

An Extended View

Heat trans.
Fluid mech.
Mech. design
Thermodyn.
Economics



Heat Transfer and Thermodynamics

Earmarks of heat transfer:

(a) Exists only in transit -- can't be stored.

(b) Always associated with Δt and always from hot $\rightarrow$ cold.

(c) As $\Delta t \to 0$, q'' Btu/(hr ft^2) $\triangleq \frac{1}{\text{Area}} \frac{dQ}{d\tau} \to 0$.

(d) Heat transfer is irreversible (for $q'' \gtrsim 0$).

It is important to make a distinction between q'' and q (at an instant) and Q (for Δ time).

$$q \triangleq \dot{Q} \text{ Btu/hr} \quad , \quad q'' \triangleq dq/dA_{trans} \text{ Btu/(hr ft}^2 \text{ of surface)} .$$
$$= dQ/d\tau$$

For

(i) Reversible $q'' > 0$, $R_{thermal} \frac{\text{hr ft}^2\ {}^\circ\text{F}}{\text{Btu}} \to 0$,

An ∞ conductivity idealization, very restrictive

(ii) Reversible $q > 0$, either

$R_{thermal} \to 0$

or $A_{trans} \to \infty$

less restrictive

(iii) Reversible $Q > 0$,

either $R_{th} \to 0$

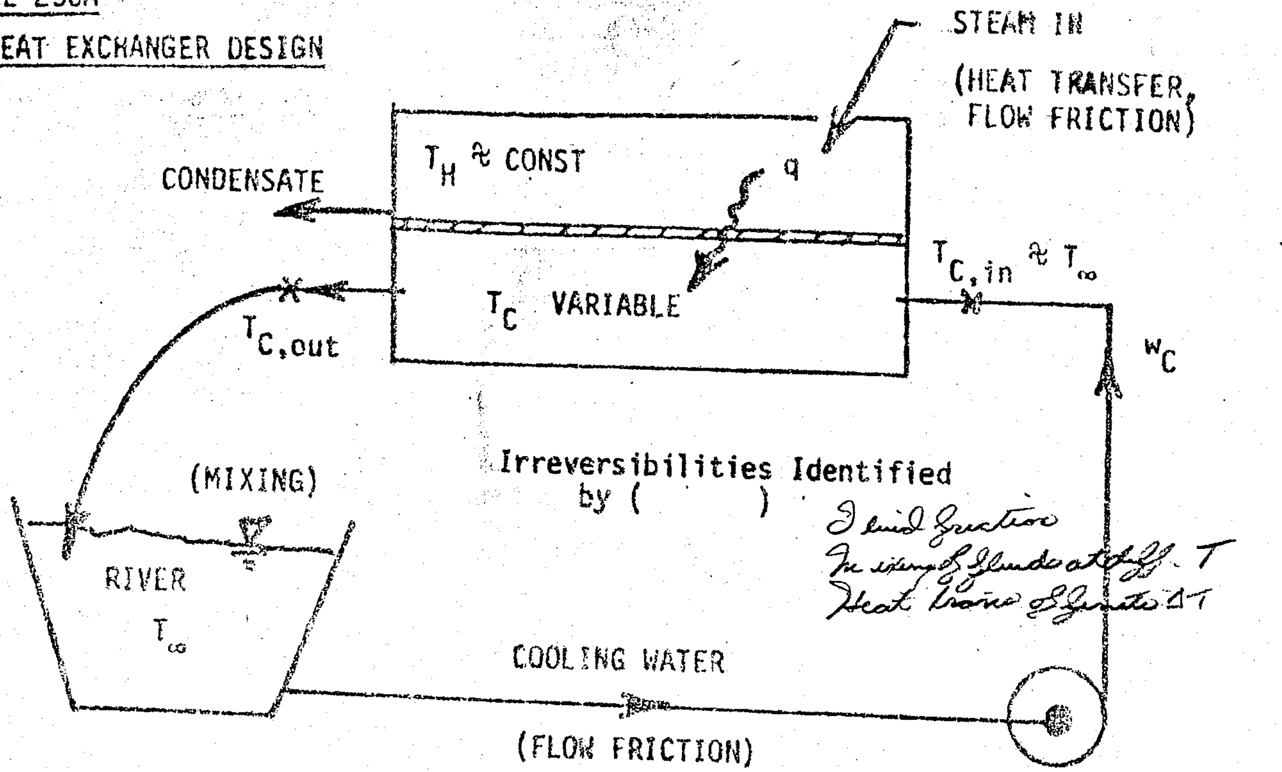
or $A_{trans} \to \infty$

or Δ time $\to \infty$

the least restrictive idealization

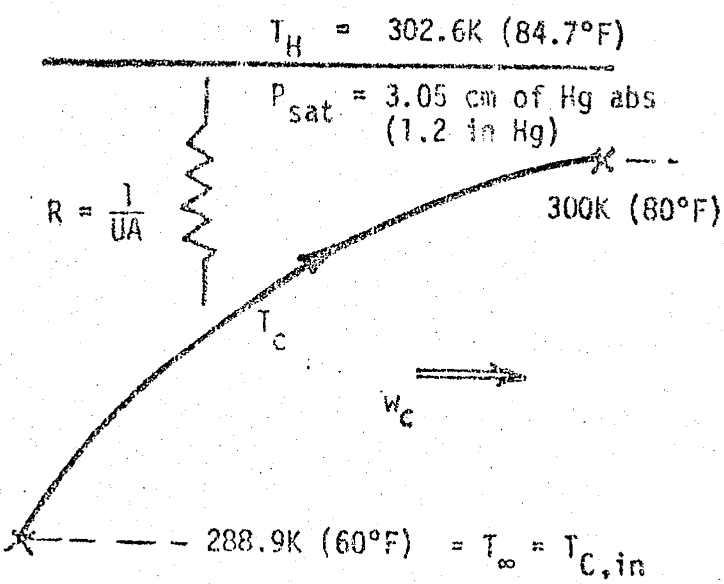
ME 250A

HEAT EXCHANGER DESIGN



Irreversibilities Identified by (   )

Fluid friction
Mixing of fluids at diff. T
Heat trans @ finite ΔT



Irreversibilities Evaluated

$$\left.\frac{\dot{IRREV}_E}{q}\right]_{ht\ trans} = 0.02660\ (57\%)$$

$$\left.\frac{\dot{IRREV}_E}{q}\right]_{\Delta P_c} = 0.00139\ (3\%)$$

$$\left.\frac{\dot{IRREV}_E}{q}\right]_{mix.} = 0.01879\ (40\%)$$

$$\frac{q}{\text{Elec Power}} = 1.12$$

Irreversibilities in a Steam-Electric Power Plant Condenser System

Options
Increase A_T
Increase w_C
Keep tubes clean
Zone Condenser.
- cond. @ two level

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3. Shah, R. K., "Compact Heat Exchanger Surface Selection Methods," Proceedings of the Sixth International Heat Transfer Conference, Vol. 4, p. 193-199, August 7-11, 1978.

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overall surface effectiveness is $\eta_o = 1 - \frac{A_f}{A}(1-\eta_f)$ assuming base has 100% efficiency

note: $\eta_o A = A_{base} + \eta_f A_{fin}$ where $A_{base} = A - A_{fin}$

$$\eta_o = 1 - .845(1-.79) = .823$$

Now, we calculate overall heat transfer coeff U

$$\frac{1}{U_{air}} = \frac{1}{\eta_o h_a} + \frac{1}{\alpha_w h_w/\alpha_a} = \frac{1}{.823 \times 40.2} + \frac{1}{92.1 \times 1560/270} = .0343$$

88% of resistance

$U_{air} = 29.2 \frac{Btu}{hr\,ft^2\,°F}$ for clean surfaces (note h_w is not that important since Air side is controlling resistance)

E-NTU method

$C_{air} = 2\times10^5 \times .244 = 48800$ $C_{min} = \dot{m} \times C_p$

$C_{water} = 4\times10^5 \times 1 = 4\times10^5$ C_{max}

$$NTU = \frac{UA}{C_{min}} = \frac{270 \times 16.9 \times 29.2}{48800} = 2.73$$ volume of heat exchanger area/volume ditto.

$\frac{C_{min}}{C_{max}} = .122 \Rightarrow \varepsilon = 90\% = .9$ — cross flow heat exchanger chart 1 side mixed

For this we can calculate output temps.

?

$$\varepsilon = \frac{C_a(T_{h_1} - T_{h_2})}{C.(T_{h_1} - T_{c_1})} \Rightarrow T_{h_2} = 80°F \Rightarrow T_{c_2} = 81.9°F$$

$$\Delta P_{AB} = \frac{G^2}{2g_c\rho_1}\left[2\left(\frac{\rho_1}{\rho_2} - 1\right) + \frac{f}{r_H}\rho_1\left(\frac{1}{\rho_m}\right)L\right]$$

if we ignore K_c and K_e (entrance, exit losses)

$$\Delta p = \frac{G^2}{2g_c\rho_1}\left[\left(\frac{\rho_1}{\rho_2} - 1\right)(1+\sigma^2) + f\frac{A}{A_c}\rho_1\frac{1}{\rho_m}\right]$$

Tot area, Cu area

$\Delta P_{airside} = 43.45\ lb/ft^2 = .3\ lb/in^2$, $\Delta P/p = .0076$

$$\Delta P_{water} \cong \frac{G^2}{2g_c\rho} f \frac{L}{R_H} = 2.1\ lb/in^2$$

11/27/78

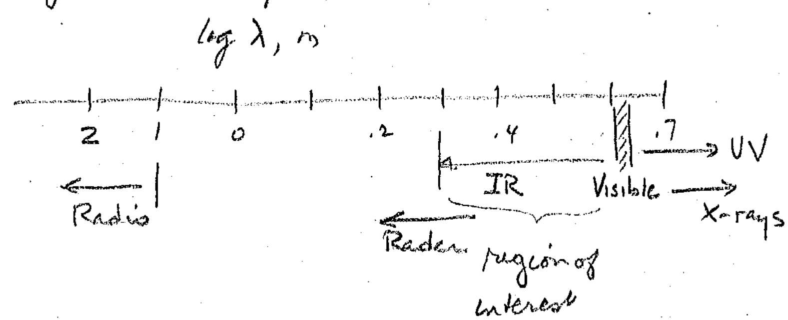
Last problem set #8

5-7, 16, 39, 60, S-6

1. Physics and properties of a black body
2. Radiation properties of materials
3. Radiation Shape Factor
4. Radiation in enclosures - black
5. " " " - gray surfaces
6. Radiation from gases (a high temp)
7. Solar Radiation

Thermal radiation - temperature rad.
Will treat rad. as a wave phenom. (but will keep in mind quantum mechanics)

- Look at general wavelength spectrum.



- The Black Body concept
Perfect absorber of incident radiation (also a perfect emitter)
- We now define the following
<u>Hemispherical spectral black body emissive power</u> - energy that leaves the black body per unit area/unit time and per unit wavelength interval around some wavelength λ

Max Planck defines $E_{b\lambda}(\lambda, T) = \dfrac{c_1}{\lambda^5 (e^{c_2/\lambda T} - 1)}$

$c_1 = 2\pi h c_0^2$ where h is Planck's const
$c_2 = h c_0 / k$ k is Boltzmann const.
c_0 is speed of light in vacuum.

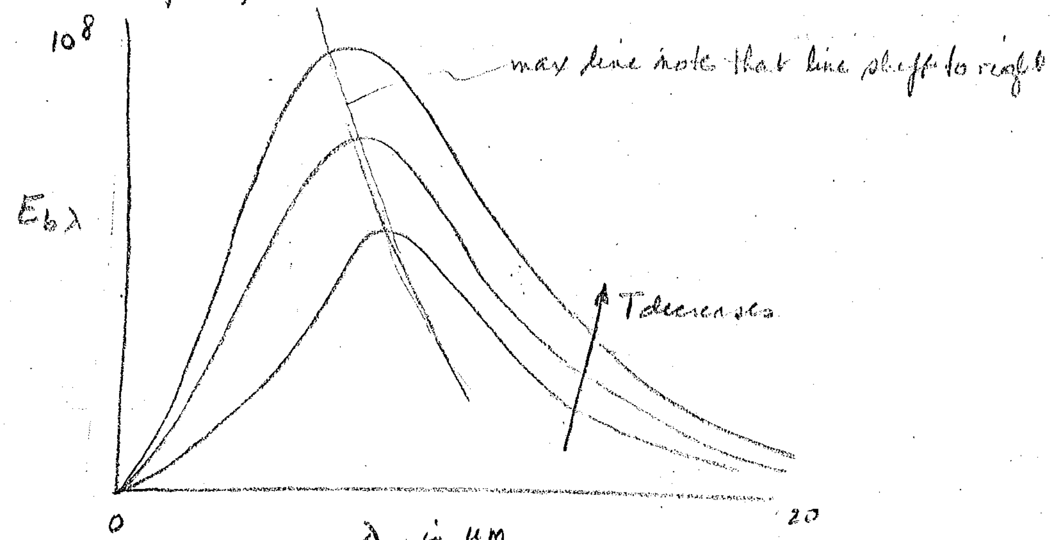
This is only good for a radiation through a gas or vacuum

For radiation through anything other than gas or vac. see Siegel & Howell "Fund of Thermal Rad." Pg 31.

To get rid of λ dependence integrate wrt λ.

$$\int_0^\infty E_{b\lambda}\,d\lambda = \int \frac{c_1}{\lambda^5\left(e^{c_2/\lambda T}-1\right)}\,d\lambda = \underbrace{\frac{c_1\pi^4}{15\,c_2^4}}_{=\,1.714\times10^{-9}\ Btu/hr\,ft^2\,°R} T^4 = \sigma T^4$$

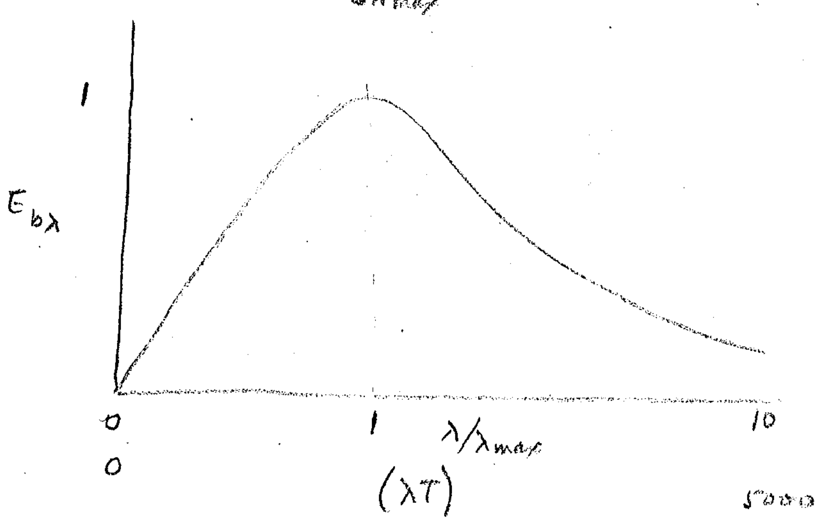
Plotting $E_{b,\lambda}$ vs. λ



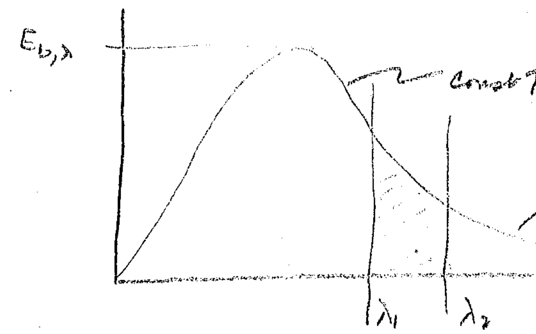
Wiens Law (p. 222) $\lambda_{max} T = const$

$$E_{b\lambda\,max} = 17.87\times10^{-6}\,T^5\ watts/m^3 \quad for\ T\ in\ °K$$

we can also plot $\dfrac{E_{b\lambda}}{E_{b\lambda\,max}}$ vs. λ/λ_{max}



how much energy is between λ_1 & λ_2 (λ_1, λ_2 arbitrary)



energy under this curve is $\int_0^\infty E_{b\lambda}\, d\lambda = \sigma T^4$

Thus we can for a given λ get $E_{b\lambda}(0,\lambda)$ @ const T

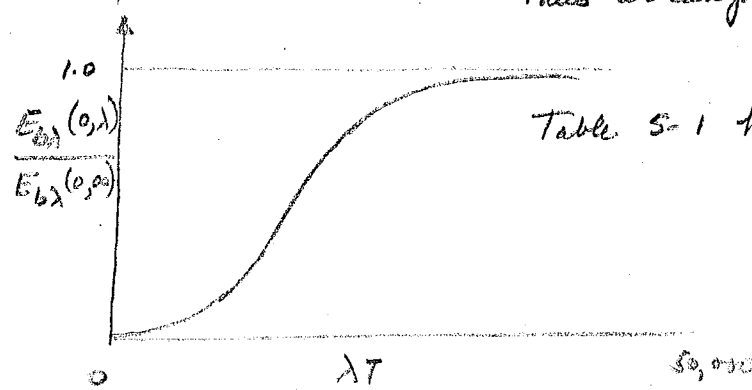
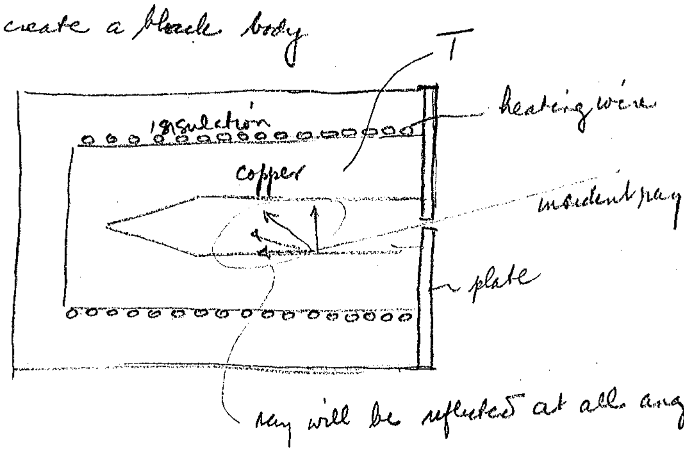


Table 5-1 has more accurate data

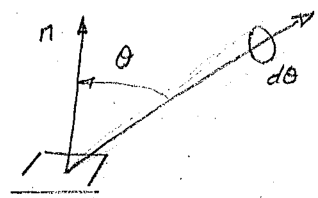
$$\therefore \quad E_{b\lambda_2}(0,\lambda_2) - E_{b\lambda_1}(0,\lambda_1)$$

Lets create a black body



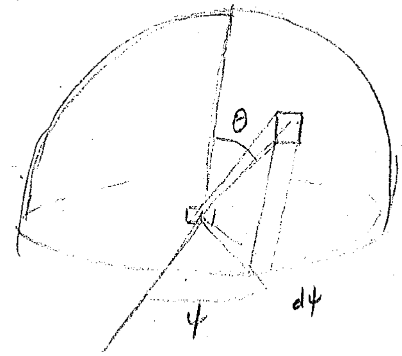
By Kirchoff's Law this body is also a radiator

Let's look at the radiation ~~of a~~ energy by a small area at some angle θ normal to the surface



The intensity is energy emitted / unit time / unit area normal to direction of radiation / unit solid angle

$$I = d\left(\frac{q}{A}\right)\frac{1}{\cos\theta}\frac{1}{d\omega}$$



$$d\omega = \frac{dA_{\text{hemisphere}}}{r^2} = \frac{r\,d\theta\,(r\sin\theta\,d\psi)}{r^2}$$

$$q/A = \int_0^{2\pi}\int_0^{\pi/2} I(\psi,\theta)\cos\theta\sin\theta\,d\theta\,d\psi$$

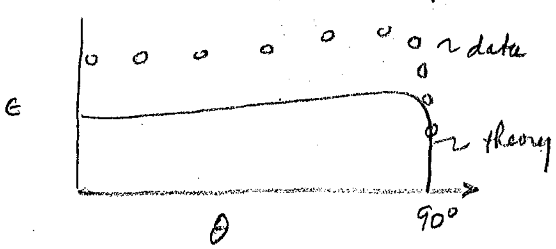
$$= \pi I_b$$

for a black body $I(\psi,\theta) = \text{const}$

again $q/A = \pi I_b = E = \sigma T^4$ 11/29/78

Assumption: surfaces are diffuse – that all parts of the surface have identical reflection properties

If we plot emissivity vs. θ (incidence angle)



ϵ is Emittance; we define the emittance of a particular wavelength as
$E_b = \epsilon_\lambda\ E_{b\lambda}$

α is the absorptance and similarly we define $G_\lambda = \alpha_\lambda G_{b\lambda}$ where G is the absorptivity

From Kirchoff's Law $\epsilon_\lambda = \alpha_\lambda$ if incidence radiation is independent of incidence angle

Thus we can write $\epsilon(T) = \dfrac{E(T)}{E_b(T)} = \dfrac{\int_0^\infty \epsilon_\lambda(T)\,E_{b\lambda}(\lambda T)\,d\lambda}{\int_0^\infty E_{b\lambda}(\lambda T)\,d\lambda}$

this is the total hemispherical emittance.

and for the total absorptance

$$\alpha(\lambda^*, T^*) = \frac{\int_0^\infty \alpha_\lambda(\lambda)\, G_\lambda(\lambda^*, T^*)\, d\lambda}{\int_0^\infty G_\lambda(\lambda^*, T^*)\, d\lambda}$$

starred quantities are evaluated at source temperature

We can then say that

$$\alpha_{TOT} = \epsilon_{TOT} \quad \text{if:}$$

1) $G_\lambda(\lambda^*, T^*) = E_{b\lambda}(\lambda, T)$ source and sink temperatures are the same

2) $\epsilon_\lambda = \alpha_\lambda$ are constant over wavelength (gray body

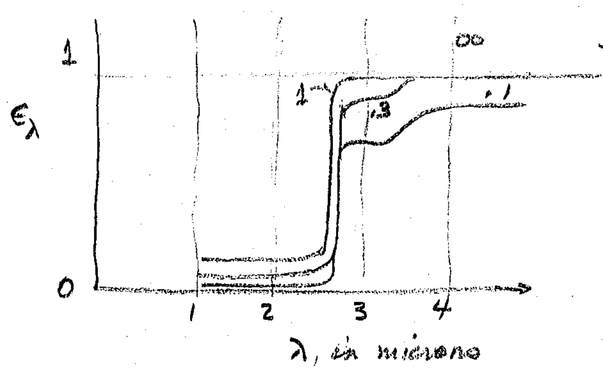
Note: G is the total amount of energy a body can absorb.
now for some metals the emittance varies with the incident ray angle.

Reflectance

$$\rho_\lambda = \frac{\text{radiant energy reflected}}{\text{per unit time - area - wavelength}} \frac{1}{G_\lambda}$$

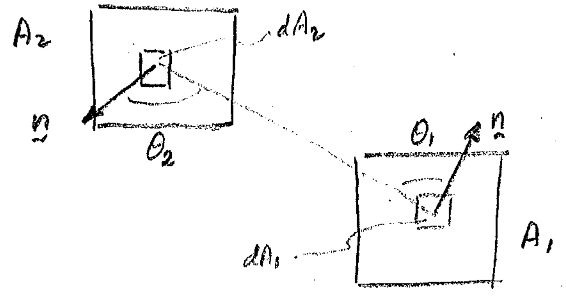
$$\rho_\lambda = \frac{\text{radiant energy reflected}}{\text{per unit time - area}} \frac{1}{\int_0^\infty G_\lambda\, d\lambda}$$

Reflectance + absorptance + transmittance = 1 or $\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1$ or $\rho + \alpha + \tau = 1$



emittance variance with wavelength for different thicknesses of glass

Shape facture



F_{1-2} = fraction of energy emitted by ① that hits ②. Note that <u>it is not the amount absorbed by ②, it is the amount that hits it only.</u>

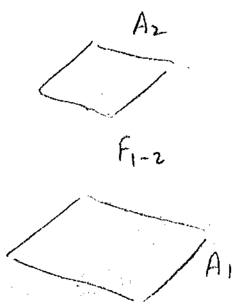
$$q_{1-2} = E_{b_1} A_1 F_{1-2}$$
$$q_{2-1} = E_{b_2} A_2 F_{2-1}$$

$$q_{net} = q_{\substack{1-2 \\ 2-1}} = E_{b_1} A_1 F_{1-2} - E_{b_2} A_2 F_{2-1}$$

if $q_{net} = 0$ $\quad E_{b_1} A_1 F_{1-2} = E_{b_2} A_2 F_{2-1}$ and $E_{b_1} = E_{b_2}$ @ same Temp.

hence $A_1 F_{1-2} = A_2 F_{2-1}$

12/1/78



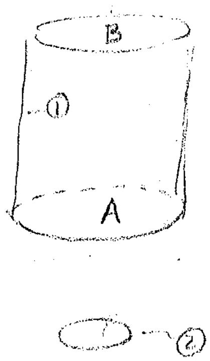
$$q_{net} = E_{b_1} A_1 F_{12} - E_2 A_2 F_{21}$$
$$A_1 F_{12} = A_2 F_{21}$$

$$dq_{1-2} = I_1 \cos\theta \, dA \, d\omega_1 \qquad d\omega_1 = \frac{dA_2 \cos\theta}{r^2} \qquad I_1 = \frac{E_{b_1}}{\pi}$$

$$\therefore dq_{1-2} = \frac{E_{b_1}}{\pi} dA_1 \frac{\cos\theta_1 \cos\theta_2}{r^2} dA_2$$
$$dq_{2-1} = \frac{E_{b_2}}{\pi} dA_2 \frac{\cos\theta_2 \cos\theta_1}{r^2} dA_1$$

$$\frac{dq_{net}}{E_{b_1} - E_{b_2}} = \underbrace{\iint_{A_1 A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1 dA_2}_{A_2 F_{21} \text{ or } F_{12} A_1}$$

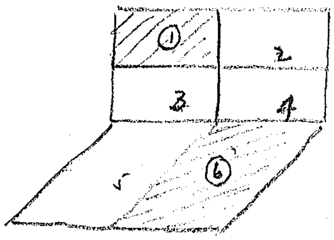
Example radiation of inside of cylinder to a disk



$$A_1 F_{12}$$

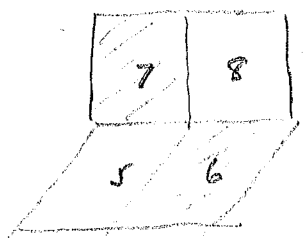
$$F_{21} = F_{2A} - F_{2B}$$ but we know from Kreith the form factor for 2 parallel plates.

Siegel and Howell – Example



find F_{1-6}

look at a simpler problem



by integrating the double integral we find that

$A_7 F_{7-6} = A_8 F_{8-5}$ (*)

The book gives results for 

now from above $F_{(5+6)-(7+8)} = F_{(5+6)-7} + F_{(5+6)-8} = \dfrac{A_7}{A_5+A_6} F_{7-(5+6)} + \dfrac{A_8}{A_5+A_6} F_{8-(5+6)}$

$$F_{(5+6)-(7+8)} = \frac{A_7}{A_5+A_6}\left(F_{7-5} + F_{7-6}\right) + \frac{A_8}{A_5+A_6}\left(F_{8-5} + F_{8-6}\right);$$ using (*) we get

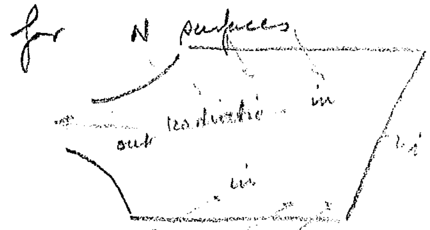
$$\therefore \quad F_{7-6} = \frac{1}{2A_7}\left(\left(A_5+A_6\right) F_{(5+6)-(7+8)} - A_7 F_{7-5} - A_8 F_{8-6}\right)$$

$$F_{1-6} = \frac{A_6}{A_1} F_{6-1} = \frac{A_6}{A_1} F_{6-(1+3)} - \frac{A_6}{A_1} F_{6-3}$$

$$F_{1-6} = \frac{A_6}{A_1}\left\{ \frac{1}{2A_6}\left[(A_1+A_2+A_3+A_4) F_{(1+2+3+4)-(5+6)} - A_6 F_{6-(2+4)} - A_5 F_{5-(1+3)}\right] - \frac{1}{2A_6}\left[(A_3+A_4) F_{(3+4)-(5+6)} - A_6 F_{6-4} - A_5 F_{5-3}\right]\right\}$$

Enclosures w/ black surfaces

for N surfaces

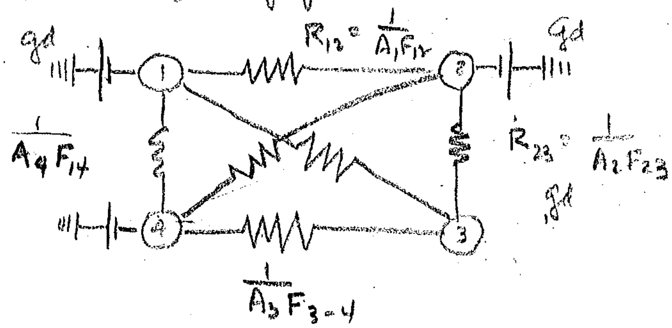


$$q_{i\,\text{net out}} = E_{b_i} A_i - \sum_{j=1}^{N} E_{b_j} F_{j-i} A_j$$

$$= A_i \left(E_{b_i} - \sum_{j=1}^{N} E_{b_j} F_{i-j} \right)$$

$$= \left(\sum_{j=1}^{N} E_{b_i} - E_{b_j} \right) F_{i-j} A_i \quad \text{since} \sum_{j=1}^{N} F_{i-j} = 1$$

The voltage diag for rad



for N=4

12/4/78

1. Assumption for Enclosures with <u>Gray</u> Surfaces
   a. for each area
      1) temp is uniform
      2) ϵ_λ, α_λ, ρ_λ are indep of wavelength and direction θ
      3) All energy is emitted/reflected diffusely
      4) Incident and hence reflected energy is uniform

   Gray surface is embedded in assumption a 2) and a 3)

Radiosity = total energy leaving a surface

$$J_i = \epsilon_i E_{b_i} + \rho_i G_i$$

Net radiation heat loss

$$q_{i,\text{net}} = A_i (J_i - G_i) = -A_i \left[\frac{J_i - \epsilon_i E_{b_i}}{\rho_i} \right] + A_i J_i$$

$$\boxed{q_{i\,\text{net}} = \frac{A_i \epsilon_i}{1-\epsilon_i} \left[E_{b_i} - J_i \right]}$$

Remember the pictures w/ n surfaces

radiation onto i is

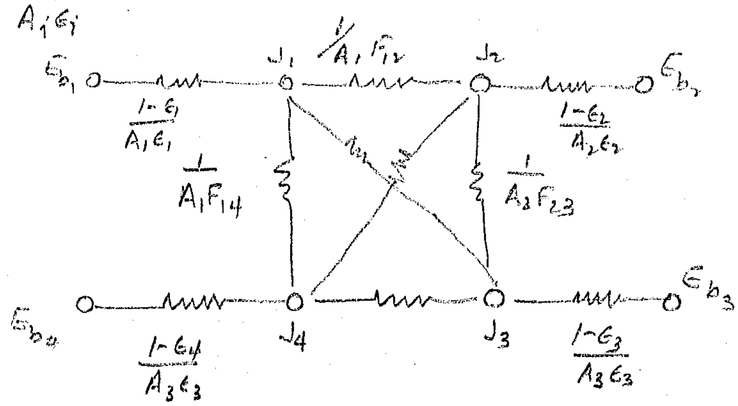
$$A_i G_i = \sum_{k=1}^{N} J_k A_k F_{k-i} = \sum_{k=1}^{N} J_k A_i F_{i-k}$$

$$q_{i\,net} = A_i \left(J_i - \sum_{k=1}^{N} J_k F_{i-k} \right)$$

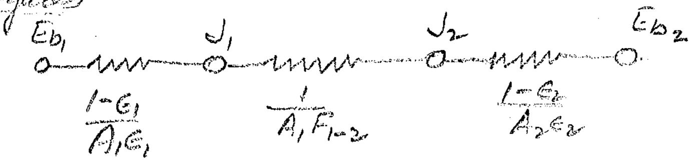
now $\sum_{k=1}^{N} J_i A_i F_{i-k} = J_i A_i \sum_{k=1}^{N} F_{i-k} = J_i A_i$

$$\therefore \boxed{q_{i,net} = \sum_{i=1}^{N} (J_i - J_k) A_i F_{i-k}}$$

$$q_{i\,net} = \frac{E_{b_i} - J_i}{\frac{1-\epsilon_i}{A_i \epsilon_i}}$$



For two surfaces



$$q_{1\to 2} = A_1 \mathcal{F} (E_{b_1} - E_{b_2})$$

$$\mathcal{F} = \frac{1}{\frac{1-\epsilon_1}{\epsilon_1} + 1 + \frac{A_1}{A_2}\left(\frac{1-\epsilon_2}{\epsilon_2}\right)}$$

very small can change by

for the case of the wire in a duct factor of 10

12/6/78

(1) $T_1 = 500°F$

$\epsilon_1 = .6$

$\epsilon_s = .2$, T_s shield

$\epsilon_2 = .06$

(2) $T_2 = 1500°F$

look at what happens when you put shield in or remove shield

Find T_s

$q_2 \rightarrow 1$

$$\frac{q_{2-1} \text{ with shield}}{q_{2,1} \text{ w/o shield}}$$

E_{b2} — J_2 — J_S — E_{bs} — J_S — J_1 — E_{b1}

$$q_{2-s} = \frac{\sigma(T_2^4 - T_s^4)}{\frac{1}{\epsilon_2} + \frac{1}{\epsilon_1} - 1} = \frac{\sigma(T_s^4 - T_1^4)}{\frac{1}{\epsilon_s} + \frac{1}{\epsilon_1} - 1}$$

$$\frac{(1960)^4 - T_s^4}{\frac{1}{.6} + \frac{1}{.2} - 1} = \frac{T_s^4 - (960)^4}{1/.2 + \frac{1}{.8} - 1}$$

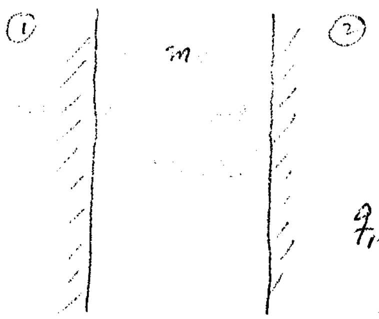
$$T_s^4 = 7.54 \times 10^8; \quad T_s = 1655°R = 1195°F$$

$$q_{2-s} = \frac{\sigma[(1960)^4 - (1655)^4]}{5.667} = 2200 \text{ Btu/hr ft}^2$$

$$q_{2-1 \text{ w/o shield}} = 12500 \text{ Btu/hr ftsq}$$

$$\frac{q_{2-1}(\text{shield})}{q_{2-1} \text{ w/o shields}} = \frac{2200}{12500} = .176$$

Radiation through a gas.

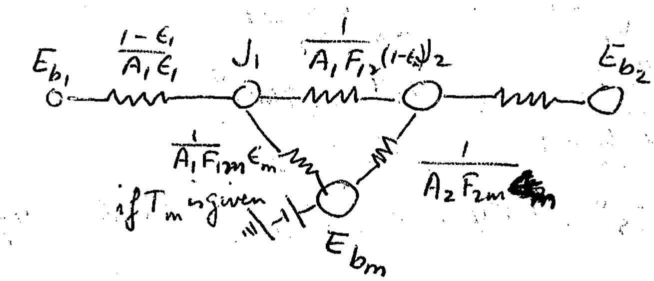


1. Assume gas is gray
2. $\tau_m + \alpha_m = 1$ and $\epsilon_m = 1$ for gas
3. $\rho_m = 0$ since for a radiating gas should not have particles to reflect radiation

$$q_{1-2} = J_1 \tau_m F_{12} A_1 = J_2 \tau_m F_{2-1} A_2$$

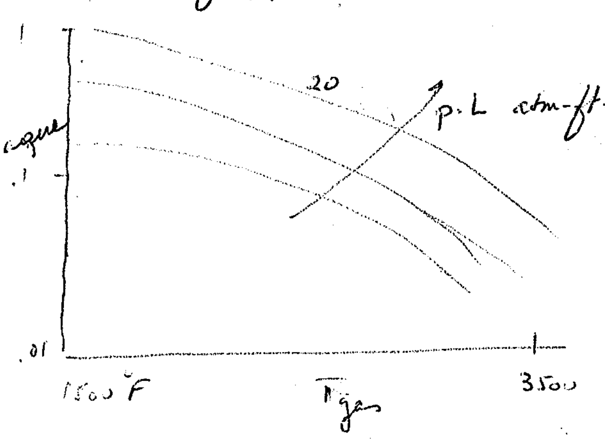
$$q_{1-2_{net}} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}(1-\epsilon_m)}}$$

$$q_{m-1_{net}} = A_m F_{m1} E_{b_m} \cdot \epsilon_m - J_1 \alpha_m A_1 F_{1m} = \frac{E_{b_m} - J_1}{\frac{1}{A_1 F_{1m} \epsilon_m}}$$

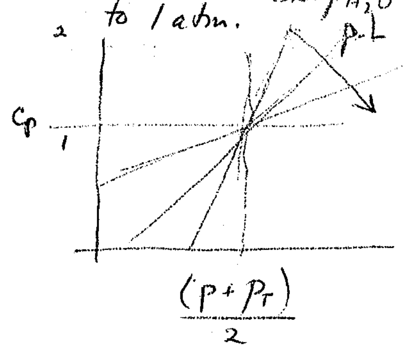


Absorptance of gas w/ a good amount of H_2O depends on # of molecules of H_2O in your path deps on beam length (L) and density of gas. [partial pressure of H_2O in the gas (p)]

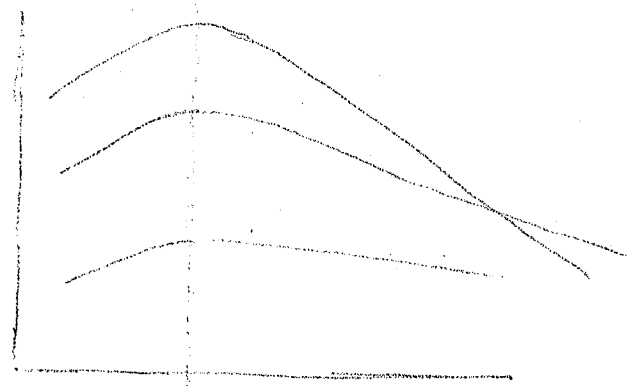
note that gas can act as opaque



plot of total pressure of gas normalized to 1 atm. and $p_{H_2O} = 0$



Radiation



Review of Course

1. Conduction — Steady, Transient; 1D, 2D, 3D (computer)
   - Lumped par
   - Charts
   - Numerical

2. Convection
   - a. BL Concepts
   - b. Flat Plate (Reynolds Anal.
   - c. Flow inside tubes
   - d. Flow over object
   - e. Natural Conv

3. Changes of Phase
   - a) boiling pool subcooled boiling in a tube
   - b) Condensation

4) Heat X-changers
   - a) LMTD $q = UA\,\Delta T_{lm}$
   - b) ε-NTU
     - flow arrangement

5) Radiation
   - a) Basic concepts - wavelength dependence
   - b) Form factors
   - c) Enclosures w/ gray bodies
   - d) Gas radiation.

Fall, 1978

ME 250A

Problem S-6

Attached are two pages from the book Fundamentals of Air Pollution by Williamson. Your attention is directed to the discussion of the greenhouse effect on page 94. Also your attention is directed to Kreith, page 220, center of page, where the greenhouse effect is mentioned.

For this special problem, your task is to determine how greenhouses work by considering radiation and convection effects. Take a typical case or two and discuss the heat transfer both qualitatively and quantitatively. Also comment on the two explanations as given by Williamson and by Kreith.

man has gained a leverage on nature. Some air pollutants at only relatively small concentrations can have an important effect on the transfer of energy between the earth and its atmosphere. In Appendix E we point out that this is because the molecules of these pollutants have the appropriate shape to strongly absorb electromagnetic radiation. Consequently, even minor constituents of the atmosphere (such as water, water vapor, and carbon dioxide) can influence the mean global temperature.

The essential features of the earth's energy balance can be understood with reference to Fig. 4.5 and can be stated simply: A large fraction of the incident solar radiation is in the spectral region where the atmosphere is transparent; although some is reflected, half passes through and is absorbed by the earth. The surface of the earth loses some energy by contact with the atmosphere, and convection of sensible and latent heat toward higher altitudes; but most of its energy loss is through thermal black body radiation. Since its surface temperature is only 288°K, the radiation will be in the long wavelength infrared portion of the spectrum, with a maximum intensity at about 10 microns. The atmosphere, however, almost completely absorbs this radiation; and a portion of the energy is then reradiated as thermal radiation back to the surface, with a smaller fraction being reradiated into space. By these processes of absorption and reradiation, the atmosphere provides an "insulating blanket" for maintaining heat near the surface of the earth. The warming of the earth's surface, resulting from the fact that the atmosphere is largely transparent to solar radiation but opaque to terrestrial, is called the *greenhouse effect*. It is this effect which maintains the surface temperature of the earth about 40°K higher than is dictated by simple energy balance considerations of the incoming and outgoing radiation from the earth, when assuming that 34% of the incoming energy is reflected without absorption.

Unfortunately the "greenhouse effect" is a misnomer when applied to the earth's atmosphere. It was once thought that the glass of a greenhouse acted in the same way to maintain the interior and ground surface warmer than the exterior. However, glass does not absorb sufficiently in the infrared portion of the spectrum to account for the magnitude of the warming effect. Both theory and experiment have shown that the correct explanation of the high temperature within a greenhouse is the suppression of convective cooling, not radiation cooling.[7,8] The glass roof forms a barrier to the convection and advection of heat away from the ground. It also discourages evaporative cooling of moist ground when the absence of circulation has permitted the air to become saturated with water vapor. Somehow these facts still appear to be insufficient to discourage the use of a catchy name, so we shall continue to use "greenhouse effect" in the well-established tradition.

A quantitative analysis of the greenhouse effect has been achieved only in general terms. No one has yet succeeded in predicting from first principles such details of the atmosphere as the global distribution of humidity or cloud cover. Often, models for the energy balance between the earth and atmosphere are based

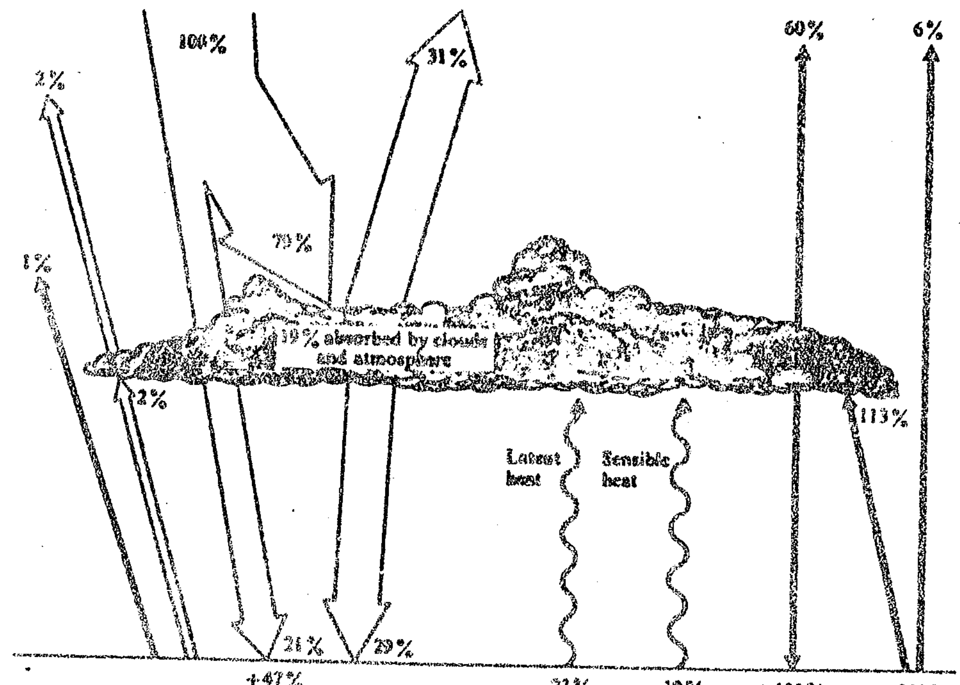


Fig. 4.8 Energy balance of the earth.

on estimated average global conditions which neglect seasonal and diurnal variations.

The results of one analysis by Houghton,[9] elaborated further by Budyko,[7] are illustrated in Fig. 4.8. The numbers in this figure indicate the average rate of energy transport by the various modes, expressed as a *percentage* of the rate at which solar radiant energy is incident downward at the tropopause. (Absorption of solar radiation at higher altitudes by ozone and oxygen leads to only a 1% difference between the solar constant and the average solar energy incident per unit area at the tropopause.) The rate at which energy is conveyed from one place to another by some modes may exceed the rate at which it is incident from the sun, so the corresponding percentage exceeds 100%.

Figure 4.8 shows that about 79% of the incident radiation is intercepted by clouds, aerosols, water vapor, and other gases, with 31% being scattered away from the earth and 29% scattered down to the surface. The remaining 19% is absorbed by constituents of the atmosphere. Thus the earth's surface receives 50% of the incident solar energy, of which only 21% is unscattered direct radiation. Most of the radiation which strikes the earth's surface is absorbed, accounting for the 47% of incident solar energy indicated in the figure.

For an energy balance, the surface must lose energy at a rate equal to this 47%. Some is removed through evaporation of water and conduction of heat to